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Reduction of Spurious Signal Upconversion in Frequency Multipliers

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Abstract: Usually many applications of radar transceivers and heterodyne frequency synthesizers assume a spurious signal power level below −60 dBc. In the case of modern synthesizers using direct digital synthesis (DDS) systems, the number of emerging spurious signal frequencies is very large, and spectral purity within −60 dBc can only be obtained in the relatively narrow tuning band of the DDS unit. For the purposes of widening this useful frequency range, the frequency multiplying operation is applied commonly. Then, during the process of frequency multiplication of the baseband signal containing inband spurious signals, the effect of the upconversion of spurious signals occurs. The paper contains an analysis of the undesirable effects of the conversion of spurious signal frequencies accompanying the process of frequency multiplication. A method of reducing the level of upconverted spurious signals is proposed. The numerical calculations and measurement results are provided. For the case of a frequency multiplier with a multiplying factor equal to N, the power ratio between the desired output signal and upconverted spurious signal drops by an additional $1/N^2$. It has been found that the application of the presented method during the design process of the frequency multiplier allows this ratio to be improved by 6 dB.

Keywords: frequency multiplying; signal synthesis; frequency conversion

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1. Introduction

The main essence of radiolocation and sensor techniques can be reduced to the following steps: generation of the desired signal with appropriately selected parameters, sending it to the tested object (space) and inferring about the characteristics of the object by analyzing changes in the parameters of the received signal. Therefore the ability to generate a sufficiently clear probe signal is crucial. The main requirements relating to the level and stability of power as well as proper spectral purity. Spectral purity should be regarded as the absence of undesirable components in the signal spectrum (in transmission or reception band) and adequately low level of phase noise.

From the point of the use of generated signals in radiolocation and sensor techniques, the ideal situation is where one can generate a signal with high spectral purity (max −100 dBc/Hz at 1 kHz in the X band) with the possibility of relatively fast and quasicontinuous (i.e., with a step of Hz) tuning in the preferably wide band, e.g., equal to several GHz.

Currently, the most effective and perspective method seems to be the direct digital synthesis (DDS, or similarly, the so-called direct mapping method) using a high-quality clock signal. The current technological possibilities of frequency synthesis by DDS systems range from zero to 4.3 GHz and a tuning step may reach 0.1 Hz or below [1–6]. In a relatively narrow frequency range, the DDS system synthesizes the output signal without the presence of additional spurious signals, i.e., for a certain reference frequency, one can select a tuning band in which the output signal spectrum will be devoid of spurious frequencies (purity level of −60 dBc and below). Unfortunately, when all the available tuning band is used, many additional components appear in the output signal spectrum with frequencies that can be described by Relationship (1) and with relative powers, in extreme cases even reaching −30 dBc [7,8].

$$f_p = n \cdot f_{clk} \pm m \cdot f_{out} \quad (1)$$

where:

n, m—integers;

f_{clk}—clock frequency (reference) of DDS;

f_{out}—frequency of the output signal generated by DDS.

Thus, the relatively narrow frequency synthesis range with high spectral purity forces the use of multiplication and upconversion techniques for the DDS output frequency. This means that in order to obtain the desired tuning band of the final signal (e.g., in radar or heterodyne), the basic DDS tuning band should be duplicated the required number of times and upconverted to the target values. If further multiplication is impossible, the desired band can be assembled from sub-bands with the frequency upconversion method using properly selected heterodyne signals [8].

As a result of the described effects associated with the direct digital synthesis, as well as with upconversion using frequency mixers or multiplication, the spectrum of the resulting signal may consist of the frequencies of undesirable, spurious signals. Their presence can be tolerated until the relative power level of these signals exceeds the assumed value, typically −60 dB, determined mainly, in the case of radars, by the dynamic range of the receiver. If these signals appear in the usable band at the initial processing stage, their level may increase due to the phenomena accompanying further signal processing towards the target frequency range and power levels.

The paper concerns the analysis of parasitic signals upconversion during the frequency multiplication process. This process causes the main signal to parasitic signal power ratio to be decreased at the multiplier output concerning the ratio at the input. For example, for the frequency tripler, the main to parasitic signal ratio drops by about 9.5 dB. This means that the initial main to spurious signal ratio of the desired value, even greater than 60 dB, may be rapidly degraded after several stages of frequency multiplication.

The aim of the work is finding the possibility to reduce the parasitic signal upconversion.

In further sections of the paper, this phenomenon has been mathematically described and its origin has been indicated. The multiplier's transient characteristic has been modeled with the use of power series. A specific condition for a decrease of parasitic signal upconversion has been found. The analytical calculations and measurement results for a case of a tripler example have been conducted and presented.

2. Analysis of the Frequency Conversion Effect of Spurious Signals

In general, the nonlinear transient characteristic of a nonlinear circuit may be described with the use of the polynomial expression:

$$y = a_1 \cdot x + a_2 \cdot x^2 + a_3 \cdot x^3 + a_4 \cdot x^4 + a_5 \cdot x^5 \quad (2)$$

where x is an input signal and y denotes an output signal. In the real circuits the input and output signals are usually in the form of voltage or current.

For the most commonly used multipliers, doubler and tripler, it may be assumed that they are described by perfect even and odd characteristics, respectively, which may be limited to the most significant components:

$$y_2 = a_2 \cdot x^2 \quad (3)$$

$$y_3 = a_1 \cdot x + a_3 \cdot x^3 \quad (4)$$

In order to investigate the phenomena corresponding to the process of frequency multiplication with the presence of additional undesired spurious signals, the following approach has been applied.

The nonlinear circuit is described by a polynomial and the input of the circuit is excited with the sum of signals, useful and parasitic, with similar frequencies and different amplitudes.

