

A Novel Mechanism of Pair Creation in Pulsar Magnetospheres

Zaza Osmanov ^{1,2,*} , George Machabeli ³ and Nino Chkheidze ³¹ School of Physics, Free University of Tbilisi, 0183 Tbilisi, Georgia² Evgeni Kharadze Georgian National Astrophysical Observatory, 0301 Abastumani, Georgia;³ Centre for Theoretical Astrophysics (ITP), Ilia State University, G. Tsereteli 3, 0162 Tbilisi, Georgia; g.machabeli@iliauni.edu.ge (G.M.); nino.chkheidze@iliauni.edu.ge (N.C.)

* Correspondence: z.osmanov@freeuni.edu.ge

Abstract: In this paper we study the possibility of efficient pair production in a pulsar's magnetosphere. It has been shown that by means of relativistic centrifugal force the electrostatic field exponentially amplifies. As a result the field approaches the Schwinger limit leading to a pair creation process in the light cylinder area where the effects of rotation are very efficient. Analysing the parameters of the normal period (~ 1 s) pulsars we found that the process is so efficient that the number density of electron–positron pairs exceeds the Goldreich–Julian density by five orders of magnitude.

Keywords: pair creation; pulsars: general; instabilities; acceleration of particles



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1. Introduction

In 1969 Gold [1] suggested that pulsars are rotating neutron stars that centrifugally energise a pulsar's magnetospheric particles to energy values high enough to produce high energy electromagnetic radiation. Generally speaking, it is evident that neutron stars rotate with an increasing period of rotation, P , characterized by the slow down rate, $\dot{P} \equiv dP/dt > 0$. The corresponding slow down power $\dot{W} = I\Omega|\dot{\Omega}|$ then becomes

$$\dot{W} \simeq 4.7 \times 10^{31} \times P^{-3} \times \frac{\dot{P}}{10^{-15}} \times \frac{M_{ns}}{1.5M_{\odot}} \text{ ergs s}^{-1}, \quad (1)$$

where Ω is the angular velocity of rotation of the pulsar, the slow down rate is normalized by the parameters of the normal period pulsars (with periods of the order of one second), $I = 2M_{ns}R_{*}^2/5$ is the moment of inertia of the neutron star, M_{ns} and $R_{*} \simeq 10^6$ cm denote its mass and radius, respectively, and $M_{\odot} \simeq 2 \times 10^{33}$ g is the solar mass. The value of \dot{W} indicates that a pulsar is a reservoir of enormous energy, which potentially might lead to very interesting processes.

Gold's idea provoked a new theory of pulsar electrodynamics proposed by [2], where the authors described the spatial distribution of charges inside the light cylinder (LC) surface (a hypothetical area where the linear velocity of rotation coincides with the speed of light) and studied efficiency of acceleration of particles escaping the inner magnetospheric area on open magnetic field lines. As has been pointed out by [3], the theory of [2] could not explain an emission pattern of radio pulsars. To study this particular problem [3] examined a general case: When the dipolar momentum of a neutron star is inclined with respect to the rotation axis. The main idea is based on the fact that, by means of interaction of very high energy curvature gamma rays and strong magnetic field of the order of 10^{12} Gauss, the high efficiency pair production process initiates at the electron and proton polar zones. This model was extended by [4], where apart from the pair production by means of the curvature photons, the author also studied the mechanism of hadronic photo-absorption. Several modifications of pair cascading were proposed in a series of papers [5–7] considering the role of the strong magnetic field.

