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Cosmic Microwave Background from Effective Field Theory †

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Abstract: In this work, we study the key role of generic Effective Field Theory (EFT) framework to quantify the correlation functions in a quasi de Sitter background for an arbitrary initial choice of the quantum vacuum state. We perform the computation in unitary gauge, in which we apply the Stückelberg trick in lowest dimensional EFT operators which are broken under time diffeomorphism. In particular, using this non-linear realization of broken time diffeomorphism and truncating the action by considering the contribution from two derivative terms in the metric, we compute the two-point and three-point correlations from scalar perturbations and two-point correlation from tensor perturbations to quantify the quantum fluctuations observed in the Cosmic Microwave Background (CMB) map. We also use equilateral limit and squeezed limit configurations for the scalar three-point correlations in Fourier space. To give future predictions from EFT setup and to check the consistency of our derived results for correlations, we use the results obtained from all classes of the canonical single-field and general single-field $P(X,\phi)$ model. This analysis helps us to fix the coefficients of the relevant operators in EFT in terms of the slow-roll parameters and effective sound speed. Finally, using CMB observations from Planck we constrain all these coefficients of EFT operators for the single-field slow-roll inflationary paradigm.

Keywords: effective field theories; cosmology of theories beyond the SM; quantum field theory of de Sitter space

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1. Introduction

The basic idea of Effective Field Theory (EFT) is very useful in many branches in theoretical physics including particle physics [1,2], condensed matter physics [3], gravity [4,5], cosmology [6–26] and hydrodynamics [27,28]. In a more technical ground, EFT framework is an approximated model-independent version of the underlying physical theory which is valid up to a specified cut-off scale at high energies, commonly known as UV cut-off scale ($\Lambda_{\rm UV}$), which is in usual practice fixed at the Planck scale M_p. EFT prescription deal with all possible relevant and irrelevant operators allowed by the underlying symmetry in the effective action and all the higher-dimensional non-renormalizable operators are accordingly suppressed by the UV cut-off scale ($\Lambda_{\rm UV} \sim M_p$). There are two possible approaches within the framework of quantum field theory (QFT) using which one can explain the origin of EFT, which are appended below:

1. **Top-down approach:**

   In this case, the usual idea is to start with a UV complete fundamental QFT framework which contain all possible degrees of freedom. Furthermore, using this setup one can finally derive the

EFT of relevant degrees of freedom at low energy scale $\Lambda_s < \Lambda_{\rm UV} \sim M_p$ by doing path integration over all irrelevant field contents [12,14]. To demonstrate this idea in a more technical ground let us consider a visible sector light scalar field ϕ which has a very small mass $m_\phi < \Lambda_{\rm UV} \sim M_p$ and heavy scalar fields $\Psi_i \forall i = 1, 2, \cdots, N$ with mass $M_{\Psi_i} > \Lambda_{\rm UV} \sim M_p$, in the hidden sector of the theory. the representative action of the theory is described by the following action [12,14]:

$$S[\phi, \Psi_i, g_{\mu\nu}] = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R + \mathcal{L}_{\rm vis}[\phi] + \sum_{i=1}^{N}\mathcal{L}^{(i)}_{\rm hid}[\Psi_i] + \sum_{j=1}^{N}\mathcal{L}^{(j)}_{\rm int}[\phi, \Psi_j]\right], \tag{1}$$

where $g_{\mu\nu}$ is the classical background metric, $\mathcal{L}_{\rm vis}[\phi]$ is the Lagrangian density of the visible sector light field, $\mathcal{L}^{(i)}_{\rm hid}[\Psi_i] \forall i = 1, 2, \cdots, N$ is the Lagrangian density of the hidden-sector heavy field and $\mathcal{L}^{(j)}_{\rm int}[\phi, \Psi_j] \forall j = 1, 2, \cdots, N$ is the Lagrangian density of the interaction between hidden-sector and visible-sector field. Furthermore, using Equation (1) one can construct an EFT by performing path integration over the contributions from all hidden-sector heavy fields and all possible high-frequency contributions as given by:

$$S_{EFT}\left[\phi, g_{\mu\nu}\right] = -i\ln\left[\prod_{j=1}^{N}\int\left[\mathcal{D}\Psi_j\right] e^{iS[\phi,\Psi_j,g_{\mu\nu}]}\right] = -i\sum_{j=1}^{N}\ln\left[\int\left[\mathcal{D}\Psi_j\right] e^{S[\phi,\Psi_j,g_{\mu\nu}]}\right]. \tag{2}$$

Finally, one can express the EFT action in terms of the systematic series expansion of visible sector light degrees of freedom and classical gravitational background as [12,14]:

$$S_{EFT}\left[\phi, g_{\mu\nu}\right] = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R + \mathcal{L}_{\rm vis}[\phi] + \sum_{\gamma}\sum_{j=1}^{N}\mathcal{C}^{(j)}_{\gamma}(g_c)\frac{\widetilde{\mathcal{O}}^{(j)}_{\gamma}[\phi]}{M^{\Delta_\gamma - 4}_{\Psi_j}}\right], \tag{3}$$

where $\mathcal{C}^{(j)}_{\gamma}(g_c) \forall \gamma,\ \forall j = 1, 2, \cdots, N$ represent dimensionless coupling constants which depend on the parameter g_c of the UV complete QFT. Also $\widetilde{\mathcal{O}}^{(j)}_{\gamma}[\phi] \forall \gamma,\ \forall j = 1, 2, \cdots, N$ represent Δ_γ mass dimensional local EFT operators suppressed by the scale $M^{\Delta_\gamma - 4}_{\Psi_j}$. In this connection one of the best possible example of UV complete field theoretic setup is string theory from which one can derive an EFT setup at the string scale Λ_s which is identified with M_{Ψ_j} in Equation (3).

2. **Bottom-up approach:**

   In this case, the usual idea is to start with a low-energy model-independent effective action allowed by the symmetry requirements. Using such a setup, the prime job is to find out the appropriate UV complete field theoretic setup allowed by the underlying symmetries [12,14]. This identification allows us to determine the coefficients of the EFT operators in terms of the model parameters of UV complete field theories. In this paper we follow this approach to write down the most generic EFT framework using which we describe the theory of quantum fluctuations observed in CMB around a quasi de Sitter inflationary background solution of Einstein's equations.

In this paper, our prime objective is to compute the expressions for the cosmological two- and three-point correlation functions in unitary gauge using the well-known Stückelberg trick [29,30] along with the arbitrary choice of initial quantum vacuum state. The working principle of the Stückelberg trick in quasi de Sitter background is to break the time diffeomorphism symmetry to generate all the required quantum fluctuations observed in CMB. This is exactly same as applicable in the context of $SU(N)$ non-abelian gauge theory to describe the spontaneous symmetry breaking. In the present context the scalar modes which are appearing from the quantum fluctuation exactly mimic the role of Goldstone mode as appearing in $SU(N)$ non-abelian gauge theory. After breaking the time diffeomorphism in the unitary gauge, scalar Goldstone-like degrees of freedom are eaten by the metric.

In unitary gauge, to write a most generic EFT in terms of operators which breaks time diffeomorphism symmetry, the following contributions play significant roles in quasi de Sitter background:

- Polynomial powers of the time fluctuation of the component in the metric, g^{00} such as, $\delta g^{00} = g^{00} + 1$,
- Polynomial powers of the time fluctuation in the extrinsic curvature at constant time surfaces, $K_{\mu\nu}$ such as, $\delta K_{\mu\nu} = \left(K_{\mu\nu} - a^2 H h_{\mu\nu}\right)$, where a is the scale factor in quasi de Sitter background.

Construction of EFT action using the Stückelberg trick also allows us to characterize all the possible contributions to the model-independent simple versions of field theoretic framework based on the models of the inflationary paradigm described by single field, where the observables are constrained by the CMB observation appearing from Planck data. It is important to note that this idea of constructing EFT action using the Stückelberg trick can also be generalized to the EFT framework guided by multiple number of scalar fields as well.

The main highlighting points of this paper are appended below pointwise:

1. We have presented all the results by restricting up to all possible contributions coming from the two derivative terms in the metric which finally give rise to a consistently truncated EFT action[1]. Consequently, we get consistent predictions for single-field slow-roll [31–42] and Generalized Single-Field $P(X,\phi)$ models of inflation [43–56]. In earlier works various efforts are made to derive cosmological three-point correlation functions by writing a consistent EFT action in the similar theoretical framework. However, the earlier results are not consistent with the single-field slow-roll inflation with effective sound speed $c_S = 1$ as it predicts vanishing three-point correlation function for scalar fluctuations. See ref. [6] for more details. The main reason for this inconsistency was ignoring specific contributions from the fluctuation in the EFT action, which give rise to improper truncation.
2. We have computed the analytical expression for the two-point and three-point correlation function for the scalar fluctuation in quasi-de Sitter inflationary background in the presence of generalized initial quantum state. Also, for the first time we have presented the result for two-point correlation function for the tensor fluctuation in this context. To simplify our results we have also presented the results for Bunch–Davies vacuum and α,β vacuums[2].

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1 In the context of EFT one can in principle consider terms containing more than two derivatives in the metric, which will give rise to appearance of many higher derivative operators, i.e., $\sum_{n=2}^{\infty} R^n$, $\sum_{m=1}^{\infty}(R_{\mu\nu}R^{\mu\nu})^m$, $\sum_{p=1}^{\infty}(R_{\mu\nu\alpha\beta}R^{\mu\nu\alpha\beta})^p$, $\sum_{q=1}^{\infty}(R_{\mu\nu\alpha\beta}R^{\mu\nu}R^{\alpha\beta})^q$ and various other terms which contain the quantum fluctuation on the trace of the extrinsic curvature terms i.e., $\sum_{m=1}^{\infty}(\delta K^{\mu}_{\mu})^{m+2}$ and other possible terms which are appearing due to all possible index contraction of extrinsic curvature terms i.e., $\sum_{m=1}^{\infty}\delta K^{\mu_1}_{\mu_2}\delta K^{\mu_2}_{\mu_3}\delta K^{\mu_3}_{\mu_4}\cdots\delta K^{\mu_{m+1}}_{\mu_{m+2}}\delta K^{\mu_{m+2}}_{\mu_1}$ in the gravity sector of the EFT action. However, in the present work our prime objective is to compute the expressions for cosmological correlation (two- and three-point) functions from quasi-de Sitter space. For this reason it is sufficient enough to consider the two derivative terms in the metric as such contributions will appear in the two- and three-point correlation functions in the leading order. If one condensed the effects of higher derivative terms in the metric it will appear at the sub-leading or sub-sub-leading-order expressions for the correlation functions, which are highly suppressed due to the validity of slow-roll approximations, i.e., $\epsilon = -\frac{\dot{H}}{H^2} << 1$ and $|\eta| = \left|\epsilon - \frac{\dot{\epsilon}}{2\epsilon H}\right| << 1$ during inflation. Thus, it implies that due to very small numerical contributions one can easily neglect the terms which contain the higher-order slow-roll contributions in the two- and three-point cosmological correlation functions. In our computations performed in this paper we have also maintained these approximations everywhere and this will give rise to the leading-order result which we have presented explicitly later. This is the main reason for which we have restricted up to two derivative terms in the metric in this paper.

2 In QFT of quasi de Sitter space we deal with a class of non-thermal quantum states, characterized by infinite family of two real parameters α and β, commonly known as α,β vacuums. It is important to note that α,β quantum states are CP invariant under the SO$(1,4)$ de Sitter isometry group. On the other hand, we fix $\beta = 0$ then we get α vacuum which is actually CPT-invariant under the SO$(1,4)$ de Sitter isometry group. Furthermore, if we fix both $\alpha = 0$ and $\beta = 0$ then we get the thermal Bunch–Davies vacuum state.

3. We have presented the exact analytical expressions for all the coefficients of EFT operators for single-field slow-roll and Generalized Single-Field $P(X,\phi)$ models of inflation in terms of the time-dependent slow-roll parameters as well the parameters which characterize the generalized initial quantum state. To give numerical estimates we have further presented the results for Bunch–Davies vacuum and α,β vacuums.

This paper is organized as follows. In Section 2, we discuss the overview of the EFT framework under consideration, which includes the construction of the EFT action under broken time diffeomorphism in quasi de Sitter background. In Section 3, we derive the expression for the two-point correlation function from EFT using scalar and tensor mode fluctuation. Furthermore, in Section 4, we derive the expression for the scalar three-point function from EFT using scalar mode fluctuation in equilateral and squeezed limit configurations. After that, in Section 5, we derive the exact analytical expressions for coefficients of EFT operators for both single-field slow-roll inflation and generalized single-field $P(X,\phi)$ models of inflation. Finally, we conclude in Section 6 with some future prospects of the present work.

2. Overview of EFT

2.1. Construction of the Generic EFT Action

In this section, our motivation is to construct the most generic EFT action in the background of quasi de Sitter space. Before going into the further technical details it is important to note that the method of implementing cosmological perturbation using a scalar field is different compared to the generic EFT framework. However, the underlying connection can be explained by interpreting the scalar (inflaton) field as a scalar under all space-time diffeomorphisms in General Relativity:

$$\textbf{Space-time diffeomorphism:}\ x^{\mu} \Longrightarrow x^{\mu} + \xi^{\mu}(t,\mathbf{x}) \quad \forall\, \mu = 0,1,2,3\,. \tag{4}$$

Consequently, in the cosmological perturbation the scalar field $\delta\phi$ transforms like a scalar under the operation of spatial diffeomorphisms; on the other hand, it transforms in non-linear fashion with respect to time diffeomorphisms. The space and time diffeomorphic transformation rules are appended bellow:

$$\begin{aligned}\textbf{Spatial diffeomorphism}: t &\Longrightarrow t,\ x^i \Longrightarrow x^i + \xi^i(t,\mathbf{x}) \quad \forall\, i = 1,2,3 \longrightarrow \delta\phi \Longrightarrow \delta\phi,\\ \textbf{Time diffeomorphism}: t &\Longrightarrow t + \xi^0(t,\mathbf{x}),\ x^i \Longrightarrow x^i \quad \forall\, i = 1,2,3 \longrightarrow \delta\phi \Longrightarrow \delta\phi + \dot{\phi}_0(t)\xi^0(t,\mathbf{x}).\end{aligned} \tag{5}$$

Here $\xi^0(t,\mathbf{x})$ and $\xi^i(t,\mathbf{x})\forall i = 1,2,3$ are the diffeomorphism parameter. In this context one can choose a specific gauge in which we set the background scalar degrees of freedom as, $\phi(t,\mathbf{x}) = \phi_0(t)$, which is consistent with the requirement that the perturbation in the scalar field vanishes:

$$\textbf{Unitary gauge fixing} \quad \Rightarrow \quad \delta\phi(t,\mathbf{x}) = 0\,, \tag{6}$$

In cosmological perturbation theory this is known as unitary gauge in which all degrees of freedom are preserved in the metric of quasi de Sitter space. This phenomenon is analogous to the spontaneous symmetry breaking as appearing in the context of $SU(N)$ gauge theory where the Goldstone mode transform in a non-linear fashion and destroyed by the $SU(N)$ gauge boson in unitary gauge to give a massive spin 1 degrees of freedom after symmetry breaking. In an alternative way one can present the framework of EFT by describing cosmological perturbation theory during inflation where time diffeomorphisms are realized in non-linear fashion.

Now to construct a most general structure of the EFT action suitable for the inflationary paradigm we need to follow the step appended below:

1. One must write down the EFT operators that are functions of the metric $g_{\mu\nu}$. Here one of the possibilities is Riemann tensor.
2. Also the EFT operators are invariant under the linearly realized time-dependent spatial diffeomorphic transformation:

$$\textbf{Spatial diffeomorphism}: \; t \Longrightarrow t, \;\; x^i \Longrightarrow x^i + \xi^i(t, \mathbf{x}) \quad \forall\, i = 1, 2, 3\,. \tag{7}$$

   For an example, one can consider an EFT operator constructed by g^{00} or its polynomials without derivatives which transform like a scalar under Equation (7).
3. Due to the reduced symmetry of the physical system many more extra contributions are allowed in the EFT action.
4. In the EFT action one can also allow geometrical quantities in a preferred space-time slice. For example, one can consider the extrinsic curvature $K_{\mu\nu}$ of surfaces at constant time, which transform like a tensor under Equation (7).

Consequently, the most general EFT action can be written in terms of all possible allowed operators by the space-time diffeomorphism as [6,20]:

$$\begin{aligned} S &= \int d^4x \sqrt{-g} \left[\frac{M_p^2}{2} R + M_p^2 \dot{H} g^{00} - M_p^2 \left(3H^2 + \dot{H}\right) + \sum_{n=2}^{\infty} \frac{M_n^4(t)}{n!} (\delta g^{00})^n \right. \\ &\left. - \sum_{q=0}^{\infty} \frac{\bar{M}_1^{3-q}(t)}{(q+2)!} \delta g^{00} \left(\delta K^\mu_\mu\right)^{q+1} - \sum_{m=0}^{\infty} \frac{\bar{M}_2^{2-m}(t)}{(m+2)!} \left(\delta K^\mu_\mu\right)^{m+2} - \sum_{m=0}^{\infty} \frac{\bar{M}_3^{2-m}(t)}{(m+2)!} [\delta K]^{m+2} + \cdots \right]. \end{aligned} \tag{8}$$

where the dots stand for higher-order fluctuations in the EFT action which contains operators with more derivatives in space-time metric. Here we use the following sets of definitions for extrinsic curvature $K_{\mu\nu}$, unit normal n_μ and induced metric $h_{\mu\nu}$:

$$\begin{aligned} K_{\mu\nu} &= h^\sigma_\mu \nabla_\sigma n_\nu = \frac{\delta^0_\mu \partial_\nu g^{00} + \delta^0_\nu \partial_\mu g^{00}}{2(-g^{00})^{3/2}} + \frac{\delta^0_\mu \delta^0_\nu g^{0\sigma} \partial_\sigma g^{00}}{2(-g^{00})^{5/2}} - \frac{g^{0\rho} \left(\partial_\mu g_{\rho\nu} + \partial_\nu g_{\rho\mu} - \partial_\rho g_{\mu\nu}\right)}{2(-g^{00})^{1/2}}, \\ h_{\mu\nu} &= g_{\mu\nu} + n_\mu n_\nu, \quad n_\mu = \frac{\partial_\mu t}{\sqrt{-g^{\mu\nu} \partial_\mu t \partial_\nu t}} = \frac{\delta^0_\mu}{\sqrt{-g^{00}}}. \end{aligned} \tag{9}$$

Here $\delta K_{\mu\nu}$ represents the variation of the extrinsic curvature of constant time surfaces with respect to the unperturbed background FLRW metric in quasi de Sitter space-time:

$$\delta g^{00} = g^{00} + 1, \quad \delta K_{\mu\nu} = K_{\mu\nu} - a^2 H h_{\mu\nu}. \tag{10}$$

Additionally, we have used a shorthand notation $[\delta K]$ to define the following tensor contraction rule useful to quantify the EFT action [20]:

$$[\delta K]^{m+2} = \delta K^{\mu_1}_{\mu_2} \delta K^{\mu_2}_{\mu_3} \delta K^{\mu_3}_{\mu_4} \cdots \delta K^{\mu_{m+1}}_{\mu_{m+2}} \delta K^{\mu_{m+2}}_{\mu_1}. \tag{11}$$

Before going into the further details let us first point out the few important characteristics of the EFT action which are appended bellow:

- In the EFT action the operators $M_p^2 \dot{H} g^{00}$ and $M_p^2 \left(3H^2 + \dot{H}\right)$ are completely specified by the Hubble parameter $H(t)$ which is the solution of Friedman's equations in unperturbed background.
- The rest of the contributions in EFT action captures the effect of quantum fluctuations, which are characterized by the perturbation around the background FLRW solution of all UV complete theories of inflation.
- The coefficients of the operators appearing in the EFT action are in general time-dependent.

Now as we are interested to compute the two- and three-point correlation function, we have restricted to the following truncated EFT action [6,20]:

$$
\begin{aligned}
S &= \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R + M_p^2\dot{H}g^{00} - M_p^2\left(3H^2+\dot{H}\right) + \frac{M_2^4(t)}{2!}\left(g^{00}+1\right)^2 + \frac{M_3^4(t)}{3!}\left(g^{00}+1\right)^3\right. \\
&\left. - \frac{\bar{M}_1^3(t)}{2}\left(g^{00}+1\right)\delta K^\mu_\mu - \frac{\bar{M}_2^2(t)}{2}(\delta K^\mu_\mu)^2 - \frac{\bar{M}_3^2(t)}{2}\delta K^\mu_\nu \delta K^\nu_\mu\right].
\end{aligned} \tag{12}
$$

where we have considered the terms in two derivatives in the metric[3].

2.2. EFT as a Theory of Goldstone Boson

2.2.1. Stückelberg Trick I: An Example from $SU(N)$ Gauge Theory with Massive-Gauge Boson in Flat Background

In the unitary gauge, the EFT action consists of graviton mode, two helicities, and scalar mode, respectively. In this context first we apply a broken time diffeomorphic transformation on the Goldstone boson. As a result, $SU(N)$ gauge symmetry [6,58] is non-linearly realized in the framework of EFT. This mechanism is commonly known as the Stückelberg trick. Let us mention two crucial roles of the Stückelberg trick in gauge theory:

1. Using this trick in $SU(N)$ gauge theory [6,58] one can study the physical implications from longitudinal components of a massive-gauge boson degrees of freedom.
2. It is expected that in the weak coupling limit the contributions from the mixing terms are very small and consequently Goldstone modes decouple from the theory.

To give a specific example of the Stückelberg trick we consider $SU(N)$ gauge theory characterized by a non-abelian gauge field A^a_μ in the background of Minkowski flat space-time. In unitary gauge this theory is described by the following action:

$$
S = \int d^4x\left[-\frac{1}{4}\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) - \frac{m^2}{2}\mathrm{Tr}(A_\mu A^\mu)\right], \tag{13}
$$

where $A_\mu = A^a_\mu T_a$ and $F^a_{\mu\nu} = \partial_{[\mu}A^a_{\nu]}$. Here the label $a = 1, 2, \cdots, N$ for $SU(N)$ gauge theory. Also T_a are the generators of the non-abelian gauge group which satisfy the following properties:

$$
\left[T^a, T^b\right] = if^{abc}T_c, \qquad \mathrm{Tr}(T^a) = 0, \qquad \mathrm{Tr}(T^aT^b) = \frac{\delta^{ab}}{2}. \tag{14}
$$

Here $f^{abc}\ \forall a, b, c = 1, 2, \cdots, N$ are the structure constants of the non-abelian $SU(N)$ gauge theory.

It is important to mention that in this context the $SU(N)$ gauge transformation on the non-abelian gauge field can be written as:

$$
A_\mu \Longrightarrow \tilde{A}_\mu = \frac{i}{g}UD_\mu U^\dagger, \quad \text{with } D_\mu = \partial_\mu - igA_\mu \tag{15}
$$

[3] As we are dealing with EFT, in principle one can consider operators which includes higher derivatives in the metric i.e., $\left(g^{00}+1\right)^2\delta K^2, \delta K^2\delta K^\nu_\mu\delta K^\mu_\nu, \delta K^3, \delta K\delta N^2$ (here $\delta N = N-1$, where N is the lapse function in ADM formalism. See ref. [57] for more details). But since we have considered the terms two derivative in the metric we have truncated the EFT action in the form presented in Equation (12) and the form of the EFT action is exactly similar to ref. [6]. In this paper our prime objective is to concentrate only on the leading-order tree-level contributions and for this reason we have not considered any sub-leading suppressed contributions or any other contributions which are coming from the quantum loop corrections. Additionally, we have also neglected the term $\left(g^{00}+1\right)^2\delta K$ in the EFT action as this term is suppressed by the contribution $H^2\epsilon << 1$ in the decoupling limit and also the higher derivatives of the Goldstone mode π after implementing the symmetry breaking through the Stückelberg trick.

where D_μ is the covariant derivative. Here g is the gauge coupling parameter for $SU(N)$ non-abelian gauge theory. Under this gauge transformation each of the terms in the action stated in Equation (13) transform as:

$$\mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}) \implies \mathrm{Tr}(\tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu}) = \mathrm{Tr}(F_{\mu\nu}F^{\mu\nu}), \tag{16}$$

$$\frac{m^2}{2}\mathrm{Tr}(A_\mu A^\mu) \implies \frac{m^2}{2}\mathrm{Tr}(\tilde{A}_\mu \tilde{A}^\mu) = \frac{m^2}{2g}\mathrm{Tr}[(D_\mu U^\dagger)(D^\mu U)], \tag{17}$$

where U is the unitary operator in $SU(N)$ non-abelian gauge theory.

Consequently, after doing $SU(N)$ gauge transformation action can be expressed as:

$$S \implies \tilde{S} = S + \underbrace{\int d^4x \left[\frac{m^2}{2}\mathrm{Tr}(A_\mu A^\mu) - \frac{m^2}{2g}\mathrm{Tr}[(D_\mu U^\dagger)(D^\mu U)]\right]}_{\textbf{Additional part which breaks SU(N) gauge symmetry}}. \tag{18}$$

where $\underbrace{}$ term signifies the gauge symmetry breaking contribution in the unitary gauge.

Furthermore, it is important to note that the $SU(N)$ gauge symmetry can be restored by defining the previously mentioned unitary operator in a following fashion:

$$U = \exp\left[iT^a\pi^a(t,\mathbf{x})\right], \tag{19}$$

where one can identify the $\pi^a \ \forall\ a = 1, 2, \cdots, N$ s with the Goldstone modes, which transform in a linear fashion under the action of the following gauge transformation:

$$U \implies \tilde{U} = \exp\left[iT^a\tilde{\pi}^a(t,\mathbf{x})\right] = \Sigma(t,\mathbf{x})\exp\left[iT^a\pi^a(t,\mathbf{x})\right] = \underbrace{\Sigma(t,\mathbf{x})}_{Local\ operator} U. \tag{20}$$

For the sake of simplicity one can rescale the Goldstone modes by absorbing the mass of the $SU(N)$ gauge field m and the $SU(N)$ gauge coupling parameter g by introducing the following canonical normalization as given by:

$$\text{Canonical normalization}: \quad \pi_c = \frac{m}{g}\pi \quad . \tag{21}$$

Consequently, the action in terms of canonically normalized field π_c can be written after $SU(N)$ gauge transformation as:

$$S \implies \tilde{S} = S + \int d^4x \left[\frac{m^2}{2}\mathrm{Tr}(A_\mu A^\mu) - \underbrace{\frac{1}{2}\mathrm{Tr}[(\partial_\mu\pi_c)(\partial^\mu\pi_c)]}_{\textbf{Kinetic term of Goldstone}} \right.$$
$$\left. \underbrace{-\frac{2g^2}{m}\mathrm{Tr}(A_\mu\partial^\mu\pi_c) + \frac{g^2}{2}\mathrm{Tr}(A_\mu A^\mu\pi_c^2) + ig\mathrm{Tr}(\pi_c A_\mu\partial^\mu\pi_c)}_{\textbf{Mixing terms after canonical normalization}}\right]. \tag{22}$$

It is important to note the important facts from Equation (22) which are appended below:

- The last two terms in Equation (22) are the mixing terms between the transverse component of the $SU(N)$ gauge field, the Goldstone boson, and its kinetic term, respectively.
- Here one can neglect all such mixing contributions at the energy scale $E_{mix} >> m$. Consequently, two sectors decouple from each other as they are weakly coupled in the energy scale $E_{mix} >> m$ and Equation (22) takes the following form:

$$S \Longrightarrow \tilde{S} \quad = S + \int d^4x \left[\frac{m^2}{2} \mathrm{Tr}(A_\mu A^\mu) - \frac{1}{2} \mathrm{Tr}[(\partial_\mu \pi_c)(\partial^\mu \pi_c)] \right]. \tag{23}$$

2.2.2. Stückelberg Trick II: Broken Time Diffeomorphism in Quasi-de Sitter Background

Here one needs to perform a time diffeomorphism with a local parameter $\xi^0(t, \mathbf{x})$, which is interpreted as a Goldstone field $\pi(t, \mathbf{x})$. These Goldstone modes shifts under the application of time diffeomorphism, as given by:

$$\textbf{Time diffeomorphism}: \; t \quad \Longrightarrow t + \xi^0(t, \mathbf{x}), \; x^i \Longrightarrow x^i \quad \forall\, i = 1, 2, 3 \longrightarrow \pi(t, \mathbf{x}) \to \pi(t, \mathbf{x}) - \xi^0(t, \mathbf{x}). \tag{24}$$

The π is the Goldstone mode which describes the scalar perturbations around the background FLRW metric. The effective action in the unitary gauge can be reproduced by gauge-fixing the time diffeomorphism as:

$$\textbf{Unitary gauge fixing} \quad \Rightarrow \quad \pi(t, \mathbf{x}) = 0 \quad \Rightarrow \quad \tilde{\pi}(t, \mathbf{x}) = -\xi^0(t, \mathbf{x}) \,. \tag{25}$$

To construct the EFT action, it is important to write down the transformation property of each operators under the application of broken time diffeomorphism, which are given by:

1. **Rule for metric:** Under broken time diffeomorphism contravariant and covariant metric transform as:

$$\begin{aligned}
\textbf{Contravariant metric}: \; g^{00} \Longrightarrow &\quad (1+\dot{\pi})^2 g^{00} + 2(1+\dot{\pi}) g^{0i} \partial_i \pi + g^{ij} \partial_i \pi \partial_j \pi, \\
g^{0i} \Longrightarrow &\quad (1+\dot{\pi}) g^{0i} + g^{ij} \partial_j \pi, \\
g^{ij} \Longrightarrow &\quad g^{ij}.
\end{aligned} \tag{26}$$

$$\begin{aligned}
\textbf{Covariant metric}: \; g_{00} &\Longrightarrow (1+\dot{\pi})^2 g_{00}, \\
g_{0i} &\Longrightarrow (1+\dot{\pi}) g_{0i} + g_{00} \dot{\pi} \partial_i \pi, \\
g_{ij} &\Longrightarrow g_{ij} + g_{0j} \partial_i \pi + g_{i0} \partial_j \pi.
\end{aligned} \tag{27}$$

2. **Rule for Ricci scalar and Ricci tensor:** Under broken time diffeomorphism Ricci scalar and the spatial component of the Ricci tensor on 3-hypersurface transform as:

$$\begin{aligned}
\textbf{Ricci scalar}: \; {}^{(3)}R &\Longrightarrow {}^{(3)}R + \frac{4}{a^2} H(\partial^2 \pi), \\
\textbf{Spatial Ricci tensor}: \; {}^{(3)}R_{ij} &\Longrightarrow {}^{(3)}R_{ij} + H(\partial_i \partial_j \pi + \delta_{ij} \partial^2 \pi).
\end{aligned} \tag{28}$$

3. **Rule for extrinsic curvature:** Under broken time diffeomorphism trace and the spatial, time and mixed component of the extrinsic curvature transform as:

$$\begin{aligned}
\textbf{Trace}: \ \delta K &\Longrightarrow \delta K - 3\pi\dot{H} - \frac{1}{a^2}(\partial^2\pi), \\
\textbf{Spatial extrinsic curvature}: \ \delta K_{ij} &\Longrightarrow \delta K_{ij} - \pi\dot{H}h_{ij} - \partial_i\partial_j\pi \\
\textbf{Temporal extrinsic curvature}: \ \delta K^0_0 &\Longrightarrow \delta K^0_0, \\
\textbf{Mixed extrinsic curvature}: \ \delta K^0_i &\Longrightarrow \delta K^0_i, \\
\textbf{Mixed extrinsic curvature}: \ \delta K^i_0 &\Longrightarrow \delta K^i_0 + 2Hg^{ij}\partial_j\pi.
\end{aligned} \tag{29}$$

4. **Rule for time-dependent EFT coefficients:** Under broken time diffeomorphism time-dependent EFT coefficients transform after canonical normalization $\pi_c = F^2(t)\pi$ as:

$$\begin{aligned}
\textbf{EFT coefficient}: \ F(t) \Longrightarrow F(t+\pi) &= \left[\sum_{n=0}^{\infty} \frac{\pi^n}{n!}\frac{d^n}{dt^n}\right] F(t) \\
&= \left[\sum_{n=0}^{\infty} \underbrace{\frac{\pi_c^n}{n!F^{2n}}}_{\textbf{Suppression}} \frac{d^n}{dt^n}\right] F(t) \approx F(t)\,.
\end{aligned} \tag{30}$$

Here $F(t)$ corresponds to all EFT coefficients mention in the EFT action.

5. **Rule for Hubble parameter:** Under broken time diffeomorphism, time-dependent EFT coefficients transform after using the following canonical normalization:

$$\textbf{Canonical normalization}: \quad \pi_c = F^2(t)\pi\,, \tag{31}$$

as given by:

$$\begin{aligned}
\textbf{Hubble parameter}: \ H(t) \Longrightarrow H(t+\pi) &= \left[\sum_{n=0}^{\infty} \frac{\pi^n}{n!}\frac{d^n}{dt^n}\right] H(t) \\
&= \left[1 \underbrace{- \pi H(t)\epsilon - \frac{\pi^2 H(t)}{2}\left(\dot{\epsilon} - 2\epsilon^2\right) + \cdots}_{\textbf{Correction terms}}\right] H(t)\,.
\end{aligned} \tag{32}$$

Here $\epsilon = -\dot{H}/H^2$ is the slow-roll parameter.

Now to construct the EFT action we need to also understand the behavior of all the operators appearing in the weak coupling regime of EFT. In this regime one can neglect the mixing contributions between the gravity and Goldstone modes. To demonstrate this explicitly let us start with the EFT operator:

$$\mathcal{O}_1(t) = -\dot{H}M_p^2 g^{00}. \tag{33}$$

Under broken time diffeomorphism, the operator $\mathcal{O}(t)$ transform as:

$$\mathcal{O}_1(t) \Longrightarrow \left[1 + \frac{\pi}{\epsilon}\left(\dot{\epsilon} - 2H\epsilon^2\right) + \cdots\right]\left[(1+\dot{\pi})^2\mathcal{O}_1(t) - \dot{H}M_p^2\left(2(1+\dot{\pi})\partial_i\pi g^{0i} + g^{ij}\partial_i\pi\partial_j\pi\right)\right]. \tag{34}$$

For further simplification the temporal component of the metric g^{00} can be written as, $g^{00} = \bar{g}^{00} + \delta g^{00}$, where the background metric is given by, $\bar{g}^{00} = -1$ and the metric fluctuation is characterized by

δg^{00} [6,20]. Using this in Equation (34) and considering only the first term in Equation (34) we get a kinetic term, $M_p^2\dot{H}\dot{\pi}^2\bar{g}^{00}$ and a mixing contribution, $M_p^2\dot{H}\dot{\pi}\delta g^{00}$ respectively. Furthermore, we use a canonical normalized metric fluctuation from the mixing contribution as given by:

$$\textbf{Canonical normalization}: \quad \delta g_c^{00} = M_p \delta g^{00}\,, \tag{35}$$

in terms of which one can write, $M_p^2\dot{H}\dot{\pi}\delta g^{00} = \sqrt{\dot{H}}\dot{\pi}_c\delta g_c^{00}$. Consequently, at above the energy scale $E_{mix} = \sqrt{\dot{H}}$, we can neglect this mixing term in the weak coupling regime.

One can also consider mixing contributions $M_p^2\dot{H}\dot{\pi}^2\delta g^{00}$ and $\pi M_p^2\ddot{H}\dot{\pi}\bar{g}^{00}$, which can be recast after canonical normalization as, $M_p^2\dot{H}\dot{\pi}^2\delta g^{00} = \dot{\pi}_c^2\delta g_c^{00}/M_p$ and $\pi M_p^2\ddot{H}\dot{\pi}\bar{g}^{00} = \ddot{H}\pi_c\dot{\pi}_c\bar{g}^{00}/\dot{H}$ with $\ddot{H}/\dot{H} << 1$. Here all higher-order terms in $\dot{\pi}$ will lead to additional Planck-suppression after canonical normalization. Consequently, we can neglect the contribution from $M_p^2\dot{H}\dot{\pi}\delta g^{00}$ term at the scale $E > E_{mix}$. Finally, in the weak coupling regime one can recast Equation (34) as:

$$\mathcal{O}_1(t) \Longrightarrow \mathcal{O}_1(t)\left[\dot{\pi}^2 - \frac{1}{a^2}(\partial_i\pi)^2\right]. \tag{36}$$

2.2.3. The Goldstone Action from EFT

Finally, in the weak coupling limit (or decoupling limit) we get the following simplified EFT action:

$$S_{EFT} = S_g + S_\pi, \tag{37}$$

where the gravitational part and the Goldstone action is given by:

$$S_g = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R - M_p^2\left(3H^2+\dot{H}\right)\right], \tag{38}$$

$$S_\pi = S_\pi^{(2)} + S_\pi^{(3)} + \cdots, \tag{39}$$

where the second and third-order Goldstone action can be written as:

$$\begin{aligned} S_\pi^{(2)} &= \int d^4x\, a^3\left[-M_p^2\dot{H}\left(\dot{\pi}^2 - \frac{1}{a^2}(\partial_i\pi)^2\right) + 2M_2^4\dot{\pi}^2\right. \\ &\left. + \frac{1}{2}\left(\bar{M}_3^2 + 3\bar{M}_2^2\right)H^2(1-\epsilon)\frac{(\partial_i\pi)^2}{a^2} - \left(\bar{M}_3^2+3\bar{M}_2^2\right)H^2\frac{(\partial_i\pi)^2}{a^2} - \bar{M}_1^3\dot{\pi}\frac{1}{a^2}(\partial_i^2\pi)\right]. \end{aligned} \tag{40}$$

$$\begin{aligned} S_\pi^{(3)} &= \int d^4x\, a^3\left[\left(2M_2^4 - \frac{4}{3}M_3^4\right)\dot{\pi}^3 - 2M_2^4\dot{\pi}\frac{1}{a^2}(\partial_i\pi)^2\right. \\ &- \bar{M}_3^2\pi\dot{H}\frac{1}{a^2}\partial_i^2\pi - 3\bar{M}_2^2\dot{H}\pi\frac{1}{a^2}(\partial_i^2\pi) + \frac{3}{2}\bar{M}_1^3\pi\dot{H}\frac{1}{a^2}(\partial_i\pi)^2 \\ &\left. - \frac{3}{2}\bar{M}_1^3\dot{H}\pi\dot{\pi}^2 - \bar{M}_1^3\dot{\pi}\frac{1}{a^2}(\partial_i\pi)^2\right]. \end{aligned} \tag{41}$$

Here we introduce EFT sound speed c_S as:

$$c_S \equiv \frac{1}{\sqrt{1 - \frac{2M_2^4}{\dot{H}M_p^2}}}. \tag{42}$$

Here if we set $M_2 = 0$ or equivalently if we say that $\frac{M_2^4}{2!}(g^{00}+1)^2$ term is absent in the effective Lagrangian then Equation (42) suggests that in that case sound speed $c_S = 1$, which is true for single-field canonical slow-roll inflation. Next using Equation (42) and applying integration by parts in the Goldstone part of the Lagrangian we get[4]:

$$S^{(2)}_{\pi} = \int d^4x\, a^3 \left(-\frac{M_p^2\dot{H}}{c_S^2}\right)\left[\dot{\pi}^2 - c_S^2\left(1 - \frac{\bar{M}_1^3 H}{M_p^2\dot{H}} - \left[\bar{M}_3^2 + 3\bar{M}_2^2\right]\frac{H^2(1+\epsilon)}{2M_p^2\dot{H}}\right)\frac{1}{a^2}(\partial_i\pi)^2\right]. \quad (47)$$

$$\begin{aligned} S^{(3)}_{\pi} &= \int d^4x\, a^3\left[\left\{\left(1-\frac{1}{c_S^2}\right)\dot{H}M_p^2 + \frac{3}{2}\bar{M}_1^3 H - \frac{4}{3}M_3^4\right\}\dot{\pi}^3\right.\\ &\qquad - \left\{\left(1-\frac{1}{c_S^2}\right)\dot{H}M_p^2 + \frac{3}{2}\bar{M}_1^3 H\right\}\frac{1}{a^2}\dot{\pi}(\partial_i\pi)^2 \\ &\qquad \left. - \frac{9}{2}\bar{M}_1^3 H^2\pi\dot{\pi}^2 + \frac{3}{2}\bar{M}_1^3 H\frac{1}{a^2}\pi\frac{d}{dt}(\partial_i\pi)^2\right]. \end{aligned} \quad (48)$$

In the present context metric fluctuation of the spatial components are given by:

$$g_{ij} = a^2(t)\left[(1+2\zeta(t,\mathbf{x}))\,\delta_{ij} + \gamma_{ij}\right] \quad \forall \quad i = 1,2,3, \quad (49)$$

where $a(t)$ is the scale factor in FLRW quasi de Sitter background space-time. Also $\zeta(t,\mathbf{x})$ is known as curvature perturbation which signifies scalar fluctuation. On the other hand, tensor fluctuations are identified with γ_{ij}, which is spin-2, transverse, and traceless rank 2 tensor. Here under the broken time diffeomorphism the scale factor $a(t)$ transforms in the following fashion:

$$a(t) \Longrightarrow a(t-\pi(t,\mathbf{x})) = a(t) - H\pi(t,\mathbf{x})a(t) + \cdots \approx a(t)\,(1 - H\pi(t,\mathbf{x}))\,. \quad (50)$$

Furthermore, using Equations (49) and (50), we get:

$$a^2(t)\,(1-H\pi(t,\mathbf{x}))^2 \approx a^2(t)\,(1-2H\pi(t,\mathbf{x})) = a^2(t)\,(1+2\zeta(t,\mathbf{x}))\,. \quad (51)$$

---

[4] Let us concentrate on the following contribution in the second- and third-order perturbed EFT action, which can be written after integration by parts as:

$$\begin{aligned} S^2_{\pi} \supset\; & -\int d^3x\, dt\, a^3\, \bar{M}_1^3\,\frac{\dot{\pi}}{a^2}\left(\partial_i^2\pi\right) = \int d^3x\, dt\, a^3\,\frac{\bar{M}_1^3}{a^2}\left[-\partial_i\left(\dot{\pi}\partial_i\pi\right) + \frac{1}{2}\frac{d}{dt}(\partial_i\pi)^2\right] \\ &= \int d^3x\, dt\, a^3\,\frac{\bar{M}_1^3}{2}\left[\frac{d}{dt}\left(\frac{(\partial_i\pi)^2}{a^2}\right) - \frac{H}{a^2}(\partial_i\pi)^2\right] \\ &= -\int d^3x\, dt\, a^3\,\frac{\bar{M}_1^3}{2}\, H\,\frac{(\partial_i\pi)^2}{a^2}. \end{aligned} \quad (43)$$

$$S^3_{\pi} \supset -\int d^3x\, dt\, a^3\, \bar{M}_1^3\,\frac{3}{2}\,\dot{H}\,\pi\,\dot{\pi}^2 = \int d^3x\, dt\, a^3\left[\frac{3}{2}\bar{M}_1^3 H\dot{\pi}^3 - \frac{9}{2}H^2\bar{M}_1^3\pi\dot{\pi}^2\right]. \quad (44)$$

$$S^3_{\pi} \supset \int d^3x\, dt\, a^3\, \bar{M}_1^3\,\frac{3}{2}\,\dot{H}\,\pi\,\frac{(\partial_i\pi)^2}{a^2} = -\int d^3x\, dt\, a^3\left[\frac{3}{2}\bar{M}_1^3\frac{H\pi}{a^2}(\partial_i\pi)^2 + \frac{\dot{\pi}}{a^2}\frac{3}{2}\bar{M}_1^3(\partial_i\pi)^2\right]. \quad (45)$$

$$S^3_{\pi} \supset -3\int d^3x\, dt\, a^3\, \bar{M}_2^2\,\dot{H}\,\pi\,\frac{\pi}{a^2}\left(\partial_i^2\pi\right) = \int d^3x\, dt\, a^3\, 3\bar{M}_2^2\left[\frac{H\pi}{a^2}(\partial_i\pi)^2 + \frac{\dot{\pi}}{a^2}(\partial_i\pi)^2\right]. \quad (46)$$

This implies that the curvature perturbation $\zeta(t,\mathbf{x})$ can be written in terms of Goldstone modes $\pi(t,\mathbf{x})$ in the following way[5]:

$$\textbf{Quantum fluctuation in terms of Goldstone mode}: \quad \zeta(t,\mathbf{x}) = -H\pi(t,\mathbf{x})\,. \tag{53}$$

Furthermore, using Equation (53), the effective action for the Goldstone part of the Lagrangian can be recast in terms of curvature perturbation $\zeta(t,\mathbf{x})$ as:

$$S^{(2)}_{\zeta} \approx \int d^4x\, a^3 \left(\frac{M_p^2\epsilon}{c_S^2}\right)\left[\dot{\zeta}^2 - c_S^2\left(1 - \frac{\bar{M}_1^3 H}{M_p^2\dot{H}} - \left[\bar{M}_3^2 + 3\bar{M}_2^2\right]\frac{H^2(1+\epsilon)}{2M_p^2\dot{H}}\right)\frac{1}{a^2}(\partial_i\zeta)^2\right]. \tag{54}$$

$$\begin{aligned} S^{(3)}_{\zeta} \approx \int d^4x\, \frac{a^3}{H^3}\Bigg[&-\left\{\left(1-\frac{1}{c_S^2}\right)\dot{H}M_p^2 + \frac{3}{2}\bar{M}_1^3 H - \frac{4}{3}M_3^4\right\}\dot{\zeta}^3 \\ &+\left\{\left(1-\frac{1}{c_S^2}\right)\dot{H}M_p^2 + \frac{3}{2}\bar{M}_1^3 H\right\}\frac{1}{a^2}\dot{\zeta}(\partial_i\zeta)^2 \\ &+\frac{9}{2}\bar{M}_1^3 H^2\zeta\dot{\zeta}^2 - \frac{3}{2}\bar{M}_1^3 H\frac{1}{a^2}\zeta\frac{d}{dt}(\partial_i\zeta)^2\Bigg]\,. \end{aligned} \tag{55}$$

For further simplification we introduce a few new parameters which are appended bellow[6]:

- First we define an effective sound speed $\tilde{c}_S$, which can be expressed in terms of the usual EFT sound speed c_S as[7]:

$$\tilde{c}_S = c_S\sqrt{1 - \frac{\bar{M}_1^3 H}{M_p^2\dot{H}} - \left[\bar{M}_3^2 + 3\bar{M}_2^2\right]\frac{H^2(1+\epsilon)}{2M_p^2\dot{H}}}. \tag{56}$$

Since the following approximations:

$$\left|\frac{\bar{M}_1^3 H}{M_p^2\dot{H}}\right| \quad << \quad 1, \qquad \left|\left[\bar{M}_3^2 + 3\bar{M}_2^2\right]\frac{H^2(1+\epsilon)}{2M_p^2\dot{H}}\right| << 1, \tag{57}$$

---

5 Here we have considered the linear relation between the curvature perturbation (ζ) and the Goldstone mode (π). In this context one can consider the following non-linear relation to compute the three-point correlation function from the present setup:

$$\zeta(t,\mathbf{x}) = -H\pi(t,\mathbf{x}) - \frac{(\epsilon-\eta)}{2}H^2\pi^2(t,\mathbf{x}) + \cdots\,, \tag{52}$$

where the slow-roll parameters are given by, $\epsilon = -\dot{H}/H^2$ and $\eta = \epsilon - \frac{1}{2}\frac{d\ln\epsilon}{d\mathcal{N}}$. Here $\mathcal{N} = \int H\,dt$, represents the number of e-foldings. However, the contribution from such non-linear term is extremely small and proportional to sub-leading terms ϵ^2, η^2 and $\epsilon\eta$ in the expression for the three-point function and the associated bispectrum. From the observational perspective such contributions also not so important and can be treated as very small correction to the leading-order result computed in this paper.

