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Role of Single-Particle Energies in Microscopic Interacting Boson Model Double Beta Decay Calculations

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Abstract: Single-particle level energies form a significant input in nuclear physics calculations where single-particle degrees of freedom are taken into account, including microscopic interacting boson model investigations. The single-particle energies may be treated as input parameters that are fitted to reach an optimal fit to the data. Alternatively, they can be calculated using a mean field potential, or they can be extracted from available experimental data, as is done in the current study. The role of single-particle level energies in the microscopic interacting boson model calculations is discussed with special emphasis on recent double beta decay calculations.

Keywords: single-particle energies; microscopic interacting boson model; neutrinoless double beta decay

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1. Introduction

The question of the nature of neutrinos, are they Dirac or Majorana particles, and what are their masses, as well as phases, in the mixing matrix, is one of the most fundamental open problems in physics today. Thus, observing neutrinoless double beta decay ($0\nu\beta\beta$) is at the moment one of the major experimental challenges [1–4], motivated also by its potential as a promising candidate for observing lepton number violation. If detected, it would offer information about the fundamental nature of neutrinos and about the absolute effective neutrino mass [5–9], as well as right-handed leptonic current coupling constants [9,10]. It would also shed light on the matter– antimatter asymmetry of the universe [11].

The half-life of $0\nu\beta\beta$ decay can be factorized as

$$[\tau_{1/2}^{0\nu}]^{-1} = G_{0\nu}|M_{0\nu}|^2|f(m_i, U_{ei})|^2, \tag{1}$$

to consist of phase space factor $G_{0\nu}$ [6,9,12], nuclear matrix element $M_{0\nu}$ and function containing physics beyond the standard model, $f(m_i, U_{ei})$, through the masses m_i and mixing matrix elements U_{ei} of neutrino species. Related to yet unobserved neutrinoless double beta decay, there is also the process allowed by the standard model and observed in several nuclei [13], where two (anti)neutrinos are emitted ($2\nu\beta\beta$). In order to access physics beyond the standard model contained in the function f in Equation (1), an accurate calculation of the nuclear matrix element, $M_{0\nu}$, is needed. The calculations of $M_{0\nu}$ are crucial when extracting the neutrino mass $\langle m_\nu \rangle$ if neutrinoless double beta decay is observed, and serve the purpose of guiding future searches if $0\nu\beta\beta$ remains undetected.

Since $0\nu\beta\beta$ decay is a unique, not yet observed process, it is a challenge also for theoretical models. Thus, information from other studies such as nucleon transfer reactions [14–20], the photonuclear reactions [21–23], the nuclear muon capture process [24–26], the study of single β [27–29], and $2\nu\beta\beta$ decays [29–35], as well as, single-charge-exchange [36–43], and pion double-charge-exchange [44–46] reactions are highly valuable in view of estimating the uncertainties of $0\nu\beta\beta$ decay calculations.

On the other hand, the energies of the single particle orbitals have a significant role in models of nuclear structure. In addition to being essential tests of the shell model for doubly magic or semi-magic nuclei, they also constitute important input parameters in many nuclear structure calculations such as the (interacting) shell model, quasiparticle random-phase approximation, microscopic interacting boson model, or any other nuclear model calculations where single-particle degrees of freedom are considered.

Experimental single-particle energies are known to change with the nucleon number primarily due to the monopole–monopole part of the neutron–proton residual interaction, which is of interest itself. Implicitly single-particle energies are of interest since they play a role in the description of various nuclear physical and astrophysical processes. These include also double beta decay (DBD), single beta decay, and double charge exchange reaction (DCE). An issue closely connected to single-particle levels is their occupancies. Ground state occupancies can be obtained experimentally by one nucleon transfer reaction. Such experiments have been carried out for several candidates participating in $0\nu\beta\beta$ decay in a series of experiments [16–20]. The obtained results offer an important test for theoretical models used to calculate nuclear properties [19,20,47–50]. The comparison of calculated occupation probabilities with experimentally obtained ones serves the purpose of assessing the goodness of the chosen single-particle energies, as well as the used wave functions.

