Supplementary Material

In elaboration of reference [80], there is extensive literature describing single particle inclusive reactions by means of the Tsallis distribution. The 'Tsallis method' is based on a statistical approach that is very different from the framework described in our paper. The Tsallis method posits that the p_T distribution is a low- p_T exponential that evolves to a power law at high p_T with terms that are related to thermodynamic quantities. On the other hand, our paper and our earlier results [9–10] find inclusive single particle and jet production in pp and pA collisions to be described quite well by a power law embodied in our A-function, even down to low p_T and $\sqrt{s}$, when expressed in terms of the modified transverse momentum P_T, demonstrating that hard parton-parton scattering is visible even at low energies.

We have shown the validity of our power law framework in several figures – such as Figures 18, 19 and 22 of this publication. Here, we include another example of our description of the A-function in terms of a power law in the modified transverse momentum, P_T, with an analysis of the K^+ data taken by the NA49 Collaboration at CERN [49] at $\sqrt{s}$ = 17.24 GeV, roughly 1.3 x10^{-3} of the 13 TeV LHC energy scale. These data were included in our Figure 20, showing the Λ_m – m relationship.

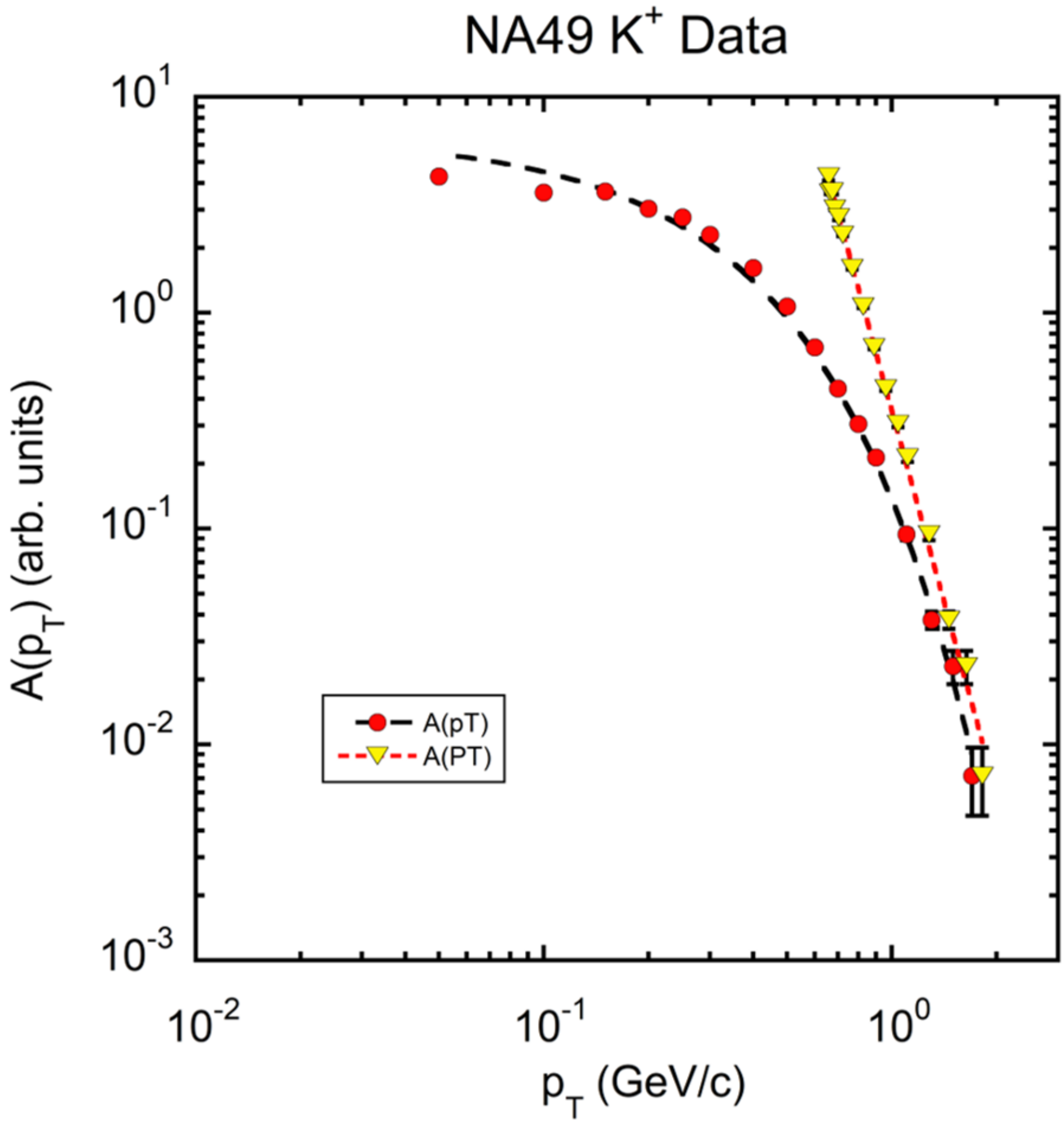

Figure S1. Shown is the A-function for K+ data taken by the NA49 Collaboration at CERN. The A-function is plotted vs. p_T (red circles) and vs. the modified transverse momentum, $P_T = \sqrt{p_T^2 + \Lambda_m^2}$ on the same axis. The black bashed line is an exponential fit to the red points (~ exp(-3.9 p_T) and the red dashed line is the standard power law fit in the modified transverse momentum P_T, with n_{pT} = 5.7 ± 0.2 and Λ_m = 0.66 ± 0.02 GeV/c.

Note that for low p_T, of order Λ_m (~ m, the mass of the particle detected), we find a power law in the modified transverse momentum $P_T = \sqrt{p_T^2 + m^2}$, rather than in p_T. When this power law spectrum is plotted as a function of p_T rather than P_T, it can be roughly be approximated by an exponential for low p_T. Thus, our formulation offers a natural explanation of the transverse momentum spectrum analyzed by the Tsallis method – namely exponential behavior at low p_T and a power-law behavior at high p_T when the modified transverse momentum P_T becomes approximately equal to p_T. Further, it is crucial to analyze the A-function as a power law in the modified P_T when p_T of order m. Hence, we have only one behavior – a power law in the modified transverse momentum $P_T \to p_T$.

The key to our method is to separately determine the p_T dependence of the inclusive invariant cross section by the extrapolation of $x_R \to 0$, which separates 'hard' scattering from the 'soft' processes of hadronization and fragmentation. Our approach connects observables to the proton PDFs, elementary parton-parton scattering cross sections and a wealth of data gained by lepton-nucleon scattering over the years. In summary, we find no need for statistical concepts to describe the inclusive single particle/jet p_T-spectra in pp and pA data in two regimes – an exponential behavior at low transverse momentum and a power law at high transverse momentum. However, our framework and the Tsallis approach may be related – especially in heavy ion collisions in the $[x_R - p_T - \sqrt{s}]$ sector where a multitude of soft physics processes may be described statistically.

See: Tsallis, C. Possible generalization of Boltzmann-Gibbs statistics. J. Stat. Phys. 1988, 52, 479–487, doi:10.1007/bf01016429. and Wilk, G.; Włodarczyk, Z. Power laws in elementary and heavy-ion collisions. Eur. Phys. J. A 2009, 40, 299, doi:10.1140/epja/i2009-10803-9.