In general, strong electric fields can also contribute to the efficient pair cascading process. In particular, according to electrodynamics, in vacuum the virtual particles and antiparticles are continuously producing and annihilating. If, on the other hand, a strong electric field is present, it can separate the virtual charges significantly, leading, via quantum tunnelling, to pair creation. This happens when the work done on the Compton wave-length, λ_c , equals the necessary energy of materialised pairs, $eE\lambda_c \simeq 2mc^2$, where m and e are the electron’s mass and charge, respectively, and c is the speed of light. Original works dedicated to this problem have been considered in a series of papers [8–10]. The role of electric fields in pair cascading was studied in the context of the relativistic ionic Coulomb fields [11] and oscillating electric fields surrounding a pulsar [12]. In the latter, the authors considered the radial electric field induced by means of the radial currents and investigated the rate of pair production. It is worth noting that the pulsar current sheets provide small induced electric fields compared to the Schwinger limit $E_s = \pi m^2 c^3 / (e\hbar) \simeq 1.4 \times 10^{14}$ statvolt cm^{-1} [10], where \hbar is the Planck constant. Therefore, it is clear that the aforementioned mechanism is not very efficient. On the other hand, in the pulsar magnetospheres the electric field might be induced by means of the neutron star’s rotation, $\mathbf{E} = \mathbf{v} \times \mathbf{B} / c$, which is maximum close to the star’s surface and for normal pulsars equals 2×10^8 statvolt cm^{-1} and still is by several orders of magnitude less than E_s .

Since the pulsars are rapidly rotating neutron stars, it is clear that the centrifugal effects might be very important [1]. Efficiency of centrifugal force depends on the configuration of magnetic field lines; the particles slide on. For example, if the force free approximation (curved field lines) is valid, efficiency will be relatively low [13], whereas for almost straight field lines, energies of centrifugally accelerated particles will be very high [14]. The transition to the force-free regime takes place in a very thin zone near the LC and most of the time particles accelerate on almost rectilinear co-rotating magnetic field lines [14]. As has been shown in [15] the centrifugal force in the LC area is different for different species of particles (magnetospheric electrons and positrons). This in turn might lead to charge separation creating the Langmuir waves. On the other hand, since the centrifugal force is time dependent [15], it acts as a parameter and the driven electrostatic field is parametrically amplified [14–18]. In a recent study [19], it was shown that the mentioned mechanism might lead to extremely efficient heating of AGN magnetospheres.

By means of the centrifugal-parametric amplification, the induced electric field exponentially increases and gradually approaches the Schwinger limit resulting in efficient pair production. This is a completely new mechanism of pair creation in the pulsar magnetosphere which we study in this paper.

The paper is organized as follows: In Section 2 we introduce the theoretical model, in Section 3 we apply the approach to pulsars and obtain results and in Section 4 we summarize them.

2. Theoretical Model

In this section we present the theory of parametric instability of centrifugally excited Langmuir waves and apply the amplified electrostatic field to the pair production process. For this purpose, for the nearby zone of the LC we consider the linearized system of equations [16,18] composed by the Euler equation

$$\frac{\partial p_\beta}{\partial t} + ikv_{\beta 0} p_\beta = v_{\beta 0} \Omega^2 r_\beta p_\beta + \frac{e_\beta}{m} E, \tag{2}$$

the continuity equation

$$\frac{\partial n_\beta}{\partial t} + ikv_{\beta 0} n_\beta + ikn_{\beta 0} v_\beta = 0 \tag{3}$$

and the Poisson equation

$$ikE = 4\pi \sum_{\beta} n_{\beta 0} e_\beta, \tag{4}$$

where β denotes the index of species (electrons and positrons), p_β is the perturbation of momentum per unit mass, $r_\beta(t) \simeq \frac{V_{0\beta}}{\Omega} \sin(\Omega t + \phi_\beta)$, $v_\beta \simeq V_{0\beta} \cos(\Omega t + \phi_\beta)$ [16] and e_β are, respectively, the radial coordinate, radial velocity perturbation ($v_{\beta 0}$ is the unperturbed value) and charge of the corresponding species, $V_{0\beta}$ is the velocity amplitude, ϕ_β denotes the phase, E denotes the induced electrostatic field of the driven wave, k is its wave number and n_β and $n_{\beta 0}$ represent, respectively, perturbed and unperturbed particle number densities. Since the magnetic field in the pulsar’s magnetosphere is very high, we assume that the particles are in the frozen-in condition $\mathbf{E}_0 + \frac{1}{c} \mathbf{v}_{0\beta} \times \mathbf{B}_0 = 0$ and are co-rotating with the field lines. The latter are assumed to be almost straight because it is believed that the emission is formed by the particles sliding along the open field lines. In this approach we assume that magnetic induction is very strong and therefore we do not use the full set of Maxwell’s equations.

