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Effective Field Theory of Loop Quantum Cosmology

Martin Bojowald

Institute for Gravitation and the Cosmos, The Pennsylvania State University, 104 Davey Lab, University Park, PA 16802, USA; bojowald@gravity.psu.edu

Received: 29 November 2018; Accepted: 15 January 2019; Published: 23 January 2019

Abstract: Quantum cosmology is traditionally formulated in a minisuperspace setting, implicitly averaging fields over space to obtain homogeneous models. For universal reasons related to the uncertainty principle, quantum corrections then depend on the size of the averaging volume. In minisuperspace truncations, the value of this volume remains an arbitrary parameter devoid of physical meaning, but in an effective field theory it is identified with the infrared scale of inhomogeneous modes. Moreover, the infrared scale is running during gravitational collapse, when regions in which homogeneity remains approximately valid shrink to increasingly smaller co-moving sizes. Conceptual implications of this infrared renormalization for perturbative inhomogeneity in quantum cosmology are presented here, mainly for the example of loop quantum cosmology. Several claims made in this framework are altered by infrared renormalization.

Keywords: quantum cosmology; effective field theory; infrared renormalization

1. Introduction

Quantum cosmology aims to perform two main tasks: to describe in a meaningful way what is indicated by general relativity as the big-bang singularity and to derive potentially observable predictions in early-universe cosmology. Both questions are of a physical nature and therefore require a detailed understanding of the relevant equations and solutions of Planck-scale physics. In the absence of a complete theory of quantum gravity, effective field theory provides important means to highlight implications that are sufficiently generic to be insensitive to quantization ambiguities or choices made in specific models. As we will see here, mainly in an application to loop quantum cosmology [1,2], an effective field theory of quantum cosmology indeed revises some claims extracted from a limited set of simplified models.

These revisions can be split into two types: those related to the background dynamics and those related to the behavior of inhomogeneity. As for the former, a central role is played by the averaging volume V_0 used to express an inhomogeneous geometry by a simplified homogeneous model. While the classical equations are invariant under changing the size of this volume, quantum corrections depend on this parameter as a consequence of uncertainty relations: The averaging volume V_0 can be made arbitrarily small, such that the geometrical volume $V = V_0a^3$ of a given region in an isotropic geometry with scale factor a has no lower bound. However, in a canonical quantization, volume fluctuations ΔV are bounded from below by the uncertainty relation $\Delta V \Delta H \geq \pi\ell_{\rm P}^2$ (noting that $H/4\pi G$ is canonically conjugate to V). Since the Hubble parameter H and the Planck length $\ell_{\rm P}$ are invariant under changing V_0, it is impossible that fluctuations $(\Delta V, \Delta H)$ scale in the same way as (V, H) under changes of V_0. Quantum corrections from fluctuations therefore generically break the classical V_0-invariance. (The fact that new quantum effects may depend on the size of a region in which the theory is formulated is well-known from the Casimir force.)

Canonical effective field theory, as reviewed in the next section, has provided a physical interpretation for the appearance of V_0 in quantum corrections of minisuperspace models [3]:

These corrections describe the infrared contribution of quantum back-reaction in the underlying inhomogeneous theory of which the minisuperspace model gives an averaged description. The main new contribution of the present paper is an elaboration of this result for perturbative inhomogeneity around the background of a minisuperspace model. Because minisuperspace quantum corrections already contain the infrared contribution from inhomogeneous modes, additional modes brought in by perturbative inhomogeneity must be restricted to (co-moving) wavelengths less than the cubic root of the averaging volume. Without this restriction, some quantum corrections would be duplicated. (As another consequence, it follows that V_0 is not an infrared regulator, which should be sent to infinity after observables have been computed, but rather an infrared scale which separates modes described in different ways—by averaging and more directly through perturbative inhomogeneity, respectively.)

If one uses a minisuperspace model amended by perturbative inhomogeneity to probe deep quantum regimes, evolving toward increasing curvature, the co-moving size of regions in which homogeneity is approximately realized shrinks due to gravitational collapse. In order to maintain the approximation implied by averaging, the value of V_0 should therefore be progressively reduced. Combined with the interpretation of V_0 as an infrared scale, reducing V_0 amounts to infrared renormalization. Based on the same interpretation, the modes of perturbative inhomogeneity have to be restricted to wavelengths less than the cubic root of V_0; therefore, the number of modes changes during evolution, as a consequence of infrared renormalization [4]. This behavior cannot be described by unitary evolution on a single Hilbert space, but effective field theory provides the appropriate setting.

