

Article

Representation Formalism and Quantum Mechanics in Curved Spacetime

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Abstract

We extend the representation frame formalism, previously introduced to account for key cosmological observations in the Einstein static universe, to non-relativistic quantum mechanics. In this framework, each inertial observer is associated with a flat *representation referential* R_{obs} , defined as the tangent space to the spatial manifold at the observer's position, in which all measurements are represented. The Euclidean structure of R_{obs} allows quantum systems to be described using the standard Schrödinger formalism, avoiding the technical ambiguities that arise when quantising directly on curved manifolds. We derive the relation between the Hamiltonian governing quantum dynamics in R_{obs} and its counterpart defined on the physical manifold U , and show that curvature effects enter as observer-dependent modifications of effective potentials. Although the resulting quantum description depends on the observer's representation frame, we show that this does not lead to contradictions between observers: consistency of measurement outcomes follows from the standard structure of quantum correlations established by physical interactions. We illustrate the formalism with explicit applications, including the hydrogen atom in an Einstein static universe and quantum systems in the vicinity of a black hole, highlighting how spacetime curvature manifests itself in the observer's quantum description.

Keywords: representation referential; quantum gravity; quantum mechanics

1. Introduction

A main challenge in relating geometry, gravitation, and quantum theory lies in the tension between curved spacetime descriptions and the operational nature of physical measurements. All empirical information available to an observer is acquired locally through measurements of distances, angles, time intervals, frequencies and energies performed in their immediate environment. These quantities are not abstract geometric objects, but operational constructs inferred from signals received and processed by the observer.

The distinction between local and global measurements is not sharply defined. There is no natural scale separating the domain where measurements remain purely local from the regime where curvature effects become relevant. In the absence of a clear criterion to establish this boundary, we assume that the observer's measurement space, while grounded in local apparatus, extends globally to encompass observations at all distances. Any framework connecting spacetime geometry with observation must therefore specify the space in which all measurements are represented.

In a previous work [1], we introduced the *representation frame* as a geometric implementation of this principle. Rather than describing observations directly on the curved



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spatial manifold of the universe, we associate to each inertial observer an intrinsic, flat referential, denoted R_{obs} , which serves as the operational space in which all measurements are encoded. This representation frame is defined as the tangent space to the spatial section of spacetime at the observer's position. By construction, R_{obs} admits a Euclidean geometry and provides the inertial observer's natural laboratory.

In this framework, the underlying physical universe is modeled as a globally static spacetime satisfying Einstein's field equations, whose spatial part defines a three-dimensional Riemannian manifold U . The representation map $T : U \rightarrow R_{\text{obs}}$ associates to each point $x \in U$ a point $r = T(x)$ in the observer's representation frame, located at the same proper distance from the observer and oriented along the initial direction of the geodesic connecting them. The map preserves radial distances but distorts transverse directions due to curvature. This distortion is quantified by a local scaling factor

$$a(x) = \frac{D_A}{d}, \quad (1)$$

where d is the proper distance from the observer to x in U and D_A the corresponding angular diameter distance [1]. To ensure consistency of signal propagation in the representation frame, the same factor applies to local time intervals,

$$\delta t_{\text{obs}} = a(x) \delta t_U, \quad (2)$$

so that photons propagate in R_{obs} with constant speed and constant wavelength as measured by the observer. Our formalism rests on three postulates:

1. Spacetime is globally static and satisfies Einstein's field equations.
2. All measurements performed by an observer are represented in R_{obs} and obey Euclidean geometry.
3. In R_{obs} , radiation propagates as in flat space, with constant speed and constant wavelength between emission and detection.

As shown in [1], these assumptions are sufficient to reproduce key cosmological observations, such as Supernovae and CMB data within a static Einstein Universe, without invoking cosmic expansion. This universe, introduced by Einstein in 1917, is the unique static solution to Einstein's field equations with a homogeneous matter distribution and positive cosmological constant Λ . Its spatial geometry is that of a 3-sphere S^3 with constant positive curvature, embedded as a hypersurface in $\mathbb{R}^4$:

$$U = S^3 = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 : \sum_{i=1}^4 x_i^2 = R^2\} \quad (3)$$

where R is the curvature radius. The metric is given by

$$ds^2 = -dt^2 + R^2[d\chi^2 + \sin^2(\chi)(d\theta^2 + \sin^2\theta d\phi^2)] \quad (4)$$

with $\chi \in [0, \pi]$. The static nature requires a specific relation between R , the matter density ρ , pressure p , and Λ , obtained from Einstein's field equations [2].