Following the method developed in [16,17] we introduce the ansatz

$$n_\beta = N_\beta e^{-\frac{iV_{0\beta}k}{\Omega} \sin(\Omega t + \phi_\beta)}, \tag{5}$$

which reduces the aforementioned set of equations to non-autonomous “mode” equations for electrons and positrons

$$\frac{d^2 N_e}{dt^2} + \omega_e^2 N_e = -\omega_e^2 N_p e^{-i\chi}, \tag{6}$$

$$\frac{d^2 N_p}{dt^2} + \omega_p^2 N_p = -\omega_p^2 N_e e^{i\chi}, \tag{7}$$

where $\omega_{e,p} \equiv \sqrt{4\pi e^2 n_{e,p} / m \gamma_{e,p}^3}$ and $\gamma_{e,p}$ are the relativistic plasma frequencies and the relativistic factors for the beam components and $\chi = b \cos(\Omega t + \phi_+)$, $b = \frac{2ck}{\Omega} \sin \phi_-$, $2\phi_\pm = \phi_p \pm \phi_e$. After making the Fourier transform of Equations (6) and (7) one straightforwardly derives the dispersion relation of the Langmuir wave

$$\omega^2 - \omega_e^2 - \omega_p^2 J_0^2(b) = \omega^2 \omega_p^2 \sum_{\mu \neq 0} \frac{J_\mu^2(b)}{(\omega + \mu\Omega)^2}. \tag{8}$$

To study the instability the frequency must be decomposed by the real and imaginary parts $\omega = \omega_r + \Delta$. The former should satisfy the resonance condition $\omega_r = \mu_r \Omega$ for the most efficient process [16–18]. Then, one obtains the following cubic equation

$$\Delta^3 = \frac{\omega_r \omega_p^2 J_{\mu_r}(b)^2}{2}, \tag{9}$$

having two complex solutions characterising a growth rate of the centrifugally excited Langmuir waves

$$\Gamma = \frac{\sqrt{3}}{2} \left(\frac{\omega_e \omega_p^2}{2} \right)^{\frac{1}{3}} J_{\mu_r}(b)^{\frac{2}{3}}, \tag{10}$$

where $\omega_r = \omega_e$ and J_{μ_r} denotes the Bessel function of the first kind. In this context, it is worth noting that energy losses, as discussed in [16,17] are negligible. In particular, the synchrotron emission does not impose any constraints on the acceleration process because the particles very rapidly lose their perpendicular (with respect to the magnetic field) momentum and slide only along the co-rotating field lines. The inverse Compton mechanism occurs in the Klein–Nishina regime when the time-scale is proportional to the Lorentz factor of electrons [20]. It is worth noting that the mentioned process is a certain kind of a two stream instability, where one stream is associated to electrons and the other to positrons. Henceforth, we denote the electrons by the subscript “1” and positrons by “2”.

The time evolution of the electrostatic field is given as follows

$$E(t) = E_0 e^{\Gamma t}, \tag{11}$$

where E_0 is its initial value which can be approximated straightforwardly

$$E_0 \simeq 4\pi n_{GJ} e \Delta r, \tag{12}$$

where

$$n_{GJ} = \frac{\Omega B}{2\pi e c} \times \frac{1}{1 - r^2/R_{lc}^2}, \tag{13}$$

is the Goldreich–Julian number density [2], $R_{lc} = c/\Omega$ denotes the LC radius and Δr is the characteristic length-scale of charge separation, which takes place close to the LC area. Its value can be derived from the fact that the process of centrifugal acceleration (CA) occurs in the LC zone, where the Lorentz factor asymptotically increases [21]

$$\gamma = \frac{\gamma_0}{(1 - r^2/R_{lc}^2)}, \tag{14}$$

Therefore, Δr should characterise a scale factor of non-uniformity, which can be expressed as $\Delta r = \frac{\gamma}{\partial\gamma/\partial r}$, finally leading to $\Delta r \simeq \gamma_0 R_{lc} / (2\gamma)$ [18]. Here γ_0 is the initial relativistic factor.

As it turns out from Equation (11) the field is exponentially increasing and in due course of time the electric field will inevitably reach the Schwinger limit and thus the probability of pair production might increase very rapidly.