Assuming the input signal as a sum of two sinusoidal components, corresponding to the main s_0 and spurious s_p signals:

$$x = s_{in} = s_0 + s_p = A_0 \cos(\omega_0 t) + A_p \cos(\omega_p t) \tag{5}$$

For the doubler (X2), this situation can be written with the use of the following relationships [9]:

$$s_{out}^{X2}(s_{in}) = a_2 \cdot s_{in}^2 = a_2 \cdot [A_0 \cos(\omega_0 t) + A_p \cos(\omega_p t)]^2 \tag{6}$$

Equation (6) gives the following results:

$$\begin{aligned} s_{out}^{X2} = \tfrac{1}{2} a_2 \{ & A_0^2 + A_p^2 + A_0^2 \cos(2\omega_0 t) + A_p^2 \cos(2\omega_p t) + \\ & 2A_0 A_p \cos[(\omega_0 + \omega_p)t] + 2A_0 A_p \cos[(\omega_p - \omega_0)t] \} \end{aligned} \tag{7}$$

Similarly, for the frequency tripler:

$$s_{out}^{X3} = a_1 \cdot \left[A_0 \cos(\omega_0 t) + A_p \cos(\omega_p t)\right] + a_3 \cdot \left[A_0 \cos(\omega_0 t) + A_p \cos(\omega_p t)\right]^3 \tag{8}$$

$$\begin{aligned} s_{out}^{X3} = {} & a_1 \cdot \left[A_0 \cos(\omega_0 t) + A_p \cos(\omega_p t)\right] + \frac{1}{4} a_3 \cdot \{ A_0^3 \cos(3\omega_0 t) + A_p^3 \cos(3\omega_p t) + \\ & 3A_0^2 A_p \cos\left[(2\omega_0 + \omega_p)t\right] + 3A_0^2 A_p \cos\left[(2\omega_0 + \omega_p)t\right] + 3A_p^2 A_o \cos\left[(2\omega_p + \omega_0)t\right] + \\ & 3A_p^2 A_0 \cos\left[(2\omega_p + \omega_o)t\right] + 3(A_0^3 + 2A_p^2 A_0) \cos(\omega_0 t) + 3(A_p^3 + 2A_0^2 A_p) \cos(\omega_p t) \} \end{aligned} \tag{9}$$

It can be seen that the nonlinear terms introduce the new frequency components at frequencies being the linear combination of the input signals frequencies.

For all the expressions above the variable ω is the angular frequency, which is equal to $2\pi f$, where f is the frequency.

For the case of the doubler, the useful signal is the signal with doubled frequency $2f_0$. At the same time, a product of frequency conversion with the sum frequency $(f_0 + f_p)$ is present. The interesting issue is the frequency relationship between the useful signal and the spurious upconverted signal (with summed frequency) and their amplitude ratio. The frequency offset Δf between frequency components $2f_0$, denoted as f(2, 0) and $(f_0 + f_p)$, denoted as f(1, 1) is equal to:

$$f(1, 1) - f(2, 0) = \Delta f \tag{10}$$

is the same as at the doubler input Δf_{IN}, i.e., between f_p and f_0.

$$\Delta f = f_p - f_0 = \Delta f_{IN} \tag{11}$$

For the following considerations, it is assumed that the frequency conversion product of the frequency equal to $mf_0 + nf_p$ is denoted as f(m, n).

This means that the parasitic signal f_p in the baseband, located in a vicinity of the main signal f_0 appears after the frequency multiplication as the upconverted new signal at the same distance (in frequency) from the useful output signal with multiplied frequency.

By introducing the ratio of the main signal power to the parasitic signal at the doubler input as the main signal to parasitic power ratio, $MSPR_{IN}$:

$$MSPR_{IN} = \left(\frac{A_0}{A_p}\right)^2 \tag{12}$$

it can be seen that the ratio of useful to parasitic signal at the doubler's output $MSPR^{X2}{}_{OUT}$ is given by:

$$MSPR^{X2}{}_{OUT} = \left(\frac{\frac{1}{2}a_2A_0^2}{\frac{1}{2}a_2 2A_0A_p}\right)^2 = \left(\frac{A_0}{2A_p}\right)^2 = \frac{1}{4}MSPR_{IN} \tag{13}$$

In the case of the frequency tripler, the useful output signal is $3f_0$ and the ratio $MSPR^{X3}{}_{OUT}$ can be presented as follows:

$$MSPR^{X3}{}_{OUT} = \left(\frac{\frac{1}{4}a_3A_0^3}{\frac{1}{4}a_3 3A_0^2A_p}\right)^2 = \left(\frac{A_0}{3A_p}\right)^2 = \frac{1}{9}MSPR_{IN} \tag{14}$$

The frequency offset between the output signal at tripled frequency and the upconverted parasitic signal at $2f_0 + f_p$ is equal to:

$$f(2,1) - f(3,0) = \Delta f_{IN} \tag{15}$$

Similar relationships appear in the case of higher-order multipliers.

By investigating Formulas (7)–(9), one may find the general relationship:

$$MSPR^{XN}{}_{OUT} = \frac{1}{N^2}MSPR_{IN} \tag{16}$$

$$f(N\ -\ 1,\ 1) - f(N,\ 0) = \Delta f_{IN} \tag{17}$$

In general case one may notice that for the input and spurious signal frequencies denoted as f_0 and f_p respectively, and multiplying factor equal to N the upconverted parasitic signal appears at $(N - 1)f_0 + f_p$. This situation is presented in Figure 1.

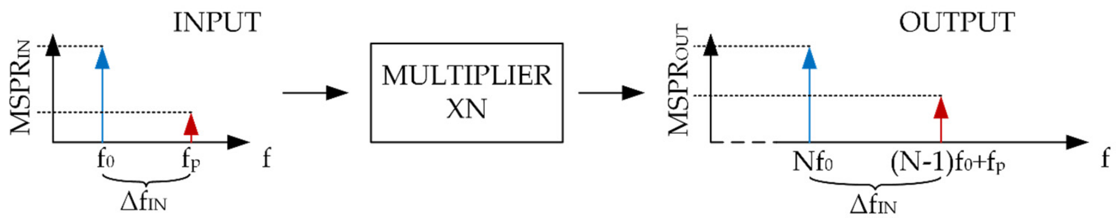

Figure 1. Frequency distribution of signals before and after the multiplication process.

The upconverted parasitic signal at $(N - 1)f_0 + f_p$ gains its amplitude after each frequency conversion taking the energy from the main signal thanks to the nonlinear phenomenon.

This situation repeats during every frequency multiplication process. It means that for the case of a cascade of frequency multipliers (obviously with proper driver amplifiers and output filters) the power ratio between the respective desired output signal and upconverted spurious signal drops at each stage by additional $1/N^2$ (Table 1). For example, for two cascaded frequency doublers, the overall main-to-spur signal ratio drops by 12 dB (2 × 6 dB). This means that the initial main to spurious signal power ratio of the desired value, even greater than 60 dB, may be rapidly degraded after several stages of frequency multiplication.

