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Inflationary Implications of the Covariant Entropy Bound and the Swampland de Sitter Conjectures

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Abstract: We present a proposal to relate the de Sitter conjecture (dSC) with the time dependence of fluxes via the covariant entropy bound (CEB). By assuming an early phase of accelerated expansion where the CEB is satisfied, we take into account a contribution from time-dependent flux compactification to the four-dimensional entropy which establishes a bound on the usual slow-roll parameters η_H and ϵ_H. We also show an explicit calculation of entropy from a toroidal flux compactification, from a transition amplitude of time-dependent fluxes which allows us to determine the conditions on which the bounds on the slow-roll parameters are in agreement to the dSC.

Keywords: fluxes; entropy; swampland; multi-field inflation

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1. Introduction

The Swampland program has reached great advances in the last few years in its pursuit of characterize those features distinguishing effective theories that can be consistently completed into a quantum gravity theory (Landscape) from those which do not (Swampland) [1–4]. The boundary between the Landscape and Swampland is usually defined by a series of bounds on quantities of the proper effective theory which, in turn, are expressed in terms of the Planck mass M_p. In the limit where gravity decouples, i.e., $M_p \to \infty$, the Swampland constraints vanish. A question to be answered is whether these boundaries can be traced back to some essential microscopic physics in the Quantum Gravity Theory, or accordingly, to some fundamental constraints in the 10-dimensional String Theory formulation.

Many different bounds have been proposed as Swampland conjectures, although it is expected that all of them are related in some way through more fundamental principles (see review [3] and lectures [4] for references). Some of them are based on solid grounds, such as the distance conjecture and the weak gravity conjecture, while others, as the de Sitter conjectures, were initially motivated by some *empirical* evidence based on string models [5,6], and appropriately refined into its final form in order to be compatible with some well known effective scenarios such as the Higgs potential [7] among others. The refined version of the conjecture was motivated by stability of de Sitter extrema [8], by cosmological arguments through the slow-roll parameter η_V [9], as well as by entropy arguments together with the distance conjecture on the dS space [10]. Many logic and solid arguments have been followed in the literature [11–20] so far with the purpose to give stronger ground basis to this conjecture. A matter of particular interest to us is the series of arguments supporting the de Sitter conjecture (dSC) based on the laws of thermodynamics [21–24]. Relating the dSC to thermodynamic principles, such as a positive temperature phase and the concavity of the entropy functional, respectively, have also been considered in the literature. Furthermore, as stochastic effects become relevant, it is found

by thermodynamic arguments that slow-roll conditions either violate the second law or the swampland dSC [25,26].

In its original form, the de Sitter conjecture states that

$$\frac{|\nabla V|}{V} \geq c \qquad \text{or} \qquad \frac{\min \nabla_i \partial_j V}{V} \leq -c', \tag{1}$$

with c and c' positive constants of order one in Planck units and V the effective scalar field potential. In addition to the well known implication of discarding the existence of dS minima, dSC also have implications on single and multi-field inflationary scenarios [27,28] which have been proven to be a matter of great interest [29–41].

In this paper, we are interested in studying the implications of the CEB [42–45] on the inflationary slow-roll parameters. We shall show that a time dependence of entropy (which in turn we propose it comes from a time-dependent flux compactification) leads to the presence of bounds on η_H and ϵ_H. Our goal is to determine if such bounds are related to the dSC inequalities.

However, it is important to emphasize that we are not generating an expanding (inflationary or not) four-dimensional Universe through fluxes. The source of an expanding Universe from a string theory point of view is beyond the scope of this paper. Our interest focuses on the compatibility of an inflationary Universe with the CEB[1].

The paper is organized as follows. In Section 2, we briefly review some basics regarding the CEB and establish a relation between the change of entropy over time and the slow-roll parameter ϵ_H Equation (9). In Section 3, we use this relation to study its implications on single field and multi-field inflationary scenarios. Our main result is that we are able to reproduce the inequalities of the dSC, as well as the mutual exclusion between them in terms of the time variation of entropy sourced from physics in extra dimensions, as well as the explicit form of the constants c and c' in terms of entropy and time derivatives of it. A simple example concerning a toroidal flux compactification is presented to show how fluxes contribute to the four-dimensional entropy. Finally, we discuss our conclusions and some final comments.

2. Covariant Entropy Bound and Inflation 2

The covariant entropy bound (CEB) is a well tested conjecture in many gravitationally scenarios, including cosmology [42–45]. Basically, it states that for a codimension 2 surface B, there exist at least two light-sheets L, defined as the codimension 1 null hypersurfaces orthogonal to B spanned by light-type trajectories terminating at a singularity or a caustic, with a negative expansion rate. These light-sheets can be future or past-directed. The general statement is that the entropy associated to the light-sheet $S(L[B])$, at a given time, is bounded by the area A of the surface B as[2]

$$S(L[B]) \leq \frac{A(B)}{4}. \tag{2}$$

In [10] the association of the CEB with an expanding de Sitter (dS) Universe establishes a bound for the entropy given by,

$$S \leq \pi R_{AH}^2, \tag{3}$$

where R_{AH} is the radius of the apparent horizon (AH). Appealing to the distance conjecture (DC), it is discussed the case in which entropy increases with the field moduli distance through the appearance of towers of massless modes ϕ. Since in a dS space $R_{AH}^2 \sim 1/V$, where V is the scalar potential, the CEB implies that[3]

$$\partial_\phi S \leq -\frac{\partial_\phi V}{V^2}. \tag{4}$$

Due to the fact that the potential V decreases as the number of massless modes grows, $\partial_\phi V < 0$. By taking $\partial_\phi S \sim c/V$, with c a number of order 1 in Planck units, the slow-roll parameter ϵ_V acquires a bound given by

$$2\epsilon_V = \frac{|\partial_\phi V|^2}{|V|^2} \geq c^2. \tag{5}$$

As noticed in [9], ϵ_V is not necessarily of order 1 if

$$\eta_V \leq -c', \tag{6}$$

with c' another constant of order 1 in Planck Units. This was also concluded in [10] by constraining the scalar potential fluctuations to remain within the AH. In this context, the refined de Sitter conjecture (dSC) follows from the dependence of entropy, and of the apparent horizon radius, on the tower of massless modes.