In general for a constant electric field the pair production rate (R —number of produced pairs per unit of time and unit of volume) is given by [10,22]

$$R \equiv \frac{dN}{dt dV} = \frac{e^2 E^2}{4\pi^3 c \hbar^2} \sum_n \frac{1}{n^2} \exp\left(-\frac{\pi m^2 c^3}{e \hbar E} n\right). \tag{15}$$

Generally speaking, if the electrostatic field is varying the pair production rate is higher, but in the considered scenario the plasma oscillation frequency can be maximum 10^4 – 10^5 Hz, which is by many orders of magnitude less than $\nu \simeq 2mc^2/h \sim 10^{20}$ Hz. Therefore, the constant field case is a physically realistic approximation and hence the aforementioned formula is valid.

3. Discussion

In this section we consider the normal period pulsars and study the possibility of pair production by means of the centrifugally driven electrostatic fields. On the LC length-scale, where the CA is most efficient, the magnetic induction [23]

$$B_{lc} \simeq 10^{12} \times \left(\frac{P}{1 \text{ s}} \times \frac{\dot{P}}{10^{-15} \text{ ss}^{-1}}\right)^{1/2} \times \left(\frac{R_{st}}{R_{lc}}\right)^3 \text{ Gauss}, \tag{16}$$

guarantees frozen-in condition, leading to a one-dimensional geometry of particles' kinematics along the magnetic field lines. Here $R_{st} \simeq 10$ km denotes the neutron star's radius.

Using the fact that CA lasts until the plasma energy density, $\gamma m c^2 n_{GJ}$, exceeds that of magnetic field, $B^2 / (8\pi)$, for the maximum Lorentz factor, one obtains

$$\gamma_M \simeq 2.6 \times 10^5 \times P^{5/4} \times \left(\frac{\gamma_0}{10^4}\right)^{1/2} \times \left(\frac{\dot{P}}{10^{-15} \text{ s}^{-1}}\right)^{1/4}, \tag{17}$$

where we have taken into account Equations (14) and (16).

To demonstrate efficiency of the Langmuir wave's amplification, we consider two streams: Electrons with two different Lorentz factors $\gamma_0 \equiv \gamma_1 = \{10^3; 10^4\}$ and positrons

with the relativistic factors up to $\gamma_2 = 350$. In Figure 1 we plot the dependence of the instability time-scale ($\tau = 1/\Gamma$) on γ_2 . The set of the rest of the parameters is: $P = 1$ s, $\dot{P} = 10^{-15}$ s s⁻¹, $R_{st} = 10$ km. It is clear that the particles do not have fixed initial energies (mono-energetic) but are always distributed by γ [2] and this initial difference plays a key role in inducing the initial charge separation leading (via the centrifugal effects) to the amplification of the Langmuir waves. We assume the equipartition approximation: When energy is uniformly distributed among all species: $\gamma_M n_{GJ} \simeq n_1 \gamma_1 \simeq n_2 \gamma_2$, where n_1 and n_2 are the number densities of the corresponding species. As is clear from the plots the instability time-scales are small compared to the escape time-scale (which is of the order of the rotation period of the pulsar $P \simeq 1$ s), indicating high efficiency of the amplification process.