Table 1. Comparison of degradation of input main signal to parasitic power ratio depending on the multiplication factor.

Multiplication Factor N	Main Output Signal	Parasitic Upconverted Signal	Degradation of Input Main Signal to Parasitic Power Ratio (dB)
2	$2f_0$	$f_0 + f_p$	6.02
3	$3f_0$	$2f_0 + f_p$	9.54
4	$4f_0$	$3f_0 + f_p$	12.04
5	$5f_0$	$4f_0 + f_p$	13.98

3. Method for Reducing the Level of Parasitic Signals in Frequency Multipliers

It is important to note that the parasitic signal after frequency conversion is the same N-th order product as the multiplied main signal. This means that both useful and undesirable signals are the result of the same coefficient of nonlinearity. This fact is of crucial importance for the analysis of the possibilities of reducing the amplitude of the upconverted spurious signal. This means that neither a change in the input power level nor a decrease in the N-th nonlinearity coefficient will increase the MSPR ratio at the frequency multiplier output. To compare, in the case of amplifiers and methods reduction of intermodulation products, the main output signal has the first order and the unwanted intermodulation signals are third-order products [10]. Then, the reduction of intermodulation products relies on a reduction of the third-order nonlinearity with minimal change of linear coefficient.

Considering the frequency tripler as an example, described only by the coefficients a_1 and a_3, one may note that the useful signal with tripled frequency and the neighboring, upconverted parasitic signal have amplitudes proportional to the nonlinearity coefficient of the third-order (coefficient a_3). The ratio of the amplitudes of these signals is independent of the value of the nonlinearity coefficient and the amplitude of the input signal. For such a nonlinearity model (only a_1 and a_3), it is impossible to have any influence on the phenomenon of degradation of the ratio of the amplitudes of the desired and spurious signals during the frequency multiplication process.

Therefore, the possibility of compensating the amplitude of the upconverted spurious signal with a minimal effect on the amplitude of the useful output signal is wanted. Assuming further the frequency tripler as the subject of considerations means the necessity of generation of additional signal components at $3f_0$ and $2f_0 + f_p$ frequencies, with amplitude reduction desirable for the $2f_0 + f_p$ parasitic signal. This situation may occur only by the introduction of a higher order of nonlinearity in the model. In order to illustrate the idea of the technique of reducing the level of spurious signals converted during multiplication, a new transient characteristic of tripler is assumed. It is extended up to the fifth-order of nonlinearity.

$$s_{out}^{X3}(s_{in}) = a_1 \cdot s_{in} + a_3 \cdot s_{in}^3 + a_5 \cdot s_{in}^5 \tag{18}$$

The tripler described by Equation (18) will produce also a frequency component being the fifth harmonic of the input frequency, but the third harmonic is still analyzed here.

When this characteristic (Equation (18)) is stimulated by the sum of sinusoidal signals (5), the relationships of the amplitudes of signals at $3f_0$ and $2f_0 + f_p$ are expressed as:

$$A(3,\ 0) = \frac{1}{4}a_3A_0^3 + \frac{5}{16}a_5A_0^5 + \frac{5}{4}a_5A_0^3A_p^2 \tag{19}$$

$$A(2,\ 1) = \frac{3}{4}a_3A_0^2A_p + \frac{5}{4}a_5A_0^4A_p + \frac{30}{16}a_5A_0^2A_p^3 \tag{20}$$

In addition, as the result of nonlinear frequency conversion process a new parasitic signal of fifth-order appears at the frequency equal to $4f_0 - f_p$ and the amplitude described by the relationship:

$$A(4,\ -1) = \frac{5}{16}a_5A_0^4A_p \tag{21}$$

It is located on the frequency axis symmetrically to $2f_0 + f_p$ with respect to the multiplied main signal at $3f_0$. This situation in general for N order multiplier is shown in Figure 2. The frequency offset between the main output signal at multiplied frequency Nf_0 and parasitic signal at $(N - 1)f_0 + f_p$ is equal to the offset between Nf_0 and parasitic signal at $(N + 1)f_0 - f_p$.

One may note that Formula (2) for the amplitude of the parasitic signal at frequency f(2, 1) contains the terms dependent on fifth-order nonlinearity coefficient a_5 and amplitudes A_0 and A_p. Thereby, there is a theoretical possibility to reduce the amplitude A(2, 1) in case the coefficients a_3 and a_5 have the opposite signs and for a specific value of amplitude A_0 and given A_0/A_p ratio at the input.

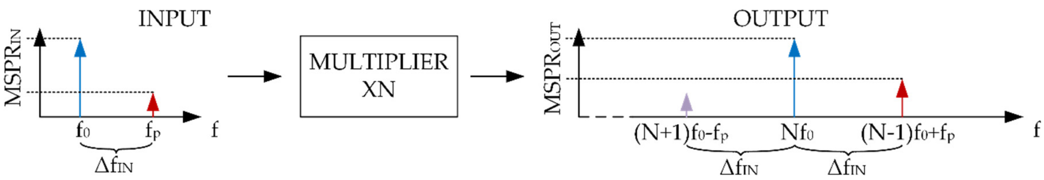

Figure 2. Frequency distribution of signals before and after the multiplication process for higher-order analysis.

On the other hand, in the same case, the increase of A_0 causes undesired reduction of A(3, 0) and an increase of the new parasitic signal amplitude A(4, −1).

For determined values of nonlinear characteristics coefficients, there is a certain input signal power value for which the amplitudes of the parasitic signals A(2, 1) and A(4, −1) become equal to each other, and then the $MSPR^{X3}{}_{OUT}$ degrades by a value smaller than the theoretical $20\log_{10}3$ (i.e., 9.54 dB). The amplitude of the spurious signal A(2, 1) decreases due to compensation resulting from the appearance of components introduced by a higher order of nonlinearity, here the fifth-order a_5.

Because the spurious signal of frequency f(2, 1) may be decreased at the cost of a simultaneous increase of the second spurious signal at f(4, −1), the search for a solution is based on the analysis of the condition for equality of their power levels (i.e., expressed as squared amplitudes):

$$[A(2,\ 1)]^2 = [A(4,\ -1)]^2 \tag{22}$$

or, in general, for different multipliers with multiplication factor N:

$$[A(N\ -\ 1,\ 1)]^2 = [A(N+1,\ -1)]^2 \tag{23}$$

and finding the optimal value of the input signal power for a determined nonlinear characteristic of the multiplier, i.e., given coefficients a_1, a_3, a_5.