In this work we are interested in a slightly different—but equivalent—approach by demanding that entropy and R_{AH} increases as time runs, via an effective scalar field ϕ derived from a string compactification with fluxes depending on time.

We now proceed to elaborate our proposal. In a flat 4-dimensional expanding Universe described by the Friedmann–Robertson–Walker (FRW) metric:

$$ds_4^2 = -dt^2 + a^2(t)(dr^2 + r^2 d\Omega^2), \tag{7}$$

It is possible to define the radius of the AH at a time t as $R_{AH} = 1/H$, with H the Hubble constant defined as $H = \dot{a}/a$, where $\dot{a}$ denotes the time derivative with respect to the proper time t. As the Universe expands, the comoving Hubble radius R_{AH}/a shrinks and the AH comoving area given by

$$A_{AH} = 4\pi R_{AH}^2 = \frac{4\pi^2}{H^2} \tag{8}$$

increases. Following [45], an infinitesimal variation of time helps us to rewrite the CEB in the form[4].

$$\gamma \equiv \frac{H\dot{S}}{2\pi} \leq \epsilon_H, \tag{9}$$

with $\epsilon_H = -\dot{H}/H^2$ the usual slow-roll inflationary parameter satisfying $\epsilon_H \ll 1$, implying that $\dot{\phi}^2$ and $\ddot{\phi}$ are much smaller than V. Notice that according to our anzatz, as the AH radius increases we also expect an increase on entropy, thus our model is valid only for $\dot{S} > 0$.

In the following we shall assume that a variation of entropy in time comes from extra dimensions within a string theory based scenario. In this context, we expect that entropy must be a function of the Hubble parameter H, time and other string parameters which we shall ignore to our present purpose.

3. Inflation and Effective Field Theory

Let us consider an N-field scenario. As shown in [57], it is more convenient to switch to the kinematic −Frenet–Serret− frame to compute the equations of motion of an effective theory in the presence of a scalar potential $V = V(\phi_a)$ with $a = 1, \ldots N$. See Appendix A for more details.

The equations of motion can be written in terms of the unitary tangential vector $\hat{e}_1$, the normal $\hat{e}_2$ and the $(N-2)$ binormal $\hat{e}_i$, as

$$(\ddot{\phi} + 3H\dot{\phi} + V_1)\hat{e}_1 + (-\dot{\phi}\Omega_1 + V_2)\hat{e}_2 + V_i\hat{e}_i = 0, \tag{10}$$

with $i = 3, \ldots, N$, $\dot{\phi}$ is the norm of the vector field $\vec{\dot{\phi}} = (\dot{\phi}^1, \ldots, \dot{\phi}^n)$ and Ω_1 is the curvature defined as $D_t \hat{e}_1 = -\Omega_1 \hat{e}_2$. Since the Frenet–Serret frame $\{\hat{e}_1, \hat{e}_2, \hat{e}_i\}$ is an orthonormal basis, we find that

$$\begin{aligned} \ddot{\phi} + 3H\dot{\phi} + V_1 &= 0, & (11)\\ -\dot{\phi}\Omega_1 + V_2 &= 0, & (12)\\ V_i &= 0. & (13) \end{aligned}$$

Notice all projections of $\vec{\nabla} V$ along normal vectors $\hat{e}_i$, denoted by V_i, vanish for $i = 3, \ldots, n$, implying that we can focus only on the field trajectory on the osculating plane.

Now, together with the Einstein–Friedmann equation

$$3H^2 = \frac{\dot{\phi}^2}{2} + V, \tag{14}$$

and by the usual definitions of the slow-roll parameters,

$$\epsilon_V \equiv \frac{1}{2}\left(\frac{\vec{\nabla} V}{V}\right)^2 = \tilde{\epsilon}_V + \hat{\epsilon}_V, \tag{15}$$

with

$$\tilde{\epsilon}_V = \frac{1}{2}\left(\frac{V_1}{V}\right)^2, \quad \hat{\epsilon}_V = \frac{1}{2}\left(\frac{V_2}{V}\right)^2, \tag{16}$$

we can construct a relation between ϵ_H and $\hat{\epsilon}_V$ given by

$$\epsilon_H^2 - (6 + \alpha)\epsilon_H + 9 = 0, \tag{17}$$

where

$$\alpha = \frac{\Omega_1^2}{H^2 \hat{\epsilon}_V} = \frac{2V^2}{H^2 \dot{\phi}^2}, \tag{18}$$

with $\hat{\epsilon}_V \neq 0$. Notice that $\alpha \to 0$ implies $\epsilon_H \to 3$ for which an accelerated expansion—and therefore inflation—is ruled out. An almost zero α means two things: one is that the kinetic energy is much bigger than V; the second possibility involves $\hat{\epsilon}_V$ growing faster than the ratio Ω_1 / H.

By using ϵ_H as a function of $\hat{\epsilon}_V$ from (17) we obtain from inequality (9)

$$\gamma \leq \left(3 + \frac{\alpha}{2}\right) \pm \sqrt{\left(3 + \frac{\alpha}{2}\right)^2 - 9}. \tag{19}$$

For $\gamma \leq 3 + \frac{\alpha}{2}$ (otherwise $\epsilon_H > 1$) we can establish a bound for ϵ_V as follows. From Equation (19) we have

$$\hat{\epsilon}_V \geq \frac{\Omega_1^2}{H^2} \frac{\gamma}{(\gamma - 3)^2}. \tag{20}$$

Then a lower bound for ϵ_V follows in the form of

$$\begin{aligned} \frac{1}{2}\frac{|\nabla V|^2}{|V|^2} &= \widetilde{\epsilon}_V + \hat{\epsilon}_V, \\ &\geq \widetilde{\epsilon}_V + \frac{\Omega_1^2}{H^2} \frac{\gamma}{(\gamma - 3)^2}, \end{aligned} \tag{21}$$

implying that

$$\frac{|\nabla V|^2}{|V|^2} \geq 2\epsilon_H + \frac{\Omega_1^2}{H^2}\frac{2\gamma}{(\gamma-3)^2}, \tag{22}$$

where according to our initial anzatz we have taken the slow-roll limit, $\epsilon_H \sim \widetilde{\epsilon}_V$.