In the current study, the role of SPEs in the microscopic interacting boson model (IBM-2) calculations is discussed. In IBM-2, valence nucleon pairs are described as bosons with angular momentum 0 or 2, denominated as *s* and *d* bosons, respectively. IBM-2 was originally introduced as a phenomenological approach to describe collective excitations in nuclei [51–53] and its relation with the shell model was established in References [54–56].

In Section 2, a brief summary of how single-particle energies (SPEs) enter interacting boson model calculations is given followed by the introduction of considered neutron/proton single-particle energies (SPEs) in Section 3. The impact of using different values of the SPEs on pair structure coefficients in general is discussed in Section 4, and in Section 5, specific results of $0\nu\beta\beta$ nuclear matrix elements, including their connection to DCE nuclear matrix elements, are considered. Finally, conclusions are presented in Section 6.

2. Role of Single-Particle Energies in IBM-2 Calculations

Formally, any problem dealing with fermions may be transformed into an equivalent problem dealing with bosons. For this transformation mapping from the original fermion space, the shell model space, onto desired space, in this case IBM-2 space, is needed. A detailed description of such mapping procedure can be found in References [54,55] and in particular concerning DBD in Reference [57]. Here, a brief review of the main aspects of the method is given. The starting points are the shell model creation operators of collective S and D pairs with angular momenta 0 and 2, respectively:

$$S_\rho^\dagger = \sum_j \alpha_{\rho,j} \sqrt{\frac{\Omega_j}{2}} \left(\rho_j^\dagger \times \rho_j^\dagger\right)^{(0)}, \tag{2}$$

$$D_{\rho,M}^\dagger = \sum_{j\leq j'} \beta_{\rho,jj'} \frac{1}{\sqrt{1+\delta_{jj'}}} \left(\rho_j^\dagger \times \rho_{j'}^\dagger\right)_M^{(2)}, \tag{3}$$

where $\Omega_j = j + 1/2$ and ρ refers to proton or neutron indices, $\rho = \pi, \nu$. For each kind of nucleon, these pairs are then used to span the subspaces, the SD fermion spaces, of the full shell model spaces. The states of each subspace have a certain number of protons or neutrons n, generalized seniority quantum number v, and angular momentum J, and are labeled accordingly as $|n, v, \alpha, J\rangle$,where α denotes additional quantum numbers required for a unique specification of the states.

There are several ways to obtain the pair structure coefficients $\alpha_{\rho,j}$ and $\beta_{\rho,jj'}$ in Equations (2) and (3) [58–63]. In the method given by [63] and followed here, $S_\rho^\dagger$ and $D_{\rho,M}^\dagger$ generate the 0^+ ground state and the first excited 2^+ two-fermion state. These states correspond to a nucleus with two-valence-particles or two-valence-holes outside a closed shell. The used method allows the inclusion of some possible renormalization effects induced by the neutron–proton interaction to be included approximately. For the effective interaction between identical nucleons, the surface delta interaction (SDI) is chosen. The associated isovector strength parameter A_1 is fitted to reproduce the energy difference between the 0^+ ground state and the first excited 2^+ in the corresponding two-valence-particle or two-valence-hole nucleus. The single-particle energies enter the SDI calculation as input.

As a result, pair structure coefficients are obtained and are normalized as

$$\sum_j \Omega_j \alpha_j^2 = \sum_j \Omega_j, \tag{4}$$

$$\sum_{j \leq j'} \beta_{jj'}^2 = 1, \tag{5}$$

where the label ρ is from now on omitted for simplicity.

The states belonging to the SD subspaces are then mapped onto sd boson states of the IBM space as

$$S^\dagger \rightarrow s^\dagger \tag{6}$$

$$D^\dagger \rightarrow d^\dagger, \tag{7}$$

and similarly the fermionic operators are mapped into bosonic operators

$$O_F \rightarrow O_B \tag{8}$$

using the Otsuka, Arima, and lachello (OAI) method [55]. In the OAI method, the matrix element of the bosonic image of the operator in question between IBM states, is made equal to the corresponding fermionic shell model matrix element. When calculating the matrix elements in the shell model using the generalized seniority scheme and making the correspondence between the generalized-seniority state vectors and boson state vectors, the commutator method of References [64,65] is employed. By using the OAI and commutator methods, one is assured that the matrix elements between fermionic states in the collective subspace are identical to the matrix elements in the bosonic space.

