Comments on P. Jordan's Cosmological Model

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Abstract: We analyse the original cosmology of P. Jordan through his 1939 key paper entitled *"Bemerkungen zur Kosmologie"* or *"Comments on cosmology"*. In this almost forgotten work, the author introduced a model of dynamical cosmology with spontaneous creation of matter, based on the Large Numbers study, initiated by Eddington and further developed by Dirac. Jordan's will to explore heuristically all possible cosmological models in order to be prepared in case of surprising future astronomical data is very compelling in this article. Since we think it is wise to learn from our predecessors and from the unsuccessful theories that were later left behind, the present article also offers an overview of Jordan's work during the 1930s through the analysis of a series of some of his other original pieces. An English translation of Jordan's key paper can be found in the appendix.

Keywords: history of cosmology; Large Numbers hypothesis; static universe

1. Introduction

During the thirties, Pascual Jordan, already famous for his work in quantum mechanics, turned to cosmology. The context of this study switch was particular. When Eddington published studies of the fine structure constant, Dirac postulated his Large Numbers hypothesis, which principle led to the variation of fundamental constants as G and the creation of matter. We will detail this more thoroughly in the first section. Moreover, cosmology was a newborn science and, even with Hubble's results [1], dynamical and static models used to compete against each other. As Jordan's model could have been considered to be stationary, in the second section we will compare static and steady state models developed in the thirties.

After this contextual setting, Jordan's 1939 paper [2] here studied, will be exposed in Jordan's perspective and development, in the third section. Indeed, this work seems to be the closure of an exploration phase, leading Jordan to work on the empirical consequences of his model.

Then, in the fourth section, we will address three engaging points of Jordan's work: his system of units, the variation of the gravitational constant and the specification of the created matter.

Jordan's cosmological model did not make it as a breakthrough, nor was it even well diffused at the time. Yet, in 1949, Max Born invited Jordan to present and publish his work in English in *Nature*. This publication will be discussed in the fifth and last section.

After his 1930s work, Jordan pursued the study of his model. However, hereby, we chose to only focus on the first phase of Jordan's work, more centred on cosmology. For more information about Jordan's geological observational consequences see Reference [3] or Reference [4]. More extensive review on variations of constants shall be found in Reference [5] in French or in Reference [6] in English, or more recently in Reference [7].

2. Large Numbers Study

Eddington worked on the fundamental meaning hidden in the fine structure constant $\alpha = \frac{2\pi e^2}{hc}$, where e is the charge of the electron [8]. This number also calls upon Planck's constant h and the speed of light c; Eddington saw in α an opportunity to harmonize quantum and relativistic theories [9,10]. Using Clifford algebras to describe the wave function of two interacting electric charges, he witnessed the emergence of the number 137, which is equal to the inverse of[1]. This part of Eddington's work is often associated with numerology, yet we must underline that it opened the way for Large Numbers hypothesis, study of coincidences and Dirac principle.

First, in 1937, in a short letter to *Nature* Editors [13], Dirac expressed the age of the universe in atomic units and found a dimensionless large number, the so-called epoch, about 10^{39}. He noted that the ratio between Coulombian and gravitational forces between a proton and an electron is about 10^{39} and the ratio between the mass of the universe and of a proton is about 10^{78}, roughly the square of 10^{39}. Too improbable to be a coincidence, Dirac wrote:

> "This suggests that the above-mentioned large numbers are to be regarded, not as constants, but as simply functions of our present epoch, expressed in atomic units." [13] (p. 323)

So the ratio of the two forces must evolve with time and the mass of the universe, expressed in units of proton mass, must increase as the square of the time. This leads to consider variable constants and a process of matter creation.

Quickly after this article, Dirac built a consistent cosmological model based on his 1937 hypothesis [14]. In this new piece, would be laid down what is currently known as Dirac principle:

> "Any two of the very large dimensionless numbers occurring in Nature are connected by a simple mathematical relation, in which the coefficients are of the order of magnitude unity." [14], (p. 201)

Dirac introduced a development to establish the law of recession of spiral nebulae, which we quote as galaxies, in the frame of the Large Numbers hypothesis. The distance between two galaxies could be expressed in atomic units and becomes a dimensionless number $f(t)$. Working in a system where $c = 1$, the time the light needs to go from one galaxy to the other is also $f(t)$. So, if light is emitted with a period of δt, it will be received with a period of $\delta t + f(t + \delta t) - f(t)$. Knowing that, the redshift, the change in period per unit period, is namely $\dot{f}(t)$. And, defining Hubble constant as the redshift per unit distance, it appears $H = \frac{\dot{f}(t)}{f(t)}$.

