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Rapidity Asymmetry in Cosmology and the Observable Cosmological Arrow

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Abstract

On background cosmological scales, after subtraction of peculiar velocities and local bound-system motions, observed cosmological signals are redshifted rather than blueshifted. Yet, redshift alone does not distinguish the past lightcone of an expanding Universe from the future lightcone of a contracting one. In practice, the identification of the observed redshifted branch with the observational past is set primarily by electromagnetic radiation, whose retarded character is independently established in controlled physics, albeit over non-cosmological scales. From that perspective, the observed cosmological arrow is not separable from the causal/radiative prescription used to interpret the signals. This effective entanglement between the cosmological and the radiative arrows should nevertheless be distinguished from the notion of arrow used in the present work. Here instead, the relevant arrow is not thermodynamic but kinematic; it is defined by the symmetry or asymmetry of background lightcone observables under $\xi \leftrightarrow -\xi$, where $\xi \equiv \ln(1+z)$ and z is the redshift—a criterion motivated directly by the time-reversal-symmetric special-relativistic longitudinal Doppler shift. Equivalently, the arrow considered here is the observed redshift/blueshift asymmetry of cosmological lightcone signals; retarded observations of an expanding FRW Universe are in the redshifted branch, whereas the opposite rapidity orientation would correspond to the blueshifted branch. This naturally suggests using rapidity-reversal symmetry as the redshift-space no-arrow condition when passing from special relativity (SR) to Friedmann–Robertson–Walker (FRW) cosmology, where the empty Milne Universe is a bridging borderline case. In fact, the viewpoint advocated here is that ξ-symmetry/asymmetry is practically more fundamental than t-symmetry/asymmetry simply because the former is more readily related to cosmological observables. It is shown here that generic non-empty FRW Universes possess an intrinsic ξ-asymmetry already at the background level, independently of entropy, coarse-graining, structure growth, or a Past Hypothesis.

Keywords: light-cone observables; rapidity parameter; past hypothesis; cosmological arrow of time

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1. Introduction

The problem of the arrow of time is one of the oldest and deepest puzzles in fundamental physics. At the microscopic level, the basic dynamical laws used in classical mechanics, quantum theory, special and general relativity, and relativistic field theory are, to a very good approximation, time-reversal symmetric, whereas at the macroscopic level, the Universe is manifestly asymmetric; entropy increases, radiation is observed in retarded rather than advanced form, we remember the past but not the future, and cosmology itself

seems to distinguish one temporal direction from the other [1–9]. The problem is then how a large-scale time arrow can emerge from underlying time-symmetric laws. Notably, the problem thus formulated is about 'time', although in cosmology at least, evolution is practically parameterized by redshift, rather than time.

This distinction underlies the present work. Specifically, cosmology does not directly observe the cosmic time parameter t. It does observe spectra, redshifts, distances, fluxes, angles, and number counts, which are organized observationally as functions of z or equivalently $\xi = \ln(1+z)$. The standard statement that the Universe is time-asymmetric already relies on an interpretation of redshift-space observations after the observed branch has been identified with the past. The central question of the present work is under what conditions the background cosmological observables themselves possess a redshift/blueshift branch symmetry under $\xi \to -\xi$?

In the literature, a few approaches have been pursued. One possible explanation is that the observed arrow is due to a very special low-entropy early state of the Universe, often referred to as a Past Hypothesis [10,11]. According to that perspective, the cosmological problem is ultimately a boundary-condition one; the Universe began in a very ordered state, e.g., refs. [8,9,12]. It was emphasized that such explanations should avoid employing a double standard between initial and final boundary conditions [5]. A related but distinct approach emphasizes gravity; following Penrose Weyl-curvature hypothesis, the early Universe was special not merely in a thermodynamic sense, but also in the specifically gravitational one, i.e., being extraordinarily smooth and having low gravitational entropy, with later structure formation providing a natural direction for a gravitational arrow [3,13]. Other proposals appeal to inflationary mechanisms, to time-symmetric cosmologies with locally emergent arrows, or to bounce scenarios in which arrows can emerge or reverse in different branches of cosmic history [14–16]. The literature is diverse; different approaches disagree not only on the origin of the arrow but also on what exactly should count as the cosmological arrow in the first place.

