

Article

Hybrid-Input Deep Learning Framework for Electromagnetic Imaging of Anisotropic Objects

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Abstract

This paper presents a hybrid-input deep learning framework for electromagnetic imaging of anisotropic objects in a two-dimensional environment. The proposed method combines transverse magnetic (TM) and transverse electric (TE) illuminations to reconstruct the directional permittivity components. Backpropagation (BP) and the direct sampling method (DSM) are first applied to the measured scattered fields to generate complementary preliminary images. These images are then superimposed to form a physically informed hybrid input for U-Net reconstruction. Numerical results show that the proposed hybrid-input strategy improves reconstruction accuracy, structural consistency, and noise robustness compared with single-input baselines using BP or DSM alone. The results demonstrate the potential of the proposed method for anisotropic electromagnetic imaging.

Keywords: anisotropic objects; backpropagation; deep learning; direct sampling method; electromagnetic imaging; hybrid input; U-Net

1. Introduction

Electromagnetic inverse scattering problems (ISPs) aim to reconstruct the internal dielectric profiles of unknown targets by analyzing the scattered fields measured from external illuminations. These techniques are widely applied in biomedical imaging, non-destructive evaluation, geophysical exploration, and smart sensing. However, due to the intrinsic nonlinearity and ill-posed nature of ISPs, achieving high-resolution and real-time imaging remains a critical challenge, particularly when dealing with high-contrast or anisotropic objects.

Traditional iterative solvers, such as the Born iterative method and Contrast Source Inversion (CSI), have been extensively studied and are well-regarded for their strong physical interpretability and theoretical foundations. Nonetheless, these conventional solvers often suffer from high computational complexity and sensitivity to measuring noise, particularly when solving nonlinear problems for large-scale or high-contrast targets. Significant efforts have been made to improve their computational efficiency. Despite these improvements, traditional optimization-based algorithms still struggle with ill-conditioning, slow convergence, and susceptibility to local minima, especially when high-permittivity scatterers are involved. These challenges motivate the search for alternative approaches capable of achieving higher robustness and computational efficiency.

Accordingly, existing electromagnetic imaging and inverse scattering solvers can be broadly categorized into two major classes: traditional physics-based algorithms [1–13] and



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deep-learning-based approaches [14–30]. Traditional algorithms [1–13] are primarily derived from electromagnetic theory and mathematical optimization, including iterative schemes, sampling-based methods, and hybrid numerical solvers. These methods emphasize physical consistency and interpretability and have demonstrated reliable performance in accurately characterizing scattering mechanisms, particularly in low-noise environments or when prior information about the target is available. However, their practical applicability is often constrained by high computational cost, slow convergence, and degraded stability when addressing large-scale, high-contrast, or strongly nonlinear inverse scattering problems, which limits their scalability in practical imaging systems.

In contrast, deep-learning-based approaches [14–30] have emerged as a powerful alternative by leveraging data-driven modeling and nonlinear approximation capabilities. By learning direct mappings between scattered fields and material properties, these methods can effectively handle high-dimensional data and significantly accelerate the reconstruction process. Recent studies further integrate physical prior or intermediate imaging results into neural network architecture, achieving improved robustness, generalization, and real-time performance in complex electromagnetic imaging scenarios. Nevertheless, the performance and generalization of purely data-driven models often depend on the availability of representative training data and appropriate incorporation of physical constraints.