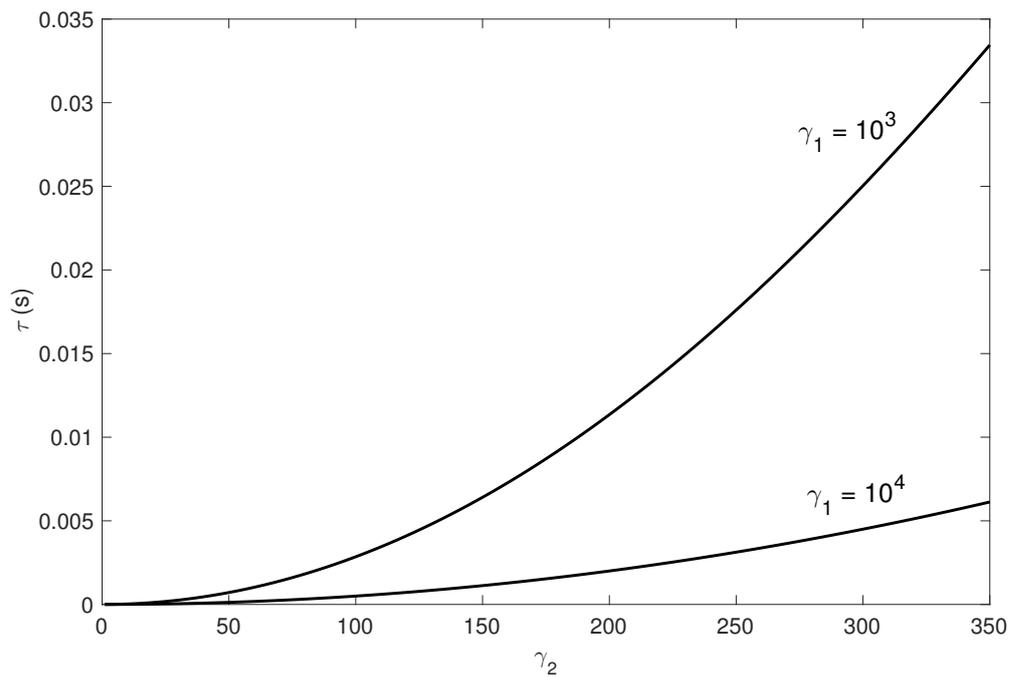


Figure 1. Behaviour of the instability time-scale versus γ_2 . The set of parameters is: $P = 1$ s, $\dot{P} = 10^{-15}$ s s⁻¹, $\gamma_1 = \{10^3; 10^4\}$.

Generally speaking, the proposed mechanism should be somehow terminated. If the co-rotation is maintained this will happen when the produced total energy density of electron positron pairs equals the energy density of the electrostatic field, leading to the following integral expression

$$\sum_n \frac{1}{n^2} \int_0^{t_0} \exp\left(-\frac{\pi m^2 c^3 n}{e \hbar E_0} e^{-\Gamma t}\right) dt = \frac{\pi^2 \hbar^2}{4 m c e^2}, \tag{18}$$

which actually is an equation for determining the value of t_0 when the pair production is terminated. After integration one can straightforwardly derive the corresponding algebraic equation

$$\begin{aligned} \sum_n \frac{1}{n^2} \left[E_1\left(-\frac{\pi m^2 c^3 n}{e \hbar E_0} e^{-\Gamma t_0}\right) - E_1\left(-\frac{\pi m^2 c^3 n}{e \hbar E_0}\right) \right] &= \\ &= \frac{\pi^2 \hbar^2}{4 m c e^2} \Gamma, \end{aligned} \tag{19}$$

where $E_1(z) \equiv \int_z^\infty \frac{e^{-x}}{x} dx$ denotes the exponential integral.

For the same set of parameters as in Figure 1, except $\gamma_2 = 100$ one can solve Equation (19) for t_0 . In particular, for $\gamma_1 = 10^3$ and $\gamma_1 = 10^4$ the corresponding termination time scales equal $\sim 1.3 \times 10^{-2}$ s and $\sim 4.7 \times 10^{-2}$ s, respectively. It is clear that for

the process to be efficient, not only the time scale of the unstable Langmuir waves must be much less than the escape time scale, but the termination time scale must satisfy the same condition.

By taking these values into account one can estimate the pair production rate from Equation (15). On Figure 2 we plot the dependence of R versus time. The set of parameters is the same as in Figure 1 except $\gamma_2 = 100$. As is clear from the plots, the pair creation rate is a continuously increasing function of time, which is a natural result of the fact that, by means of the parametric instability, the electrostatic field exponentially amplifies (see Equation (11)). From the plots it is also evident that only at time scales very close to the termination time scale the pair creation becomes extremely efficient; therefore, only those cases are physically interesting where t_0 is not more than P . One can straightforwardly check that for $\gamma_1 = 10^3$ the maximum value of γ_2 when the mentioned condition is satisfied equals 370 and for $\gamma_1 = 10^4$ the critical relativistic factor equals 865. The similar dependence as in Figure 2 but for the critical value of γ_2 corresponding to $\gamma_1 = 10^3$ is shown in Figure 3 and, as is clear, the termination occurs for the escape time scale. For higher values of γ_2 the termination time will be larger and therefore the process will not be efficient. The results for $\gamma_1 = 10^4$ are not distinguished significantly from the ones shown in Figure 3 and therefore we do not show them.