An important issue is the relation between the cosmological arrow and the other familiar arrows of time. Traditionally, the cosmological arrow is often identified with the direction of cosmic expansion and then treated as the large-scale source from which electromagnetic, thermodynamic, and even quantum arrows derive [1,4,9,17]. For example, while the argument of [1] concerns the cosmological conditions that support the retarded radiative and thermodynamic arrows, the present work focuses on a different question, i.e., whether the background cosmological lightcone relations themselves possess a rapidity-reversal symmetry before any appeal to entropy production or radiative irreversibility. According to alternative views, the cosmological arrow is in contrast regarded as secondary; what genuinely requires explanation is the low-entropy boundary condition, and cosmic expansion by itself is insufficient [5,8,12].

Beyond approaches that attempt to identify the arrow with expansion or with special low-entropy boundary conditions, there is also a line of thought emphasizing epistemic asymmetry, memory, and the role of records in fixing the temporal orientation accessible to observers. A recent study of the epistemic arrow of time has explicitly connected temporal asymmetry with memory and knowledge [18], while studies of two-time boundary conditions and locally opposed arrows have shown that temporally nonstandard global structures can in principle coexist with local asymmetric phenomenology [19–21].

Whereas these works do not formulate the present cosmological question in terms of redshift-space observables, they do help locate the present operational approach within a broader context in which the physically relevant arrow is related to accessible records and observations rather than to a directly observed global time orientation. This is important because the arrow relevant to any local observation needs not be *a priori* identical to a global

cosmological arrow. In the literature, both top-down views—according to which the global cosmological arrow is taken to set the local arrows [9]—and alternative settings—in which opposite or region-dependent arrows may coexist [16,19–21]—are found. The present work does not address that issue. Rather, it attempts to address the following question: do cosmological observables themselves, when written in terms of redshift or rapidity, exhibit an intrinsic sign asymmetry independently of any prior identification of the observed branch with the past? Central to the present work is the assumption that ζ-symmetry/asymmetry is observationally more fundamental in cosmology than t-symmetry/asymmetry; it is found that the well-known ζ-symmetry in empty space is invariably broken once the Universe is not empty. In other words, whereas physics described in ideally empty spacetime is fundamentally t-symmetric as well as ζ-symmetric, non-empty spacetime is generically ζ-asymmetric while remaining t-symmetric, and consequently, the cosmological arrow formulated in terms of ζ rather than t is innate to a non-empty Universe.

Focusing on redshift or redshift-related quantities rather than time itself is also motivated by ideas of how cosmological redshift itself should be interpreted. In the empty Milne model—a special Friedmann–Robertson–Walker (FRW) cosmology—the redshift admits a special-relativistic interpretation, and more generally, the relation between Doppler, gravitational, and geometric interpretations of cosmological redshift has been studied in [22,23]. These discussions show that the variable $\zeta = \ln(1+z)$ is especially natural; in the Milne limit, it coincides with the special-relativistic 'rapidity parameter', while in general FRW cosmology it remains an additive parameter of the observed redshift even when the latter is no longer purely kinematical. This makes ζ an ideally suited variable in terms of which an observational no-arrow condition can be formulated; in an observationally arrowless cosmology, one would expect no preferred sign of ζ, and therefore no intrinsic asymmetry under $\zeta \leftrightarrow -\zeta$ transformation.

More fundamentally, the practical identification of the observed branch with the past is not set by redshift alone. In observational cosmology, this identification is anchored mainly by electromagnetic observations, whose retarded character is independently established in laboratory and astrophysical practice. Present gravitational-wave observations do not yet provide a comparably independent calibration of advanced versus retarded sectors. The question relevant here is as follows: Once that evidential input is set aside, are background cosmological relations themselves symmetric under $\zeta \leftrightarrow -\zeta$, or not?

From this perspective, the question addressed here is different from the standard one. We are not interested in whether the fundamental laws are time-asymmetric, nor whether entropy increase, coarse-graining, or a Past Hypothesis must ultimately determine the observed time direction. Rather, our question is under what conditions the observable lightcone relations remain symmetric under rapidity reversal. We show that exact symmetry is highly exceptional within the standard FRW component repertoire. Within the extended KΛCDM model (which allows for non-vanishing spatial curvature, K, in addition to the more standard cold dark matter, baryons, cosmological constant, Λ, and radiation), pure Milne spacetime is the only permanently rapidity-symmetric case; other restorations require special pairings and apply to chosen reference epochs only.