Beyond its cosmological applications, the representation frame provides a natural setting for quantum mechanics. Quantum theory is fundamentally operational: its basic objects (wave functions, probability amplitudes, transition rates) refer to outcomes of measurements performed in the observer's laboratory. Within the present formalism, this laboratory is precisely the representation space R_{obs} . Postulates 2 and 3 ensure that all operational data entering the quantum formalism (photon energies, atomic transitions, arrival times, interference patterns) are registered in a flat Euclidean structure, even though

the underlying manifold U is curved. Consequently, the standard flat-space machinery of quantum mechanics is not a mere approximation, but the natural framework for describing microscopic physics as it appears to the observer.

This framework leads to two complementary but distinct levels of description. On the physical manifold U , matter and radiation are described by continuous, classical fields that source curvature through Einstein's equations. These fields represent macroscopic, coarse-grained quantities and define a fixed, static spacetime geometry. In contrast, the discrete, particle-like behaviour revealed in laboratory experiments pertains to the observer's representation frame R_{obs} , where physical measurements are operationally defined and quantum systems are described.

The two descriptions are related by the representation map T , which encodes how the background curvature of U affects the propagation of signals. When these signals are expressed in R_{obs} , the map ensures that they reproduce the locally flat behaviour required by Postulate 3. In this sense, while the energy carried by matter and radiation contributes to the classical gravitational field on U , their observable quantum behaviour unfolds entirely within the observer's representation frame. Microscopic quantum events do not backreact on the geometry of U , which remains fixed by construction. This separation should not be viewed as a fundamental principle, but rather as defining the regime of validity of the formalism: quantum mechanics is formulated operationally on a static spacetime background sourced by classical fields. Extending the framework beyond static spacetimes would require a theory of quantum backreaction and lies beyond the scope of this work.

From this perspective, attempting to formulate quantum mechanics directly on the curved manifold U is neither necessary nor conceptually well motivated within this regime. Such approaches are known to suffer from ambiguities in the choice of measure, Laplacian, and operator ordering [3,4]. These difficulties arise precisely because U is not the space in which measurements are operationally represented. By contrast, the representation frame R_{obs} is flat by construction, allowing the standard Schrödinger formalism to be applied without modification or ambiguity.

The purpose of the present work is to develop this observer-centered quantum extension of the representation frame formalism in the context of static spacetimes. We show how quantum systems can be consistently defined in each observer's representation frame and related to their classical description on U through the map T . We derive the corresponding transformation laws for wave functions and effective potentials and illustrate the construction with explicit examples, including the hydrogen atom in an Einstein static universe and quantum systems in the vicinity of a black hole.

An important conceptual consequence of this approach is that quantum states and Hamiltonians are intrinsically observer dependent. However, this dependence does not lead to inconsistencies. Agreement between observers arises through standard quantum mechanical correlations established by physical interaction, without the need for an observer-independent quantum state defined on U . In this sense, observer dependence is not a defect of the theory, but a direct reflection of the operational and relational character of quantum mechanics.

2. Construction of the Quantum System

Given the previous considerations, the state of a spinless, non-relativistic quantum particle can be represented in this formalism by a wave function $\psi_{\text{obs}}(r, t_{\text{obs}})$ defined on R_{obs} , whose evolution follows the standard Schrödinger equation:

$$i\hbar \frac{\partial \psi_{\text{obs}}}{\partial t_{\text{obs}}} = \hat{H}_{\text{obs}} \psi_{\text{obs}}. \quad (5)$$

Here, $\hat{H}_{\text{obs}}$ denotes the usual quantum Hamiltonian operator, defined in the presence of a scalar potential as

$$\hat{H}_{\text{obs}} = -\frac{\hbar^2}{2m} \nabla_r^2 + V_{\text{obs}}, \quad (6)$$

where V_{obs} is the effective scalar potential perceived by the observer in the Euclidean space R_{obs} . For a scalar potential V_U defined in U , it transforms as

$$V_{\text{obs}}(r) = \frac{V_U(x)}{a(x)}. \quad (7)$$

This scaling arises directly from the time scaling relation (2), which ensures consistency of signal propagation in the representation referential [1]. Since energy is conjugate to time, this time dilation implies a corresponding rescaling of energy

$$E_{\text{obs}} = E_U / a(x),$$

which also holds for potentials. This follows from the operational requirement that measured energy scales reflect the time dilation in Equation (2).