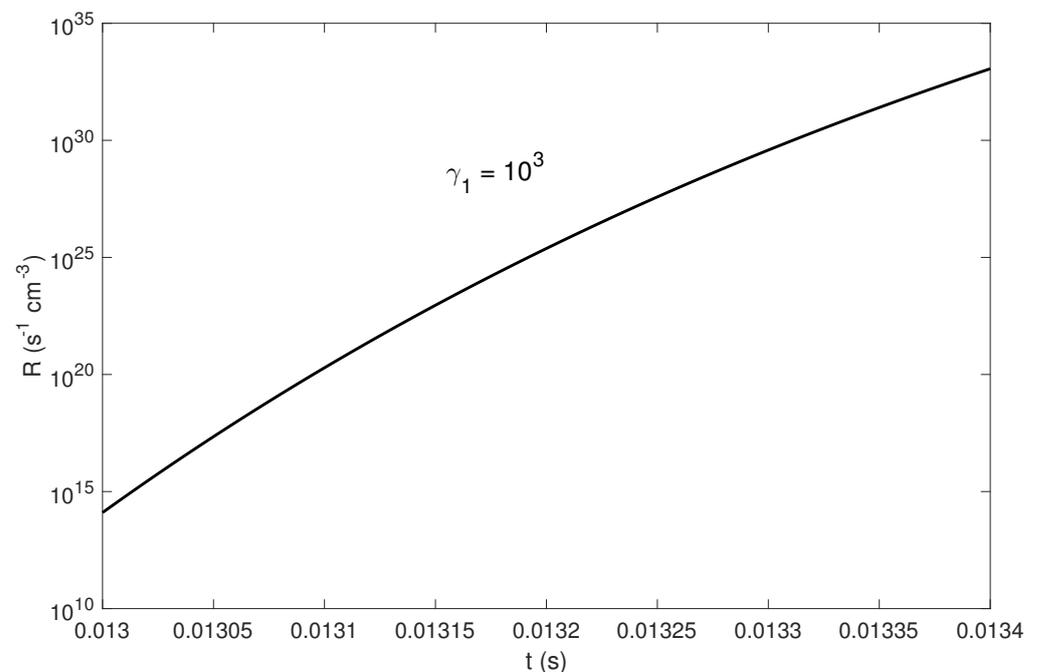


Figure 2. Cont.

