

Article

Stability Analysis of Magnetized Quark Matter in Tsallis Statistics

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Abstract

In this work, we employ the nonextensive Nambu–Jona-Lasinio model to analyze the thermodynamic properties of magnetized quark matter. The nonequilibrium state is described in Tsallis distribution by a dimensionless parameter q . We find that within a reasonable temperature range, the system undergoes a crossover transition at the critical chemical potential, which is decreased by the increase of both the temperature and q value. In contrast to the enhanced stability by magnetic field in Boltzmann statistics, it is found that the stability of chiral restored matter in Tsallis statistics would be reduced by an increase of the magnetic field. Conversely, the increase of the q would enhance the stability of quark matter. Finally, we display the different magnetic effects on the stability in the chiral broken and restored regions.

Keywords: Tsallis statistics; the magnetic field; quark matter; NJL model; crossover transition; stability

1. Introduction

As a core component of the Standard Model of particle physics, quantum chromodynamics (QCD) not only explains the internal structure of hadrons but also provides a theoretical framework for the evolution of the early Universe and heavy ion collisions [1]. For many years, the theory has been used to predict and has confirmed the state of matter within a few microseconds after the Big Bang in the Relativistic Heavy Ion Collider (RHIC) and the Large Hadron Collider (LHC) experiments [2,3]. That is to say, quarks and gluons are liberated from confinement at high temperatures and high densities, forming quark gluon plasma (QGP) that only exists for a short time and exhibits the properties of a perfect fluid [4]. For the low energy region, in 1974, Kenneth G. Wilson proposed the lattice QCD (LQCD)-simulation method, which is one of the most effective methods in the nonperturbation theory. It would become the continuum field theory in the thermodynamic limit and the continuum limit [5]. At present, the QCD theory is utilized to achieve precise prediction of the matter construction and its progress has a profound impact on particle physics, nuclear physics, cosmology and even quantum information.

The phase diagram and phase transition of QCD are among the most interesting and challenging fields in modern physics. The critical temperature and chemical potential of transitions play a very important role in our understanding of the phase diagram and the theory itself [6,7]. Currently, the HotQCD Collaboration has provided precise pseudocritical temperature of the chiral phase transition by calculating different quantities, such as chiral order parameter and chiral susceptibility [8,9]. When the mass of a quark is infinite, the quark cannot be excited in vacuum. At this time, the phase transition of the system is the



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confinement–deconfinement transition, which is another independent phase transition in QCD [10]. Recently, the large $\mathcal{N}_c$ argument indicates that there is an area in the phase diagram called quarkyonic matter, which could be discovered in heavy ion collisions and compact stars [11]. Due to the existence of strong magnetic fields, the thermodynamic properties of quark matter would be influenced, thereby causing the phase structures to become more abundant. The boundary line is confirmed to be changed by the magnetic fields. It is somewhat difficult to find the exact location of the critical end point (CEP), at which phase transitions of strong interacting matter occur. A mainstream view is that the CEP exists and there is only one, but in some of the literature the authors believe that there is no CEP or that there might be two CEPs, etc. [12–16].

Generally speaking, the most reliable source for the equation of state is the lattice gauge theory and its application to QCD, which provides relevant nonperturbation information from first principle calculations [17–19]. However, the lattice study at finite baryon chemical potential has been severely hindered due to the sign problem; other approaches such as effective theories need to be used for calculation, which have the properties and symmetries of QCD. In 1961, Yoichiro Nambu and G. Jona-Lasinio proposed the Nambu–Jona-Lasinio (NJL) model to describe the interaction between nucleons [20,21]. Later, the model was also employed to research questions such as the mass and decay constant of scalar mesons, and became a crucial theoretical model [22,23]. Nevertheless, the role of gluons is ignored in general NJL mean fields, resulting in the model’s inability to demonstrate the color confinement of quarks. In order to make up for the shortcoming, the polyakov loop, which is the dynamic gluon field, has been added on this basis [24,25].