Quantum cosmology has another effect on the behavior of inhomogeneity, related to the form of quantum space–time structure. In particular in the context of theories such as loop quantum gravity that indicate some kind of spatial or space–time discreteness, effective field theory is relevant also for conceptual questions. For instance, while it is easy in minisuperspace models to avoid divergences such as the big-bang singularity by modifying classical equations in terms of bounded functions, this common practice raises several questions: A modification in minisuperspace equations ignores consistency conditions imposed by the requirement that equations for inhomogeneity be covariant. If the theory resulting from such a modification is not consistent with covariance, one has to demonstrate that one can still avoid low-energy problems [5]. Moreover, the physical nature of any mechanism that helps to avoid the big-bang singularity often remains unclear in minisuperspace models. For instance, some studies in loop quantum cosmology have claimed that the big bang is replaced by a "bounce" [6]. However, these models modify the gravitational terms in the Friedmann equation while keeping the matter terms largely unchanged. How, then, can it be possible that one evades singularity theorems without violating energy conditions? These theorems do not use Einstein's equation or the Friedmann equation and should therefore remain valid after a modification of the latter—provided the modified theory has the same (Riemannian) space–time structure as classical gravity, a property which is often assumed implicitly in models of loop quantum cosmology [7–10].

As we will discuss, a canonical effective field theory of quantum cosmology provides insights into space–time structures and is thereby able to answer the conceptual questions posed here. The two main topics of this paper—averaging and covariance—are related by the presence of an infrared scale, which is implied by averaging and has an effect on possible covariant formulations.

2. Averaging Volume

Going back to [11], minisuperspace models have traditionally been used in quantum cosmology, in which one eliminates the spatial dependence of all fields by averaging them over a specified region in space. The resulting model has only temporal dependence, a fact which trivializes many consistency conditions imposed by covariance. However, it turns out that minisuperspace models remain sensitive to the covariance problem in a subtle way, which is related to the coordinate volume of the region used to define the spatial averaging.

This result has been illustrated in [3] by a model of a minisuperspace model, using a scalar field theory in flat space–time with Lagrangian

$$L = \int \mathrm{d}^3x \left(\frac{1}{2}\dot{\phi}^2 - \frac{1}{2}|\nabla\phi|^2 - W(\phi) \right) . \tag{1}$$

This theory has a minisuperspace model in which ϕ is assumed spatially constant, such that the Lagrangian can be integrated over some region with finite coordinate volume $V_0 = \int \mathrm{d}^3x$:

$$L_{\mathrm{mini}} = V_0 \left(\frac{1}{2}\dot{\phi}^2 - W(\phi) \right) . \tag{2}$$

We obtain the minisuperspace momentum

$$p = \frac{\partial L_{\mathrm{mini}}}{\partial \dot{\phi}} = V_0 \dot{\phi} \tag{3}$$

and the Hamiltonian

$$H_{\mathrm{mini}} = \frac{1}{2}\frac{p^2}{V_0} + V_0 W(\phi) . \tag{4}$$

The minisuperspace Hamiltonian can straightforwardly be quantized to

$$\hat{H}_{\mathrm{mini}} = \frac{1}{2}\frac{\hat{p}^2}{V_0} + V_0 W(\hat{\phi}) . \tag{5}$$

In a semiclassical analysis, for instance using [12,13], the minisuperspace potential is found to have first-order corrections given by

$$W_{\mathrm{eff}}^{\mathrm{mini}}(\phi) = W(\phi) + \frac{1}{2V_0}\hbar\sqrt{W''(\phi)} . \tag{6}$$

The full scalar theory, on the other hand, has first-order corrections given by the Coleman–Weinberg potential [14]

$$W_{\mathrm{eff}}(\phi) = W(\phi) + \frac{1}{2}\hbar \int \frac{\mathrm{d}^4k}{(2\pi)^4} \log\left(1 + \frac{W''(\phi)}{||\mathbf{k}||^2} \right) \tag{7}$$

with an explicit k^0-integration [15]

$$W_{\mathrm{eff}}(\phi) = W(\phi) + \frac{1}{2}\hbar \int \frac{\mathrm{d}^3k}{(2\pi)^3} \left(\sqrt{|\vec{k}|^2 + W''(\phi)} - |\vec{k}| \right) . \tag{8}$$

The minisuperspace potential Equation (6) is therefore obtained as the infrared contribution from quantum field theory. Integrating

$$\frac{1}{2}\hbar \int \frac{\mathrm{d}^3k}{(2\pi)^3} \left(\sqrt{|\vec{k}|^2 + W''(\phi)} - |\vec{k}| \right) \tag{9}$$

over $|\vec{k}| \leq k_{\mathrm{max}} = 2\pi/V_0^{1/3}$ results in

$$W_{\mathrm{eff}}(\phi) \approx W(\phi) + \frac{\hbar}{12\pi^2}k_{\mathrm{max}}^3\sqrt{W''(\phi)} = W(\phi) + \frac{2\pi}{3V_0}\hbar\sqrt{W''(\phi)} . \tag{10}$$

2.1. Infrared Renormalization

We conclude that a minisuperspace truncation can capture some quantum effects of the full theory, as long as $V_0 \not\to \infty$. If one sends V_0 to infinity, as sometimes done following a misinterpretation of V_0 as an infrared regulator [16], all quantum corrections are erased. For finite V_0, on the other hand, quantum corrections, unlike the classical equations, depend on the seemingly arbitrary V_0. While minisuperspace models could not explain the relevance of V_0, the connection with the quantum-field theory Equation (1) makes it clear: V_0 represents the infrared scale of modes included in a minisuperspace model through averaging. Quantum effects of long-wavelength modes are therefore captured by minisuperspace effective potentials. If short-wavelength modes are physically relevant, they should be added onto the minisuperspace model, either as a full quantum-field theory or as an effective field theory of perturbative inhomogeneity. In order to avoid duplicating quantum corrections already contained in the infrared contribution, perturbative inhomogeneity must be restricted to modes with wavelengths less than $V_0^{1/3}$.