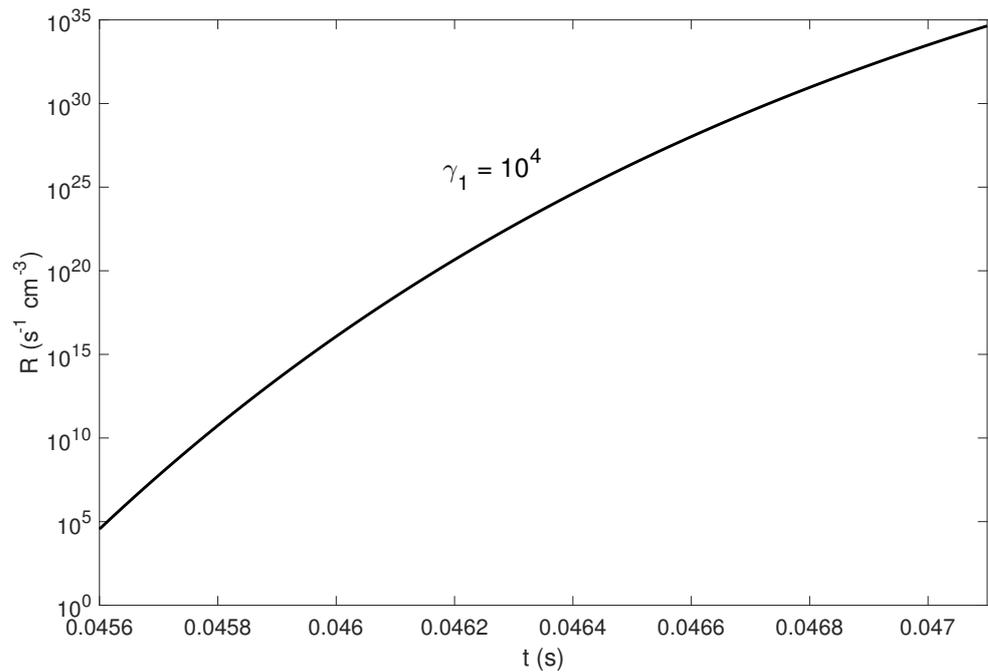


Figure 2. Two plots for the behaviour of the pair production rate versus time are shown. The set of parameters is the same as in Figure 1 except $\gamma_2 = 100$. On the top panel we show the plot for $\gamma_1 = 10^3$ and on the bottom panel we show the results for $\gamma_1 = 10^4$.

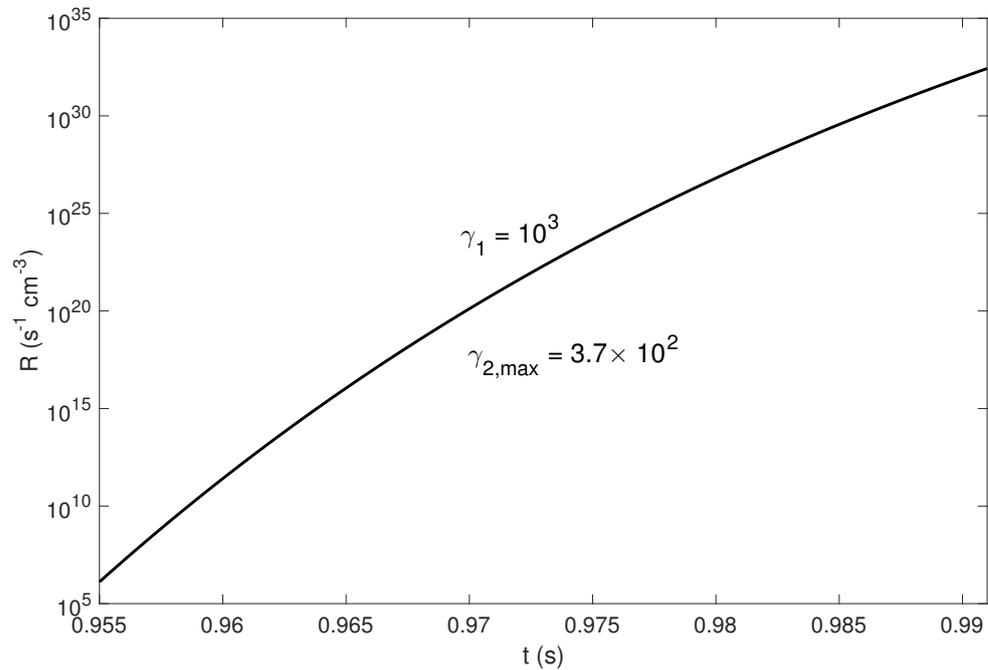


Figure 3. The behaviour of the pair production rate versus time for the critical case. The set of parameters is the same as in Figure 2 (top panel) except the critical value $\gamma_{2,max} = 370$.

One has to note that the absolute values of R are unrealistically high, indicating that some additional mechanism of termination of the process must exist. The proposed mechanism strongly depends on the effects of relativistic centrifugal force, which takes place only if the energy density of particles is less than that of the magnetic field. However, in due

course of time the number density of magnetospheric electron–positron pairs, n_p , increases very rapidly and as soon as the following condition

$$2\gamma_{rot}mc^2n_p < \frac{B_{lc}^2}{8\pi} \tag{20}$$

is violated, the co-rotation stops and consequently the process is terminated. Here $\gamma_{rot} = (1 - r^2/R_{lc}^2)^{-1/2}$ is the Lorentz factor corresponding to rotation. After taking into account Equation (14), one can straightforwardly derive the maximum value density of pairs, n_p on the LC zone

$$n_p \simeq \frac{B^{7/4}}{16\pi mc^2} \times \left(\frac{4mc\gamma_0\Omega}{e}\right)^{1/4} \simeq 4.17 \times 10^5 \times P^{-37/8} \times \left(\frac{\dot{P}}{10^{-15}}\right)^{7/8} \times \left(\frac{\gamma_0}{10^4}\right)^{1/4}. \tag{21}$$

On Figure 4 we plot the behaviour of the multiplicity factor n_p/n_{GJ} versus the initial relativistic factor. The set of parameters is the same as in Figure 2 except γ_0 , which varies in the interval $[10^3-10^4]$. As one can see from the plot the multiplicity factor is a continuously increasing function of the initial Lorentz factor and for 10^4 reaches the value of the order of 1.3×10^5 , indicating the process is highly efficient.