Considering the average density of the universe, with galaxies flying away from each other, it could be established that $\rho \propto f(t)^{-3}$. But, on the other hand, Hubble's constant and average density could be made dimensionless, so with the Large Numbers hypothesis, there is a simple relation between them $\rho \propto H$. So, solving $f(t)^{-3} \propto \frac{f'(t)}{f(t)}$, Dirac arrived to the law for the rate of recession of galaxies $f(t) \propto t^{\frac{1}{3}}$[2] and the velocities of recession are not constant, but vary $\propto t^{\frac{-2}{3}}$.

Dirac also studied the curvature of hyper-surfaces defined by a constant time, denoted t-space. Without considering local irregularities, the curvature of three dimensional space, with t fixed, must be constant. This curvature, k, could be positive, null or negative. It is easy to rule out the positive k case. Indeed, if k is positive, the hyper-surface is closed and contains a finite mass. This finite total mass divided by the mass of the proton gives a large dimensionless number which is a constant. That is in direct contradiction with the large numbers hypothesis. Considering a curvature negative, the same

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[1] We would like to point out that Eddington originally arrived at the result 136 and disregarded the data: "*The experimental value of $\frac{hc}{2\pi e^2}$ is 137. According to the theory proposed in this paper it should be the integer* 136" [11]. Thereupon, he found the value 137 in his theory: "*I appear to have made such a mistake, and the new prediction is* 137." [12].

[2] By comparison, in Einstein- de Sitter model, with null pressure, $f(t) \propto t^{\frac{2}{3}}$; a universe of radiation goes with $f(t) \propto t^{\frac{1}{2}}$ and de Sitter model has $f(t) \propto \exp(t)$.

reasoning could be applied to a sphere with radius equal to the radius of curvature. Thus, only the flat *t*-spaces could satisfy the Large Numbers hypothesis.

The space time could be divided in flat *t*-spaces and a satisfactory theory of cosmology can be built. Of course, this model required process of spontaneous matter creation (or annihilation). As other cosmological models are consistent with data and do not demand a such exotic process, Dirac temporarily gave up[3] on his attempt at a cosmological model. Moreover, the contemporary models, such as Lemaître's one [18], were satisfying.

Pleading the scientific curiosity and the importance of considering a maximum of theoretical possibilities, Pascual Jordan developed his own cosmological model. Seeing the fine structure constant as a translation of a link between quantum and relativistic theories, developing the ideas of variable constants and of matter creation; he could be considered in perfect continuity with Eddington and Dirac.

3. Static Versus Steady Universes

Posing the foundations of cosmology, Einstein had in mind a static universe. This could be seen as a product of Copernician principle: humanity did not rise in a specific space-time configuration of the universe; the world is static. Einstein built a cylindrical cosmological model[4] [19]. Naturally, at that time, the conception of the universe came down to our only galaxy, the Milky Way.

Quickly, dynamical cosmological models were conceived for example, Reference [20], Reference [21] or Reference [18]. In parallel, the rising efficiency of telescopes led to identify other, external, galaxies and to determine a relation between their distance and their velocity [1], which could be interpreted as a motion at the universe scale. Accordingly, the universe was dynamic.

No more static model of the universe could be built, but still, the universe could be steady, or in a steady state. Even if the universe evolves ,a constant will remain or a certain mathematical relation will be conserved. Recently, O'Raifeartaigh discovered that Einstein may have been the first to consider this kind of model [22]. Indeed, in 1931, Albert Einstein worked on a draft about a steady universe [23]. In this attempt, Einstein considered a constant density of matter in the universe. Unfortunately, this density tended to be null. The aborted Einstein's steady universe was, in fact, the empty de Sitter's universe.

Dirac's model, presented in 1938, could be called steady state, even if it was a pure product of the large numbers analysis. Indeed, by preserving the Dirac principle through time, Dirac's model entailed some kind of steadiness.

As for Jordan, he built a cosmological model without any steady state consideration. Yet, his universe is also a steady one. The most acclaimed steady model was presented in 1948 by Hoyle [24]. A comparison of Jordan's and Hoyle's versions was published in a Nature publication, later discussed in the fifth section.

For a further study of the difference between static and steady state universes, and the diversity of their motivations, we invite you to refer to our previous article [25].