Based on Equation (10), a minisuperspace approximation is expected to be reliable for large V_0 because it implicitly replaces the integration in Equation (9) with the k-volume multiplied with the integrand at $|\vec{k}| = 0$. Geometrically, the averaging is justified only if there is no strong inhomogeneity on scales less than V_0. This condition is realized at late times in cosmology, thanks to large-scale homogeneity. However, if one tries to use the same models to explain the big-bang singularity, the Belinskii–Khalatnikov–Lifshitz (BKL) scenario [17] shows that generically one has to extend them to small V_0: This scenario indicates that homogeneous dynamics may be relevant even close to a space-like singularity, but only because the geometries at different spatial points decouple from one another even while inhomogeneity grows on all scales. The homogeneous dynamics indicated by BKL, viewed in a minisuperspace model, is therefore compatible only with small V_0. If one evolves from small curvature to large curvature, the averaging volume V_0 must be progressively reduced in order to keep up with the shrinking scales of approximate homogeneity. Adjusting the infrared scale V_0 amounts to infrared renormalization [4].

Infrared renormalization has two main implications. First, any quantum treatment of perturbative inhomogeneity must be such that modes are restricted to wavelenths less than $V_0^{1/3}$. As V_0 changes during infrared renormalization, the number of field-theory modes changes as well. This behavior cannot be modeled by unitary evolution on a fixed Hilbert space. At this point, effective field theory is essential. Secondly, regarding the background dynamics in minisuperspace models, small V_0 are relevant near the big bang. It turns out that models of loop quantum cosmology are especially sensitive to this conclusion.

2.2. Models of Loop Quantum Gravity

In isotropic models of loop quantum cosmology [18], one uses an isotropic connection $A_a^i = c\delta_a^i$ and a densitized triad $E_j^b = p\delta_j^b$ parameterized by a pair of canonical variables, $c = \gamma\dot{a}$ and $|p| = a^2$. In this form, the flat-space classical Friedmann equation is written as

$$-\frac{c^2}{\gamma^2|p|} + \frac{8\pi G}{3}\rho = 0\,. \tag{11}$$

Loop quantization, however, does not provide an operator directly for c but only for matrix elements of holonomies [19,20], given in this model by $\exp(i\mu c)$ with a real number μ [21]. In order to become loop quantizable, the Friedmann equation is therefore modified by using "holonomies", replacing $c^2/|p|$ with $\sin(\ell c/\sqrt{|p|})^2/\ell^2$ in Equation (11), with some length parameter ℓ. (The p-dependence of the argument of the sine was first evaluated in a cosmological setting in [22] and is motivated by lattice refinement [23,24].) Taken in isolation, holonomy modifications indicate a "bounce" of isotropic models: The equation

$$\frac{\sin(\ell c/\sqrt{|p|})^2}{\gamma^2\ell^2} = \frac{8\pi G}{3}\rho \tag{12}$$

implies a bounded energy density which, for macroscopic ρ, remains true in terms of expectation values in any nearly semiclassical state.

Gravitational effective actions are usually written in higher-curvature form; see for instance [25,26]. This well-known result is a combination of two properties, the fact that quantum corrections generically imply higher-derivative terms together with the condition of space–time covariance. Covariance will be discussed in more detail in the next section, and for now we focus on higher-derivative corrections or their canonical analog. The canonical approach to effective theory [12,13], which is suitable for canonical quantum cosmology, realizes higher-derivative corrections through auxiliary degrees of freedom, which have a useful physical interpretation as moments of an evolving quantum state, back-reacting on the trajectory of its basic expectation values. These terms therefore originate from quantum-cosmological analogs of effective potentials such as Equation (6). As we just learned, these terms depend on the averaging volume V_0, as can be seen also in explicit derivations such as [27–29].

An important consequence of the V_0-dependence can be seen easily if the modified Friedmann Equation (12) is rewritten as [30]

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_{\rm QG}}\right) \tag{13}$$

with

$$\rho_{\rm QG} = \frac{3}{8\pi G\ell^2}\,. \tag{14}$$

This equation is obtained by expressing the canonical momentum c in terms of $\dot{a} = \frac{1}{2}\dot{p}/\sqrt{|p|}$ using an equation of motion generated by Equation (12), and is therefore equivalent to Equation (12).