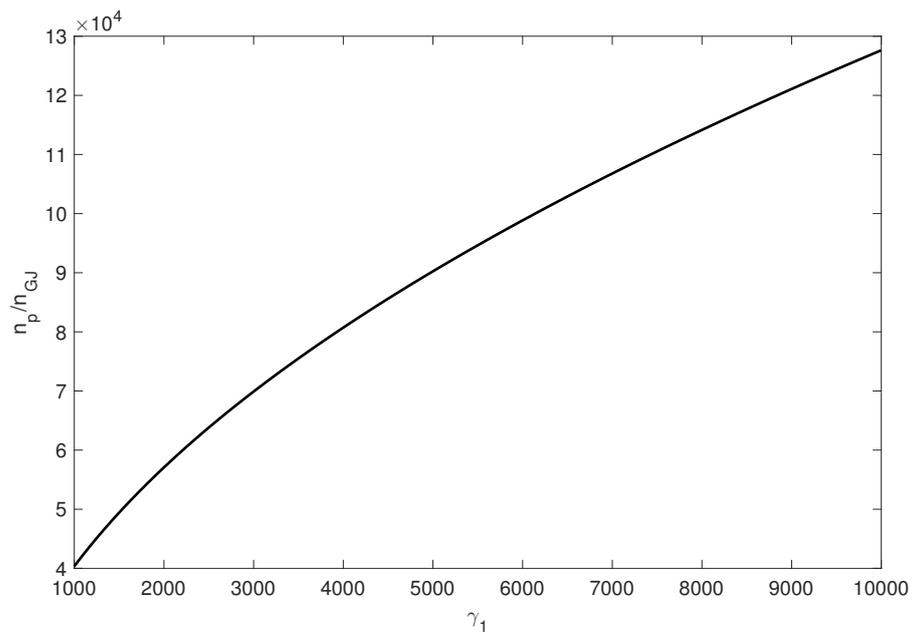


Figure 4. Dependence of the multiplicity factor versus the initial Lorentz factor. The set of parameters is the same as in Figure 2 except γ_1 which changes in the interval $[10^3-10^4]$.

4. Summary

We have examined a centrifugally accelerated pulsar’s magnetospheric particles and studied the parametric amplification of the electrostatic field.

Considering the typical pulsar parameters, it has been shown that the electric field exponentially increases and gradually reaches the Schwinger limit, when efficient pair creation might occur.

Analysing constraints imposed on the process we have found that the mechanism terminates when the energy density of produced particles exceeds that of the magnetic field. As a result, it has been shown that this process leads to the multiplicity factor compared to the GJ number density of the order of 10^5 .

A similar problem can be extended to milliseconds and young pulsars as well by taking into account characteristic features of particle dynamics in these objects.

Given that the novel mechanism of pair production substantially changes the number density of electron–positron plasmas in the pulsar’s magnetospheres, it might significantly influence the physical processes there. In particular, it is evident that the processes of particle acceleration strongly depend on the plasma density. Consequently, the emission spectral pattern will be influenced as well by the efficient pair creation. This in turn, might give rise to coherent radio emission, which on the one hand seems to be quite promising in the light of modern enigma—fast radio bursts—and on the other hand is a completely new view concerning the generation of radio emission in pulsars, where it is usually believed that the process of generation takes place close to the magnetic axis. Since all these problems are beyond the intended scope of the paper we are going to consider them very soon.

Author Contributions: Z.O., G.M. and N.C. developed the analytical model, obtained the results and wrote the manuscript. All authors have read and agreed to the published version of the manuscript.

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Abbreviations

The following abbreviations are used in this manuscript:

CA	centrifugal acceleration
LC	Light cylinder

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