4. Jordan's Work

Referring to Eddington's and Dirac's works, Pascual Jordan suggested his own model, which he developed in several articles, mainly in References [2,26,27]. As the last one is the most accomplished of this period, we decided to introduce Jordan's cosmological model through the translation of Jordan's *"Bemerkungen zur Kosmologie"* : *"Comments on cosmology"* [2]. After this theoretical research, Jordan dedicated his cosmological work to the experimental and observational aspects, in astrophysics

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[3] During the seventies, Dirac came back to his Large Numbers hypothesis and suggested the study of two kinds of matter creation processes [15–17].
[4] It is a cylindrical universe with S^3 spatial hyper-surfaces.

and geophysics, e.g., in [28]. H. Kragh preferred to divide Jordan's cosmological career not in two, theoretical and then observational, but in three, intuitive, deductive and then devoted to the consequences [4].

Jordan's progression is quite similar to Dirac's one [3]. First, both of them made themselves known with their work in quantum mechanics. Secondly, they both shifted to cosmology via Eddington's numerical work. Eventually, none of them both has been remembered for their cosmological works, in spite of how long they worked on this subject.

As evidence that Jordan's cosmology was not well received, we have taken two mixed reviews on, expressed during the fifties. In 1950, Paul Couderc classified Jordan's model as heterodox, the same way he did Hoyle's, Lyttleton's, Bondi and Gold's, and Milne's [29]. Couderc highlighted the work accomplished but, unfortunately, did not find a scientific value in it. Later, in a review of Jordan's *Schwerkraft und Weltall* [30], McCrea acknowledged the pedagogical qualities of Jordan's introduction to Riemann-Einstein theory, wishing this book would be used as an introductory text for students. However, McCrea was not convinced by Jordan's ideas on cosmology, as Jordan failed to find a global mathematical treatment for his ideas [31]

5. Walk in Jordan's Paper

The article *Bemerkungen zur Kosmologie*, written as a synthesis of cosmological ideas during the thirties, is quite self sufficient. We chose to give a deeper commentary on three important points. First, we will consider the system of units used by Jordan, which was exclusively built from cosmological constants and permits to consider cosmology as a complete field of science, without any requirement of links with quantum mechanics. Secondly, we will analyse the variation of G that Jordan developed, in parallel with the static universe suggested by Sambursky. Finally, we will characterize the matter spontaneously created in Jordan's model. Indeed, he considered creation of stars with a specific mass-radius ratio, thereupon these stars turned out to be compact objects.

5.1. System of Units in Jordan's Work

Five numbers characterized the cosmological knowledge at the time—c, the velocity of light; κ encoding the gravitational constraint from general relativistic theory; μ, the average mass density of the universe; α, Hubble's constant[5]; and A, the age of the universe.

From these five values, Jordan built two dimensionless numbers $\alpha.A$ and $\frac{\alpha}{c\sqrt{\kappa\mu}}$, both of them are in the order of one. This results from Jordan's choice of units, given the characteristic sizes of the cosmological problem. Actually, from the five characteristic constants, Jordan brought out a mass-element $\frac{1}{\sqrt{\kappa\mu}}$ and time-element A. This approach is similar to Planck's in quantum mechanics in 1899 [32], when he defined his, now illustrious, system of natural units. Thus, Jordan built a purely gravitational and cosmological system of units without any link to quantum mechanics (through Planck constant) nor to statistical mechanics (through Boltzmann constant). Furthermore, in his system, the cosmological constant emerges also as purely cosmical $\Lambda \simeq \frac{3\alpha^2}{c^2}$.

5.2. Variation of the Gravitational Constant

We value expounding Samuel Sambursky's approach as examined by Jordan. What Sambursky wrote [33] is worth to be scrutinised in this paper, since Jordan was the only other author to quote him[6].

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[5]　　To be consistent with Jordan's piece in the Appendix A, in this paragraph, the authors chose to keep the original notation α for Hubble's constant, usually α is the fine structure constant and Hubble's constant is written H or H_0.

[6]　　Except Sambursky himself in his following work [34].

For Sambursky, the homogeneous spatial distribution of nebulae indicated that the universe is static. To retain a constant radius of the universe, the radius of the electron and the other universal lengths have to shrink with time.

> "The dynamics of expansion are transferred into the dimensions of atomistic phenomena." [33] (p. 335)

Since there are two universal lengths, $\frac{e^2}{mc^2}$ and $\frac{\hbar}{mc}$, whose ratio is the fine structure constant, usually denoted α; and assuming that α and c are constant, then h ought to decrease with time and e^2 diminish at the same rate as h does. Therefore, Sambursky suggested a static universe with h decreasing, equivalent to an expanding universe with a constant value of h.

Thereby, Sambursky somehow explained the measurement of redshift. By preserving the Planck-Einstein relation $\varepsilon = h\nu$, it becomes manifest that the old stars emitted light when h was larger, so the frequency ν was smaller and was transmitted to us without undergoing any change. Thus, the observations interpreted as a redshift of the emitted light is no more than the true frequency of emission, evidence of the variability of h.