The significance of this result is conceptual and operational. Once the problem is formulated in the observable variable ζ, exact rapidity symmetry is seen to be non-generic. A non-empty FRW Universe does not generically preserve it; at best, a tuned radiation–Λ pairing can restore it relative to a particular instant of time, whereas any nonzero non-relativistic (NR) matter forbids exact restoration at any epoch. In the only evidential sense directly available to cosmology, the relevant no-arrow condition is rapidity reversal symmetry of lightcone observables, and standard FRW cosmology generically does not

satisfy it and fails to do so already at the background level, with no recourse to large-scale structure evolution, coarse-graining arguments, etc.

The paper is organized as follows. In Section 2, we motivate the use of the rapidity variable $\xi = \ln(1+z)$ in the special-relativistic framework and its empty Milne Universe reparameterization. In Section 3, it is shown that exact background rapidity reversal symmetry is highly exceptional observationally when FRW cosmology is formulated in terms of lightcone observables such as $r(\xi)$, with pure Milne as the only permanently symmetric FRW case. Section 4 discusses how this asymmetry could be related to the observed cosmological arrow, followed by a conclusion in Section 5.

Throughout, natural units are adopted in which the speed of light, c, is set to unity.

2. The Rapidity Parameter and Its Special-Relativistic Origin

A key conceptual point is that cosmology does not directly observe cosmic time itself. It observes light received on the observer's past lightcone, including spectroscopic redshifts, angular scales, fluxes, and inferred background relations such as the angular distance $D_A(z)$, the luminosity distance $D_L(z)$, and the Hubble function $H(z)$. These are naturally expressed as functions of z, or equivalently of $\xi \equiv \ln(1+z)$. In terms of ξ the redshifted and blueshifted branches occupy the symmetric domain $\xi \in (-\infty, \infty)$. The discrete symmetry test $\xi \to -\xi$ is thus the exchange of the redshift and blueshift branches of the observable relation.

Indeed, the emergence of an observational cosmic asymmetry is not inferred from direct access to time itself but rather from the loss of rapidity-reversal symmetry in observable-redshift relations. Minkowski spacetime is exactly time-reversal-symmetric. For longitudinal motion in special relativity (SR), the longitudinal Doppler frequency shift is

$$1 + z = \sqrt{\frac{1+\beta}{1-\beta}}, \tag{1}$$

where β is the velocity in units of the speed of light. Introducing the rapidity parameter ξ,

$$\beta \equiv \tanh \xi, \tag{2}$$

the Doppler factor becomes

$$1 + z = e^{\xi}. \tag{3}$$

The rapidity range, $\xi \in (-\infty, +\infty)$, is symmetric around zero, while $z \in (-1, +\infty)$ is asymmetric. This latter asymmetry is genuinely parametric. In fact, the swap $\xi \leftrightarrow -\xi$ corresponds to $\beta \leftrightarrow -\beta$, which is exactly the operational definition of time reversal, where all momenta are simultaneously reversed. SR *per se* has no preference to any particular temporal direction; i.e., it does not by itself favor past lightcone over future lightcone observations, and it correspondingly does not favor any particular ξ branch. In this sense, the present construction is kinematic. The ξ-criterion is introduced prior to any appeal to irreversible radiation, entropy increase, thermodynamic considerations, or special boundary data. It asks only whether the background observable relations preserve the SR-motivated rapidity-reversal symmetry. In that respect, it differs from Gold-type analyses [1], in which the practical arrow is connected to thermodynamic and electromagnetic asymmetry under cosmological expansion.

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The natural bridge between the special-relativistic and FRW notions of redshift is the Milne Universe, a special case of the FRW spacetime described by the infinitesimal line interval

$$ds^2 = -dt^2 + a^2(t)\left[\frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right], \tag{4}$$

with $a(t) = \sqrt{-K}t$, where $K < 0$. Milne spacetime is simply a reparameterized wedge of the lightcone interior of empty Minkowski spacetime. Nevertheless, in its FRW representation it still obeys the standard cosmological relation $1 + z = 1/a$. In this limiting case, the cosmological redshift admits a strict special-relativistic reinterpretation.