The detailed analysis for the tripler starts from Equation (22) with the substitution of adequate products amplitudes expanded in Equations (20) and (21). Then, the condition for equal parasitic signals powers takes the form of Equation (24):

$$\left(\frac{3}{4}a_3A_0^2A_p + \frac{5}{4}a_5A_0^4A_p + \frac{30}{16}a_5A_0^2A_p^3\right)^2 = \left(\frac{5}{16}a_5A_0^4A_p\right)^2 \tag{24}$$

Further solving gives four possible results for A_0, where finally the one is useful:

$$A_0 = 0.4\cdot\sqrt{MSPR_{IN}}\cdot\frac{\sqrt{-15\cdot a_3\cdot a_5\cdot(5\cdot MSPR_{IN}+6)}}{a_5(5\cdot MSPR_{IN}+6)} \tag{25}$$

For the adequate higher values of $MSPR_{IN}$ the expression $5\cdot MSPR_{IN}$ + 6 may be treated as an approximate value of $5\cdot MSPR_{IN}$ and further simplification of Equation (26) may be derived. Assuming that $MSPR_{IN}$ is higher than about 30 dB the approximate formula may be expressed as:

$$A_0 = \frac{2\cdot\sqrt{-75\cdot a_3\cdot a_5}}{25\cdot a_5} \tag{26}$$

This gives the detailed condition for the relationship between the tripler nonlinear coefficients and the input signal amplitude.

Similar considerations may be done for another type of multiplier, for example, the doubler.

4. Results

The proposed method of parasitic signal reduction has been verified during the theoretical calculations and measurements of a diode-based model.

4.1. Calculation of The Doubler and Tripler Cases

Designing a frequency tripler is often accomplished by the use of Schottky diodes. In order to achieve odd characteristics of output versus input signal, a pair of antiparallel Schottky diodes is used. A single diode, i.e., its junction, may be described by the known exponential expression [11]:

$$I(U) = I_S \cdot (e^{\alpha \cdot U} - 1) \tag{27}$$

A pair of antiparallel diodes exhibit a characteristic in the form of the hyperbolic sine.

$$I(U) = I_S \cdot (e^{\alpha \cdot U} - e^{-\alpha \cdot U}) = 2 \cdot I_S \cdot \sin h(\alpha \cdot U) \tag{28}$$

Assuming that the tripler is a microwave two-port network, the diode pair may be connected to the microwave path in series (cascaded way between input and output ports) or in parallel, when it shunts the microwave path between input and output ports [12]. For purposes of further explanation, it is omitted that the diodes are loaded by 50 Ohm system impedance or other transformed values depending on the given circuit design. Then, in general, it is possible to describe the overall tripler transient characteristic using two models:

Aturating-shape function (like hyperbolic tangent or arcus tangent);

Nonsaturating-shape function in the form of hyperbolic sine.

Both of which are odd functions concerning their input arguments.

Taking into consideration the hyperbolic sine and expanding it into power series at zero, one may obtain:

$$\sin h(x) = x + \frac{x^3}{6} + \frac{x^5}{120} + \frac{x^7}{5040} + \ldots \tag{29}$$

This expansion shows that all the polynomial coefficients have positive values. It means that the hyperbolic sine, or in general nonsaturating function, does not fulfill the requirements necessary to obtain the equalization of the parasitic signal.

For the case of hyperbolic tangent the power series expansion gives:

$$\tan h(x) = x - \frac{x^3}{3} + \frac{2x^5}{15} - \frac{17x^7}{315} + \ldots \tag{30}$$

Here, the polynomial coefficients have positive and negative values alternately. Therefore, it is possible to calculate the result of Equation (22) and to solve the condition for parasitic signals equalization (i.e., the optimal value of the input signal amplitude A_0).

The calculated value of the amplitude $A_0 = 1.0954$ of the main input signal at f_0 is used to calculate the amplitudes A(3, 0), A(2, 1) and A(4, −1). It allows the $MSPR^{X3}{}_{OUT}$ to be found. The value of $MSPR^{X3}{}_{OUT}$ for the hyperbolic tangent is equal to (31) when the reduction method is used.

$$MSPR^{X3}{}_{OUT} = \frac{4 \cdot (MSPR_{IN} - 3)^2}{9 \cdot MSPR_{IN}} \approx \frac{4}{9} \tag{31}$$

Equation (31) clearly shows that for the high value of the input MSPR, the output MSPR is multiplied by factor 4 (6 dB). The example of calculated value for $MSPR_{IN}$ = 60 dB is $MSPR^{X3}{}_{OUT}$ = 56.478 dB (50.4576 dB without the reduction method).

Using the same logical approach one may present a doubler transient characteristic as the hyperbolic cosine function. The power series expansion shows only positive polynomial coefficients. Though, a function without saturation parts for higher input variable absolute values, does not fulfill requirements necessary to obtain the equalization of the parasitic signal.

The example of calculation results for the tripler described by the hyperbolic tangent transient function (Equation (30)) is shown in Figure 3. The transient characteristic is assumed to be in voltage notation, i.e., input signal amplitude and output signal amplitude, are expressed in Volts.

The corresponding input and adequate output signals powers are calculated with the assumption of 50 Ohm source and load impedance. The output signals at f(3, 0), f(2, 1) and f(4, −1) have been calculated with the use of Equations (19)–(21). The nonlinearity coefficients up to the fifth-order have been used for calculations. The input MSPR equal to 30 dB is used in this example.