Recent years have seen continued development of traditional inverse scattering algorithms aimed at enhancing performance in complex scattering environments. In 2021, Menshov proposed a modified CSI method for two-dimensional microwave tomography by introducing a focusing medium in the form of a Veselago lens. By properly positioning the sources and sensors at the focal points of the lens, the proposed configuration alleviated rank deficiency in the inverse problem formulation and improved convergence behavior. Numerical results demonstrate enhanced reconstruction accuracy and faster convergence compared with conventional CSI methods [1]. In 2022, Narendra introduced a combined inverse source and scattering technique designed to reconstruct lossless dielectric profiles that achieve desired far-field field shaping. The method first solves an inverse source problem to retrieve equivalent aperture fields, which are then used in a secondary inversion to obtain the underlying permittivity distribution. This approach enables tailored electromagnetic responses, though it requires careful regularization to ensure physical realizability and stability [5]. In 2023, Sun proposed a fast Cross-Correlated Contrast Source Inversion (CC-CSI) algorithm to improve the computational efficiency of nonlinear inverse scattering solvers. By assuming a homogeneous background medium, the most time-consuming linear system computations were accelerated using Fast Fourier Transforms (FFT) and Inverse FFTs (IFFT)-based implementations for TM, TE, and three-dimensional cases. Experimental and numerical results showed that the proposed approach achieved at least an order-of-magnitude speedup over LU decomposition and biconjugate gradient methods while maintaining acceptable memory usage [7]. In 2024, Baharian presented a state-space model combined with nonuniform frequency sampling to solve inverse scattering problems in coupled nonuniform transmission lines. The proposed formulation, based on Peano-Baker series and Chebyshev expansion, bypasses nonlinear integral equations and enables efficient inversion of frequency-domain scattering parameters. The method exhibited stability and accuracy in 1D settings, though its scalability to higher-dimensional systems remains a topic for future work [9]. In 2025, Batista released EISPY2D, an open-source Python library tailored for the simulation, evaluation, and benchmarking of two-dimensional electromagnetic inverse scattering algorithms. The platform features modular algorithmic components, standardized datasets, and performance metrics to facilitate reproducible research and fair comparison. While limited to 2D configurations, EISPY2D offers a unified environment for developing and testing classical and learning-based inversion methods [11].

Meanwhile, deep learning has emerged as a powerful alternative to traditional inverse scattering methods, offering improved accuracy, robustness, and real-time performance through data-driven modeling. In 2021, Zhou proposed a Modified Contrast Scheme (MCS) to mitigate the strong nonlinearity in two- and three-dimensional inverse scattering problems. By introducing a local-wave amplification coefficient, the conventional contrast formulation was reformulated into a unity-bounded representation that is more suitable for deep learning-based inversion. This physics-informed preprocessing improved training stability and reconstruction accuracy without compromising physical fidelity. Numerical results showed that the MCS-based framework achieved superior performance over dominant conventional schemes, particularly for high-contrast scatterers [14]. In 2022, Luo proposed a Cascaded Complex U-Net (CCU-Net) framework to solve inverse scattering problems with phaseless measurement data. The method employed a two-step architecture that jointly optimized phase retrieval and image reconstruction in the complex domain using complex-valued neural networks. Numerical and experimental results demonstrated robust and accurate reconstructions for high-contrast scatterers, outperforming conventional phaseless inversion approaches [18]. In 2023, Wang proposed an early fusion deep learning framework for solving electromagnetic inverse scattering problems by integrating scattered-field information and backpropagation-based prior reconstructions. The framework fused features from a scattered-field encoder and a backpropagation encoder, enabling complementary external and internal scattering representations. Numerical and experimental results demonstrated faster convergence and more stable reconstructions compared with conventional learning-based approaches [20]. In 2024, Zhou introduced a physics-guided deep unfolding parallel network with a self-supervised learning strategy, referred to as a Self-Supervised Inverse Scattering Problem Network (SSISP-Net), to address nonlinear inverse scattering problems without requiring labeled data. By unfolding traditional iterative solvers into network layers and enforcing physical consistency through crossing losses, the method enabled effective training using only scattered-field measurements. Numerical and experimental results showed that SSISP-Net achieved competitive reconstruction performance compared with supervised learning methods and outperformed traditional solvers under similar conditions [23]. In 2025, Huang proposed a meta-learning-assisted untrained neural network, termed Meta-uSOM, for electromagnetic inverse scattering problems. The method employed meta-learned initialization to accelerate the convergence of an untrained deep unrolling network and incorporated an edge-preserving total variation loss to enhance reconstruction quality. Numerical results demonstrated significantly improved convergence speed and reconstruction accuracy compared with existing untrained neural network approaches [28]. In 2025, Wang proposed PUP-Net, a twofold physical model embedded 3-D U-Net with polarization fusion for solving inverse scattering problems using sparse planar arrays. The framework combined traditional physical inversion methods to obtain preliminary reconstructions and a U-Net to refine quantitative contrast images, following a divide-and-conquer strategy. Numerical and experimental results demonstrated superior reconstruction accuracy and generalization ability compared with conventional iterative and deep learning methods [29]. In 2025, Du proposed a physics-assisted deep learning scheme incorporating a globally perceptive module to model multiple scattering effects in inverse scattering problems. By combining global self-attention with convolutional layers, the proposed network captured both global interactions and local features from physics-based pre-solutions. Validation results demonstrated improved reconstruction accuracy, robustness, and generalization compared with conventional convolution-based learning frameworks [30].