If quantum back-reaction is included, there is an additional coupling term to quantum fluctuations and correlations which can be written as [27,28]

$$\frac{\dot{a}^2}{a^2} = \frac{8\pi G}{3}\rho\left(1 - \frac{\rho}{\rho_{\rm QG}} + \sigma\right) \tag{15}$$

where

$$\sigma = \frac{(\Delta V)^2 - C + (2\pi G\ell\hbar/3)^2}{(V + 2\pi G\ell\hbar/3)^2} \tag{16}$$

and $V = V_0 a^3$. The fluctuation ΔV of V appears explicitly in Equation (16), and C depends on the quantum correlation C_{VH} between the volume and its canonical momentum, the Hubble parameter H. Not much is known about the quantum state of the universe, which makes it difficult to provide an estimate of C_{VH}. However, for given fluctuations, quantum correlations cannot be arbitrarily large owing to uncertainty relations. We may therefore consider the term C in Equation (16) less significant than (ΔV). The expression Equation (16) can be used whenever higher-order moments of the state are small compared with the second-order moments that appear in Equation (16), that is, if the state is sufficiently semiclassical. More generally, one can extend (16) to a series in higher moments, as given in [27,28].

For $V \gg 2\pi G\ell\hbar/3$ (close to the Planck volume if $\ell \sim \ell_{\rm P}$) and a semiclassical state, we have $\sigma \ll 1$ and the σ-term in Equation (15) can be ignored. However, when the term $\rho/\rho_{\rm QG}$ in Equation (15) is relevant, we expect to be close to a classical singularity. We may then be justified in using homogeneous (although anisotropic) models to analyze the local dynamics, appealing to the BKL scenario. This scenario is asymptotic and does not set any lower bound, not even the Planck volume, on the size V_0 of regions of approximate homogeneity. Generically, we should therefore use small V_0 in this regime. If $V \ll 2\pi G\ell\hbar/3$, even a semiclassical state results in $\sigma \approx 1$, and quantum back-reaction cannot be ignored. Unfortunately, small-V_0 solutions have so far been neglected in loop quantum cosmology, following an influential claim [6] that argues for large V_0 but overlooks infrared renormalization, the central lesson of the BKL scenario for quantum cosmology.

The process of reducing the value of V_0 is related (but not equivalent) to coarse-graining in which one views a continuum description as an averaged microscopic formulation of many discrete patches.

In coarse-graining, one is led to consider small V_0 even in late-time cosmological eras in which one is not forced to do so by inhomogeneity. In the context of loop quantum cosmology, coarse graining has been studied, for instance, in [31]. In [32], it has been observed that it is possible to construct coherent states which, at least in some models, are insensitive to coarse-graining in that their moments behave additively when several such states are patched together. Such a behavior could be of interest also in the context of changing V_0. However, even though the coherent state in such a model is adapted to coarse-graining, it would still give rise to V_0-dependent quantum corrections. In the example of [32], for instance, the analog of the volume operator is called $\hat{z}$, and the scale V_0 is replaced by a Casimir quantum number j. The expectation value $\langle\hat{z}\rangle$ as well as the variance $(\Delta z)^2$ of $\hat{z}$ scale like j, such that a collection of N coherent states behaves like a single coherent state with scale Nj. However, the same proportionality implies that $\langle\hat{z}^2\rangle = \langle\hat{z}\rangle^2 + (\Delta z)^2$ does not have a clear scaling behavior since it is the sum of two terms, one of which scales like j^2 and one like j. Therefore, quantum corrections violate the classical scaling behavior of z^2 even in a coherent state. (This violation can be traced back to generic properties of uncertainty relations mentioned in the introduction.)

3. Covariance

Another set of quantum corrections in gravitational models is, generically, given by higher-curvature contributions. In loop quantum cosmology, even if we do not include higher-curvature corrections in the classical Friedmann Equation (11), from which we set out to quantize a cosmological model, such terms are expected in the semiclassical or effective dynamics that describes an evolving wave function through the time dependence of its basic expectation values. As explicitly shown in [33], such a dynamics generically contains higher-derivative terms. If these terms are to descend from a covariant action, they must be of a higher-curvature type. (In loop quantum gravity, the usual assumption is that covariance should not be broken by quantum space–time effects, although it may be deformed. More generally, it could be possible to construct theories that are not covariant in the ultraviolet, as in [34], but have covariance restored in the infrared; see for instance [35]. In such models, the covariance-breaking terms in an action can usually be written as a non-invariant higher-curvature contribution, for instance using the extrinsic curvature of a preferred foliation. Such theories are therefore included in the broad setting of higher-curvature effective actions.)