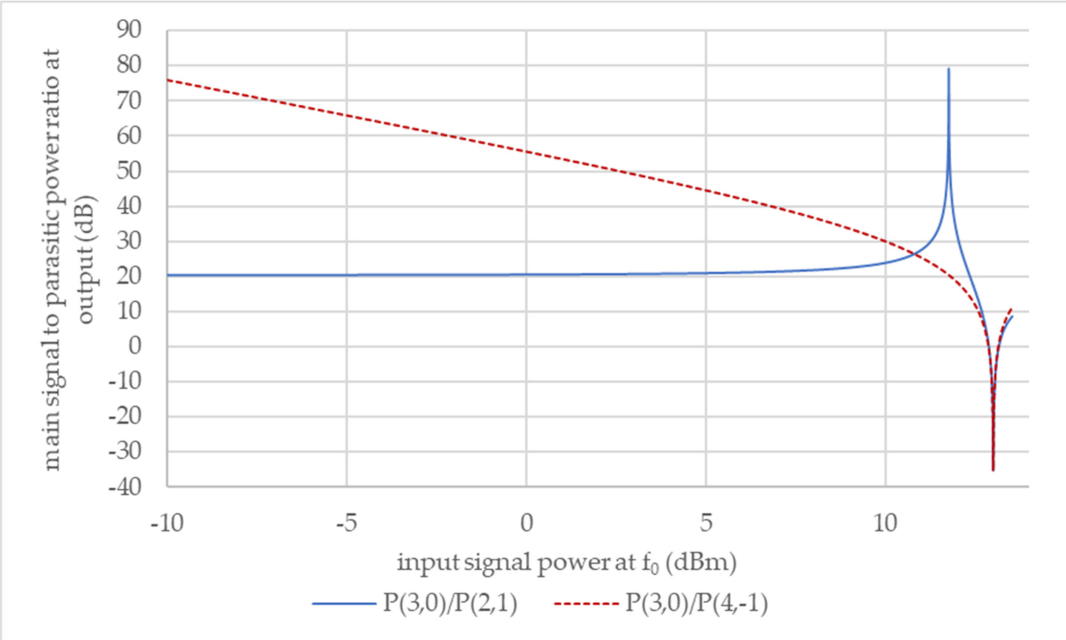

Figure 3. Example of calculation results for main signal to parasitic power ratio $MSPR^{X3}{}_{OUT}$ versus input power level at f_0 for input MSPR = 30 dB: MSPR for f(2, 1) component—blue, and for f(4, –1) component—dashed red.

The calculation results show that the initial $MSPR^{X3}{}_{OUT}$ for lower values of input power is about 20 dB, which corresponds with the theoretical degradation of the input MSPR = 30 dB by the value of 9.54 dB ($20\log_{10}3$). For higher input power levels, the output MSPR increases and reaches its optimal value for both parasitic components f(N − 1, 1) and f(N + 1, −1), equal to 26.452 dB for 10.786 dBm (which corresponds to input main signal amplitude A_0 = 1.0948 V). It can be seen that there is an input power range where the output $MSPR^{X3}$ is almost constant. It means that for this input power range there are no changes or improvement of the output main signal to parasitic power ratio. The power of the useful signal at $3f_0$ and the parasitic signal at $2f_0 + f_p$ arises with the increase of input power level but their power ratio is constant. The power of the parasitic component at $4f_0$-f_p arises for increased input power level, therefore, the $MSPR^{X3}{}_{OUT}$ for this component decreases. Both parasitic components reach the same power values for a specific input power value. For this point, the input MSPR equal to 30 dB is degraded to 26.452 dB, instead of the value of 20.452 dB that appears for lower input powers. Therefore, there is a gain equal to 6 dB compared to the low-power case.

4.2. Calculation of the General Case

The theoretical values of the power ratio of multiplied N times signal to parasitic signals after the upconversion $MSPR^{XN}{}_{OUT}$, in the situation of using the reduction effect, can be found by calculating the amplitudes of the respective frequency components in the case of two-tone stimulation of the nonlinear system described by a polynomial characteristic. In this case, there is a calculation inaccuracy associated with the adoption of the finite order of the polynomial.

The aim is to analyze the odd and even functions to have coherent results for deducing the possibility of spurious signals reduction in multiplier general models of order: two, three, four and five. Moreover, these functions are supposed to be of one kind and should have saturation regions.

To accomplish this, one may use a selected fragment of the sine function to model the odd characteristic, and the cosine one to model the even characteristic (even order multiplier, e.g., a doubler). As a result, the two-tone harmonic stimulation of the system described by the sine or cosine function allows expressing the amplitudes of the resulting harmonic components in the form of the Bessel functions of the first kind. This kind of analytic approach is used in the case of electrooptic modulators described by the cosine-like transfer function [13–16]. Then, the amplitudes of useful signals with multiplied frequency and parasitic signals may be expressed as in Table 2.

Table 2. Amplitudes of output useful harmonic signal and adjacent parasitic signal for common frequency multipliers.

Multiplier	Useful Output Signal Nf_0	Parasitic Signals f(N − 1, 1) f(N + 1, −1))	A(N,0)	A(N − 1, 1)	A(N + 1, −1)
Doubler, X2	$2f_0$	$f_0 + f_p$ $3f_0 - f_p$	$2J_2(A_0)J_0(A_p)$	$2J_1(A_0)J_1(A_p)$	$2J_3(A_0)J_1(A_p)$
Tripler, X3	$3f_0$	$2f_0 + f_p$ $4f_0 - f_p$	$2J_3(A_0)J_0(A_p)$	$2J_2(A_0)J_1(A_p)$	$2J_4(A_0)J_1(A_p)$
Quadrupler, X4	$4f_0$	$3f_0 + f_p$ $5f_0 - f_p$	$2J_4(A_0)J_0(A_p)$	$2J_3(A_0)J_1(A_p)$	$2J_5(A_0)J_1(A_p)$
Quintupler, X5	$5f_0$	$4f_0 + f_p$ $6f_0 - f_p$	$2J_5(A_0)J_0(A_p)$	$2J_4(A_0)J_1(A_p)$	$2J_6(A_0)J_1(A_p)$

The further analysis follows Formulas (22) and (23), i.e., verifying the condition, calculation of optimal A_0 value, and substituting it to formulas for spurious signals amplitudes.

It has been noticed that the results of the above-mentioned analysis are consistent. In all cases of multipliers, i.e., for multiplying factors from two to five, the gain due to the application of the reduction method is the same and equal to 6 dB in power notation (factor of two for the amplitude).

Therefore the general case may be expressed with the use of the following relationship:

$$\mathrm{MSPR}^{XN}{}_{OUT} = \mathrm{MSPR}_{IN} \cdot \left(\frac{2}{N}\right)^2 \tag{32}$$

The aggregated results are presented in Table 3:

Table 3. Comparison of the analysis results for common frequency multipliers: main signal to parasitic power ratio at the multiplier output without and with the reduction effect.