In the conventional description of the QCD transition, equilibrium is typically based on Boltzmann–Gibbs (BG) statistics. However, in heavy ion collision experiments, the spatial configuration of the system is far from uniform, which prevents the establishment of global thermal equilibrium. In the development process of thermodynamic statistical models, the introduction of the Tsallis distribution better describes the nonequilibrium statistical phenomena in the high energy region. In 1988, Constantino Tsallis first proposed the concept of nonextensive entropy and formed a generalized distribution [26]. In the practice of the PHENIX and STAR at RHIC in BNL, the Tsallis distribution has developed into more general forms, including the Boltzmann, the Fermi–Dirac and the Bose–Einstein distribution [27,28]. As mentioned earlier, the Tsallis statistic is parameterized by a nonequilibrium factor q , and when q approaches 1, the distribution degenerates into the standard Boltzmann–Gibbs distribution [29]. In recent years, many studies have taken the Tsallis statistic into account to describe high energy physics [30–34]. Here, it should be emphasized that the strong magnetic fields existing in relativistic heavy ion collisions and neutron stars do play an important role in the thermodynamics of the system [35,36].

The paper is organized as follows. In Section 2, the thermodynamics of the nonextensive NJL model is reviewed in a strong magnetic field. In Section 3, the numerical results and discussion concentrate on the stability of quark matter affected by the magnetic field and nonextensive parameters. Finally we present our conclusions in Section 4.

2. Nonextensive NJL Model in the Strong Magnetic Field

We next present the main formulas necessary for the application of the nonextensive statistics to the NJL model. As mentioned above, the Tsallis entropy was proposed [30,37]

$$S_q = k \frac{1 - \sum p_i^q}{q - 1}, \quad (1)$$

where k is a conventional positive constant, p_i represents the probability related to the event, q can be any real number. The concept is delicate, and the entropy satisfies all properties of Boltzmann–Gibbs entropy, except the additivity, so

$$S_q(A + B) = S_q(A) + S_q(B) + (1 - q)S_q(A)S_q(B), \quad (2)$$

where A and B refer to two independent systems. The nonextensive phenomenon is considered to be related to the long-range interacting mechanic [38].

The function $\exp_q(x)$ is usually defined as [26,37,39,40]

$$\exp_q(x) \equiv \begin{cases} (1 + (q - 1)x)^{1/(q-1)} & x > 0 \\ (1 + (1 - q)x)^{1/(1-q)} & x \leq 0 \end{cases} \quad (3)$$

where $x = \frac{\omega_i - \mu}{T}$ is defined, ω_i is the energy of the particle given by $\omega_i = \sqrt{p^2 + M^2}$ and μ is the chemical potential. In the limit $q \rightarrow 1$, the standard exponential $\lim_{q \rightarrow 1} \exp_q(x) \rightarrow \exp(x)$ is recovered. The function of the q -logarithm as well given by

$$\ln_q(x) \equiv \begin{cases} \frac{x^{q-1} - 1}{q - 1} & x > 0 \\ \frac{x^{1-q} - 1}{1 - q} & x \leq 0 \end{cases}. \quad (4)$$

By using the above formula, we can obtain the distribution function of fermions at the temperature T

$$f_+(x) = \begin{cases} \frac{1}{1 + (1 + (q - 1)x)^{\frac{1}{q-1}}} & x > 0 \\ \frac{1}{1 + (1 + (1 - q)x)^{\frac{1}{1-q}}} & x \leq 0 \end{cases}, \quad (5)$$

where $x = \frac{\omega_i - \mu}{T}$, $\omega_i = \sqrt{p_z^2 + M^2 + 2k_i|q_i|B}$ gives the effective energy and q_i represents the charge of quark in strong magnetic fields [41,42].

In order to model dynamical chiral symmetry breaking, the Lagrangian density of the two-flavor and three-color NJL model in the presence of strong external magnetic fields is [43]

$$\mathcal{L}_{NJL} = \bar{\psi}(i\mathcal{D} - m)\psi + G[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2], \quad (6)$$

where ψ represents the quark fields and m is the bare mass of fermions. The coupling of quarks to electromagnetic fields is given by $\mathcal{D} = \gamma^\mu D_\mu$ and the covariant derivative $D_\mu = \partial_\mu + iQA_\mu$, which implies the sum of the flavor and color degrees of freedom. G is the four-fermion interaction coupling constant and $\vec{\tau}$ is isospin Pauli matrice.