A detailed description for obtaining factors required for the mapping of combinations of s and d operators relevant in the description of DBD in IBM-2 is given in Reference [57].

3. Considered Sets of Single-Particle Energies

The single-particle energies may be considered as input parameters to be fitted to reach an optimal correspondence with the data, or alternatively they can be calculated using a mean field potential, or they can be extracted from available experimental data. In Reference [50] the single-particle and single-hole energies for protons and neutrons were extracted from experimental data and discussed in detail. The underlying motivation in [50] was to estimate the validity of the single-particle energies and check the reliability of the used IBM-2 wave functions by calculating occupancies of the appropriate single-particle levels. These kinds of tests are particularly important in the case of nuclei involved in DBD, as they directly affect the evaluation of the nuclear matrix elements and thus their reliability [66]. In Reference [50], single-particle energies for several major shells were updated to values given in Tables 1–4 and marked as set (I). These single-particle energy sets were then used to calculate the occupancies of several nuclei of interest in neutrinoless double beta decay. Finally, the results were compared with experimental occupancies, when available, as well as other theoretical calculations, and good correspondence was obtained. The comparison set (II) in Tables 1–4 [57] refers to values used in previous IBM-2 double beta decay calculations.

3.1. Single-Particle Energies for the 28-50 Shell

In Table 1, the single-particle energies for the orbitals of the 28-50 shell for proton particles and holes are given. The proton particle energies are appropriate for $A \sim 76, 82$. The updated values of set (I) were obtained by interpolating linearly between proton particle SPEs of set (II) in Table 1 and proton hole SPEs of set (II) (but inverted to particle energies). The proton hole energies in set (I) are appropriate for $A \sim 100, 116$ and $N \sim 60$ and were obtained from the spectrum of ^{107}In. For set (II), the energies were taken, without any interpolation, from the spectrum of ^{57}Cu for proton particles, and from isotones N = 50 for proton holes, suitable for $A \sim 100$ and neutron number $N < 50$.

The neutron hole energies of set (I) in Table 2 are appropriate for $A \sim 76, 82$ and $Z \sim 40$ and were obtained from the spectrum of ^{89}Zr. For set (II) the energies were taken from the spectrum of ^{57}Ni.

As can be seen from Tables 1 and 2, in shell 28-50 for proton particles the biggest changes in SPEs are for $1g_{9/2}$ and $1f_{5/2}$, which both are lowered when going from set (II) to set (I). For proton holes, as well as neutron holes, all other orbitals are lowered in energy with respect to the lowest orbital $1g_{7/2}$.

Table 1. Considered energies of proton single-particle orbitals and A_1 isovector surface delta interaction (SDI) strength parameters in MeV in the 28-50 shell (set (I) [50], set (II) [57]).

Orbital	Protons (I) (Particles) $A \sim 76, 82$ $A_1 = 0.299$	Protons (II) (Particles) $A_1 = 0.366$	Protons (I) (Holes) $A \sim 100, 116$ $A_1 = 0.239$	Protons (II) (Holes) $A_1 = 0.264$
$2p_{1/2}$	1.179	1.106	0.678	0.931
$2p_{3/2}$	0.000	0.000	1.107	2.198
$1f_{5/2}$	0.340	1.028	1.518	2.684
$1g_{9/2}$	2.640	3.009	0.000	0.000

Table 2. Considered energies of neutron single-particle orbitals and A_1 isovector SDI strength parameters in MeV in the 28-50 shell (set (I) [50], set (II) [57]).