Since our quantum system is defined on the flat space R_{obs} , the different phenomena specific to the quantum world, such as the collapse of the wave function and the dynamics of the particle after measurement, entanglement and superposition of quantum states, are accounted for exactly as in standard quantum mechanics.

Now that we have defined a quantum system in R_{obs} , it appears natural to define a wave function $\Psi(x, t_U)$ on U by applying to $\psi_{\text{obs}}(r, t_{\text{obs}})$, for any t_{obs} , the unitary transformation is as follows:

$$\Psi(x, t_U) := (\mathcal{U}_T^{-1} \psi_{\text{obs}})(x, t_U) := [\sqrt{g(x)} J(r)]^{-1/2} \psi_{\text{obs}}(r, t_{\text{obs}}), \quad (8)$$

where $g(x)$ is the determinant of the metric in U , $J(r) = |\det(\partial x / \partial r)|$ is the Jacobian of T^{-1} , so that $d^3x = J(r) d^3r$. By construction, $\mathcal{U}_T^{-1}$ is a unitary pullback and maps probability densities on R_{obs} to those in U .

Defining such a quantum system on U would be meaningful if all observers would agree on this Universal quantum system, but we will see that it is not the case in general.

Differentiating Equation (8) with respect to time and using Equations (2) and (5) yields

$$i\hbar \partial_{t_U} \Psi = a \mathcal{U}_T^{-1} \hat{H}_{\text{obs}} \mathcal{U}_T \Psi. \quad (9)$$

Therefore, if we define the Hamiltonian:

$$\hat{H}_U := a \mathcal{U}_T^{-1} \hat{H}_{\text{obs}} \mathcal{U}_T, \quad (10)$$

the state Ψ satisfies a Schrödinger type equation on U :

$$i\hbar \partial_{t_U} \Psi = \hat{H}_U \Psi. \quad (11)$$

This system would acquire a universal status if this construction was observer independent, i.e., we obtain the same $\hat{H}_U$, yielding the same dynamics for Ψ , for any observer. To see why this cannot be the case, we can decompose it in a kinetic and potential part as:

$$\hat{H}_U = -a \mathcal{U}_T^{-1} \frac{\hbar^2}{2m} \nabla_r^2 \mathcal{U}_T + V_U := \hat{K}_U + V_U. \quad (12)$$

Although the potential part V_U is independent of the observer, the kinetic part $\hat{K}_U$ depends in a non-linear way on the scaling factors associated with the different observers

and the local geometry in U through the unitary transformation $\mathcal{U}_T$. As a consequence, one cannot define a unique quantum system on U that remains consistent for all observational frames, making the quantum description intrinsically observer dependent. This perspective is broadly compatible with relational approaches to quantum theory, notably Relational Quantum Mechanics [5,6], in which quantum states and observables do not represent observer-independent properties but instead encode information defined relative to a given observing system. As we will see in the next section, in such a relational framework, agreement between different observers about the outcome of a measurement arises from the physical correlations established through their interaction, not from the assumption that they are accessing the same underlying quantum state, which do not exist.

3. Observer Agreement and Relational Consistency

A potential concern in an observer-centered formulation of quantum mechanics is whether different observers, each equipped with their own representation space, nevertheless agree on the outcomes of measurements. Since quantum states and Hamiltonians are explicitly observer dependent in our construction, it is legitimate to ask whether this observer dependence could lead to inconsistencies when measurement results are compared. We show in this section that no such inconsistency arises. Agreement between observers follows from standard quantum mechanical correlations established through physical interaction, in close analogy with the principles of Relational Quantum Mechanics (RQM) [5,6]. Importantly, this agreement does not rely on the existence of an observer-independent quantum state, but only on the internal consistency of quantum mechanics within each representation frame.