Any effective dynamics of loop quantum cosmology, such as a complete version of Equation (12), should therefore contain isotropic reductions of higher-curvature terms. No such terms are included in Equation (12), as can easily be seen from the observation that they would imply higher-derivative corrections (or auxiliary fields) that are absent in Equation (12). (This statement should not be confused with the existence of higher-curvature analogs that can mimic the modified isotropic dynamics of loop quantum cosmology [36–38] but differ in the presence of anisotropies or perturbations [39–41].) Since models of loop quantum cosmology are not based on a derivation from a covariant quantum theory, suitable higher-curvature corrections have not been derived yet in this framework. As long as they remain unknown, it is not justified to trust the full function $\sin^2(\ell c/\sqrt{|p|})/\ell^2$ which, as a power series, contains contributions of arbitrarily high order in $\ell^2 c^2/|p|$. Instead, one should use only the first term in the expansion, such as

$$\frac{\sin(\ell c/\sqrt{|p|})^2}{\ell^2} \sim \frac{c^2}{|p|}\left(1 - \frac{1}{3}\ell^2\frac{c^2}{|p|} + \cdots\right). \tag{17}$$

If $\ell \sim \ell_{\mathrm{P}}$, the leading corrections are $\ell_{\mathrm{P}}^2 c^2/|p| \sim \rho/\rho_{\mathrm{P}}$, which is indeed of the same order as expected for higher-curvature terms. Once suitable higher-curvature corrections have been derived from loop quantum cosmology, they may well change the high-density behavior; see for instance [42].

3.1. Problem of States

Higher-derivative terms are a general feature in effective equations of quantum mechanics, and they are state-dependent. The low-energy limit of effective actions often used in particle physics or for semiclassical gravity [25,26] results in a unique effective action, but only because an expansion around the ground state is assumed. In quantum gravity, it is not clear whether there is a suitable ground state. Even if there is a candidate, it would likely not be the right choice for Planckian physics near a spacelike singularity.

This issue is exacerbated by the problem of time: Cosmology is dynamical and therefore requires a reliable understanding of time. The classical equations of cosmological models are time-reparameterization-invariant, which provides a simplified setting in which covariance can be studied in quantum cosmology. Moreover, the classical cosmological dynamics is constrained.

Consider a quantum constraint of the form $\hat{C} = \hat{p}_\phi^2 - \hat{H}^2$. If H does not depend on ϕ, we can write the constraint equation $\hat{C}\psi = 0$ for states as an "evolution" equation

$$-\hat{p}_\phi \psi = i\hbar \frac{\partial \psi}{\partial \phi} = \pm |\hat{H}|\psi. \tag{18}$$

However, the choice of ϕ as time affects quantum corrections [43], and it is unclear how one should choose relevant initial states for ϕ-"evolution". Moreover, in a cosmological model, the classical constraint is strongly restricted by its reduction from a covariant theory. Consistency conditions are needed to find corresponding restrictions on quantum corrections in a minisuperspace model, which does not have a clear relationship with a covariant quantum theory.

Dealing with the problem of states requires parameterizations of a large class of models, which again calls for an application of effective field theory. In a canonical version, moments as auxiliary variables in an effective field theory describe the freedom contained in the choice of states. In the absence of a distinguished state, quantum corrections, such as higher-derivative terms from quantum back-reaction, depend on the choice of state. As a consequence, it is not clear whether the "bounce" claimed in loop quantum cosmology is generic if higher-derivative terms are uncontrolled.

3.2. Space–Time Structure

While the dynamical high-density behavior of models in loop quantum cosmology remains ambiguous, the space–time structure at high density turns out to be more controlled. Canonical quantization does not presuppose the space–time structure of a potential covariant quantum theory of gravity. Formulated in terms of an action, both the Lagrangian density and the measure in

$$S[g] = \frac{1}{16\pi G} \int \mathrm{d}^4x \sqrt{|\det g|}\, (R[g] + \cdots) \tag{19}$$

may then be subject to quantum corrections. If this possibility is realized, quantum-field theory on curved space–time, which presupposes a Riemannian structure of space–time, is different from quantum gravity, in which the very structure of space–time is likely modified by quantum effects.

Formal aspects of these statements may be illustrated for perturbative inhomogeneity, assuming that we are perturbing the basic field as $A(x) = \bar{A} + \delta A(x)$, which appears in a sample Hamiltonian $h[A] := A(x)^2$. As in models of loop quantum gravity, a modified background dynamics is then imposed by replacing $\bar{A}$ with $\ell^{-1}\sin(\ell\bar{A})$.

The classical, unmodified theory has a perturbative Hamiltonian

$$h[\bar{A}, \delta A] = \bar{A}^2 + 2\bar{A}\delta A(x) + \delta A(x)^2\,. \tag{20}$$