Orbital	Neutrons (I) (Holes) $A \sim 76, 82$ $A_1 = 0.237$	Neutrons (II) (Holes) $A_1 = 0.280$
$2p_{1/2}$	0.588	1.896
$2p_{3/2}$	1.095	3.009
$1f_{5/2}$	1.451	2.240 0
$1g_{9/2}$	0.000	0.000

3.2. Single-Particle Energies for the 50-82 Shell

In Table 3, the single-particle energies for the orbitals of the 50-82 shell for proton particles are given, appropriate for $A \sim 128, 130, 136$. The energies in set (I) were taken from the spectrum of ^{133}Sb, the exception being the $3s_{1/2}$ level, where the energy was obtained from systematics of odd $N = 82$ nuclei [67]. For set (II), the proton particle energies were taken from the spectrum of ^{133}Sb without any exceptions.

The energies for neutron particles and holes in the 50-82 shell are shown in Table 4. In set (I) for neutron particles, suitable for $A \sim 100, 116$, the energies of $3s_{1/2}$, $2d_{3/2}$, and $1g_{7/2}$ orbitals were obtained from the spectra of ^{97}Pd, ^{95}Ru, and ^{101}Sn, respectively. For the $1h_{11/2}$ orbital, the energy was taken from systematics of odd $N = 51$ nuclei. For set (II), the neutron particle energies were taken from the spectra of ^{91}Zr. The neutron hole energies, appropriate for $A \sim 128, 130, 136$ were obtained from the spectrum of ^{131}Sn for both set (I) and set (II), so there were no changes in these single-particle energies.

In shell 50-82 for proton particles and neutron holes, there are only minor changes in SPEs, as shown in Tables 3 and 4. For neutron particles, in Table 4, the $1h_{11/2}$ orbital is raised, whereas $3s_{1/2}$, $2d_{3/2}$, and $1g_{7/2}$ are lowered.

Table 3. Considered energies of proton single-particle orbitals and A_1 isovector SDI strength parameters in MeV in the 50-82 shell (set (I) [50], set (II) [57]).

Orbital	Protons (I) (Particles) $A \sim 128, 130, 136$ $A_1 = 0.222$	Protons (II) (Particles) $A_1 = 0.221$
$3s_{1/2}$	2.990	2.990
$2d_{3/2}$	2.440	2.690
$2d_{5/2}$	0.962	0.960
$1g_{7/2}$	0.000	0.000
$1h_{11/2}$	2.792	2.760

Table 4. Considered energies of neutron single-particle orbitals and A_1 isovector SDI strength parameters in MeV in the 50-82 shell (set (I) [50], set (II) [57]).

Orbital	Neutrons (I) (Particles) $A \sim 100, 116$ $A_1 = 0.242$	Neutrons (II) (Particles) $A_1 = 0.269$	Neutrons (I) (Holes) $A \sim 128, 130, 136$ $A_1 = 0.163$	Neutrons (II) (Holes) $A_1 = 0.163$
$3s_{1/2}$	0.775	1.205	0.332	0.332
$2d_{3/2}$	1.142	2.042	0.000	0.000
$2d_{5/2}$	0.000	0.000	1.654	1.655
$1g_{7/2}$	0.172	2.200	2.434	2.434
$1h_{11/2}$	2.868	2.170	0.069	0.070

4. Impact of Single-Particle Energies on Pair Structure Coefficients

In the definition of the pair operators Equations (2) and (3), the pair structure coefficients α and β appear. The method used for obtaining the coefficients α and β is by diagonalizing the SDI (for details see, e.g., in [68]), where inputs are the single-particle energies and values of A_1. The obtained pair structure coefficients for different shells are given in Tables 5–8.

In shell 28-50 for proton particles, Table 5, the obtained α with set (I) SPEs are smaller in magnitude than the ones obtained with set (II), the exception being $\alpha_{5/2}$, and β are larger, the exception being $\beta_{3/23/2}$. For proton holes, Table 5, as well as for neutron holes, Table 6, α and β are larger, the exceptions being $\alpha_{9/2}$ and $\beta_{9/29/2}$. In hole energies, $1g9/2$ is the lowest orbital and as already noted compared to set (II) in set (I), other orbitals are lowered in energy with respect to the lowest orbital.