Sambursky propounded a value for $\dot{h}$ of -1.03×10^{-43} erg[7], working with an expansion speed, based on Ten Bruggencate's work [35], of a value of $486 \frac{\text{km}}{\text{s Mpc}}$. S. Sambursky rejected the idea of a complete linear shrinkage and suggested that h should vanish asymptotically for $t = \infty$. By posing $h = h_0 \exp(-kt)$, the ratio[8] $\frac{\dot{h}}{h} = -k$ enables to evaluate the Hubble factor.

From a strictly dimensional point of view, G can be written as $G = \frac{2\pi e^2}{M^2 mc^2}\dot{h}$. And so, it can be witnessed that

$$\frac{GMm}{e^2} = \frac{2\pi\dot{h}}{Mc^2}.$$

Noting that, it becomes obvious that the relation of the gravitational energy of the hydrogen atom to its Coulombian energy (the left hand of the equality) could be expressed by the rest energy of the atom and $\dot{h}$. So, G is not a constant anymore, it is proportional to $e^2 h$ which decreases as h^2, since e^2 behaves like h. And, as the creation of stars and stellar systems is determined by the product GM (where M is the mass of the system), the masses of the stars that arose back in time must be smaller given that G was greater.

As Sambursky's ideas have been delineated, here comes the time to go back to Jordan's paper and his reaction to Sambursky's work. Since Reference [27], P. Jordan has been echoing Sambursky's approach. In Jordan's heuristic interest, Sambursky's procedure is completely acceptable. Going back and forth between the expanding universe with constant h and the static universe with variable h is always possible. However, Jordan regretted that Sambursky's idea led to abandon the clear relation between the element of length and the standard measure, like the Platinum rod.

Jordan displayed a new way to reach the variability of G. As $\kappa \cong \frac{R}{M}$ from (A5) and because R divided by the element of length, Λ, is equal to γ, the epoch[9], and with M divided by m_p, the proton mass, is γ^2; it could be written that $\kappa \cong \gamma^{-1}\frac{\Lambda}{m_p}$. The relativistic gravitational constant κ is not constant anymore but shrinks as the inverse power law of the epoch and so does G[10].

5.3. Spontaneous Creation of Compact Objects

A direct consequence of Jordan's assumptions was the spontaneous creation of matter. As observations showed older and younger stars, he deduced that the matter emerges directly in

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[7]   That is equivalent to -1.03×10^{-36} J.
[8]   Pay attention, in the original paper, the minus symbol is missing.
[9]   Since Reference [13], the epoch γ is a dimensionless number defined as the age of the universe divided by a time unit, in most case the atomic time.
[10] Jordan preferred the notation f for the Newtonian gravitational constant.

the form of stars. To keep the total energy of the universe, Jordan suggested that the created matter equilibrates its rest energy with its gravitational potential energy.

$$\text{Rest energy} + \text{gravitational potential energy} = 0$$

$$M_\star c^2 - \frac{3}{5}\frac{GM_\star^2}{R_\star} = 0$$

$$R_\star = \frac{3}{40\pi}\kappa M_\star.$$

Such created stars were characterised by $\dfrac{R_\star}{M_\star} = \dfrac{3\kappa}{40\pi}$.

This result was based upon the gravitational binding energy in Newtonian gravity $U = \dfrac{3GM^2}{5R}$, which is the energy to provide to destroy a gravitationally bound system, under the assumption that it is a spherical mass of homogeneous density. Jordan's idea was to equal the rest energy (Mc^2) with the gravitational binding energy. Unfortunately, this gravitational binding energy in the strong field regime of general relativity, *id est* of compact objects, is still an open question nowadays. Jordan concealed his use of a Newtonian concept while working in relativity.

Moreover, Jordan came to the creation of stellar objects with a certain ratio between their mass and their radius, without any condition of their order of magnitude. With a modern eye, the compactness[11] of these created stars could be computed. These stars have a compactness of $\dfrac{5}{3}$, making them not luminous at all since their compactness is larger than the one of black holes [36]. In some way, Jordan developed a model of creation of black holes, which foreshadowed primordial black holes in cosmology.

The absence of comment from Jordan on the mechanism of this creation process is regrettable. Indeed, he established the relation between the mass and the radius of a possible created star without explaining the creation in itself.