Notice that for $\gamma \sim \epsilon_H \ll 1$, the above expression reduces to

$$\epsilon_V \sim \epsilon_H\left(1 + \frac{\Omega_1^2}{9H^2}\right). \tag{23}$$

Therefore, in a multi-field scenario, inflation is allowed with ϵ_V of order 1, while $\gamma < \epsilon_H \ll 1$. This is precisely the statement by Achúcarro and Palma in [28]. A stringy example in which it is possible to fulfill the Swampland constraint in a multi-field scenario by considering fat inflations was studied in [58]. Observe that for single field inflation, i.e., with $\Omega_1 = 0$, expression (22) allows ϵ_V to be smaller than 1.

3.1. Refined Swampland de Sitter Conjecture

Our initial anzatz assumes the validity of $\gamma \leq \epsilon_H \ll 1$. However, from Equation (22), a very small value for γ opens up the possibility for ϵ_V be smaller than $\mathcal{O}(1)$. It is then of interest to construct a bound for η_H, which according to the dSC, must be of the form $\eta_H < -c'$. First of all, let us assume that γ is defined in a way such that we can write

$$\dot{\gamma} \leq \mathcal{O}(1)\dot{\epsilon}_H. \tag{24}$$

This is a reasonable assumption since γ must be smaller than ϵ_H during a finite time interval in which an exponential accelerated expansion occurs. Since we are assuming $\gamma/\epsilon_H \leq 1$, we can restrict to a time interval during which

$$\dot{\gamma}/\gamma \leq \dot{\epsilon}_H/\epsilon_H. \tag{25}$$

According to [10] this means that fluctuations of the scalar potential must remain inside the AH. Actually this is equivalent to take a double time derivative on the CEB while preserving the inequality, i.e.,

$$\ddot{S} \leq 2\pi\frac{d}{dt}\left(\frac{\epsilon_H}{H}\right). \tag{26}$$

Hence, using the expression for η_H in terms of the Hubble constant as $\eta_H = -\ddot{H}/(2H\dot{H})$, expression (25) reduces to

$$\eta_H \leq -\frac{\ddot{S}}{4\pi\epsilon_H} + \frac{3}{2}\epsilon_H. \tag{27}$$

On the other hand, by taking the derivative of the Einstein–Friedmann equations of motion[5], it is found that

$$\eta_H + \epsilon_H = \frac{D_t V_1}{3H^2\dot{\phi}}, \tag{28}$$

which together with

$$\frac{D_t V_1}{\dot{\phi}} = Q_1 - \Omega_1^2 \tag{29}$$

where Q_1 is a quadratic form given by $Q_1 = V_{ab}e_1^a e_1^b$ with $e_1^a = \dot{\phi}^a/\dot{\phi}$ (see Appendix A), it follows that

$$\eta_V \approx \frac{\lambda_{min} - \Omega_1^2}{V} \leq -\frac{1}{4\pi}\frac{\ddot{S}}{\epsilon_H} + \frac{5}{2}\epsilon_H. \tag{30}$$

In this last expression, we have used that under the condition of orthonormality $\sum_a |e_1^a|^2 = 1$, the quadratic form Q_1 has a minimum value given by the smallest eigenvalue λ_{min} of the Hessian matrix V_{ab} in the basis $\{e_i^a\}$, as well as that $3H^2 \approx V$.

Notice the following:

1. $\ddot{S} \leq 0$ if, and only if, $\eta_V \geq \frac{5}{2}\epsilon_H$. Therefore, $\eta_V < 0$ only if $\ddot{S} > 0$;
2. For a single field effective theory, i.e., $\Omega_1 = 0$, λ_{min} can be positive only if $\ddot{S} < 10\pi\epsilon_H^2$. Since $\epsilon_H \ll 1$, $\ddot{S}$ can be a (less than one) positive number or a negative number. For $\ddot{S} \geq 10\pi\epsilon_H^2$ the minimal squared mass is negative;
3. For $\Omega_1 \neq 0$, it is possible to have a negative η_V of order 1 even though λ_{min} is positive if $\Omega_1 \gg 1$. On the other hand, λ_{min} would be negative only if $\ddot{S} > \frac{4\pi\epsilon_H\Omega_1^2}{V}$.

In summary, it is the profile of $\ddot{S}$ which would determine whether the dSC would be satisfied or not in both single and multi-field scenarios.

3.2. Lyth's Bound and the Swampland Distance Conjecture

It has been found that, for a canonical single-field slow-roll model of inflation, generally the overall field displacement $\Delta\varphi$ experienced by the inflation during the quasi-de Sitter phase must satisfy a lower bound, which is known as the *Lyth bound* [59]

$$\Delta\phi \simeq \sqrt{2\epsilon_H}\Delta N, \tag{31}$$

where ΔN is the effective number of e-folds during inflation needed to be at least 60. So, we can rewrite (31) as,

$$\Delta\phi \simeq \sqrt{2}\cdot 60\sqrt{\epsilon_H} = \kappa\sqrt{\epsilon_H}, \tag{32}$$

where κ is a number of $\mathcal{O}(10)$. Hence, during inflation field displacement is expected to be of order 1 in Planck units, although larger orders have been considered in the literature and known as trans-Planckian distances. However, recently it has been conjectured (see [3,4] and references therein) that field displacements have an upper bound, probably connecting to other Swampland conjectures, as the Distance conjecture.