A version of quantum-field theory on modified space–time, as proposed in models of loop quantum cosmology in different guises [7–10], has the perturbative Hamiltonian

$$h_\ell^{\rm QFT}[\bar{A}, \delta A] = \ell^{-2}\sin(\ell\bar{A})^2 + 2\bar{A}\delta A(x) + \delta A(x)^2 \tag{21}$$

in which classical equations are used for perturbations but not for the background. From an effective field theory of cosmological perturbations within loop quantum cosmology [44–46], however, one obtains

$$h_\ell^{\rm QG}[\bar{A},\delta A] = \ell^{-2}\sin(\ell\bar{A})^2 + F_\ell(\bar{A})\delta A(x) + G_\ell(\bar{A})\delta A(x)^2 \tag{22}$$

with two functions F_ℓ and G_ℓ, which are constrained by the classical limit, $\lim_{\ell\to 0} F_\ell(\bar{A}) = 2\bar{A}$ and $\lim_{\ell\to 0} G_\ell(\bar{A}) = 1$, as well as covariance conditions. The latter are rather lengthy, but they imply that $F_\ell/\bar{A}$ and G_ℓ are comparable to $(\ell\bar{A})^{-2}\sin(\ell\bar{A})^2$. Therefore, they cannot be ignored if the background dynamics is modified, presenting a crucial difference between quantum-field theory on a modified background and effective quantum cosmology. In particular, quantum-field theory on a modified background, as used in [7–10], is inconsistent because it ignores terms that are of the same order as crucial terms included in the modification.

3.3. Covariance from Hypersurface Deformations

In canonical gravity, the generators $D[N^a]$ of deformations along a vector field $N^a(x)$ tangential to a spatial slice together with the generators $H[N]$ of normal deformations by a displacement of $N(x)$ obey brackets [47]

$$[D[N^a], D[M^b]] = -D[\mathcal{L}_{M^b} N^a] \tag{23}$$

$$[H[N], D[M^b]] = -H[\mathcal{L}_{M^b} N] \tag{24}$$

$$[H[N_1], H[N_2]] = D[q^{ab}(N_1\partial_b N_2 - N_2\partial_b N_1)] \tag{25}$$

of a Lie algebroid [48] in which the inverse of the induced metric q_{ab} on spatial slice appears in "structure functions." Covariance in canonical quantum gravity then requires an anomaly-free representation of these brackets by operators $\hat{D}$, $\hat{H}$, and $\hat{q}$, such that the commutators of smeared $\hat{D}$ and $\hat{H}$ are still proportional to constraints and equal Equations (23)–(25) in a suitable classical limit.

Although there has been some recent progress [49–55], both the anomaly problem and the question of the classical limit remain open in loop quantum gravity. It is, however, possible to analyze potential outcomes by using methods of effective constraints [56,57] which, like effective Hamiltonians, are defined as expectation values of constraint operators written as functions of basic expectation values and moments. These variables are subject to a Poisson bracket that replaces the commutator. In the classical limit, these effective constraints should therefore obey the hypersurface-deformation brackets

$$\{D[N^a], D[M^b]\} = -D[\mathcal{L}_{M^b} N^a] \tag{26}$$

$$\{H[N], D[M^b]\} = -H[\mathcal{L}_{M^b} N] \tag{27}$$

$$\{H[N_1], H[N_2]\} = D[q^{ab}(N_1\partial_b N_2 - N_2\partial_b N_1)] \tag{28}$$

in Poisson-bracket form.

It is important to note that this condition constitutes an off-shell property that cannot be tested in gauge-fixed models. (See also [58].) It turns out that this condition is stronger than formal anomaly-freedom in a reformulated system. For instance, it is possible to write the constraints in spherically symmetric and polarized Gowdy models such that the $\{H,H\}$-bracket with structure functions is replaced by an Abelian bracket of the form $\{H+D, H+D\} = 0$ [59,60]. However, these formally consistent models are not always covariant in the sense just defined [61,62]. Similarly, in some versions of the full theory, it is possible to rewrite the bracket with structure functions in the form $\{H,H\} = \{D',D'\}$ and quantize this theory [52] which, however, does not imply that the covariance condition is maintained.

3.4. Model

Generic consequences of covariance in the presence of holonomy modifications can be illustrated by another scalar model [63], now for a canonical pair of a field $\phi(x)$ with momentum $p(x)$ in one spatial dimension. The Hamiltonian and diffeomorphism constraints

$$H[N] = \int \mathrm{d}x N \left(f(p) - \frac{1}{4}(\phi')^2 - \frac{1}{2}\phi\phi'' \right) \quad , \quad D[w] = \int \mathrm{d}x w \phi p' \tag{29}$$

can be chosen such that the latter generates spatial diffeomorphisms,

$$\delta_w \phi = \{\phi, D[w]\} = -(w\phi)' \quad , \quad \delta_w p = \{p, D[w]\} = -wp' \tag{30}$$

(showing that the field ϕ has density weight one), while the former has a bracket

$$\{H[N], H[M]\} = D[\beta(p)(N'M - NM')] \tag{31}$$

with $\beta(p) = \frac{1}{2}\mathrm{d}^2 f/\mathrm{d}p^2$ that mimics results from detailed evaluations of covariance in models of loop quantum cosmology [45,64].