6. Publication in Nature

In 1948, two articles published in the *Monthly Notices of the Royal Astronomical Society* suggested a cosmological model with matter creation. There was the birth of the Steady State Theory. In the first founding paper, due to H. Bondi and T.Gold [37], the idea is to enlarge the cosmological principle. This is the idea that the universe, at large scale, is homogeneous and isotropic. This hypothesis is needed to permit the study of the universe as a whole. Bondi and Gold suggested to strengthen this hypothesis, adding a constancy regarding to time. This is known as the perfect cosmological principle. In their work, there is a direct reference to Eddington's work and a more subtle to Dirac—"*A further point to be mentioned in relations to the stationary property of the universe is the coincidence of numbers pointed out by Eddington [38]. Two non-dimensional numbers which can be constructed from observation are both found to be of the order* 10^{39}. [37] (p. 259)" They granted the paternity of the study of large numbers to Eddington but, referencing the epoch 10^{39}, there is a clear link to Dirac works.

On the other hand, Hoyle, in the second founding paper of steady-state theory [24], developed a steady-state theory on a more mathematical basis. He suggested a modification of Einstein's equations, $R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R + C_{\mu\nu} = -\kappa T_{\mu\nu}$[12]. By adding the creation tensor, $C_{\mu\nu}$ in the left hand side of the equation,

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[11] In current notation, compactness is defined as $\Xi = \dfrac{GM}{c^2 R} = \dfrac{R_s}{2R} \simeq \dfrac{R_s}{R}$, where R_s the Schwarzschild radius. With this convention, the compactness of a black hole is 0.5 and of a neutron star is 0.1.

[12] In this model, the scale factor is $f(t) = \left(C_1 + C_2 e^{\frac{3}{2}C_3 ct}\right)^{\frac{2}{3}}$, where C_1 and C_2 are integration constants, C_3 is linked to the rate of creation.

Hoyle's model is very similar to one with a cosmological constant. For a further details on his model, the reader could see [25]. Hoyle arrived to the perfect cosmological principle as a consequence of his modification. Hoyle made a tiny mention of Dirac's work—"*More recently Dirac* [13] *has pointed out that continuous creation of matter can be related to the wider questions of cosmology.* [24] (p. 372)".

Following the infatuation around the idea of permanent creation of matter, Max Born invited Jordan to publish in *Nature* [39]. In this paper, Pascual Jordan compared his model to Hoyle's. Both agreed on the idea of stationary cosmology requiring a process of matter creation to counterbalance the universe dynamics, but they achieved it in very different ways. On the one side, Jordan suggested a spontaneous creation of stars in global energy balance. On the other side, Hoyle proposed the creation of helium saving the energy conservation law in the border of the observable universe. This is why Jordan wrote down:

> "Several decisive ideas of Hoyle's are in full harmony with my own theory [...]. But there are also considerable differences between Hoyle's theory and my own." [39] (p. 640)

7. Conclusions

Currently, Jordan's name is associated with Jordan's frame and Brans-Dicke's theory [40]. Yet, his cosmological model tends to be forgotten, while it would deserve a brighter place in the relevant literature.

This paper put into light that Jordan was part of the continuity of Eddington's and Dirac's works. Jordan suggested his own cosmological model with the influence of Large Numbers hypothesis and previous works on variation of the constants, such as Sambursky's. Per se, Jordan joined the precursors of all the modified gravity models working on varying constants. His approach could be linked with Wetterich's work [41]. From another point of view, with the spontaneous creation of stellar objects, and more precisely of compact objects, Jordan could be seen as a forerunner of primordial black holes study, later initiated by S. Hawking, B. Carr and others, and nowadays still studied [42].

We hope that this article will put Jordan at the position that is rightly his in the historical development of cosmology, and perhaps arouse an interest for his vast series of publications on his model.

Appendix A

Bemerkungen zur Kosmologie[13]—by P. Jordan[14].

The purpose of this study is to develop, roughly and logically, the cosmological theory, which follows when we accept for bases Dirac's principle on the one hand and, on the other hand, the homogeneity of the world (in expansion). It follows (with the principle of energy conservation) a constant growth of matter by production on the model of stars and nebulae explosion.

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[13]　Translated by E.-A. Dubois with the helpful collaboration of D. Bertrand, the original article is [2].
[14]　Deceased in 1980.

Appendix A.1

The cosmological representation sketched in what follows is essentially based on Dirac principle[15], according to which we can interpret the huge dimensionless quantity of terrestrial and cosmical physics as functions of the age of the universe. The special interpretation, like Dirac granted to this principle[16] does not seem sufficient to me; then it is preferable to put up for discussion a quite different theory whose mainlines I partly developed in previous publications[17].

