

Scale-Independent Relations Between Neutrino Mass Parameters

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Abstract

Theories of flavor operate at various scales. Recently it has been pointed out that in the context of modular flavor symmetries, certain combinations of observables are highly constrained, or even uniquely fixed, by modular invariance and holomorphicity. We find that even in the absence of supersymmetry, these combinations are surprisingly immune against quantum corrections.

Keyword: flavor physics; neutrinos; renormalization

1. Introduction

Theories of flavor accommodate, or even predict, fermion masses, mixing angles and $\mathcal{CP}$ phases, which constitute a significant fraction of the standard model (SM) parameters. The scale of new physics underlying the corresponding models, which we will denote by Λ_{flavor} , generally are different from scales at which experimental measurements are made. This means that quantum corrections to the model predictions have to be taken into account. This raises the question of whether there are predictions that do not depend on the scale Λ_{flavor} at which the model is defined.

In the context of modular flavor symmetries [1] (for reviews, see, e.g., [2–7]), it has recently been pointed out that there are certain combinations of entries of the Weinberg operator that are independent of the modulus τ [8]. In addition, these combinations are known to be renormalization group (RG)-invariant at one-loop [9]. This latter statement holds both in the SM and minimal supersymmetric standard model (MSSM).

The purpose of this analysis is to discuss the impact of quantum corrections on the above invariance in the absence of supersymmetry (SUSY). This is also motivated by the recent proposal of non-holomorphic modular flavor symmetries [10–12], in non-supersymmetric setups.

2. Neutrino Masses Described by the Weinberg Operator

We consider scenarios in which neutrino masses are described by the Weinberg operator. In the supersymmetric context, the superpotential of the lepton sector is then given by

$$\mathcal{W}_{\text{lepton mass}} = Y_e^{gf} L_g \cdot H_d E_f + \frac{1}{2} \kappa_{gf} L_g \cdot H_u L_f \cdot H_u. \quad (1)$$

Here, the superfields L^f and E^f denote the three generations of the $SU(2)_L$ charged lepton doublets and singlets, respectively. The flavor indices are f and g . The superfields $H_{u/d}$ stand for the MSSM Higgs doublets. In (1), “ $\cdot$ ” indicates contractions with the Levi–Civita symbol. $m_\nu = v_u^2 \kappa$ is the neutrino mass matrix, with κ being the effective neutrino mass



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operator. Finally, Y_e denotes the charged lepton Yukawa couplings. In models based on modular flavor symmetries, κ and Y_e are given in terms of the modular forms.

In the SM amended by the Weinberg operator, the lepton masses are described by

$$\mathcal{L}_{\text{lepton mass}} = -Y_e^{gf} \overline{\ell_{L,g}} e_{R,g} \cdot \phi - \frac{1}{4} \kappa_{gf} \ell^g \cdot \phi \ell^f \cdot \phi + \text{h.c.} . \quad (2)$$

Here, $\ell_{L,f}$ denote the lepton doublets, $e_{R,g}$ the right-handed charged leptons, and ϕ the SM Higgs.

Apart from the charged lepton masses, $m_f = y_f v_{EW}$ with v_{EW} denoting the vacuum expectation value (VEV) of the electroweak Higgs ϕ , the lepton sector has nine flavor parameters,

$$\{\xi_i\} = \{m_1, m_2, m_3, \theta_{12}, \theta_{13}, \theta_{23}, \delta, \varphi_1, \varphi_2\} . \quad (3)$$

Out of these parameters, two mass squared differences, $\Delta m_{ij}^2 := m_i^2 - m_j^2$, and the mixing angles θ_{ij} have been measured with relatively good precision; see, e.g., [13]. The recent JUNO results [14] have significantly reduced the error bars of θ_{12} and $\Delta m_{12}^2 := m_2^2 - m_1^2$; see, e.g., [15]. On the other hand, the absolute neutrino mass scale and the Dirac phase δ are subject to constraints but not determined precisely. We currently do not know whether neutrinos are Majorana fermions and thus have no knowledge of the values of the Majorana phases φ_i .

3. Lepton Flavor Parameters and Quantum Corrections

Equations (1) and (2) contain the Weinberg operator,

$$\mathcal{L}_\kappa = -\frac{1}{4} \kappa_{gf} \ell^g \cdot \phi \ell^f \cdot \phi + \text{h.c.} . \quad (4)$$

κ is a symmetric matrix of mass dimension -1 .