We all know that the thermodynamic potential is a function of natural variables and all other thermodynamic quantities are differentiated from it. In the finite temperature field theory and mean-field approximation, it can be expressed as

$$\Omega = \frac{(M - m_i)^2}{4G} + \sum_{i=u,d} \Omega_i, \quad (7)$$

where the first term is the interaction term, and Ω_i is defined as $\Omega_i = \Omega_i^{\text{vac}} + \Omega_i^{\text{mag}} + \Omega_i^{\text{med}}$ in the second term. The vacuum, the magnetic field, and the medium contributions at finite temperature and chemical potential are

$$\Omega_i^{\text{vac}} = \frac{N_c}{8\pi^2} \left[M^4 \ln\left(\frac{\Lambda + \epsilon_\Lambda}{M}\right) - \epsilon_\Lambda \Lambda (\Lambda^2 + \epsilon_\Lambda^2) \right], \quad (8)$$

$$\Omega_i^{\text{mag}} = -\frac{N_c(|q_i|B)^2}{2\pi^2} \left[\zeta'(-1, x_i) - \frac{1}{2}(x_i^2 - x_i) \ln(x_i) + \frac{x_i^2}{4} \right], \quad (9)$$

$$\begin{aligned} \Omega_i^{\text{med}} &= -\sum_{k_i=0}^{\infty} a_{k_i} \frac{|q_i|BN_c T}{4\pi^2} \int dp \left[\ln_q(1 + e^{-(\omega_i - \mu)/T}) + \ln_q(1 + e^{-(\omega_i + \mu)/T}) \right] \\ &= \begin{cases} -\sum_{k_i=0}^{\infty} a_{k_i} \frac{|q_i|BN_c T}{4\pi^2} \int dp \left\{ \frac{[1 + (1 - \varphi^+)^{\frac{1}{1-q}}]^{q-1} - 1}{q-1} + \frac{[1 + (1 - \varphi^-)^{\frac{1}{1-q}}]^{q-1} - 1}{q-1} \right\} & x > 0 \\ -\sum_{k_i=0}^{\infty} a_{k_i} \frac{|q_i|BN_c T}{4\pi^2} \int dp \left\{ \frac{[1 + (1 + \varphi^+)^{\frac{1}{q-1}}]^{q-1} - 1}{q-1} + \frac{[1 + (1 + \varphi^-)^{\frac{1}{q-1}}]^{q-1} - 1}{q-1} \right\} & x \leq 0 \end{cases}, \quad (10) \end{aligned}$$

where the quantity ϵ_Λ is defined as $\epsilon_\Lambda = \sqrt{\Lambda^2 + M^2}$. The $\zeta(a, x) = \sum_{n=0}^{\infty} \frac{1}{(a+n)^x}$ is the Riemann–Hurwitz zeta function. The $\zeta'(-1, x) = \frac{d\zeta(z, x)}{dz}|_{z=-1}$ is defined in the magnetic field term [44]. Then, $a_{k_i} = 2 - \delta_{k_i,0}$ and k_i are the degeneracy label and the Landau quantum number, respectively. The function $\varphi^\pm$ in the medium term is expressed as $\varphi^\pm = \frac{(q-1)(\omega_i \mp \mu)}{-T}$ as a matter of convenience.

In the interaction term, M refers to the effective dynamical mass of the quark. The condensation that can be regarded as the order parameter is formed due to the pairing of quarks–antiquarks with the same chirality in the mean-field approximation [45]. So we have the gap equation

$$M_i = m_i - 2G\langle\bar{\psi}\psi\rangle, \quad (11)$$

where the quark condensates include u and d quark contributions as $\langle\bar{\psi}\psi\rangle = \phi = \sum_{i=u,d} \phi_i$. Here we use $M = M_u \approx M_d$ as the representation of the current mass. Similarly, the contributions from the quark condensates with flavor i include the vacuum, the magnetic field, and the medium term as [46–48]

$$\phi_i = \phi_i^{\text{vac}} + \phi_i^{\text{mag}} + \phi_i^{\text{med}}, \quad (12)$$

where each term reads

$$\phi_i^{\text{vac}} = -\frac{MN_c}{2\pi^2} \left[\Lambda \sqrt{\Lambda^2 + M^2} - M^2 \ln\left(\frac{\Lambda + \sqrt{\Lambda^2 + M^2}}{M}\right) \right], \quad (13)$$

$$\phi_i^{\text{mag}} = -\frac{M|q_i|BN_c}{2\pi^2} \left\{ \ln[\Gamma(x_i)] - \frac{1}{2} \ln(2\pi) + x_i - \frac{1}{2}(2x_i - 1) \ln(x_i) \right\}, \quad (14)$$

$$\phi_i^{\text{med}} = \sum_{k_i=0}^{\infty} a_{k_i} \frac{M|q_i|BN_c}{4\pi^2} \int \frac{f_+(\omega_i) + f_-(\omega_i)}{\omega_i} dp. \quad (15)$$