Now, in the case of multifield inflation, the field excursion $\Delta\phi$ has an extra contribution coming from non-trivial angular motion Ω_1 and entropy mass M [28],

$$\Delta\phi \gtrsim \kappa\sqrt{\frac{\epsilon_H}{\beta}} \tag{33}$$

where β is a function depending on Ω_1 and M. Now following [28], if the fluctuating mode crosses the horizon in the linear regime, i.e., if the condition $(1 - c_s^2)H < c_s M$ is satisfied, then $\beta \simeq c_s$, where c_s is the propagating speed of adiabatic perturbation also known as the speed of sound, given by

$$c_s = \left(1 + \frac{4\Omega_1^2}{M^2}\right)^{-1/2} \tag{34}$$

where M^2 is given by,

$$M^2 = Q_2 + \epsilon_H H^2 \mathbb{R} - \Omega_1^2, \tag{35}$$

with the quadratic form Q_2 given by

$$Q_2 = e_2^a e_2^b V_{ab}, \tag{36}$$

where e_2^a are the normal unitary directions in the Frenet–Serret frame, and $\mathbb{R} = \mathbb{R}_{abcd} e_1^a e_2^b e_1^c e_2^d$. Notice that, the maximum and minimum values of Q_2 are given by the eigenvalues of the Hessian matrix V_{ab} denoted $\lambda_{\min}$ or $\lambda_{\max}$, with $\mathbb{R}_{abcd}$ and $\mathbb{R}$ the Riemann tensor and Ricci scalar of the scalar field manifold.

Hence the Lyth bound (33) can be written as,

$$\Delta\phi \geq \kappa\sqrt{\epsilon_H}\left(1 + \frac{4\Omega_1^2}{M^2}\right)^{1/4} = \kappa\sqrt{\epsilon_H}\left(1 + \frac{4\Omega_1^2}{\lambda - \Omega_1^2 + \epsilon_H H^2 \mathbb{R}}\right)^{1/4}, \tag{37}$$

where $\lambda \in [\lambda_{min}, \lambda_{max}]$.

Let us also comment on different scenarios, by considering the CEB (9),

1. If $c_s \simeq 1$, it looks like the bound can be made even stronger than in (31). This is because $c_s = 1$ implies that either $\Omega_1 = 0$ −which holds for the case of a single field− or $M^2 \gg \Omega_1^2$. For $\epsilon_H \ll 1$ the last case implies that λ cannot be negative, indicating that the second inequality of the dSC does not hold and that $\lambda \gg \Omega_1^2$. Additionally, for inflation to take place, we need $\gamma \ll 1$ which makes the bound for $\Delta\phi$ stronger compared to the bound fixed by ϵ_H;
2. If $c_s < 1$, Ω_1 cannot be zero. This case then applies to multi-field scenarios. Notice that the bound on $\Delta\phi$ is now weaker than the bound for single field models (31) if $\lambda < \Omega_1^2$. As shown in [60], if $\Delta\phi \geq 10$ in Planck units, there is still room to satisfy the dSC. So now we have to see under which conditions the ratio $\sqrt{\gamma/c_s}$ remains unaltered, allowing the dSC to be satisfied. If $\lambda - \Omega_1^2 > 0$ the bound can become stronger.

Summing up, we have presented an anzatz in which a contribution to entropy allegedly from string theory, leads us to the possibility to reproduce both inequalities in the dSC. We now proceed to consider a simple toy example in which the entropy contribution from a flux string scenario is computed.

3.3. Entropy from a Flux Compactification on T^6/Z_2

In this section, we shall present a very simple example in which we show that a contribution to the 4-dimensional entropy from string compactification is a possibility to be considered. Before explicitly describing our model, it is important to contextualize it within different scenarios already studied in the literature. The first thing to have in mind is that we are interested in models with an implicit time dependence.

A possible simple scenario may consist of an internal metric with a time-dependent warping factor. However, as shown in [61], those scenarios are in conflict with cosmological models with an inflationary stage. Concretely a single scalar field with a non-stabilized modulus constructed from a theory with extra dimensions is not compatible with inflation if null energy conditions (NEC) are fulfilled.

We shall take into account another option[6], which has not been vastly explored in the literature by allowing the fluxes to depend on time during a quantum transition. For this, we start from a well-known scenario in which NS–NS and RR 3-form fluxes are turned on while fulfilling the usual constraints as tadpole cancellation, Bianchi identities, and Dirac quantization. Given a flux configuration specified by a specific set of RR and NS–NS fluxes some of the moduli are fixed at their specific vevs while the Kähler moduli remain a runaway direction. Therefore, in this case we have a single field inflationary scenario[7].

Now, we allow the potential forms to behave as quantum fields in order to compute the transition amplitude of a given flux configuration into itself during a time interval. Throughout this transition, the fluxes can get any value, which does not necessarily obey any of the aforementioned restrictions inherent to the consistent stringy scenario (one can think about these fluxes as *virtual* ones). The amplitude is then given in terms of the associated path integral

$$\mathcal{Z} \sim \int \mathcal{D}[F(t)] e^{-i\mathcal{S}_{\rm IIB}(F)} \tag{38}$$

where $F(t)$ denotes all possible flux configurations in the type IIB action depending on time and $[\mathcal{D}(F)]$ denotes the Feynman's measure along all possible trajectories for different flux configurations, modulo those which are equivalent by the same tadpole contribution.

In the following, we shall compute the partition function $\mathcal{Z}$ for a simple toy model and, in consequence, we shall compute the entropy contribution as a function of time, which, in turn, will lead us to estimate the value of $\dot{S}$ and $\ddot{S}$. In this context, we shall be able to test the possibility to obtain the dSC according to our initial anzatz, this is, by proposing that the source of the bounds come from time-dependent internal fluxes.

3.4. A Toy Model: Isotropic Toroidal Compactification with Fluxes in Type IIB

Our model consists on a compactification on an isotropic six-dimensional torus T^6 threaded by fluxes with an orientifold $O3$-plane. We are not considering the presence of $D3$-branes for which we are taking RR potential $C_4 = 0$[8]. We will follow the notation of [63] in this subsection.