Specifically, choosing the present epoch to satisfy $a(t_0) = 1$, i.e., $t_0 = 1/\sqrt{-K}$, and defining the conformal time relative to this epoch, $\eta(t) - \eta(t_0) = \int_{t_0}^{t} dt'/a(t') = \ln\left(\sqrt{-K}t\right)/\sqrt{-K}$. Since $1 + z = 1/a = 1/(\sqrt{-K}t)$, this gives $\sqrt{-K}\,[\eta(t) - \eta(t_0)] = -\xi$. For a radial null ray in the Milne metric, $\chi \equiv \eta(t_0) - \eta(t) = \int_0^r \frac{dr'}{\sqrt{1-Kr'^2}} = \frac{1}{\sqrt{-K}}\sinh^{-1}(\sqrt{-K}r)$. Combining these results leads to

$$\tanh\xi = \frac{\sqrt{-K}r}{\sqrt{1-Kr^2}}. \tag{5}$$

On scales small compared with the curvature radius, this reduces to $\tanh\xi \simeq \sqrt{-K}r$. Since in the Milne model $H_0 = t_0^{-1} = \sqrt{-K}$, this is just the local Hubble law written in the rapidity variable.

This is precisely the reason that ξ- rather than time-reflection symmetry (around any fiducial time) more faithfully informs us about past–future asymmetry. This is nicely illustrated in the Milne limit. The Milne model is expanding in its FRW representation, and therefore the cosmic-time description distinguishes increasing from decreasing t relative to a chosen observer epoch. Nevertheless, its redshift–distance relation remains symmetric under the rapidity reflection $\xi \to -\xi$. Thus, Milne provides the borderline case showing that expansion, or an apparent asymmetry in the time parametrization, is not by itself the observable redshift/blueshift arrow defined in the present work. The latter appears generically only once non-curvature stress energy is present. In this sense, the present criterion separates the usual time-orientation question from the redshift-space symmetry question.

Thus, Milne spacetime is the unique intermediate case in which the FRW redshift law and the SR Doppler description essentially coincide. In other words, the expanding Milne spacetime (lightcone interior of Minkowski spacetime) is both t-asymmetric and ξ-symmetric. For this reason, $\xi = \ln(1+z)$ is not an ad hoc import from SR; in the empty FRW spacetime limit, it is the SR rapidity, while in general FRW spacetimes, it remains the natural additive observable variable even after the redshift becomes genuinely dynamical rather than purely kinematical. As will be shown below, Milne spacetime is singled out as the unique permanently ξ-symmetric FRW Universe. Any departure from emptiness manifestly breaks this symmetry, except for special tuned pairings such as radiation with Λ, which can restore it transiently only relative to a single epoch.

3. Observables in FRW Cosmology and Rapidity Symmetry Breaking

A general FRW spacetime Equation (4) has to satisfy the Friedmann equation

$$H^2 + \frac{K}{a^2} = \frac{8\pi G\rho(a)}{3}, \tag{6}$$

where $H \equiv \dot{a}/a$ is the expansion rate, G is the Universal gravitational constant, and $\rho(a)$ is the total energy density. For a general FRW spacetime with radiation, dust and a cosmological constant $\rho(a) = \rho_{r,0}/a^4 + \rho_{m,0}/a^3 + \rho_\Lambda$, where $\rho_{i,0}$ is the present day energy density of the i'th species. Equation (5) can be employed to replace the t-dependence in Equation (4) by a corresponding ξ-dependence. Specifically, $a = (1+z)^{-1} = e^{-\xi}$ implies $d\xi = -\frac{da}{a} = -Hdt$, and therefore $dt^2 = \frac{d\xi^2}{H^2(\xi)}$. The Friedmann equation gives

$$H(\xi) = H_0\sqrt{\Omega_r e^{4\xi} + \Omega_m e^{3\xi} + \Omega_k e^{2\xi} + \Omega_\Lambda}, \tag{7}$$