In order to obtain a good approximation, we choose the dimensionless quantity x_i , defined as $x_i = M^2/(2|q_i|B)$ in the renormalized expression. In Equation (15), $a_{k_i} = 2 - \delta_{k_i,0}$ and k_i are the degeneracy label and the Landau quantum number, respectively. In nonzero magnetic fields, the concepts of Landau quantization and magnetic catalysis, where the magnetic field is assumed to be oriented along z direction, should be implemented. So we can use the mapping $\int \frac{d^3p}{(2\pi)^3} \rightarrow \frac{|q_i|B}{2\pi} \sum_k \int \frac{dp_z}{2\pi} (2 - \delta_{k_i,0})$ to compute the integral over the three momenta [49,50]. Here the distribution function of the system is in the form of nonextensive statistics because the global equilibrium is difficult to establish. It should be noted that the NJL model, as a non-renormalization model, needs to be regularized

to remove the divergence of the vacuum. In the literature, there are many works that have discussed the renormalization of the free energy of charged fermions in a magnetic field. The appropriate scheme was proposed to regularize the integral. Schwinger's proper time formalism regularized all integrals without separating the divergent (vacuum) piece from the convergent (thermomagnetic) contribution. The magnetic field independent regularization (MFIR) makes a full separation of the finite magnetic contributions and the divergencies.

Generally, the pressure can be obtained by $P = -\Omega$. In this context, we take care of the normalized pressure by subtracting vacuum pressure contribution at zero chemical potential from the total pressure given as

$$P_{\text{eff}} = -\Omega - P(\mu = 0, T = 0). \quad (16)$$

In terms of thermodynamic relations of the system, the free energy density reads

$$F = -P_{\text{eff}} + \mu \sum_i n_i, \quad (17)$$

where n_i is the number number of particles, it can be obtained through the relation

$$n_i = -\left. \frac{\partial \Omega_i}{\partial \mu} \right|_T. \quad (18)$$

The detailed formulas are as follows

$$n_i = \begin{cases} \sum_{k_i=0}^{\infty} a_{k_i} \frac{|q_i| B N_c}{4\pi^2} \int dp \left\{ [1 + (1 - \varphi^+)^{\frac{1}{1-q}}]^{q-2} (1 - \varphi^+)^{\frac{q}{1-q}} - [1 + (1 - \varphi^-)^{\frac{1}{1-q}}]^{q-2} (1 - \varphi^-)^{\frac{q}{1-q}} \right\} & x > 0 \\ \sum_{k_i=0}^{\infty} a_{k_i} \frac{|q_i| B N_c}{4\pi^2} \int dp \left\{ [1 + (1 + \varphi^+)^{\frac{1}{q-1}}]^{q-2} (1 + \varphi^+)^{\frac{2-q}{q-1}} - [1 + (1 + \varphi^-)^{\frac{1}{q-1}}]^{q-2} (1 + \varphi^-)^{\frac{2-q}{q-1}} \right\} & x \leq 0 \end{cases}. \quad (19)$$

3. Numerical Result and Discussion

In the framework of the SU(2) NJL model, the following parameters are adopted: $m_u = m_d = 5.5$ MeV for the up and down quarks, $\Lambda = 587.9$ MeV and $G = 2.44/\Lambda^2$ for the momentum cutoff and the coupling constant [22]. It is worth noting that the maximum deviation of q should not exceed 20% [28,51–54]. Following this, we chose three different q values in the following Tsallis statistic for comparison.

In the literature, the dynamical mass of quarks serves as an order parameter to reflect the chiral crossover in the NJL model. Figure 1 depicts the dynamical mass of u and d quarks in the chiral phase transition at the magnetic field $eB = 0.2$ GeV² and the nonextensive parameter $q = 1.1$. Here we have chosen a typical value of the magnetic field, which is helpful for understanding the magnetic effects in heavy ion collisions and neutron stars [55]. Three different temperatures are denoted by black, red and blue curves, respectively. The dotted line corresponds to the dynamical mass of nonmagnetized quarks. It is obvious that the magnetic field and the temperature play competing roles to influence the quark mass at $\mu = 0$. For given finite temperatures, the curves exhibit a decline to lower values at high chemical potentials, indicating that the quarks undergo a chiral restoration. As the temperature increases, the crossover occurs at lower chemical potential, which means that high temperatures favor the chiral restoration at low chemical potentials. Moreover, compared with the zero magnetic field, a strong magnetic field leads to an increase of the critical chemical potential.

