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Editorial

Editorial: Special Issue "The 13th Bolyai–Gauss–Lobachevsky Conference on Non-Euclidean Geometry and Its Applications"

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This Special Issue, *The 13th Bolyai–Gauss–Lobachevsky Conference on Non-Euclidean Geometry and Its Applications*, brings together seven peer-reviewed contributions originating from presentations delivered at the 13th Bolyai–Gauss–Lobachevsky (BGL2025) Conference, held in Saïdia, Morocco, from 26 to 29 May 2025. The conference was organized by the Laboratory of Physics of Matter and Radiation, Mohammed First University, Oujda, Morocco. Continuing a well-established scientific tradition, the BGL conference series provides an international forum where mathematicians and physicists exchange ideas on the role of geometry in modern science. Inspired by the pioneering work of János Bolyai, Carl Friedrich Gauss, and Nikolai Lobachevsky, the conference highlights the profound influence of non-Euclidean geometry on contemporary mathematics, gravitation, cosmology, and theoretical physics [1–3].

The emergence of Einstein's General Theory of Relativity transformed geometry from a purely mathematical discipline into one of the fundamental languages of physics [4–10]. Since then, differential geometry and its generalizations have become indispensable tools for understanding gravitation, black holes, quantum field theory, cosmology, and quantum gravity. Conversely, many of the most challenging problems in modern physics continue to motivate the development of new mathematical structures and geometrical concepts. The articles collected in this Special Issue illustrate this fruitful interaction by addressing problems that range from modified theories of gravity and black hole physics to cosmology, quantum vacuum phenomena, dark matter, and the conceptual foundations of quantum spacetime.

A central theme emerging from several contributions is the study of gravitational theories extending Einstein's General Relativity. Juhász and Gergely [11] extend previous investigations of scalar-field theories by constructing exact Kerr–Schild solutions sourced by kinetic gravity braiding. Their work establishes the conditions under which perturbative Kerr–Schild configurations become exact solutions and it introduces a hierarchy of pp-wave geometries generated through successive Kerr–Schild mappings. Besides enriching the mathematical theory of exact solutions, this construction provides new perspectives on scalar–tensor theories and modified gravity.

Alternative theories of gravitation are further explored by Blaga and Blaga [12]. Employing the Kosambi–Cartan–Chern (KCC) theory together with Lyapunov stability analysis, the authors investigate the stability of timelike circular geodesics around spherically symmetric Weyl black holes. Their results reveal how free parameters characterizing conformally invariant Weyl gravity influence the dynamical behavior of particle

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orbits, offering new insights into the geometric stability of black hole spacetimes beyond General Relativity.

Black hole physics is also examined from the perspective of holography. Ladghami and Ouali [13] investigate charged Anti-de Sitter black holes within the framework of conformal holographic extended thermodynamics. By treating the conformal rescaling factor of the boundary conformal field theory as a thermodynamic variable, they demonstrate how variations in the CFT central charge modify the thermodynamic properties, stability, and phase structure of higher-dimensional black holes. Their analysis reveals Van der Waals-like critical phenomena and further illustrates the deep interplay between gravitational dynamics in the bulk and quantum degrees of freedom on the boundary.

Modern cosmology constitutes another major theme of this Special Issue. Chung et al. [14] revisit one of the fundamental assumptions underlying Type Ia supernova cosmology. Their analysis identifies a statistically significant correlation between standardized supernova luminosities and progenitor age, implying the presence of a systematic redshift-dependent bias. After accounting for this effect, the revised supernova data exhibit improved agreement with recent DESI baryon acoustic oscillation measurements while simultaneously revealing a significant tension with the standard ΛCDM cosmological model when combined with BAO and CMB observations. These results provide an important contribution to the ongoing discussion concerning the nature of dark energy and the evolution of the Universe.

Dark matter phenomenology is addressed from a complementary perspective by Gorkavenko et al. [15]. The authors investigate how the internal vortex structure of ultralight dark matter modifies dynamical friction acting on globular clusters. Their study demonstrates that co-rotating vortex configurations can substantially suppress orbital decay, naturally alleviating the long-standing Fornax timing problem. The work highlights how the microscopic properties of ultralight dark matter may leave observable signatures on galactic scales.

Gorkavenko et al. [16] present quantum field theory from nontrivial geometrical and topological backgrounds by studying vacuum polarization generated by a finite-thickness magnetic topological defect under general Robin boundary conditions. Remarkably, they show that the induced vacuum energy depends explicitly on the curvature-coupling parameter even in flat spacetime, emphasizing the nontrivial role of boundary conditions in quantum vacuum phenomena and illustrating how geometry and topology continue to influence quantum field theoretical effects.