The learnings from spectroscopy achieved from faraway objects justify the fact of considering the world geometry as Riemannian (with integrable length transmission), and more distant[18], of considering the dimensionless number $\frac{e^2}{hc}$ maybe as a cosmological constant ; the length element $\Lambda = \frac{e^2}{m_o c^2} \sim 2 \times 10^{-13}$ cm also stays in a strong relation with the length $\frac{h}{m_o c}$ together with lengths defined by a Cd-spectral-ray or with a Pt-rod.

Besides, we accept that the proton mass m_p is in a constant cosmological relation with the electron mass and that the forces binding the nucleus are cosmological constants. The fact of accepting this reasoning is empirically founded[19]. The functioning of radioactive clocks is in a strong relation with the time-element $\frac{\Lambda}{c}$.

However, the cosmological constancy or inconstancy of beta-forces, is uncertain; the Fermi's constant for beta-decay is perhaps proportional to $\kappa^{\frac{1}{4}}$, where κ is the (relativistic) gravitational constant; the possibility of spontaneous decay probability of the mesotron seems, according to Blackett, visibly proportional to $\kappa^{\frac{1}{2}}$. Nevertheless, this plays no essential role in the following.

Appendix A.2

Without reference to the size of the length elements (which has only the existence of unstable length measurements), we can enunciate the following astrophysical fundamental constants:

1.　$c = 3 \times 10^{10} cm/s^{-1}$
2.　$\kappa = \dfrac{8\pi f}{c^2} = 1.87 \times 10^{-27} g^{-1}/cm$
3.　$\mu = 10^{-30} g\, cm^{-3}$
4.　$\alpha = 1.8 \times 10^{-17} s^{-1}$ [20]
5.　$A \cong 10^{10} \text{years} = 3 \times 10^{17} s$

Here, A is the age of the universe, established from radioactive clocks and sustained by some other astrophysical facts and reflections. Hereafter, α is the Hubble's constant, μ the average mass-density of the universe and κ the relativistic gravitational constant. Given that this, hereinafter, (according to Dirac) is not considered as a true constant but as a slowly unstable value, it is essential to accept, in the planetary system, the κ value as approaching the average current value of κ.

From the above-mentioned values, two dimensionless numbers could be built.

$$\alpha A = 5.4 \tag{A1}$$

$$\frac{\alpha}{c\sqrt{\kappa\mu}} = 15 \tag{A2}$$

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[15]　P.A.M Dirac, Nature **139**. S.323, 1001. 1937 [13].
[16]　P.A.M Dirac, Proc. Roy. Soc. A. **165**. S.199. 1938 [14].
[17]　P. Jordan, Naturw. **25**. S.513. 1937 [26]; **26**. S.417. 1938 [27].
[18]　P. Jordan Ztschr.f.Phys. (to appear).
[19]　(cf. ibid)
[20]　Here there is a typo in the original paper, the value $1.8 \times 10^{-7} s^{-1}$ is written.

The fact that these dimensionless numbers are close to 1 makes the fact plausible that simple relations exist. After this, it permits the clear definition of two numbers R and M with length and mass dimension:

$$\begin{cases} R = \dfrac{1}{\sqrt{\kappa\mu}} = 2.5 \times 10^{28} cm = 2.5 \times 10^{10} \text{light-years}; \\ M = \dfrac{1}{\sqrt{\kappa^3\mu}} = 1.3 \times 10^{55} g; \end{cases} \tag{A3}$$

Or, using (A2), without κ (with α instead), we establish:

$$\begin{cases} R \cong \dfrac{c}{\alpha}, \\ M \cong \mu\dfrac{c^3}{\alpha^3} \cong \mu R^3. \end{cases} \tag{A4}$$

A satisfactory theory must

(a)　make the relations (A1) and (A2) comprehensible,

(b)　offer a clear analysis of the numbers (A3) respectively (A4),

(c)　put in harmony, in a simple way, Hubble's effect with the principle of the non-existence of speed larger than c.

Moreover, the interdiction of exceeding the speed of light is applied to Hubble's redshift, even if we should try to treat the analysis of Hubble's effect as a Doppler effect (which anyway seems to us forced or artificial).

These requirements are met, when we leave (A2) on one side, and take us up to numbers and relations independent of κ, the easiest way with the representation of a Riemannian space with a radius R and a mass M. The radius grows at light speed, and has initially (at time A) a very little value. In the cosmological model suggested by Dirac, the flat infinite space is accepted with an infinite mass, such as R and M lose their meaning.