6 Here we have used a few choices for the simplifications of the further computation of the two- and three-point correlation function in the EFT coefficients which are partly motivated by ref. [59]. Also it is important to note that since we are restricted our computation up to tree-level and not considering any quantum effects through loop correction, we have discussed the radiative stability or naturalness of these choices under quantum corrections.

7 Here it is important to point out that in the case when $M_2 = 0$ we have the EFT sound speed $c_S = 1$ exactly, which is true for all canonical slow-roll models of inflation driven by a single field. But since here the EFT coefficients are sufficiently small $\bar{M}_i \forall i = 1,2,3 (\sim \mathcal{O}(10^{-2} - 10^{-3}))$ it is expected that $\tilde{c}_S \approx c_S$ and for the situation $c_S = 1$ one can approximately fix $\tilde{c}_S \approx 1$. Thus, for the canonical slow-roll model one can easily approximate the redefined sound speed $\tilde{c}_S$ with the usual EFT sound speed c_S without losing any generality. But such small EFT coefficients $\bar{M}_i \forall i = 1,2,3 (\sim \mathcal{O}(10^{-2} - 10^{-3}))$ play significant roles in the computation of the three-point function and the associated bispectrum as in the absence of these coefficients the amplitude of the bispectrum f_{NL} is zero. This also implies that for the canonical slow-roll model of single-field inflation the amount of non-Gaussianity is not very large and this completely consistent with the previous finding that in that case the amplitude of the bispectrum $f_{NL} \propto \epsilon$ (where ϵ is the slow-roll parameter), at the leading order of the computation. See ref. [31] for details.

are valid in the present context of discussion, one can recast the effective sound speed in the following simplified form as:

$$\tilde{c}_S \approx c_S \left\{1 + \frac{1}{2\epsilon H M_p^2}\left[\bar{M}_1^3 + \left(\bar{M}_3^2 + 3\bar{M}_2^2\right)\frac{H(1+\epsilon)}{2}\right]\right\}. \tag{58}$$

- Secondly, we introduce the following connecting relationship between M_3 and M_2 given by:

$$M_3^4 c_S^2 = -\tilde{c}_3 M_2^4. \tag{59}$$

When $M_2 = 0$ then from Equation (42) we can see that the sound speed $c_S = 1$ and Equation (59) also implies that $M_3 = 0$ in that case.

- Next we define the following connecting relationship between M_3 and $\bar{M}_1$ given by:

$$M_3^4 \tilde{c}_4 = -H\bar{M}_1^3 \tilde{c}_3. \tag{60}$$

When $M_2 = 0$ then from Equation (42) we can see that the sound speed $c_S = 1$ (which is actually the result for single-field canonical slow-roll models of inflation) and Equations (59) and (60) also implies the following possibilities:

1. $M_3 = 0$, $\bar{M}_1 \neq 0$ and $\frac{\tilde{c}_3}{\tilde{c}_4} \to 0$. We will look into this possibility in detail during our computation for $c_S = 1$ case as this will finally give rise to non-vanishing three-point function (non-Gaussianity).
2. $M_3 = 0$, $\bar{M}_1 = 0$ and $\frac{\tilde{c}_3}{\tilde{c}_4} \neq 0$. We do not consider this possibility for $c_S = 1$ case because for this case third ($S^{(3)}_{\zeta}$) action for curvature perturbation vanishes, which will give rise to zero three-point function (non-Gaussianity).

- For further simplification one can also assume that:

$$\bar{M}_3^2 + 3\bar{M}_2^2 = \frac{\bar{M}_1^3}{H\tilde{c}_5} \tag{61}$$

so that one can write:

$$\frac{1}{\epsilon H M_p^2}\left[\bar{M}_1^3 + \left(\bar{M}_3^2 + 3\bar{M}_2^2\right)\frac{H(1+\epsilon)}{2}\right] = \frac{\bar{M}_1^3}{\epsilon H M_p^2}\left[1 + \frac{(1+\epsilon)}{2\tilde{c}_5}\right]. \tag{62}$$

For $c_S = 1$ this implies the following two possibilities:

1. $\bar{M}_1 \neq 0$ and $\tilde{c}_5 = -\frac{1}{2}(1+\epsilon)$. We will look into this possibility in detail during our computation for $c_S = 1$ case as this will finally give rise to non-vanishing three-point function (non-Gaussianity).
2. $\bar{M}_1 = 0$. We do not consider this possibility for $c_S = 1$ case because for this case third ($S^{(3)}_{\zeta}$) action for curvature perturbation vanishes, which will give rise to zero three-point function (non-Gaussianity).

Consequently, the effective sound speed can be recast as:

$$\tilde{c}_S = c_S\sqrt{1 + \frac{\Delta \bar{M}_1^3}{2\epsilon H M_p^2}} \approx c_S\left\{1 + \frac{\Delta \bar{M}_1^3}{4\epsilon H M_p^2}\right\} \tag{63}$$

where Δ is defined as, $\Delta = 2 + \frac{1+\epsilon}{\tilde{c}_5}$. Here $\Delta = 0$ for $\tilde{c}_5 = -\frac{1}{2}(1+\epsilon)$ when $c_S = 1$. Consequently, we have $\tilde{c}_S = c_S = 1$ in that case.

- For further simplification one can also assume that:

$$\bar{M}_3^2 \approx \bar{M}_2^2 = \frac{\bar{M}_1^3}{4H\tilde{c}_5}. \tag{64}$$

Here $c_S = 1$ this implies the following two possibilities:

1. $\bar{M}_3^2 \approx \bar{M}_2^2 \neq 0, \bar{M}_1 \neq 0$ and $\tilde{c}_5 = -\frac{1}{2}(1+\epsilon)$ as mentioned earlier. We will investigate this possibility in detail during our computation for $c_S = 1$ case as this will finally give rise to non-vanishing non-Gaussianity.
2. $\bar{M}_3^2 \approx \bar{M}_2^2 = 0, \bar{M}_1 = 0$. As mentioned earlier here we do not consider this possibility for $c_S = 1$ case because for this case second ($S_\zeta^{(2)}$) and third-order ($S_\zeta^{(3)}$) action for curvature perturbation vanishes, which will give rise to zero non-Gaussianity.

- Next we define the following connecting relationship between M_4 and M_3 given by:

$$M_4^4\tilde{c}_6 = M_3^4\tilde{c}_4 \quad = \quad -H\bar{M}_1^3\tilde{c}_3. \tag{65}$$

When $M_2 = 0$ then from Equation (42) we can see that the sound speed $c_S = 1$ and Equations (59) and (65) also implies the following possibilities:

1. $M_4 \neq 0$, $M_3 = 0$, $\bar{M}_1 \neq 0$ and $\frac{\tilde{c}_3}{\tilde{c}_4} \to 0$. We will look into this possibility in detail during our computation for $c_S = 1$ case as this will finally give rise to non-vanishing three-point function (non-Gaussianity).
2. $M_4 = 0$, $M_3 = 0$, $\bar{M}_1 = 0$ and $\frac{\tilde{c}_3}{\tilde{c}_4} \neq 0$. We do not consider this possibility for $c_S = 1$ case because for this case third ($S_\zeta^{(3)}$) order action for curvature perturbation vanishes, which will give rise to zero three-point function (non-Gaussianity).

Furthermore, using all such new defined parameters the EFT action for the Goldstone boson can be recast as[8]:

For $\mathbf{c_S = 1}$:

$$S_\zeta^{(2)} \approx \int d^4x\, a^3\, M_p^2\epsilon \left[\dot{\zeta}^2 - \frac{1}{a^2}(\partial_i\zeta)^2\right]. \tag{66}$$

$$\begin{aligned} S_\zeta^{(3)} \approx \int d^4x\, \frac{a^3}{H^3}\Bigg[&-\left\{\frac{3}{2}\bar{M}_1^3 H\right\}\dot{\zeta}^3 + \left\{\frac{3}{2}\bar{M}_1^3 H\right\}\frac{1}{a^2}\dot{\zeta}(\partial_i\zeta)^2 \\ &+ \frac{9}{2}\bar{M}_1^3 H^2\zeta\dot{\zeta}^2 - \frac{3}{2}\bar{M}_1^3 H\frac{1}{a^2}\zeta\frac{d}{dt}(\partial_i\zeta)^2\Bigg]. \end{aligned} \tag{67}$$

---

[8] Here it is important to note that for the case $c_S = 1$ we have written an approximated form of the second and third-order action by assuming that $\tilde{c}_S \approx c_S \sim 1$, which is true for all canonical slow-roll models of inflation driven by a single field. Here the EFT coefficients are sufficiently small $\bar{M}_i \forall i = 1,2,3 (\sim \mathcal{O}(10^{-2} - 10^{-3}))$ for which it is expected that $\tilde{c}_S \approx c_S$ and for the situation $c_S = 1$ one can approximately fix $\tilde{c}_S \approx 1$.

For $\mathbf{c_S < 1}$:

$$S_{\zeta}^{(2)} \approx \int d^4x\, a^3 \left(\frac{M_p^2\epsilon}{c_S^2}\right)\left[\dot{\zeta}^2 - \tilde{c}_S^2\frac{1}{a^2}(\partial_i\zeta)^2\right]. \tag{68}$$

$$\begin{aligned} S_{\zeta}^{(3)} \approx \int d^4x\, a^3\, \frac{\epsilon M_p^2}{H}\left(1-\frac{1}{c_S^2}\right)\Bigg[\left\{1+\frac{3\tilde{c}_4}{4c_S^2}+\frac{2\tilde{c}_3}{3c_S^2}\right\}\dot{\zeta}^3 - \left\{1+\frac{3\tilde{c}_4}{4c_S^2}\right\}\frac{1}{a^2}\dot{\zeta}(\partial_i\zeta)^2 \\ -\frac{9H\tilde{c}_4}{4c_S^2}\zeta\dot{\zeta}^2 + \frac{3\tilde{c}_4}{4c_S^2}\frac{1}{a^2}\zeta\frac{d}{dt}(\partial_i\zeta)^2\Bigg]. \end{aligned} \tag{69}$$

3. Two-Point Correlation Function from EFT

3.1. For Scalar Modes

3.1.1. Mode Equation and Solution for Scalar Perturbation

Here we compute the two-point correlation from scalar perturbation. For this purpose we consider the second-order perturbed action as given by[9]:

$$S_{\zeta}^{(2)} \approx \int d^4x\, a^3 \left(\frac{M_p^2\epsilon}{c_S^2}\right)\left[\dot{\zeta}^2 - c_S^2\left(1-\frac{\bar{M}_1^3 H}{M_p^2\dot{H}} - \left[\bar{M}_3^2+3\bar{M}_2^2\right]\frac{H^2(1+\epsilon)}{2M_p^2\dot{H}}\right)\frac{1}{a^2}(\partial_i\zeta)^2\right], \tag{70}$$

which can be recast for $c_S = 1$ and $c_S < 1$ case as:

$$\textbf{For } \mathbf{c_S = 1}: \quad S_{\zeta}^{(2)} \approx \int d^4x\, a^3\, M_p^2\epsilon\left[\dot{\zeta}^2 - \frac{1}{a^2}(\partial_i\zeta)^2\right], \tag{71}$$

$$\textbf{For } \mathbf{c_S < 1}: \quad S_{\zeta}^{(2)} \approx \int d^4x\, a^3 \left(\frac{M_p^2\epsilon}{c_S^2}\right)\left[\dot{\zeta}^2 - \tilde{c}_S^2\frac{1}{a^2}(\partial_i\zeta)^2\right], \tag{72}$$

where the effective sound speed $\tilde{c}_S$ is defined earlier.

Next we define Mukhanov–Sasaki variable $v(\eta, \mathbf{x})$ which is defined as:

$$\textbf{Mukhanov˘Sasaki variable}: \quad v(\eta,\mathbf{x}) = z\,\zeta(\eta,\mathbf{x})\,M_p = -z\,H\,\pi(\eta,\mathbf{x})\,M_p\,. \tag{73}$$

In general, the parameter z is defined for the present EFT setup as, $z = \frac{a\sqrt{2\epsilon}}{\tilde{c}_S}$. Now in terms of $v(\eta, \mathbf{x})$ the second-order action for the curvature perturbation can be recast as:

$$S_{\zeta}^{(2)} \approx \int d^3x\, d\eta\, \left[v'^2 - \tilde{c}_S^2(\partial_i v)^2\frac{1}{a^2}(\partial_i\zeta)^2 - m_{eff}^2(\eta)v^2\right], \tag{74}$$

where the effective mass parameter $m_{eff}(\eta)$ is defined as, $m_{eff}^2(\eta) = -\frac{1}{z}\frac{d^2z}{d\eta^2}$. Here η is the conformal time which can be expressed in terms of physical time t as, $\eta = \int \frac{dt}{a(t)}$. The conformal time described here is negative and lying within $-\infty < \eta < 0$. During inflation, the scale factor and the parameter z can be expressed in terms of the conformal time η as:

$$a(\eta) = \begin{cases} -\frac{1}{H\eta} & \textbf{for dS} \\ -\frac{1}{H\eta}(1+\epsilon) & \textbf{for qdS}. \end{cases} \tag{75}$$

[9] See also ref. [31,44], where similar computations have been performed for canonical single-field slow-roll and generalized slow-roll models of inflation in the presence of Bunch–Davies vacuum state.

and

$$z = \frac{a\sqrt{2\epsilon}}{c_S} = \begin{cases} -\frac{1}{H\eta}\frac{\sqrt{2\epsilon}}{c_S} & \textbf{for dS} \\ -\frac{1}{H\eta}\frac{\sqrt{2\epsilon}}{c_S}(1+\epsilon) & \textbf{for qdS}. \end{cases} \tag{76}$$

Additionally, it is important to note that for de Sitter and quasi de Sitter case the relation between conformal time η and physical time t can be expressed as, $t = -\frac{1}{H}\ln(-H\eta)$. Within this setup inflation ends when the conformal time $\eta \sim 0$.

Now further doing the Fourier transform:

$$v(\eta, \boldsymbol{x}) = \int \frac{d^3k}{(2\pi)^3}\, v_{\mathbf{k}}(\eta)\, e^{i\mathbf{k}.\mathbf{x}} \tag{77}$$

one can write down the equation of motion for scalar fluctuation as:

$$\textbf{Mukhanov–Sasaki Eqn for scalar mode:} \qquad v''_{\mathbf{k}} + \left(\tilde{c}_S^2 k^2 + m^2_{eff}(\eta)\right) v_{\mathbf{k}} = 0\,. \tag{78}$$

Here it is important to note that for de Sitter and quasi de Sitter case the effective mass parameter can be expressed as:

$$m^2_{eff}(\eta) = \begin{cases} -\frac{2}{\eta^2} & \textbf{for dS} \\ -\frac{\left(\nu^2 - \frac{1}{4}\right)}{\eta^2} & \textbf{for qdS}. \end{cases} \tag{79}$$

Here in the de Sitter and quasi de Sitter case the parameter ν can be written as:

$$\nu = \begin{cases} \frac{3}{2} & \textbf{for dS} \\ \frac{3}{2} + 3\epsilon - \eta + \frac{s}{2} & \textbf{for qdS}, \end{cases} \tag{80}$$

where ϵ, η and s are the slow-roll parameter defined as:

$$\epsilon = -\frac{\dot{H}}{H^2}, \quad \eta = 2\epsilon - \frac{\dot{\epsilon}}{2H\epsilon}, \quad s = \frac{\dot{c_S}}{Hc_S}. \tag{81}$$

In the slow-roll regime of inflation $\epsilon << 1$ and $|\eta| << 1$ and at the end of inflation, the slow-roll condition breaks when any of the criteria satisfy (1) $\epsilon = 1$ or $|\eta| = 1$, (2) $\epsilon = 1 = |\eta|$.

The general solution for $v_{\mathbf{k}}(\eta)$ thus can be written as:

$$v_{\mathbf{k}}(\eta) = \begin{cases} \sqrt{-\eta}\left[C_1 H^{(1)}_{\frac{3}{2}}(-k\tilde{c}_S\eta) + C_2 H^{(2)}_{\frac{3}{2}}(-k\tilde{c}_S\eta)\right] & \textbf{for dS} \\ \sqrt{-\eta}\left[C_1 H^{(1)}_{\nu}(-k\tilde{c}_S\eta) + C_2 H^{(2)}_{\nu}(-k\tilde{c}_S\eta)\right] & \textbf{for qdS}. \end{cases} \tag{82}$$

Here C_1 and C_2 are the arbitrary integration constants and the numerical values depend on the choice of the initial vacuum. In the present context we consider the following choice of the vacuum for the computation:

1. **Bunch–Davies vacuum:** In this case, we choose, $C_1 = 1, C_2 = 0$.
2. $\boldsymbol{\alpha, \beta}$ **vacuum:** In this case, we choose $C_1 = \cosh\alpha, C_2 = e^{i\beta}\sinh\alpha$. Here β is a phase factor.

For the most general solution as stated in Equation (82) one can consider the limiting physical situations, as given by, I. **Superhorizon regime:** $k\tilde{c}_S\eta << -1$, II. **Horizon crossing:** $k\tilde{c}_S\eta = -1$, III. **Subhorizon regime:** $k\tilde{c}_S\eta >> -1$.

Finally, considering the behavior of the mode function in the subhorizon regime and superhorizon regime one can write the expression in de Sitter and quasi de Sitter case as:

$$v_{\mathbf{k}}(\eta) = \begin{cases} \frac{1}{i\eta}\frac{1}{\sqrt{2}\,(k\tilde{c}_S)^{\frac{3}{2}}}\left[C_1 e^{-ik\tilde{c}_S\eta}\,(1+ik\tilde{c}_S\eta)\,e^{-i\pi} - C_2 e^{ikc_S\eta}\,(1-ik\tilde{c}_S\eta)\,e^{i\pi}\right] & \textbf{for dS} \\ 2^{\nu-\frac{3}{2}}\frac{1}{i\eta}\frac{1}{\sqrt{2}\,(k\tilde{c}_S)^{\frac{3}{2}}}(-k\tilde{c}_S\eta)^{\frac{3}{2}-\nu}\left|\frac{\Gamma(\nu)}{\Gamma\left(\frac{3}{2}\right)}\right|\left[C_1 e^{-ik\tilde{c}_S\eta}\,(1+ik\tilde{c}_S\eta)\,e^{-\frac{i\pi}{2}\left(\nu+\frac{1}{2}\right)}\right. & \\ \qquad\left. - C_2 e^{ikc_S\eta}\,(1-ik\tilde{c}_S\eta)\,e^{\frac{i\pi}{2}\left(\nu+\frac{1}{2}\right)}\right] & \textbf{for qdS}. \end{cases}$$

Furthermore, using Equation (83) one can write down the expression for the curvature perturbation $\zeta(\eta,\mathbf{k}) = \frac{v_{\mathbf{k}}(\eta)}{z\,M_p}$ as:

$$\zeta(\eta,\mathbf{k}) = \begin{cases} \frac{iH\tilde{c}_S}{2\,M_p\sqrt{\epsilon}\,(k\tilde{c}_S)^{\frac{3}{2}}}\left[C_1 e^{-ik\tilde{c}_S\eta}\,(1+ik\tilde{c}_S\eta)\,e^{-i\pi} - C_2 e^{ikc_S\eta}\,(1-ik\tilde{c}_S\eta)\,e^{i\pi}\right] & \textbf{for dS} \\ 2^{\nu-\frac{3}{2}}\frac{iH\tilde{c}_S}{2\,M_p\sqrt{\epsilon}(1+\epsilon)(\tilde{c}_S k)^{\frac{3}{2}}}(-k\tilde{c}_S\eta)^{\frac{3}{2}-\nu}\left|\frac{\Gamma(\nu)}{\Gamma\left(\frac{3}{2}\right)}\right|\left[C_1 e^{-ik\tilde{c}_S\eta}\,(1+ik\tilde{c}_S\eta)\,e^{-\frac{i\pi}{2}\left(\nu+\frac{1}{2}\right)}\right. & \\ \qquad\left. - C_2 e^{ikc_S\eta}\,(1-ik\tilde{c}_S\eta)\,e^{\frac{i\pi}{2}\left(\nu+\frac{1}{2}\right)}\right] & \textbf{for qdS}. \end{cases}$$

One can further compute the two-point function for scalar fluctuation as:

$$\langle\zeta(\eta,\mathbf{k})\zeta(\eta,\mathbf{q})\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{q})P_\zeta(k,\eta)\ , \tag{83}$$

where $P_\zeta(k,\eta)$ is the power spectrum at time η for scalar fluctuations and in the present context it is defined as:

$$P_\zeta(k,\eta) = \frac{|v_{\mathbf{k}}(\eta)|^2}{z^2 M_p^2} = \begin{cases} \frac{H^2}{4\,M_p^2\epsilon\tilde{c}_S}\frac{1}{k^3}\left|C_1 e^{-ik\tilde{c}_S\eta}\,(1+ik\tilde{c}_S\eta)\,e^{-i\pi} - C_2 e^{ikc_S\eta}\,(1-ik\tilde{c}_S\eta)\,e^{i\pi}\right|^2 & \textbf{for dS} \\ 2^{2\nu-3}\frac{H^2}{4\,M_p^2\epsilon(1+\epsilon)^2\tilde{c}_S}\frac{1}{k^3}(-k\tilde{c}_S\eta)^{3-2\nu}\left|\frac{\Gamma(\nu)}{\Gamma\left(\frac{3}{2}\right)}\right|^2 & \\ \left|C_1 e^{-ik\tilde{c}_S\eta}\,(1+ik\tilde{c}_S\eta)\,e^{-\frac{i\pi}{2}\left(\nu+\frac{1}{2}\right)} - C_2 e^{ikc_S\eta}\,(1-ik\tilde{c}_S\eta)\,e^{\frac{i\pi}{2}\left(\nu+\frac{1}{2}\right)}\right|^2 & \textbf{for qdS}. \end{cases} \tag{84}$$

3.1.2. Primordial Power Spectrum for Scalar Perturbation

Finally, at the horizon crossing one can furthermore write the two-point correlation function as[10]:

$$\langle\zeta(\mathbf{k})\zeta(\mathbf{q})\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{q})P_\zeta(k)\ , \tag{85}$$

where $P_\zeta(k)$ is the power spectrum at time η for scalar fluctuations and it is defined as:

$$P_\zeta(k) = \left[\frac{|v_{\mathbf{k}}(\eta)|^2}{z^2 M_p^2}\right]_{|k\tilde{c}_S\eta|=1} = P_\zeta(k_*)\frac{1}{k^3} = \begin{cases} \frac{H^2}{4\,M_p^2\epsilon\tilde{c}_S}\frac{1}{k^3}\left[|C_1|^2+|C_2|^2-(C_1^*C_2+C_1C_2^*)\right] & \textbf{for dS} \\ 2^{2\nu-3}\frac{H^2}{4\,M_p^2\epsilon(1+\epsilon)^2\tilde{c}_S}\frac{1}{k^3}\left|\frac{\Gamma(\nu)}{\Gamma\left(\frac{3}{2}\right)}\right|^2\left[|C_1|^2+|C_2|^2\right. & \\ \left.-\left(C_1^*C_2 e^{i\pi\left(\nu+\frac{1}{2}\right)} + C_1C_2^* e^{-i\pi\left(\nu+\frac{1}{2}\right)}\right)\right] & \textbf{for qdS}, \end{cases} \tag{86}$$

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[10] See also ref. [8,31,44], where similar computation have been performed for canonical single-field slow-roll and generalized slow-roll models of inflation in the presence of Bunch–Davies vacuum state and general initial state.

where $P_\zeta(k_*)$ is power spectrum for scalar fluctuation at the pivot scale $k = k_*$. For simplicity one can keep $k^3/2\pi^2$ dependence outside and further define amplitude of the power spectrum $\Delta_\zeta(k_*)$ at the pivot scale $k = k_*$ as:

$$\Delta_\zeta(k_*) = \frac{k^3}{2\pi^2}P_\zeta(k) = \frac{1}{2\pi^2}P_\zeta(k_*) = \begin{cases} \frac{H^2}{8\pi^2\, M_p^2\epsilon\tilde{c}_S}\left[|C_1|^2+|C_2|^2-(C_1^*C_2+C_1C_2^*)\right] & \textbf{for dS} \\ 2^{2\nu-3}\frac{H^2}{8\pi^2\, M_p^2\epsilon(1+\epsilon)^2\tilde{c}_S}\left|\frac{\Gamma(\nu)}{\Gamma\left(\frac{3}{2}\right)}\right|^2\left[|C_1|^2+|C_2|^2\right. & \\ \qquad\left.-\left(C_1^*C_2e^{i\pi\left(\nu+\frac{1}{2}\right)}+C_1C_2^*e^{-i\pi\left(\nu+\frac{1}{2}\right)}\right)\right] & \textbf{for qdS}. \end{cases} \tag{87}$$

For **Bunch–Davies** and $\boldsymbol{\alpha,\beta}$ vacuum power spectrum can be written as:

- **For Bunch–Davies vacuum** :
In this case, by setting $C_1 = 1$ and $C_2 = 0$ we get the following expression for the power spectrum:

$$P_\zeta(k) = \begin{cases} \frac{H^2}{4\, M_p^2\epsilon\tilde{c}_S}\frac{1}{k^3} & \textbf{for dS} \\ 2^{2\nu-3}\frac{H^2}{4\, M_p^2\epsilon(1+\epsilon)^2\tilde{c}_S}\frac{1}{k^3}\left|\frac{\Gamma(\nu)}{\Gamma\left(\frac{3}{2}\right)}\right|^2 & \textbf{for qdS}. \end{cases} \tag{88}$$

Also, the power spectrum $\Delta_\zeta(k_*)$ at the pivot scale $k = k_*$ as:

$$\Delta_\zeta(k_*) = \begin{cases} \frac{H^2}{8\pi^2\, M_p^2\epsilon\tilde{c}_S} & \textbf{for dS} \\ 2^{2\nu-3}\frac{H^2}{8\pi^2\, M_p^2\epsilon(1+\epsilon)^2\tilde{c}_S}\left|\frac{\Gamma(\nu)}{\Gamma\left(\frac{3}{2}\right)}\right|^2 & \textbf{for qdS}. \end{cases} \tag{89}$$

- **For $\boldsymbol{\alpha,\beta}$ vacuum** :
In this case, by setting $C_1 = \cosh\alpha$ and $C_2 = e^{i\beta}\sinh\alpha$ we get the following expression for the power spectrum:

$$P_\zeta(k) = \begin{cases} \frac{H^2}{4\, M_p^2\epsilon\tilde{c}_S}\frac{1}{k^3}\left[\cosh 2\alpha - \sinh 2\alpha\cos\beta\right] & \textbf{for dS} \\ 2^{2\nu-3}\frac{H^2}{4\, M_p^2\epsilon(1+\epsilon)^2\tilde{c}_S}\frac{1}{k^3}\left|\frac{\Gamma(\nu)}{\Gamma\left(\frac{3}{2}\right)}\right|^2 & \\ \left[\cosh 2\alpha - \sinh 2\alpha\cos\left(\pi\left(\nu+\frac{1}{2}\right)+\beta\right)\right] & \textbf{for qdS}. \end{cases} \tag{90}$$

Also, the power spectrum $\Delta_\zeta(k_*)$ at the pivot scale $k = k_*$ as:

$$\Delta_\zeta(k_*) = \begin{cases} \frac{H^2}{8\pi^2\, M_p^2\epsilon\tilde{c}_S}\left[\cosh 2\alpha - \sinh 2\alpha\cos\beta\right] & \textbf{for dS} \\ 2^{2\nu-3}\frac{H^2}{8\pi^2\, M_p^2\epsilon(1+\epsilon)^2\tilde{c}_S}\left|\frac{\Gamma(\nu)}{\Gamma\left(\frac{3}{2}\right)}\right|^2 & \\ \left[\cosh 2\alpha - \sinh 2\alpha\cos\left(\pi\left(\nu+\frac{1}{2}\right)+\beta\right)\right] & \textbf{for qdS}. \end{cases} \tag{91}$$

Finally, at the horizon crossing we get the following expression for the spectral tilt for scalar fluctuation at the pivot scale $k = k_*$ as:

$$n_\zeta(k_*) - 1 = \left[\frac{d\ln\Delta_\zeta(k)}{d\ln k}\right]_{|k\tilde{c}_S\eta|=1} = 2\eta - 4\epsilon - \tilde{s}, \tag{92}$$

where $\tilde{s}$ is defined as, $\tilde{s} = \frac{\dot{\tilde{c}}_S}{H\tilde{c}_S}$.

3.2. For Tensor Modes

3.2.1. Mode Equation and Solution for Tensor Perturbation

Here we compute the two-point correlation from tensor perturbation. For this purpose we consider the second-order perturbed action as given by[11]:

$$S^{(2)}_{\gamma} \approx \int d^4x\, a^3\, \frac{M_p^2}{8}\left[\left(1-\frac{\bar{M}_3^2}{M_p^2}\right)\dot{\gamma}_{ij}\dot{\gamma}_{ij}-\frac{1}{a^2}(\partial_m\gamma_{ij})^2\right]=\int d^3x\, d\eta\, a^2\,\frac{M_p^2}{8}\left[\left(1-\frac{\bar{M}_3^2}{M_p^2}\right)\gamma'^2_{ij}-(\partial_m\gamma_{ij})^2\right]. \quad (93)$$

In Fourier space one can write $\gamma_{ij}(\eta,\mathbf{x})$ as:

$$\gamma_{ij}(\eta,\mathbf{x})=\sum_{\lambda=\times,+}\int\frac{d^3k}{(2\pi)^{\frac{3}{2}}}\epsilon^{\lambda}_{ij}(k)\,\gamma_{\lambda}(\eta,\mathbf{k})\,e^{i\mathbf{k}.\mathbf{x}}, \quad (94)$$

where the rank-2 polarization tensor ϵ^{λ}_{ij} satisfies the properties, $\epsilon^{\lambda}_{ii}=k^i\epsilon^{\lambda}_{ij}=0$, $\sum_{i,j}\epsilon^{\lambda}_{ij}\epsilon^{\lambda'}_{ij}=2\delta_{\lambda\lambda'}$. Similar to scalar fluctuation here we also define a new variable $u_{\lambda}(\eta,\mathbf{k})$ in Fourier space as:

$$u_{\lambda}(\eta,\mathbf{k})=\frac{a}{\sqrt{2}}M_p\,\gamma_{\lambda}(\eta,\mathbf{k})=\begin{cases}-\frac{1}{\sqrt{2}H\eta}\,M_p\,\gamma_{\lambda}(\eta,\mathbf{k}) & \textbf{for dS}\\ -\frac{1}{\sqrt{2}H\eta}\,(1+\epsilon)\;M_p\,\gamma_{\lambda}(\eta,\mathbf{k}) & \textbf{for qdS}.\end{cases} \quad (95)$$

Using $u_{\lambda}(\eta,\mathbf{k})$ one can further write Equation (93) as:

$$S^{(2)}_{\gamma}\approx\int d^3x\,d\eta\,a^2\,\frac{M_p^2}{4}\left[\left(1-\frac{\bar{M}_3^2}{M_p^2}\right)u'^2_{\lambda}(\eta,\mathbf{k})-\left(k^2-\frac{a''}{a}\right)(u_{\lambda}(\eta,\mathbf{k}))^2\right]. \quad (96)$$

From this action one can find out the mode equation for tensor fluctuation as:

$$\textbf{Mukhanov–Sasaki Eqn for tensor mode:}\qquad u''_{\lambda}(\eta,\mathbf{k})+\frac{\left(k^2-\frac{a''}{a}\right)}{\left(1-\frac{\bar{M}_3^2}{M_p^2}\right)}u_{\lambda}(\eta,\mathbf{k})=0\,. \quad (97)$$

Furthermore, we introduce a new parameter c_T defined as:

$$c_T=\frac{1}{\sqrt{1-\frac{\bar{M}_3^2}{M_p^2}}}. \quad (98)$$

The general solution for the mode equation for graviton fluctuation can finally written as:

$$u_{\lambda}(\eta,\mathbf{k})=\begin{cases}\sqrt{-\eta}\left[D_1H^{(1)}_{\frac{1}{2}\sqrt{1+8c_T^2}}(-kc_T\eta)+D_2H^{(2)}_{\frac{1}{2}\sqrt{1+8c_T^2}}(-kc_T\eta)\right] & \textbf{for dS}\\ \sqrt{-\eta}\left[D_1H^{(1)}_{\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}}(-kc_T\eta)+D_2H^{(2)}_{\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}}(-kc_T\eta)\right] & \textbf{for qdS}.\end{cases} \quad (99)$$

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11 See also ref. [8,31,44], where similar computation have been performed for canonical single-field slow-roll and generalized slow-roll models of inflation in the presence of Bunch–Davies vacuum state and general initial state.

Here D_1 and D_2 are the arbitrary integration constants and the numerical values depend on the choice of the initial vacuum. In the present context we consider the following choice of the vacuum for the computation:

1. **Bunch–Davies vacuum:** In this case, we choose, $D_1 = 1, \;\; D_2 = 0$.
2. **α, β vacuum:** In this case, we choose $D_1 = \cosh\alpha, \;\; D_2 = e^{i\beta}\sinh\alpha$. Here β is a phase factor.

For the most general solution as stated in Equation (99) one can consider the limiting physical situations, as given by, I. **Superhorizon regime:** $|kc_T\eta| << 1$, II. **Horizon crossing:** $|kc_T\eta| = 1$, III. **Subhorizon regime:** $|kc_T\eta| >> 1$.

Finally, considering the behavior of the mode function in the subhorizon regime and superhorizon regime we get:

$$u_\lambda(\eta,\mathbf{k}) = \begin{cases} 2^{\frac{1}{2}\sqrt{1+8c_T^2}-\frac{3}{2}}\dfrac{1}{i\eta}\dfrac{1}{\sqrt{2}\,(kc_T)^{\frac{3}{2}}}(-kc_T\eta)^{\frac{3}{2}-\frac{1}{2}\sqrt{1+8c_T^2}}\left|\dfrac{\Gamma\left(\frac{1}{2}\sqrt{1+8c_T^2}\right)}{\Gamma\left(\frac{3}{2}\right)}\right| & \\ \left[D_1 e^{-ikc_T\eta}\,(1+ikc_T\eta)\,e^{-\frac{i\pi}{2}\left(\frac{1}{2}\sqrt{1+8c_T^2}+\frac{1}{2}\right)} - D_2 e^{ikc_T\eta}\,(1-ikc_T\eta)\,e^{\frac{i\pi}{2}\left(\frac{1}{2}\sqrt{1+8c_T^2}+\frac{1}{2}\right)}\right] & \textbf{for dS} \\ 2^{\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}-\frac{3}{2}}\dfrac{1}{i\eta}\dfrac{1}{\sqrt{2}\,(kc_T)^{\frac{3}{2}}}(-kc_T\eta)^{\frac{3}{2}-\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}}\left|\dfrac{\Gamma\left(\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}\right)}{\Gamma\left(\frac{3}{2}\right)}\right| & \\ \left[D_1 e^{-ikc_T\eta}\,(1+ikc_T\eta)\,e^{-\frac{i\pi}{2}\left(\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}+\frac{1}{2}\right)} - D_2 e^{ikc_T\eta}\,(1-ikc_T\eta)\,e^{\frac{i\pi}{2}\left(\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}+\frac{1}{2}\right)}\right] & \textbf{for qdS}. \end{cases} \tag{100}$$

Furthermore, using Equation (83) one can write down the expression for the curvature perturbation $\zeta(\eta,\mathbf{k})$ as:

$$h_\lambda(\eta,\mathbf{k}) = \frac{u_\lambda(\eta,\mathbf{k})}{a\,M_p} = \begin{cases} 2^{\frac{1}{2}\sqrt{1+8c_T^2}-\frac{3}{2}}\dfrac{iH}{M_p}\dfrac{1}{(kc_T)^{\frac{3}{2}}}(-kc_T\eta)^{\frac{3}{2}-\frac{1}{2}\sqrt{1+8c_T^2}}\left|\dfrac{\Gamma\left(\frac{1}{2}\sqrt{1+8c_T^2}\right)}{\Gamma\left(\frac{3}{2}\right)}\right| & \\ \left[D_1 e^{-ikc_T\eta}\,(1+ikc_T\eta)\,e^{-\frac{i\pi}{2}\left(\frac{1}{2}\sqrt{1+8c_T^2}+\frac{1}{2}\right)} - D_2 e^{ikc_T\eta}\,(1-ikc_T\eta)\,e^{\frac{i\pi}{2}\left(\frac{1}{2}\sqrt{1+8c_T^2}+\frac{1}{2}\right)}\right] & \textbf{for dS} \\ 2^{\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}-\frac{3}{2}}\dfrac{iH}{M_p\,(1+\epsilon)}\dfrac{1}{(kc_T)^{\frac{3}{2}}}(-kc_T\eta)^{\frac{3}{2}-\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}} & \\ \left|\dfrac{\Gamma\left(\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}\right)}{\Gamma\left(\frac{3}{2}\right)}\right|\left[D_1 e^{-ikc_T\eta}\,(1+ikc_T\eta)\,e^{-\frac{i\pi}{2}\left(\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}+\frac{1}{2}\right)}\right. & \\ \qquad\left. - D_2 e^{ikc_T\eta}\,(1-ikc_T\eta)\,e^{\frac{i\pi}{2}\left(\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}+\frac{1}{2}\right)}\right] & \textbf{for qdS}. \end{cases} \tag{101}$$

3.2.2. Primordial Power Spectrum for Tensor Perturbation

One can further compute the two-point function for tensor fluctuation as:

$$\langle h(\eta,\mathbf{k})h(\eta,\mathbf{q})\rangle = \sum_{\lambda,\lambda'}\langle h_\lambda(\eta,\mathbf{k})h_{\lambda'}(\eta,\mathbf{q})\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{q})P_h(k,\eta)\,, \tag{102}$$

where $P_h(k,\eta)$ is the power spectrum at time η for tensor fluctuations and in the present context it is defined as:

$$P_h(k,\eta) = \frac{4|h_\lambda(\eta,\mathbf{k})|^2}{a^2 M_p^2} = \begin{cases} 2^{\sqrt{1+8c_T^2}-3}\frac{4H^2}{M_p^2}\frac{1}{(kc_T)^3}(-kc_T\eta)^{3-\sqrt{1+8c_T^2}}\left|\frac{\Gamma\left(\frac{1}{2}\sqrt{1+8c_T^2}\right)}{\Gamma\left(\frac{3}{2}\right)}\right|^2 \\ \left|D_1 e^{-ikc_T\eta}(1+ikc_T\eta)\,e^{-\frac{i\pi}{2}\left(\frac{1}{2}\sqrt{1+8c_T^2}+\frac{1}{2}\right)} - D_2 e^{ikc_T\eta}(1-ikc_T\eta)\,e^{\frac{i\pi}{2}\left(\frac{1}{2}\sqrt{1+8c_T^2}+\frac{1}{2}\right)}\right|^2 & \textbf{for dS} \\ 2^{\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}-3}\frac{4H^2}{M_p^2(1+\epsilon)^2}\frac{1}{(kc_T)^3}(-kc_T\eta)^{3-\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}} \\ \left|\frac{\Gamma\left(\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}\right)}{\Gamma\left(\frac{3}{2}\right)}\right|^2 \left|D_1 e^{-ikc_T\eta}(1+ikc_T\eta)\,e^{-\frac{i\pi}{2}\left(\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}+\frac{1}{2}\right)}\right. \\ \left. -\, D_2 e^{ikc_T\eta}(1-ikc_T\eta)\,e^{\frac{i\pi}{2}\left(\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}+\frac{1}{2}\right)}\right|^2 & \textbf{for qdS}. \end{cases} \tag{103}$$

Finally, at the horizon crossing we get the following two-point correlation function for tensor perturbation as:

$$\langle h(\mathbf{k})h(\mathbf{q})\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{q})P_h(k)\,, \tag{104}$$

where $P_h(k)$ is known as the power spectrum at the horizon crossing for tensor fluctuations and in the present context it is defined as:

$$P_h(k) = P_h(k_*)\frac{1}{k^3} = \begin{cases} 2^{\sqrt{1+8c_T^2}-3}\frac{4H^2}{M_p^2 c_T^3}\frac{1}{k^3}\left|\frac{\Gamma\left(\frac{1}{2}\sqrt{1+8c_T^2}\right)}{\Gamma\left(\frac{3}{2}\right)}\right|^2\left[|D_1|^2+|D_2|^2\right. \\ \left.-\left(D_1^* D_2 e^{i\pi\left(\frac{1}{2}\sqrt{1+8c_T^2}+\frac{1}{2}\right)} + D_1 D_2^* e^{-i\pi\left(\frac{1}{2}\sqrt{1+8c_T^2}+\frac{1}{2}\right)}\right)\right] & \textbf{for dS} \\ 2^{\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}-3}\frac{4H^2}{M_p^2(1+\epsilon)^2 c_T^3}\frac{1}{k^3}\left|\frac{\Gamma\left(\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}\right)}{\Gamma\left(\frac{3}{2}\right)}\right|^2\left[|D_1|^2+|D_2|^2\right. \\ \left.-\left(D_1^* D_2 e^{i\pi\left(\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}+\frac{1}{2}\right)} + D_1 D_2^* e^{-i\pi\left(\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}+\frac{1}{2}\right)}\right)\right] & \textbf{for qdS}. \end{cases} \tag{105}$$

where $P_h(k_*)$ is power spectrum for tensor fluctuation at the pivot scale $k = k_*$. For simplicity one can keep $k^3/2\pi^2$ dependence outside and further define amplitude of the power spectrum $\Delta_h(k_*)$ at the pivot scale $k = k_*$ as:

$$\begin{aligned} \Delta_h(k_*) &= \frac{k^3}{2\pi^2}P_h(k) = \frac{1}{2\pi^2}P_h(k_*) \\ &= \begin{cases} 2^{\sqrt{1+8c_T^2}-3}\frac{2H^2}{\pi^2 M_p^2 c_T^3}\left|\frac{\Gamma\left(\frac{1}{2}\sqrt{1+8c_T^2}\right)}{\Gamma\left(\frac{3}{2}\right)}\right|^2\left[|D_1|^2+|D_2|^2\right. \\ \left.-\left(C_1^* C_2 e^{i\pi\left(\frac{1}{2}\sqrt{1+8c_T^2}+\frac{1}{2}\right)} + D_1 D_2^* e^{-i\pi\left(\frac{1}{2}\sqrt{1+8c_T^2}+\frac{1}{2}\right)}\right)\right] & \textbf{for dS} \\ 2^{\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}-3}\frac{2H^2}{\pi^2 M_p^2(1+\epsilon)^2 c_T^3}\left|\frac{\Gamma\left(\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}\right)}{\Gamma\left(\frac{3}{2}\right)}\right|^2\left[|D_1|^2+|D_2|^2\right. \\ \left.-\left(D_1^* D_2 e^{i\pi\left(\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}+\frac{1}{2}\right)} + D_1 D_2^* e^{-i\pi\left(\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}+\frac{1}{2}\right)}\right)\right] & \textbf{for qdS}. \end{cases} \end{aligned} \tag{106}$$

For **Bunch–Davies** and $\boldsymbol{\alpha}, \boldsymbol{\beta}$ vacuums we get:

- **For Bunch–Davies vacuum** :
  In this case, by setting $D_1 = 1$ and $D_2 = 0$ we get the following expression for the power spectrum:

$$P_h(k) = \begin{cases} 2^{\sqrt{1+8c_T^2}-3}\frac{4H^2}{M_p^2 c_T^3}\frac{1}{k^3}\left|\frac{\Gamma\left(\frac{1}{2}\sqrt{1+8c_T^2}\right)}{\Gamma\left(\frac{3}{2}\right)}\right|^2 & \textbf{for dS} \\ 2^{\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}-3}\frac{4H^2}{M_p^2(1+\epsilon)^2 c_T^3}\frac{1}{k^3}\left|\frac{\Gamma\left(\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}\right)}{\Gamma\left(\frac{3}{2}\right)}\right|^2 & \textbf{for qdS}. \end{cases} \tag{107}$$

Also, the power spectrum $\Delta_\zeta(k_*)$ at the pivot scale $k = k_*$ as:

$$\Delta_h(k_*) = \begin{cases} 2^{\sqrt{1+8c_T^2}-3}\frac{2H^2}{\pi^2 M_p^2 c_T^3}\left|\frac{\Gamma\left(\frac{1}{2}\sqrt{1+8c_T^2}\right)}{\Gamma\left(\frac{3}{2}\right)}\right|^2 & \textbf{for dS} \\ 2^{\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}-3}\frac{2H^2}{\pi^2 M_p^2(1+\epsilon)^2 c_T^3}\left|\frac{\Gamma\left(\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}\right)}{\Gamma\left(\frac{3}{2}\right)}\right|^2 & \textbf{for qdS}. \end{cases} \tag{108}$$

- **For α, β vacuum** :
  In this case, by setting $D_1 = \cosh\alpha$ and $D_2 = e^{i\beta}\sinh\alpha$ we get the following expression for the power spectrum:

$$P_h(k) = \begin{cases} 2^{\sqrt{1+8c_T^2}-3}\frac{4H^2}{M_p^2 c_T^3}\frac{1}{k^3}\left|\frac{\Gamma\left(\frac{1}{2}\sqrt{1+8c_T^2}\right)}{\Gamma\left(\frac{3}{2}\right)}\right|^2 \\ \left[\cosh 2\alpha - \sinh 2\alpha\cos\left(\pi\left(\frac{1}{2}\sqrt{1+8c_T^2}+\frac{1}{2}\right)+\beta\right)\right] & \textbf{for dS} \\ 2^{\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}-3}\frac{4H^2}{M_p^2(1+\epsilon)^2 c_T^3}\frac{1}{k^3}\left|\frac{\Gamma\left(\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}\right)}{\Gamma\left(\frac{3}{2}\right)}\right|^2 \\ \left[\cosh 2\alpha - \sinh 2\alpha\cos\left(\pi\left(\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}+\frac{1}{2}\right)+\beta\right)\right] & \textbf{for qdS}. \end{cases} \tag{109}$$

  Also, the power spectrum $\Delta_\zeta(k_*)$ at the pivot scale $k = k_*$ as:

$$\Delta_h(k_*) = \begin{cases} 2^{\sqrt{1+8c_T^2}-3}\frac{2H^2}{\pi^2 M_p^2 c_T^3}\left|\frac{\Gamma\left(\frac{1}{2}\sqrt{1+8c_T^2}\right)}{\Gamma\left(\frac{3}{2}\right)}\right|^2 \\ \left[\cosh 2\alpha - \sinh 2\alpha\cos\left(\pi\left(\frac{1}{2}\sqrt{1+8c_T^2}+\frac{1}{2}\right)+\beta\right)\right] & \textbf{for dS} \\ 2^{\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}-3}\frac{2H^2}{\pi^2 M_p^2(1+\epsilon)^2 c_T^3}\left|\frac{\Gamma\left(\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}\right)}{\Gamma\left(\frac{3}{2}\right)}\right|^2 \\ \left[\cosh 2\alpha - \sinh 2\alpha\cos\left(\pi\left(\frac{1}{2}\sqrt{1+4c_T^2\left(\nu^2-\frac{1}{4}\right)}+\frac{1}{2}\right)+\beta\right)\right] & \textbf{for qdS}. \end{cases} \tag{110}$$

Now let us consider a special case for tensor fluctuation where $c_T = 1$ and it implies the following two possibilities:

1. $\bar{M}_3 = 0$. But for this case as we have assumed earlier $\bar{M}_3^2 \approx \bar{M}_3^2 = \bar{M}_1^3/4H\tilde{c}_5$, then $\bar{M}_1 = 0$ which is not our matter of interest in this work as this leads to zero three-point function for scalar fluctuation. But if we assume that $\bar{M}_3^2 \neq \bar{M}_3^2$ but $\bar{M}_3^2 = \bar{M}_1^3/4H\tilde{c}_5$ then by setting $\bar{M}_3 = 0$ one can get $\bar{M}_1 \neq 0$, which is necessarily required for non-vanishing three-point function for scalar fluctuation.