Throughout this study, we will work in a basis in which the fields are canonically normalized and Y_e is diagonal and positive,

$$Y_e = \text{diag}(y_e, y_\mu, y_\tau) \quad \text{with } y_f > 0 \text{ for } f \in \{e, \mu, \tau\} . \quad (5)$$

In this basis, all the renormalizable interactions in the lepton sector are diagonal in flavor space.

3.1. Invariants

In the basis chosen as given in (5), we define the invariants

$$I_{fg} := \frac{(m_\nu)_{ff} (m_\nu)_{gg}}{((m_\nu)_{fg})^2} = \frac{\kappa_{ff} \kappa_{gg}}{(\kappa_{fg})^2} , \quad (6)$$

where no summation over the flavor indices f and g is implied. We are interested in quantum corrections to these combinations. In order to obtain the second equality in (6), we have to assume that the normalizations of the three lepton doublets coincide at a given scale. The focus of this study is on the RG stability of I_{fg} (6).

A key feature of these expressions is that they can be entirely expressed in terms of observable flavor parameters. Explicitly,

$$I_{12} = \frac{a_0 [\tilde{m}_1 (c_{23}s_{12} + e^{-i\delta} c_{12}s_{13}s_{23})^2 + \tilde{m}_2 (c_{12}c_{23} - e^{-i\delta} s_{12}s_{13}s_{23})^2 + m_3 c_{13}^2 s_{23}^2]}{[\tilde{m}_1 c_{12} (c_{23}s_{12} + e^{-i\delta} c_{12}s_{13}s_{23}) - \tilde{m}_2 s_{12} (c_{12}c_{23} - e^{-i\delta} s_{12}s_{13}s_{23}) - e^{i\delta} m_3 s_{13}s_{23}]}^2 \quad (7a)$$

$$I_{13} = \frac{a_0 [\tilde{m}_1 (e^{-i\delta} c_{12}c_{23}s_{13} - s_{12}s_{23})^2 + \tilde{m}_2 (e^{-i\delta} c_{23}s_{12}s_{13} + c_{12}s_{23})^2 + m_3 c_{13}^2 c_{23}^2]}{[\tilde{m}_1 c_{12} (e^{-i\delta} c_{12}c_{23}s_{13} - s_{12}s_{23}) + \tilde{m}_2 s_{12} (e^{-i\delta} c_{23}s_{12}s_{13} + c_{12}s_{23}) - e^{i\delta} m_3 c_{13}c_{23}]}^2 \quad (7b)$$

$$I_{23} = \left[m_3 c_{13}^2 s_{23}^2 + \tilde{m}_1 (c_{23}s_{12} + e^{-i\delta} c_{12}s_{13}s_{23})^2 + \tilde{m}_2 (c_{12}c_{23} - e^{-i\delta} s_{12}s_{13}s_{23})^2 \right] \times \frac{4 [m_3 c_{13}^2 c_{23}^2 + \tilde{m}_2 (e^{-i\delta} c_{23}s_{12}s_{13} + c_{12}s_{23})^2 + \tilde{m}_1 (e^{-i\delta} c_{12}c_{23}s_{13} - s_{12}s_{23})^2]}{[(\tilde{m}_1 a_1 + \tilde{m}_2 a_2) - m_3 \sin(2\theta_{23}) c_{13}^2]^2}, \quad (7c)$$

$$s_{ij} := \sin \theta_{ij}, c_{ij} := \cos \theta_{ij}, t_{ij} := \tan \theta_{ij}, \text{ and}$$

$$a_0 := \tilde{m}_1 c_{12}^2 + \tilde{m}_2 s_{12}^2 + e^{2i\delta} m_3 t_{13}^2 \quad (8a)$$

$$a_1 := \left[(s_{12}^2 - e^{-2i\delta} c_{12}^2 s_{13}^2) \sin(2\theta_{23}) - e^{-i\delta} \cos(2\theta_{23}) \sin(2\theta_{12}) s_{13} \right], \quad (8b)$$

$$a_2 := \left[e^{-i\delta} \cos(2\theta_{23}) \sin(2\theta_{12}) s_{13} + (c_{12}^2 - e^{-2i\delta} s_{12}^2 s_{13}^2) \sin(2\theta_{23}) \right]. \quad (8c)$$