Consider a factorizable torus $T^6 = T^2 \times T^2 \times T^2$ with coordinates as in Figure 1, and let us take the ten-dimensional metric given by

$$ds_{10}^2 = ds_4^2 + \sum_i (dy^i)^2 \,, \tag{39}$$

where ds_4^2 is the homogeneous and isotropic FRW metric (7). Since we are interested in computing an amplitude transition of a given flux configuration of N units of RR flux F_3 and M units of NS–NS flux H_3 which satisfy the supergravity equations of motion and the stringy constraints, we shall consider that during transition the fluxes are time dependent. Let us start by considering a specific example in which the closed string potentials C_2 and B_2 depending on time, are given by

$$C_2 = f(t) N y^6 dy^2 \wedge dy^4, \quad \text{and} \quad B_2 = f(t) M y^5 dy^1 \wedge dy^3, \tag{40}$$

because the function $f(t)$ depends on time. The corresponding action in the string frame

$$\mathcal{A}_{\rm flux} = -\frac{1}{2} \int d^{10}x \sqrt{-g_{10}} \left(F_3^2 + e^{-2\phi} H_3^2 \right), \tag{41}$$

reduces to

$$\mathcal{A} = \frac{1}{2} \int d^{10}x \, a^3(t) \left[N^2 (\dot{f}(t)^2 (y^6)^2 - f^2) + M^2 e^{-2\phi} (\dot{f}(t)^2 (y^5)^2 - f^2) \right] \tag{42}$$

where the 3-form fluxes are defined as

$$\begin{aligned} F_3 &= N(\dot{f} y^6 dt + f dy^6) \wedge dy^2 \wedge dy^4, \\ H_3 &= M(\dot{f} y^5 dt + f dy^5) \wedge dy^1 \wedge dy^3, \end{aligned} \tag{43}$$

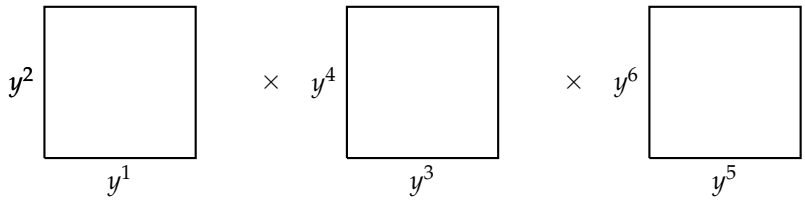

Figure 1. Coordinates of the compact space.

Since we are considering the isotropic case, we take all the internal coordinates y_i on equal footing (y), and the above integral can be written as

$$\mathcal{A}_{\text{flux}} = \frac{1}{2}\int d^{10}x\, a^3(t)(N^2 + e^{-2\phi}M^2)\Big((\dot{f}(t))^2 y^2 - f(t)^2\Big), \tag{44}$$

which mimics the behavior of a harmonic oscillator in a compact space. Thus, the partition function related to the above action can be found explicitly. The first step is to integrate over the internal coordinates from 0 to $2\pi R$ and external space coordinate r from 0 to r_*, for some finite r_* from which one obtains

$$\mathcal{A}_{\text{flux}} = -\int_0^{t_*} dt\, f(t)\hat{D}f(t), \tag{45}$$

where

$$\hat{D} \;=\; \alpha(t)\left(\frac{d^2}{dt^2} + 3H\frac{d}{dt} + \omega_0^2\right), \tag{46}$$

with $\omega_0^2 = 3/(2\pi R)^2$ and

$$\alpha(t) = \frac{(2\pi)^9 R^8 r_*^3}{9} a^3(t)(N^2 + e^{-2\phi}M^2), \tag{47}$$

with the boundary condition $f(0) = f(t_*)$, which states that at time $t = 0$ there is a given distribution of fluxes with N units of RR flux and M units of NS–NS flux. Between time 0 and t_* this distribution of fluxes changes such that at time t_*, the original distribution of fluxes is recovered. It is important to notice that $\alpha(t)$ is finite due to the tadpole cancellation condition and the fact that the dilaton has been fixed by the fluxes[9].

Therefore, the partition function can be given by:

$$\mathcal{Z}_{(N,M)}(0,t_*) = \int \mathcal{D}[f(t)]e^{-\frac{i}{2}\int_0^{t_*} dt\, \alpha(t)f(t)\hat{D}f(t)}, \tag{48}$$

which can be used to compute the transition amplitude between an initial flux configuration (N, M) and the final one specified by the same flux configuration. After performing the integral of (48) (and replacing t_* by t), and by taking the case in which $\Omega^2 = \frac{9H^2}{4} - \omega_0^2 < 0$, one obtains

$$\mathcal{Z}_{(N,M)}(0,t) = \left(\frac{-i\alpha(t)\Omega}{2\pi\sin(\Omega t)}\right)^{1/2}. \tag{49}$$

Notice that, our model behaves like a harmonic oscillator with a time-dependent mass $\alpha(t)$. For $\Omega^2 > 0$ a similar expression is obtained by replacing $\sin(\Omega t)$ by $\sinh(\Omega t)$.

Let us concentrate on the first case. After using standard methods in Euclidean time, it is possible to compute the entropy $S(t)$ from $\mathcal{Z}_{(N,M)}$ which reads

$$S(t) = \frac{1}{2}\log\left[-\frac{i}{2\pi}\alpha(t)\Omega\csc(\Omega t)\right] + \frac{1}{2}(\dot{\Omega}t + \Omega)t\cot(\Omega t) - \frac{t}{2}\left(\frac{\dot{\alpha}}{\alpha} + \frac{\dot{\Omega}}{\Omega}\right) \tag{50}$$

where

$$\frac{\dot{\alpha}}{\alpha} = 3H, \qquad \text{and} \qquad \frac{\dot{\Omega}}{\Omega} = \frac{9}{4}\frac{H\dot{H}}{\Omega^2} = -\left(\frac{3H}{2\Omega}\right)^2 H\epsilon_H, \tag{51}$$

while time variation of S is given by

$$\begin{aligned} \dot{S} &= -\frac{1}{2}[\Lambda H\epsilon_H t - 1]^2\Omega^2 t\,\csc^2(\Omega t) \\ &\quad +\Lambda\epsilon_H H\left[\frac{H}{2}((\epsilon_H + 2\eta_H) - \Lambda\epsilon_H)t - 1\right]\Omega t\,\cot(\Omega t) \\ &\quad +\frac{1}{2}[3\epsilon_H + \Lambda\epsilon_H(2\Lambda\epsilon_H - \epsilon_H - 2\eta_H)]H^2 t, \end{aligned} \tag{52}$$

where $\Lambda = \left(\frac{3H}{2\Omega}\right)^2$. Notice that entropy S and its derivatives with respect to time, $\dot{S}$ and $\ddot{S}$, are now functions of the slow-roll parameters η_H and ϵ_H, for which the inequalities (9) and (27) implicitly depend on them.