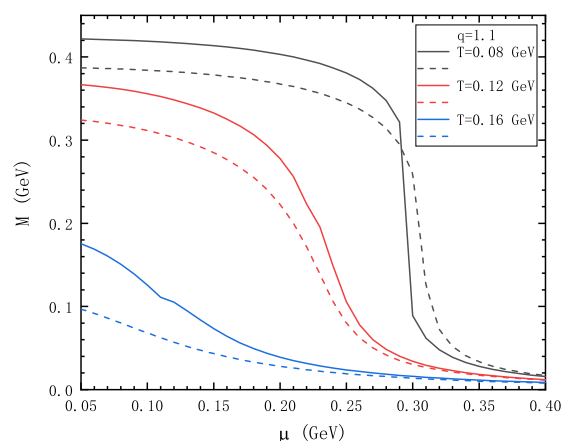


Figure 1. The dynamical mass of nonextensive quark matter as a function of chemical potential at different temperatures.

In a nonequilibrium system, the influence of the parameter q on quark matter is beyond doubt. At a fixed temperature and magnetic field, the variation of quark mass with chemical potential induced by q is shown in Figure 2. On the one hand, within a specific range, the increase in the q value reduces the critical chemical potential, which is determined by the peak of the derivative of the dynamical mass with respect to the chemical potential in the phase transition. On the other hand, the presence of strong magnetic fields shifts the critical chemical potential to higher values, reflecting the magnetic catalytic effect observed in the lattice QCD. Consequently, it can be concluded that the parameter q and the strong magnetic field exhibit opposite effects on the change of the critical chemical potential in nonextensive case.

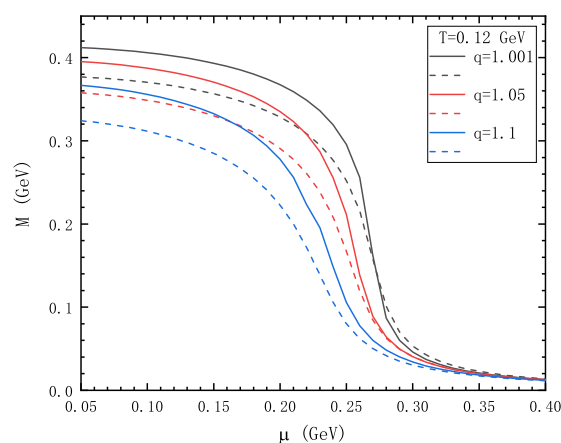


Figure 2. The dynamical mass of quark matter as a function of chemical potential for different nonextensive parameters at $T = 0.12$ GeV.

In Figure 3, the normalized pressure at the finite chemical potential was calculated for different temperatures. The pressure is shown as a monotonically increasing function of quark chemical potential for all given temperatures. Furthermore, the lower the temperature, the smaller the effective pressure as expected. In addition, there are intersection points between the solid and dashed lines in each group. This implies that the effective pressure of the magnetized system exceeds that in the absence of a magnetic field at high chemical potentials. This similar phenomenon occurs for three sets of q values in Figure 4. The positions of the intersection points vary slightly by changing the q value. Roughly speaking,

the magnetized pressure is larger than the nonmagnetized one in the chiral restoration. In the chiral broken phase at low chemical potential, one is in the opposite situation.

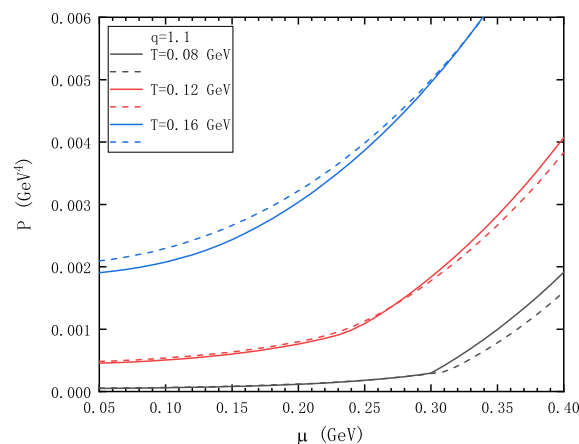


Figure 3. The behavior of the pressure for $q = 1.1$ as a function of chemical potential at different temperatures.