The invariants I_{fg} depend on m_1, m_2, φ_1 and φ_2 only via the combinations $\tilde{m}_1 := m_1 e^{i\varphi_1}$ and $\tilde{m}_2 := m_2 e^{i\varphi_2}$. As I_{fg} are complex, each of them contains two real flavor parameters. This means that, unless there are degeneracies, six independent linear combinations out of the nine flavor parameters ξ_i in (3) are described by I_{fg} . As discussed in [8], the zeros and poles of I_{fg} correspond to the texture zeros of the Weinberg operator.

Why are we interested in these invariants, I_{fg} ? There are two main reasons. First of all, they are RG-invariant at the one-loop level [9], as we shall discuss in more detail in Section 3.2. Additionally, they turn out to have remarkable properties in the framework of modular flavor symmetries. For instance, in the Feruglio model [1], $I_{12} = -2$, and $I_{13} I_{23} = -32$, independently of the value of the modulus [8]. That is, these invariants carry a large amount of information on modular symmetries. As we shall see next, they are not only independent of the modulus but also, for all practical purposes, insensitive to the definition flavor scale Λ_{flavor} of the model.

3.2. Renormalization Group Equations

In [9], it was found that I_{fg} defined in (6) are independent of the renormalization scale at one-loop. In the supersymmetric context, one may view this as being a simple consequence of the non-renormalization theorem. Only the wave function renormalization constants depend on the scale, and the latter cancel in the I_{fg} expressions [16]. The RG invariance of I_{fg} thus holds at all loop levels in supersymmetric models. So in the following, we will focus on the non-supersymmetric case.

Since κ is a symmetric matrix, its renormalization group equation (RGE) has the form

$$\frac{d}{dt} \kappa = \sum_k \kappa^{(k)} := \sum_k \left(\frac{1}{16\pi^2} \right)^k \left[\alpha^{(k)} \kappa + P^{(k)} \kappa + \kappa (P^{(k)})^T + Q^{(k)} \kappa (Q^{(k)})^T \right]. \quad (9)$$

Here, the superscript “(k)” indicates the loop order; $\alpha^{(k)}$ denotes the scalar contribution, which includes gauge couplings and parameters of the Higgs potential; and $P^{(k)}$ and $Q^{(k)}$

are implicitly defined by (9). We will provide explicit expressions below in (13). The t -derivative is the logarithmic derivative with respect to the renormalization scale μ ,

$$\frac{d}{dt} := \mu \frac{d}{d\mu}, \quad (10)$$

i.e., $t = \ln(\mu/\mu_0)$ with some reference scale μ_0 . In (9), k indicates the loop level, and $\alpha^{(k)}$ are flavor-independent coefficients. The matrices $P^{(k)}$ and $Q^{(k)}$ are composed of the renormalizable couplings of the theory and diagonal,

$$P^{(k)} = \text{diag}(P_1^{(k)}, P_2^{(k)}, P_3^{(k)}) , \quad (11a)$$

$$Q^{(k)} = \text{diag}(Q_1^{(k)}, Q_2^{(k)}, Q_3^{(k)}) . \quad (11b)$$

At one-loop, $P^{(1)} = C_e Y_e Y_e^\dagger = C_e \text{diag}(y_e^2, y_\mu^2, y_\tau^2)$ with Y_e being the charged lepton Yukawa matrix (5). $C_e = -3/2$ in the SM [17] and two-Higgs models [18], and $C_e = 1$ in the MSSM [19,20] (see, e.g., [21,22] for reviews). At the one-loop level in (9), there is only one matrix in flavor space, $P^{(1)}$, and we can choose $Q^{(1)} = \mathbb{1}$.