2. $\bar{M}_3 << M_p$. In this case if we assume $\bar{M}_3^2 \approx \bar{M}_3^2 = \bar{M}_1^3/4H\tilde{c}_5$, then $\bar{M}_1^3/4H\tilde{c}_5 M_p^2 << 1$ and $\bar{M}_2 << M_p$. This is perfectly ok of generating non-vanishing three-point function for scalar fluctuation.

If we set $c_T = 1$ then for **Bunch–Davies** and **ff, fi** vacuum power spectrum can be recast into the following simplified form:

- **For Bunch–Davies vacuum** :
  In this case, by setting $D_1 = 1$ and $D_2 = 0$ we get the following expression for the power spectrum:

$$P_h(k) = \begin{cases} \frac{4H^2}{M_p^2}\frac{1}{k^3} & \textbf{for dS} \\ 2^{2\nu-3}\frac{4H^2}{M_p^2(1+\epsilon)^2}\frac{1}{k^3}\left|\frac{\Gamma(\nu)}{\Gamma\left(\frac{3}{2}\right)}\right|^2 & \textbf{for qdS}. \end{cases} \tag{111}$$

  Also, the power spectrum $\Delta_\zeta(k_*)$ at the pivot scale $k = k_*$ as:

$$\Delta_h(k_*) = \begin{cases} \frac{2H^2}{\pi^2 M_p^2} & \textbf{for dS} \\ 2^{\nu-3}\frac{2H^2}{\pi^2 M_p^2(1+\epsilon)^2}\left|\frac{\Gamma(\nu)}{\Gamma\left(\frac{3}{2}\right)}\right|^2 & \textbf{for qdS}. \end{cases} \tag{112}$$

- **For α, β vacuum** :
  In this case, by setting $D_1 = \cosh\alpha$ and $D_2 = e^{i\beta}\sinh\alpha$ we get the following expression for the power spectrum:

$$P_h(k) = \begin{cases} \frac{4H^2}{M_p^2}\frac{1}{k^3}\left[\cosh 2\alpha - \sinh 2\alpha\cos\beta\right] & \textbf{for dS} \\ 2^{2\nu-3}\frac{4H^2}{M_p^2(1+\epsilon)^2}\frac{1}{k^3}\left|\frac{\Gamma(\nu)}{\Gamma\left(\frac{3}{2}\right)}\right|^2\left[\cosh 2\alpha - \sinh 2\alpha\cos\left(\pi\left(\nu+\frac{1}{2}\right)+\beta\right)\right] & \textbf{for qdS}. \end{cases} \tag{113}$$

  Also, the power spectrum $\Delta_\zeta(k_*)$ at the pivot scale $k = k_*$ as:

$$\Delta_h(k_*) = \begin{cases} \frac{2H^2}{\pi^2 M_p^2}\left[\cosh 2\alpha - \sinh 2\alpha\cos\beta\right] & \textbf{for dS} \\ 2^{2\nu-3}\frac{2H^2}{\pi^2 M_p^2(1+\epsilon)^2}\left|\frac{\Gamma(\nu)}{\Gamma\left(\frac{3}{2}\right)}\right|^2 & \\ \left[\cosh 2\alpha - \sinh 2\alpha\cos\left(\pi\left(\nu+\frac{1}{2}\right)+\beta\right)\right] & \textbf{for qdS}. \end{cases} \tag{114}$$

4. Scalar Three-Point Correlation Function from EFT

4.1. Basic Setup

Here we compute the three-point correlation function for perturbations from scalar modes. For this purpose we consider the third-order perturbed action for the scalar modes as given by[12]:

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[12] Here it is important to note that the $\underbrace{\quad}$ terms are the new contribution in the EFT action considered in this paper, which are not present in ref. [6]. From the EFT action itself it is clear that for effective sound speed $c_S = 1$ three-point correlation function and the associated bispectrum vanishes if we do not contribution these **red** colored terms. This is obviously true if we fix $c_S = 1$ in the result obtained in ref. [6]. On the other hand, if we consider these **red** colored terms then the result is consistent with ref. [31] with $c_S = 1$ and with ref. [44] with $c_S \neq 1$. This implies that $c_S = 1$ is not fully radiatively stable in single-field slow-roll inflation. However, if we include the effects produced by quantum correction through loop effects, then a small deviation in the effective sound speed $1 - c_S \sim \epsilon\left(H/M_p\right)^2$ can be produced. See ref. [6] where this fact is clearly pointed. But for inflation we know that in the inflationary regime the slow-roll parameter $\epsilon < 1$ and the scale of inflation is $H/M_p << 1$, which imply this deviation is also very small and not very interesting for our purpose studied in this paper. Also see ref. [8] for more details.

$$
\begin{aligned}
S_{\zeta}^{(3)} \approx \int d^4x\, \frac{a^3}{H^3} \Bigg[& -\left\{\left(1-\frac{1}{c_S^2}\right)\dot{H}M_p^2+\frac{3}{2}\bar{M}_1^3 H-\frac{4}{3}M_3^4\right\}\dot{\zeta}^3 \\
& +\left\{\left(1-\frac{1}{c_S^2}\right)\dot{H}M_p^2+\frac{3}{2}\bar{M}_1^3 H\right\}\frac{1}{a^2}\dot{\zeta}(\partial_i\zeta)^2 \\
& \underbrace{+\frac{9}{2}\bar{M}_1^3 H^2\zeta\dot{\zeta}^2-\frac{3}{2}\bar{M}_1^3 H\frac{1}{a^2}\zeta\frac{d}{dt}(\partial_i\zeta)^2}_{\textbf{New contributions in EFT action}}\Bigg],
\end{aligned}
\tag{115}
$$

which can be recast for $c_S = 1$ and $c_S < 1$ case as:

$\mathbf{For}\ \mathbf{c_S = 1}:$

$$
\begin{aligned}
S_{\zeta}^{(3)} \approx \int d^4x\, \frac{a^3}{H^3} \Bigg[& -\left\{\frac{3}{2}\bar{M}_1^3 H\right\}\dot{\zeta}^3+\left\{\frac{3}{2}\bar{M}_1^3 H\right\}\frac{1}{a^2}\dot{\zeta}(\partial_i\zeta)^2 \\
& +\frac{9}{2}\bar{M}_1^3 H^2\zeta\dot{\zeta}^2-\frac{3}{2}\bar{M}_1^3 H\frac{1}{a^2}\zeta\frac{d}{dt}(\partial_i\zeta)^2\Bigg],
\end{aligned}
\tag{116}
$$

$\mathbf{For}\ \mathbf{c_S < 1}:$

$$
\begin{aligned}
S_{\zeta}^{(3)} \approx \int d^4x\, a^3\, \frac{\epsilon M_p^2}{H}\left(1-\frac{1}{c_S^2}\right) \Bigg[& \left\{1+\frac{3\tilde{c}_4}{4c_S^2}+\frac{2\tilde{c}_3}{3c_S^2}\right\}\dot{\zeta}^3-\left\{1+\frac{3\tilde{c}_4}{4c_S^2}\right\}\frac{1}{a^2}\dot{\zeta}(\partial_i\zeta)^2 \\
& -\frac{9H\tilde{c}_4}{4c_S^2}\zeta\dot{\zeta}^2+\frac{3\tilde{c}_4}{4c_S^2}\frac{1}{a^2}\zeta\frac{d}{dt}(\partial_i\zeta)^2\Bigg],
\end{aligned}
\tag{117}
$$

To extract further information from third-order action, first one needs to start with the Fourier transform of the curvature perturbation $\zeta(\eta,\mathbf{x})$ defined as:

$$
\zeta(\eta,\mathbf{x}) = \int \frac{d^3k}{(2\pi)^3}\zeta_{\mathbf{k}}(\eta)\exp(i\mathbf{k}.\mathbf{x}),
\tag{118}
$$

where $\zeta_{\mathbf{k}}(\eta)$ is the time-dependent part of the curvature fluctuation after Fourier transform and can be expressed in terms of the normalized time-dependent scalar mode function $v_{\mathbf{k}}(\eta)$ as:

$$
\hat{\zeta}(\eta,\mathbf{k}) = \frac{v_{\mathbf{k}}(\eta)}{zM_p} = \frac{\zeta\left(\eta,\mathbf{k}\right)a\left(\mathbf{k}\right)+\zeta^*\left(\eta,-\mathbf{k}\right)a^{\dagger}\left(-\mathbf{k}\right)}{zM_p}
\tag{119}
$$

where z is explicitly defined earlier and $a(\mathbf{k}), a^{\dagger}(\mathbf{k})$ are the creation and annihilation operator satisfies the following commutation relations:

$$
\left[a(\mathbf{k}),a^{\dagger}(-\mathbf{k}')\right] = (2\pi)^3\delta^3(\mathbf{k}+\mathbf{k}'),\quad \left[a(\mathbf{k}),a(\mathbf{k}')\right]=0,\quad \left[a^{\dagger}(\mathbf{k}),a^{\dagger}(\mathbf{k}')\right]=0.
\tag{120}
$$

4.2. Computation of Scalar Three-Point Function in Interaction Picture

Presently our prime objective is to compute the three-point function of the curvature fluctuation in momentum space from S_{ζ}^2 with respect to the arbitrary choice of vacuum, which leads to important result in the context of primordial cosmology. Furthermore, using the interaction picture the three-point function of the curvature fluctuation in momentum space can be expressed as:

$$
\langle\zeta(\mathbf{k_1})\zeta(\mathbf{k_2})\zeta(\mathbf{k_3})\rangle = -i\int_{\eta_i=-\infty}^{\eta_f=0} d\eta\; a(\eta)\;\langle 0|\left[\hat{\zeta}(\eta_f,\mathbf{k_1})\hat{\zeta}(\eta_f,\mathbf{k_2})\hat{\zeta}(\eta_f,\mathbf{k_3}),H_{int}(\eta)\right]|0\rangle\,,
\tag{121}
$$

where $a(\eta)$ is the scale factor defined in the earlier section in terms of Hubble parameter H and conformal time scale η. Here $|0\rangle$ represents any arbitrary vacuum state and for discussion we will only derive the results for Bunch–Davies vacuum and α, β vacuum. In the interaction picture the Hamiltonian can written as[13]:

$$\begin{aligned} H_{int}(\eta) &= -\frac{1}{H^3}\left[-\left\{\left(1-\frac{1}{c_S^2}\right)\dot{H}M_p^2 + \frac{3}{2}\bar{M}_1^3 H - \frac{4}{3}M_3^4\right\}\dot{\zeta}^3 \right. \\ &\quad + \left\{\left(1-\frac{1}{c_S^2}\right)\dot{H}M_p^2 + \frac{3}{2}\bar{M}_1^3 H\right\}\frac{1}{a^2}\dot{\zeta}(\partial_i\zeta)^2 \\ &\quad \left. + \frac{9}{2}\bar{M}_1^3 H^2 \zeta\dot{\zeta}^2 - \frac{3}{2}\bar{M}_1^3 H \frac{1}{a^2}\zeta\frac{d}{dt}\left(\partial_i\zeta\right)^2\right]. \end{aligned} \tag{122}$$

which gives the primary information to compute the explicit expression for the three-point function in the present context. After substituting the Hamiltonian interaction, we finally get the following expression for the three-point function for the scalar fluctuation:

$$\begin{aligned} \langle\zeta(\mathbf{k_1})\zeta(\mathbf{k_2})\zeta(\mathbf{k_3})\rangle &= -i\int_{\eta_i=-\infty}^{\eta_f=0} d\eta\, a(\eta)\int\frac{d^3x}{H^3}\int\int\int\frac{d^3k_4}{(2\pi)^3}\frac{d^3k_5}{(2\pi)^3}\frac{d^3k_6}{(2\pi)^3}\, e^{i(\mathbf{k_4}+\mathbf{k_5}+\mathbf{k_6}).\mathbf{x}} \\ &\quad \left\{\alpha_1\langle 0|\left[\hat{\zeta}(\eta_f,\mathbf{k_1})\hat{\zeta}(\eta_f,\mathbf{k_2})\hat{\zeta}(\eta_f,\mathbf{k_3}),\hat{\zeta}'(\eta,\mathbf{k_4})\hat{\zeta}'(\eta,\mathbf{k_5})\hat{\zeta}'(\eta,\mathbf{k_6})\right]|0\rangle\right. \\ &\quad -\alpha_2(\mathbf{k_5}.\mathbf{k_6})\langle 0|\left[\hat{\zeta}(\eta_f,\mathbf{k_1})\hat{\zeta}(\eta_f,\mathbf{k_2})\hat{\zeta}(\eta_f,\mathbf{k_3}),\hat{\zeta}'(\eta,\mathbf{k_4})\hat{\zeta}(\eta,\mathbf{k_5})\hat{\zeta}(\eta,\mathbf{k_6})\right]|0\rangle \\ &\quad +\alpha_3\, a(\eta)\,\langle 0|\left[\hat{\zeta}(\eta_f,\mathbf{k_1})\hat{\zeta}(\eta_f,\mathbf{k_2})\hat{\zeta}(\eta_f,\mathbf{k_3}),\hat{\zeta}(\eta,\mathbf{k_4})\hat{\zeta}'(\eta,\mathbf{k_5})\hat{\zeta}'(\eta,\mathbf{k_6})\right]|0\rangle \\ &\quad -\alpha_4(\mathbf{k_5}.\mathbf{k_6})\langle 0|\left[\hat{\zeta}(\eta_f,\mathbf{k_1})\hat{\zeta}(\eta_f,\mathbf{k_2})\hat{\zeta}(\eta_f,\mathbf{k_3}),\hat{\zeta}(\eta,\mathbf{k_4})\hat{\zeta}'(\eta,\mathbf{k_5})\hat{\zeta}(\eta,\mathbf{k_6})\right]|0\rangle \\ &\quad \left.-\alpha_5(\mathbf{k_5}.\mathbf{k_6})\langle 0|\left[\hat{\zeta}(\eta_f,\mathbf{k_1})\hat{\zeta}(\eta_f,\mathbf{k_2})\hat{\zeta}(\eta_f,\mathbf{k_3}),\hat{\zeta}(\eta,\mathbf{k_4})\hat{\zeta}(\eta,\mathbf{k_5})\hat{\zeta}'(\eta,\mathbf{k_6})\right]|0\rangle\right\}, \end{aligned} \tag{123}$$

where the coefficients $\alpha_j \forall j = 1,2,3,4,5$ are defined as[14]:

$$\alpha_1 = \left\{\left(1-\frac{1}{c_S^2}\right)\dot{H}M_p^2 + \frac{3}{2}\bar{M}_1^3 H - \frac{4}{3}M_3^4\right\}, \tag{124}$$

$$\alpha_2 = -\left\{\left(1-\frac{1}{c_S^2}\right)\dot{H}M_p^2 + \frac{3}{2}\bar{M}_1^3 H\right\}, \tag{125}$$

$$\alpha_3 = -\frac{9}{2}\bar{M}_1^3 H^2, \tag{126}$$

$$\alpha_4 = \frac{3}{2}\bar{M}_1^3 H, \tag{127}$$

$$\alpha_5 = \frac{3}{2}\bar{M}_1^3 H. \tag{128}$$

Now let us evaluate the coefficients of $\alpha_1, \alpha_2, \alpha_3, \alpha_4$ in the present context using Wick's theorem:

---

13 See also ref. [31,44], where similar computations have been performed for canonical single-field slow-roll and generalized slow-roll models of inflation in the presence of Bunch–Davies vacuum state and general initial state.

14 Here it is clearly observed that for canonical single-field slow-roll model, which is described by $c_S = 1$ we have $M_3 = 0$ and other EFT coefficients are sufficiently small, $\bar{M}_i \forall i = 1,2,3 (\sim \mathcal{O}()10^{-3} - 10^{-2})$. This directly implies that the contribution in the three-point function and in the associated bispectrum is very small and consistent with the previous result as obtained in ref. [31]. Additionally, it is important to mention that in momentum space the bispectrum contains additional terms in the presence of any arbitrary choice of the quantum vacuum initial state. Also, if we compare with ref. [6].

1. **Coefficient of** α_1:

$$\begin{aligned}
&\langle 0|\left[\hat{\zeta}(\eta_f,\mathbf{k_1})\hat{\zeta}(\eta_f,\mathbf{k_2})\hat{\zeta}(\eta_f,\mathbf{k_3}),\hat{\zeta}'(\eta,\mathbf{k_4})\hat{\zeta}'(\eta,\mathbf{k_5})\hat{\zeta}'(\eta,\mathbf{k_6})\right]|0\rangle \\
&= \langle 0|a(\mathbf{k_1})a(\mathbf{k_2})a(\mathbf{k_3})a^\dagger(-\mathbf{k_4})a^\dagger(-\mathbf{k_5})a^\dagger(-\mathbf{k_6})|0\rangle \\
&\bar{v}(\eta_f,\mathbf{k_1})\bar{v}(\eta_f,\mathbf{k_2})\bar{v}(\eta_f,\mathbf{k_3})\bar{v}^{'*}(\eta_f,-\mathbf{k_4})\bar{v}^{'*}(\eta_f,-\mathbf{k_5})\bar{v}^{'*}(\eta_f,-\mathbf{k_6}) \\
&+\langle 0|a(\mathbf{k_4})a(\mathbf{k_5})a(\mathbf{k_6})a^\dagger(-\mathbf{k_1})a^\dagger(-\mathbf{k_2})a^\dagger(-\mathbf{k_3})|0\rangle \\
&\bar{v}^*(\eta_f,-\mathbf{k_1})\bar{v}^*(\eta_f,-\mathbf{k_2})\bar{v}^*(\eta_f,-\mathbf{k_3})\bar{v}'(\eta,\mathbf{k_4})\bar{v}'(\eta,\mathbf{k_5})\bar{v}'(\eta,\mathbf{k_6}).
\end{aligned} \tag{129}$$

2. **Coefficient of** α_2:

$$\begin{aligned}
&(\mathbf{k_5}.\mathbf{k_6})\langle 0|\left[\hat{\zeta}(\eta_f,\mathbf{k_1})\hat{\zeta}(\eta_f,\mathbf{k_2})\hat{\zeta}(\eta_f,\mathbf{k_3}),\hat{\zeta}'(\eta,\mathbf{k_4})\hat{\zeta}(\eta,\mathbf{k_5})\hat{\zeta}(\eta,\mathbf{k_6})\right]|0\rangle \\
= \quad &(\mathbf{k_5}.\mathbf{k_6})\langle 0|a(\mathbf{k_1})a(\mathbf{k_2})a(\mathbf{k_3})a^\dagger(-\mathbf{k_4})a^\dagger(-\mathbf{k_5})a^\dagger(-\mathbf{k_6})|0\rangle \\
&\bar{v}(\eta_f,\mathbf{k_1})\bar{v}(\eta_f,\mathbf{k_2})\bar{v}(\eta_f,\mathbf{k_3})\bar{v}^{'*}(\eta_f,-\mathbf{k_4})\bar{v}^*(\eta_f,-\mathbf{k_5})\bar{v}^*(\eta_f,-\mathbf{k_6}) \\
&+(\mathbf{k_5}.\mathbf{k_6})\langle 0|a(\mathbf{k_4})a(\mathbf{k_5})a(\mathbf{k_6})a^\dagger(-\mathbf{k_1})a^\dagger(-\mathbf{k_2})a^\dagger(-\mathbf{k_3})|0\rangle \\
&\bar{v}^*(\eta_f,-\mathbf{k_1})\bar{v}^*(\eta_f,-\mathbf{k_2})\bar{v}^*(\eta_f,-\mathbf{k_3})\bar{v}'(\eta_f,\mathbf{k_4})\bar{v}(\eta_f,\mathbf{k_5})\bar{v}(\eta_f,\mathbf{k_6}).
\end{aligned} \tag{130}$$

3. **Coefficient of** α_3:

$$\begin{aligned}
&\langle 0|\left[\hat{\zeta}(\eta_f,\mathbf{k_1})\hat{\zeta}(\eta_f,\mathbf{k_2})\hat{\zeta}(\eta_f,\mathbf{k_3}),\hat{\zeta}(\eta,\mathbf{k_4})\hat{\zeta}'(\eta,\mathbf{k_5})\hat{\zeta}'(\eta,\mathbf{k_6})\right]|0\rangle \\
= \quad &\langle 0|a(\mathbf{k_1})a(\mathbf{k_2})a(\mathbf{k_3})a^\dagger(-\mathbf{k_4})a^\dagger(-\mathbf{k_5})a^\dagger(-\mathbf{k_6})|0\rangle \\
&\bar{v}(\eta_f,\mathbf{k_1})\bar{v}(\eta_f,\mathbf{k_2})\bar{v}(\eta_f,\mathbf{k_3})\bar{v}^*(\eta_f,-\mathbf{k_4})\bar{v}^{'*}(\eta_f,-\mathbf{k_5})\bar{v}^{'*}(\eta_f,-\mathbf{k_6}) \\
&+\langle 0|a(\mathbf{k_4})a(\mathbf{k_5})a(\mathbf{k_6})a^\dagger(-\mathbf{k_1})a^\dagger(-\mathbf{k_2})a^\dagger(-\mathbf{k_3})|0\rangle \\
&\bar{v}^*(\eta_f,-\mathbf{k_1})\bar{v}^*(\eta_f,-\mathbf{k_2})\bar{v}^*(\eta_f,-\mathbf{k_3})\bar{v}(\eta_f,\mathbf{k_4})\bar{v}'(\eta_f,\mathbf{k_5})\bar{v}'(\eta_f,\mathbf{k_6}).
\end{aligned} \tag{131}$$

4. **Coefficient of** α_4:

$$\begin{aligned}
&(\mathbf{k_5}.\mathbf{k_6})\langle 0|\left[\hat{\zeta}(\eta_f,\mathbf{k_1})\hat{\zeta}(\eta_f,\mathbf{k_2})\hat{\zeta}(\eta_f,\mathbf{k_3}),\hat{\zeta}(\eta,\mathbf{k_4})\hat{\zeta}'(\eta,\mathbf{k_5})\hat{\zeta}(\eta,\mathbf{k_6})\right]|0\rangle \\
= \quad &(\mathbf{k_5}.\mathbf{k_6})\langle 0|a(\mathbf{k_1})a(\mathbf{k_2})a(\mathbf{k_3})a^\dagger(-\mathbf{k_4})a^\dagger(-\mathbf{k_5})a^\dagger(-\mathbf{k_6})|0\rangle \\
&\bar{v}(\eta_f,\mathbf{k_1})\bar{v}(\eta_f,\mathbf{k_2})\bar{v}(\eta_f,\mathbf{k_3})\bar{v}^*(\eta_f,-\mathbf{k_4})\bar{v}^{'*}(\eta_f,-\mathbf{k_5})\bar{v}^*(\eta_f,-\mathbf{k_6}) \\
&+(\mathbf{k_5}.\mathbf{k_6})\langle 0|a(\mathbf{k_4})a(\mathbf{k_5})a(\mathbf{k_6})a^\dagger(-\mathbf{k_1})a^\dagger(-\mathbf{k_2})a^\dagger(-\mathbf{k_3})|0\rangle \\
&\bar{v}^*(\eta_f,-\mathbf{k_1})\bar{v}^*(\eta_f,-\mathbf{k_2})\bar{v}^*(\eta_f,-\mathbf{k_3})\bar{v}(\eta_f,\mathbf{k_4})\bar{v}'(\eta_f,\mathbf{k_5})\bar{v}(\eta_f,\mathbf{k_6}).
\end{aligned} \tag{132}$$

5. **Coefficient of** α_5:

$$\begin{aligned}
&(\mathbf{k_5}.\mathbf{k_6})\langle 0|\left[\hat{\zeta}(\eta_f,\mathbf{k_1})\hat{\zeta}(\eta_f,\mathbf{k_2})\hat{\zeta}(\eta_f,\mathbf{k_3}),\hat{\zeta}(\eta,\mathbf{k_4})\hat{\zeta}(\eta,\mathbf{k_5})\hat{\zeta}'(\eta,\mathbf{k_6})\right]|0\rangle \\
= \quad &\langle 0|a(\mathbf{k_1})a(\mathbf{k_2})a(\mathbf{k_3})a^\dagger(-\mathbf{k_4})a^\dagger(-\mathbf{k_5})a^\dagger(-\mathbf{k_6})|0\rangle \\
&\bar{v}(\eta_f,\mathbf{k_1})\bar{v}(\eta_f,\mathbf{k_2})\bar{v}(\eta_f,\mathbf{k_3})\bar{v}^*(\eta_f,-\mathbf{k_4})\bar{v}^*(\eta_f,-\mathbf{k_5})\bar{v}^{'*}(\eta_f,-\mathbf{k_6}) \\
&+\langle 0|a(\mathbf{k_4})a(\mathbf{k_5})a(\mathbf{k_6})a^\dagger(-\mathbf{k_1})a^\dagger(-\mathbf{k_2})a^\dagger(-\mathbf{k_3})|0\rangle \\
&\bar{v}^*(\eta_f,-\mathbf{k_1})\bar{v}^*(\eta_f,-\mathbf{k_2})\bar{v}^*(\eta_f,-\mathbf{k_3})\bar{v}(\eta_f,\mathbf{k_4})\bar{v}(\eta_f,\mathbf{k_5})\bar{v}'(\eta_f,\mathbf{k_6}).
\end{aligned} \tag{133}$$

where we define $\bar{v}$ as:

$$\bar{v}(\eta,\mathbf{k}) = \frac{v(\eta,\mathbf{k})}{zM_p}. \tag{134}$$

Furthermore, we also use the following result in simplify the coefficients of $\alpha_1, \alpha_2, \alpha_3, \alpha_4$:

$$\begin{aligned}
& \langle 0|a(\mathbf{k_1})a(\mathbf{k_2})a(\mathbf{k_3})a^\dagger(-\mathbf{k_4})a^\dagger(-\mathbf{k_5})a^\dagger(-\mathbf{k_6})|0\rangle \\
= & \langle 0|a(\mathbf{k_4})a(\mathbf{k_5})a(\mathbf{k_6})a^\dagger(-\mathbf{k_1})a^\dagger(-\mathbf{k_2})a^\dagger(-\mathbf{k_3})|0\rangle \\
= & (2\pi)^9 \left\{\delta^{(3)}(\mathbf{k_4}+\mathbf{k_1})\left[\delta^{(3)}(\mathbf{k_5}+\mathbf{k_2})\delta^{(3)}(\mathbf{k_6}+\mathbf{k_3}) + \delta^{(3)}(\mathbf{k_5}+\mathbf{k_3})\delta^{(3)}(\mathbf{k_6}+\mathbf{k_2})\right]\right. \\
& +\delta^{(3)}(\mathbf{k_4}+\mathbf{k_2})\left[\delta^{(3)}(\mathbf{k_5}+\mathbf{k_1})\delta^{(3)}(\mathbf{k_6}+\mathbf{k_3}) + \delta^{(3)}(\mathbf{k_5}+\mathbf{k_3})\delta^{(3)}(\mathbf{k_6}+\mathbf{k_1})\right] \\
& \left.+\delta^{(3)}(\mathbf{k_4}+\mathbf{k_3})\left[\delta^{(3)}(\mathbf{k_5}+\mathbf{k_1})\delta^{(3)}(\mathbf{k_6}+\mathbf{k_3}) + \delta^{(3)}(\mathbf{k_5}+\mathbf{k_2})\delta^{(3)}(\mathbf{k_6}+\mathbf{k_1})\right]\right\}. \qquad (135)
\end{aligned}$$

Finally, one can write the following expression for the three-point function for the scalar fluctuation[15]:

$$\langle \zeta(\mathbf{k_1})\zeta(\mathbf{k_2})\zeta(\mathbf{k_3})\rangle = (2\pi)^3\delta^{(3)}(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3})B_{EFT}(k_1,k_2,k_3)\,. \qquad (136)$$

where $B_{EFT}(k_1,k_2,k_3)$ is the bispectrum for scalar fluctuation. In the present computation, one can further write down the expression for the bispectrum as:

$$B_{EFT}(k_1,k_2,k_3) = \sum_{j=1}^{5}\alpha_j\Theta_j(k_1,k_2,k_3)\,, \qquad (137)$$

where $\Theta_j(k_1,k_2,k_3)\forall j = 1,2,3,4,5$ is defined in the next subsections. Here it is important to note that we have derived the expression for the three-point function and the associated bispectrum for effective sound speed $c_S = 1$ and $c_S < 1$ with a choice of general quantum vacuum state.

4.2.1. Function $\Theta_1(k_1,k_2,k_3)$

Here we can write the function $\Theta_1(k_1,k_2,k_3)$ as:

$$\begin{aligned}
\Theta_1(k_1,k_2,k_3) = & \; 6i\int_{\eta_i=-\infty}^{\eta_f=0} d\eta\,\frac{a(\eta)}{H^3}\left[\bar{v}(\eta_f,\mathbf{k_1})\bar{v}(\eta_f,\mathbf{k_2})\bar{v}(\eta_f,\mathbf{k_3})\bar{v}^{'*}(\eta,\mathbf{k_1})\bar{v}^{'*}(\eta,\mathbf{k_2})\bar{v}^{'*}(\eta,\mathbf{k_3})\right. \\
& \left.+\,\bar{v}^{*}(\eta_f,\mathbf{k_1})\bar{v}^{*}(\eta_f,\mathbf{k_2})\bar{v}^{*}(\eta_f,\mathbf{k_3})\bar{v}^{'}(\eta,-\mathbf{k_1})\bar{v}^{'}(\eta,-\mathbf{k_2})\bar{v}^{'}(\eta,-\mathbf{k_3})\right]. \qquad (138)
\end{aligned}$$

Furthermore, using the integrals from the Appendix C we finally get the following simplified expression for the three-point function for the scalar fluctuations[16]:

$$\begin{aligned}
\Theta_1(k_1,k_2,k_3) = & \frac{3H^2}{16\epsilon^3 M_p^6}\frac{1}{k_1k_2k_3}\left[\left\{\frac{1}{K^3}\left[(C_1-C_2)^3\left(C_1^{*3}+C_2^{*3}\right)+(C_1^*-C_2^*)^3\left(C_1^3+C_2^3\right)\right]\right.\right. \\
& \left.\left.+\left[(C_1-C_2)^3C_1^*C_2^*(C_1^*-C_2^*)+(C_1^*-C_2^*)^3C_1C_2(C_1-C_2)\right]\sum_{i=1}^{3}\frac{1}{(2k_i-K)^3}\right\}\right], \qquad (140)
\end{aligned}$$

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[15] See also ref. [8,31,44], where similar computations have been performed for canonical single-field slow-roll and generalized slow-roll models of inflation in the presence of Bunch–Davies vacuum and general initial state.

[16] Here it is important to point out that in de Sitter space if we consider the Bunch–Davies vacuum state then here only the term with $1/K^3$ will appear explicitly in the expression for the three-point function and in the associated bispectrum. On the other hand, if we consider all other non-trivial quantum vacuum states in our computation, then the rest of the contribution will explicitly appear. From the perspective of observation, this is obviously important information as for the non-trivial quantum vacuum state we get additional contribution in the bispectrum which may enhance the amplitude of the non-Gaussianity in squeezed limiting configuration. Additionally, it is important to mention that in quasi de Sitter case we get extra contributions $1/\tilde{c}_S^{6\nu-9}$ and $1/(1+\epsilon)^5$. Also, the factor $1/(k_1k_2k_3)$ will be replaced by $1/(k_1k_2k_3)^{2(\nu-1)}$. Consequently, in quasi de Sitter case this contribution in the bispectrum can be recast as:

$$\begin{aligned}
\Theta_1(k_1,k_2,k_3) = & \frac{3H^2}{16\epsilon^3 M_p^6\tilde{c}_S^{6\nu-9}(1+\epsilon)^5}\frac{1}{(k_1k_2k_3)^{2(\nu-1)}}\left[\left\{\frac{1}{K^3}\left[(C_1-C_2)^3\left(C_1^{*3}+C_2^{*3}\right)+(C_1^*-C_2^*)^3\left(C_1^3+C_2^3\right)\right]\right.\right. \\
& \left.\left.+\left[(C_1-C_2)^3C_1^*C_2^*(C_1^*-C_2^*)+(C_1^*-C_2^*)^3C_1C_2(C_1-C_2)\right]\sum_{i=1}^{3}\frac{1}{(2k_i-K)^3}\right\}\right], \qquad (139)
\end{aligned}$$

Finally, for Bunch–Davies and α, β vacuum we get the following contribution in the three-point function for scalar fluctuations:

- **For Bunch–Davies vacuum:**
  After setting $C_1 = 1$ and $C_2 = 0$ we get:

$$\Theta_1(k_1, k_2, k_3) = \frac{6H^2}{16\epsilon^3 M_p^6} \frac{1}{k_1 k_2 k_3} \frac{1}{K^3}, \tag{141}$$

- **For $\boldsymbol{\alpha, \beta}$ vacuum:**
  After setting $C_1 = \cosh\alpha$ and $C_2 = e^{i\beta}\ \sinh\alpha$ we get:

$$\begin{aligned}\Theta_1(k_1, k_2, k_3) \quad &= \frac{3H^2}{16\epsilon^3 M_p^6} \frac{1}{k_1 k_2 k_3} \left\{ \frac{1}{K^3} \left[\left(\cosh\alpha - e^{i\beta}\sinh\alpha\right)^3 \left(\cosh^3\alpha + e^{-3i\beta}\sinh^3\alpha\right) \right.\right. \\ &\left. + \left(\cosh\alpha - e^{-i\beta}\sinh\alpha\right)^3 \left(\cosh^3\alpha + e^{3i\beta}\sinh^3\alpha\right) \right] \\ &+ \frac{1}{2} \left[\left(\cosh\alpha - e^{i\beta}\sinh\alpha\right)^3 e^{-i\beta}\sinh 2\alpha \left(\cosh\alpha - e^{-i\beta}\sinh\alpha\right) \right. \\ &\left.\left. + \left(\cosh\alpha - e^{-i\beta}\sinh\alpha\right)^3 e^{i\beta}\sinh 2\alpha \left(\cosh\alpha - e^{i\beta}\sinh\alpha\right) \right] \sum_{i=1}^{3} \frac{1}{(2k_i - K)^3} \right\}.\end{aligned} \tag{142}$$

4.2.2. Function $\Theta_2(k_1, k_2, k_3)$

Here we can write the function $\Theta_2(k_1, k_2, k_3)$ as:

$$\begin{aligned}\Theta_2(k_1, k_2, k_3) \quad = \quad & i \int_{\eta_i=-\infty}^{\eta_f=0} d\eta\ \frac{a(\eta)}{H^3} \left\{ 2(\mathbf{k_2}.\mathbf{k_3}) \left[\bar{v}(\eta_f, \mathbf{k_1}) \bar{v}(\eta_f, \mathbf{k_2}) \bar{v}(\eta_f, \mathbf{k_3}) \bar{v}^{'*}(\eta, \mathbf{k_1}) \bar{v}^*(\eta, \mathbf{k_2}) \bar{v}^*(\eta, \mathbf{k_3}) \right.\right. \\ & \left. + \bar{v}^*(\eta_f, -\mathbf{k_1}) \bar{v}^*(\eta_f, -\mathbf{k_2}) \bar{v}^*(\eta_f, -\mathbf{k_3}) \bar{v}^{'}(\eta, -\mathbf{k_1}) \bar{v}(\eta, -\mathbf{k_2}) \bar{v}(\eta, -\mathbf{k_3}) \right] \\ & + 2(\mathbf{k_3}.\mathbf{k_1}) \left[\bar{v}(\eta_f, \mathbf{k_1}) \bar{v}(\eta_f, \mathbf{k_2}) \bar{v}(\eta_f, \mathbf{k_3}) \bar{v}^{'*}(\eta, \mathbf{k_2}) \bar{v}^*(\eta, \mathbf{k_1}) \bar{v}^*(\eta, \mathbf{k_3}) \right. \\ & \left. + \bar{v}^*(\eta_f, -\mathbf{k_1}) \bar{v}^*(\eta_f, -\mathbf{k_2}) \bar{v}^*(\eta_f, -\mathbf{k_3}) \bar{v}^{'}(\eta, -\mathbf{k_2}) \bar{v}(\eta, -\mathbf{k_1}) \bar{v}(\eta, -\mathbf{k_3}) \right] \\ & + 2(\mathbf{k_1}.\mathbf{k_2}) \left[\bar{v}(\eta_f, \mathbf{k_1}) \bar{v}(\eta_f, \mathbf{k_2}) \bar{v}(\eta_f, \mathbf{k_3}) \bar{v}^{'*}(\eta, \mathbf{k_3}) \bar{v}^*(\eta, \mathbf{k_1}) \bar{v}^*(\eta, \mathbf{k_2}) \right. \\ & \left.\left. + \bar{v}^*(\eta_f, -\mathbf{k_1}) \bar{v}^*(\eta_f, -\mathbf{k_2}) \bar{v}^*(\eta_f, -\mathbf{k_3}) \bar{v}^{'}(\eta, -\mathbf{k_3}) \bar{v}(\eta, -\mathbf{k_1}) \bar{v}(\eta, -\mathbf{k_2}) \right] \right\}. \end{aligned} \tag{143}$$

Using the results derived in Appendix C we finally get the following simplified expression for the three-point function for the scalar fluctuations[17]:

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[17] Here it is important to point out that in de Sitter space if we consider the Bunch–Davies vacuum state then here only the term with $1/K^3$ will appear explicitly in the expression for the three-point function and in the associated bispectrum. On the other hand, if we consider all other non-trivial quantum vacuum states in our computation, then the rest of the contribution will explicitly appear. From the perspective of observation, this is obviously important information as for the non-trivial quantum vacuum state we get additional contribution in the bispectrum which may enhance the amplitude of the non-Gaussianity in squeezed limiting configuration. Additionally, it is important to mention that in quasi de Sitter case we get extra contributions $1/\tilde{c}_S^{6\nu-7}$ and $1/(1+\epsilon)^5$. Also, the factor $1/(k_1k_2k_3)^3$ will be replaced by $1/(k_1k_2k_3)^{2\nu}$. Consequently, in quasi de Sitter case this contribution in the bispectrum can be recast as:

$$\begin{aligned}\Theta_2(k_1, k_2, k_3) \quad &= \frac{H^2}{32\epsilon^3 M_p^6 \tilde{c}_S^{6\nu-7}(1+\epsilon)^5} \frac{1}{(k_1k_2k_3)^{2\nu}} \left[k_1^2(\mathbf{k_2}.\mathbf{k_3}) G_1(k_1, k_2, k_3) \right. \\ &\left. + k_2^2(\mathbf{k_1}.\mathbf{k_3}) G_2(k_1, k_2, k_3) + k_3^2(\mathbf{k_1}.\mathbf{k_2}) G_3(k_1, k_2, k_3) \right],\end{aligned} \tag{144}$$

$$\begin{aligned}\Theta_2(k_1,k_2,k_3) &= \frac{H^2}{32\epsilon^3 M_p^6 \tilde{c}_S^2}\frac{1}{(k_1k_2k_3)^3}\Big[k_1^2(\mathbf{k_2}.\mathbf{k_3})G_1(k_1,k_2,k_3) \\ &\quad + k_2^2(\mathbf{k_1}.\mathbf{k_3})G_2(k_1,k_2,k_3) + k_3^2(\mathbf{k_1}.\mathbf{k_2})G_3(k_1,k_2,k_3)\Big], \end{aligned} \tag{145}$$

where the momentum dependent functions $G_1(k_1,k_2,k_3)$, $G_2(k_1,k_2,k_3)$ and $G_3(k_1,k_2,k_3)$ are defined as:

$$\begin{aligned} G_1(k_1,k_2,k_3) &= \frac{1}{K^3}\Big[K^2+2k_2k_3+K(K-k_1)\Big]\Big[(C_1-C_2)^3(C_1^{*3}+C_2^{*3})+(C_1-C_2)^3(C_1^{*3}+C_2^{*3})\Big] \\ &+\Big\{\frac{1}{(2k_1-K)^3}\Big[K^2+2k_2k_3+K(K-5k_1)-2(K-k_1)k_1+4k_1^2\Big] \\ &+\frac{1}{(2k_2-K)^3}\Big[K^2-4k_2k_3+K(k_3-5k_2)+6k_2^2\Big] \\ &+\frac{1}{(2k_1-K)^3}\Big[K^2-4k_2k_3+K(k_2-5k_3)+6k_3^2\Big]\Big\} \\ &\quad\Big[(C_1-C_2)^3C_1^*C_2^*(C_1^*+C_2^*)+(C_1^*-C_2^*)^3C_1C_2(C_1+C_2)\Big]. \end{aligned} \tag{146}$$

$$\begin{aligned} G_2(k_1,k_2,k_3) &= \frac{1}{K^3}\Big[K^2+2k_1k_3+K(K-k_2)\Big]\Big[(C_1-C_2)^3(C_1^{*3}+C_2^{*3})+(C_1-C_2)^3(C_1^{*3}+C_2^{*3})\Big] \\ &+\Big\{\frac{1}{(2k_2-K)^3}\Big[K^2+2k_1k_3+K(K-5k_2)-2(K-k_2)k_2+4k_2^2\Big] \\ &+\frac{1}{(2k_1-K)^3}\Big[K^2-4k_1k_3+K(k_3-5k_1)+6k_1^2\Big] \\ &+\frac{1}{(2k_3-K)^3}\Big[K^2-4k_1k_3+K(k_1-5k_3)+6k_3^2\Big]\Big\} \\ &\quad\Big[(C_1-C_2)^3C_1^*C_2^*(C_1^*+C_2^*)+(C_1^*-C_2^*)^3C_1C_2(C_1+C_2)\Big]. \end{aligned} \tag{147}$$

$$\begin{aligned} G_3(k_1,k_2,k_3) &= \frac{1}{K^3}\Big[K^2+2k_1k_2+K(K-k_3)\Big]\Big[(C_1-C_2)^3(C_1^{*3}+C_2^{*3})+(C_1-C_2)^3(C_1^{*3}+C_2^{*3})\Big] \\ &+\Big\{\frac{1}{(2k_3-K)^3}\Big[K^2+2k_1k_2+K(K-5k_3)-2(K-k_3)k_3+4k_3^2\Big] \\ &+\frac{1}{(2k_2-K)^3}\Big[K^2-4k_1k_2+K(k_1-5k_2)+6k_2^2\Big] \\ &+\frac{1}{(2k_1-K)^3}\Big[K^2-4k_1k_2+K(k_2-5k_1)+6k_1^2\Big]\Big\} \\ &\quad\Big[(C_1-C_2)^3C_1^*C_2^*(C_1^*+C_2^*)+(C_1^*-C_2^*)^3C_1C_2(C_1+C_2)\Big]. \end{aligned} \tag{148}$$

Here $\sum_{i=1}^{3}\mathbf{k_i}=\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3}=0$. Consequently, one can write:

$$\mathbf{k_1}.\mathbf{k_2} = \frac{1}{2}\left(k_3^2-k_2^2-k_1^2\right),\ \mathbf{k_1}.\mathbf{k_3}=\frac{1}{2}\left(k_2^2-k_1^2-k_3^2\right),\ \mathbf{k_2}.\mathbf{k_3}=\frac{1}{2}\left(k_1^2-k_2^2-k_3^2\right), \tag{149}$$

and using these results one can further recast the three-point function for the scalar fluctuation as:

$$\begin{aligned}\Theta_2(k_1,k_2,k_3) &= \frac{H^2}{64\epsilon^3 M_p^6 \tilde{c}_S^2}\frac{1}{(k_1k_2k_3)^3}\Big[k_1^2\left(k_1^2-k_2^2-k_3^2\right)G_1(k_1,k_2,k_3) \\ &\quad + k_2^2\left(k_2^2-k_1^2-k_3^2\right)G_2(k_1,k_2,k_3) + k_3^2\left(k_3^2-k_2^2-k_1^2\right)G_3(k_1,k_2,k_3)\Big]. \end{aligned} \tag{150}$$

Finally, for Bunch–Davies and α,β vacuum we get the following contribution in the three-point function for scalar fluctuations:

- **For Bunch–Davies vacuum:**
After setting $C_1 = 1$ and $C_2 = 0$ we get:

$$G_1(k_1, k_2, k_3) = \frac{2}{K^3}\left[K^2 + 2k_2k_3 + K(K - k_1)\right]. \tag{151}$$
$$G_2(k_1, k_2, k_3) = \frac{2}{K^3}\left[K^2 + 2k_1k_3 + K(K - k_2)\right]. \tag{152}$$
$$G_3(k_1, k_2, k_3) = \frac{2}{K^3}\left[K^2 + 2k_1k_2 + K(K - k_3)\right]. \tag{153}$$

Consequently, we get:

$$\begin{aligned}\Theta_2(k_1, k_2, k_3) = \frac{H^2}{32\epsilon^3 M_p^6 \tilde{c}_S^2}\frac{1}{(k_1k_2k_3)^3}\frac{1}{K^3}\Big[&k_1^2\left(k_1^2 - k_2^2 - k_3^2\right)\left[K^2 + 2k_2k_3 + K(K - k_1)\right] \\ &+ k_2^2\left(k_2^2 - k_1^2 - k_3^2\right)\left[K^2 + 2k_1k_3 + K(K - k_2)\right] \\ &+ k_3^2\left(k_3^2 - k_2^2 - k_1^2\right)\left[K^2 + 2k_1k_2 + K(K - k_3)\right]\Big].\end{aligned} \tag{154}$$

- **For α, β vacuum:**
After setting $C_1 = \cosh\alpha$ and $C_2 = e^{i\beta}\,\sinh\alpha$ we get:

$$\begin{aligned}G_1(k_1, k_2, k_3) = &\frac{1}{K^3}\left[K^2 + 2k_2k_3 + K(K - k_1)\right]J_1(\alpha, \beta) \\ &+ \left\{\frac{1}{(2k_1 - K)^3}\left[K^2 + 2k_2k_3 + K(K - 5k_1) - 2(K - k_1)k_1 + 4k_1^2\right]\right. \\ &+ \frac{1}{(2k_2 - K)^3}\left[K^2 - 4k_2k_3 + K(k_3 - 5k_2) + 6k_2^2\right] \\ &\left.+ \frac{1}{(2k_1 - K)^3}\left[K^2 - 4k_2k_3 + K(k_2 - 5k_3) + 6k_3^2\right]\right\}J_2(\alpha, \beta).\end{aligned} \tag{155}$$

$$\begin{aligned}G_2(k_1, k_2, k_3) = &\frac{1}{K^3}\left[K^2 + 2k_1k_3 + K(K - k_2)\right]J_1(\alpha, \beta) \\ &+ \left\{\frac{1}{(2k_2 - K)^3}\left[K^2 + 2k_1k_3 + K(K - 5k_2) - 2(K - k_2)k_2 + 4k_2^2\right]\right. \\ &+ \frac{1}{(2k_1 - K)^3}\left[K^2 - 4k_1k_3 + K(k_3 - 5k_1) + 6k_1^2\right] \\ &\left.+ \frac{1}{(2k_3 - K)^3}\left[K^2 - 4k_1k_3 + K(k_1 - 5k_3) + 6k_3^2\right]\right\}J_2(\alpha, \beta).\end{aligned} \tag{156}$$

$$\begin{aligned}G_3(k_1, k_2, k_3) = &\frac{1}{K^3}\left[K^2 + 2k_1k_2 + K(K - k_3)\right]J_1(\alpha, \beta) \\ &+ \left\{\frac{1}{(2k_3 - K)^3}\left[K^2 + 2k_1k_2 + K(K - 5k_3) - 2(K - k_3)k_3 + 4k_3^2\right]\right. \\ &+ \frac{1}{(2k_2 - K)^3}\left[K^2 - 4k_1k_2 + K(k_1 - 5k_2) + 6k_2^2\right] \\ &\left.+ \frac{1}{(2k_1 - K)^3}\left[K^2 - 4k_1k_2 + K(k_2 - 5k_1) + 6k_1^2\right]\right\}J_2(\alpha, \beta).\end{aligned} \tag{157}$$

where $J_1(\alpha, \beta)$ and $J_2(\alpha, \beta)$ are defined as:

$$\begin{aligned}J_1(\alpha, \beta) = &\Big[\left(\cosh\alpha - e^{i\beta}\sinh\alpha\right)^3\left(\cosh^3\alpha + e^{-3i\beta}\sinh^3\alpha\right) \\ &+ \left(\cosh\alpha - e^{-i\beta}\sinh\alpha\right)^3\left(\cosh^3\alpha + e^{3i\beta}\sinh^3\alpha\right)\Big],\end{aligned} \tag{158}$$

$$J_2(\alpha,\beta) = \frac{1}{2}\left[\left(\cosh\alpha - e^{i\beta}\sinh\alpha\right)^3 e^{-i\beta}\sinh 2\alpha\left(\cosh\alpha - e^{-i\beta}\sinh\alpha\right) + \left(\cosh\alpha - e^{-i\beta}\sinh\alpha\right)^3 e^{i\beta}\sinh 2\alpha\left(\cosh\alpha - e^{i\beta}\sinh\alpha\right)\right]. \quad (159)$$

Consequently the three-point function for the scalar fluctuation can also be written after substituting all the momentum dependent functions $G_1(k_1,k_2,k_3)$, $G_2(k_1,k_2,k_3)$ and $G_3(k_1,k_2,k_3)$ for α,β vacuum.

4.2.3. Function $\Theta_3(k_1,k_2,k_3)$

Here we can write the function $\Theta_3(k_1,k_2,k_3)$ as:

$$\begin{aligned}\Theta_3(k_1,k_2,k_3) = 2i\int_{\eta_i=-\infty}^{\eta_f=0} d\eta\,\frac{a^2(\eta)}{H^3}\Big\{&\Big[\bar{v}(\eta_f,\mathbf{k_1})\bar{v}(\eta_f,\mathbf{k_2})\bar{v}(\eta_f,\mathbf{k_3})\bar{v}^*(\eta,\mathbf{k_1})\bar{v}^{'*}(\eta,\mathbf{k_2})\bar{v}^{'*}(\eta,\mathbf{k_3}) \\ &+ \bar{v}^*(\eta_f,-\mathbf{k_1})\bar{v}^*(\eta_f,-\mathbf{k_2})\bar{v}^*(\eta_f,-\mathbf{k_3})\bar{v}(\eta,-\mathbf{k_1})\bar{v}^{'}(\eta,-\mathbf{k_2})\bar{v}^{'}(\eta,-\mathbf{k_3})\Big] \\ &+ \Big[\bar{v}(\eta_f,\mathbf{k_1})\bar{v}(\eta_f,\mathbf{k_2})\bar{v}(\eta_f,\mathbf{k_3})\bar{v}^*(\eta,\mathbf{k_2})\bar{v}^{'*}(\eta,\mathbf{k_1})\bar{v}^{'*}(\eta,\mathbf{k_3}) \\ &+ \bar{v}^*(\eta_f,-\mathbf{k_1})\bar{v}^*(\eta_f,-\mathbf{k_2})\bar{v}^*(\eta_f,-\mathbf{k_3})\bar{v}(\eta,-\mathbf{k_2})\bar{v}^{'}(\eta,-\mathbf{k_1})\bar{v}^{'}(\eta,-\mathbf{k_3})\Big] \\ &+ \Big[\bar{v}(\eta_f,\mathbf{k_1})\bar{v}(\eta_f,\mathbf{k_2})\bar{v}(\eta_f,\mathbf{k_3})\bar{v}^*(\eta,\mathbf{k_3})\bar{v}^{'*}(\eta,\mathbf{k_1})\bar{v}^{'*}(\eta,\mathbf{k_2}) \\ &+ \bar{v}^*(\eta_f,-\mathbf{k_1})\bar{v}^*(\eta_f,-\mathbf{k_2})\bar{v}^*(\eta_f,-\mathbf{k_3})\bar{v}(\eta,-\mathbf{k_3})\bar{v}^{'}(\eta,-\mathbf{k_1})\bar{v}^{'}(\eta,-\mathbf{k_2})\Big]\Big\}. \end{aligned} \quad (160)$$

Using the results obtained in the Appendix C we finally get the following simplified expression for the three-point function for the scalar fluctuations[18]:

$$\Theta_3(k_1,k_2,k_3) = \frac{H}{32\epsilon^3 M_p^6}\frac{1}{(k_1k_2k_3)^3}\Big[(k_2k_3)^2 M_1(k_1,k_2,k_3) + (k_1k_3)^2 M_2(k_1,k_2,k_3) + (k_1k_2)^2 M_3(k_1,k_2,k_3)\Big], \quad (162)$$

where the momentum dependent functions $M_1(k_1,k_2,k_3)$, $M_2(k_1,k_2,k_3)$ and $M_3(k_1,k_2,k_3)$ are defined as:

$$\begin{aligned} M_1(k_1,k_2,k_3) = &\frac{1}{K^2}(K+k_1)\left[(C_1-C_2)^3(C_1^{*3}+C_2^{*3}) + (C_1-C_2)^3(C_1^{*3}+C_2^{*3})\right] \\ &+ \left\{\frac{(K-3k_1)}{(2k_1-K)^2} + \frac{(K+k_1-2k_2)}{(2k_2-K)^2} + \frac{(K+k_1-2k_3)}{(2k_3-K)^2}\right\} \\ &\left[(C_1-C_2)^3 C_1^* C_2^*(C_1^*+C_2^*) + (C_1^*-C_2^*)^3 C_1C_2(C_1+C_2)\right]. \end{aligned} \quad (163)$$

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[18] Here it is important to point out that in de Sitter space if we consider the Bunch–Davies vacuum state then here only the term with $1/K^2$ will appear explicitly in the expression for the three-point function and in the associated bispectrum. On the other hand, if we consider all other non-trivial quantum vacuum states in our computation, then the rest of the contribution will explicitly appear. From the perspective of observation, this is obviously important information as for the non-trivial quantum vacuum state we get additional contribution in the bispectrum which may enhance the amplitude of the non-Gaussianity in squeezed limiting configuration. Additionally, it is important to mention that in quasi de Sitter case we get extra contributions $1/\tilde{c}_S^{6\nu-9}$ and $1/(1+\epsilon)^3$. Also, the factor $1/(k_1k_2k_3)^3$ will be replaced by $1/(k_1k_2k_3)^{2\nu}$. Consequently, in quasi de Sitter case this contribution in the bispectrum can be recast as:

$$\Theta_3(k_1,k_2,k_3) = \frac{H}{32\epsilon^3 M_p^6 \tilde{c}_S^{6\nu-9}(1+\epsilon)^3}\frac{1}{(k_1k_2k_3)^{2\nu}}\Big[(k_2k_3)^2 M_1(k_1,k_2,k_3) + (k_1k_3)^2 M_2(k_1,k_2,k_3) + (k_1k_2)^2 M_3(k_1,k_2,k_3)\Big], \quad (161)$$

$$\begin{aligned} M_2(k_1,k_2,k_3) &= \frac{1}{K^2}(K+k_2)\left[(C_1-C_2)^3(C_1^{*3}+C_2^{*3})+(C_1-C_2)^3(C_1^{*3}+C_2^{*3})\right] \\ &+\left\{\frac{(K-3k_2)}{(2k_2-K)^2}+\frac{(K+k_2-2k_1)}{(2k_1-K)^2}+\frac{(K+k_2-2k_3)}{(2k_3-K)^2}\right\} \\ &\left[(C_1-C_2)^3C_1^*C_2^*(C_1^*+C_2^*)+(C_1^*-C_2^*)^3C_1C_2(C_1+C_2)\right]. \end{aligned} \tag{164}$$

$$\begin{aligned} M_3(k_1,k_2,k_3) &= \frac{1}{K^2}(K+k_3)\left[(C_1-C_2)^3(C_1^{*3}+C_2^{*3})+(C_1-C_2)^3(C_1^{*3}+C_2^{*3})\right] \\ &+\left\{\frac{(K-3k_3)}{(2k_3-K)^2}+\frac{(K+k_3-2k_2)}{(2k_2-K)^2}+\frac{(K+k_3-2k_1)}{(2k_1-K)^2}\right\} \\ &\left[(C_1-C_2)^3C_1^*C_2^*(C_1^*+C_2^*)+(C_1^*-C_2^*)^3C_1C_2(C_1+C_2)\right]. \end{aligned} \tag{165}$$

Finally, for Bunch–Davies and α, β vacuum we get the following contribution in the three-point function for scalar fluctuations:

- **For Bunch–Davies vacuum:**
  After setting $C_1 = 1$ and $C_2 = 0$ we get:

$$M_1(k_1,k_2,k_3) = \frac{2}{K^2}(K+k_1). \tag{166}$$

$$M_2(k_1,k_2,k_3) = \frac{2}{K^2}(K+k_2). \tag{167}$$

$$M_3(k_1,k_2,k_3) = \frac{2}{K^2}(K+k_3). \tag{168}$$

  Consequently, we get the following contribution:

$$\Theta_3(k_1,k_2,k_3) = \frac{H}{16\epsilon^3 M_p^6}\frac{1}{(k_1k_2k_3)^3}\frac{1}{K^2}\left[(k_2k_3)^2(K+k_1)]+(k_1k_3)^2(K+k_2)+(k_1k_2)^2(K+k_3)\right], \tag{169}$$

- **For α, β vacuum:**
  After setting $C_1 = \cosh\alpha$ and $C_2 = e^{i\beta}\ \sinh\alpha$ we get:

$$M_1(k_1,k_2,k_3) = \frac{(K+k_1)J_1(\alpha,\beta)}{K^2}+\left\{\frac{(K-3k_1)}{(2k_1-K)^2}+\frac{(K+k_1-2k_2)}{(2k_2-K)^2}+\frac{(K+k_1-2k_3)}{(2k_3-K)^2}\right\}J_2(\alpha,\beta). \tag{170}$$

$$M_2(k_1,k_2,k_3) = \frac{(K+k_2)J_1(\alpha,\beta)}{K^2}+\left\{\frac{(K-3k_2)}{(2k_2-K)^2}+\frac{(K+k_2-2k_1)}{(2k_1-K)^2}+\frac{(K+k_2-2k_3)}{(2k_3-K)^2}\right\}J_2(\alpha,\beta). \tag{171}$$

$$M_3(k_1,k_2,k_3) = \frac{(K+k_3)J_1(\alpha,\beta)}{K^2}+\left\{\frac{(K-3k_3)}{(2k_3-K)^2}+\frac{(K+k_3-2k_2)}{(2k_2-K)^2}+\frac{(K+k_3-2k_1)}{(2k_1-K)^2}\right\}J_2(\alpha,\beta). \tag{172}$$

  where $J_1(\alpha,\beta)$ and $J_2(\alpha,\beta)$ are defined earlier. Consequently the three-point function for the scalar fluctuation can also be written after substituting all the momentum dependent functions $M_1(k_1,k_2,k_3)$, $M_2(k_1,k_2,k_3)$ and $M_3(k_1,k_2,k_3)$ for α, β vacuum.

4.2.4. Function $\Theta_4(k_1,k_2,k_3)$

Here we can write the function $\Theta_4(k_1,k_2,k_3)$ as:

$$\Theta_4(k_1,k_2,k_3) = i\int_{\eta_i=-\infty}^{\eta_f=0} d\eta\ \frac{a(\eta)}{H^3}\left\{(\mathbf{k_2}.\mathbf{k_3})X_1(k_1,k_2,k_3)+(\mathbf{k_1}.\mathbf{k_3})X_2(k_1,k_2,k_3)+(\mathbf{k_1}.\mathbf{k_2})X_3(k_1,k_2,k_3)\right\}, \tag{173}$$

where the momentum dependent functions $X_1(k_1,k_2,k_3)$, $X_2(k_1,k_2,k_3)$ and $X_3(k_1,k_2,k_3)$ can be expressed in terms of the various combinations of the scalar mode functions as:

$$\begin{aligned}
X_1(k_1,k_2,k_3) \quad = \quad & \bar{v}(\eta_f,\mathbf{k_1})\bar{v}(\eta_f,\mathbf{k_2})\bar{v}(\eta_f,\mathbf{k_3})\bar{v}^*(\eta,\mathbf{k_1})\bar{v}^{'*}(\eta,\mathbf{k_2})\bar{v}^*(\eta,\mathbf{k_3}) \\
& + \bar{v}^*(\eta_f,-\mathbf{k_1})\bar{v}^*(\eta_f,-\mathbf{k_2})\bar{v}^*(\eta_f,-\mathbf{k_3})\bar{v}(\eta,-\mathbf{k_1})\bar{v}^{'}(\eta,-\mathbf{k_2})\bar{v}(\eta,-\mathbf{k_3}) \\
& + \bar{v}(\eta_f,\mathbf{k_1})\bar{v}(\eta_f,\mathbf{k_2})\bar{v}(\eta_f,\mathbf{k_3})\bar{v}^*(\eta,\mathbf{k_1})\bar{v}^{'*}(\eta,\mathbf{k_3})\bar{v}^*(\eta,\mathbf{k_2}) \\
& + \bar{v}^*(\eta_f,-\mathbf{k_1})\bar{v}^*(\eta_f,-\mathbf{k_2})\bar{v}^*(\eta_f,-\mathbf{k_3})\bar{v}(\eta,-\mathbf{k_1})\bar{v}^{'}(\eta,-\mathbf{k_3})\bar{v}(\eta,-\mathbf{k_2}), \qquad (174)\\
X_2(k_1,k_2,k_3) \quad = \quad & \bar{v}(\eta_f,\mathbf{k_1})\bar{v}(\eta_f,\mathbf{k_2})\bar{v}(\eta_f,\mathbf{k_3})\bar{v}^*(\eta,\mathbf{k_2})\bar{v}^{'*}(\eta,\mathbf{k_1})\bar{v}^*(\eta,\mathbf{k_3}) \\
& + \bar{v}^*(\eta_f,-\mathbf{k_1})\bar{v}^*(\eta_f,-\mathbf{k_2})\bar{v}^*(\eta_f,-\mathbf{k_3})\bar{v}(\eta,-\mathbf{k_2})\bar{v}^{'}(\eta,-\mathbf{k_1})\bar{v}(\eta,-\mathbf{k_3}) \\
& + \bar{v}(\eta_f,\mathbf{k_1})\bar{v}(\eta_f,\mathbf{k_2})\bar{v}(\eta_f,\mathbf{k_3})\bar{v}^*(\eta,\mathbf{k_2})\bar{v}^{'*}(\eta,\mathbf{k_3})\bar{v}^*(\eta,\mathbf{k_1}) \\
& + \bar{v}^*(\eta_f,-\mathbf{k_1})\bar{v}^*(\eta_f,-\mathbf{k_2})\bar{v}^*(\eta_f,-\mathbf{k_3})\bar{v}(\eta,-\mathbf{k_2})\bar{v}^{'}(\eta,-\mathbf{k_3})\bar{v}(\eta,-\mathbf{k_1}), \qquad (175)\\
X_3(k_1,k_2,k_3) \quad = \quad & \bar{v}(\eta_f,\mathbf{k_1})\bar{v}(\eta_f,\mathbf{k_2})\bar{v}(\eta_f,\mathbf{k_3})\bar{v}^*(\eta,\mathbf{k_3})\bar{v}^{'*}(\eta,\mathbf{k_1})\bar{v}^*(\eta,\mathbf{k_2}) \\
& + \bar{v}^*(\eta_f,-\mathbf{k_1})\bar{v}^*(\eta_f,-\mathbf{k_2})\bar{v}^*(\eta_f,-\mathbf{k_3})\bar{v}(\eta,-\mathbf{k_3})\bar{v}^{'}(\eta,-\mathbf{k_1})\bar{v}(\eta,-\mathbf{k_2}) \\
& + (\mathbf{k_1}.\mathbf{k_2})\bar{v}(\eta_f,\mathbf{k_1})\bar{v}(\eta_f,\mathbf{k_2})\bar{v}(\eta_f,\mathbf{k_3})\bar{v}^*(\eta,\mathbf{k_3})\bar{v}^{'*}(\eta,\mathbf{k_2})\bar{v}^*(\eta,\mathbf{k_1}) \\
& + \bar{v}^*(\eta_f,-\mathbf{k_1})\bar{v}^*(\eta_f,-\mathbf{k_2})\bar{v}^*(\eta_f,-\mathbf{k_3})\bar{v}(\eta,-\mathbf{k_3})\bar{v}^{'}(\eta,-\mathbf{k_2})\bar{v}^{'}(\eta,-\mathbf{k_1}), \qquad (176)
\end{aligned}$$

Using the results obtained in the Appendix C we finally get the following simplified expression for the three-point function for the scalar fluctuations[19]:

$$\begin{aligned}
\Theta_4(k_1,k_2,k_3) \quad = & -\frac{H^2}{64\tilde{c}_S^2\epsilon^3 M_p^6}\frac{1}{(k_1k_2k_3)^3}\Big[k_2^2(\mathbf{k_2}.\mathbf{k_3})\mathcal{F}_1(k_1,k_2,k_3)+k_3^2(\mathbf{k_2}.\mathbf{k_3})\mathcal{F}_2(k_1,k_2,k_3) \\
& + k_1^2(\mathbf{k_1}.\mathbf{k_3})\mathcal{F}_3(k_1,k_2,k_3)+k_3^2(\mathbf{k_1}.\mathbf{k_3})\mathcal{F}_4(k_1,k_2,k_3) \qquad (178)\\
& + k_1^2(\mathbf{k_1}.\mathbf{k_2})\mathcal{F}_5(k_1,k_2,k_3)+k_2^2(\mathbf{k_1}.\mathbf{k_2})\mathcal{F}_6(k_1,k_2,k_3)\Big],
\end{aligned}$$

where the momentum dependent functions $\mathcal{F}_i(k_1,k_2,k_3)\forall i=1,2,\cdots,6$ are defined as:

$$\begin{aligned}
\mathcal{F}_1(k_1,k_2,k_3) \quad = \quad & \frac{1}{K^3}\Big[K^2+2k_1k_3+K(K-k_2)\Big]\Big[(C_1-C_2)^3(C_1^{*3}+C_2^{*3})+(C_1-C_2)^3(C_1^{*3}+C_2^{*3})\Big] \\
& + \Bigg\{\frac{1}{(2k_1-K)^3}\Big[K^2-4k_1k_3+K(k_3-5k_1)+6k_1^2\Big] \\
& + \frac{1}{(2k_2-K)^3}\left[(K-2k_2)(K-2k_2+k_1)+(K+2k_1-2k_2)k_3\right] \\
& + \frac{1}{(2k_3-K)^3}\Big[K^2-4k_1k_3+K(k_1-5k_3)+6k_3^2\Big]\Bigg\} \\
& \Big[(C_1-C_2)^3C_1^*C_2^*(C_1^*+C_2^*)+(C_1^*-C_2^*)^3C_1C_2(C_1+C_2)\Big]. \qquad (179)
\end{aligned}$$

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[19] Here it is important to point out that in de Sitter space if we consider the Bunch–Davies vacuum state then here only the term with $1/K^3$ will appear explicitly in the expression for the three-point function and in the associated bispectrum. On the other hand, if we consider all other non-trivial quantum vacuum states in our computation, then the rest of the contribution will explicitly appear. From the perspective of observation, this is obviously important information as for the non-trivial quantum vacuum state we get additional contribution in the bispectrum which may enhance the amplitude of the non-Gaussianity in squeezed limiting configuration. Additionally, it is important to mention that in quasi de Sitter case we get an extra contribution $1/(1+\epsilon)^5$. Also the factor $1/(k_1k_2k_3)^3$ will be replaced by $1/(k_1k_2k_3)^{2\nu}$ and $1/\tilde{c}_S^2$ is replaced by $1/\tilde{c}_S^{6\nu-7}$. Consequently, in quasi de Sitter case this contribution in the bispectrum can be recast as:

$$\begin{aligned}
\Theta_4(k_1,k_2,k_3) \quad = & -\frac{H^2}{64\tilde{c}_S^{6\nu-7}\epsilon^3 M_p^6(1+\epsilon)^5}\frac{1}{(k_1k_2k_3)^{2\nu}}\left[k_2^2(\mathbf{k_2}.\mathbf{k_3})\mathcal{F}_1(k_1,k_2,k_3)+k_3^2(\mathbf{k_2}.\mathbf{k_3})\mathcal{F}_2(k_1,k_2,k_3)\right. \qquad (177)\\
& \left. + k_1^2(\mathbf{k_1}.\mathbf{k_3})\mathcal{F}_3(k_1,k_2,k_3)+k_3^2(\mathbf{k_1}.\mathbf{k_3})\mathcal{F}_4(k_1,k_2,k_3)+k_1^2(\mathbf{k_1}.\mathbf{k_2})\mathcal{F}_5(k_1,k_2,k_3)+k_2^2(\mathbf{k_1}.\mathbf{k_2})\mathcal{F}_6(k_1,k_2,k_3)\right],
\end{aligned}$$

$$
\begin{aligned}
\mathcal{F}_2(k_1,k_2,k_3) \;=\;& \frac{1}{K^3}\left[K^2+2k_1k_2+K(K-k_3)\right]\left[(C_1-C_2)^3(C_1^{*3}+C_2^{*3})+(C_1-C_2)^3(C_1^{*3}+C_2^{*3})\right] \\
&+\left\{\frac{1}{(2k_1-K)^3}\left[K^2-4k_1k_2+K(k_2-5k_1)+6k_1^2\right]\right. \\
&+\frac{1}{(2k_3-K)^3}\left[(K-2k_3)(K-2k_3+k_1)+(K+2k_1-2k_3)k_2\right] \\
&\left.+\frac{1}{(2k_2-K)^3}\left[K^2-4k_1k_2+K(k_1-5k_2)+6k_2^2\right]\right\} \\
&\quad\left[(C_1-C_2)^3C_1^*C_2^*(C_1^*+C_2^*)+(C_1^*-C_2^*)^3C_1C_2(C_1+C_2)\right]. \qquad (180)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{F}_3(k_1,k_2,k_3) \;=\;& \frac{1}{K^3}\left[K^2+2k_2k_3+K(K-k_1)\right]\left[(C_1-C_2)^3(C_1^{*3}+C_2^{*3})+(C_1-C_2)^3(C_1^{*3}+C_2^{*3})\right] \\
&+\left\{\frac{1}{(2k_2-K)^3}\left[K^2-4k_2k_3+K(k_3-5k_2)+6k_2^2\right]\right. \\
&+\frac{1}{(2k_1-K)^3}\left[(K-2k_1)(K-2k_1+k_2)+(K+2k_2-2k_1)k_2\right] \\
&\left.+\frac{1}{(2k_3-K)^3}\left[K^2-4k_2k_3+K(k_2-5k_3)+6k_3^2\right]\right\} \\
&\quad\left[(C_1-C_2)^3C_1^*C_2^*(C_1^*+C_2^*)+(C_1^*-C_2^*)^3C_1C_2(C_1+C_2)\right]. \qquad (181)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{F}_4(k_1,k_2,k_3) \;=\;& \frac{1}{K^3}\left[K^2+2k_1k_2+K(K-k_3)\right]\left[(C_1-C_2)^3(C_1^{*3}+C_2^{*3})+(C_1-C_2)^3(C_1^{*3}+C_2^{*3})\right] \\
&+\left\{\frac{1}{(2k_2-K)^3}\left[K^2-4k_1k_2+K(k_1-5k_2)+6k_2^2\right]\right. \\
&+\frac{1}{(2k_3-K)^3}\left[(K-2k_3)(K-2k_3+k_2)+(K+2k_2-2k_3)k_2\right] \\
&\left.+\frac{1}{(2k_1-K)^3}\left[K^2-4k_1k_2+K(k_2-5k_1)+6k_1^2\right]\right\} \\
&\quad\left[(C_1-C_2)^3C_1^*C_2^*(C_1^*+C_2^*)+(C_1^*-C_2^*)^3C_1C_2(C_1+C_2)\right]. \qquad (182)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{F}_5(k_1,k_2,k_3) \;=\;& \frac{1}{K^3}\left[K^2+2k_2k_3+K(K-k_1)\right]\left[(C_1-C_2)^3(C_1^{*3}+C_2^{*3})+(C_1-C_2)^3(C_1^{*3}+C_2^{*3})\right] \\
&+\left\{\frac{1}{(2k_3-K)^3}\left[K^2-4k_2k_3+K(k_2-5k_3)+6k_3^2\right]\right. \\
&+\frac{1}{(2k_1-K)^3}\left[(K-2k_1)(K-2k_1+k_2)+(K+2k_2-2k_1)k_3\right] \\
&\left.+\frac{1}{(2k_2-K)^3}\left[K^2-4k_2k_3+K(k_3-5k_2)+6k_2^2\right]\right\} \\
&\quad\left[(C_1-C_2)^3C_1^*C_2^*(C_1^*+C_2^*)+(C_1^*-C_2^*)^3C_1C_2(C_1+C_2)\right]. \qquad (183)
\end{aligned}
$$

$$
\begin{aligned}
\mathcal{F}_6(k_1,k_2,k_3) \;=\;& \frac{1}{K^3}\left[K^2+2k_1k_3+K(K-k_2)\right]\left[(C_1-C_2)^3(C_1^{*3}+C_2^{*3})+(C_1-C_2)^3(C_1^{*3}+C_2^{*3})\right] \\
&+\left\{\frac{1}{(2k_3-K)^3}\left[K^2-4k_1k_3+K(k_1-5k_3)+6k_3^2\right]\right. \\
&+\frac{1}{(2k_2-K)^3}\left[(K-2k_2)(K-2k_2+k_3)+(K+2k_3-2k_2)k_1\right] \\
&\left.+\frac{1}{(2k_1-K)^3}\left[K^2-4k_1k_3+K(k_3-5k_1)+6k_1^2\right]\right\} \\
&\quad\left[(C_1-C_2)^3C_1^*C_2^*(C_1^*+C_2^*)+(C_1^*-C_2^*)^3C_1C_2(C_1+C_2)\right]. \qquad (184)
\end{aligned}
$$

Furthermore, after simplification one can recast the three-point function for the scalar fluctuation as:

$$\begin{aligned}\Theta_4(k_1,k_2,k_3) &= -\frac{H^2}{128\tilde{c}_5^2\epsilon^3 M_p^6}\frac{1}{(k_1k_2k_3)^3}\Big[k_2^2\left(k_1^2-k_2^2-k_3^2\right)\mathcal{F}_1(k_1,k_2,k_3)+k_3^2\left(k_1^2-k_2^2-k_3^2\right)\mathcal{F}_2(k_1,k_2,k_3)\\ &\quad +k_1^2\left(k_2^2-k_1^2-k_3^2\right)\mathcal{F}_3(k_1,k_2,k_3)+k_3^2\left(k_2^2-k_1^2-k_3^2\right)\mathcal{F}_4(k_1,k_2,k_3)\\ &\quad +k_1^2\left(k_3^2-k_2^2-k_1^2\right)\mathcal{F}_5(k_1,k_2,k_3)+k_2^2\left(k_3^2-k_2^2-k_1^2\right)\mathcal{F}_6(k_1,k_2,k_3)\Big],\end{aligned} \tag{185}$$

Finally, for Bunch–Davies and α,β vacuum we get the following contribution in the three-point function for scalar fluctuations:

- **For Bunch–Davies vacuum:**
  After setting $C_1 = 1$ and $C_2 = 0$ we get:

$$\mathcal{F}_1(k_1,k_2,k_3) = \frac{1}{K^3}\left[K^2+2k_1k_3+K(K-k_2)\right]. \tag{186}$$

$$\mathcal{F}_2(k_1,k_2,k_3) = \frac{1}{K^3}\left[K^2+2k_1k_2+K(K-k_3)\right]. \tag{187}$$

$$\mathcal{F}_3(k_1,k_2,k_3) = \frac{1}{K^3}\left[K^2+2k_2k_3+K(K-k_1)\right]. \tag{188}$$

$$\mathcal{F}_4(k_1,k_2,k_3) = \frac{1}{K^3}\left[K^2+2k_1k_2+K(K-k_3)\right]. \tag{189}$$

$$\mathcal{F}_5(k_1,k_2,k_3) = \frac{1}{K^3}\left[K^2+2k_2k_3+K(K-k_1)\right]. \tag{190}$$

$$\mathcal{F}_6(k_1,k_2,k_3) = \frac{1}{K^3}\left[K^2+2k_1k_3+K(K-k_2)\right]. \tag{191}$$

  Consequently, the three-point function for the scalar fluctuation can be expressed as:

$$\begin{aligned}\Theta_4(k_1,k_2,k_3) &= -\frac{H^2}{128\tilde{c}_5^2\epsilon^3 M_p^6}\frac{1}{(k_1k_2k_3)^3}\Big[k_2^2\left(k_1^2-k_2^2-k_3^2\right)\left[K^2+2k_1k_3+K(K-k_2)\right]\\ &\quad +k_3^2\left(k_1^2-k_2^2-k_3^2\right)\left[K^2+2k_1k_2+K(K-k_3)\right]\\ &\quad +k_1^2\left(k_2^2-k_1^2-k_3^2\right)\left[K^2+2k_2k_3+K(K-k_1)\right]\\ &\quad +k_3^2\left(k_2^2-k_1^2-k_3^2\right)\left[K^2+2k_1k_2+K(K-k_3)\right]\\ &\quad +k_1^2\left(k_3^2-k_2^2-k_1^2\right)\left[K^2+2k_2k_3+K(K-k_1)\right]\\ &\quad +k_2^2\left(k_3^2-k_2^2-k_1^2\right)\left[K^2+2k_1k_3+K(K-k_2)\right]\Big],\end{aligned} \tag{192}$$

- **For α,β vacuum:**
  After setting $C_1 = \cosh\alpha$ and $C_2 = e^{i\beta}\ \sinh\alpha$ we get:

$$\begin{aligned}\mathcal{F}_1(k_1,k_2,k_3) &= \frac{1}{K^3}\left[K^2+2k_1k_3+K(K-k_2)\right]J_1(\alpha,\beta)\\ &\quad +\Bigg\{\frac{1}{(2k_1-K)^3}\left[K^2-4k_1k_3+K(k_3-5k_1)+6k_1^2\right]\\ &\quad +\frac{1}{(2k_2-K)^3}\left[(K-2k_2)(K-2k_2+k_1)+(K+2k_1-2k_2)k_3\right]\\ &\quad +\frac{1}{(2k_3-K)^3}\left[K^2-4k_1k_3+K(k_1-5k_3)+6k_3^2\right]\Bigg\}J_2(\alpha,\beta).\end{aligned} \tag{193}$$

$$\begin{aligned}\mathcal{F}_2(k_1,k_2,k_3) &= \frac{1}{K^3}\left[K^2+2k_1k_2+K(K-k_3)\right]J_1(\alpha,\beta)\\&+\left\{\frac{1}{(2k_1-K)^3}\left[K^2-4k_1k_2+K(k_2-5k_1)+6k_1^2\right]\right.\\&+\frac{1}{(2k_3-K)^3}\left[(K-2k_3)(K-2k_3+k_1)+(K+2k_1-2k_3)k_2\right]\\&\left.+\frac{1}{(2k_2-K)^3}\left[K^2-4k_1k_2+K(k_1-5k_2)+6k_2^2\right]\right\}J_2(\alpha,\beta).\end{aligned} \tag{194}$$

$$\begin{aligned}\mathcal{F}_3(k_1,k_2,k_3) &= \frac{1}{K^3}\left[K^2+2k_2k_3+K(K-k_1)\right]J_1(\alpha,\beta)\\&+\left\{\frac{1}{(2k_2-K)^3}\left[K^2-4k_2k_3+K(k_3-5k_2)+6k_2^2\right]\right.\\&+\frac{1}{(2k_1-K)^3}\left[(K-2k_1)(K-2k_1+k_2)+(K+2k_2-2k_1)k_2\right]\\&\left.+\frac{1}{(2k_3-K)^3}\left[K^2-4k_2k_3+K(k_2-5k_3)+6k_3^2\right]\right\}J_2(\alpha,\beta).\end{aligned} \tag{195}$$

$$\begin{aligned}\mathcal{F}_4(k_1,k_2,k_3) &= \frac{1}{K^3}\left[K^2+2k_1k_2+K(K-k_3)\right]J_1(\alpha,\beta)\\&+\left\{\frac{1}{(2k_2-K)^3}\left[K^2-4k_1k_2+K(k_1-5k_2)+6k_2^2\right]\right.\\&+\frac{1}{(2k_3-K)^3}\left[(K-2k_3)(K-2k_3+k_2)+(K+2k_2-2k_3)k_2\right]\\&\left.+\frac{1}{(2k_1-K)^3}\left[K^2-4k_1k_2+K(k_2-5k_1)+6k_1^2\right]\right\}J_2(\alpha,\beta).\end{aligned} \tag{196}$$

$$\begin{aligned}\mathcal{F}_5(k_1,k_2,k_3) &= \frac{1}{K^3}\left[K^2+2k_2k_3+K(K-k_1)\right]J_1(\alpha,\beta)\\&+\left\{\frac{1}{(2k_3-K)^3}\left[K^2-4k_2k_3+K(k_2-5k_3)+6k_3^2\right]\right.\\&+\frac{1}{(2k_1-K)^3}\left[(K-2k_1)(K-2k_1+k_2)+(K+2k_2-2k_1)k_3\right]\\&\left.+\frac{1}{(2k_2-K)^3}\left[K^2-4k_2k_3+K(k_3-5k_2)+6k_2^2\right]\right\}J_2(\alpha,\beta).\end{aligned} \tag{197}$$

$$\begin{aligned}\mathcal{F}_6(k_1,k_2,k_3) &= \frac{1}{K^3}\left[K^2+2k_1k_3+K(K-k_2)\right]J_1(\alpha,\beta)\\&+\left\{\frac{1}{(2k_3-K)^3}\left[K^2-4k_1k_3+K(k_1-5k_3)+6k_3^2\right]\right.\\&+\frac{1}{(2k_2-K)^3}\left[(K-2k_2)(K-2k_2+k_3)+(K+2k_3-2k_2)k_1\right]\\&\left.+\frac{1}{(2k_1-K)^3}\left[K^2-4k_1k_3+K(k_3-5k_1)+6k_1^2\right]\right\}J_2(\alpha,\beta).\end{aligned} \tag{198}$$

where $J_1(\alpha,\beta)$ and $J_2(\alpha,\beta)$ are defined earlier. Consequently the three-point function for the scalar fluctuation can also be written after substituting all the momentum dependent functions $\mathcal{F}_i(k_1,k_2,k_3)\forall i=1,2,\cdots,6$ for α,β vacuum.