Consider two inertial observers O_1 and O_2 in the same static spacetime, located at distinct events and equipped with representation frames R_{O_1} and R_{O_2} , respectively. Each observer defines quantum systems exclusively in their own representation space, where the standard flat-space Schrödinger formalism applies. Suppose that observer O_1 performs a measurement on a quantum system S localised near O_1 . In R_{O_1} , this measurement is described as a unitary interaction between S and a macroscopic apparatus $A^{(1)}$, leading to an entangled state of the form

$$|\Psi\rangle_{SA^{(1)}}^{(O_1)} = \sum_i c_i |s_i\rangle |A_i^{(1)}\rangle, \quad (13)$$

where the states $|A_i^{(1)}\rangle$ represent orthogonal pointer states encoding the possible outcomes, which are stable and mutually exclusive. Right after the measurement by O_1 , a definite outcome i is realised relative to O_1 . At this stage, the outcome is a well-defined physical fact for O_1 , but it has no meaning for O_2 , which has not yet interacted with either the system or the apparatus.

From the perspective of O_2 , the situation is described differently. The interaction between S and $A^{(1)}$ near O_1 is described entirely unitarily in R_{O_2} . After this interaction has occurred, the composite system $S + A^{(1)}$ continues to evolve according to the Schrödinger equation generated by the observer-dependent Hamiltonian $\hat{H}_{O_2}$ according to the time t_{O_2} . Crucially, the collapse of the wave function that occurred for O_1 does *not* immediately induce a corresponding collapse for O_2 . Rather, O_2 continues to describe the system as a superposition until O_2 physically interacts with the measurement record. This absence of collapse relative to O_2 reflects the relational character of state assignment rather than a breakdown of dynamics. The consistency of this description relies on a key property: the pointer states $|A_i^{(1)}\rangle$ correspond to macroscopically distinct and operationally distinguishable configurations of the apparatus—different macroscopic arrangements that are environmentally decohered and stable. These states are therefore represented by orthogonal

subspaces not only in R_{O_1} but also in R_{O_2} , since macroscopic orthogonality is preserved under the coordinate transformations relating different representation frames. Unitary evolution in R_{O_2} preserves this orthogonality in the absence of further interactions.

Agreement between observers can only be meaningfully discussed once they physically interact. Suppose that O_2 later interacts with O_1 by receiving a signal, reading a measurement record, or otherwise correlating with the apparatus $A^{(1)}$. In the representation frame R_{O_2} , this process is described by a unitary interaction between the degrees of freedom of O_1 carrying the record and a local apparatus $A^{(2)}$ belonging to O_2 :

$$|A_i^{(1)}\rangle |A_0^{(2)}\rangle \longmapsto |A_i^{(1)}\rangle |A_i^{(2)}\rangle. \quad (14)$$

By linearity, the joint state after interaction is

$$|\Psi\rangle_{SA_1A_2}^{(O_2)} = \sum_i c_i |s_i\rangle |A_i^{(1)}\rangle |A_i^{(2)}\rangle. \quad (15)$$

When O_2 performs this measurement, collapse occurs in O_2 's description, but this is the collapse due to O_2 's own measurement interaction, not a non-local propagation of O_1 's earlier collapse. Tracing out the microscopic system S , one obtains the reduced density matrix for the records of the two observers:

$$\rho_{A^{(1)}A^{(2)}} = \sum_i |c_i|^2 |A_i^{(1)}A_i^{(2)}\rangle \langle A_i^{(1)}A_i^{(2)}|. \quad (16)$$

This is a classical mixture of correlated pointer states: with probability $|c_i|^2$, both apparatuses register outcome i . The absence of off-diagonal terms (coherences between different i, j) reflects the decoherence of macroscopic pointer states. This density matrix implies perfect correlation between the outcomes recorded by O_1 and O_2 .