Multiplier	Useful Output Signal Nf_0	Main Signal to Parasitic Power Ratio at the Output $\frac{\mathrm{MSPR}^{XN}{}_{OUT}}{\mathrm{MSPR}_{IN}}$ Degradation:	Main Signal to Parasitic Power Ratio at the Output $\frac{\mathrm{MSPR}^{XN}{}_{OUT}}{\mathrm{MSPR}_{IN}}$ Degradation with Reduction Effect:	Gain from the Use of the Reduction Method
Doubler, X2	$2f_0$	1/4 (−6.02 dB)	1 (0 dB)	4 (6 dB)
Tripler, X3	$3f_0$	1/9 (−9.54 dB)	4/9 (−3.54 dB)	4 (6 dB)
Quadrupler, X4	$4f_0$	1/16 (−12.04 dB)	4/16 (−6.04 dB)	4 (6 dB)
Quintupler, X5	$5f_0$	1/25 (−13.98 dB)	4/25 (−7.98 dB)	4 (6 dB)

To sum up, the results presented above prove that it is possible to design the multiplier system which allows the signals power ratio degradation to be reduced by 6 dB. This is due to the application of the reduction method presented here. The interpretation of the results above is as follows: for a multiplier with the multiplication factor N the inherent output MSPR degradation is equal to $20\log_{10}(N)$. It means that the output MSPR is equal to input MSPR decreased by $20\log_{10}(N)$. After the application of the proposed method, the output of MSPR degradation is equal to $20\log_{10}(N)$ −6dB. This method works theoretically for any value of input MSPR, however, in real frequency synthesizer circuits for radar applications, the input MSPR, i.e., at the output of the initial signal source before any frequency conversion, is higher than 60 dBc.

4.3. Measurements of a Tripler

In order to verify the reduction of the upconverted spurious signal level, which results from the analytical calculations, an experiment for a simple tripler circuit has been conducted. The nonlinear circuit of a tripler consists of a pair of antiparallel Schottky diodes shunt-connected to a 50 Ohm microstrip line (made on FR4 laminate).

The experiments have been carried out in the arrangement shown in Figure 4 whose exact diagram is presented in Figure 5. Two generators are used to generate the useful (f_0 = 1.5 GHz) and spurious (f_p = 1.51 GHz) signals. An amplifier is added to the output of the useful signal generator to provide the desired $MSPR_{IN}$ at the tripler's input port. The signals from both generators are combined with the use of a power combiner and then filtered by a low-pass filter to eliminate any harmonic products that may arise in the generators or the amplifier. The first step is to carry out calibration to measure the power of signals reaching the tripler input port. During this measurement, the power of the spurious signal has been also adjusted to ensure constant $MSPR_{IN}$ at a value of 30 dB. This measurement has been performed by connecting the low-pass filter output directly to the attenuator at the input of the spectrum analyzer (calibration path in Figure 5). The attenuator in this system is necessary to protect the analyzer's input against too high input signal power. This measurement allows the selection of the power range in which the following measurements can be made on the available equipment. Moreover, it has confirmed that there are no additional undesirable harmonics in the tested power and frequency range that could adversely affect the measurement results of the multiplier.

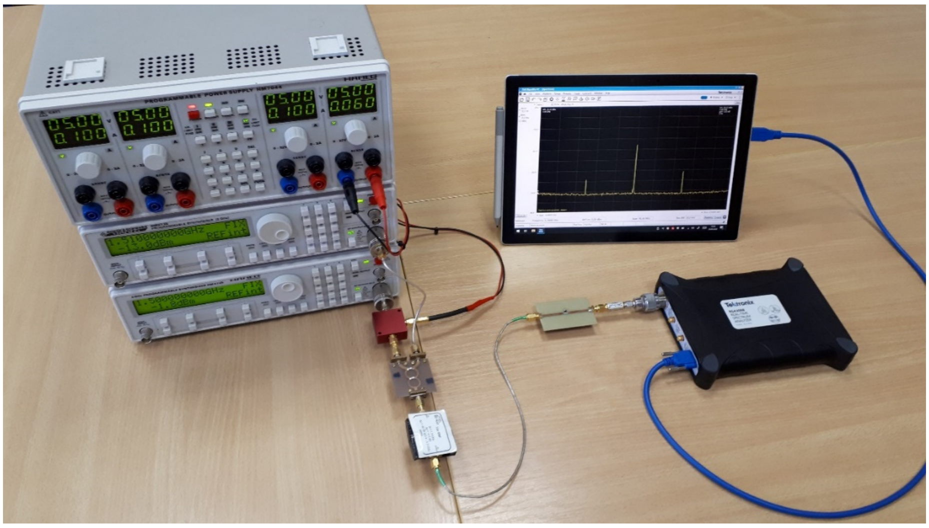

Figure 4. View of measuring bench.

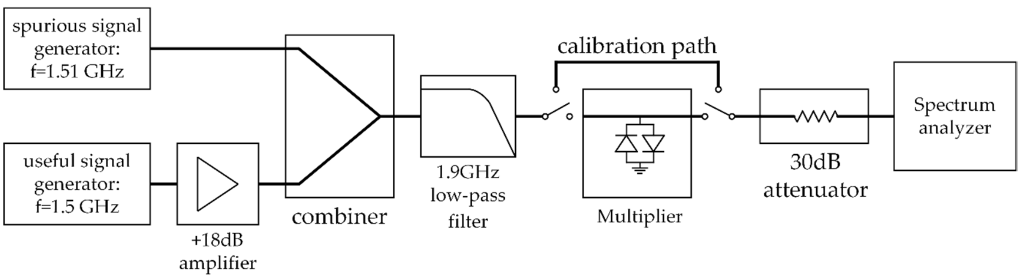

Figure 5. Diagram of the measuring system.