We have Lorentzian-type hypersurface deformations for $f(p) = p^2$. With "holonomy" modifications $f(p) = p_0^2 \sin^2(p/p_0)$, however,

$$\beta(p) = \frac{1}{2}\mathrm{d}^2 f/\mathrm{d}p^2 = \cos(2p/p_0) \tag{32}$$

can be negative. In particular, at the maximum of $f(p)$,

$$\{H[N], H[M]\} = D[-(N'M - NM')] \tag{33}$$

implies Euclidean signature: linear N and M give boosts for $\beta(p) = 1$, such that $\Delta x = v\Delta t$, but rotations if $\beta(p) = -1$, such that $\Delta x = -\theta\Delta y$ if y is transversal to hypersurfaces. The opposite sign is also obtained if hypersurface-deformation brackets are derived for Euclidean gravity, and using the methods of [65,66], one can show that the brackets imply elliptic field equations if $\beta(p) < 0$ [67].

In the scalar model, $\{H[N], D[w]\}$ does not close, but there are several consistent gravity versions, including spherical symmetry and cosmological perturbations [45,68]. The role of p in the model is then played by some curvature variable K, usually a component of extrinsic curvature of space in the canonical splitting of space–time. Replacing $K^2 \longrightarrow f(K)$ in the Hamiltonian constraint then modifies the bracket such that

$$\{H[N_1], H[N_2]\} = D[\beta q^{ab}(N_1\partial_b N_2 - N_2\partial_b N_1)] \tag{34}$$

with

$$\beta(K) = \frac{1}{2}\mathrm{d}^2 f(K)/\mathrm{d}K^2 = \cos(2\ell K) \tag{35}$$

for $f(K) = \ell^{-2}\sin^2(\ell K)$, with a free parameter ℓ often related to the Planck length. For small ℓK, $\beta(K) \sim 1$, and the classical hypersurface-deformation brackets are approximately realized. In cosmological models, the small-ℓK limit is usually achieved dynamically at late times, but, more in the spirit of coarse-graining or renormalization, one could interpret it as an infrared limit of the theory, akin to [35].

If ℓK is not small, the structure of space–time can be subject to strong modifications. In particular, we obtain signature change, $\beta(K) < 0$, around any local maximum of $f(K)$. In the same regime, we would conclude, in a purely homogeneous setting, that we obtain an upper bound of the energy density Equation (12). Since this point is surrounded by four-dimensional Euclidean-type space, however, any "bounce" obtained in this way is indeterministic. (It is possible to have holonomy modifications without signature change in models that use self-dual connections [69–71] or implement only the

Euclidean version of the Hamiltonian constraint [72]. Given the special form of simplified constraints in these models, the genericness of this outcome remains unclear.)

These conclusions about modified structure functions are not undone by quantum back-reaction or higher time derivatives [73]. They are distinct from higher-curvature corrections, which would not alter the geometry of hypersurface deformations [74]. In general, effects of holonomy modifications cannot be described by an effective line element on a standard space–time, as postulated in [10], because $\mathrm{d}x^a$ in

$$\mathrm{d}s^2_{\mathrm{eff}} = \tilde{q}_{ab}\mathrm{d}x^a\mathrm{d}x^b \tag{36}$$

do not transform by changes dual to deformed gauge transformations $\{\tilde{q}_{ab}, H[N] + D[w]\}$. Field redefinitions to a standard q_{ab} are possible as long as β does not change sign [75,76]. With signature change, however, we have a new model of non-classical space–time.

This result explains one of the questions posed in the introduction: Bounce models of loop quantum cosmology can evade singularity theorems even without violating energy conditions because they can be embedded in inhomogeneous settings only with non-Riemannian space–time structures. Since singularity theorems are formulated in the Riemannian setting, there is no reason why they should be applicable. While this observation resolves a conceptual question behind these bounce models, it also sheds doubt on the usual deterministic interpretation of such a bounce: The modified space–time structure implies signature change at high density, such that the bounce regime is contained in a four-dimensional space without time or a well-posed initial-value problem.

3.5. Signature Change

A well-posed formulation of a mixed-type partial differential equation such as

$$-\frac{\partial^2 u}{\partial t^2} + \beta(\mathcal{H})\Delta u = 0 \tag{37}$$

requires data on a characteristic where the equation is hyperbolic and on an arc in the elliptic regime [77]. The arc implies that we need future data for well-posedness, so there is no deterministic evolution. Mixed-type partial differential equations such as Equation (37) are more familiar from hydrodynamical descriptions of transonic flow, in which case they give rise to the well-known sonic boom. Mathematically, this phenomenon corresponds to a root-like pole, which generically appears in solutions of Equation (37) at the end of the elliptic arc [77]. The same consequence should be expected in cosmological applications [78], implying a cosmic boom. While solutions u of Equation (37) are finite, their derivatives may diverge at isolated points. This behavior is more well-behaved than the usual big-bang singularity, which implies diverging derivatives of the space–time metric everywhere on a spatial slice. Nevertheless, the existence of cosmic booms indicates that cosmological perturbations are not sufficient for reliable solutions. The existence of signature change, on the other hand, is more robust because it happens also in spherically symmetric models with non-perturbative inhomogeneity [68,76,79–82]. Signature change in models of loop qantum cosmology means that instead of a "bounce" we have a non-singular beginning.