In shell 50-82 for proton particles, Table 7, and for neutron holes, Table 8, the obtained α and β with set (I) remain essentially the same. For neutron particles, Table 8, α and β are smaller for $5/2$ and $11/2$, and larger for others. $2d_{5/2}$ is the lowest orbital and $1h_{11/2}$ is the highest orbital, which is raised even higher in set (I) compared to set (II).

Table 5. Obtained pair structure coefficients with different single-particle energies given in Table 1 for protons of the 28-50 shell.

	Protons (I) (Particles) $A \sim 76, 82$	**Protons (II) (Particles)**	**Protons (I) (Holes) $A \sim 100, 116$**	**Protons (II) (Holes)**
$\alpha_{1/2}$	−0.701	−0.850	0.765	0.689
$\alpha_{3/2}$	−1.650	−1.867	0.602	0.408
$\alpha_{5/2}$	−1.187	−0.884	0.500	0.352
$\alpha_{9/2}$	0.409	0.439	−1.337	−1.401
$\beta_{1/23/2}$	−0.742	−0.322	−0.149	−0.092
$\beta_{3/23/2}$	−0.280	−0.866	−0.088	−0.048
$\beta_{1/25/2}$	−0.280	−0.234	−0.154	−0.099
$\beta_{3/25/2}$	0.381	0.222	0.071	0.040
$\beta_{5/25/2}$	−0.373	−0.182	−0.088	−0.052
$\beta_{9/29/2}$	0.096	0.093	0.966	0.988

Table 6. Obtained pair structure coefficients with different single-particle energies given in Table 2 for neutrons of the 28-50 shell.

	Neutrons (I) (Holes) $A \sim 76, 82$	**Neutrons (II) (Holes)**
$\alpha_{1/2}$	0.807	0.468
$\alpha_{3/2}$	0.603	0.336
$\alpha_{5/2}$	0.512	0.418
$\alpha_{9/2}$	−1.329	−1.416
$\beta_{1/23/2}$	−0.157	−0.063
$\beta_{3/23/2}$	−0.089	−0.037
$\beta_{1/25/2}$	−0.164	−0.091
$\beta_{3/25/2}$	0.073	0.039
$\beta_{5/25/2}$	−0.092	−0.064
$\beta_{9/29/2}$	0.963	0.990

Table 7. Obtained pair structure coefficients with different single-particle energies given in Table 3 for protons of the 50-82 shell.

	Protons (I) (Particles) $A \sim 128, 130, 136$	**Protons (II) (Particles)**
$\alpha_{1/2}$	0.384	0.382
$\alpha_{3/2}$	0.449	0.414
$\alpha_{5/2}$	0.818	0.817
$\alpha_{7/2}$	1.765	1.769
$\alpha_{11/2}$	−0.405	−0.406
$\beta_{1/23/2}$	−0.058	−0.054
$\beta_{3/23/2}$	0.045	0.040
$\beta_{1/25/2}$	0.094	0.092
$\beta_{3/25/2}$	0.058	0.053
$\beta_{5/25/2}$	0.134	0.131
$\beta_{3/27/2}$	0.190	0.170
$\beta_{5/27/2}$	−0.133	−0.131
$\beta_{7/27/2}$	0.951	0.957
$\beta_{11/211/2}$	−0.076	−0.075

Table 8. Obtained pair structure coefficients with different single-particle energies given in Table 4 for neutrons of the 50-82 shell.