The next measurement has been performed with the tripler circuit, which contains the Schottky diode antiparallel pair NXP PMBD353. The diode pair is connected between a 50 Ω microstrip line and ground. The whole circuit forms a symmetrical and reciprocal two-port network. The tripler circuit is not optimized and has no special matching circuits, filters nor idler resonators. The structure is maximally simple to verify the behavior of the nonlinear circuit compared to theoretical calculation results, avoiding the influence of additional circuits that have not been taken into account during calculations. The measurements have been carried out for the useful signal f_0 power range from 5 dBm to 18 dBm (respectively, spurious signal power ranges from −25 dBm to −12 dBm). The spectrum analyzer used for the measurement is Tektronix, type RSA306B. All the measurements have been done with RBW = 10 kHz. For $P(f_0)$ = 5 dBm, the power level of the component f(4, −1) is less or equal to the displayed noise level, measured with the use of RSA306B with 30 dB attenuator at the input port and the RBW equal to 10kHz. The measured $MSPR^{X3}{}_{OUT}$ value is 20.36 dB, which coincides with theoretical calculations and presents the case without $MSPR_{OUT}$ compensation. Figure 6 shows an example for $P(f_0)$ = 7 dBm, where the appearance of the component f(4, −1) can be observed. In this case, the measured $MSPR^{X3}{}_{OUT}$ is equal to 22.18 dB, which is a little improved compared to the case without any compensation.

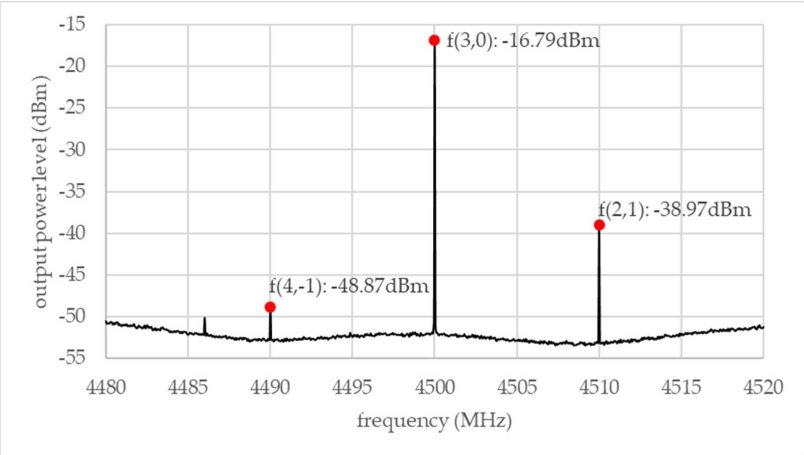

Figure 6. Signals at the tripler output for input power level $P(f_0)$ = 7 dBm.

Figure 7 presents the case with the largest $MSPR^{X3}{}_{OUT}$ ratio obtained at $P(f_0)$ = 16 dBm and $P(f_p)$ = −14 dBm. A significant increase in the component f(4, −1) power can be observed. In comparison to the case presented in Figure 6, the power value of f(4, 1) is increased by 11.6 dB. The main frequency component f(3, 0) is increased by 8.54 dB, which is similar to the increment of the input f_0 power. On the other hand, the power level of the f(2, 1) increased only by 4.79 dB. In this case, $MSPR^{X3}{}_{OUT}$ reaches the value of 25.93 dB, which is 5.57 dB better than in the case without compensation. It is also 4.08 dB worse than input MSPR equal to 30 dB.

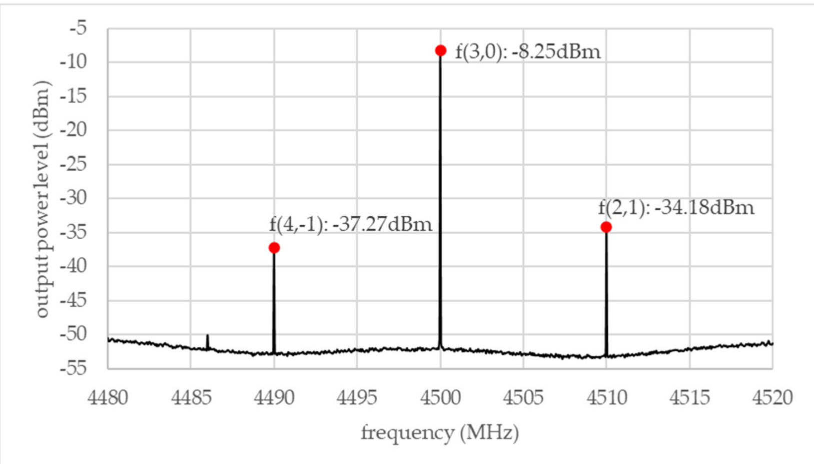

Figure 7. Signals at the tripler output for input power level $P(f_0)$ = 16 dBm.

Figure 8 shows changes in the levels of individual components as a function of the input signal power at f_0. It can be seen that from a certain input power level (around $P(f_0) = 14$ dBm to the end of the measuring range $P(f_0) = 18$ dBm) the components f(2, 1) and f(4, −1) increase evenly. A consistent level of $MSPR^{X3}{}_{OUT}$ measured between levels P(f(3, 0)) and P(f(2, 1)) is also observed for this range, which is shown in Figure 9.

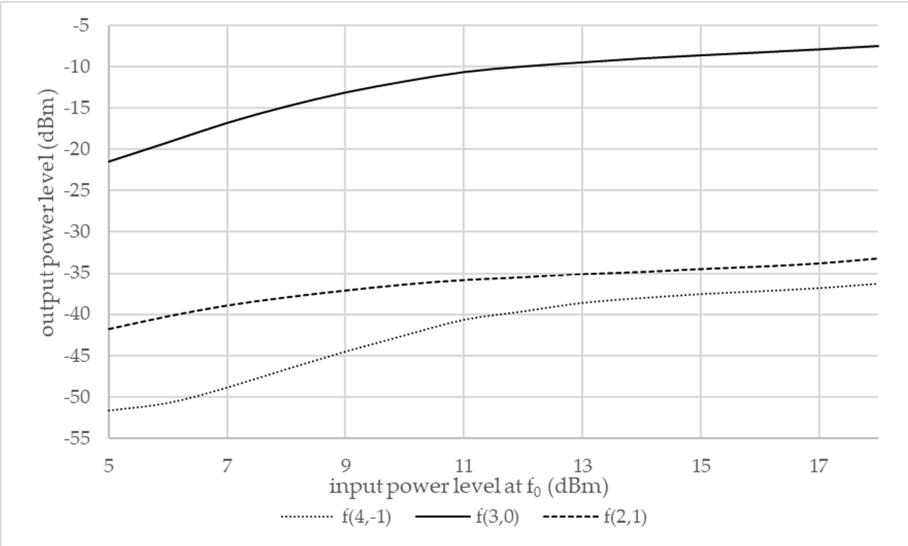

Figure 8. Levels of selected components at the tripler output as a function of input signal power level.