4.2.5. Function $\Theta_5(k_1,k_2,k_3)$

$$\Theta_5(k_1,k_2,k_3) = i\int_{\eta_i=-\infty}^{\eta_f=0} d\eta\,\frac{a(\eta)}{H^3}\left\{(\mathbf{k_2}.\mathbf{k_3})Y_1(k_1,k_2,k_3)+(\mathbf{k_1}.\mathbf{k_3})Y_2(k_1,k_2,k_3) + (\mathbf{k_1}.\mathbf{k_2})Y_3(k_1,k_2,k_3)\right\}, \quad (199)$$

where the momentum dependent functions $Y_1(k_1,k_2,k_3)$, $Y_2(k_1,k_2,k_3)$ and $Y_3(k_1,k_2,k_3)$ can be expressed in terms of the various combinations of the scalar mode functions as:

$$\begin{aligned}
Y_1(k_1,k_2,k_3) &= \bar{v}(\eta_f,\mathbf{k_1})\bar{v}(\eta_f,\mathbf{k_2})\bar{v}(\eta_f,\mathbf{k_3})\bar{v}^*(\eta,\mathbf{k_1})\bar{v}^*(\eta,\mathbf{k_2})\bar{v}^{*'}(\eta,\mathbf{k_3})\\
&+\bar{v}^*(\eta_f,-\mathbf{k_1})\bar{v}^*(\eta_f,-\mathbf{k_2})\bar{v}^*(\eta_f,-\mathbf{k_3})\bar{v}(\eta,-\mathbf{k_1})\bar{v}(\eta,-\mathbf{k_2})\bar{v}^{'}(\eta,-\mathbf{k_3})\\
&+\bar{v}(\eta_f,\mathbf{k_1})\bar{v}(\eta_f,\mathbf{k_2})\bar{v}(\eta_f,\mathbf{k_3})\bar{v}^*(\eta,\mathbf{k_1})\bar{v}^*(\eta,\mathbf{k_3})\bar{v}^{*'}(\eta,\mathbf{k_2})\\
&+\bar{v}^*(\eta_f,-\mathbf{k_1})\bar{v}^*(\eta_f,-\mathbf{k_2})\bar{v}^*(\eta_f,-\mathbf{k_3})\bar{v}(\eta,-\mathbf{k_1})\bar{v}(\eta,-\mathbf{k_3})\bar{v}^{'}(\eta,-\mathbf{k_2}), \quad (200)\\
Y_2(k_1,k_2,k_3) &= \bar{v}(\eta_f,\mathbf{k_1})\bar{v}(\eta_f,\mathbf{k_2})\bar{v}(\eta_f,\mathbf{k_3})\bar{v}^*(\eta,\mathbf{k_2})\bar{v}^*(\eta,\mathbf{k_1})\bar{v}^{*'}(\eta,\mathbf{k_3})\\
&+\bar{v}^*(\eta_f,-\mathbf{k_1})\bar{v}^*(\eta_f,-\mathbf{k_2})\bar{v}^*(\eta_f,-\mathbf{k_3})\bar{v}(\eta,-\mathbf{k_2})\bar{v}(\eta,-\mathbf{k_1})\bar{v}^{'}(\eta,-\mathbf{k_3})\\
&+\bar{v}(\eta_f,\mathbf{k_1})\bar{v}(\eta_f,\mathbf{k_2})\bar{v}(\eta_f,\mathbf{k_3})\bar{v}^*(\eta,\mathbf{k_2})\bar{v}^*(\eta,\mathbf{k_3})\bar{v}^{*'}(\eta,\mathbf{k_1})\\
&+\bar{v}^*(\eta_f,-\mathbf{k_1})\bar{v}^*(\eta_f,-\mathbf{k_2})\bar{v}^*(\eta_f,-\mathbf{k_3})\bar{v}(\eta,-\mathbf{k_2})\bar{v}(\eta,-\mathbf{k_3})\bar{v}^{'}(\eta,-\mathbf{k_1}), \quad (201)\\
Y_3(k_1,k_2,k_3) &= \bar{v}(\eta_f,\mathbf{k_1})\bar{v}(\eta_f,\mathbf{k_2})\bar{v}(\eta_f,\mathbf{k_3})\bar{v}^*(\eta,\mathbf{k_3})\bar{v}^*(\eta,\mathbf{k_1})\bar{v}^{'*}(\eta,\mathbf{k_2})\\
&+\bar{v}^*(\eta_f,-\mathbf{k_1})\bar{v}^*(\eta_f,-\mathbf{k_2})\bar{v}^*(\eta_f,-\mathbf{k_3})\bar{v}(\eta,-\mathbf{k_3})\bar{v}(\eta,-\mathbf{k_1})\bar{v}^{'}(\eta,-\mathbf{k_2})\\
&+\bar{v}(\eta_f,\mathbf{k_1})\bar{v}(\eta_f,\mathbf{k_2})\bar{v}(\eta_f,\mathbf{k_3})\bar{v}^*(\eta,\mathbf{k_3})\bar{v}^*(\eta,\mathbf{k_2})\bar{v}^{*'}(\eta,\mathbf{k_1})\\
&+\bar{v}^*(\eta_f,-\mathbf{k_1})\bar{v}^*(\eta_f,-\mathbf{k_2})\bar{v}^*(\eta_f,-\mathbf{k_3})\bar{v}(\eta,-\mathbf{k_3})\bar{v}^{'}(\eta,-\mathbf{k_2})\bar{v}^{'}(\eta,-\mathbf{k_1}), \quad (202)
\end{aligned}$$

Here we get the following contribution in the three-point function for scalar fluctuations[20]:

$$\Theta_5(k_1,k_2,k_3) = \Theta_4(k_1,k_2,k_3), \quad (204)$$

where $\Theta_4(k_1,k_2,k_3)$ is defined earlier. Here the result is exactly same as derived for the coefficient α_4.

4.3. Limiting Configurations of Scalar Bispectrum

To analyze the features of the bispectrum computed from the present setup here we further consider the following two configurations:

[20] Here it is important to point out that in de Sitter space if we consider the Bunch–Davies vacuum state then here only the term with $1/K^3$ will appear explicitly in the expression for the three-point function and in the associated bispectrum. On the other hand, if we consider all other non-trivial quantum vacuum states in our computation, then the rest of the contribution will explicitly appear. From the perspective of observation, this is obviously important information as for the non-trivial quantum vacuum state we get additional contribution in the bispectrum which may enhance the amplitude of the non-Gaussianity in squeezed limiting configuration. Additionally, it is important to mention that in quasi de Sitter case we get an extra contribution $1/(1+\epsilon)^5$. Also the factor $1/(k_1k_2k_3)^3$ will be replaced by $1/(k_1k_2k_3)^{2\nu}$ and $1/\tilde{c}_S^2$ is replaced by $1/\tilde{c}_S^{6\nu-7}$. Consequently, in quasi de Sitter case this contribution in the bispectrum can be recast as:

$$\begin{aligned}\Theta_5(k_1,k_2,k_3) &= \Theta_4(k_1,k_2,k_3) = -\frac{H^2}{64\tilde{c}_S^{6\nu-7}\epsilon^3M_p^6(1+\epsilon)^5}\frac{1}{(k_1k_2k_3)^{2\nu}}\left[k_2^2(\mathbf{k_2}.\mathbf{k_3})\mathcal{F}_1(k_1,k_2,k_3)+k_3^2(\mathbf{k_2}.\mathbf{k_3})\mathcal{F}_2(k_1,k_2,k_3)\right.\\
&\left.+k_1^2(\mathbf{k_1}.\mathbf{k_3})\mathcal{F}_3(k_1,k_2,k_3)+k_3^2(\mathbf{k_1}.\mathbf{k_3})\mathcal{F}_4(k_1,k_2,k_3)+k_1^2(\mathbf{k_1}.\mathbf{k_2})\mathcal{F}_5(k_1,k_2,k_3)+k_2^2(\mathbf{k_1}.\mathbf{k_2})\mathcal{F}_6(k_1,k_2,k_3)\right],\end{aligned} \quad (203)$$

4.3.1. Equilateral Limit Configuration

Equilateral limit configuration is characterized by the condition, $k_1 = k_2 = k_3 = k$, where $k_i = |\mathbf{k_i}| \forall i = 1,2,3$. Consequently, we have, $K = 3k$.

For this case, the bispectrum can be written as:

$$B_{EFT}(k,k,k) = \sum_{j=1}^{5} \alpha_j \Theta_j(k,k,k) \,, \tag{205}$$

where $\alpha_j \forall j = 1,2,\cdots,5$ are defined earlier and $\Theta_j(k,k,k) \forall j = 1,2,\cdots,5$ are given by:

$$\begin{aligned}
\Theta_1(k,k,k) &= \frac{3H^2}{16\epsilon^3 M_p^6} \frac{1}{k^6} \left[\frac{1}{27} U_1 - 3U_2\right], & (206)\\
\Theta_2(k,k,k) &= -\frac{3H^2}{64\epsilon^3 M_p^6 \tilde{c}_S^2} \frac{1}{k^6} \left[\frac{17}{27} U_1 - 3U_2\right], & (207)\\
\Theta_3(k,k,k) &= \frac{3H}{32\epsilon^3 M_p^6} \frac{1}{k^6} \left[\frac{10}{9} U_1 - \frac{22}{49} U_2\right], & (208)\\
\Theta_4(k,k,k) &= \frac{3H^2}{64\tilde{c}_S^2 \epsilon^3 M_p^6} \frac{1}{k^6} \left[\frac{17}{27} U_1 - 3U_2\right] = \Theta_5(k,k,k), & (209)
\end{aligned}$$

where U_1 and U_2 are defined as:

$$\begin{aligned}
U_1 &= \left[(C_1 - C_2)^3 \left(C_1^{*3} + C_2^{*3}\right) + (C_1^* - C_2^*)^3 \left(C_1^3 + C_2^3\right)\right], \\
U_2 &= \left[(C_1 - C_2)^3 C_1^* C_2^* \left(C_1^* - C_2^*\right) + (C_1^* - C_2^*)^3 C_1 C_2 \left(C_1 - C_2\right)\right]. & (210)
\end{aligned}$$

Furthermore, substituting the explicit expressions for $\alpha_j \forall j = 1,2,\cdots,5$ and $\Theta_j(k,k,k) \forall j = 1,2,\cdots,5$ we get the following expression for the bispectrum for scalar fluctuations:

$$B_{EFT}(k,k,k) = \frac{3H^2}{16\epsilon^3 \tilde{c}_S M_p^6} \frac{1}{k^6} \sum_{p=1}^{2} f_p U_p \,, \tag{211}$$

where $f_p \forall p = 1,2$ are defined as:

$$\begin{aligned}
f_1 &= \frac{\tilde{c}_s}{27} \left\{ \left(1 - \frac{1}{c_S^2}\right) \dot{H} M_p^2 + \frac{3}{2} \bar{M}_1^3 H - \frac{4}{3} M_3^4 \right\} + \frac{17}{108 \tilde{c}_s} \left\{ \left(1 - \frac{1}{c_S^2}\right) \dot{H} M_p^2 + \frac{3}{2} \bar{M}_1^3 H \right\} \\
&\quad - \frac{5}{2} \bar{M}_1^3 H \tilde{c}_s + \frac{17}{36 \tilde{c}_s} \bar{M}_1^3 H, & (212)\\
f_2 &= -3\tilde{c}_s \left\{ \left(1 - \frac{1}{c_S^2}\right) \dot{H} M_p^2 + \frac{3}{2} \bar{M}_1^3 H - \frac{4}{3} M_3^4 \right\} - \frac{3}{4\tilde{c}_s} \left\{ \left(1 - \frac{1}{c_S^2}\right) \dot{H} M_p^2 + \frac{3}{2} \bar{M}_1^3 H \right\} \\
&\quad + \frac{99}{98} \bar{M}_1^3 H \tilde{c}_s - \frac{9}{4\tilde{c}_s} \bar{M}_1^3 H. & (213)
\end{aligned}$$

- **For Bunch–Davies vacuum:**
  After setting $C_1 = 1$ and $C_2 = 0$ we get:

$$U_1 = 2, \quad U_2 = 0. \tag{214}$$

Consequently, we get the following expression for the bispectrum for scalar fluctuations:

$$B_{EFT}(k,k,k) = \frac{H^2}{4\epsilon\tilde{c}_S M_p^2}\frac{1}{M_p^4\epsilon^2}\frac{1}{k^6}\left[\frac{\tilde{c}_s}{18}\left\{\left(1-\frac{1}{c_S^2}\right)\dot{H}M_p^2+\frac{3}{2}\bar{M}_1^3 H-\frac{4}{3}M_3^4\right\}\right. \\ \left. +\frac{17}{72\tilde{c}_s}\left\{\left(1-\frac{1}{c_S^2}\right)\dot{H}M_p^2+\frac{3}{2}\bar{M}_1^3 H\right\}-\frac{15}{4}\bar{M}_1^3 H\tilde{c}_s+\frac{17}{24\tilde{c}_s}\bar{M}_1^3 H\right]. \quad (215)$$

For $\tilde{c}_S = 1 = c_S$ case we know that $M_2 = 0$ and $M_3 = 0$ which we have already shown earlier. As a result the bispectrum for scalar fluctuation can be expressed in the following simplified form:

$$B_{EFT}(k,k,k) = -\frac{H^2}{4\epsilon M_p^2}\frac{1}{M_p^4\epsilon^2}\frac{1}{k^6}\frac{125}{48}\bar{M}_1^3 H. \quad (216)$$

For $\tilde{c}_S < 1$ and $c_S < 1$ case one can also recast the bispectrum for scalar fluctuations in the following simplified form:

$$B_{EFT}(k,k,k) = \frac{H^2}{4\epsilon\tilde{c}_S M_p^2}\frac{1}{M_p^4\epsilon^2}\frac{1}{k^6}\bar{M}_1^3 H\left[\frac{\tilde{c}_S}{18}\left\{\frac{3}{2}+\frac{4}{3}\frac{\tilde{c}_3}{\tilde{c}_4}+\frac{2c_S^2}{\tilde{c}_4}\right\}\right. \\ \left. +\frac{17}{72\tilde{c}_S}\left\{\frac{2c_S^2}{\tilde{c}_4}+\frac{3}{2}\right\}-\frac{15}{4}\tilde{c}_S+\frac{17}{24\tilde{c}_S}\right]. \quad (217)$$

- **For α, β vacuum:**
After setting $C_1 = \cosh\alpha$ and $C_2 = e^{i\beta}\,\sinh\alpha$ we get:

$$U_1 = J_1(\alpha,\beta), \quad U_2 = J_2(\alpha,\beta), \quad (218)$$

where $J_1(\alpha,\beta)$ and $J_2(\alpha,\beta)$ are defined earlier.

Consequently, we get the following expression for the bispectrum for scalar fluctuations:

$$B_{EFT}(k,k,k) = \frac{H^2}{4\epsilon\tilde{c}_S M_p^2}\frac{1}{M_p^4\epsilon^2}\frac{1}{k^6}\left[\left(\frac{\tilde{c}_s}{36}\left\{\left(1-\frac{1}{c_S^2}\right)\dot{H}M_p^2+\frac{3}{2}\bar{M}_1^3 H-\frac{4}{3}M_3^4\right\}\right.\right. \\ +\frac{17}{144\tilde{c}_s}\left\{\left(1-\frac{1}{c_S^2}\right)\dot{H}M_p^2+\frac{3}{2}\bar{M}_1^3 H\right\}-\frac{15}{8}\bar{M}_1^3 H\tilde{c}_s+\left.\frac{17}{48\tilde{c}_s}\bar{M}_1^3 H\right)J_1(\alpha,\beta) \\ +\left(-\frac{9}{4}\tilde{c}_s\left\{\left(1-\frac{1}{c_S^2}\right)\dot{H}M_p^2+\frac{3}{2}\bar{M}_1^3 H-\frac{4}{3}M_3^4\right\}\right. \\ \left.\left.-\frac{9}{16\tilde{c}_s}\left\{\left(1-\frac{1}{c_S^2}\right)\dot{H}M_p^2+\frac{3}{2}\bar{M}_1^3 H\right\}+\frac{297}{392}\bar{M}_1^3 H\tilde{c}_s-\frac{27}{16\tilde{c}_s}\bar{M}_1^3 H\right)J_2(\alpha,\beta)\right]. \quad (219)$$

For $\tilde{c}_S = 1 = c_S$ case we know that $M_2 = 0$ and $M_3 = 0$ which we have already shown earlier. As a result the bispectrum for scalar fluctuation can be expressed in the following simplified form:

$$B(k,k,k) = -\frac{H^2}{4\epsilon M_p^2}\frac{1}{M_p^4\epsilon^2}\frac{1}{k^6}\bar{M}_1^3 H\left[\frac{125}{96}J_1(\alpha,\beta)+\frac{8073}{1568}J_2(\alpha,\beta)\right]. \quad (220)$$

For $\tilde{c}_S < 1$ and $c_S < 1$ case one can also recast the bispectrum for scalar fluctuations in the following simplified form:

$$\begin{aligned} B_{EFT}(k,k,k) &= \frac{H^2}{4\epsilon\tilde{c}_S M_p^2}\frac{1}{M_p^4\epsilon^2}\frac{1}{k^6}\bar{M}_1^3 H\left[\left(\frac{\tilde{c}_S}{36}\left\{\frac{3}{2}+\frac{4}{3}\frac{\tilde{c}_3}{\tilde{c}_4}+\frac{2c_S^2}{\tilde{c}_4}\right\}+\frac{17}{144\tilde{c}_S}\left\{\frac{2c_S^2}{\tilde{c}_4}+\frac{3}{2}\right\}\right.\right. \\ &\left. -\frac{15}{8}\tilde{c}_S+\frac{17}{48\tilde{c}_S}\right)J_1(\alpha,\beta)+\left(-\frac{9}{4}\tilde{c}_S\left\{\frac{3}{2}+\frac{4}{3}\frac{\tilde{c}_3}{\tilde{c}_4}+\frac{2c_S^2}{\tilde{c}_4}\right\}-\frac{9}{16\tilde{c}_S}\left\{\frac{2c_S^2}{\tilde{c}_4}+\frac{3}{2}\right\}\right. \\ &\left.\left. +\frac{297}{392}\tilde{c}_S-\frac{27}{16\tilde{c}_S}\right)J_2(\alpha,\beta)\right]. \end{aligned} \tag{221}$$

4.3.2. Squeezed Limit Configuration

Squeezed limit configuration is characterized by the condition, $k_1 \approx k_2(=k_L) >> k_3(=k_S)$, where $k_i = |\mathbf{k_i}| \forall i = 1,2,3$. Also k_L and k_S characterize long and short mode momentum, respectively. Consequently, we have, $K = 2k_L + k_S$. For this case, the bispectrum can be written as:

$$B_{EFT}(k_L,k_L,k_S) = \sum_{j=1}^{5}\alpha_j\Theta_j(k_L,k_L,k_S)\,, \tag{222}$$

where $\alpha_j \forall j = 1,2,\cdots,5$ are defined earlier and $\Theta_j(k_L,k_L,k_S)\forall j = 1,2,\cdots,5$ are given by:

$$\Theta_1(k_L,k_L,k_S) \approx \frac{3H^2}{128\epsilon^3 M_p^6}\frac{1}{k_L^5 k_S}\left[U_1 - 16U_2\left(\frac{k_L}{k_S}\right)^3\right], \tag{223}$$

$$\begin{aligned} \Theta_2(k_L,k_L,k_S) &= -\frac{H^2}{64\epsilon^3 M_p^6\tilde{c}_S^2}\frac{1}{k_L^5 k_S}\left\{\frac{3}{4}\left[U_1-3U_2\right]+\frac{3}{4}\left[U_1-U_2\left(1+\frac{8}{3}\left(\frac{k_L}{k_S}\right)^2\right)\right]\right. \\ &\left. +\frac{5}{4}\left(2-\left(\frac{k_S}{k_L}\right)^2\right)\left[U_1-U_2\left(1-\frac{8}{5}\left(\frac{k_L}{k_S}\right)^3\right)\right]\right\}, \end{aligned} \tag{224}$$

$$\Theta_3(k_L,k_L,k_S) = \frac{H}{64\epsilon^3 M_p^6}\frac{1}{k_L^5 k_S}\left\{3\left[U_1-U_2\right]+\left(\frac{k_L}{k_S}\right)^2\left[U_1-\left(1+8\left(\frac{k_L}{k_S}\right)\right)U_2\right]\right\}, \tag{225}$$

$$\begin{aligned} \Theta_4(k_L,k_L,k_S) &= -\frac{H^2}{64\tilde{c}_S^2\epsilon^3 M_p^6}\left\{\frac{3}{4}\left(\frac{1}{k_L^5 k_S}+\frac{k_S}{k_L^7}\right)\left[U_1-U_2\left(1+\frac{4}{3}\left(\frac{k_L}{k_S}\right)^2\right)\right]\right. \\ &+\frac{5}{4}\left(\frac{1}{k_L^5 k_S}+\frac{k_S}{k_L^7}\right)\left[U_1-U_2\left(1-\frac{16}{5}\left(\frac{k_L}{k_S}\right)^2\right)\right] \\ &\left. +\frac{3}{2k_L^3 k_S^3}\left(2-\left(\frac{k_S}{k_L}\right)^2\right)\left[U_1-U_2\left(1+\frac{8}{3}\left(\frac{k_L}{k_S}\right)+\frac{4}{3}\left(\frac{k_L}{k_S}\right)^2\right)\right]\right\} \\ &= \Theta_5(k_L,k_L,k_S), \end{aligned} \tag{226}$$

where U_1 and U_2 are defined earlier.

Furthermore, substituting the explicit expressions for $\alpha_j \forall j = 1,2,\cdots,5$ and $\Theta_j(k_L,k_L,k_S)\forall j = 1,2,\cdots,5$ we get the following expression for the bispectrum for scalar fluctuations:

$$B_{EFT}(k_L,k_L,k_S) = \frac{H^2}{64\epsilon^3\tilde{c}_S M_p^6}\sum_{p=1}^{2} g_p(k_L,k_S)U_p, \tag{227}$$

where $g_p(k_L, k_S) \forall p = 1, 2$ are defined as:

$$\begin{aligned} g_1(k_L, k_S) &= \frac{3\tilde{c}_S}{2k_L^5 k_S} \left\{ \left(1 - \frac{1}{c_S^2}\right) \dot{H} M_p^2 + \frac{3}{2}\bar{M}_1^3 H - \frac{4}{3} M_3^4 \right\} \\ &\quad + \frac{1}{\tilde{c}_S k_L^5 k_S} \left\{ \left(1 - \frac{1}{c_S^2}\right) \dot{H} M_p^2 + \frac{3}{2}\bar{M}_1^3 H \right\} \left(2 - \frac{5}{4}\left(\frac{k_S}{k_L}\right)^2\right) \\ &\quad - \frac{9}{2}\bar{M}_1^3 H \frac{\tilde{c}_S}{k_L^5 k_S} \left(3 + \left(\frac{k_L}{k_S}\right)^2\right) + \frac{3}{\tilde{c}_S}\bar{M}_1^3 H \left\{ 2\left(\frac{1}{k_L^5 k_S} + \frac{k_S}{k_L^7}\right) + \frac{3}{2k_L^3 k_S^3}\left(2 - \left(\frac{k_S}{k_L}\right)^2\right) \right\}, \end{aligned} \tag{228}$$

$$\begin{aligned} g_2(k_L, k_S) &= \frac{24\tilde{c}_S}{k_L^5 k_S} \left\{ \left(1 - \frac{1}{c_S^2}\right) \dot{H} M_p^2 + \frac{3}{2}\bar{M}_1^3 H - \frac{4}{3} M_3^4 \right\} \left(\frac{k_L}{k_S}\right)^2 \\ &\quad + \frac{1}{\tilde{c}_S k_L^5 k_S} \left\{ \left(1 - \frac{1}{c_S^2}\right) \dot{H} M_p^2 + \frac{3}{2}\bar{M}_1^3 H \right\} \left\{ 3 + 2\left(\frac{k_S}{k_L}\right)^2 \right. \\ &\quad \left. + \frac{5}{4}\left(2 - \frac{5}{4}\left(\frac{k_S}{k_L}\right)^2\right)\left(1 - \frac{8}{5}\left(\frac{k_S}{k_L}\right)^3\right) \right\} - \frac{9}{2}\bar{M}_1^3 H \frac{\tilde{c}_S}{k_L^5 k_S} \left\{ 3 + \left(\frac{k_L}{k_S}\right)^2 \left(1 + 8\left(\frac{k_L}{k_S}\right)\right) \right\} \\ &\quad + \frac{3}{\tilde{c}_S}\bar{M}_1^3 H \left\{ 2\left(\frac{1}{k_L^5 k_S} + \frac{k_S}{k_L^7}\right) + \frac{3}{2k_L^3 k_S^3}\left(2 - \left(\frac{k_S}{k_L}\right)^2\right) \right\}. \end{aligned} \tag{229}$$

- **For Bunch–Davies vacuum:**
  After setting $C_1 = 1$ and $C_2 = 0$ we get:

$$U_1 = 2, \quad U_2 = 0. \tag{230}$$

Consequently, we get the following expression for the bispectrum for scalar fluctuations:

$$\begin{aligned} B_{EFT}(k_L, k_L, k_S) &= \frac{H^2}{4\epsilon \tilde{c}_S M_p^2} \frac{1}{8M_p^4 \epsilon^2} \left[\frac{3\tilde{c}_S}{2k_L^5 k_S} \left\{ \left(1 - \frac{1}{c_S^2}\right) \dot{H} M_p^2 + \frac{3}{2}\bar{M}_1^3 H - \frac{4}{3} M_3^4 \right\} \right. \\ &\quad + \frac{1}{\tilde{c}_S k_L^5 k_S} \left\{ \left(1 - \frac{1}{c_S^2}\right) \dot{H} M_p^2 + \frac{3}{2}\bar{M}_1^3 H \right\} \left(2 - \frac{5}{4}\left(\frac{k_S}{k_L}\right)^2\right) \\ &\quad - \frac{9}{2}\bar{M}_1^3 H \frac{\tilde{c}_S}{k_L^5 k_S} \left(3 + \left(\frac{k_L}{k_S}\right)^2\right) \\ &\quad \left. + \frac{3}{\tilde{c}_S}\bar{M}_1^3 H \left\{ 2\left(\frac{1}{k_L^5 k_S} + \frac{k_S}{k_L^7}\right) + \frac{3}{2k_L^3 k_S^3}\left(2 - \left(\frac{k_S}{k_L}\right)^2\right) \right\} \right]. \end{aligned} \tag{231}$$

For $\tilde{c}_S = 1 = c_S$ case we know that $M_2 = 0$ and $M_3 = 0$ which we have already shown earlier. As a result the bispectrum for scalar fluctuation can be expressed in the following simplified form:

$$\begin{aligned} B_{EFT}(k_L, k_L, k_S) &= \frac{H^2}{4\epsilon M_p^2} \frac{1}{8M_p^4 \epsilon^2} \bar{M}_1^3 H \left[\frac{9}{4k_L^5 k_S} + \frac{3}{2k_L^5 k_S} \left(2 - \frac{5}{4}\left(\frac{k_S}{k_L}\right)^2\right) \right. \\ &\quad \left. - \frac{9}{2}\frac{1}{k_L^5 k_S} \left(3 + \left(\frac{k_L}{k_S}\right)^2\right) + 3\left\{ 2\left(\frac{1}{k_L^5 k_S} + \frac{k_S}{k_L^7}\right) + \frac{3}{2k_L^3 k_S^3}\left(2 - \left(\frac{k_S}{k_L}\right)^2\right) \right\} \right]. \end{aligned} \tag{232}$$

For $\tilde{c}_S < 1$ and $c_S < 1$ case one can also recast the bispectrum for scalar fluctuations in the following simplified form:

$$\begin{aligned} B_{EFT}(k_L, k_L, k_S) \quad = \quad & \frac{H^2}{4\epsilon \tilde{c}_S M_p^2} \frac{1}{8M_p^4\epsilon^2} \bar{M}_1^3 H \left[\frac{3\tilde{c}_S}{2k_L^5 k_S} \left\{ \frac{3}{2} + \frac{4}{3}\frac{\tilde{c}_3}{\tilde{c}_4} + \frac{2c_S^2}{\tilde{c}_4} \right\} \right. \\ & + \frac{1}{\tilde{c}_S k_L^5 k_S} \left\{ \frac{2c_S^2}{\tilde{c}_4} + \frac{3}{2} \right\} \left(2 - \frac{5}{4}\left(\frac{k_S}{k_L}\right)^2 \right) \\ & - \frac{9}{2}\frac{\tilde{c}_S}{k_L^5 k_S} \left(3 + \left(\frac{k_L}{k_S}\right)^2 \right) \\ & \left. + \frac{3}{\tilde{c}_S} \left\{ 2\left(\frac{1}{k_L^5 k_S} + \frac{k_S}{k_L^7} \right) + \frac{3}{2k_L^3 k_S^3} \left(2 - \left(\frac{k_S}{k_L}\right)^2 \right) \right\} \right]. \end{aligned} \tag{233}$$

- **For α, β vacuum:**
After setting $C_1 = \cosh\alpha$ and $C_2 = e^{i\beta}\ \sinh\alpha$ we get:

$$U_1 = J_1(\alpha, \beta), \quad U_2 = J_2(\alpha, \beta), \tag{234}$$

where $J_1(\alpha, \beta)$ and $J_2(\alpha, \beta)$ are defined earlier.

Consequently, we get the following expression for the bispectrum for scalar fluctuations:

$$B_{EFT}(k_L, k_L, k_S) \quad = \quad \frac{H^2}{4\epsilon \tilde{c}_S M_p^2} \frac{1}{16M_p^4\epsilon^2} \left[g_1(k_L, k_S) J_1(\alpha, \beta) + g_2(k_L, k_S) J_2(\alpha, \beta) \right]. \tag{235}$$

For $\tilde{c}_S = 1 = c_S$ case we know that $M_2 = 0$ and $M_3 = 0$ which we have already shown earlier. As a result the factors $g_1(k_L, k_S)$ and $g_2(k_L, k_S)$ appearing in the expression for bispectrum for scalar fluctuation can be expressed in the following simplified form:

$$\begin{aligned} g_1(k_L, k_S) \quad = \quad & \frac{9}{4k_L^5 k_S} \bar{M}_1^3 H + \frac{3}{2k_L^5 k_S} \bar{M}_1^3 H \left(2 - \frac{5}{4}\left(\frac{k_S}{k_L}\right)^2 \right) \\ & - \frac{9}{2} \bar{M}_1^3 H \frac{1}{k_L^5 k_S} \left(3 + \left(\frac{k_L}{k_S}\right)^2 \right) \\ & + 3\bar{M}_1^3 H \left\{ 2\left(\frac{1}{k_L^5 k_S} + \frac{k_S}{k_L^7} \right) + \frac{3}{2k_L^3 k_S^3} \left(2 - \left(\frac{k_S}{k_L}\right)^2 \right) \right\}, \end{aligned} \tag{236}$$

$$\begin{aligned} g_2(k_L, k_S) \quad = \quad & \frac{36}{k_L^5 k_S} \bar{M}_1^3 H \left(\frac{k_L}{k_S}\right)^2 + \frac{3}{2k_L^5 k_S} \bar{M}_1^3 H \left\{ 3 + 2\left(\frac{k_S}{k_L}\right)^2 \right. \\ & \left. + \frac{5}{4}\left(2 - \frac{5}{4}\left(\frac{k_S}{k_L}\right)^2 \right) \left(1 - \frac{8}{5}\left(\frac{k_S}{k_L}\right)^3 \right) \right\} \\ & - \frac{9}{2} \bar{M}_1^3 H \frac{1}{k_L^5 k_S} \left\{ 3 + \left(\frac{k_L}{k_S}\right)^2 \left(1 + 8\left(\frac{k_L}{k_S}\right) \right) \right\} \\ & + 3\bar{M}_1^3 H \left\{ 2\left(\frac{1}{k_L^5 k_S} + \frac{k_S}{k_L^7} \right) + + \frac{3}{2k_L^3 k_S^3} \left(2 - \left(\frac{k_S}{k_L}\right)^2 \right) \right\}. \end{aligned} \tag{237}$$

For $\tilde{c}_S < 1$ and $c_S < 1$ case one can also recast the factors $g_1(k_L, k_S)$ and $g_2(k_L, k_S)$ as appearing in the expression for bispectrum for scalar fluctuations in the following simplified form:

$$\begin{aligned} g_1(k_L, k_S) &= \frac{3\tilde{c}_S}{2k_L^5 k_S}\left\{\frac{3}{2}+\frac{4}{3}\frac{\tilde{c}_3}{\tilde{c}_4}+\frac{2c_S^2}{\tilde{c}_4}\right\} \\ &\quad+\frac{1}{\tilde{c}_S k_L^5 k_S}\left\{\frac{2c_S^2}{\tilde{c}_4}+\frac{3}{2}\right\}\left(2-\frac{5}{4}\left(\frac{k_S}{k_L}\right)^2\right) \\ &\quad-\frac{9}{2}\bar{M}_1^3 H\frac{\tilde{c}_S}{k_L^5 k_S}\left(3+\left(\frac{k_L}{k_S}\right)^2\right) \\ &\quad+\frac{3}{\tilde{c}_S}\bar{M}_1^3 H\left\{2\left(\frac{1}{k_L^5 k_S}+\frac{k_S}{k_L^7}\right)+\frac{3}{2k_L^3 k_S^3}\left(2-\left(\frac{k_S}{k_L}\right)^2\right)\right\}, \end{aligned} \tag{238}$$

$$\begin{aligned} g_2(k_L, k_S) &= \frac{24\tilde{c}_S}{k_L^5 k_S}\left\{\frac{3}{2}+\frac{4}{3}\frac{\tilde{c}_3}{\tilde{c}_4}+\frac{2c_S^2}{\tilde{c}_4}\right\}\left(\frac{k_L}{k_S}\right)^2 \\ &\quad+\frac{1}{\tilde{c}_S k_L^5 k_S}\left\{\frac{2c_S^2}{\tilde{c}_4}+\frac{3}{2}\right\}\left\{3+2\left(\frac{k_S}{k_L}\right)^2+\frac{5}{4}\left(2-\frac{5}{4}\left(\frac{k_S}{k_L}\right)^2\right)\left(1-\frac{8}{5}\left(\frac{k_S}{k_L}\right)^3\right)\right\} \\ &\quad-\frac{9}{2}\bar{M}_1^3 H\frac{\tilde{c}_S}{k_L^5 k_S}\left\{3+\left(\frac{k_L}{k_S}\right)^2\left(1+8\left(\frac{k_L}{k_S}\right)\right)\right\} \\ &\quad+\frac{3}{\tilde{c}_S}\bar{M}_1^3 H\left\{2\left(\frac{1}{k_L^5 k_S}+\frac{k_S}{k_L^7}\right)++\frac{3}{2k_L^3 k_S^3}\left(2-\left(\frac{k_S}{k_L}\right)^2\right)\right\}. \end{aligned} \tag{239}$$

5. Determination of EFT Coefficients and Future Predictions

In this section, we compute the exact analytical expression for the EFT coefficients for two specific cases—1. Canonical single-field slow-roll inflation and 2. General single-field $P(X, \phi)$ models of inflation, where $X = -\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ is the kinetic term. To determine the EFT coefficients for the canonical single-field slow-roll model or from general single-field $P(X, \phi)$ model of inflation we will follow the following strategy:

1. First, we will start with the general expression for the three-point function and the bispectrum for scalar perturbations with an arbitrary choice of quantum vacuum. Then we take the Bunch–Davies and α, β vacuum to match with the standard results of scalar three-point function.
2. Next we take the equilateral limit and squeezed limit configuration of the bispectrum obtained from the single-field slow-roll model and general single-field $P(X, \phi)$ model.
3. Furthermore, we equate the equilateral limit and squeezed limit configuration of the bispectrum computed from the EFT of inflation with the single-field slow-roll or from the general single-field $P(X, \phi)$ model.
4. Finally, for sound speed $c_S = 1$ and $c_S < 1$ we get the analytical expressions for all the EFT coefficients for canonical single-field slow-roll models or from generalized single-field $P(X, \phi)$ models of inflation.

5.1. For Canonical Single-Field Slow-Roll Inflation

Here our prime objective is to derive the EFT coefficients by computing the most general expression for the three-point function for scalar fluctuations from the canonical single-field slow-roll model of inflation for arbitrary vacuum. Then we give specific example for Bunch–Davies and α, β vacuum for completeness.

5.1.1. Basic Setup

Let us start with the action for single scalar field (inflaton) which has canonical kinetic term as given by:

$$\textbf{Canonical model}: \; S = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R + X - V(\phi)\right], \tag{240}$$

where $V(\phi)$ is the potential which satisfies the slow-roll condition for inflation.

It is important to mention here that perturbations to the homogeneous situation discussed above are introduced in the ADM formalism where the metric takes the form [31]:

$$\textbf{ADM metric}: \; ds^2 = -N^2dt^2 + g_{ij}\left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right), \tag{241}$$

where g_{ij} is the metric on the spatial three-surface characterized by t, lapse N and shift N_i. Here we choose synchronous gauge to maintain diffeomorphism invariance of the theory where the gauge-fixing conditions are given by:

$$\textbf{Synchronous gauge}: \quad N = 1, \quad N^i = 0\,, \tag{242}$$

and the perturbed metric is given by:

$$g_{ij} = a^2(t)\left[(1+2\zeta(t,\mathbf{x}))\delta_{ij} + \gamma_{ij}\right], \quad \gamma_{ii} = 0\,, \tag{243}$$

where $\zeta(t,\mathbf{x})$ and γ_{ij} are defined earlier. Here it is important to note that the structure of g_{ij} is exactly same that we have mentioned in the case of EFT framework discussed in this paper. Please note that in the context of ADM formalism one can treat the scalar field ϕ, induced metric g_{ij} as the dynamical variables. On the other hand, N and N^i mimics the role of Lagrange multipliers in ADM formalism. Consequently, one needs to impose the equations of motion of N, N^i as additional constraints in the synchronous gauge where the gauge condition as stated in Equation (242) holds good perfectly. More precisely, in this context the equations of motion of N and N^i correspond to time and spatial reparametrization invariance.

Furthermore, using the ADM metric as stated in Equation (241), the action for the single scalar field Equation (240) can be recast as [31]:

$$S = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}N\,{}^{(3)}R - NV + \frac{1}{2N}\left(E_{ij}E^{ij} - E^2\right) + \frac{1}{2N}\left(\dot{\phi} - N^i\partial_i\phi\right)^2 - Ng^{ij}\partial_i\phi\partial_j\phi\right], \tag{244}$$

where ${}^{(3)}R$ is the Ricci scalar curvature of the spatial slice. Also, here E_{ij} and E is defined as [31]:

$$E_{ij} := \frac{1}{2}\left(\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i\right) = NK_{ij}, \tag{245}$$

$$E := E^i_i = g_{ij}E^{ij} = g_{ij}g^{im}g^{jn}E_{mn} = Ng_{ij}g^{im}g^{jn}K_{mn}. \tag{246}$$

Here the covariant derivative ∇_i, is taken with respect to the 3-metric g_{ij}. Also, in this context the extrinsic curvature K_{ij} is defined as [31]:

$$K_{ij} = \frac{1}{N}E_{ij} = \frac{1}{2N}\left(\dot{g}_{ij} - \nabla_i N_j - \nabla_j N_i\right). \tag{247}$$

Additionally, we choose the following two gauges:

$$\textbf{Gauge I}: \quad \delta\phi(t,\mathbf{x}) = 0, \quad \zeta(t,\mathbf{x}) \neq 0, \quad \partial_i\gamma_{ij} = 0, \quad \gamma_{ii} = 0\,. \tag{248}$$

$$\textbf{Gauge II}: \quad \delta\phi(t,\mathbf{x}) \neq 0, \quad \zeta(t,\mathbf{x}) = 0, \quad \partial_i\gamma_{ij} = 0, \quad \gamma_{ii} = 0\,. \tag{249}$$

For our present computations, we will work in **Gauge I** as this is exactly same as the unitary gauge that we have used in the context of EFT framework. Also, the tensor perturbation γ_{ij} is exactly same for the unitary gauge that we have used for EFT setup.

5.1.2. Scalar Three-Point Function for Single-Field Slow-Roll inflation

Before computing the three-point function for scalar mode fluctuation here it is important to note that the two-point function for single-field slow-roll inflation is exactly same with the results obtained for EFT of inflation with sound speed $c_S = 1$ and $\tilde{c}_S = 1$, which can be obtained by setting the EFT coefficients, $M_2 = 0$, $M_3 = 0$, $\bar{M}_1 \neq 0$, $M_4 \neq 0$, $\bar{M}_2 \neq 0$, $\bar{M}_3 \neq 0$, $\tilde{c}_5 = -\frac{1}{2}(1+\epsilon)$[21]. Using three-point function we can able to fix all of these coefficients.

We here now proceed to calculate the three-point function for the scalar fluctuation $\zeta(t, \mathbf{x})$ in the interacting picture with arbitrary vacuum. Then we cite results for Bunch–Davies and α, β vacuum. For single-field slow-roll inflation, the third-order term in the action Equation (244) is given by [31]:

$$S^{(3)}_{\zeta} = \int d^4x \left[a^3\epsilon^2\tilde{\zeta}\dot{\tilde{\zeta}}^2 + a\epsilon^2\tilde{\zeta}(\partial\tilde{\zeta})^2 - 2a^3\epsilon\dot{\tilde{\zeta}}\partial_i\tilde{\zeta}\partial_i(\epsilon\partial^{-2}\dot{\tilde{\zeta}})\right], \tag{251}$$

which is derived from Equation (244) and here after neglecting all the contribution from the terms which are sub-leading in the slow-roll parameters. Additionally, here we use the following field redefinition:

$$\zeta = \tilde{\zeta} + \left\{\epsilon - \frac{\eta}{2}\right\}\tilde{\zeta}^2, \tag{252}$$

where ϵ, η, δ and s are slow-roll parameters which are defined in the context of single-field slow-roll inflation as:

$$\epsilon \sim \frac{1}{2M_p^2}\frac{\dot{\phi}^2}{H^2}, \quad \eta \sim \epsilon - \delta, \quad \delta = \frac{\ddot{\phi}}{H\dot{\phi}}, \quad s = 0. \tag{253}$$

Here one can also express the slow-roll parameters ϵ and η in terms of the slowly varying potential $V(\phi)$ as, $\epsilon \sim \epsilon_V, \eta \sim \eta_V - \epsilon_V, \delta \sim 2\epsilon_V - \eta_V$. where the new slow-roll parameter ϵ_V and η_V are defined as, $\epsilon_V = \frac{M_p^2}{2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2, \quad \eta_V = M_p^2\left(\frac{V''(\phi)}{V(\phi)}\right)$. Here $'$ represents $d/d\phi$.

Now it is important to note that in the present context of discussion we are interested in the three-point function for the scalar fluctuation field ζ, not for the redefined scalar field fluctuation $\tilde{\zeta}$ and

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[21] In the case of single-field slow-roll inflation amplitude of power spectrum and spectral tilt for scalar fluctuation can be written at the horizon crossing $|k\eta| = 1$ as:

$$\textbf{For Bunch\text{\textasciicaron}Davies vacuum}: \quad \Delta_{\zeta}(k_*) = \begin{cases} \dfrac{V(\phi_*)}{24\pi^2 M_p^4\epsilon_V} & \textbf{for dS} \\ 2^{6\epsilon_V - 2\eta_V}\dfrac{V(\phi_*)}{24\pi^2 M_p^4\epsilon_V(1+\epsilon_V)^2}\left|\dfrac{\Gamma(\frac{3}{2}+4\epsilon_V-\eta_V)}{\Gamma\left(\frac{3}{2}\right)}\right|^2 & \textbf{for qdS}. \end{cases}$$

$$\textbf{For ff, fi vacuum}: \quad \Delta_{\zeta}(k_*) = \begin{cases} \dfrac{V(\phi_*)}{24\pi^2 M_p^4\epsilon_V}\left[\cosh 2\alpha - \sinh 2\alpha\cos\beta\right] & \textbf{for dS} \\ 2^{6\epsilon_V - 2\eta_V}\dfrac{V(\phi_*)}{24\pi^2 M_p^4\epsilon_V(1+\epsilon_V)^2}\left|\dfrac{\Gamma(\frac{3}{2}+4\epsilon_V-\eta_V)}{\Gamma\left(\frac{3}{2}\right)}\right|^2 & \\ \left[\cosh 2\alpha - \sinh 2\alpha\cos\left(\pi\left(2+4\epsilon_V-\eta_V\right)+\beta\right)\right] & \textbf{for qdS}. \end{cases}$$

and

$$n_{\zeta}(k_*) - 1 = 2\eta_V - 6\epsilon_V. \tag{250}$$

for this reason one can write down the exact connection between the three-point function for the scalar function field ζ and redefined scalar fluctuation field $\tilde{\zeta}$ in position space as:

$$\begin{aligned}\langle\zeta(\mathbf{x_1})\zeta(\mathbf{x_2})\zeta(\mathbf{x_3})\rangle \;&=\; \langle\tilde{\zeta}(\mathbf{x_1})\tilde{\zeta}(\mathbf{x_2})\tilde{\zeta}(\mathbf{x_3})\rangle + (2\epsilon-\eta)\left[\langle\zeta(\mathbf{x_1})\zeta(\mathbf{x_2})\rangle\langle\zeta(\mathbf{x_1})\zeta(\mathbf{x_3})\rangle\right. \\ &\quad \left. + \langle\zeta(\mathbf{x_2})\zeta(\mathbf{x_1})\rangle\langle\zeta(\mathbf{x_2})\zeta(\mathbf{x_3})\rangle + \langle\zeta(\mathbf{x_3})\zeta(\mathbf{x_1})\rangle\langle\zeta(\mathbf{x_3})\zeta(\mathbf{x_2})\rangle\right].\end{aligned} \tag{254}$$

After taking the Fourier transform of the scalar function field ζ and redefined scalar fluctuation field $\tilde{\zeta}$ one can express connection between three-point function in momentum space and this is also our main point of interest.