Signature change also has implications for black-hole models, and in particular for the information-loss problem [63]. A non-singular, non-rotating black-hole model can be envisioned by evolving through the classical singularity using quantum evolution of the homogeneous interior. In an inhomogenous completion, there is then no event horizon [83,84]. However, consistent quantum space–time structure again tells us that the high-curvature region is Euclidean. A well-posed problem requires arbitrary boundary values at the top boundary of the Euclidean region, which affect the future space–time. In addition to the event horizon, there is now a Cauchy horizon [63].

In contrast to these cautionary results, dynamical signature change from holonomy modifications may be beneficial in a variety of scenarios. For instance, non-commutative geometry [85] gives rise to discrete spectra in Euclidean signature, which can help to regularize some of its features [86]. Dynamical signature change is surprisingly productive [87–89] in an application to the no-boundary proposal [90]. An analysis of the Lorentzian path integral has recently revealed severe stability problems of perturbative inhomogeneity with no-boundary initial conditions [91–93]. Signature change from holonomy modifications of loop quantum gravity changes the form of the relevant off-shell instantons such that stability is found [88]. "Off-shell" here means that one does not impose the Friedmann equation (or the Hamiltonian constraint) but only the equations of motion it generates. The relationship Equation (12) between holonomy modifications and the energy density is therefore lost, and as a consequence it is conceivable that signature change occurs at lower than Planckian densities. This result is in fact borne out by a detailed analysis [88], making signature change applicable at the sub-Planckian densities used in the no-boundary proposal.

There is a final question which links the last result with our first statements about the averaging volume. Even though off-shell instantons do not impose the Friedmann equations, the background equations of motion they do use should contain quantum corrections in which the averaging volume appears. In contrast to our previous discussion of bounce models, infrared renormalization now appears in a different form. In the no-boundary proposal, spatial slices are assumed compact, such that they can foliate a four-dimensional Euclidean sphere rounding off space–time at the big bang. Near the pole of this four-sphere (replacing the big-bang singularity), the universe is nearly homogeneous because it "started" in a single point, the pole, and inhomogeneous perturbations are now under control thanks to the effects described in [88]. There is then no need for infrared renormalization, and any constant V_0 may be used, such as the full coordinate volume $2\pi^2$ of a unit three-sphere. Therefore, V_0 does not become smaller and smaller as in the BKL picture, and quantum corrections, just like perturbative inhomogeneity, remain under control.

4. Further Directions

In order to ensure reliable approximations, minisuperspace models can be applied in the following way: Starting at low curvature, where our universe indicates large-scale homogeneity and a large value of V_0 is consistent with its role as an infrared scale, one progressively evolves toward larger curvature. As the scale of homogeneity within a co-moving region is reduced by gravitational collapse, the value of V_0 must be set smaller and smaller, with implications for perturbative inhomogeneity as discussed in this paper.

First, a fixed infrared scale means that modes of perturbative inhomogeneity must be restricted to wave lengths less than $V_0^{1/3}$. Secondly, a fixed infrared scale breaks covariance, but for consistency one must still ensure that the infrared-fixed modes are obtained from a covariant theory. This condition can be enforced by following the canonical procedure of implementing the hypersurface-deformation algebroid, as derived for cosmological perturbations in loop quantum cosmology in [44,45]. The resulting covariant equations should then be evolved with a running infrared scale, a task which has not been completed yet. The changing number of modes implies that effective field theory is essential at this point and cannot be replaced by unitary evolution on a fixed Hilbert space.

As V_0 becomes smaller and smaller, quantum corrections in the background dynamics are magnified and, through their state dependence, the problem of states gains in relevance. However, in such a regime, the minisuperspace approximation is much less justified because an infrared regulator is pushed into the ultraviolet. An effective field theory based on minisuperspace approximations should break down at these scales, at the latest. It might, of course, break down earlier, for instance in the presence of energy cascades as in hydrodynamics: If energy is pumped into the system at large distance scales, and dissipates due to friction at small distance scales, the distance scales do not sufficiently decouple from one another to allow a reliable effective evolution over long time scales. There may be a similar picture in quantum cosmology, where expansion provides the analog of pumping energy into inhomogeneous modes on large scales (particle production), while a modified Planck-scale dynamics can often lead to new friction phenomena on small scales [94] or repulsive contributions to the usually attractive gravitational force [95]. The analysis of energy cascades in quantum cosmology requires a deeper understanding of inhomogeneity, but even if they are absent, it remains doubtful whether a minisuperspace treatment, even one combined with perturbative inhomogeneity and infrared renormalization, can reliably describe the fate of the big-bang singularity. At least for the approach to high curvature, a well-defined formulation is available.

Funding: This research was funded by NSF grant number PHY-1607414.

Acknowledgments: The author thanks the referees for several useful suggestions.

Conflicts of Interest: The author declares no conflict of interest.

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