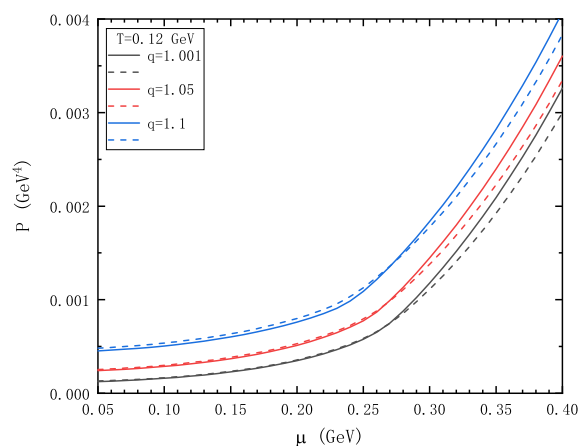


Figure 4. The behavior of the pressure for three sets of q value as a function of chemical potential at $T = 0.12$ GeV.

The free energy per baryon is defined according to the partition function, providing a good theoretical perspective for understanding thermodynamic stability. The lower free energy of quark matter corresponds to its more stable state. The free energy per baryon is shown at the temperature $T = 0.08, 0.12, 0.16$ GeV in Figure 5. With the increase of quark chemical potential, the free energy per baryon monotonically grows. On the contrary, the free energy per baryon is lower when the temperature rises. The zero magnetic field and strong magnetic field are marked by the dotted and solid lines, respectively. The two lines intersect at a moderate chemical potential value of up to 0.4 GeV. This indicates that the stability on both sides of the crossing point is determined by the presence or absence of the magnetic field. The inflection corresponds to the critical chemical potential at which the phase transition occurs. At high temperature $T = 0.16$ GeV, the system remains in the chiral restoration phase regardless of the magnitude of the chemical potential. Consequently, no inflection behavior is observed along the blue line. The magnetic field has weak influence on the free energy at low temperatures. At high temperatures, it is observed that the free energy without a magnetic field is lower than that of magnetized matter.

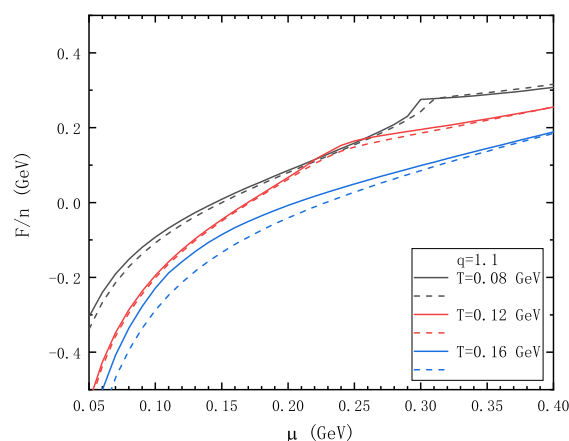


Figure 5. The variation of the free energy per baryon as a function of chemical potential at different temperatures.

Similarly, as depicted in Figure 6, we investigated the influence of the nonextensive parameter q on the free energy per baryon. For a given q value, the free energy exhibits monotonic growth with the chemical potential, and the larger the q value is, the chemical potential of the inflection point shifts towards a lower value. In our previous calculation, we reached the conclusion that the existence of a strong magnetic field could potentially reduce the stability of quark matter in the lower chemical potential region. However, the increase of the parameter q contributes to enhancing its stability. The intertwining of the three groups of curves in the figure can be attributed to the fact that, within the phase transition region, the free energy of quark matter with a higher q value is the greatest.

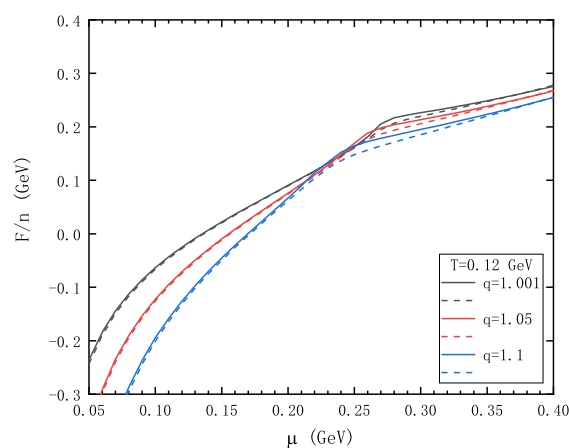


Figure 6. The variation of the free energy per baryon as a function of chemical potential for different nonextensive parameters.