The interaction Hamiltonian for the redefined scalar fluctuation $\tilde{\zeta}$ can be expressed as:

$$H_{int} = \int d^3x \left[a\,\epsilon^2\tilde{\zeta}\tilde{\zeta}'^2 + a\,\epsilon^2\tilde{\zeta}(\partial\tilde{\zeta})^2 - 2a\epsilon\tilde{\zeta}'\partial_i\tilde{\zeta}\partial_i(\epsilon\partial^{-2}\tilde{\zeta}')\right]. \tag{255}$$

Furthermore, following the in-in formalism in interaction picture the expression for the three-point function for the redefined scalar fluctuation $\tilde{\zeta}$ and then transforming the final result in terms of the scalar fluctuation ζ in momentum one can write the following expression:

$$\begin{aligned}\langle\zeta(\mathbf{k_1})\zeta(\mathbf{k_2})\zeta(\mathbf{k_3})\rangle \;&= -i\int_{\eta_i=-\infty}^{\eta_f=0} d\eta\; a(\eta)\;\langle 0|\left[\zeta(\eta_f,\mathbf{k_1})\zeta(\eta_f,\mathbf{k_2})\zeta(\eta_f,\mathbf{k_3}), H_{int}(\eta)\right]|0\rangle \\ &= (2\pi)^3\delta^{(3)}(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3})B_{SFSR}(k_1,k_2,k_3)\,,\end{aligned} \tag{256}$$

where $B_{SFSR}(k_1,k_2,k_3)$ represents the bispectrum of scalar fluctuation ζ, which is computed from single-field slow-roll inflation. Here the final expression for the bispectrum of scalar fluctuation for arbitrary vacuum is given by:

$$\begin{aligned}B_{SFSR}(k_1,k_2,k_3) \;&= \frac{H^4}{32\epsilon^2M_p^4}\frac{1}{(k_1k_2k_3)^3}\left[2(2\epsilon-\eta)\left(|C_1|^2+|C_2|^2\right)^2\sum_{i=1}^{3}k_i^3\right. \\ &\quad +\epsilon\left(|C_1|^2-|C_2|^2\right)^2\left(-\sum_{i=1}^{3}k_i^3+\sum_{i,j=1,i\neq j}^{3}k_ik_j^2+\frac{8}{K}\sum_{i,j=1,i>j}^{3}k_i^2k_j^2\right) \\ &\quad +\epsilon\,(C_1^*C_2+C_1C_2^*)^2\left(-\sum_{i=1}^{3}k_i^3+\sum_{i,j=1,i\neq j}^{3}k_ik_j^2\right. \\ &\quad \left.\left. +\,8\sum_{i,j=1,i>j}^{3}k_i^2k_j^2\sum_{m=1}^{3}\frac{1}{K-2k_m}\right)\right].\end{aligned} \tag{257}$$

For Bunch–Davies and α,β vacuum we get the following simplified expression for the bispectrum for scalar fluctuation:

- **For Bunch–Davies vacuum**:
  After setting $C_1=1$ and $C_2=0$ we get [31]:

$$\begin{aligned}B_{SFSR}(k_1,k_2,k_3) \;&= \frac{H^4}{32\epsilon^2M_p^4}\frac{1}{(k_1k_2k_3)^3}\left[2(2\epsilon-\eta)\sum_{i=1}^{3}k_i^3\right. \\ &\quad \left. +\epsilon\left(-\sum_{i=1}^{3}k_i^3+\sum_{i,j=1,i\neq j}^{3}k_ik_j^2+\frac{8}{K}\sum_{i,j=1,i>j}^{3}k_i^2k_j^2\right)\right].\end{aligned} \tag{258}$$

- **For α, β vacuum**:
After setting $C_1 = \cosh\alpha$ and $C_2 = e^{i\beta}\sinh\alpha$ we get [33]:

$$\begin{aligned} B_{SFSR}(k_1,k_2,k_3) &= \frac{H^4}{32\epsilon^2 M_p^4}\frac{1}{(k_1k_2k_3)^3}\left[2(2\epsilon-\eta)\cosh^2 2\alpha\sum_{i=1}^{3}k_i^3\right.\\ &+\epsilon\left(-\sum_{i=1}^{3}k_i^3+\sum_{i,j=1,i\neq j}^{3}k_ik_j^2+\frac{8}{K}\sum_{i,j=1,i>j}^{3}k_i^2k_j^2\right)\\ &\left.+\epsilon\sinh^2 2\alpha\cos^2\beta\left(-\sum_{i=1}^{3}k_i^3+\sum_{i,j=1,i\neq j}^{3}k_ik_j^2+8\sum_{i,j=1,i>j}^{3}k_i^2k_j^2\sum_{m=1}^{3}\frac{1}{K-2k_m}\right)\right]. \end{aligned} \tag{259}$$

Furthermore, we consider equilateral limit and squeezed limit in which we finally get:

1. Equilateral limit configuration:
Here the bispectrum for scalar perturbations in the presence of arbitrary quantum vacuum can be expressed as:

$$\begin{aligned} B_{SFSR}(k,k,k) &= \frac{H^4}{32\epsilon^2 M_p^4}\frac{1}{k^6}\left[6(2\epsilon-\eta)\left(|C_1|^2+|C_2|^2\right)^2+11\epsilon\left(|C_1|^2-|C_2|^2\right)^2\right.\\ &\left.+27\epsilon\,(C_1^*C_2+C_1C_2^*)^2\right]. \end{aligned} \tag{260}$$

Now for Bunch–Davies and α, β vacuum we get the following simplified expression for the bispectrum for scalar fluctuation:

- **For Bunch–Davies vacuum**:
After setting $C_1 = 1$ and $C_2 = 0$ we get:

$$B_{SFSR}(k,k,k) = \frac{H^4}{32\epsilon^2 M_p^4}\frac{1}{k^6}\left[23\epsilon-6\eta\right]. \tag{261}$$

- **For α, β vacuum**:
After setting $C_1 = \cosh\alpha$ and $C_2 = e^{i\beta}\sinh\alpha$ we get:

$$B_{SFSR}(k,k,k) = \frac{H^4}{32\epsilon^2 M_p^4}\frac{1}{k^6}\left[6(2\epsilon-\eta)\cosh^2 2\alpha+11\epsilon+27\epsilon\sinh^2 2\alpha\cos^2\beta\right]. \tag{262}$$

2. Squeezed limit configuration:
Here the bispectrum for scalar perturbations in the presence of arbitrary quantum vacuum can be expressed as:

$$B_{SFSR}(k_L,k_L,k_S) = \frac{H^4}{32\epsilon^2 M_p^4}\frac{1}{k_L^3k_S^3}\sum_{j=-1}^{3}a_j\left(\frac{k_S}{k_L}\right)^j, \tag{263}$$

where the expansion coefficients $a_j \forall j = -1, \cdots, 3$ for arbitrary vacuum are defined as:

$$\begin{aligned} a_{-1} &= 16\epsilon\,(C_1^*C_2+C_1C_2^*)^2,\\ a_0 &= 4(2\epsilon-\eta)\left(|C_1|^2+|C_2|^2\right)^2+4\epsilon\left(|C_1|^2-|C_2|^2\right)^2+4\epsilon\,(C_1^*C_2+C_1C_2^*)^2,\\ a_1 &= 34\epsilon\,(C_1^*C_2+C_1C_2^*)^2,\quad a_2 = 10\epsilon\left(|C_1|^2-|C_2|^2\right)^2+10\epsilon\,(C_1^*C_2+C_1C_2^*)^2,\\ a_3 &= 2(2\epsilon-\eta)\left(|C_1|^2+|C_2|^2\right)^2-5\epsilon\left(|C_1|^2-|C_2|^2\right)^2-\epsilon\,(C_1^*C_2+C_1C_2^*)^2. \end{aligned}$$

Now for Bunch–Davies and α, β vacuum we get the following simplified expression for the bispectrum for scalar fluctuation:

- **For Bunch–Davies vacuum**:
  After setting $C_1 = 1$ and $C_2 = 0$, we get the following expression for the expansion coefficients $a_j \forall j = -1, \cdots, 3$:

$$a_{-1} = 0, \; a_0 = 4(3\epsilon - \eta), \; a_1 = 0, \; a_2 = 10\epsilon, \; a_3 = -(\epsilon + 2\eta). \tag{264}$$

  Consequently, the bispectrum can be recast as:

$$B_{SFSR}(k_L, k_L, k_S) = \frac{H^4}{32\epsilon^2 M_p^4} \frac{1}{k_L^3 k_S^3} \left[4(3\epsilon - \eta) + 10\epsilon \left(\frac{k_S}{k_L} \right)^2 - (\epsilon + 2\eta) \left(\frac{k_S}{k_L} \right)^3 \right]. \tag{265}$$

- **For α, β vacuum**:
  After setting $C_1 = \cosh\alpha$ and $C_2 = e^{i\beta} \sinh\alpha$, we get the following expression for the expansion coefficients $a_j \forall j = -1, \cdots, 3$:

$$\begin{aligned} a_{-1} &= 16\epsilon \sinh^2 2\alpha \cos^2 \beta, \; a_0 = 4(2\epsilon - \eta) \cosh^2 2\alpha + 4\epsilon + 4\epsilon \sinh^2 2\alpha \cos^2 \beta, \\ a_1 &= 34\epsilon \sinh^2 2\alpha \cos^2 \beta, \; a_2 = 10\epsilon + 10\epsilon \sinh^2 2\alpha \cos^2 \beta, \\ a_3 &= 2(2\epsilon - \eta) \cosh^2 2\alpha - 5\epsilon - \epsilon \sinh^2 2\alpha \cos^2 \beta. \end{aligned}$$

  Consequently, the bispectrum can be recast as:

$$\begin{aligned} B_{SFSR}(k_L, k_L, k_S) &= \frac{H^4}{32\epsilon^2 M_p^4} \frac{1}{k_L^3 k_S^3} \left[16\epsilon \sinh^2 2\alpha \cos^2 \beta \left(\frac{k_S}{k_L} \right)^{-1} \right. \\ &+ \Big(4(2\epsilon - \eta) \cosh^2 2\alpha + 4\epsilon + 4\epsilon \sinh^2 2\alpha \cos^2 \beta \Big) \\ &\quad + 34\epsilon \sinh^2 2\alpha \cos^2 \beta \left(\frac{k_S}{k_L} \right) + \Big(10\epsilon + 10\epsilon \sinh^2 2\alpha \cos^2 \beta \Big) \left(\frac{k_S}{k_L} \right)^2 \\ &\quad \left. + \Big(2(2\epsilon - \eta) \cosh^2 2\alpha - 5\epsilon - \epsilon \sinh^2 2\alpha \cos^2 \beta \Big) \left(\frac{k_S}{k_L} \right)^3 \right]. \end{aligned} \tag{266}$$

5.1.3. Expression for EFT Coefficients for Single-Field Slow-Roll Inflation

Here our prime objective is to derive the analytical expressions for EFT coefficients for single-field slow-roll inflation. To serve this purpose we start with a claim that the three-point function and the associated bispectrum for the scalar fluctuations computed from single-field slow-roll inflation is exactly same as that we have computed from EFT setup for consistent UV completion. Here we use the equilateral limit and squeezed limit configurations to extract the analytical expression for the EFT coefficients. In the two limiting cases the results are as follows:

1. Equilateral limit configuration:
   For this case with arbitrary vacuum one can write:

$$B_{EFT}(k, k, k) = B_{SFSR}(k, k, k), \tag{267}$$

which implies that:

$$
\begin{aligned}
\bar{M}_1 &= \left\{ \frac{HM_p^2\epsilon\Big[6(\eta-2\epsilon)\big(|C_1|^2+|C_2|^2\big)^2-11\epsilon\big(|C_1|^2-|C_2|^2\big)^2-27\epsilon(C_1^*C_2+C_1C_2^*)^2\Big]}{\left[\frac{125}{12}U_1+\frac{8073}{196}U_2\right]} \right\}^{\frac{1}{3}}, \\
\bar{M}_2 &\approx \bar{M}_3 = \sqrt{\frac{\bar{M}_1^3}{4H\tilde{c}_5}} = \left\{ \frac{2M_p^2\epsilon\Big[6(2\epsilon-\eta)\big(|C_1|^2+|C_2|^2\big)^2+11\epsilon\big(|C_1|^2-|C_2|^2\big)^2+27\epsilon(C_1^*C_2+C_1C_2^*)^2\Big]}{(1+\epsilon)\left[\frac{125}{3}U_1+\frac{8073}{49}U_2\right]} \right\}^{\frac{1}{2}}, \\
\tilde{c}_5 &= -\frac{1}{2}(1+\epsilon), \quad M_2=0, \quad M_3=0, \\
M_4 &= \left(-\frac{\tilde{c}_3}{\tilde{c}_6}H\bar{M}_1^3\right)^{\frac{1}{4}} = \left\{ \frac{\tilde{c}_3H^2M_p^2\epsilon\Big[6(2\epsilon-\eta)\big(|C_1|^2+|C_2|^2\big)^2+11\epsilon\big(|C_1|^2-|C_2|^2\big)^2+27\epsilon(C_1^*C_2+C_1C_2^*)^2\Big]}{\tilde{c}_6\left[\frac{125}{12}U_1+\frac{8073}{196}U_2\right]} \right\}^{\frac{1}{4}}.
\end{aligned} \tag{268}
$$

where for arbitrary vacuum U_1 and U_2 are defined as:

$$U_1 = \left[(C_1-C_2)^3\left(C_1^{*3}+C_2^{*3}\right)+(C_1^*-C_2^*)^3\left(C_1^3+C_2^3\right)\right], \tag{269}$$

$$U_2 = \left[(C_1-C_2)^3 C_1^*C_2^*(C_1^*-C_2^*)+(C_1^*-C_2^*)^3 C_1C_2(C_1-C_2)\right]. \tag{270}$$

To constraint all these coefficients of EFT operators using CMB observations from Planck TT+low P data we use [32]:

$$
\begin{aligned}
\epsilon &< 0.012 \;\; (95\%\,\mathrm{CL}), \; \eta = -0.0080^{+0.0088}_{-0.0146} \;\; (68\%\,\mathrm{CL}), \; c_S = 1 \;\; (95\%\,\mathrm{CL}), \\
H = H_{inf} &\leq 1.09\times10^{-4}\,M_p\,\sqrt{\epsilon\,c_S},
\end{aligned} \tag{271}
$$

where $M_p = 2.43\times10^{18}$ GeV is the reduced Planck mass. Now for Bunch–Davies and α,β vacuum we get the following simplified expression for the bispectrum for scalar fluctuation:

- **For Bunch–Davies vacuum**:
  After setting $C_1 = 1$ and $C_2 = 0$ we get:

$$
\begin{aligned}
\bar{M}_1 &= \left\{\frac{6}{125}HM_p^2\epsilon\,[6\eta-23\epsilon]\right\}^{\frac{1}{3}}, \; \bar{M}_2 \approx \bar{M}_3 = \sqrt{\frac{\bar{M}_1^3}{4H\tilde{c}_5}} = \left\{\frac{3}{125(1+\epsilon)}M_p^2\epsilon\,[23\epsilon-6\eta]\right\}^{\frac{1}{2}}, \\
\tilde{c}_5 &= -\frac{1}{2}(1+\epsilon), \;\; M_2=0, \;\; M_3=0, \;\; M_4 = \left(-\frac{\tilde{c}_3}{\tilde{c}_6}H\bar{M}_1^3\right)^{\frac{1}{4}} = \left\{\frac{6\tilde{c}_3}{125\tilde{c}_6}H^2M_p^2\epsilon\,[23\epsilon-6\eta]\right\}^{\frac{1}{4}}.
\end{aligned} \tag{272}
$$

  Furthermore, using the constraint stated in Equation (271) we finally get the following constraints on the coefficients of EFT operators:

$$
\begin{aligned}
&1.23\times10^{-3}\,M_p < |\bar{M}_1| < 1.41\times10^{-3}\,M_p, \;\; 8.79\times10^{-3}\,M_p < |\bar{M}_2| \approx |\bar{M}_3| < 1.08\times10^{-2}\,M_p, \\
&M_2=0, \;\; M_3=0, \;\; 3.86\times10^{-4}\,M_p < M_4\times(-\tilde{c}_6/\tilde{c}_3)^{1/4} < 4.29\times10^{-4}\,M_p.
\end{aligned} \tag{273}
$$

- **For α,β vacuum**:
  After setting $C_1 = \cosh\alpha$ and $C_2 = e^{i\beta}\sinh\alpha$ we get:

$$
\begin{aligned}
\bar{M}_1 &= \left\{\frac{HM_p^2\epsilon\Big[6(\eta-2\epsilon)\cosh^2 2\alpha-11\epsilon-27\epsilon\sinh^2 2\alpha\cos^2\beta\Big]}{\left[\frac{125}{12}J_1(\alpha,\beta)+\frac{8073}{196}J_2(\alpha,\beta)\right]}\right\}^{\frac{1}{3}}, \\
\bar{M}_2 &\approx \bar{M}_3 = \sqrt{\frac{\bar{M}_1^3}{4H\tilde{c}_5}} = \left\{\frac{2M_p^2\epsilon\Big[6(2\epsilon-\eta)\cosh^2 2\alpha+11\epsilon+27\epsilon\sinh^2 2\alpha\cos^2\beta\Big]}{(1+\epsilon)\left[\frac{125}{3}J_1(\alpha,\beta)+\frac{8073}{49}J_2(\alpha,\beta)\right]}\right\}^{\frac{1}{2}}, \\
\tilde{c}_5 &= -\frac{1}{2}(1+\epsilon), \quad M_2=0, \quad M_3=0, \\
M_4 &= \left(-\frac{\tilde{c}_3}{\tilde{c}_6}H\bar{M}_1^3\right)^{\frac{1}{4}} = \left\{\frac{\tilde{c}_3H^2M_p^2\epsilon\Big[6(2\epsilon-\eta)\cosh^2 2\alpha+11\epsilon+27\epsilon\sinh^2 2\alpha\cos^2\beta\Big]}{\tilde{c}_6\left[\frac{125}{12}J_1(\alpha,\beta)+\frac{8073}{196}J_2(\alpha,\beta)\right]}\right\}^{\frac{1}{4}}.
\end{aligned} \tag{274}
$$

Furthermore, using the constraint stated in Equation (271) we finally get the following constraints on the coefficients of EFT operators for a given value of the parameters α and β (say for $\alpha = 0.1$ and $\beta = 0.1$):

$$9.1 \times 10^{-4}\, M_p < |\bar{M}_1| < 1.1 \times 10^{-3}\, M_p,\;\; 1.11 \times 10^{-2}\, M_p < |\bar{M}_2| \approx |\bar{M}_3| < 1.5 \times 10^{-2}\, M_p,$$
$$M_2 = 0,\;\; M_3 = 0,\;\; 3.06 \times 10^{-4}\, M_p < M_4 \times (-\tilde{c}_6/\tilde{c}_3)^{1/4} < 3.54 \times 10^{-4}\, M_p. \tag{275}$$

2. Squeezed limit configuration:
For this case with arbitrary vacuum one can write:

$$B_{EFT}(k_L, k_L, k_S) \;\; = \;\; B_{SFSR}(k_L, k_L, k_S), \tag{276}$$

which implies that:

$$\begin{aligned}
\bar{M}_1 \;\; &= \left\{ 2HM_p^2\epsilon \frac{\sum_{j=-1}^{3} a_j \left(\frac{k_S}{k_L}\right)^j}{\sum_{m=-1}^{3} b_m \left(\frac{k_S}{k_L}\right)^m} \right\}^{\frac{1}{3}},\; \bar{M}_2 \approx \bar{M}_3 = \sqrt{\frac{\bar{M}_1^3}{4H\tilde{c}_5}} = \left\{ -\frac{M_p^2\epsilon}{(1+\epsilon)} \frac{\sum_{j=-1}^{3} a_j \left(\frac{k_S}{k_L}\right)^j}{\sum_{m=-1}^{3} b_m \left(\frac{k_S}{k_L}\right)^m} \right\}^{\frac{1}{2}}, \\
\tilde{c}_5 \;\; &= -\frac{1}{2}\left(1+\epsilon\right), \quad M_2 = 0, \quad M_3 = 0,\; M_4 = \left(-\frac{\tilde{c}_3}{\tilde{c}_6} H\bar{M}_1^3\right)^{\frac{1}{4}} = \left\{ \frac{2H^2M_p^2\epsilon\;\tilde{c}_3}{\tilde{c}_6} \frac{\sum_{j=-1}^{3} a_j \left(\frac{k_S}{k_L}\right)^j}{\sum_{m=-1}^{3} b_m \left(\frac{k_S}{k_L}\right)^m} \right\}^{\frac{1}{4}}.
\end{aligned} \tag{277}$$

where the expansion coefficients $a_j \forall j = -1, \cdots, 3$ are defined earlier and here the coefficients $b_m \forall m = -1, \cdots, 3$ for arbitrary vacuum are defined as:

$$b_{-1} = -36U_2,\;\; b_0 = \frac{9}{2}\left(U_1 + 9U_2\right),\;\; b_1 = 0,\;\; b_2 = \left(\frac{27}{4}U_1 + \frac{17}{2}U_2\right),\;\; b_3 = 0\,, \tag{278}$$

where U_1 and U_2 are already defined earlier.

Now for Bunch–Davies and α, β vacuum we get the following simplified expression for the bispectrum for scalar fluctuation:

- **For Bunch–Davies vacuum**:
After setting $C_1 = 1$ and $C_2 = 0$, we get $U_1 = 2$ and $U_2 = 0$. Consequently, the expansion coefficients can be recast as:

$$a_{-1} = 0,\;\; a_0 = 4(3\epsilon - \eta),\;\; a_1 = 0,\;\; a_2 = 10\epsilon,\;\; a_3 = -(\epsilon + 2\eta)\,, \tag{279}$$

and

$$b_{-1} = 0,\;\; b_0 = 9,\;\; b_1 = 0,\;\; b_2 = \frac{27}{2},\;\; b_3 = 0\,, \tag{280}$$

Finally, the EFT coefficients for scalar fluctuation can be written as:

$$\begin{aligned}
\bar{M}_1 \;\; &= \left\{ \frac{2HM_p^2\epsilon\left[4(3\epsilon-\eta)+10\epsilon\left(\frac{k_S}{k_L}\right)^2-(\epsilon+2\eta)\left(\frac{k_S}{k_L}\right)^3\right]}{\left[18-\frac{27}{2}\left(\frac{k_S}{k_L}\right)^2\right]} \right\}^{\frac{1}{3}}, \\
\bar{M}_2 \;\; &\approx \bar{M}_3 = \sqrt{\frac{\bar{M}_1^3}{4H\tilde{c}_5}} = \left\{ \frac{M_p^2\epsilon\left[4(\eta-3\epsilon)-10\epsilon\left(\frac{k_S}{k_L}\right)^2+(\epsilon+2\eta)\left(\frac{k_S}{k_L}\right)^3\right]}{(1+\epsilon)\left[18-\frac{27}{2}\left(\frac{k_S}{k_L}\right)^2\right]} \right\}^{\frac{1}{2}}, \\
\tilde{c}_5 \;\; &= -\frac{1}{2}\left(1+\epsilon\right), \quad M_2 = 0, \quad M_3 = 0, \\
M_4 \;\; &= \left(-\frac{\tilde{c}_3}{\tilde{c}_6} H\bar{M}_1^3\right)^{\frac{1}{4}} = \left\{ \frac{2H^2M_p^2\epsilon\;\tilde{c}_3}{\tilde{c}_6} \frac{\left[4(\eta-3\epsilon)-10\epsilon\left(\frac{k_S}{k_L}\right)^2+(\epsilon+2\eta)\left(\frac{k_S}{k_L}\right)^3\right]}{\left[18-\frac{27}{2}\left(\frac{k_S}{k_L}\right)^2\right]} \right\}^{\frac{1}{4}}.
\end{aligned} \tag{281}$$

Furthermore, using the constraint stated in Equation (271) we finally get the following constraints on the coefficients of EFT operators for a given value of the parameter k_S/k_L (say for $k_S/k_L = 0.1$):

$$\begin{aligned} &1.22\times 10^{-3}\,M_p < |\bar{M}_1| < 1.56\times 10^{-3}\,M_p,\;\; 8.67\times 10^{-3}\,M_p < |\bar{M}_2| \approx |\bar{M}_3| < 1.25\times 10^{-2}\,M_p, \\ &M_2 = 0,\;\; M_3 = 0,\;\; 3.75\times 10^{-4}\,M_p < M_4 \times (-\tilde{c}_6/\tilde{c}_3)^{1/4} < 4.51\times 10^{-4}\,M_p. \end{aligned} \tag{282}$$

- **For α,β vacuum**:
After setting $C_1 = \cosh\alpha$ and $C_2 = e^{i\beta}\sinh\alpha$, we get $U_1 = J_1(\alpha,\beta)$ and $U_2 = J_2(\alpha,\beta)$. Consequently, the expansion coefficients can be recast as:

$$\begin{aligned} a_{-1} &= 16\epsilon\sinh^2 2\alpha\cos^2\beta,\;\; a_0 = 4(2\epsilon-\eta)\cosh^2 2\alpha + 4\epsilon + 4\epsilon\sinh^2 2\alpha\cos^2\beta, \\ a_1 &= 34\epsilon\sinh^2 2\alpha\cos^2\beta,\;\; a_2 = 10\epsilon + 10\epsilon\sinh^2 2\alpha\cos^2\beta, \\ a_3 &= 2(2\epsilon-\eta)\cosh^2 2\alpha - 5\epsilon - \epsilon\sinh^2 2\alpha\cos^2\beta. \end{aligned} \tag{283}$$

and

$$\begin{aligned} b_{-1} &= -36 J_2(\alpha,\beta),\;\; b_0 = \frac{9}{2}\left(J_1(\alpha,\beta) + 9J_2(\alpha,\beta)\right), \\ b_2 &= \left(\frac{27}{4}J_1(\alpha,\beta) + \frac{17}{2}J_2(\alpha,\beta)\right),\;\; b_3 = 0 = b_1. \end{aligned} \tag{284}$$

Finally, the EFT coefficients for scalar fluctuation can be written as:

$$\begin{aligned} \bar{M}_1 &= \left\{2HM_p^2\epsilon\left[-36J_2(\alpha,\beta)\left(\frac{k_S}{k_L}\right)^{-1} + 9\left(J_1(\alpha,\beta) + \frac{J_2(\alpha,\beta)}{2}\right)\right.\right. \\ &\left.-\frac{3}{4}\left(9J_1(\alpha,\beta) - 7J_2(\alpha,\beta)\right)\left(\frac{k_S}{k_L}\right)^2\right]^{-1} \\ &\left[16\epsilon\sinh^2 2\alpha\cos^2\beta\left(\frac{k_S}{k_L}\right)^{-1} + \left(4(2\epsilon-\eta)\cosh^2 2\alpha + 4\epsilon + 4\epsilon\sinh^2 2\alpha\cos^2\beta\right)\right. \\ &+34\epsilon\sinh^2 2\alpha\cos^2\beta\left(\frac{k_S}{k_L}\right) + \left(10\epsilon + 10\epsilon\sinh^2 2\alpha\cos^2\beta\right)\left(\frac{k_S}{k_L}\right)^2 \\ &\left.\left.+\left(2(2\epsilon-\eta)\cosh^2 2\alpha - 5\epsilon - \epsilon\sinh^2 2\alpha\cos^2\beta\right)\left(\frac{k_S}{k_L}\right)^3\right]\right\}^{\frac{1}{3}}, \\ \bar{M}_2 &\approx \bar{M}_3 = \sqrt{\frac{\bar{M}_1^3}{4H\tilde{c}_5}} = \left\{\frac{M_p^2\epsilon}{(1+\epsilon)}\left[36J_2(\alpha,\beta)\left(\frac{k_S}{k_L}\right)^{-1} - 9\left(J_1(\alpha,\beta) + \frac{J_2(\alpha,\beta)}{2}\right)\right.\right. \\ &\left.+\frac{3}{4}\left(9J_1(\alpha,\beta) - 7J_2(\alpha,\beta)\right)\left(\frac{k_S}{k_L}\right)^2\right]^{-1} \\ &\left[16\epsilon\sinh^2 2\alpha\cos^2\beta\left(\frac{k_S}{k_L}\right)^{-1} + \left(4(2\epsilon-\eta)\cosh^2 2\alpha + 4\epsilon + 4\epsilon\sinh^2 2\alpha\cos^2\beta\right)\right. \\ &+34\epsilon\sinh^2 2\alpha\cos^2\beta\left(\frac{k_S}{k_L}\right) + \left(10\epsilon + 10\epsilon\sinh^2 2\alpha\cos^2\beta\right)\left(\frac{k_S}{k_L}\right)^2 \\ &\left.\left.+\left(2(2\epsilon-\eta)\cosh^2 2\alpha - 5\epsilon - \epsilon\sinh^2 2\alpha\cos^2\beta\right)\left(\frac{k_S}{k_L}\right)^3\right]\right\}^{\frac{1}{2}}, \\ \tilde{c}_5 &= -\frac{1}{2}(1+\epsilon),\quad M_2 = 0,\quad M_3 = 0, \\ M_4 &= \left(-\frac{\tilde{c}_3}{\tilde{c}_6}H\bar{M}_1^3\right)^{\frac{1}{4}} = \left\{\frac{2H^2M_p^2\epsilon\,\tilde{c}_3}{\tilde{c}_6}\left[36J_2(\alpha,\beta)\left(\frac{k_S}{k_L}\right)^{-1} - 9\left(J_1(\alpha,\beta) + \frac{J_2(\alpha,\beta)}{2}\right)\right.\right. \\ &\left.+\frac{3}{4}\left(9J_1(\alpha,\beta) - 7J_2(\alpha,\beta)\right)\left(\frac{k_S}{k_L}\right)^2\right]^{-1} \\ &\left[16\epsilon\sinh^2 2\alpha\cos^2\beta\left(\frac{k_S}{k_L}\right)^{-1} + \left(4(2\epsilon-\eta)\cosh^2 2\alpha + 4\epsilon + 4\epsilon\sinh^2 2\alpha\cos^2\beta\right)\right. \\ &+34\epsilon\sinh^2 2\alpha\cos^2\beta\left(\frac{k_S}{k_L}\right) + \left(10\epsilon + 10\epsilon\sinh^2 2\alpha\cos^2\beta\right)\left(\frac{k_S}{k_L}\right)^2 \\ &\left.\left.+\left(2(2\epsilon-\eta)\cosh^2 2\alpha - 5\epsilon - \epsilon\sinh^2 2\alpha\cos^2\beta\right)\left(\frac{k_S}{k_L}\right)^3\right]\right\}^{\frac{1}{4}}. \end{aligned} \tag{285}$$

Furthermore, using the constraint stated in Equation (271) we finally get the following constraints on the coefficients of EFT operators for a given value of the parameters α, β and k_S/k_L (say for $\alpha = 0.1$, $\beta = 0.1$ and $k_S/k_L = 0.1$):

$$6.05 \times 10^{-4}\, M_p < |\bar{M}_1| < 7.15 \times 10^{-4}\, M_p,\;\; 3.03 \times 10^{-3}\, M_p < |\bar{M}_2| \approx |\bar{M}_3| < 3.89 \times 10^{-3}\, M_p,$$
$$M_2 = 0,\;\; M_3 = 0,\;\; 2.22 \times 10^{-3}\, M_p < M_4 \times (-\tilde{c}_6/\tilde{c}_3)^{1/4} < 2.51 \times 10^{-3}\, M_p. \tag{286}$$

5.2. For General Single-Field $P(X,\phi)$ Inflation

Here our prime objective is to derive the EFT coefficients by computing the most general expression for the three-point function for scalar fluctuations from the general single-field $P(X,\phi)$ model of inflation for arbitrary vacuum. Then we give specific example for Bunch–Davies and α,β vacuum for completeness.

5.2.1. Basic Setup

Let us start with the action for single scalar field (inflaton) which is described by the general function $P(X,\phi)$, contains non-canonical kinetic term in general and it is minimally coupled to the gravity [33,44]:

$$S = \int d^4x\sqrt{-g}\left[\frac{M_p^2}{2}R + P(X,\phi)\right]. \tag{287}$$

In the case of general structure of $P(X,\phi)$ the pressure p, the energy density ρ and effective speed of sound parameter c_S can be written as [44]:

$$p = P(X,\phi), \quad \rho = 2XP_{,X}(X,\phi) - P(X,\phi), \quad c_S = \sqrt{\frac{P_{,X}(X,\phi)}{P_{,X}(X,\phi) + 2XP_{,XX}(X,\phi)}}. \tag{288}$$

In the case of general $P(X,\phi)$ theory the slow-roll parameters can be expressed as [44]:

$$\begin{aligned}
\epsilon &= \frac{XP_X(X,\phi)}{H^2M_p^2}, \quad \eta = \epsilon - \frac{\delta}{P_{,X}(X,\phi)}\left[P_{,X}(X,\phi) + XP_{,XX}(X,\phi)\right], \\
s &= \frac{2X\delta}{P_{,X}(X,\phi)}\frac{\left[XP^2_{,XX}(X,\phi) - P_{,X}(X,\phi)P_{,XX}(X,\phi) - XP_{,X}(X,\phi)P_{,XXX}(X,\phi)\right]}{\left[P_{,X}(X,\phi) + XP_{,XX}(X,\phi)\right]}.
\end{aligned} \tag{289}$$

In the case of **single-field slow-roll inflation** we have:

$$P(X,\phi) = X - V(\phi), \tag{290}$$

where $V(\phi)$ is the single-field slowly varying potential. For this case if we compute the effective sound speed then it turns out to be $c_S = 1$, which is consistent with our result obtained in the previous section. Also, if we compute the expressions for the slow-roll parameters ϵ, η, δ and s the results also perfectly match the results obtained in Equation (253).

Similarly, in case **DBI inflationary model** one can identify the function $P(X,\phi)$ as [45]:

$$P(X,\phi) = -\frac{1}{f(\phi)}\sqrt{1 - 2Xf(\phi)} + \frac{1}{f(\phi)} - V(\phi), \tag{291}$$

where the inflaton ϕ is identified to be the position of a D3 barne which is moving in warped throat geometry and $f(\phi)$ characterize the warp factor[22]. For the effective potential $V(\phi)$ one can consider following mathematical structures of the potentials in the UV and IR regime [45]:

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[22] For AdS-like throat geometry, $f(\phi) \approx \frac{\lambda}{\phi^4}$, where λ is the parameter in string theory which depends on the flux number.

- **UV regime**: In this case, the inflaton moves from the UV regime of the warped geometric space to the IR regime under the influence of the effective potential, $V(\phi) \simeq \frac{1}{2}m^2\phi^2$, where the inflaton mass satisfies the constraint $m >> M_p\sqrt{\lambda}$. In this specific situation the inflaton starts rolling very far away from the origin of the effective potential and then rolls down in a relativistic fashion to the minimum of potential situated at the origin.
- **IR regime**: In this case, the inflaton started moving from the IR regime of the warped space geometry to the UV regime under the influence of the effective potential, $V(\phi) \simeq V_0 - \frac{1}{2}m^2\phi^2$, where the inflaton mass is comparable to the scale of inflation, as given by, $m \approx H$. In this specific situation, the inflaton starts rolling down near the origin of the effective potential and rolls down in a relativistic fashion away from it.

In the case of the DBI model the pressure p and the energy density ρ can be written as [45]:

$$p = \frac{1}{f(\phi)}(1-c_S) - V(\phi), \quad \rho = \frac{1}{f(\phi)}\left(\frac{1}{c_S}-1\right) + V(\phi), \quad c_S = \sqrt{1-2Xf(\phi)} = \sqrt{1-\dot{\phi}^2 f(\phi)}, \tag{292}$$

where $X = \dot{\phi}^2/2$. In this context the slow-roll parameter [45]:

$$\epsilon = \frac{3\dot{\phi}^2}{2\left[c_S V(\phi) + \frac{1}{f(\phi)}(1-c_S)\right]} \approx \frac{3}{2\left[1 + c_S f(\phi)V(\phi)\right]}. \tag{293}$$

is not small and as a result the effective sound speed is very small, $c_S << 1$. Consequently, the inflaton speed during inflation is given by the expression, $\dot{\phi} = \pm\frac{1}{\sqrt{f(\phi)}}$. Additionally, it is important to note that in the context of DBI inflation the other slow-roll parameters η and s can be computed as:

$$\begin{aligned}
\eta &\approx \frac{\left[3\sqrt{1+c_S f(\phi)V(\phi)} + \frac{\sqrt{3f(\phi)c_S}}{2}M_p\dot{\phi}c_S\left\{f(\phi)V^{'}(\phi) + V(\phi)f^{'}(\phi) - \frac{1}{2c_S^2}\left(2\ddot{\phi}f(\phi) + \dot{\phi}^2 f^{'}(\phi)\right)\right\}\right]}{\left[1+c_S f(\phi)V(\phi)\right]^{\frac{3}{2}}}, \\
s &= -\frac{\sqrt{3f(\phi)c_S}}{2c_S^2}M_p\dot{\phi}\left[2\ddot{\phi}f(\phi) + \dot{\phi}^2 f^{'}(\phi)\right].
\end{aligned} \tag{294}$$

In the slow-roll regime to validate slow-roll approximation along with $c_S << 1$ we need to satisfy the constraint condition for DBI inflation, $2c_S f(\phi)V(\phi) >> 1$.

5.2.2. Scalar Three-Point Function for General Single-Field $P(X,\phi)$ Inflation

Before computing the three-point function for scalar mode fluctuation here it is important to note that the two-point function for general single-field $P(X,\phi)$ inflation is exactly same with the results obtained for EFT of inflation with sound speed $c_S << 1$ and $\tilde{c}_S << 1$, which can be obtained by setting the EFT coefficients, $M_2 \neq 0$, $M_3 \neq 0$, $\bar{M}_1 \neq 0$, $M_4 \neq 0$, $\bar{M}_2 \neq 0$, $\bar{M}_3 \neq 0$, $\tilde{c}_5 \neq -\frac{1}{2}(1+\epsilon)$[23] . Using three-point function we can able to fix all of these coefficients.

Now here before going into the details of the computation for three-point function, just using the knowledge of the two-point function, we can easily identify the exact analytical expression for the EFT coefficient M_2. For this, we need to identify the effective sound speed computed from general

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[23] In the case of general single-field $P(X,\phi)$ inflation amplitude of power spectrum and spectral tilt for scalar fluctuation can be written at the horizon crossing $|k\tilde{c}_S\eta| = 1$ as:

$$\textbf{For Bunch–Davies vacuum:} \quad \Delta_\zeta(k_*) = \begin{cases} \dfrac{2X_* P_{,X}(X_*,\phi_*) - P(X_*,\phi_*)}{24\pi^2 M_p^4 \tilde{c}_S \epsilon} & \textbf{for dS} \\ 2^{3\epsilon-\eta+\frac{s}{2}}\dfrac{2X_* P_{,X}(X_*,\phi_*) - P(X_*,\phi_*)}{24\pi^2 M_p^4 \epsilon(1+\epsilon)^2}\left|\dfrac{\Gamma\left(\frac{3}{2}+3\epsilon-\eta+\frac{s}{2}\right)}{\Gamma\left(\frac{3}{2}\right)}\right|^2 & \textbf{for qdS}. \end{cases}$$

single-field $P(X,\phi)$ inflation with the result obtained for the proposed EFT set up. Consequently, we get:

$$M_2 = \left(-\frac{XP_{,XX}(X,\phi)}{P_{,X}(X,\phi)}\dot{H}M_p^2\right)^{\frac{1}{4}} = \begin{cases} 0 & \textbf{for single-field slow-roll} \\ \left[\left(\frac{\dot{\phi}^2 f(\phi)}{\dot{\phi}^2 f(\phi)-1}\right)\frac{\dot{H}M_p^2}{2}\right]^{\frac{1}{4}} & \textbf{for DBI.} \end{cases} \quad (295)$$

We here now proceed to calculate the three-point function for the scalar fluctuation $\zeta(t,\mathbf{x})$ in the interacting picture with arbitrary vacuum in the case of general single-field $P(X,\phi)$ inflation. Then we cite results for Bunch–Davies and α,β vacuum and give a specific example for the DBI model of inflation.

Here we introduce two new parameters [44]:

$$\Sigma_1(X,\phi) = XP_{,X}(X,\phi) + 2X^2P_{,XX}(X,\phi) = \frac{\epsilon H^2 M_p^2}{c_S^2}, \quad (296)$$

$$\Sigma_2(X,\phi) = X^2P_{,XX}(X,\phi) + \frac{2}{3}X^3P_{,XXX}(X,\phi). \quad (297)$$

which will appear in the expression for three-point function for the scalar fluctuation. For **single-field slow-roll inflation** and **DBI inflation** we get the following expressions for these parameters [44]:

$$\Sigma_1(X,\phi) = \begin{cases} X = \epsilon H^2 M_p^2 & \textbf{for single-field slow-roll} \\ \frac{X}{(1-2Xf(\phi))^{\frac{3}{2}}} = \frac{\epsilon H^2 M_p^2}{c_S^2} & \textbf{for DBI.} \end{cases} \quad (298)$$

$$\Sigma_2(X,\phi) = \begin{cases} 0 & \textbf{for single-field slow-roll} \\ \frac{X^2 f(\phi)}{(1-2Xf(\phi))^{\frac{5}{2}}} & \textbf{for DBI.} \end{cases} \quad (299)$$

For general single-field $P(X,\phi)$ inflation the third-order term in the action Equation (287) is given by [44]:

$$\begin{aligned} S_\zeta^{(3)} &= \int d^4x \left[-a^3\left\{\Sigma_1(X,\phi)\left(1-\frac{1}{c_S^2}\right)+2\Sigma_2(X,\phi)\right\}\frac{\dot{\tilde{\zeta}}^3}{H^3} + \frac{a^3\epsilon\left(\epsilon-3+3c_S^2\right)}{c_S^4}\tilde{\zeta}\dot{\tilde{\zeta}}^2 \right. \\ &\left. + \frac{a\epsilon\left(\epsilon-2s+1-c_S^2\right)}{c_S^2}\tilde{\zeta}(\partial\tilde{\zeta})^2 - 2a^3\epsilon\dot{\tilde{\zeta}}\partial_i\tilde{\zeta}\partial_i\left(\frac{\epsilon}{c_S^2}\partial^{-2}\dot{\tilde{\zeta}}\right)\right], \end{aligned} \quad (300)$$

$$\textbf{For ff, fi vacuum}: \quad \Delta_\zeta(k_*) = \begin{cases} \frac{2X_*P_{,X}(X_*,\phi_*)-P(X_*,\phi_*)}{24\pi^2 M_p^4\tilde{c}_S\epsilon}\left[\cosh 2\alpha - \sinh 2\alpha\cos\beta\right] & \textbf{for dS} \\ 2^{6\epsilon-2\eta+s}\frac{2X_*P_{,X}(X_*,\phi_*)-P(X_*,\phi_*)}{24\pi^2 M_p^4\epsilon(1+\epsilon)^2}\left|\frac{\Gamma(\frac{3}{2}+3\epsilon-\eta+\frac{s}{2})}{\Gamma(\frac{3}{2})}\right|^2 & \\ \left[\cosh 2\alpha - \sinh 2\alpha\cos\left(\pi\left(2+3\epsilon-\eta+\frac{s}{2}\right)+\beta\right)\right] & \textbf{for qdS.} \end{cases}$$

and

$$n_\zeta(k_*) - 1 = 2\eta - 6\epsilon - s.$$

which is derived from Equation (244) and here after neglecting all the contribution from the terms which are sub-leading in the slow-roll parameters. Additionally, here we use the following field redefinition:

$$\zeta = \tilde{\zeta} + \frac{1}{c_S^2}\left\{\epsilon - \frac{\eta}{2}\right\}\tilde{\zeta}^2, \tag{301}$$

where ϵ, η, δ and s are already defined earlier for general single-field $P(X,\phi)$ inflation.