In this case, the volume of the internal manifold is bounded as

$$R^6 \sim \mathrm{Vol}(T^6) \lesssim \frac{1}{27\pi^6 H^6} \sim R_{AH}^6, \tag{53}$$

implying that the AH radius is always greater than the size of the internal torus. This suggests that, if correct, the internal volume is smaller than the observable universe when inflation takes place. For the second case, with $\Omega^2 > 0$ we have that

$$R^6 \sim \mathrm{Vol}(T^6) \gtrsim \frac{1}{27\pi^6 H^6} \sim R_{AH}^6. \tag{54}$$

Notice that the above bounds for the internal volume do not imply a stabilization for the Kähler modulus, but only characterize the validity of our analysis: if the internal volume is too small, smaller than R_{AH}^6, we must take into account the case in which $\Omega^2 < 0$ and see if the flux contribution to entropy is important enough and conversely for an internal volume larger than R_{AH}^6.

Swampland Implications

With the purpose to verify whether inequalities (9) and (27) (which we rewrite here for accessibility)

$$\begin{aligned} \dot{S}(\epsilon_H, \eta_H) &\leq \frac{2\pi}{H}\epsilon_H \\ \ddot{S}(\epsilon_H, \eta_H) &\leq 4\pi\epsilon_H\left(\frac{3}{2}\epsilon_H - \eta_H\right), \end{aligned}$$

hold for positives[10] $\dot{S}$ and $\ddot{S}$, we present in Figure 2 the contour plots for selected times for which (9) and (27) are satisfied[11]. The red contours represent the regions where $\dot{S} < 0$ whereas the blue ones are those for which $\dot{S} > 0$.

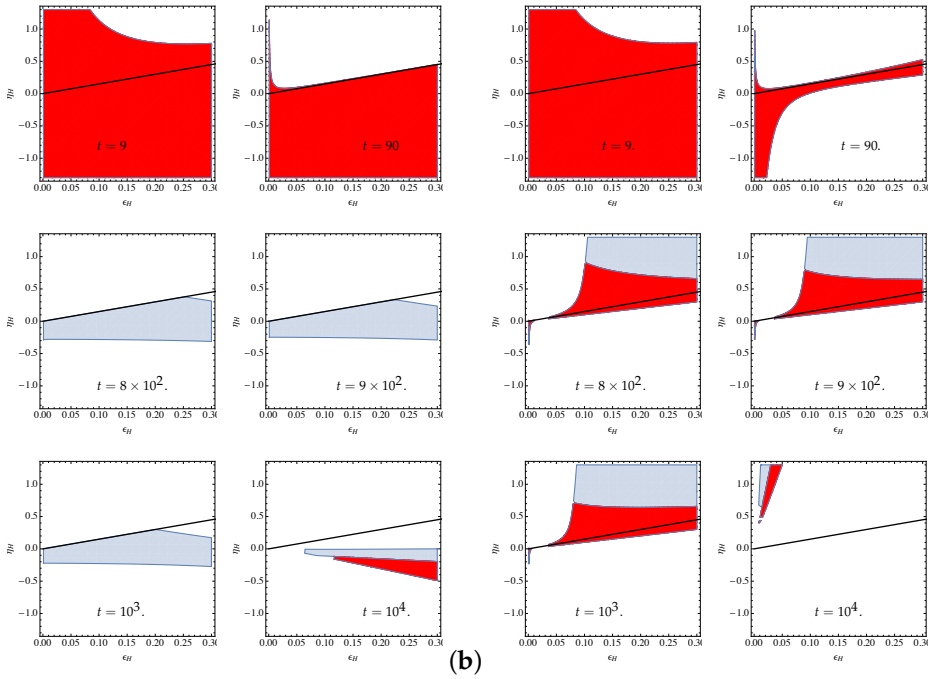

Figure 2. Plots for different times showing the values of η_H and ϵ_H for which inequalities (9) and (27) are fulfilled with $\ddot{S} > 0$. Red regions are discarded since they are related to a negative value for $\dot{S}$ whereas blue regions are related to positive $\dot{S}$. In both cases we take $H = 2.5 \times 10^{-3}$ [64]. (**a**) $\Omega^2 < 0$ (with $\Omega^2 = -10^{-2}$), (**b**) $\Omega^2 > 0$ (with $\Omega^2 = 3 \times 10^{-4}$).

As it is observed, there are conditions in time for which the inequalities are satisfied, but since our expressions for $\dot{S}$ and $\ddot{S}$ implicitly depend on η_H and ϵ_H, we find that not always it is possible to find conditions for which $\epsilon \ll 1$ and $\eta_H < 0$. Specifically, we observe that,

1. For $\Omega^2 < 0$, at late times, it is possible to fulfill the dSC with the constant c' of order 0.5 or less. This implies that, at least for this simple model, there is a contribution to the effective four-dimensional entropy from time-dependent fluxes which allows the dSC to be satisfied. For this to happen, the size of the AH radius must be larger than each of the torus radius. For initial times all the inequalities are satisfied only if $\dot{S} < 0$. As time evolves the regions where all the inequalities are satisfied are reduced in size. It is interesting to note that for $t \gg 1$ the regions where all inequalities are satisfied exclude small values of η_H for $\epsilon < 0.3$. In summary, this model is consistent with the RdS conjecture only for late times;
2. For $\Omega^2 > 0$ we also find that at late times there are conditions for the inequalities to be fulfilled but with positive values for η_H which are not in agreement with the dSC. However, this case implies that the radius of the AH is smaller than the internal torus. It is possible that this is not an available condition during inflation.