However, the relation (A2) need to be interpreted too, which, when we define R and M with (A4), as

$$R \cong \kappa M, \tag{A5}$$

could be expressed as an approximate consensus between the value of the geometrical universe radius and the value of the gravitational universe radius. We interpret this - following a Hass's remark-as the expression of the energy principle written in the form $\frac{\kappa M^2}{R} \sim M$, it means the added potential energies Mc^2 of all material particles are precisely compensated by the negative gravitational energy, so that the whole universe energy stays constant (namely virtually null). In a possible way, maybe it could be appropriate to take also notice of the kinetic energy of the nebulae flux, which will get the same order of magnitude, like the sum of the potential energies.[21] Especially, the matter concentration for the stars, the nebulae[22], makes necessary a more precise conception of the energetic assessment with which, again, the orders of magnitude are unchanged.

Appendix A.3

If we divide R by the length element Λ—or in the same way, the universe age A by the time element- we find back a value of size around $\gamma \sim 10^{40}$, this will be called in the following shortly as

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[21]　This is equivalent to the other statement, that Hubble's flow the current values of μ and κ, are exactly enough to prevent a conditional concentration due to the gravitation of cosmic masses. Gamow and Teller (Phys. Rev. **55**. S.654 1939 [43]) discussed, in an interesting way, the fact that taking into account more precisely the numerical values of the effect of the flux is remarkably stronger than the gravitational effect. However, in our thoughts, it is right to deal only with questions which stay when the numbers of order of magnitude 1 are replaced by 1 in the summarized thought, without taking so fine proportions into account.

[22]　Jordan used the world *Nebeln* surely to speak of the galaxies.

the age of the universe. A division of M by $m_p = 1,6510^{-24}g$ gives, like Eddington and Hass found it, something close to γ^2; and with the conclusion of Dirac's principle, it leads to an empirical law-which is not yet theoretically founded- of Nature

$$\frac{M}{m_p} = \left(\frac{R}{\Lambda}\right)^2; \tag{A6}$$

or in Hass's new writing

$$Mc \cong h\frac{R^2}{\Lambda^3}. \tag{A7}$$

The surprising conclusions are:

(a)　M is not constant but grows proportionally to γ^2;
(b)　κ also is not constant, but with (A5), is inversely proportional to γ :

$$\kappa \cong \gamma^{-1}\frac{\Lambda}{m_p}. \tag{A8}$$

This law (A8) has been brought out by Dirac. In an other side, Dirac showed that the hypothesis of a world mass growing could be avoided, nevertheless, we can hold Dirac's principle only at the cost of the hypothesis of an infinitely large universe in volume and mass. In the following, the idea of a world mass temporally growing must be followed to win elements for a future decision. Surely, the hypothesis of a new and constant production of mass in space is disconcerting. Our knowledge about cosmological questions is however currently yet so limited that it could be heuristically useful to conceive the different solutions of the cosmological problem that seem plausible, if possible in a systematic way. Furthermore, the present considerations have nothing more than a heuristic value.

Appendix A.4

The application of Dirac's principle is, due to the fact that e^2/hc and m_p/m_0 are already noticeably different of 1, linked to important incertitudes. The value $m_p\Lambda^{-3} \cong 10^{14}$ g/cm^3 could be estimated as the maximum of the physically possible mass. This order of magnitude is present in atomic nuclei, the well-known Baade and Zwicky's Super-Novae-Theory assign an as large approximative density to the star[23]. Although this density value is much larger than the white dwarf density $\sim 10^5$ or the hydrogen density 1, this difference only lays on factors founded on atomic physics and, consequently, are cosmologically constant. So, it is right to not relate them to γ in the mean of Dirac's principle: we obtain a density of 1 as order of magnitude, when we replace in $m_p\Lambda^{-3}$ the length element by the Bohr's hydrogen radius.

The new dimensionless constants appear now when we compare the radii and the masses of the stars and the spiral nebulae with Λ and m_p [24].

In spite of the difficulties just touched upon, it is certainly possible to judge reliably with respect to the star how to apply Dirac's principle here. The significant differences existing between the different stars types lay on atomic physic factors; to agree, we can say this:

(a)　Eddington's theory of lighter stars[25],
(b)　Kothari's theory of dwarf stars[26] and
(c)　Zwicky's theory of neutron star[27] with R and M

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[23]　See also F. Zwicky, Phys. Rev **55**. S.726. 1939 [44].
[24]　cf. D.S. Kothari, Nature **142**. S.354. 1938 [45].
[25]　cf. A.S. Eddington, Der innerre Aufbau der Sterne. Berlin 1928 [46].
[26]　D.S. Kothari, Proc. Roy. Soc. A. **165** S.486. 1938 [47].
[27]　cf. ibid.