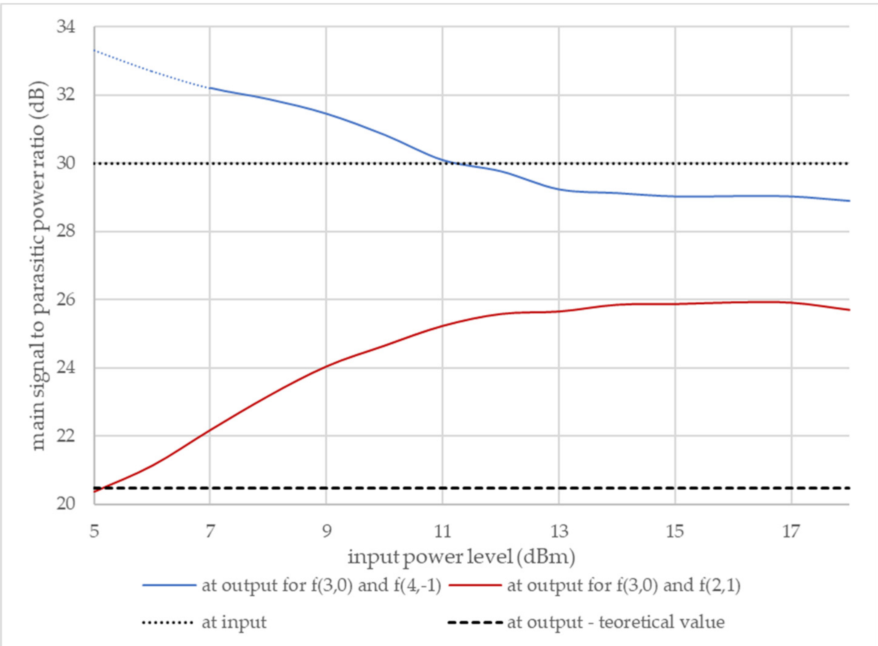

Figure 9. Comparison of MSPR output changes as a function of input signal power level.

Figure 9 shows how $MSPR^{X3}{}_{OUT}$ changes in the whole range of f_0 input signal powers. The $MSPR^{X3}{}_{OUT}$ is defined as the ratio between the multiplication result f(3, 0) and the f(2, 1) parasitic signal (red line), and f(4, −1) parasitic signal (blue line). The dotted fragment of the blue line represents theoretical values where the power level of f(4, −1) has been below the noise level so that the actual value of $MSPR^{X3}{}_{OUT}$ could not be read.

For comparison, Figure 9 also shows the $MSPR_{IN}$ level equal to 30 dB (black dotted line) and $MSPR^{X3}{}_{OUT}$ without compensation (black dashed line). It can be seen that the measured $MSPR^{X3}{}_{OUT}$

for f(3, 0) and f(2, 1) and $MSPR^{X3}{}_{OUT}$ for f(3, 0) and f(4, –1) for the input power level greater than about $P(f_0)$ = 14 dBm changes in a parallel manner with each other. This means that an improvement of the measured $MSPR^{X3}{}_{OUT}$ compared to the theoretical $MSPR^{X3}{}_{OUT}$ can be obtained in a certain range of input power levels.

5. Discussion

Frequency synthesis systems have to meet high requirements for the purity of the synthesized signals spectrum. For radar applications and heterodyne frequency synthesis, there is a need for the relative power level of spurious and harmonic signals in the spectrum depending on the desired dynamic range value of the receiver channel. Typically, in many applications, a value of ≤−60 dBc is assumed. In produced systems, this requirement is difficult to meet. In the case of the older generation of matrix synthesis systems, which use many multiplications and summations of basic signals, the problem of the presence in the output spectrum of many frequency components being a linear combination of the frequency of signals used for synthesis arises. In the case of modern synthesizers using DDS units, the number of emerging spurious signal frequencies is very big, and spectral purity at the level of −60 dBc can be obtained only in the relatively narrow tuning band of the DDS system. In practice, one should take into account the possibility of the appearance of spurious signals with a level of −50 to −30 dBc. Despite these drawbacks, DDS synthesizers are characterized by short switching/tuning times, the frequency resolution of fractions of a Hertz, and very high tuning linearity.

Multiplication of the useful signal frequency when an additional unwanted signal exists in the multiplier input band, causes that the multiplied output spectrum consists of an upconverted new additional unwanted frequency components. The ratio of useful signal to spurious signal power drops $1/N^2$ times at the output of the multiplier, where N is the multiplication factor. The process of converting parasitic signals resembles a phase noise upconversion; in both cases. The relative power level increases by $20\log_{10}N$ after multiplication. For example, for two cascaded frequency doublers, the overall main to spurious signal power ratio drops by 12 dB (2 × 6 dB). This means that the initial main to spurious signal power ratio of the desired value, even greater than 60 dB, may be rapidly degraded after several stages of frequency multiplication.

This phenomenon may be especially surprising for an engineer. Due to the fact that in real practice during direct measurement in baseband a spurious signal seems to be not present, due to being hidden in displayed noise floor for wider RBW/VBW settings in a spectrum analyzer.

The application of the reduction method presented here relies on finding the optimal value of the input signal power for a determined nonlinear characteristic of the multiplier, i.e., given nonlinearity coefficients $a_1, \ldots, a_n$. The second approach is shaping the multiplier characteristic for a given value of the input power during the design process of the circuit. It may be accomplished by a change of diode polarization voltage, adding more devices (diodes), or transforming the load impedance seen by diodes. To apply the method presented here, the general rule must be achieved. The multiplier transient characteristic has to contain a saturation region. The power series expansion of this characteristic should give the opposite signs of subsequent coefficients.

The use of the spurious component reduction effect presented in the paper allows the power ratio of useful signal to spurious signal at the multiplier output to be improved by max. 6 dB.

In the case of signal synthesis systems in radiolocation applications, where it is critical to maintain a high power ratio between the main and the parasitic signals, the implementation of this reduction method may be extremely convenient. In the case when a spurious signal in baseband is unavoided the further frequency multiplication causes its presence in the upconverted form up to the final band. Moreover, its amplitude increases and the output main signal to parasitic signal power ratio decreases. In that case, the method proposed in the paper seems to be the "last-chance" solution to gain the 6 dB improvement, which may be critical in certain applications.

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