	Neutrons (I) (Particles) $A \sim 100, 116$	Neutrons (II) (Particles)	Neutrons (I) (Holes) $A \sim 128, 130, 136$	Neutrons (II) (Holes)
$\alpha_{1/2}$	0.888	0.852	−0.998	−0.999
$\alpha_{3/2}$	0.749	0.614	−1.394	−1.395
$\alpha_{5/2}$	1.463	1.921	−0.469	−0.469
$\alpha_{7/2}$	1.280	0.584	−0.357	−0.357
$\alpha_{11/2}$	−0.431	−0.589	−1.288	1.287
$\beta_{1/23/2}$	−0.193	−0.118	−0.402	−0.402
$\beta_{3/23/2}$	0.121	0.068	0.490	0.492
$\beta_{1/25/2}$	0.395	0.324	0.159	0.159
$\beta_{3/25/2}$	0.173	0.115	0.098	0.098
$\beta_{5/25/2}$	0.550	0.899	0.078	0.078
$\beta_{3/27/2}$	0.392	0.149	0.176	0.176
$\beta_{5/27/2}$	−0.267	−0.088	−0.037	−0.037
$\beta_{7/27/2}$	0.472	0.098	0.065	0.065
$\beta_{11/211/2}$	−0.111	−0.124	−0.722	−0.721

5. Impact of the SPEs on IBM-2 Calculations

5.1. Neutrinoless Double Beta Decay

In the current calculation, the closure approximation is assumed. Short range correlations (SRC) are taken into account using the Jastrow function with Argonne parametrization [69]. The details of the $0\nu\beta\beta$ calculation in IBM-2, including form factors, neutrino potential, form factor charges, etc., are given in [70]. In Table 9, the $0\nu\beta\beta$ decay nuclear matrix elements calculated using SPEs of set (II), labeled as "old", and SPEs of set (I), labeled as "new", are shown. The full matrix element is divided into Fermi ($\mathcal{M}_F$), Gamow–Teller ($\mathcal{M}_{GT}$) and tensor ($\mathcal{M}_T$) components as

$$\mathcal{M}_\nu = g_A^2\left[-\left(\frac{g_V}{g_A}\right)^2\mathcal{M}_F + \mathcal{M}_{GT} - \mathcal{M}_T\right]. \tag{9}$$

Conservative quenched value $g_A = 1$ is chosen simply to allow straightforward use of other values of g_A using Equation (9) for the full matrix element. The quenching of g_A is still an open question, which, however, is beyond the scope of the current study. Note that a negative sign of the tensor nuclear matrix element (NME) relative to that of GT NME, as shown in Equation (9), was derived in Reference [70] in contrary to previous papers [30,71].

As was shown in Table 1 for proton particles, in set (II), the high-j orbitals are at higher excitation energy than in set (I). In addition, neutron hole energies in Table 2 are more packed for set (I) than set (II). This leads to generally smaller α and larger β in Tables 5 and 6. Eventually, also the calculated $0\nu\beta\beta$ decay nuclear matrix elements for nuclei ^{76}Ge and ^{82}Se, where proton particles and neutron holes occupy the shell 28-50, are larger when set (I) SPEs are employed, as shown in Table 9. For ^{100}Mo and ^{116}Cd, proton holes occupy the shell 28-50 and neutron particles occupy the shell 50-82. In these cases, the energies are more compressed in set (I) than in set (II). Thus α and β in Table 5 are generally larger, and $0\nu\beta\beta$ NMEs, as well, are larger. In the description of $0\nu\beta\beta$ decay in the framework of IBM-2 (see Reference [57] for details), α and β are raised to exponents depending on the number of bosons (pairs), and appear in products. Thus, the increase of NMEs is shown especially when both proton and neutron energies are affected and the number of bosons (valence particles outside closed shells) is higher. The biggest increase in NMEs are for ^{76}Ge, ^{82}Se, and ^{100}Mo, and is mainly due to an increase in the GT component. The case $A = 116$ is less affected because of the low number of protons outside the closed shell.

In shell 50-82 for proton particles, Table 7, and for neutron holes, Table 8, the SPEs remain essentially the same, as do α and β, and thus also NMEs in Table 9 for ^{128}Te, ^{130}Te, and ^{136}Xe remain essentially the same. The minor change in NMEs in these cases is due to updated form factor charge values used in [70] compared to [30].

Compared to NMEs obtained with other nuclear models and taking into account the sign of the tensor matrix element, the current results are generally very close to QRPA-Tü [72] and QRPA-Jy [73] results, and 1.5–2 times larger than the ones obtained with deformed QRPA [74] and ISM [75].