In order to have nonzero RG effects, we consider two-loop RGEs [23–26]. The two-loop contribution in (9) in the SM is given by [27]

$$\kappa^{(2)} = \left(\frac{1}{16\pi^2} \right)^2 \left[\alpha^{(2)} \kappa + P^{(2)} \kappa + \kappa (P^{(2)})^\top + Q^{(2)} \kappa (Q^{(2)})^\top \right], \quad (12)$$

where there is now a nontrivial Q -matrix,

$$P^{(2)} = \left(-\frac{57}{16}g_1^2 + \frac{33}{16}g_2^2 + \frac{5}{4}T \right) Y_e Y_e^\dagger + \frac{19}{4} Y_e Y_e^\dagger Y_e Y_e^\dagger, \quad (13a)$$

$$Q^{(2)} = \sqrt{2} Y_e Y_e^\dagger. \quad (13b)$$

Here, g_1 and g_2 are the running gauge coupling constants, and $T := \text{Tr}[Y_e Y_e^\dagger + 3Y_u Y_u^\dagger + 3Y_d Y_d^\dagger]$, with Y_u and Y_d being the Yukawa coupling matrices for the up-type quarks and the down-type quarks, respectively.

Analogously to (9), we can write the loop expansion of the I_{fg} as

$$\dot{I}_{fg} := \frac{d}{dt} I_{fg} = \sum_k I_{fg}^{(k)}, \quad (14)$$

where

$$I_{fg}^{(k)} = \frac{\kappa_{ff}^{(k)} \kappa_{gg}^{(k)}}{(\kappa_{fg})^2} + \frac{\kappa_{ff} \kappa_{gg}^{(k)}}{(\kappa_{fg})^2} - 2 \frac{\kappa_{ff} \kappa_{gg}}{(\kappa_{fg})^3} \kappa_{fg}^{(k)}. \quad (15)$$

Truncating (9) at the two-loop level, i.e., $k = 2$, and inserting this truncation into (14), we obtain

$$\begin{aligned}
\frac{d}{dt} I_{fg} &= \frac{(\kappa_{ff}^{(1)} + \kappa_{ff}^{(2)}) \kappa_{gg}}{\kappa_{fg}^2} + \frac{\kappa_{ff} (\kappa_{gg}^{(1)} + \kappa_{gg}^{(2)})}{\kappa_{fg}^2} - 2 \frac{\kappa_{ff} \kappa_{gg}}{\kappa_{fg}^3} (\kappa_{fg}^{(1)} + \kappa_{fg}^{(2)}) \\
&= \left[\frac{\kappa_{ff}^{(1)} \kappa_{gg}}{\kappa_{fg}^2} + \frac{\kappa_{ff} \kappa_{gg}^{(1)}}{\kappa_{fg}^2} - 2 \frac{\kappa_{ff} \kappa_{gg}}{\kappa_{fg}^3} \kappa_{fg}^{(1)} \right] + \left[\frac{\kappa_{ff}^{(2)} \kappa_{gg}}{\kappa_{fg}^2} + \frac{\kappa_{ff} \kappa_{gg}^{(2)}}{\kappa_{fg}^2} - 2 \frac{\kappa_{ff} \kappa_{gg}}{\kappa_{fg}^3} \kappa_{fg}^{(2)} \right] \\
&=: I_{fg}^{(1)} + I_{fg}^{(2)} .
\end{aligned} \tag{16}$$

Then, we can calculate $I_{fg}^{(k)}$ as

$$\begin{aligned}
I_{fg}^{(k)} &= \frac{\kappa_{ff} \kappa_{gg}}{(16\pi^2)^k \kappa_{fg}^2} \left[\alpha^{(k)} + 2P_{ff}^{(k)} + (Q_{ff}^{(k)})^2 + \alpha^{(k)} + 2P_{gg}^{(k)} + (Q_{gg}^{(k)})^2 \right. \\
&\quad \left. - 2(\alpha^{(k)} + P_{ff}^{(k)} + P_{gg}^{(k)} + Q_{ff}^{(k)} Q_{gg}^{(k)}) \right] \\
&= \frac{\kappa_{ff} \kappa_{gg}}{(16\pi^2)^k \kappa_{fg}^2} (Q_{ff}^{(k)} - Q_{gg}^{(k)})^2 .
\end{aligned} \tag{17}$$

This shows that the α - and P -type terms in (9) do not affect the RGEs of the invariants. At one-loop, $Q_{ff}^{(1)} = 0$, so there is no correction to I_{fg} at this order. At two-loop, $Q^{(2)}$ is a diagonal matrix with diagonal elements given by

$$Q_{ff}^{(2)} = \sqrt{2} (Y_e Y_e^\dagger)_{ff} = \sqrt{2} y_f^2 , \tag{18}$$

where we use $y_f > 0$. Therefore, I_{fg} up to two-loop using Equation (17) is explicitly given by

$$\frac{dI_{fg}}{dt} = \frac{2(y_f^2 - y_g^2)^2}{(16\pi^2)^2} I_{fg} . \tag{19}$$