Now it is important to note that in the present context of discussion we are interested in the three-point function for the scalar fluctuation field ζ, not for the redefined scalar field fluctuation $\tilde{\zeta}$ and for this reason one can write down the exact connection between the three-point function for the scalar function field ζ and redefined scalar fluctuation field $\tilde{\zeta}$ in position space as:

$$\begin{aligned}\langle\zeta(\mathbf{x_1})\zeta(\mathbf{x_2})\zeta(\mathbf{x_3})\rangle \quad &= \quad \langle\tilde{\zeta}(\mathbf{x_1})\tilde{\zeta}(\mathbf{x_2})\tilde{\zeta}(\mathbf{x_3}) + \frac{(2\epsilon-\eta)}{c_S^2}\left[\langle\zeta(\mathbf{x_1})\zeta(\mathbf{x_2})\rangle\langle\zeta(\mathbf{x_1})\zeta(\mathbf{x_3})\rangle\right.\\ &\quad \left. + \langle\zeta(\mathbf{x_2})\zeta(\mathbf{x_1})\rangle\langle\zeta(\mathbf{x_2})\zeta(\mathbf{x_3})\rangle + \langle\zeta(\mathbf{x_3})\zeta(\mathbf{x_1})\rangle\langle\zeta(\mathbf{x_3})\zeta(\mathbf{x_2})\rangle\right].\end{aligned} \tag{302}$$

After taking the Fourier transform of the scalar fluctuation field ζ and redefined scalar fluctuation field $\tilde{\zeta}$ one can express connection between three-point function in momentum space and this is also our main point of interest.

The interaction Hamiltonian for the redefined scalar fluctuation $\tilde{\zeta}$ can be expressed as:

$$\begin{aligned}H_{int} \quad = \quad &\int d^3x\,\left[-\left\{\Sigma_1(X,\phi)\left(1-\frac{1}{c_S^2}\right)+2\Sigma_2(X,\phi)\right\}\frac{\tilde{\zeta}'^3}{H^3} + \frac{a\,\epsilon\left(\epsilon-3+3c_S^2\right)}{c_S^4}\tilde{\zeta}\tilde{\zeta}'^2\right.\\ &\left. + \frac{a\,\epsilon\left(\epsilon-2s+1-c_S^2\right)}{c_S^2}\tilde{\zeta}(\partial\tilde{\zeta})^2 - 2a\tilde{\zeta}'\partial_i\tilde{\zeta}\partial_i\left(\frac{\epsilon}{c_S^2}\partial^{-2}\tilde{\zeta}'\right)\right].\end{aligned} \tag{303}$$

Furthermore, following the in-in formalism in interaction picture the expression for the three-point function for the redefined scalar fluctuation $\tilde{\zeta}$ and then transforming the final result in terms of the scalar fluctuation ζ in momentum one can write the following expression:

$$\begin{aligned}\langle\zeta(\mathbf{k_1})\zeta(\mathbf{k_2})\zeta(\mathbf{k_3})\rangle \quad &= -i\int_{\eta_i=-\infty}^{\eta_f=0} d\eta\; a(\eta)\;\langle 0|\left[\zeta(\eta_f,\mathbf{k_1})\zeta(\eta_f,\mathbf{k_2})\zeta(\eta_f,\mathbf{k_3}), H_{int}(\eta)\right]|0\rangle\\ &= (2\pi)^3\delta^{(3)}(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3})B_{GSF}(k_1,k_2,k_3),\end{aligned} \tag{304}$$

where $B_{GSF}(k_1,k_2,k_3)$ represents the bispectrum of scalar fluctuation ζ, which is computed from general single-field $P(X,\phi)$ inflation. Here the final expression for the bispectrum of scalar fluctuation for arbitrary vacuum is given by:

$$\begin{aligned} B_{GSF}(k_1,k_2,k_3) &= \frac{H^4}{32\epsilon^2 M_p^4}\frac{1}{(k_1k_2k_3)^3}\left[\frac{3}{2}\left(\frac{1}{c_S^2}-1-\frac{2\Sigma_2(X,\phi)}{\Sigma_1(X,\phi)}\right)\left(|C_1|^2+|C_2|^2\right)^2\frac{(k_1k_2k_3)^2}{K^3}\right. \\ &+\left(\frac{1}{c_S^2}-1\right)\left(|C_1|^2+|C_2|^2\right)^2\left(\sum_{i=1}^{3}k_i^3+\frac{4}{K^2}\sum_{i,j=1,i\neq j}^{3}k_i^2k_j^3-\frac{8}{K}\sum_{i,j=1,i>j}^{3}k_i^2k_j^2\right) \\ &+\frac{s}{c_S^2}\left(|C_1|^2+|C_2|^2\right)^2\left(-2\sum_{i=1}^{3}k_i^3+\frac{4}{K^2}\sum_{i,j=1,i\neq j}^{3}k_i^2k_j^3-\frac{8}{K}\sum_{i,j=1,i>j}^{3}k_i^2k_j^2\right) \\ &+2(2\epsilon-\eta)\left(|C_1|^2+|C_2|^2\right)^2\sum_{i=1}^{3}k_i^3 \\ &+\epsilon\left(|C_1|^2-|C_2|^2\right)^2\left(-\sum_{i=1}^{3}k_i^3+\sum_{i,j=1,i\neq j}^{3}k_ik_j^2+\frac{8}{K}\sum_{i,j=1,i>j}^{3}k_i^2k_j^2\right) \\ &+\epsilon\left(C_1^*C_2+C_1C_2^*\right)^2\left(-\textstyle\sum_{i=1}^{3}k_i^3+\sum_{i,j=1,i\neq j}^{3}k_ik_j^2\right. \\ &\qquad\left.\left.+8\sum_{i,j=1,i>j}^{3}k_i^2k_j^2\sum_{m=1}^{3}\frac{1}{K-2k_m}\right)+\mathcal{O}(\epsilon)\right], \end{aligned} \tag{305}$$

where $\mathcal{O}(\epsilon)$ characterizes the sub-leading corrections in the three-point function for the scalar fluctuation computed from general single-field $P(X,\phi)$ inflation.

Furthermore, we consider a very specific class of models, where the following constraint condition[24] $P_{,X\phi}(X,\phi)=0$ perfectly holds good. In this case one can write down the following simplified expression for the bispectrum of scalar fluctuation for arbitrary vacuum as:

$$\begin{aligned} B_{GSF}(k_1,k_2,k_3) &= \frac{H^4}{32\epsilon^2 M_p^4}\frac{1}{(k_1k_2k_3)^3}\left[\left(\frac{1}{c_S^2}-1-\frac{s\epsilon}{3\epsilon_X}\right)\left(|C_1|^2+|C_2|^2\right)^2\frac{(k_1k_2k_3)^2}{K^3}\right. \\ &+\left(\frac{1}{c_S^2}-1\right)\left(|C_1|^2+|C_2|^2\right)^2\left(\sum_{i=1}^{3}k_i^3+\frac{4}{K^2}\sum_{i,j=1,i\neq j}^{3}k_i^2k_j^3-\frac{8}{K}\sum_{i,j=1,i>j}^{3}k_i^2k_j^2\right) \\ &+2(2\epsilon-\eta)\left(|C_1|^2+|C_2|^2\right)^2\sum_{i=1}^{3}k_i^3 \\ &+\epsilon\left(|C_1|^2-|C_2|^2\right)^2\left(-\sum_{i=1}^{3}k_i^3+\sum_{i,j=1,i\neq j}^{3}k_ik_j^2+\frac{8}{K}\sum_{i,j=1,i>j}^{3}k_i^2k_j^2\right) \\ &\left.+\epsilon\left(C_1^*C_2+C_1C_2^*\right)^2\left(-\sum_{i=1}^{3}k_i^3+\sum_{i,j=1,i\neq j}^{3}k_ik_j^2+8\sum_{i,j=1,i>j}^{3}k_i^2k_j^2\sum_{m=1}^{3}\frac{1}{K-2k_m}\right)\right], \end{aligned} \tag{306}$$

where the new parameter ϵ_X is defined as[25]:

$$\epsilon_X=-\frac{\dot{X}H_{,X}}{H^2}. \tag{308}$$

For Bunch–Davies and α,β vacuum we get the following simplified expression for the bispectrum for scalar fluctuation:

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[24] Strictly speaking, the DBI model is one of the exceptions where this condition is not applicable. On the other hand, in the case of single-field slow-roll inflation this condition is applicable. But in that case one can set $c_S=1$ and get back all the results derived in the earlier section. Additionally, it is important to mention that here we consider those models also where $c_S<<1$ along with this given constraint.

[25] For single-field slow-roll inflation the newly introduced parameter ϵ_X is computed as:

$$\epsilon_X=\epsilon(\eta-\epsilon)\approx\epsilon_V\left(\eta_V-2\epsilon_V\right). \tag{307}$$

- **For Bunch–Davies vacuum**:
After setting $C_1 = 1$ and $C_2 = 0$ we get [44]:

$$\begin{aligned} B_{GSF}(k_1,k_2,k_3) &= \frac{H^4}{32\epsilon^2 M_p^4}\frac{1}{({}_1k_2k_3)^3}\left[\frac{3}{2}\left(\frac{1}{c_S^2}-1-\frac{2\Sigma_2(X,\phi)}{\Sigma_1(X,\phi)}\right)\frac{(k_1k_2k_3)^2}{K^3}\right. \\ &+\left(\frac{1}{c_S^2}-1\right)\left(\sum_{i=1}^{3}k_i^3+\frac{4}{K^2}\sum_{i,j=1,i\neq j}^{3}k_i^2k_j^3-\frac{8}{K}\sum_{i,j=1,i>j}^{3}k_i^2k_j^2\right) \\ &+\frac{s}{c_S^2}\left(-2\sum_{i=1}^{3}k_i^3+\frac{4}{K^2}\sum_{i,j=1,i\neq j}^{3}k_i^2k_j^3-\frac{8}{K}\sum_{i,j=1,i>j}^{3}k_i^2k_j^2\right) \\ &+2(2\epsilon-\eta)\sum_{i=1}^{3}k_i^3 \\ &\left.+\epsilon\left(-\sum_{i=1}^{3}k_i^3+\sum_{i,j=1,i\neq j}^{3}k_ik_j^2+\frac{8}{K}\sum_{i,j=1,i>j}^{3}k_i^2k_j^2\right)\right]. \end{aligned} \tag{309}$$

Furthermore, for restricted classes of the general single-field $P(X,\phi)$ model, which satisfies the constraint $P_{,X\phi}(X,\phi) = 0$, one can further write down the following expression for the bispectrum:

$$\begin{aligned} B_{GSF}(k_1,k_2,k_3) &= \frac{H^4}{32\epsilon^2 M_p^4}\frac{1}{(k_1k_2k_3)^3}\left[\left(\frac{1}{c_S^2}-1-\frac{s\epsilon}{3\epsilon_X}\right)\frac{(k_1k_2k_3)^2}{K^3}\right. \\ &+\left(\frac{1}{c_S^2}-1\right)\left(\sum_{i=1}^{3}k_i^3+\frac{4}{K^2}\sum_{i,j=1,i\neq j}^{3}k_i^2k_j^3-\frac{8}{K}\sum_{i,j=1,i>j}^{3}k_i^2k_j^2\right) \\ &+2(2\epsilon-\eta)\sum_{i=1}^{3}k_i^3 \\ &\left.+\epsilon\left(-\sum_{i=1}^{3}k_i^3+\sum_{i,j=1,i\neq j}^{3}k_ik_j^2+\frac{8}{K}\sum_{i,j=1,i>j}^{3}k_i^2k_j^2\right)\right], \end{aligned} \tag{310}$$

- **For α,β vacuum**:
After setting $C_1 = \cosh\alpha$ and $C_2 = e^{i\beta}\sinh\alpha$ we get [33]:

$$\begin{aligned} B_{GSF}(k_1,k_2,k_3) &= \frac{H^4}{32\epsilon^2 M_p^4}\frac{1}{(k_1k_2k_3)^3}\left[\frac{3}{2}\left(\frac{1}{c_S^2}-1-\frac{2\Sigma_2(X,\phi)}{\Sigma_1(X,\phi)}\right)\cosh^2 2\alpha\frac{(k_1k_2k_3)^2}{K^3}\right. \\ &+\left(\frac{1}{c_S^2}-1\right)\cosh^2 2\alpha\left(\sum_{i=1}^{3}k_i^3+\frac{4}{K^2}\sum_{i,j=1,i\neq j}^{3}k_i^2k_j^3-\frac{8}{K}\sum_{i,j=1,i>j}^{3}k_i^2k_j^2\right) \\ &+\frac{s}{c_S^2}\cosh^2 2\alpha\left(-2\sum_{i=1}^{3}k_i^3+\frac{4}{K^2}\sum_{i,j=1,i\neq j}^{3}k_i^2k_j^3-\frac{8}{K}\sum_{i,j=1,i>j}^{3}k_i^2k_j^2\right) \\ &+2(2\epsilon-\eta)\cosh^2 2\alpha\sum_{i=1}^{3}k_i^3+\epsilon\left(-\sum_{i=1}^{3}k_i^3+\sum_{i,j=1,i\neq j}^{3}k_ik_j^2+\frac{8}{K}\sum_{i,j=1,i>j}^{3}k_i^2k_j^2\right) \\ &\left.+\epsilon\sinh^2 2\alpha\cos^2\beta\left(-\sum_{i=1}^{3}k_i^3+\sum_{i,j=1,i\neq j}^{3}k_ik_j^2+8\sum_{i,j=1,i>j}^{3}k_i^2k_j^2\sum_{m=1}^{3}\frac{1}{K-2k_m}\right)\right]. \end{aligned} \tag{311}$$

Furthermore, for restricted classes of the general single-field $P(X,\phi)$ model, which satisfies the constraint $P_{,X\phi}(X,\phi) = 0$, one can further write down the following expression for the bispectrum:

$$\begin{aligned} B_{GSF}(k_1,k_2,k_3) &= \frac{H^4}{32\epsilon^2 M_p^4}\frac{1}{(k_1k_2k_3)^3}\left[\left(\frac{1}{c_S^2}-1-\frac{s\epsilon}{3\epsilon_X}\right)\cosh^2 2\alpha\frac{(k_1k_2k_3)^2}{K^3}\right. \\ &+\left(\frac{1}{c_S^2}-1\right)\cosh^2 2\alpha\left(\sum_{i=1}^{3}k_i^3+\frac{4}{K^2}\sum_{i,j=1,i\neq j}^{3}k_i^2k_j^3-\frac{8}{K}\sum_{i,j=1,i>j}^{3}k_i^2k_j^2\right) \\ &+2(2\epsilon-\eta)\cosh^2 2\alpha\sum_{i=1}^{3}k_i^3+\epsilon\left(-\sum_{i=1}^{3}k_i^3+\sum_{i,j=1,i\neq j}^{3}k_ik_j^2+\frac{8}{K}\sum_{i,j=1,i>j}^{3}k_i^2k_j^2\right) \\ &\left.+\epsilon\sinh^2 2\alpha\cos^2\beta\left(-\sum_{i=1}^{3}k_i^3+\sum_{i,j=1,i\neq j}^{3}k_ik_j^2+8\sum_{i,j=1,i>j}^{3}k_i^2k_j^2\sum_{m=1}^{3}\frac{1}{K-2k_m}\right)\right], \end{aligned} \tag{312}$$

Furthermore, we consider equilateral limit and squeezed limit in which we finally get:

1. Equilateral limit configuration:
Here the bispectrum for scalar perturbations in the presence of arbitrary quantum vacuum can be expressed as:

$$\begin{aligned} B_{GSF}(k,k,k) &= \frac{H^4}{32\epsilon^2 M_p^4}\frac{1}{k^6}\left[\frac{1}{18}\left(\frac{1}{c_S^2}-1-\frac{2\Sigma_2(X,\phi)}{\Sigma_1(X,\phi)}\right)\left(|C_1|^2+|C_2|^2\right)^2\right. \\ &-\frac{7}{3}\left(\frac{1}{c_S^2}-1\right)\left(|C_1|^2+|C_2|^2\right)^2-\frac{34}{3}\frac{s}{c_S^2}\left(|C_1|^2+|C_2|^2\right)^2 \\ &\left.+6(2\epsilon-\eta)\left(|C_1|^2+|C_2|^2\right)^2+11\epsilon\left(|C_1|^2-|C_2|^2\right)^2+27\epsilon\,(C_1^*C_2+C_1C_2^*)^2\right]. \end{aligned} \tag{313}$$

Furthermore, for restricted classes of the general single-field $P(X,\phi)$ model, which satisfies the constraint $P_{,X\phi}(X,\phi) = 0$, one can further write down the following expression for the bispectrum:

$$\begin{aligned} B_{GSF}(k_1,k_2,k_3) &= \frac{H^4}{32\epsilon^2 M_p^4}\frac{1}{k^6}\left[\frac{1}{27}\left(\frac{1}{c_S^2}-1-\frac{s\epsilon}{3\epsilon_X}\right)\left(|C_1|^2+|C_2|^2\right)^2-\frac{7}{3}\left(\frac{1}{c_S^2}-1\right)\left(|C_1|^2+|C_2|^2\right)^2\right. \\ &\left.+6(2\epsilon-\eta)\left(|C_1|^2+|C_2|^2\right)^2+11\epsilon\left(|C_1|^2-|C_2|^2\right)^2+27\epsilon\,(C_1^*C_2+C_1C_2^*)^2\right], \end{aligned} \tag{314}$$

Now for Bunch–Davies and α,β vacuum we get the following simplified expression for the bispectrum for scalar fluctuation:

- **For Bunch–Davies vacuum**:
After setting $C_1 = 1$ and $C_2 = 0$ we get:

$$B_{GSF}(k,k,k) = \frac{H^4}{32\epsilon^2 M_p^4}\frac{1}{k^6}\left[\frac{1}{18}\left(\frac{1}{c_S^2}-1-\frac{2\Sigma_2(X,\phi)}{\Sigma_1(X,\phi)}\right)-\frac{7}{3}\left(\frac{1}{c_S^2}-1\right)-\frac{34}{3}\frac{s}{c_S^2}+23\epsilon-6\eta\right]. \tag{315}$$

Furthermore, for restricted classes of the general single-field $P(X,\phi)$ model, which satisfies the constraint $P_{,X\phi}(X,\phi) = 0$, we get:

$$B_{GSF}(k_1,k_2,k_3) = \frac{H^4}{32\epsilon^2 M_p^4}\frac{1}{k^6}\left[\frac{1}{27}\left(\frac{1}{c_S^2}-1-\frac{s\epsilon}{3\epsilon_X}\right)-\frac{7}{3}\left(\frac{1}{c_S^2}-1\right)+23\epsilon-6\eta\right], \tag{316}$$

- **For α, β vacuum**:
  After setting $C_1 = \cosh\alpha$ and $C_2 = e^{i\beta}\sinh\alpha$ we get:

$$\begin{aligned}
B_{GSF}(k,k,k) &= \frac{H^4}{32\epsilon^2 M_p^4}\frac{1}{k^6}\left[\frac{1}{18}\left(\frac{1}{c_S^2} - 1 - \frac{2\Sigma_2(X,\phi)}{\Sigma_1(X,\phi)}\right)\cosh^2 2\alpha\right.\\
&\quad -\frac{7}{3}\left(\frac{1}{c_S^2} - 1\right)\cosh^2 2\alpha - \frac{34}{3}\frac{s}{c_S^2}\cosh^2 2\alpha\\
&\quad \left. +6(2\epsilon - \eta)\cosh^2 2\alpha + 11\epsilon + 27\epsilon\sinh^2 2\alpha\cos^2\beta\right].
\end{aligned} \tag{317}$$

  Furthermore, for restricted classes of the general single-field $P(X,\phi)$ model, which satisfies the constraint $P_{,X\phi}(X,\phi) = 0$, we get:

$$\begin{aligned}
B_{GSF}(k,k,k) &= \frac{H^4}{32\epsilon^2 M_p^4}\frac{1}{k^6}\left[\frac{1}{27}\left(\frac{1}{c_S^2} - 1 - \frac{s\epsilon}{3\epsilon_X}\right)\cosh^2 2\alpha - \frac{7}{3}\left(\frac{1}{c_S^2} - 1\right)\cosh^2 2\alpha\right.\\
&\quad \left. + 6(2\epsilon - \eta)\cosh^2 2\alpha + 11\epsilon + 27\epsilon\sinh^2 2\alpha\cos^2\beta\right].
\end{aligned} \tag{318}$$

2. Squeezed limit configuration:
   Here the bispectrum for scalar perturbations in the presence of arbitrary quantum vacuum can be expressed as:

$$B_{GSF}(k_L, k_L, k_S) = \frac{H^4}{32\epsilon^2 M_p^4}\frac{1}{k_L^3 k_S^3}\sum_{j=-1}^{3} t_j \left(\frac{k_S}{k_L}\right)^j, \tag{319}$$

   where the expansion coefficients $t_j \forall j = -1, \cdots, 3$ for arbitrary vacuum are defined as:

$$\begin{aligned}
t_{-1} &= 16\epsilon\left(C_1^* C_2 + C_1 C_2^*\right)^2,\\
t_0 &= \left(4(2\epsilon - \eta) - \frac{6s}{c_S^2}\right)\left(|C_1|^2 + |C_2|^2\right)^2 + 4\epsilon\left(|C_1|^2 - |C_2|^2\right)^2 + 4\epsilon\left(C_1^* C_2 + C_1 C_2^*\right)^2,\\
t_1 &= 34\epsilon\left(C_1^* C_2 + C_1 C_2^*\right)^2,\\
t_2 &= \left\{\frac{3}{16}\left(\frac{1}{c_S^2} - 1 - \frac{2\Sigma_2(X,\phi)}{\Sigma_1(X,\phi)}\right) - 6\left(\frac{1}{c_S^2} - 1\right) - \frac{6s}{c_S^2}\right\}\left(|C_1|^2 + |C_2|^2\right)^2\\
&\quad + 10\epsilon\left(|C_1|^2 - |C_2|^2\right)^2 + 10\epsilon\left(C_1^* C_2 + C_1 C_2^*\right)^2,\\
t_3 &= \left\{2(2\epsilon - \eta) - \frac{9}{32}\left(\frac{1}{c_S^2} - 1 - \frac{2\Sigma_2(X,\phi)}{\Sigma_1(X,\phi)}\right) + 5\left(\frac{1}{c_S^2} - 1\right) + \frac{2s}{c_S^2}\right\}\left(|C_1|^2 + |C_2|^2\right)^2\\
&\quad - 5\epsilon\left(|C_1|^2 - |C_2|^2\right)^2 - \epsilon\left(C_1^* C_2 + C_1 C_2^*\right)^2.
\end{aligned} \tag{320}$$

Furthermore, for restricted classes of the general single-field $P(X,\phi)$ model, which satisfies the constraint $P_{,X\phi}(X,\phi)=0$, we get:

$$\begin{aligned}
t_{-1} &= 16\epsilon\,(C_1^*C_2+C_1C_2^*)^2\,,\\
t_0 &= 4(2\epsilon-\eta)\left(|C_1|^2+|C_2|^2\right)^2+4\epsilon\left(|C_1|^2-|C_2|^2\right)^2+4\epsilon\,(C_1^*C_2+C_1C_2^*)^2\,,\\
t_1 &= 34\epsilon\,(C_1^*C_2+C_1C_2^*)^2\,,\\
t_2 &= \left\{\frac{1}{8}\left(\frac{1}{c_S^2}-1-\frac{s\epsilon}{3\epsilon_X}\right)-6\left(\frac{1}{c_S^2}-1\right)\right\}\left(|C_1|^2+|C_2|^2\right)^2\\
&\quad+10\epsilon\left(|C_1|^2-|C_2|^2\right)^2+10\epsilon\,(C_1^*C_2+C_1C_2^*)^2\,,\\
t_3 &= \left\{2(2\epsilon-\eta)-\frac{3}{16}\left(\frac{1}{c_S^2}-1-\frac{s\epsilon}{3\epsilon_X}\right)+5\left(\frac{1}{c_S^2}-1\right)\right\}\left(|C_1|^2+|C_2|^2\right)^2\\
&\quad-5\epsilon\left(|C_1|^2-|C_2|^2\right)^2-\epsilon\,(C_1^*C_2+C_1C_2^*)^2\,.
\end{aligned}\tag{321}$$

Now for Bunch–Davies and α,β vacuum we get the following simplified expression for the bispectrum for scalar fluctuation:

- **For Bunch–Davies vacuum**:
  After setting $C_1=1$ and $C_2=0$, we get the following expression for the expansion coefficients $t_j\forall j=-1,\cdots,3$:

$$\begin{aligned}
t_{-1} &= 0,\\
t_0 &= 4(3\epsilon-\eta)-\frac{6s}{c_S^2},\\
t_1 &= 0,\\
t_2 &= 10\epsilon+\left\{\frac{3}{16}\left(\frac{1}{c_S^2}-1-\frac{2\Sigma_2(X,\phi)}{\Sigma_1(X,\phi)}\right)-6\left(\frac{1}{c_S^2}-1\right)-\frac{6s}{c_S^2}\right\},\\
t_3 &= -(\epsilon+2\eta)+\left\{5\left(\frac{1}{c_S^2}-1\right)+\frac{2s}{c_S^2}-\frac{9}{32}\left(\frac{1}{c_S^2}-1-\frac{2\Sigma_2(X,\phi)}{\Sigma_1(X,\phi)}\right)\right\}.
\end{aligned}\tag{322}$$

  Consequently, the bispectrum can be recast as:

$$\begin{aligned}
B_{GSF}(k_L,k_L,k_S) &= \frac{H^4}{32\epsilon^2M_p^4}\frac{1}{k_L^3k_S^3}\left[\left\{4(3\epsilon-\eta)-\frac{6s}{c_S^2}\right\}\right.\\
&\quad+\left(10\epsilon+\left\{\frac{3}{16}\left(\frac{1}{c_S^2}-1-\frac{2\Sigma_2(X,\phi)}{\Sigma_1(X,\phi)}\right)-6\left(\frac{1}{c_S^2}-1\right)-\frac{6s}{c_S^2}\right\}\right)\left(\frac{k_S}{k_L}\right)^2\\
&\quad+\left(-(\epsilon+2\eta)+\left\{5\left(\frac{1}{c_S^2}-1\right)+\frac{2s}{c_S^2}\right.\right.\\
&\quad\left.\left.\left.-\frac{9}{32}\left(\frac{1}{c_S^2}-1-\frac{2\Sigma_2(X,\phi)}{\Sigma_1(X,\phi)}\right)\right\}\right)\left(\frac{k_S}{k_L}\right)^3\right].
\end{aligned}\tag{323}$$

  Furthermore, for restricted classes of the general single-field $P(X,\phi)$ model, which satisfies the constraint $P_{,X\phi}(X,\phi)=0$, we get:

$$\begin{aligned}
t_{-1} &= 0, \\
t_0 &= 4(3\epsilon - \eta), \\
t_1 &= 0, \\
t_2 &= \left\{\frac{1}{8}\left(\frac{1}{c_S^2} - 1 - \frac{s\epsilon}{3\epsilon_X}\right) - 6\left(\frac{1}{c_S^2} - 1\right)\right\} + 10\epsilon, \\
t_3 &= \left\{5\left(\frac{1}{c_S^2} - 1\right) - \frac{3}{16}\left(\frac{1}{c_S^2} - 1 - \frac{s\epsilon}{3\epsilon_X}\right)\right\} - (\epsilon + 2\eta)
\end{aligned} \tag{324}$$

for such case bispectrum is given by:

$$\begin{aligned}
B_{GSF}(k_L, k_L, k_S) &= \frac{H^4}{32\epsilon^2 M_p^4} \frac{1}{k_L^3 k_S^3} \left[4(3\epsilon - \eta)\right. \\
&\quad + \left(\left\{\frac{1}{8}\left(\frac{1}{c_S^2} - 1 - \frac{s\epsilon}{3\epsilon_X}\right) - 6\left(\frac{1}{c_S^2} - 1\right)\right\} + 10\epsilon\right)\left(\frac{k_S}{k_L}\right)^2 \\
&\quad \left. + \left(-(\epsilon + 2\eta) + \left\{5\left(\frac{1}{c_S^2} - 1\right) - \frac{3}{16}\left(\frac{1}{c_S^2} - 1 - \frac{s\epsilon}{3\epsilon_X}\right)\right\}\right)\left(\frac{k_S}{k_L}\right)^3\right].
\end{aligned} \tag{325}$$

- **For α, β vacuum**:
  After setting $C_1 = \cosh\alpha$ and $C_2 = e^{i\beta}\sinh\alpha$, we get the following expression for the expansion coefficients $a_j \forall j = -1, \cdots, 3$:

$$\begin{aligned}
t_{-1} &= 16\epsilon \sinh^2 2\alpha \cos^2\beta, \\
t_0 &= \left(4(2\epsilon - \eta) - \frac{6s}{c_S^2}\right)\cosh^2 2\alpha + 4\epsilon + 4\epsilon \sinh^2 2\alpha \cos^2\beta, \\
t_1 &= 34\epsilon \sinh^2 2\alpha \cos^2\beta, \\
t_2 &= \left\{\frac{3}{16}\left(\frac{1}{c_S^2} - 1 - \frac{2\Sigma_2(X,\phi)}{\Sigma_1(X,\phi)}\right) - 6\left(\frac{1}{c_S^2} - 1\right) - \frac{6s}{c_S^2}\right\}\cosh^2 2\alpha \\
&\qquad + 10\epsilon + 10\epsilon \sinh^2 2\alpha \cos^2\beta, \\
t_3 &= \left\{2(2\epsilon - \eta) - \frac{9}{32}\left(\frac{1}{c_S^2} - 1 - \frac{2\Sigma_2(X,\phi)}{\Sigma_1(X,\phi)}\right) + 5\left(\frac{1}{c_S^2} - 1\right) + \frac{2s}{c_S^2}\right\}\cosh^2 2\alpha \\
&\qquad - 5\epsilon - \epsilon \sinh^2 2\alpha \cos^2\beta.
\end{aligned} \tag{326}$$

Consequently, the bispectrum can be recast as:

$$\begin{aligned}
B_{GSF}(k_L,k_L,k_S) &= \frac{H^4}{32\epsilon^2 M_p^4}\frac{1}{k_L^3 k_S^3}\left[16\epsilon\sinh^2 2\alpha\cos^2\beta\left(\tfrac{k_S}{k_L}\right)^{-1}\right.\\
&+\left(\left(4(2\epsilon-\eta)-\tfrac{6s}{c_S^2}\right)\cosh^2 2\alpha+4\epsilon+4\epsilon\sinh^2 2\alpha\cos^2\beta\right)\\
&\quad+34\epsilon\sinh^2 2\alpha\cos^2\beta\left(\tfrac{k_S}{k_L}\right)\\
&+\left(\left\{\tfrac{3}{16}\left(\tfrac{1}{c_S^2}-1-\tfrac{2\Sigma_2(X,\phi)}{\Sigma_1(X,\phi)}\right)-6\left(\tfrac{1}{c_S^2}-1\right)-\tfrac{6s}{c_S^2}\right\}\cosh^2 2\alpha\right.\\
&\quad\left.+10\epsilon+10\epsilon\sinh^2 2\alpha\cos^2\beta\right)\left(\tfrac{k_S}{k_L}\right)^2\\
&+\left(\left\{2(2\epsilon-\eta)-\tfrac{9}{32}\left(\tfrac{1}{c_S^2}-1-\tfrac{2\Sigma_2(X,\phi)}{\Sigma_1(X,\phi)}\right)+5\left(\tfrac{1}{c_S^2}-1\right)+\tfrac{2s}{c_S^2}\right\}\cosh^2 2\alpha\right.\\
&\quad\left.\left.-5\epsilon-\epsilon\sinh^2 2\alpha\cos^2\beta\right)\left(\tfrac{k_S}{k_L}\right)^3\right].
\end{aligned}\tag{327}$$

Furthermore, for restricted classes of the general single-field $P(X,\phi)$ model, which satisfies the constraint $P_{,X\phi}(X,\phi)=0$, we get:

$$\begin{aligned}
t_{-1} &= 16\epsilon\sinh^2 2\alpha\cos^2\beta,\\
t_0 &= 4(2\epsilon-\eta)\cosh^2 2\alpha+4\epsilon+4\epsilon\sinh^2 2\alpha\cos^2\beta,\\
t_1 &= 34\epsilon\sinh^2 2\alpha\cos^2\beta,\\
t_2 &= \left\{\frac{1}{8}\left(\frac{1}{c_S^2}-1-\frac{s\epsilon}{3\epsilon_X}\right)-6\left(\frac{1}{c_S^2}-1\right)\right\}\cosh^2 2\alpha\\
&\qquad+10\epsilon+10\epsilon\sinh^2 2\alpha\cos^2\beta,\\
t_3 &= \left\{2(2\epsilon-\eta)-\frac{3}{16}\left(\frac{1}{c_S^2}-1-\frac{s\epsilon}{3\epsilon_X}\right)+5\left(\frac{1}{c_S^2}-1\right)\right\}\cosh^2 2\alpha\\
&\qquad-5\epsilon-\epsilon\sinh^2 2\alpha\cos^2\beta.
\end{aligned}\tag{328}$$

Consequently, the bispectrum can be recast as:

$$\begin{aligned}
B_{GSF}(k_L,k_L,k_S) &= \frac{H^4}{32\epsilon^2 M_p^4}\frac{1}{k_L^3 k_S^3}\left[16\epsilon\sinh^2 2\alpha\cos^2\beta\left(\tfrac{k_S}{k_L}\right)^{-1}\right.\\
&+\left(4(2\epsilon-\eta)\cosh^2 2\alpha+4\epsilon+4\epsilon\sinh^2 2\alpha\cos^2\beta\right)\\
&\quad+34\epsilon\sinh^2 2\alpha\cos^2\beta\left(\tfrac{k_S}{k_L}\right)\\
&+\left(\left\{\tfrac{1}{8}\left(\tfrac{1}{c_S^2}-1-\tfrac{s\epsilon}{3\epsilon_X}\right)-6\left(\tfrac{1}{c_S^2}-1\right)\right\}\cosh^2 2\alpha\right.\\
&\quad\left.+10\epsilon+10\epsilon\sinh^2 2\alpha\cos^2\beta\right)\left(\tfrac{k_S}{k_L}\right)^2\\
&+\left(\left\{2(2\epsilon-\eta)-\tfrac{3}{16}\left(\tfrac{1}{c_S^2}-1-\tfrac{s\epsilon}{3\epsilon_X}\right)+5\left(\tfrac{1}{c_S^2}-1\right)\right\}\cosh^2 2\alpha\right.\\
&\quad\left.\left.-5\epsilon-\epsilon\sinh^2 2\alpha\cos^2\beta\right)\left(\tfrac{k_S}{k_L}\right)^3\right].
\end{aligned}\tag{329}$$

5.2.3. Expression for EFT Coefficients for General Single-Field $P(X,\phi)$ Inflation

Here our prime objective is to derive the analytical expressions for EFT coefficients for general single-field $P(X,\phi)$ inflation. To serve this purpose here we start with a claim that the three-point function and the associated bispectrum for the scalar fluctuations computed from general single-field $P(X,\phi)$ inflation is exactly same as that we have computed from EFT setup. Here we use the equilateral limit and squeezed limit configurations to extract the analytical expression for the EFT coefficients. In the two limiting cases the results are as follows:

1. Equilateral limit configuration:
   For this case with arbitrary vacuum one can write:

$$B_{EFT}(k,k,k) = B_{GSF}(k,k,k), \tag{330}$$

which implies that[26]:

$$\begin{aligned}
\bar{M}_1 &= \left\{\tfrac{\hat{A}}{2\hat{B}H}\left[-1+\sqrt{1+\tfrac{4\hat{B}\hat{C}}{\hat{A}^2}}\right]\right\}^{\frac{1}{3}}, \quad \bar{M}_2 \approx \bar{M}_3 = \sqrt{\tfrac{\bar{M}_1^3}{4H\tilde{c}_5}} = \sqrt{\tfrac{\hat{A}}{8\hat{B}H^2\tilde{c}_5}\left[-1+\sqrt{1+\tfrac{4\hat{B}\hat{C}}{\hat{A}^2}}\right]},\\
M_2 &= \left(-\tfrac{XP_{,XX}(X,\phi)}{P_{,X}(X,\phi)}\dot{H}M_p^2\right)^{\frac{1}{4}},\\
M_3 &= \left\{-\tfrac{\hat{A}}{2\hat{B}}\tfrac{\tilde{c}_3}{\tilde{c}_4}\left[-1+\sqrt{1+\tfrac{4\hat{B}\hat{C}}{\hat{A}^2}}\right]\right\}^{\frac{1}{4}},\\
M_4 &= \left(-\tfrac{\tilde{c}_3}{\tilde{c}_6}H\bar{M}_1^3\right)^{\frac{1}{4}} = \left(-\tfrac{\hat{A}}{2\hat{B}}\tfrac{\tilde{c}_3}{\tilde{c}_6}\left[-1+\sqrt{1+\tfrac{4\hat{B}\hat{C}}{\hat{A}^2}}\right]\right)^{\frac{1}{4}}.
\end{aligned} \tag{332}$$

where the factors $\hat{A}$, $\hat{B}$ and $\hat{C}$ are defined as:

$$\begin{aligned}
\hat{A} &= \left(\frac{3}{2}+\frac{4}{3}\frac{\tilde{c}_3}{\tilde{c}_4}+\frac{2c_S^2}{\tilde{c}_4}\right)\left[\frac{U_1}{27}-3U_2\right]-\frac{5}{2}U_1+\frac{99}{98}U_2\\
&\quad -\frac{\Delta}{4}\left[\frac{1}{18}\left(\frac{1}{c_S^2}-1-\frac{2\Sigma_2(X,\phi)}{\Sigma_1(X,\phi)}\right)\left(|C_1|^2+|C_2|^2\right)^2\right.\\
&\quad -\frac{7}{3}\left(\frac{1}{c_S^2}-1\right)\left(|C_1|^2+|C_2|^2\right)^2-\frac{34}{3}\frac{s}{c_S^2}\left(|C_1|^2+|C_2|^2\right)^2\\
&\quad \left.+6(2\epsilon-\eta)\left(|C_1|^2+|C_2|^2\right)^2+11\epsilon\left(|C_1|^2-|C_2|^2\right)^2+27\epsilon\,(C_1^*C_2+C_1C_2^*)^2\right],\\
\hat{B} &= \left\{\left(\frac{3}{2}+\frac{4}{3}\frac{\tilde{c}_3}{\tilde{c}_4}+\frac{2c_S^2}{\tilde{c}_4}\right)\left[\frac{U_1}{27}-3U_2\right]-\frac{5}{2}U_1+\frac{99}{98}U_2\right\}\frac{\Delta c_S^2}{2\epsilon H^2M_p^2},\\
\hat{C} &= \frac{H^2M_p^2\epsilon c_S^2}{2}\left[\frac{1}{18}\left(\frac{1}{c_S^2}-1-\frac{2\Sigma_2(X,\phi)}{\Sigma_1(X,\phi)}\right)\left(|C_1|^2+|C_2|^2\right)^2\right.\\
&\quad -\frac{7}{3}\left(\frac{1}{c_S^2}-1\right)\left(|C_1|^2+|C_2|^2\right)^2-\frac{34}{3}\frac{s}{c_S^2}\left(|C_1|^2+|C_2|^2\right)^2\\
&\quad \left.+6(2\epsilon-\eta)\left(|C_1|^2+|C_2|^2\right)^2+11\epsilon\left(|C_1|^2-|C_2|^2\right)^2+27\epsilon\,(C_1^*C_2+C_1C_2^*)^2\right].
\end{aligned} \tag{333}$$

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[26] Here we also get another solution:

$$\bar{M}_1 = \left\{\frac{A}{2BH}\left[-1-\sqrt{1+\frac{4BC}{A^2}}\right]\right\}^{\frac{1}{3}}, \tag{331}$$

which is redundant in the present context as this solution is not consistent with the $c_S = 1$ and $\tilde{c}_S = 1$ limit result as computed in the earlier section for single-field slow-roll inflation.

Furthermore, for restricted classes of the general single-field $P(X,\phi)$ model, which satisfies the constraint $P_{,X\phi}(X,\phi)=0$, we get the following expression for the factors $\hat{A}$, $\hat{B}$ and $\hat{C}$ as:

$$\begin{aligned}
\hat{A} &= \left(\frac{3}{2}+\frac{4}{3}\frac{\tilde{c}_3}{\tilde{c}_4}+\frac{2c_S^2}{\tilde{c}_4}\right)\left[\frac{U_1}{27}-3U_2\right]-\frac{5}{2}U_1+\frac{99}{98}U_2 \\
&\quad -\frac{\Delta}{4}\left[\frac{1}{27}\left(\frac{1}{c_S^2}-1-\frac{s\epsilon}{3\epsilon_X}\right)\left(|C_1|^2+|C_2|^2\right)^2-\frac{7}{3}\left(\frac{1}{c_S^2}-1\right)\left(|C_1|^2+|C_2|^2\right)^2\right. \\
&\quad \left.+6(2\epsilon-\eta)\left(|C_1|^2+|C_2|^2\right)^2+11\epsilon\left(|C_1|^2-|C_2|^2\right)^2+27\epsilon\left(C_1^*C_2+C_1C_2^*\right)^2\right], \\
\hat{B} &= \left\{\left(\frac{3}{2}+\frac{4}{3}\frac{\tilde{c}_3}{\tilde{c}_4}+\frac{2c_S^2}{\tilde{c}_4}\right)\left[\frac{U_1}{27}-3U_2\right]-\frac{5}{2}U_1+\frac{99}{98}U_2\right\}\frac{\Delta c_S^2}{2\epsilon H^2M_p^2}, \qquad (334)\\
\hat{C} &= \frac{H^2M_p^2\epsilon c_S^2}{2}\left[\frac{1}{27}\left(\frac{1}{c_S^2}-1-\frac{s\epsilon}{3\epsilon_X}\right)\left(|C_1|^2+|C_2|^2\right)^2-\frac{7}{3}\left(\frac{1}{c_S^2}-1\right)\left(|C_1|^2+|C_2|^2\right)^2\right. \\
&\quad \left.+6(2\epsilon-\eta)\left(|C_1|^2+|C_2|^2\right)^2+11\epsilon\left(|C_1|^2-|C_2|^2\right)^2+27\epsilon\left(C_1^*C_2+C_1C_2^*\right)^2\right].
\end{aligned}$$

where for arbitrary vacuum U_1 and U_2 are defined as:

$$U_1 = \left[(C_1-C_2)^3\left(C_1^{*3}+C_2^{*3}\right)+(C_1^*-C_2^*)^3\left(C_1^3+C_2^3\right)\right], \qquad (335)$$

$$U_2 = \left[(C_1-C_2)^3\,C_1^*C_2^*\,(C_1^*-C_2^*)+(C_1^*-C_2^*)^3\,C_1C_2\,(C_1-C_2)\right]. \qquad (336)$$

If we take $c_S=1$ and $\tilde{c}_S=1$ then we get then we get back all the results obtained for single-field slow-roll inflation in the previous section.