The Special Issue concludes with a broader conceptual contribution. Lake [17] revisits Felix Klein's celebrated Erlangen Program [18] from the perspective of quantum reference frames and quantum geometry. By proposing a generalized notion of quantum reference frames and demonstrating how classical spacetime symmetries may remain mathematically preserved while becoming operationally broken, the author develops a novel framework for describing Planck-scale spacetime. This work establishes an intriguing bridge between geometry, symmetry, and quantum gravity, opening promising directions for future research on the foundations of spacetime.

Although these seven contributions address diverse problems using different mathematical methods and physical motivations, they share a common objective: to deepen our understanding of the geometrical structures underlying the physical Universe. Whether through exact gravitational solutions, black hole thermodynamics, cosmological observations, dark matter phenomenology, quantum vacuum effects, or the conceptual foundations of quantum spacetime, all contributions demonstrate the continuing relevance of non-Euclidean geometry as a unifying framework across modern theoretical physics and mathematics.

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We hope that this Special Issue will stimulate further advances at the intersection of geometry, gravitation, cosmology, and quantum theory. More broadly, we trust that it will continue the scientific legacy of the Bolyai–Gauss–Lobachevsky conference series by encouraging new collaborations and fostering innovative research inspired by the enduring power of non-Euclidean geometry.

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References

1. Gauss, C.F. *General Investigations of Curved Surfaces*; Princeton University Press: Princeton, NJ, USA, 1902; For English translation see Disquisitiones Generales Circa Superficies Curvas. Dieterichische Buchhandlung: Mainz, Germany.
2. Bolyai, J. The Science Absolute of Space. In *Tentamen*; Akademiai Kiado: Budapest, Hungary, 1832.
3. Lobachevsky, N.I. *Theory of Parallels*; Open Court Publishing Company: Chicago, IL, USA, 1891.
4. Einstein, A. The Foundation of the General Theory of Relativity. *Ann. Phys.* **1916**, *49*, 769–822. [CrossRef]
5. Weyl, H. Gravitation und Elektrizität. In *Sitzungsberichte der Königlich Preußischen Akademie der Wissenschaften zu Berlin*; Königlich Preußische Akademie der Wissenschaften zu Berlin: Berlin, Germany, 1918; pp. 465–480.
6. Cartan, É. Sur une généralisation de la notion de courbure de Riemann et les espaces à torsion. In *Comptes Rendus de l'Académie des Sciences*; Académie des Sciences: Paris, France, 1922.
7. Cartan, É. Sur les variétés à connexion affine et la théorie de la relativité généralisée (Première partie). *Ann. Sci. L'éCole Norm. Supérieure* **1923**, *40*, 325–412. [CrossRef]
8. Cartan, É. Sur les variétés à connexion affine et la théorie de la relativité généralisée (Deuxième partie). *Ann. Sci. L'éCole Norm. Supérieure* **1924**, *41*, 1–25. [CrossRef]
9. Kibble, T.W.B. Lorentz Invariance and the Gravitational Field. *J. Math. Phys.* **1961**, *2*, 212–221. [CrossRef]
10. Hehl, F.W.; von der Heyde, P.; Kerlick, G.D.; Nester, J.M. General Relativity with Spin and Torsion: Foundations and Prospects. *Rev. Mod. Phys*. **1976**, *48*, 393–416. [CrossRef]
11. Juhász, B.; Gergely, L.Á. Exact Kerr–Schild Spacetimes from Linearized Kinetic Gravity Braiding. *Universe* **2025**, *11*, 314. [CrossRef]
12. Blaga, C.; Blaga, P.A. Jacobi Stability of Circular Orbits Around Conformally Invariant Weyl Gravity Black Holes. *Universe* **2026**, *12*, 114. [CrossRef]
13. Ladghami, Y.; Ouali, T. Holographic Thermodynamics of Higher-Dimensional AdS Black Holes with CFT Rescaling. *Universe* **2025**, *11*, 337. [CrossRef]
14. Chung, C.; Lee, Y.-W.; Son, J.; Park, S.; Cho, H. Strong Progenitor Age Bias in Supernova Cosmology and Alignment with DESI BAO. *Universe* **2026**, *12*, 22. [CrossRef]
15. Gorkavenko, V.; Barabash, O.; Gorkavenko, T.; Korshynska, K.; Teslyk, O.; Zaporozhchenko, A.; Gorbar, E. The Vortex State of Ultralight Dark Matter and the Fornax Timing Problem. *Universe* **2026**, *12*, 39. [CrossRef]
16. Gorkavenko, V.; Barabash, O.; Nakaznyi, P.; Tsarenkova, M.; Yakovenko, N.; Zaporozhchenko, A. Effects of Curvature–Scalar Coupling on Vacuum Energy in Flat (3+1)-Dimensional Space-Time. *Universe* **2026**, *12*, 47. [CrossRef]
17. Lake, M.J. Towards a Quantum Erlangen Program. *Universe* **2026**, *12*, 84. [CrossRef]
18. Klein, F. *A Comparative Review of Recent Researches in Geometry*; Macmillan: London, UK, 1893.

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