where the parameters Ω_r, Ω_m and Ω_Λ are the present-day energy densities in critical density units, $\rho_c \equiv 3H_0^2/(8\pi G)$, of radiation, NR matter, and the cosmological constant, respectively, and $\Omega_k \equiv -K/H_0^2$ is the effective energy density associated with spatial curvature, H_0 is the current expansion rate, and the Ω_i's are subject to the constraint $\Omega_r + \Omega_m + \Omega_k + \Omega_\Lambda = 1$. Equation (4) can then be recasted in the form

$$ds^2 = e^{-2\xi}\left[-\frac{d\xi^2}{H_0^2F^2(\xi)} + \frac{dr^2}{1-Kr^2} + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right], \tag{8}$$

where

$$F(\xi) \equiv \frac{H(\xi)}{H_0}e^{-\xi} = \sqrt{\Omega_r e^{2\xi} + \Omega_m e^{\xi} + \Omega_\Lambda e^{-2\xi} + \Omega_k}. \tag{9}$$

Equation (8) is useful conceptually because it provides a reformulation of FRW spacetime in terms of ξ rather than t. Whereas the t-based formulation, Equation (4), is manifestly t-reversal-symmetric, the ξ-based counterpart is evidently not. While, in arriving at Equation (8), use has been made of Equation (7) and $dt = d\xi/H(\xi)$ in eliminating the t-dependence from Equation (4), we could also go the other way around, starting with Equation (8), where H_0 and the various Ω_i's are treated as free parameters, then employing the Einstein equation; Equation (4) is then recovered with the identification that the 'parameters' Ω_i' are actually the dimensionless energy density parameters and H_0 is the present day expansion rate.

Equation (8) should not be read as a claim that ξ is a preferred time coordinate. General covariance permits many parametrizations of the same background solution. The issue considered here is instead the discrete redshift-space reflection $\xi \to -\xi$. Reparametrization invariance allows ξ to be used as a parameter, but it does not imply that the observable map is invariant under this reflection. Thus, the relevant question is whether the resulting background lightcone relations possess a redshift/blueshift branch symmetry.

Integration of Equation (8) along the incoming radial direction subject to the constraint $ds^2 = 0$ results in the comoving distance $r(z) = S_K(\Delta\eta)$, where

$$S_K(\Delta\eta) = \begin{cases} \sin(\sqrt{|\Omega_k|}\Delta\eta)/\sqrt{|\Omega_k|}, & \Omega_k < 0, \\ \Delta\eta, & \Omega_k = 0, \\ \sinh(\sqrt{\Omega_k}\Delta\eta)/\sqrt{\Omega_k}, & \Omega_k > 0. \end{cases} \tag{10}$$

and

$$\Delta\eta(\xi) = \frac{1}{H_0}\int_0^\xi \frac{d\xi'}{F(\xi')}. \tag{11}$$

Since $r(\xi) = r[\eta(\xi)]$, it follows that rapidity-reversal asymmetry obtains once the Universe is non-empty; only the $\Omega_k = 1$ Milne Universe is ξ-blind. More generally, any

homogeneous background observable may be written as $O(\eta) = O[\eta(\xi)] \equiv O(\xi)$, and the same redshift-space symmetry test applies. A legitimate objection is that cosmological redshift is measured spectroscopically, whereas distance is inferred separately, so on the face of it, the sign of ξ could be considered without reference to $r(\xi)$. However, within a fixed FRW spacetime, this separation is only instrumental, not physical. Once the background Ω_i parameters are specified, ξ and distance are related via $r(z) = S_K(\Delta\eta)$ and Equations (10) and (11).

The precise symmetry criterion used here is the following. The background is a rapidity-reflection symmetric if the kernel governing the conformal-distance integral is invariant under rapidity reflection, namely if $F(\xi) = F(-\xi)$. In that case, the redshifted and blueshifted sides have equal conformal-distance intervals, $\int_0^{\xi} \frac{d\xi'}{F(\xi')} = \int_{-\xi}^{0} \frac{d\xi'}{F(\xi')}$. Thus the no-arrow condition is that the two branches define equal lightcone-distance intervals on the two sides of $\xi = 0$. In this sense, the redshifted and blueshifted branches are physically equivalent after reversing the orientation of the radial light ray.