The key point is that this correlation structure appears identical to the flat spacetime case because pointer states, which encode macroscopic and distinct outcomes, remain orthogonal in all representation frames. This orthogonality is preserved because: (i) signal propagation (Postulate 3) preserves the classical distinguishability of macroscopically different configurations, (ii) decoherence ensures macroscopic states do not develop quantum coherences during propagation, and (iii) the interaction with O_2 's apparatus occurs locally where $a(x) \approx 1$, not requiring transport across curved space. Once the observers have physically compared their records, they necessarily agree on the outcome of the measurement with probability one.

The above argument takes place entirely within the flat representation spaces R_{O_1} and R_{O_2} . The curvature of the physical manifold U and the observer dependence of the induced Hamiltonians on U play no role in the establishment of agreement. This is because agreement is a statement about correlations between macroscopic records, which are created and compared through local interactions governed by standard quantum mechanics in each R_{obs} . Postulate 3 ensures that communication signals propagate in every representation frame as in flat space so that the physical interaction required to compare outcomes is consistently represented for all observers. While the mapping of these processes back to U depends on the observer through the representation map T , this dependence does not affect the correlation structure of the recorded outcomes.

As measurement, observation or any kind of interaction is always local and done with respect to a singled out system; the atomic system O_1 itself defines the natural rest frame where observable features such as emission lines are determined by standard quantum mechanics. When an other inertial observer O_2 measures photons emitted by an atom (at rest with respect to O_1), the measurement occurs locally in R_{O_2} , where $a = 1$, and the

received signal already carries the information of the measurement outcome at O_1 through correlations, as demonstrated in the discussion. All observers thus agree on the emission lines of atoms, and distortion effects such as Doppler shift then emerge from standard signal propagation between moving frames rather than transformation of quantum states between frames. From this perspective, no connection between quantum descriptions in different frames is required. This generalises to curved spacetime: observed frequencies and energies incorporate redshift through signal propagation, already encoded via the representation map T and scaling factor $a(x)$. While connections are central to covariant formulations of quantum field theory on curved manifolds, they are not required in our framework where quantum mechanics lives entirely in flat representation spaces.

The absence of an observer-independent quantum state on U is therefore not a defect but a natural feature of the theory. Quantum states encode information relative to a given observer and their representation frame. Agreement between observers is not guaranteed by access to a common underlying state, but by the relational structure of quantum mechanics: outcomes become mutually consistent precisely when observers interact and establish correlations, in alignment with Relational Quantum Mechanics [5,6].

4. Examples

Here, we discuss two relevant example in the context of a static universe.

4.1. Hydrogen Atom in Einstein Universe

This example is particularly relevant, as the representation formalism has been shown to reproduce key cosmological observations when applied to the Einstein static universe. In spherical geometry with curvature radius R , the Coulomb potential is given by [7]:

$$V_U(r_H) = \frac{\kappa}{R} \cot\left(\frac{r_H}{R}\right), \quad (17)$$

where $\kappa = \frac{e^2}{4\pi\epsilon_0}$ and r_H denotes the geodesic distance from the proton, while in S^3 ,

$$a = \frac{R \sin\left(\frac{d}{R}\right)}{d},$$

where d is the geodesic distance from $\mathcal{O}$. The resulting V_{obs} is not symmetric around the proton unless the latter is centered at O , in which case $d = r_H := r$ and

$$V_{\text{obs}}(r) = -\frac{\kappa r \cos\left(\frac{r}{R}\right)}{R^2 \sin^2\left(\frac{r}{R}\right)}. \quad (18)$$

This reduces to the standard Coulomb potential in the flat limit. We know no simple closed form for the solutions of Equation (3) with this effective potential, even when expanding to second order in r/R , though approximate or numerical methods may be applied.

We emphasise that our approach fundamentally differs from direct quantisation on S^3 . Quantum mechanics is formulated in the flat tangent space R_{obs} (reflecting operational measurement), not on the curved manifold U .

4.2. Hydrogen Atom near a Black Hole

The Schwarzschild spacetime, being static and spherically symmetric, provides another illustrative application of the representation framework. Consider a hydrogen atom (proton plus electron) located at Schwarzschild radial coordinate r_s from a black hole of mass M , with Schwarzschild radius $R_s = 2GM/c^2$, and an observer at coordinate $r_{\text{obs}} > r_s$.