give as a result a proportionality with $\kappa^{-1/2}$ or $\kappa^{-3/2}$, so

$$R_{St} \sim \gamma^{1/2}; M_{St} \sim \gamma^{3/2}. \tag{A9}$$

On the other hand, in the spiral nebulae, the application of Dirac's principle is facilitated by the larger similitude of these objects, according to Chandrasekhar and Kothari[28], it seems that

$$R_s \sim \gamma^{3/4}; M_{Sp} \sim \gamma^{7/4}. \tag{A10}$$

The increase of values in question with the age of the universe will not naturally mean the individual growth of the entities, but only the increasing of the values seeming maximal-at the youngest constructions. Possibilities of empirical tests from here are already expressed elsewhere by Zwicky. The fact that in the three cases-stars, spiral nebulae and cosmos-the mass is proportional with $\gamma \times radius$, could be publicly expressed, that the ratio (A5) holds as well for the stars and spiral nebulae as for the universe, atomic physical factors. This is close to the consideration which gives a closer explanation to the growth process of the mass of the universe M. In an Euclidian free-mass space, the spontaneous creation of a spherical mass M_0 of constant density and with a radius R_0 requires none energy, if

$$R_0 = \frac{3}{40\pi} \kappa M_0. \tag{A11}$$

Because, to scatter this sphere entirely against the gravitation, the same energy $M_0 c^2$ would be necessary, that could be represent by these scattered masses. We want to represent ourself the production of cosmic mass necessary by reason of the proportionality between M and γ^2 occurs by the spontaneous creation of simple stars. These ones have, at the beginning, approximately the density $m_p \Lambda^{-3}$, whose the radius and the mass, expressed respectively in elementary unit of length Λ and mass m_p, are of the orders of magnitude of $\gamma^{1/2}$ and $\gamma^{3/2}$.

In fact, the rest energy $M_0 c^2$ of a spontaneously dawning star must be balanced only by its own negative gravitational energy. In the neighbourhood of a spontaneous appearing star, the apparition of other stars is facilitated in an energetic point of view; and, because of the validity of (A5) for the simple star as for the spiral nebula and the cosmos, the energy used for the mass production is balanced in a similar order of magnitude by

(a)   the gravitation of the simple star,
(b)   the gravitational interaction inside the dawning spiral nebula and
(c)   the gravitational interaction with the other spiral nebulae.

We will make the link between this hypothesis of spontaneous apparition of spiral nebulae and the empiric fact that it is undeniable to have young and old spiral nebular[29]. Furthermore, it is well suitable with this that the spiral nebulea are empirically composed by simple stars[30], and not by continuously propagating matter. So that the representation from Kant and Laplace's ideas of a star building by an addition of concentration of little masses gravitationally bended do not find empirical base.

The spontaneous creation of a whole star with $\gamma^{3/2}$—an elementary part in a unique elementary act is surely a representation with a rough exaggeration. Maybe it is the place to indicate in this context Heisenberg's explosion shower, whose reality became little by little likely and, in these cases, happens in normal conditions with a very large number of produced particles by an indivisible act.

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[28]   cf. D.S. Kothari ibid.
[29]   See also the comments in Naturwiss. **26**. S.417. 1938 [27].
[30]   See, for example, E. Hubble, Das Reich der Nebel. Braunschweig 1938 [48].

Naturally, our own first considerations above-mentioned can not offer any substitute to energetic assessment of stars creation for the missing dynamics for these process, that is why the observational equipment could provide us a vast empirical base, for example the relation to the clusters of stars.

Appendix A.5

The spatial energy density of light in inter-spiral nebulae space is only of a factor $\sim 10^{-6}$, and the cosmic radiation energy density is only $\sim 10^{-4}$ times more little than μc^2 [31] These factors could become clearer in an atomic physics context, so that the ratio of radiation and matter are cosmologically constant. We are far away of considering the production of cosmic rays as a side process of the production of cosmological matter; through which a complementary participation of Baade and Zwicky's Super-Novae process could not be excluded.

The estimate apparition rate of a Super-nova per nebula per thousand years , seems to be $\gamma^{1/2}$ super-nova in the universe per time element, in consequence:

- the number of available stars is $\sim \gamma^{1/2}$,
- for all stars, the transition into a Super-Nova probability $\sim \gamma^{-1}$, per time element;
- and a total radiation production going with $\sim \gamma$, proportional to the cosmic mass.

The superior limit (which can not be defined precisely) of the energies appearing as particles of the cosmic rays, expressed in multiple of $m_0 c^2$ or $m_p c^2$, is a large number on the other hand, maybe of the order of magnitude of $\gamma^{1/4}$. It is likely that the cosmic ray strength grows without end like the age of the universe.

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[31] See, for example, A. Haas, Kosmologische Probleme der Physik. Leipzig. 1934 [49].

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