The component-pairing rule is transparent in this form. A perfect fluid component with equation of state $w = P/\rho$, where P is the pressure, contributes to $F(\xi)$ as $\Omega_w e^{(1+3w)\xi}$. Under $\xi \to -\xi$, it is paired with a component satisfying $1 + 3w' = -(1 + 3w)$, i.e., $w' = -\frac{2}{3} - w$. Therefore, spatial curvature, corresponding effectively to $w = -1/3$, is self-paired. Radiation, $w = 1/3$, pairs with a cosmological constant, $w = -1$. NR matter, $w = 0$, would pair with an exotic $w = -2/3$ component. Since such a component is not accomodated by the KΛCDM inventory, NR matter breaks exact rapidity-reflection symmetry in the standard model.

There do exist, however, special transient symmetric cases. The most general ones are

$$\Omega_m = 0, \qquad \Omega_r = \Omega_\Lambda = \frac{1 - \Omega_k}{2}, \tag{12}$$

in which case

$$\Delta\eta(\xi) = \frac{1}{H_0} \int_0^{\xi} \frac{d\xi'}{\sqrt{\Omega_k + (1 - \Omega_k)\cosh(2\xi)}}, \tag{13}$$

is ξ-symmetric around the specific time where Equation (11) applies, but not at any other time.

More generally, background ξ-symmetry requires a precise balancing of the component scalings under $\xi \to -\xi$. Curvature is self-paired. Radiation and Λ may pair with one another, but only through a special tuning, and such a restoration is necessarily epoch-dependent rather than permanent. NR matter is different; it carries no partner within the standard FRW component set, so any nonzero Ω_m breaks exact symmetry altogether. In this sense, any non-empty FRW Universe is generically ξ-asymmetric, while pure Milne is the unique permanently symmetric FRW case.

4. From Observable Asymmetry to the Observable Cosmological Arrow

The result of Section 3 has two distinct implications. First, generic non-empty FRW cosmologies are intrinsically ξ-asymmetric already at the level of background lightcone observables. Second, the empirical identification of the realized $\xi > 0$ branch with the observational past is made in practice by appealing to the retarded electromagnetic wave prescription. The first result is a statement about background cosmological observables. The second is an observational statement about access to these observables. Therefore, the observationally inferred cosmological arrow, insofar as it is inferred from astronomical signals, is operationally entangled with the electromagnetic arrow aside from the underlying ξ-asymmetry of the FRW background. The distinction may be stated more sharply as follows. In contrast to the discussion in [1], which is related to the practical

asymmetry of radiation and to the boundary conditions that support retarded wave propagation, the present ξ-arrow is a background diagnostic in redshift space, defined entirely by whether FRW lightcone observables are symmetric or asymmetric under $\xi \to -\xi$. The two notions may be ultimately related, but they are conceptually different and should not be automatically identified.

Local—non-cosmological—gravitational redshift does not change this conclusion. In an inhomogeneous Universe, local gravitational potentials and peculiar velocities can shift spectral lines and may even produce local blueshifts. These effects are perturbative or environmental contributions on top of the background relation. In an exact FRW spacetime, there is no static central potential producing a separate gravitational redshift; the cosmological redshift may be decomposed into Doppler, gravitational, or geometrical contributions. The present criterion concerns the total background observable redshift-distance relation expressed in terms of ξ.

In this sense, the cosmological arrow considered here is precisely the observed redshift/blueshift asymmetry of cosmological lightcone signals; an observer receives the redshifted branch associated with the expanding FRW solution, whereas the corresponding blueshifted branch would represent the opposite rapidity orientation. The origin of the asymmetry is not a microscopic violation of time reversal, but the non-empty FRW energy content, which renders the two rapidity branches physically inequivalent.

It should be recapitulated that redshifted observations alone do not uniquely distinguish the past lightcone of an expanding Universe from the future lightcone of a contracting one. What the background analysis establishes is only that the observable FRW relations are generically not symmetric under $\xi \to -\xi$. Thus, the realized branch is not selected from a generic ξ-symmetric cosmological background, even though the direction assigned to that branch remains observationally related to the electromagnetic arrow.

This restriction to the background level is deliberate. Growth of inhomogeneity or clumpiness, by itself, does not remove the observational ambiguity just mentioned. If similar regularity is imposed at the remote observed end in the two competing interpretations, then *linear* overdensity may increase toward the observer in both the expanding-past and contracting-future descriptions. Therefore, clumpiness growth is not, by itself, what fixes the observed direction of the cosmological arrow.