Table 9. Light neutrino exchange nuclear matrix elements for selected nuclei calculated with set (I) single-particle energies (SPEs) [70] (new) and with set (II) [30] (old) using $g_A = 1.0$ and the convention $\mathcal{M}_\nu > 0$. The "old" Fermi, Gamow–Teller, and tensor nuclear matrix elements (NMEs) are combined in the NMEs $\tilde{\mathcal{M}}_\nu^{\rm old}$ using the negative sign of the tensor NME relative to that of the GT NME (in contrary to [30], where a positive sign was used). All NMEs are in dimensionless units.

Isotope	$\mathcal{M}_F^{\rm old}$	$\mathcal{M}_{GT}^{\rm old}$	$\mathcal{M}_T^{\rm old}$	$\tilde{\mathcal{M}}_\nu^{\rm old}$	$\mathcal{M}_F$	$\mathcal{M}_{GT}$	$\mathcal{M}_T$	$\mathcal{M}_\nu$
^{76}Ge	−0.68	4.49	−0.23	5.40	−0.78	5.58	−0.28	6.64
^{82}Se	−0.60	3.59	−0.23	4.42	−0.67	4.52	−0.27	5.46
^{100}Mo	−0.48	3.73	0.19	4.02	−0.51	5.08	0.32	5.27
^{116}Cd	−0.33	2.76	0.14	2.95	−0.34	2.89	0.12	3.11
^{128}Te	−0.72	3.80	−0.15	4.67	−0.72	3.97	−0.12	4.80
^{130}Te	−0.65	3.43	−0.13	4.21	−0.65	3.59	−0.16	4.40
^{136}Xe	−0.52	2.83	−0.10	3.45	−0.52	2.96	−0.12	3.60

5.2. Double Charge Exchange Reaction

It has been recently proposed that the nuclear matrix elements involved in double charge exchange reactions may resemble, at least for their geometrical structure, those involved in neutrinoless double beta decay [76], even though mediated by different interactions, strong and weak, respectively. Furthermore, in Reference [77], a hypothesis of linear correlation between double charge exchange reaction and neutrinoless double beta decay NMEs was suggested. This hypothesis was further studied in Reference [78], where a correlation between the $0\nu\beta\beta$ decay nuclear matrix element and DCE nuclear matrix element in IBM-2 for cases ^{76}Ge, ^{82}Se, ^{116}Cd, and ^{128}Te was found. In particular, linear dependence for GT NMEs was found to be [78]

$$\mathcal{M}_{GT}^{0\nu\beta\beta} = -0.07 + 1.36\mathcal{M}_{T,GT}^{DCE}, \tag{10}$$

where $\mathcal{M}_{T,GT}^{DCE}$ refers to matrix elements for the target. In these DCE calculations, SPEs of set (II) were used and thus the comparison was made with IBM-2 $0\nu\beta\beta$ decay NMEs from [71]. However, the linear dependence can also be found for updated single-particle energies and change in constant coefficients is anticipated to be very mild. When finding the constant coefficients, the important thing is to use the same SPEs in both calculations, DBD and DCE, in order to avoid unnecessary uncertainty coming from different input parameters.

6. Conclusions

In this article, the impact of using different values of the SPEs on pair structure coefficients, crucial for IBM-2 description of double beta decay, was discussed, and specific results of $0\nu\beta\beta$ decay nuclear matrix elements, including their connection to double charge exchange reaction nuclear matrix elements, were considered. The single-particle energies may be considered as input parameters to be fitted to reach an optimal correspondence with the data, or alternatively they can be calculated using a mean field potential, or they can be extracted from available experimental data, as has become customary in the connection of IBM-2 wave functions.The observed increase of the $0\nu\beta\beta$ decay IBM-2 matrix elements can be explained by the changes in the single-particle energies. In those cases where the updated single-particle energies are generally decreased and compressed compared to the previous set, generally larger values for the pair structure coefficients α and β of the S and D pairs are produced. This then leads to larger NMEs, especially when (1) both proton and neutron single-particle energies are affected and (2) the number of valence particles outside closed shells is high.

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