Specifically,

$$\frac{dI_{12}}{dt} = \frac{2(y_e^2 - y_\mu^2)^2}{(16\pi^2)^2} I_{12} , \tag{20a}$$

$$\frac{dI_{13}}{dt} = \frac{2(y_e^2 - y_\tau^2)^2}{(16\pi^2)^2} I_{13} , \tag{20b}$$

$$\frac{dI_{23}}{dt} = \frac{2(y_\mu^2 - y_\tau^2)^2}{(16\pi^2)^2} I_{23} . \tag{20c}$$

Interestingly, these results show that if I_{fg} vanishes at some scale, it will stay zero at all scales. Since the coefficients on the right-hand sides of (20) are real, this statement applies separately to the real and imaginary parts of I_{fg} . That is, if $\text{Re } I_{fg}$ or $\text{Im } I_{fg}$ vanishes at some scale, it will remain zero at all scales.

Given the hierarchy $y_\tau \gg y_\mu \gg y_e$, we see that the corrections of I_{12} are even more suppressed than the RG effects on I_{13} and I_{23} . Even the latter are basically RG-stable. Since $y_\tau \sim 10^{-2}$, the coefficient is of the order 10^{-10} . Multiplying this by $\ln(\Lambda_{\text{flavor}}/v_{\text{EW}})$ still leads to RG effects at most of the order 10^{-8} . This means that, for all practical purposes, I_{fg} are invariant under the renormalization group in the SM and thus not sensitive to the flavor scale Λ_{flavor} .

Using Equation (19), we can estimate benchmark values to the quantum corrections, given by

$$\Delta I_{fg} \approx \frac{2(y_f^2 - y_g^2)^2}{(16\pi^2)^2} I_{fg} \Delta t. \quad (21)$$

We chose $y_e \approx 2 \times 10^{-6}$, $y_\mu \approx 5 \times 10^{-4}$, and $y_\tau \approx 7 \times 10^{-3}$, and between the energy scale $\mu = 10^3 - 10^6$ GeV and for $\text{Re } I_{fg}$, we obtained $I_{12} \approx -35$, $I_{13} \approx -10$, and $I_{23} \approx 2$ from REAP [28]. The corrections to I_{fg} using Equation (21) are then given by

$$\Delta I_{12} \approx -1.2 \times 10^{-15} \quad (22a)$$

$$\Delta I_{13} \approx -1.3 \times 10^{-11} \quad (22b)$$

$$\Delta I_{23} \approx 2.6 \times 10^{-12} \quad (22c)$$

Clearly, these changes are far smaller than the experimental error bars, including the latest results reported by the JUNO collaboration [14].

We used a modified version of REAP [28] to verify that, when running the invariants at two-loop over a few orders of magnitude, they remain practically unchanged. A more detailed numerical study will be presented elsewhere.

Let us comment on two-Higgs doublet models (2HDMs). Usually one imposes symmetries to make sure that the charged leptons only couple to one of the Higgs doublets in order to avoid flavor-changing neutral currents (FCNCs) [29,30], cf. the discussion in [18]. In these models, y_τ may be of the order unity. Even in this case, the corrections (20) remain well below the percent level.

3.3. Limitations

In our analysis, we focused on the case in which the model gives rise to the SM, MSSM or a 2HDM below its definition scale. If there are additional renormalizable couplings that are sensitive to specific lepton flavors, our analysis may no longer apply. Studying such scenarios is beyond the scope of this work.

4. Summary

Motivated by the analytic properties of certain combinations of the neutrino mass matrix I_{fg} in the context of modular flavor symmetries, we studied the stability of these expressions under the renormalization group. While I_{fg} receive corrections at the two-loop level, for all practical purposes, they remain RG-invariant in the SM and 2HDM, i.e., in the absence of SUSY. This leads to predictions that are insensitive to the scale Λ_{flavor} at which the model is defined. The conclusions drawn from the analytical properties of I_{fg} can therefore be confronted to data without the need of a detailed renormalization group analysis. In other words, experimental measurements can directly probe high-scale physics.

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