It should be stressed that the present argument does not assert a breaking of the fundamental time-reversal invariance of the underlying local laws. Our statement is purely operational; once cosmological observables are formulated in terms of the redshift variable $\xi = \ln(1+z)$, the observable lightcone relations need not remain invariant under rapidity reversal transformation. The result is therefore an observational symmetry breaking, clearly not a microscopic violation of time-reversal symmetry. Cosmology does not directly observe t; it observes lightcone quantities such as $r(z)$, and more generally $\mathcal{O}(z)$, or equivalently $\mathcal{O}(\xi)$. In the absence of an observational arrow, no preferred sign of ξ would be expected in the relevant lightcone relations. In the empty Milne limit this is indeed the case; the observable structure is exactly ξ-symmetric. More generally, however, exact symmetry is highly exceptional. Once the FRW Universe is non-empty, the observable relations are generically not symmetric under $\xi \to -\xi$; only a special tuned pairing of radiation and Λ can restore the rapidity symmetry, and even then only relative to a chosen epoch. If NR matter is present, even that possibility is lost.

5. Conclusions

The starting point of the present work is that ξ is operationally more fundamental than cosmic time. Cosmological observations directly provide redshifts and lightcone relations as functions of z, or equivalently $\xi = \ln(1+z)$, whereas cosmic time is reconstructed

only after adopting a background model and a radiative interpretation of the observed branch. The directly observable cosmological asymmetry is therefore not a primitive $t \to -t$ asymmetry but a redshift/blueshift branch asymmetry in observable relations. In this operational sense, the result is conceptually striking; the observable redshift/blueshift arrow does not require entropy production, coarse-graining, structure formation, or a Past Hypothesis. It already arises at the homogeneous background level from the mere presence of non-curvature stress-energy in the Universe. Thus, the observable cosmological arrow, in the restricted sense used here, is in principle the redshift/blueshift asymmetry of cosmological lightcone signals. Its origin is not a microscopic violation of time-reversal invariance but the non-empty FRW energy content, which makes the two rapidity branches inequivalent. This redshift/blueshift asymmetry is a separate issue from the fact that only retarded electromagnetic waves are observed, and any cosmological observations are essentially inseparable from the radiative arrow.

A natural objection is that the asymmetry is formulated in the rapidity variable ξ rather than in cosmic time t. However, cosmological observations do not access t directly but infer distances along the past lightcone as functions of redshift. Any discussion of an *observed* arrow should therefore be phrased as lightcone observables such as $r(z)$, and parameterizations thereof. Moreover, z itself has an asymmetric domain $z \in (-1, \infty)$. In contrast, adopting the special-relativistic notion of rapidity, $\xi = \ln(1+z)$, the rapidity variable $\xi \in (-\infty, \infty)$ is symmetric.

This work is limited exclusively to the cosmological arrow as is manifested in observable redshift–distance relations. No direct observation gives access to a cosmic time arrow. For that reason, the present work treats the observable ξ-asymmetry of cosmological lightcone relations as evidence for the existence of a cosmological arrow, while not addressing the question of how the direction assigned to the realized branch is related to the electromagnetic, thermodynamic, or other familiar arrows. This does not deny that the thermodynamic, electromagnetic, and other familiar arrows may be related in the actual Universe. In fact, a common view often expressed in the literature is that the low-entropy early Universe underlies not only large-scale cosmological asymmetry but also the ordinary irreversible phenomena of everyday life, such as radiative retardation, mixing processes, and irreversibility of processes in macroscopic systems. The point of the present work is that whatever the underlying relation among these arrows is, the cosmological arrow in its directly observable form can already be formulated at the level of lightcone observables, without deriving it from entropy increase, coarse-graining, gravitational clumping, or a Past Hypothesis.

Thus the familiar puzzle should be separated into two questions. The first is whether the fundamental dynamical laws are invariant under time reversal. The second is whether the cosmological observables available to us are symmetric under the exchange of redshift and blueshift branches. The present work concerns only the second question and shows that standard non-empty FRW backgrounds generically fail this observational symmetry test already at the homogeneous background level.

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