Figure 7 explains the impact of different magnetic fields on the chiral phase transition and thermodynamic properties at finite temperature $T = 0.12$ GeV. In the left panel, the smooth change of the quark mass still exhibits the crossover in QCD community. The increase in the magnetic field leads to an increase in the pseudocritical chemical potential of the crossover, that is, the magnetic catalysis effect in the density direction. In the middle panel, the pressure function is slightly decreased by an increase of the magnetic field in the low chemical potential region. Inversely in the larger chemical potential region, the stronger magnetic field would result in larger pressure. Finally, the trend of free energy per baryon is displayed on the right panel. Similar to the behavior of pressure, the free energy

per baryon exhibits different magnetic field effects at lower and higher chemical potential. It can be concluded that an increased magnetic field is favorable to the stability of chiral broken matter. Conversely, when the density increases, a strong magnetic field would reduce the stability of the chiral restored matter. The inflections on the curve correspond to the pseudocritical chemical potential of the phase transition in the left panel.

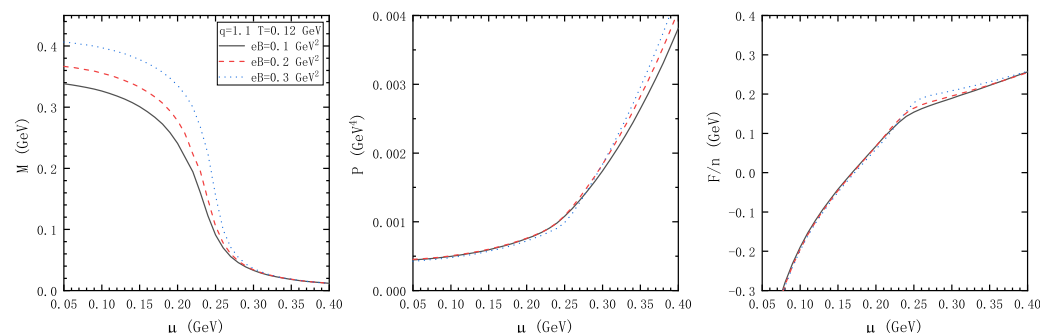


Figure 7. The thermodynamic quantities of nonextensive quark matter at finite temperatures are shown at different magnetic fields, from left to right as the dynamical mass, the normalized pressure and the free energy per baryon.

4. Summary

In this paper, we have investigated the chiral transition and the stability of quark matter in the nonextensive NJL model. Here, we focus on the influence of the finite chemical potential, nonextensive parameter q and magnetic fields on quark matter. At the fixed temperature, the chiral transition takes place at a critical chemical potential, which is decreased by an increase of the nonextensive parameter q and increased by an increase of the magnetic field. The stability is represented by the free energy per baryon. The increased temperature can result in lower free energy and therefore enhance the stability of the chiral broken matter. On the contrary, the chiral restored phase would have larger free energy at high temperature. Correspondingly, it is shown that the magnetic field also has different effects on the stability of quark matter. In particular, the stability of the chiral broken matter is enhanced by an increase of the magnetic field. Conversely, the chiral restored phase would have a larger free energy per baryon and result in a decrease in stability.

For heavy ion collision experiments, researchers investigate potential quark matter by generating extremely high temperature and high density conditions. The stability of matter directly influences the existence of new states of matter, such as quark gluon plasma, and affects the interpretation of experimental data, thereby verifying the theory of QCD phase diagrams, particularly in identifying their critical end points and phase boundaries. On the other hand, as stellar objects partially or entirely composed of strange quark matter (SQM), neutron stars may provide extreme environments that enable quark matter to exist stably [56]. According to the literature, if the strange quark matter is more stable than ordinary nuclear matter at high density, the conversion from two flavor to SQM occurs via a decay of the down quark to the strange quark ($u + d \rightarrow s + u$) [57]. The study of the stability of matter is directly related to characteristics of neutron stars, including the core structure, cooling mechanism and gravitational wave signals. Therefore, we hope that our conclusion will contribute to the investigation of nonequilibrium quark matter and serve as a bridge between experiments and astrophysical observations.

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