4. Conclusions and Final Comments

We have proposed a relation between the de Sitter conjecture (dSC) with the time dependence of fluxes through the covariant entropy bound (CEB). As an alternative of considering the appearance of towers of massless modes via the distance conjecture, we have focused on the possibility that a string time-dependent flux compactification contributes to entropy, such that the CEB also leads us to a bound on the slow-roll parameters of an expanding four-dimensional Universe.

For that we have assumed that $\epsilon_H \ll 1$ and that −as a result of the CEB− the time variation of entropy is positive and precisely bounded by it (9). Under these assumptions, we look for conditions on which the entropy profile (as a function on time) determines a bound on the second slow-roll parameter η_H. We have performed this analysis in a single

and multi-field inflationary scenarios. For the latter we have reproduced the result by Achúcarro and Palma in [28] by which it is possible to fulfill the first inequality of the dSC while allowing the small-roll parameter ϵ_V to be of order one in Planck units. We also have constructed a bound for η_V in terms of the second time derivative of entropy (27), and studied the implications on the Lyth's bound were we have observed that in general the bound becomes stronger. It is important to notice that for a null variation on the entropy from extra dimensions, the bounds on ϵ_V and η_V allow them to have values much less than one. Whether the dSC can be fulfilled by non-zero contributions of $\dot{S}$ and $\ddot{S}$ or not, will be determined by a specific model.

Finally, in order to have a concrete example, we have presented a simple toy model where we have computed the contribution to entropy from a time dependent flux compactification on a six-dimensional isotropic torus. By treating fluxes as quantum fields, we have computed the transition amplitude of a given flux configuration fulfilling all constraints as the Tadpole cancellation condition, Bianchi identities, and Dirac quantization. As a result, there is an entropy contribution as a function of time. In this case, entropy S and its derivatives with respect to time, depend on H, ϵ_H and η_H. We have plotted values for the slow-roll parameters for different values of time in order to test whether the dSC conjecture is fulfilled, meaning that if $\epsilon_H \ll 1$ then $\eta_H < 0$.

We have found that for the case in which the internal toroidal volume is smaller than R_{AH}^6, where R_{AH} is the size of the apparent horizon during inflation, there are conditions to satisfy the dSC. Further study is required to consider more realistic scenarios in which a time dependence of fluxes can be considered.

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Appendix A. Frenet–Serret Frame

In order to have a more general description we shall consider N fields. For that, let us define a map

$$\vec{\phi} : \mathbf{I} \to \mathbf{R}^n, \tag{A1}$$

such that

$$\vec{\phi}(t) = (\phi^1(t), \ldots, \phi^n(t)) \tag{A2}$$

where the entries are given in terms of a time independent basis $\hat{u}_a$, i.e.,

$$\vec{\phi}(t) = \phi^a(t)\hat{u}_a, \tag{A3}$$

with

$$||\vec{\phi}(t)||^2 = \phi^a \phi^b h_{ab} = \phi^2, \tag{A4}$$

where h_{ab} is the metric of the moduli space we are assuming to be explicitly independent of time.

As time runs, the trajectory in the field space has a tangential vector defined as

$$\hat{e}_1(t) = \frac{\dot{\vec{\phi}}}{\dot{\phi}} = \frac{\dot{\phi}^a}{\dot{\phi}}\hat{u}_a = T^a\hat{u}_a, \tag{A5}$$

with $\dot{\vec{\phi}}(t) = \dot{\phi}^a(t)\hat{u}_a$ and $||\dot{\vec{\phi}}(t)||^2 = \dot{\phi}^a\dot{\phi}^b h_{ab} = \dot{\phi}^2$. The normal unitary vector can be constructed as follows. Consider

$$\ddot{\vec{\phi}}(t) = \ddot{\phi}^a(t)\hat{u}_a, \tag{A6}$$

and the normal vector $\vec{e}_2(t)$ given by

$$\vec{e}_2(t) = \ddot{\vec{\phi}}(t) - \langle\ddot{\vec{\phi}}(t), \hat{e}_1\rangle\hat{e}_1(t). \tag{A7}$$

After some calculations, the normalized normal vector is given by

$$\hat{e}_2(t) = N^a\hat{u}_a, \tag{A8}$$

with

$$N^a = \frac{\dot{\phi}^2\ddot{\phi}^a - \dot{\phi}^a\dot{\phi}^b\ddot{\phi}_b}{\dot{\phi}(\dot{\phi}^2\ddot{\phi}^2 - (\dot{\phi}^b\ddot{\phi}_b)^2)^{1/2}}. \tag{A9}$$

Now, in terms of an orthonormal basis at each point of the curve $\vec{\phi}(t)$, i.e., the FS frame, we have that

$$D_t\begin{bmatrix}\hat{e}_1\\ \hat{e}_2\\ \hat{e}_3\\ \vdots\\ \hat{e}_{n-1}\\ \hat{e}_n\end{bmatrix} = \begin{bmatrix}0 & -\Omega_1 & 0 & & & & \\ \Omega_1 & 0 & -\Omega_2 & & & & \\ 0 & \Omega_2 & 0 & -\Omega_3 & & & \\ & & & \ddots & & & \\ & & & \Omega_{n-2} & 0 & -\Omega_{n-1} & \\ & & & 0 & \Omega_{n-1} & 0 & \end{bmatrix}\begin{bmatrix}\hat{e}_1\\ \hat{e}_2\\ \hat{e}_3\\ \vdots\\ \hat{e}_{n-1}\\ \hat{e}_n\end{bmatrix} \tag{A10}$$

and, in consequence, $D_t\hat{e}_1 = -\Omega_1\hat{e}_2$. From the corresponding equations of motion

$$D_t\dot{\phi}^a + 3H\dot{\phi}^a + V^a = 0, \tag{A11}$$

we can deduce the expression for Ω_1, where

$$D_t\dot{\phi}^a = \ddot{\phi}^a + \Gamma^a_{bc}\dot{\phi}^b\dot{\phi}^c, \tag{A12}$$

$$V^a = \vec{\nabla}V\cdot\hat{u}_a. \tag{A13}$$

and $\hat{u}_a$ is a time independent basis for the N-dimensional field space.