The scaling factor a quantifying the distortion between the curved manifold U and the observer's representation frame R_{obs} depends on the gravitational lensing properties of the Schwarzschild geometry. In general, it can be computed from

$$a = \frac{\sqrt{|D_+ D_-|}}{d} \quad (19)$$

where D_+ and D_- are the optical scalars characterizing the Jacobi matrix of the geodesic bundle, given by Equations (91) and (92) in [8], and d is the proper (radar) distance defined in [1].

Along the spatial geodesic connecting the central singularity through the source to the observer (a radial configuration), this simplifies to

$$a = \frac{r_{\text{obs}} - r_s}{(1 - R_s/r_{\text{obs}})[r_{\text{obs}} - r_s + R_s \log((r_{\text{obs}} - R_s)/(r_s - R_s))]} \quad (20)$$

where we require $r_{\text{obs}} > r_s > R_s$ for both the atom and observer to be outside the horizon.

The Coulomb potential in Schwarzschild geometry has been derived by Linet [9]. For an electron at coordinate r from a proton at the origin in Schwarzschild coordinates, the potential differs from the flat-space $-\kappa/r$ form due to the metric structure. The observed potential in R_{obs} is then

$$V_{\text{obs}} = \frac{V_U(r)}{a(r)}, \quad (21)$$

where both V_U and a depend on the precise configuration of atom, observer, and black hole.

The resulting effective potential V_{obs} breaks the spherical symmetry and introduces strong position dependence through both the gravitational modification of the Coulomb interaction in U and the scaling factor a . Near the horizon ($r_s \rightarrow R_s$), the scaling factor exhibits singular behaviour as gravitational lensing effects diverge, leading to dramatic modifications of atomic structure as seen by distant observers. A closed-form analytical solution to the Schrödinger equation (3) with such a potential is beyond the scope of this work, but numerical methods or WKB approximations could be employed to study the spectral modifications.

5. Conclusions

We have used the representation referential formalism to construct a well-defined quantum system that can be consistently mapped back onto the physical universe U . The framework developed here applies to inertial observers in a static spacetime, where the metric is time-independent, and to non-relativistic particles, for which the Schrödinger equation remains a valid description of quantum dynamics. Within this approach, curvature effects manifest as effective potentials in the observer's representation space, often breaking spatial symmetries and introducing non-linearities that make analytical solutions difficult to obtain. At microscopic scales, the deviations from standard quantum mechanics are expected to be extremely small, and therefore unlikely to be experimentally observable in the near future. However, at cosmological scales, this formalism has shown to reproduce key observational features such as supernovae data and CMB features [1] in the context of a static Universe of spherical geometry.

The examples in Section 4 illustrate the formal structure of the framework and how curvature enters the quantum description, but do not predict observable deviations in laboratory spectroscopy. Agreement between observers on local measurement outcomes is ensured by standard quantum correlations established through physical interaction (Section 3), independent of the global spacetime geometry.

A key conceptual result is that quantum states and Hamiltonians are intrinsically observer dependent, reflecting the fact that quantum theory is formulated within the observer's representation frame. Despite this, the theory remains operationally consistent: observers necessarily agree on measurement outcomes once they interact and compare records. This agreement does not rely on an observer-independent quantum state on U , but follows from standard quantum mechanical correlations and from the dynamical stability of macroscopic records within each representation frame. Therefore, the observer-centered formulation adopted here does not diminish the physical reality of quantum phenomena. Although quantum states are defined in the representation frame R_{obs} , the local nature of measurement and the relational structure of quantum mechanics ensures the objectivity of atomic spectra, stability of atoms or other quantum phenomena.

This approach differs from the vierbein formalism [10,11], which also introduces local orthonormal frames to facilitate spinor field definitions and covariant operations, but in which quantum fields are still defined on the curved manifold U . In contrast, our quantum mechanics is formulated exclusively within the flat tangent space R_{obs} , which serves as the observer's operational measurement space. The curved manifold U governs signal propagation, but quantum theory itself lives entirely in R_{obs} . Overall, it provides a mathematically consistent, observer-centered reconciliation of quantum mechanics and gravitational geometry. Extending this construction to dynamical spacetimes and relativistic regimes, for which a Dirac-like equation needs to be derived, is a natural direction for future work.

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