Now for Bunch–Davies and α,β vacuum we get the following simplified expression for the bispectrum for scalar fluctuation:

- **For Bunch–Davies vacuum**:
  After setting $C_1=1$ and $C_2=0$ we get the following expression for the factors $\hat{A}$, $\hat{B}$ and $\hat{C}$ as:

$$\begin{aligned}
\hat{A} &= \frac{2}{27}\left(\frac{3}{2}+\frac{4}{3}\frac{\tilde{c}_3}{\tilde{c}_4}+\frac{2c_S^2}{\tilde{c}_4}\right)-5-\frac{\Delta}{4}\left[\frac{1}{18}\left(\frac{1}{c_S^2}-1-\frac{2\Sigma_2(X,\phi)}{\Sigma_1(X,\phi)}\right)\right. \\
&\quad \left.-\frac{7}{3}\left(\frac{1}{c_S^2}-1\right)-\frac{34}{3}\frac{s}{c_S^2}+(23\epsilon-6\eta)\right], \\
\hat{B} &= \left\{\frac{2}{27}\left(\frac{3}{2}+\frac{4}{3}\frac{\tilde{c}_3}{\tilde{c}_4}+\frac{2c_S^2}{\tilde{c}_4}\right)-5\right\}\frac{\Delta c_S^2}{2\epsilon H^2M_p^2}, \qquad (337)\\
\hat{C} &= \frac{H^2M_p^2\epsilon c_S^2}{2}\left[\frac{1}{18}\left(\frac{1}{c_S^2}-1-\frac{2\Sigma_2(X,\phi)}{\Sigma_1(X,\phi)}\right)-\frac{7}{3}\left(\frac{1}{c_S^2}-1\right)-\frac{34}{3}\frac{s}{c_S^2}+(23\epsilon-6\eta)\right].
\end{aligned}$$

  Furthermore, for restricted classes of the general single-field $P(X,\phi)$ model, which satisfies the constraint $P_{,X\phi}(X,\phi)=0$, we get the following expression for the factors A, B and C as:

$$\begin{aligned}
\hat{A} &= \frac{2}{27}\left(\frac{3}{2}+\frac{4}{3}\frac{\tilde{c}_3}{\tilde{c}_4}+\frac{2c_S^2}{\tilde{c}_4}\right)-5-\frac{\Delta}{4}\left[\frac{1}{27}\left(\frac{1}{c_S^2}-1-\frac{s\epsilon}{3\epsilon_X}\right)-\frac{7}{3}\left(\frac{1}{c_S^2}-1\right)+(23\epsilon-6\eta)\right], \\
\hat{B} &= \left\{\frac{2}{27}\left(\frac{3}{2}+\frac{4}{3}\frac{\tilde{c}_3}{\tilde{c}_4}+\frac{2c_S^2}{\tilde{c}_4}\right)-5\right\}\frac{\Delta c_S^2}{2\epsilon H^2 M_p^2}, \\
\hat{C} &= \frac{H^2M_p^2\epsilon c_S^2}{2}\left[\frac{1}{27}\left(\frac{1}{c_S^2}-1-\frac{s\epsilon}{3\epsilon_X}\right)-\frac{7}{3}\left(\frac{1}{c_S^2}-1\right)+(23\epsilon-6\eta)\right].
\end{aligned} \tag{338}$$

- **For α, β vacuum**:
  After setting $C_1 = \cosh\alpha$ and $C_2 = e^{i\beta}\sinh\alpha$ we get the following expression for the factors $\hat{A}$, $\hat{B}$ and $\hat{C}$ as:

$$\begin{aligned}
\hat{A} &= \left(\frac{3}{2}+\frac{4}{3}\frac{\tilde{c}_3}{\tilde{c}_4}+\frac{2c_S^2}{\tilde{c}_4}\right)\left[\frac{J_1(\alpha,\beta)}{27}-3J_2(\alpha,\beta)\right]-\frac{5}{2}J_1(\alpha,\beta)+\frac{99}{98}J_2(\alpha,\beta) \\
&\quad -\frac{\Delta}{4}\left[\left\{\frac{1}{18}\left(\frac{1}{c_S^2}-1-\frac{2\Sigma_2(X,\phi)}{\Sigma_1(X,\phi)}\right)-\frac{7}{3}\left(\frac{1}{c_S^2}-1\right)-\frac{34}{3}\frac{s}{c_S^2}\right\}\cosh^2 2\alpha\right. \\
&\quad \left.+6(2\epsilon-\eta)\cosh^2 2\alpha+11\epsilon+27\epsilon\sinh^2\alpha\cos^2\beta\right], \\
\hat{B} &= \left\{\left(\frac{3}{2}+\frac{4}{3}\frac{\tilde{c}_3}{\tilde{c}_4}+\frac{2c_S^2}{\tilde{c}_4}\right)\left[\frac{J_1(\alpha,\beta)}{27}-3J_2(\alpha,\beta)\right]-\frac{5}{2}J_1(\alpha,\beta)+\frac{99}{98}J_2(\alpha,\beta)\right\}\frac{\Delta c_S^2}{2\epsilon H^2M_p^2}, \\
\hat{C} &= \frac{H^2M_p^2\epsilon c_S^2}{2}\left[\left\{\frac{1}{18}\left(\frac{1}{c_S^2}-1-\frac{2\Sigma_2(X,\phi)}{\Sigma_1(X,\phi)}\right)-\frac{7}{3}\left(\frac{1}{c_S^2}-1\right)-\frac{34}{3}\frac{s}{c_S^2}\right\}\cosh^2 2\alpha\right. \\
&\quad \left.+6(2\epsilon-\eta)\cosh^2 2\alpha+11\epsilon+27\epsilon\sinh^2\alpha\cos^2\beta\right].
\end{aligned} \tag{339}$$

Furthermore, for restricted classes of the general single-field $P(X,\phi)$ model, which satisfies the constraint $P_{,X\phi}(X,\phi) = 0$, we get the following expression for the factors $\hat{A}$, $\hat{B}$ and $\hat{C}$ as:

$$\begin{aligned}
\hat{A} &= \left(\frac{3}{2}+\frac{4}{3}\frac{\tilde{c}_3}{\tilde{c}_4}+\frac{2c_S^2}{\tilde{c}_4}\right)\left[\frac{J_1(\alpha,\beta)}{27}-3J_2(\alpha,\beta)\right]-\frac{5}{2}J_1(\alpha,\beta)+\frac{99}{98}J_2(\alpha,\beta) \\
&\quad -\frac{\Delta}{4}\left[\left\{\frac{1}{27}\left(\frac{1}{c_S^2}-1-\frac{s\epsilon}{3\epsilon_X}\right)-\frac{7}{3}\left(\frac{1}{c_S^2}-1\right)+6(2\epsilon-\eta)\right\}\cosh^2 2\alpha\right. \\
&\qquad \left.+11\epsilon+27\epsilon\sinh^2\alpha\cos^2\beta\right], \\
\hat{B} &= \left\{\left(\frac{3}{2}+\frac{4}{3}\frac{\tilde{c}_3}{\tilde{c}_4}+\frac{2c_S^2}{\tilde{c}_4}\right)\left[\frac{J_1(\alpha,\beta)}{27}-3J_2(\alpha,\beta)\right]-\frac{5}{2}J_1(\alpha,\beta)+\frac{99}{98}J_2(\alpha,\beta)\right\}\frac{\Delta c_S^2}{2\epsilon H^2M_p^2}, \\
\hat{C} &= \frac{H^2M_p^2\epsilon c_S^2}{2}\left[\left\{\frac{1}{27}\left(\frac{1}{c_S^2}-1-\frac{s\epsilon}{3\epsilon_X}\right)-\frac{7}{3}\left(\frac{1}{c_S^2}-1\right)+6(2\epsilon-\eta)\right\}\cosh^2 2\alpha\right. \\
&\qquad \left.+11\epsilon+27\epsilon\sinh^2\alpha\cos^2\beta\right].
\end{aligned} \tag{340}$$

2. Squeezed limit configuration:
   For this case with arbitrary vacuum one can write:

$$B_{EFT}(k_L,k_L,k_S) = B_{GSF}(k_L,k_L,k_S), \tag{341}$$

which implies that:

$$\begin{aligned}
\bar{M_1} &= \left\{\frac{\hat{A}}{2\hat{B}H}\left[-1+\sqrt{1+\frac{4\hat{B}\hat{C}}{\hat{A}^2}}\right]\right\}^{\frac{1}{3}}, \quad \bar{M_2} \approx \bar{M_3} = \sqrt{\frac{\bar{M}_1^3}{4H\tilde{c}_5}} = \sqrt{\frac{\hat{A}}{8\hat{B}H^2\tilde{c}_5}\left[-1+\sqrt{1+\frac{4\hat{B}\hat{C}}{\hat{A}^2}}\right]},\\
M_2 &= \left(-\frac{XP_{,XX}(X,\phi)}{P_{,X}(X,\phi)}\dot{H}M_p^2\right)^{\frac{1}{4}}, \quad M_3 = \left\{-\frac{\hat{A}}{2\hat{B}}\frac{\tilde{c}_3}{\tilde{c}_4}\left[-1+\sqrt{1+\frac{4\hat{B}\hat{C}}{\hat{A}^2}}\right]\right\}^{\frac{1}{4}}, \qquad (342)\\
M_4 &= \left(-\frac{\tilde{c}_3}{\tilde{c}_6}H\bar{M}_1^3\right)^{\frac{1}{4}} = \left(-\frac{\hat{A}}{2\hat{B}}\frac{\tilde{c}_3}{\tilde{c}_6}\left[-1+\sqrt{1+\frac{4\hat{B}\hat{C}}{\hat{A}^2}}\right]\right)^{\frac{1}{4}}.
\end{aligned}$$

where the factors $\hat{A}$, $\hat{B}$ and $\hat{C}$ are defined as[27]:

$$\hat{A} = \hat{P}_1+\hat{P}_2-\sum_{j=-1}^{3}t_j\left(\frac{k_S}{k_L}\right)^j, \quad \hat{B} = \frac{\hat{P}_1\Delta}{2\epsilon H^2M_p^2}, \quad \hat{C} = 2\epsilon H^2M_p^2\sum_{j=-1}^{3}t_j\left(\frac{k_S}{k_L}\right)^j, \qquad (344)$$

where the expansion coefficients $t_j \forall j = -1,\cdots,3$ are defined earlier for the general $P(X,\phi)$ model and also for restricted classes of the model where $P_{,X\phi}(X,\phi) = 0$ constraint is satisfied.

Here the factors $\hat{P}_1$ and $\hat{P}_2$ are defined as:

$$\hat{P}_1 = \sum_{m=-1}^{3}\hat{e}_m\left(\frac{k_S}{k_L}\right)^m, \quad \hat{P}_2 = \sum_{m=-1}^{3}\hat{h}_m\left(\frac{k_S}{k_L}\right)^m, \qquad (345)$$

where the expansion coefficients $\hat{e}_m \forall m = -1,\cdots,3$ and $\hat{h}_m \forall m = -1,\cdots,3$ for arbitrary vacuum are defined as:

$$\begin{aligned}
\hat{e}_{-1} &= -36U_2, \;\; \hat{e}_0 = \left[-\frac{9}{2}U_1+\left(24\left\{\frac{3}{2}+\frac{4}{3}\frac{\tilde{c}_3}{\tilde{c}_4}+\frac{2c_S^2}{\tilde{c}_4}\right\}-\frac{9}{2}\right)U_2\right],\\
\hat{e}_1 &= 0, \;\; \hat{e}_2 = \left[-\frac{27}{2}U_2+\left(\frac{3}{2}\left\{\frac{3}{2}+\frac{4}{3}\frac{\tilde{c}_3}{\tilde{c}_4}+\frac{2c_S^2}{\tilde{c}_4}\right\}-\frac{27}{2}\right)U_1\right], \;\; \hat{e}_3 = 0, \qquad (346)
\end{aligned}$$

and

$$\begin{aligned}
\hat{h}_{-1} &= 0, \;\; \hat{h}_0 = \frac{9}{c_S^2}\left(U_1+U_2\right), \;\; \hat{h}_1 = 0,\\
\hat{h}_2 &= \left[\left(15+2\left\{\frac{3}{2}+\frac{2c_S^2}{\tilde{c}_4}\right\}\right)U_1+\left(\frac{45}{2}+3\left\{\frac{3}{2}+\frac{2c_S^2}{\tilde{c}_4}\right\}\right)U_2\right], \;\; \hat{h}_3 = 0, \qquad (347)
\end{aligned}$$

where U_1 and U_2 are already defined earlier.

Now for Bunch–Davies and α,β vacuum we get the following simplified expression for the bispectrum for scalar fluctuation:

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[27] Here we also get another solution:

$$\bar{M_1} = \left\{\frac{\hat{A}}{2\hat{B}H}\left[-1-\sqrt{1+\frac{4\hat{B}\hat{C}}{\hat{A}^2}}\right]\right\}^{\frac{1}{3}}, \qquad (343)$$

which is redundant in the present context as this solution is not consistent with the $c_S = 1$ and $\tilde{c}_S = 1$ limit result as computed in the earlier section for general single-field $P(X,\phi)$ inflation.

- **For Bunch–Davies vacuum**:
  After setting $C_1 = 1$ and $C_2 = 0$, we get $U_1 = 2$ and $U_2 = 0$. Consequently, the expansion coefficients can be recast as:

$$\hat{e}_{-1} = 0, \; \hat{e}_0 = -9, \; \hat{e}_1 = 0, \; \hat{e}_2 = -27, \; \hat{e}_3 = 0, \tag{348}$$

  and

$$\hat{h}_{-1} = 0, \; \hat{h}_0 = \frac{18}{c_S^2}, \; \hat{h}_1 = 0, \; \hat{h}_2 = \left(30 + 4\left\{\frac{3}{2} + \frac{2c_S^2}{\tilde{c}_4}\right\}\right), \; \hat{h}_3 = 0, \tag{349}$$

- **For α, β vacuum**:
  After setting $C_1 = \cosh\alpha$ and $C_2 = e^{i\beta}\sinh\alpha$, we get $U_1 = J_1(\alpha,\beta)$ and $U_2 = J_2(\alpha,\beta)$. Consequently, the expansion coefficients can be recast as:

$$\begin{aligned}
\hat{e}_{-1} &= -36 J_2(\alpha,\beta), \; \hat{e}_0 = \left[-\frac{9}{2}J_1(\alpha,\beta) + \left(24\left\{\frac{3}{2} + \frac{4}{3}\frac{\tilde{c}_3}{\tilde{c}_4} + \frac{2c_S^2}{\tilde{c}_4}\right\} - \frac{9}{2}\right)J_2(\alpha,\beta)\right], \\
\hat{e}_1 &= 0, \; \hat{e}_2 = \left[-\frac{27}{2}J_2(\alpha,\beta) + \left(\frac{3}{2}\left\{\frac{3}{2} + \frac{4}{3}\frac{\tilde{c}_3}{\tilde{c}_4} + \frac{2c_S^2}{\tilde{c}_4}\right\} - \frac{27}{2}\right)J_1(\alpha,\beta)\right], \; \hat{e}_3 = 0,
\end{aligned} \tag{350}$$

  and

$$\begin{aligned}
\hat{h}_{-1} &= 0, \; \hat{h}_0 = \frac{9}{c_S^2}\left(U_1 + J_2(\alpha,\beta)U_2\right), \; \hat{h}_1 = 0, \\
\hat{h}_2 &= \left[\left(15 + 2\left\{\frac{3}{2} + \frac{2c_S^2}{\tilde{c}_4}\right\}\right)J_1(\alpha,\beta) + \left(\frac{45}{2} + 3\left\{\frac{3}{2} + \frac{2c_S^2}{\tilde{c}_4}\right\}\right)J_2(\alpha,\beta)\right], \; \hat{h}_3 = 0.
\end{aligned} \tag{351}$$

6. Conclusions

To summarize, in this paper, we have addressed the following issues:

- We have derived the analytical expressions for the two-point correlation function for scalar and tensor fluctuations and three-point correlation function for scalar fluctuations from EFT framework in quasi de Sitter background in a model-independent way. For this computation, we use an arbitrary quantum state as the initial choice of vacuum. Such a choice finally gives rise to the most general expressions for the two-point and three-point correlation functions for primordial fluctuation in EFT. Furthermore, we have simplified our results by considering the Bunch–Davies vacuum and α, β vacuum states.
- During our computation, we have truncated the EFT action by considering the all possible two derivative terms in the metric. This allows us to derive correct expressions for the two-point and three-point correlation functions for EFT which are consistent with both the single-field slow-roll model and generalized non-canonical $P(X, \phi)$ single-field models minimally coupled with gravity[28].
- Furthermore, we have derived the analytical expressions for the coefficients of all relevant EFT operators for the single-field slow-roll model and generalized non-canonical $P(X, \phi)$ single-field models. We have derived the results in terms of slow-roll parameters, effective sound speed parameter, and the constants which are fixed by the choice of arbitrary initial vacuum state. Next,

[28] This is really an important outcome as the earlier derived results for the three-point function for EFT in quasi de Sitter background was not consistent with the known result for the single-field slow-roll model, where effective sound speed is fixed at $\tilde{c}_S = 1$.

we have simplified our results also presented the results by considering Bunch–Davies vacuum and α, β vacuum state.

- Finally, using the CMB observation from Planck we constrain all these EFT coefficients for various single-field slow-roll models and generalized non-canonical $P(X, \phi)$ models of inflation.

The future directions of this paper are appended below pointwise:

- One can further carry forward this work to compute four-point scalar correlation function from EFT framework using an arbitrary initial choice of the quantum vacuum state. The present work can also be extended for the computation of the three-point correlation from tensor fluctuation, and other three-point cross correlations between scalar and tensor mode fluctuation in the context of EFT with arbitrary initial vacuum.
- In the present EFT framework we have not considered the effects of any additional heavy fields ($m >> H$) in the effective action. One can redo the analysis with such additional effects in the EFT framework to study the quantum entanglement, cosmological decoherence and Bell's inequality violation in the context of primordial cosmology. Once can also further generalize this computation for any arbitrary spin fields which are consistent with the unitarity bound.
- The analyticity property of response functions and scattering amplitudes in QFT implies significant connection between observables in IR regime and the underlying dynamics valid in the short-distance scale. Such analytic property is directly connected to the causality and unitarity of the QFT under consideration. Following this idea one can also study the analyticity property in the present version of EFT or including the effective of massive fields ($m >> H$) in the effective action.
- There are other open issues as well which one can study within the framework of EFT:

  1. The role of out-of-time-ordered correlations from open quantum system [60–62].
  2. EFT framework in a quantum dissipative system and its application to cosmology [63–66].
  3. Thermalization, quantum critical quench and its application to the phenomena of reheating in early universe cosmology [67].

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Appendix A. Brief Overview on Schwinger-Keldysh (In-In) Formalism

To compute the any n-point correlation function in quasi de Sitter space we use Schwinger-Keldysh (In-In) formalism. In this framework the expectation value of a product of operators $\mathcal{O}(t)$ at time t can be written as:

$$\langle \mathcal{O}(t) \rangle = \left\langle \left(T\ \exp\left[-i \int_{-\infty}^{t} H_{int}(t^{'}) dt^{'} \right] \right)^{\dagger} \mathcal{O}(t) \left(T\ \exp\left[-i \int_{-\infty}^{t} H_{int}(t^{''}) dt^{''} \right] \right) \right\rangle , \tag{A1}$$

where it is important to note that all the fields appearing in the right-hand side belong to the Heisenberg picture. Here correlation function is computed with respect to the initial quantum vacuum state $|in\rangle$, which in general can be any arbitrary vacuum state. In cosmological literature concept of Bunch–Davies and α, β vacuum are commonly used. To mention the mathematical structure of the quantum vacuum state $|in\rangle$ we first consider an arbitrary state $|\Omega(t)\rangle$, which can be expanded in terms of the eigen basis state $|m\rangle$ of the free Hamiltonian as:

$$|\Omega(t)\rangle = \sum |m\rangle \langle m|\Omega(t)\rangle. \tag{A2}$$

Furthermore, the time evolved quantum state from time $t = t_1$ to $t = t_2$ can be written as:

$$|\Omega(t_2)\rangle \quad = \quad T\ \exp\left[-i \int_{t_1}^{t_2} H_{int}(t^{'}) dt^{'} \right] |\Omega(t_1)\rangle = \underbrace{|0\rangle\langle 0|\Omega\rangle}_{\textbf{Free part}} + \underbrace{\sum_{m=1}^{\infty} \exp\left[iE_m(t_2 - t_1) \right] |m\rangle\langle m|\Omega(t_1)\rangle}_{\textbf{Interacting part}} . \tag{A3}$$

It is clearly observed that we have expressed any arbitrary quantum state in terms of the free part and the interacting part of the theory. Furthermore, for further computation we set $t_2 = -\infty(1 - i\epsilon)$ which clearly projects all excited quantum states. Using this we have the following connecting relation between the interacting vacuum and the free vacuum state, as given by:

$$|\Omega(-\infty(1 - i\epsilon))\rangle \equiv |0\rangle\langle 0|\Omega\rangle. \tag{A4}$$

Finally, at any arbitrary time the interacting vacuum can be written as:

$$|in\rangle \quad = \quad T\ \exp\left[-i \int_{-\infty(1-i\epsilon)}^{t} H_{int}(t^{'}) dt^{'} \right] |\Omega(-\infty(1 - i\epsilon))\rangle = T\ \exp\left[-i \int_{-\infty(1-i\epsilon)}^{t} H_{int}(t^{'}) dt^{'} \right] |0\rangle\langle 0|\Omega\rangle. \tag{A5}$$

For our computations, initially we have written the expression which is valid for any arbitrary choice of quantum vacuum state. But for simplicity further we consider two specific choices of vacuum state—Bunch–Davies vacuum and α, β vacuum, which are commonly used in cosmological physics. Now in this context the total Hamiltonian of the theory can be written in terms of the free and interacting part as, $H = H_0 + H_{int}$, where interaction Hamiltonian is described by H_{int} and the free field Hamiltonian is described by H_0.

In the context of cosmological perturbation theory one can follow the same formalism where one usually starts with the Einstein–Hilbert gravity action with the any matter content in the effective action. For this purpose one uses the well-known ADM formalism to derive an action which contains only dynamical degrees of freedom. From this action one needs to perform the following steps:

- First one needs to construct the canonically conjugate momenta and the Hamiltonian for the system.
- Then we need to separate out the quadratic part from the higher-order contributions in the Hamiltonian.

Now in this context let us consider a part of the effective action which contains the third-order contribution and all other higher-order contribution in cosmological perturbation theory, represented by L_{int}. In this case the usual expression for the interaction Hamiltonian is given by, $H_{int} = -L_{int}$.

Furthermore, to make a direct connection to the in-out formalism in QFT used in the computation of S-matrix, one can further insert complete sets of states labelled by α and β in Equation (A1) and finally get:

$$\begin{aligned}\langle\mathcal{O}(t)\rangle &= \int d\alpha\int d\beta \\ &\langle 0|\left(T\,\exp\left[-i\int_{-\infty}^{t}H_{int}(t^{'})dt^{'}\right]\right)^{\dagger}|\alpha\rangle\,\overbrace{\langle\alpha|\mathcal{O}(t)|\beta\rangle}^{=\mathcal{O}_{\alpha\beta}(t)}\langle\beta|\left(T\,\exp\left[-i\int_{-\infty}^{t}H_{int}(t^{''})dt^{''}\right]\right)|0\rangle,\end{aligned} \tag{A6}$$

Here the in-in quantum correlation is interpreted as the product of the vacuum transition amplitudes and in the matrix element $\langle\alpha|\mathcal{O}(t)|\beta\rangle\equiv\mathcal{O}_{\alpha\beta}(t)$, where one needs to sum over all possible quantum out states. Furthermore, to compute the quantum correlations using Schwinger-Keldysh (In-In) formalism one needs to consider the following steps:

- First of all one needs to define the time integration in the time evolution operator $U(t)$ to go over a contour in the complex plane i.e.,

$$U(t)=T\,\exp\left[-i\int_{-\infty}^{t}H_{int}(t^{'})dt^{'}\right]\Rightarrow T\,\exp\left[-i\int_{-\infty(1+i\epsilon)}^{t}H_{int}(t^{'})dt^{'}\right], \tag{A7}$$

  where we have redefined the time interval by including small imaginary contribution as given by $t\to t(1\pm i\epsilon)$. With this specific choice, one can finally write the following expression for the n-point correlation function:

$$\langle\mathcal{O}(t)\rangle=\left\langle\left(T\,\exp\left[-i\int_{-\infty(1-i\epsilon)}^{t}H_{int}(t^{'})dt^{'}\right]\right)^{\dagger}\mathcal{O}(t)\left(T\,\exp\left[-i\int_{-\infty(1+i\epsilon)}^{t}H_{int}(t^{''})dt^{''}\right]\right)\right\rangle. \tag{A8}$$

  Here it is important to note that complex conjugation of the time evolution operator $U(t)$ signifies the fact that the time-ordered contour does not at all coincide with the time-backward contour.
- Next we analytically continue the expression for the interaction Hamiltonian as appearing in the time evolution operator $U(t)$ i.e., $H_{ini}(t)\to H_{int}(t(1\pm i\epsilon))$.
- Next we consider the following Dyson Swinger series:

$$T\,\exp\left[-i\int_{-\infty(1+i\epsilon)}^{t}H_{int}(t^{'})dt^{'}\right]=1+\sum_{N=1}^{\infty}\frac{(-i)^N}{N!}\prod_{i=1}^{N}\int_{-\infty(1+i\epsilon)}^{t_i}dt_i\,H_{int}(t_i), \tag{A9}$$

  using which finally we get the following simplified expression for the n-point correlation function:

$$\langle\mathcal{O}(t)\rangle=\sum_{N=0}^{\infty}\frac{(-i)^N}{N!}\prod_{i=1}^{N}\int_{-\infty(1+i\epsilon)}^{t_i}dt_i\,\langle 0|\left[H_{int}(t_i),\mathcal{O}(t)\right]|0\rangle=\sum_{n=0}^{\infty}\langle\mathcal{O}(t)\rangle^{(n)}. \tag{A10}$$

  where $|0\rangle$ is the initial quantum vacuum state under consideration. Here expanding in the powers of interacting Hamiltonian $H_{int}(t)$ we finally get:

  1. Zeroth order term $\langle\mathcal{O}(t)\rangle^{(0)}$ in Dyson Swinger series:
     Here the zeroth order term in Dyson Swinger series can be expressed as:

$$\langle\mathcal{O}(t)\rangle^{(0)} = \langle 0|\mathcal{O}(t)|0\rangle. \tag{A11}$$

  2. First order term $\langle\mathcal{O}(t)\rangle^{(1)}$ in Dyson Swinger series:
     Here the first order term in Dyson Swinger series can be expressed as:

$$\langle\mathcal{O}(t)\rangle^{(1)} = 2\mathrm{Re}\left[-i\int_{-\infty(1+i\epsilon)}^{t}dt^{'}\,\langle 0|\mathcal{O}(t)H_{int}(t^{'})|0\rangle\right]. \tag{A12}$$

3. Second-order term $\langle\mathcal{O}(t)\rangle^{(2)}$ in Dyson Swinger series:
   Here the second-order term in Dyson Swinger series can be expressed as:

$$\begin{aligned}\langle\mathcal{O}(t)\rangle^{(2)} &= -2\mathrm{Re}\left[\int_{-\infty(1+i\epsilon)}^{t_1} dt_1 \int_{-\infty(1+i\epsilon)}^{t_2} dt_2 \langle 0|\mathcal{O}(t)H_{int}(t_1)H_{int}(t_2)|0\rangle\right] \\ &\quad + \int_{-\infty(1+i\epsilon)}^{t_1} dt_1 \int_{-\infty(1+i\epsilon)}^{t_2} dt_2 \langle 0|H_{int}(t_1)\mathcal{O}(t)H_{int}(t_2)|0\rangle. \qquad \text{(A13)}\end{aligned}$$

   Following this trick one can easily write down the expression for any n-point correlation function of the given operator $\mathcal{O}(t)$.

Appendix B. Choice of Initial Quantum Vacuum State

In general, one can consider an arbitrary initial quantum vacuum state which is specified by the two sets of constants (C_1, C_2) and (D_1, D_2) as appearing in solution of the scalar and tensor mode fluctuation. In general in this context a quantum state is described by this two number as $|C_1, C_2\rangle$ and $|D_1, D_2\rangle$ and defined as, $C(\mathbf{k})|C_1, C_2\rangle = 0 \;\forall\; \mathbf{k}, D(\mathbf{k})|D_1, D_2\rangle = 0 \;\forall\; \mathbf{k}$, where $C(\mathbf{k})$ and $D(\mathbf{k})$ are the annihilation operators for scalar and tensor mode fluctuations as appearing in cosmological perturbation theory.

In general, ground one can write down the most general state $|C_1, C_2\rangle$ in terms of the well-known Bunch–Davies vacuum state as:

$$\begin{aligned}|C_1, C_2\rangle &= \prod_{\mathbf{k}} \frac{1}{\sqrt{|C_1|}} \exp\left[\frac{C_2^*}{2C_1^*} C^\dagger(\mathbf{k})C^\dagger(-\mathbf{k})\right]|0\rangle \\ &= \frac{1}{\mathcal{N}_C} \exp\left[\frac{C_2^*}{2C_1^*} \sum_{\mathbf{k}} C^\dagger(\mathbf{k})C^\dagger(-\mathbf{k})\right]|0\rangle = \frac{1}{\mathcal{N}_C} \exp\left[\frac{C_2^*}{2C_1^*} \int \frac{d^3k}{(2\pi)^3} C^\dagger(\mathbf{k})C^\dagger(-\mathbf{k})\right]|0\rangle, \qquad \text{(A14)}\end{aligned}$$

where $\mathcal{N}_C = \sqrt{|C_1|}$ are the overall normalization constant for scalar and tensor mode fluctuations. For the tensor modes the calculation is similar.

Here it is important to mention that the quantum vacuum state $|C_1, C_2\rangle$ satisfies the following constraint equation:

$$\begin{aligned}\hat{\mathbf{P}}_C|C_1, C_2\rangle &= \int \frac{d^3p}{(2\pi)^3}\, \mathbf{p}\, C^\dagger(\mathbf{p})C(\mathbf{p})|C_1, C_2\rangle = \prod_{\mathbf{k}} \int \frac{d^3p}{(2\pi)^3} \frac{\mathbf{p}\, C^\dagger(\mathbf{p})C(\mathbf{p})}{\sqrt{|C_1|}} \exp\left[\frac{C_2^*}{2C_1^*} C^\dagger(\mathbf{k})C^\dagger(-\mathbf{k})\right]|0\rangle \\ &= \int \frac{d^3p}{(2\pi)^3} \frac{\mathbf{p}\, C^\dagger(\mathbf{p})C(\mathbf{p})}{\sqrt{|C_1|}} \exp\left[\frac{C_2^*}{2C_1^*} \sum_{\mathbf{k}} C^\dagger(\mathbf{k})C^\dagger(-\mathbf{k})\right]|0\rangle \qquad \text{(A15)} \\ &= \int \frac{d^3p}{(2\pi)^3} \frac{\mathbf{p}\, C^\dagger(\mathbf{p})C(\mathbf{p})}{\sqrt{|C_1|}} \exp\left[\frac{C_2^*}{2C_1^*} \int \frac{d^3k}{(2\pi)^3} C^\dagger(\mathbf{k})C^\dagger(-\mathbf{k})\right]|0\rangle = 0,\end{aligned}$$

which is true for the quantum vacuum state $|D_1, D_2\rangle$ for tensor modes also. Since scalar modes are exactly similar to tensor modes we will not speak about tensor modes in the next part.

Additionally, here it is important to note that the annihilation and creation operators for Bunch–Davies vacuum $(a(\mathbf{k}), a^\dagger(\mathbf{k}))$ and the arbitrary quantum vacuum $|C_1, C_2\rangle$ state $(C(\mathbf{k}), C^\dagger(\mathbf{k}))$ are connected via the following sets of Bogoliubov transformations:

$$C(\mathbf{k}) = C_1^*\, a(\mathbf{k}) - C_2^*\, a^\dagger(-\mathbf{k}), \qquad \text{(A16)}$$
$$a(\mathbf{k}) = C_1\, C(\mathbf{k}) + C_2^*\, C^\dagger(-\mathbf{k}). \qquad \text{(A17)}$$

A very well-known feature of QFT is that it makes itself particularly manifest in the context of curved space-time backgrounds, physically represents particle excitation. Consequently, a thermal quantum state depends sensitively on the proper choice of quantum mechanical vacuum state. But we know that for a generalized curved space-time, no canonical or even preferred quantum vacuum state exists. In the context of QFT in curved space-time, there is a huge class of quantum mechanical states over a background de Sitter space which are invariant under all the $\mathbf{SO(1,4)}$ isometries[29,30], which is commonly known as the α, β-vacuum and represent the excited quantum states. By fixing the parameter $\beta = 0$ one can explicitly show that α-vacuum is CPT-invariant, where α plays the role of super-selection real parameter. Furthermore, fixing the real parameter $\alpha = 0$ one can get back the well-known *Bunch–Davies* quantum vacuum state for de Sitter space where the Cosmological Constant, $\Lambda > 0$. In the limit, $\Lambda \to 0$ one can further show that with $\alpha = 0$ we can get back the unique Minkowski quantum vacuum state. One can also choose a quantum mechanical vacuum α state as an initial condition, which at late time scale will give rise to long-range (Hubble scale) quantum correlations. In this context the long-range quantum correlations are manifestation of entanglement associated with the quantum mechanical vacuum state which is here identified as the initial state.

Among these classes of quantum mechanical vacuum, there is a specific type of vacuum state whose associated Green's functions verify the well-known *Hadamard condition* behaving on the light-cone as in flat Minkowski space. This quantum mechanical state is usually known as the *Bunch–Davies vacuum* or *Euclidean vacuum*. The *Bunch–Davies vacuum* can also be described as being generated by an infinite time-trace operation from the condition that the associated scale of quantum fluctuations is much smaller than the cosmological Hubble scale.

The *Bunch–Davies* vacuum state is treated as the zero-particle ground state in the context of QFT of curved space-time which is actually observed by a geodesic observer. This quantum mechanical vacuum state is very useful which explains the origin of quantum mechanical fluctuations in the context of inflationary models.

1. Bunch–Davies vacuum:
   Bunch–Davies vacuum is specified by fixing the coefficients to, $C_1 = 1 = D_1, \quad C_2 = 0 = D_2$, in the solution of the scalar and tensor mode fluctuation as derived earlier. In this case, the quantum vacuum state $|0\rangle$ is defined as the state that gets annihilated by the annihilation operator, as given by, $a(\mathbf{k})|0\rangle = 0 \ \forall \ \mathbf{k}$. Here the creation and annihilation operators $a(\mathbf{k})$ and $a^\dagger(\mathbf{k})$ satisfy the following canonical commutation relations:

$$\left[a(\mathbf{k}), a(\mathbf{k}')\right] = 0, \quad \left[a^\dagger(\mathbf{k}), a^\dagger(\mathbf{k}')\right] = 0, \quad \left[a(\mathbf{k}), a^\dagger(\mathbf{k}')\right] = (2\pi)^3 \delta^{(3)}(\mathbf{k} + \mathbf{k}'). \tag{A18}$$

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[29] Due to isometries, a time-like Killing vector field provides a natural physical explanation of partitioning the frequency modes into positive and negative categories, which is similar to the standard procedure performed in Minkowski flat space-time. Furthermore, one can associate these positive and negative frequency modes with annihilation and creation operators in the present context. Then a quantum mechanical vacuum state can be described by imposing the constraint condition that the state be annihilated by all the annihilation operators. In the absence of time-like Killing vector, there exists no natural choice of quantum mechanical vacuum state. In such a situation, one can apply different conditions to choose a particular quantum vacuum state. In this specific situation, a natural simplest possible choice is to consider a physical region of space-time for which a time-like Killing vector does exist, which can be further used to construct the corresponding quantum mechanical vacuum state. Similarly, if the space-time asymptotically matches with the Minkowski flat case, then in that specific situation there exists another possibility to use the most generalized *Poincar'e* quantum mechanical vacuum state. In an alternative prescription, one can consider a physical situation where the quantum mechanical vacuum state be annihilated by the physical generators of some specific symmetry group. On the other hand, a quantum mechanical vacuum state can also be treated as an un-physical if it fails to satisfy certain necessary physical constraints. To demonstrate this explicitly one can consider an example, where the expectation value of the stress energy momentum tensor diverges at a non-singular point in space-time, such as at a horizon. In that situation one can easily discard the possibility of the corresponding quantum mechanical vacuum state existing. In our description we need a particular universal form for the cosmological correlation function and the associated spectrum which is actually dictated by the conservation of the stress energy.

[30] Here it is important to note that under the application of an arbitrary de Sitter transformation, which is commonly identified as isometries, each positive frequency modes of de Sitter mix among themselves and each negative frequency modes mix among themselves, but precisely they do not mix among each other. This physically implies that the *Bunch–Davies vacuum* state is invariant under the de Sitter isometry **SO(1,4)** group. On the other hand, for massive scalar fields (or may be the scalar field have very tiny non-zero mass), if we set the parameter $\beta = 0$ then we get a one-parameter family of de Sitter invariant vacuums, which is commonly known as α-vacuum, which physically represents the squeezed states. It is important to note that for our discussion we always consider non-zero mass of the scalar fields because of the fact that for massless scalar degrees of freedom quantum mechanical vacuum states are not invariant under the de Sitter isometry **SO(1,4)** group.

2. α, β vacuum:
α, β vacuum is specified by fixing the coefficients to, $C_1 = \cosh\alpha = D_1, \quad C_2 = e^{i\beta}\sinh\alpha = D_2$, in the solution of the scalar and tensor mode fluctuation as derived earlier. In this case the quantum vacuum state $|\alpha, \beta\rangle$ is defined as the state that gets annihilated by the annihilation operator, as given by, $b(\mathbf{k})|\alpha, \beta\rangle = 0 \, \forall \, \mathbf{k}$. Here the creation and annihilation operators $b(\mathbf{k})$ and $b^\dagger(\mathbf{k})$ satisfy the following canonical commutation relations:

$$\left[b(\mathbf{k}), b(\mathbf{k}^{'})\right] = 0, \quad \left[b^\dagger(\mathbf{k}), b^\dagger(\mathbf{k}^{'})\right] = 0, \quad \left[b(\mathbf{k}), b^\dagger(\mathbf{k}^{'})\right] = (2\pi)^3\delta^{(3)}(\mathbf{k}+\mathbf{k}^{'}). \tag{A19}$$

Here one can write the Bunch–Davies vacuum state $|0\rangle$ as a special class of $|\alpha, \beta\rangle$ vacuum state. Also using Bogoliubov transformation one can write down $|\alpha, \beta\rangle$ vacuum state in terms of the Bunch–Davies vacuum state $|0\rangle$, as given by:

$$\begin{aligned} |\alpha, \beta\rangle &= \prod_{\mathbf{k}} \frac{1}{\sqrt{|\cosh\alpha|}} \exp\left[-\frac{i}{2}e^{-i\beta}\tanh\alpha \, a^\dagger(\mathbf{k})a^\dagger(-\mathbf{k})\right]|0\rangle \\ &= \frac{1}{\mathcal{N}} \exp\left[-\frac{i}{2}e^{-i\beta}\tanh\alpha \sum_{\mathbf{k}} a^\dagger(\mathbf{k})a^\dagger(-\mathbf{k})\right]|0\rangle \\ &= \frac{1}{\mathcal{N}} \exp\left[-\frac{i}{2}e^{-i\beta}\tanh\alpha \int \frac{d^3k}{(2\pi)^3} \, a^\dagger(\mathbf{k})a^\dagger(-\mathbf{k})\right]|0\rangle, \end{aligned} \tag{A20}$$

where $\mathcal{N} = \sqrt{|\cosh\alpha|}$ is the overall normalization constant. Here it is important to mention that the $|\alpha, \beta\rangle$ vacuum state satisfies the following constraint equation:

$$\begin{aligned} \hat{\mathbf{P}}|\alpha, \beta\rangle &= \int \frac{d^3p}{(2\pi)^3} \, \mathbf{p} \, a^\dagger(\mathbf{p})a(\mathbf{p})|\alpha, \beta\rangle \\ &= \prod_{\mathbf{k}} \int \frac{d^3p}{(2\pi)^3} \frac{\mathbf{p} \, a^\dagger(\mathbf{p})a(\mathbf{p})}{\sqrt{|\cosh\alpha|}} \exp\left[-\frac{i}{2}e^{-i\beta}\tanh\alpha \, a^\dagger(\mathbf{k})a^\dagger(-\mathbf{k})\right]|0\rangle \\ &= \int \frac{d^3p}{(2\pi)^3} \frac{\mathbf{p} \, a^\dagger(\mathbf{p})a(\mathbf{p})}{\sqrt{|\cosh\alpha|}} \exp\left[-\frac{i}{2}e^{-i\beta}\tanh\alpha \, \sum_{\mathbf{k}} a^\dagger(\mathbf{k})a^\dagger(-\mathbf{k})\right]|0\rangle \\ &= \int \frac{d^3p}{(2\pi)^3} \frac{\mathbf{p} \, a^\dagger(\mathbf{p})a(\mathbf{p})}{\sqrt{|\cosh\alpha|}} \exp\left[-\frac{i}{2}e^{-i\beta}\tanh\alpha \, \int \frac{d^3k}{(2\pi)^3} a^\dagger(\mathbf{k})a^\dagger(-\mathbf{k})\right]|0\rangle = 0, \end{aligned} \tag{A21}$$

Additionally, here it is important to note that the creation and annihilation operators for Bunch–Davies vacuum and $|\alpha, \beta\rangle$ vacuum state are connected via the following sets of Bogoliubov transformations:

$$b(\mathbf{k}) = \cosh\alpha \, a(\mathbf{k}) + i \, e^{-i\beta}\sinh\alpha \, a^\dagger(-\mathbf{k}), \tag{A22}$$

$$a(\mathbf{k}) = \cosh\alpha \, b(\mathbf{k}) - i \, e^{-i\beta}\sinh\alpha \, b^\dagger(-\mathbf{k}). \tag{A23}$$

Appendix C. Useful Integrals as Appearing in Scalar Three-Point Function

All the useful integrals appearing in the scalar three-point function are appended bellow:

$$1.\quad \int_{-\infty}^{0} d\eta\, \eta^2\, e^{\pm iK\tilde{c}_S\eta} = \mp\frac{2}{iK^3\tilde{c}_S^3}, \tag{A24}$$

$$2.\quad \int_{-\infty}^{0} d\eta\, \eta^2\, e^{\mp i(2k_a-K)\tilde{c}_S\eta} = \pm\frac{2}{i(2k_a-K)^3\tilde{c}_S^3}, \tag{A25}$$

$$3.\quad \int_{\eta_i=-\infty}^{\eta_f=0} d\eta\, (1\mp ik_b\tilde{c}_S\eta)(1\mp ik_c\tilde{c}_S\eta)\, e^{\pm iK\tilde{c}_S\eta} = \frac{1}{iK^3\tilde{c}_S}\left[K^2+2k_bk_c+K(K-k_a)\right], \tag{A26}$$

$$\begin{aligned} 4.\quad & \int_{-\infty}^{0} d\eta\, (1-ik_b\tilde{c}_S\eta)(1-ik_c\tilde{c}_S\eta)\, e^{i(K-2k_a)\tilde{c}_S\eta} = -\int_{-\infty}^{0} d\eta\, (1+ik_b\tilde{c}_S\eta)(1+ik_c\tilde{c}_S\eta)\, e^{-i(K-2k_a)\tilde{c}_S\eta} \\ = \quad & -\frac{1}{i(2k_a-K)^3\tilde{c}_S}\left[K^2+2k_bk_c+K(K-5k_a)-2(K-k_a)k_a+4k_a^2\right], \end{aligned} \tag{A27}$$

$$\begin{aligned} 5.\quad & \int_{-\infty}^{0} d\eta\, (1-ik_b\tilde{c}_S\eta)(1\mp ik_c\tilde{c}_S\eta)\, e^{i(K-2k_b)\tilde{c}_S\eta} = -\int_{-\infty}^{0} d\eta\, (1\mp ik_b\tilde{c}_S\eta)(1\pm ik_c\tilde{c}_S\eta)\, e^{\mp i(K-2k_b)\tilde{c}_S\eta} \\ = \quad & -\frac{1}{i(2k_b-K)^3\tilde{c}_S}\left[K^2-4k_bk_c+K(k_c-5k_b)+6k_b^2\right], \end{aligned} \tag{A28}$$

$$6.\quad \int_{-\infty}^{0} d\eta\, (1\mp ik_a\tilde{c}_S\eta)\, e^{\pm iK\tilde{c}_S\eta} = \pm\frac{1}{iK^2\tilde{c}_S}(K+k_a), \tag{A29}$$

$$7.\quad \int_{-\infty}^{0} d\eta\, (1\pm ik_a\tilde{c}_S\eta)\, e^{\pm i(K-2k_a)\tilde{c}_S\eta} = \pm\frac{(K-3k_a)}{i(2k_a-K)^2\tilde{c}_S}, \tag{A30}$$

$$8.\quad \int_{-\infty}^{0} d\eta\, (1\mp ik_a\tilde{c}_S\eta)\, e^{\pm i(K-2k_b)\tilde{c}_S\eta} = \pm\frac{(K+k_a-2k_b)}{i(2k_b-K)^2\tilde{c}_S}, \tag{A31}$$

$$\begin{aligned} 9.\quad & \int_{-\infty}^{0} d\eta\, (1-ik_a\tilde{c}_S\eta)(1-ik_c\tilde{c}_S\eta)\, e^{i(K-2k_b)\tilde{c}_S\eta} \\ = \quad & -\frac{1}{i(2k_b-K)^3\tilde{c}_S}\left[(K-2k_b)(K+k_a-2k_b)+(K+2k_a-2k_b)k_c\right]. \end{aligned} \tag{A32}$$

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