The N equations can be written in a vector notation as

$$(\ddot{\phi} + 3H\dot{\phi})\hat{e}_1 + \dot{\phi}(D_t\hat{e}_1) + \vec{\nabla}V = 0, \tag{A14}$$

where $\hat{e}_1$ is the unitary tangent vector to the field trajectory. Since $D_t\hat{e}_1 = -\Omega_1\hat{e}_2$, where $\hat{e}_2$ is the unitary normal vector to the trajectory, we have

$$(\ddot{\phi} + 3H\dot{\phi})\hat{e}_1 - \dot{\phi}\Omega_1\hat{e}_2 + \vec{\nabla}V = 0, \tag{A15}$$

Now we want to express components of $\vec{\nabla}V$ in terms of the FS basis, this is

$$\vec{\nabla}V\cdot\hat{e}_j(t) = V_a e^a_j = V_j, \tag{A16}$$

where we have used a as an index for the basis $\hat{u}_a$ and j for the index in the FS system.

Notes

1 The source of entropy in extra dimensions may rely on some basic and fundamental quantities in string theory, such as the presence of NS-NS and R-R fluxes. They play a crucial role in many phenomenological applications such as generating the correct hierarchies of scales in four-dimensional effective field theories by warping the geometry and particularly in inflationary model building [46–50] among others. The Tadpole constraint also leads to important implications on effective scenarios [51–56].

2 In terms of universal constants, $S \leq \frac{c^3 \kappa_B}{4\hbar G} A$. In this paper, we take all constants -including $M_{\rm Pl}$ to be equal to 1.

3 For any area, bigger or smaller than $A_{AH} = 4\pi R_{AH}^2$, it is conjectured that the CEB will always be fulfilled. In particular, since we are considering an expanding Universe, smaller areas are past-directed and the corresponding light-sheets are represented by the expansion rate $\theta_{-,\pm}$, with expansion rates denoted by $\theta_{(\pm,\pm)} = \pm\dot{a} \pm (1/r)$. Hence, for $t_1 < t_2$, the area $A(t_2)$ is connected to a smaller area $A(t_1)$ by a light-sheet with $\theta_{-,\pm}$ which are referred to as an anti-trapped surface [42]. The CEB can be strengthened by considering the entropy associated to the light-sheet between the surfaces $A(t_2)$ and $A(t_1)$, as shown in [45], with the entropy production bounded as $\Delta S \leq (A(t_2) - A(t_1))/4$, and where ΔS is thought to be the von Neumann entropy related to the difference between matter associated to light-sheet and the entropy of vacuum. Following [45], let us consider the AH surface at different times, t_2 and t_1 with $t_2 > t_1$. We then have the entropy production between that interval, satisfying the bound

$$\Delta S \leq \frac{A_{AH}(t_2) - A_{AH}(t_1)}{4},$$

where ΔS is the difference between the entropy at time t_2 and t_1. This implies that, in this case, the function $(S - A_{AH})(t)$ is a monotonically decreasing function. Therefore, an infinitesimal variation of time helps us to rewrite this bound as $\frac{dS}{dt} \leq \frac{1}{4}\frac{dA}{dt}$. By parametrizing the scalar potential in terms of moduli, whose number changes over time, the above condition holds only if the moduli increase accordingly, allowing that $\partial_\phi S \leq \partial_\phi A_{AH}/4$. This is actually what distance conjecture claims.

4 Although the CEB was proved to be consistent only for scenarios in which the null energy condition (NEC) holds, it was shown in [45] that Buosso bound can also be satisfied independently of the NEC.

5 This expression is consistent with the well known result, $\eta_H + \epsilon_H \approx \eta_V$ for $\eta_H = -\frac{\ddot{\phi}}{H\dot{\phi}}$ and $\eta_V = \frac{D_t V_1}{3H^2\dot{\phi}}$. In the single field scenario, $\eta_H \approx -\frac{1}{2}\left(\frac{\partial_\phi V}{V}\right)^2 + \frac{\partial_\phi^2 V}{V}$.

6 Non-constant fluxes depending on moduli, were studied in [62].

7 As remarked in the introduction, we are not driving inflation by the fluxes we are turning on. Our purpose is to compute their contribution to the entropy.

8 Since we work on a no-scale model, Kähler modulus is not stabilized. Therefore, the value of the RR potential C_4 and the internal volume are not fixed. This implies that our choice of having no D3-branes, or equivalently taking $C_4 = 0$ does not correspond to a stable point in the scalar potential since it is flat along this Kähler direction. Then, corrections due to the presence of D3-branes are expected.

9 The contribution from the 3-form fluxes to $d * F_5$ is given by

$$H_3 \wedge F_3 = -\frac{MN}{2} df \wedge dy^1 \wedge dy^2 \wedge dy^3 \wedge dy^4 \wedge \left(y^5 dy^6 - y^6 dy^5\right) - MNf^2 dVol_6,$$

where $d\,Vol_6 = dy^1 \wedge dy^2 \wedge dy^3 \wedge dy^4 \wedge dy^5 \wedge dy^6$. In the case of an isotropic torus all internal coordinates have equal footing for which the factor $\left(y^5 dy^6 - y^6 dy^5\right)$ vanishes. Then, the tadpole contribution from the fluxes, $\mathcal{N}_{flux} = -MNf^2$ must be cancelled by the $O3$-plane. By Dirac quantization, this restricts the function $f(t)$ to be integer valued at $t = 0$ and $t = t_*$. For other type of compactifications, fulfilling the Bianchi identity for F_5 would impose an extra constraint on the function $f(t)$.

10 As remarked in Section 2, our anzatz is only valid under the assumption of a positive $\dot{S}$.

11 We have taken $\dot{\eta}_H = 0$.

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