In addition, polarization diversity and polarimetric imaging have been investigated for electromagnetic target characterization and imaging, demonstrating the importance of polarization-dependent scattering information in complex target analysis [31,32].

The majority of current hybrid-deep-learning-based inversion frameworks have been developed under TM conditions, with the simplifying assumption of constant out-of-plane permittivity. However, in many realistic scenarios, materials exhibit directional anisotropy, with distinct permittivity components along the z , x and y axes. While the reconstruction of anisotropic dielectric components (ε_z , ε_x , ε_y) has been explored in prior studies, the literature remains sparse, particularly when only two-dimensional scattering measurements are available.

In this paper, we propose a hybrid-deep-learning framework designed to reconstruct anisotropic dielectric constant. The method combines TM and TE wave illuminations to capture directional dependencies and utilizes DSM and BP preprocessing to extract spatial and dielectric features. These are fused into a hybrid input and fed into a U-Net architecture to simultaneously predict ε_z , ε_x and ε_y . Simulation results indicate that the proposed method achieves stable and high-fidelity reconstructions in the presence of noise contamination, thus serving as a practical tool for anisotropic imaging with full permittivity components.

Figure 1 illustrates the overall reconstruction workflow of the BP-U-Net baseline and the proposed hybrid-input U-Net framework. In the BP-U-Net baseline, the measured scattered-field data are first processed using the BP scheme to generate a preliminary BP reconstruction image. This BP image is then directly used as the input of the U-Net to reconstruct the anisotropic permittivity components. In contrast, the proposed hybrid-input U-Net framework uses both BP and DSM preprocessing. The BP reconstruction image provides quantitative dielectric-contrast information, whereas the DSM indicator image provides spatial localization and boundary-related information. These two preliminary images are independently generated from the same measured scattered-field data and then superimposed at the input-image level to construct the hybrid input. The resulting hybrid image is fed into the same U-Net backbone for reconstructing ε_z , ε_x , and ε_y . Therefore, the proposed framework improves the input representation by incorporating complementary physics-based information while keeping the neural-network architecture unchanged. The key contributions of this paper are summarized as follows:

1. A physically informed hybrid-input framework is developed for anisotropic electromagnetic imaging. In anisotropic media, the directional permittivity components are inherently coupled in the measured scattering responses under different incident-wave and polarization conditions. Such coupling may deteriorate the quality of preliminary reconstructions, resulting in boundary ambiguity, spatial distortion, and inaccurate dielectric-contrast estimation. To mitigate these effects, the proposed framework integrates BP-derived quantitative dielectric information with DSM-derived spatial localization information, thereby forming a more physically representative input for reconstructing anisotropic permittivity components.
2. A hybrid-input construction strategy is introduced to incorporate complementary physical priors before the U-Net reconstruction stage. In this strategy, BP-derived images provide quantitative dielectric-contrast information associated with permittivity estimation, whereas DSM-derived images provide spatial localization and boundary-related information of the scatterers. By integrating these two types of information at the input-image level, the resulting hybrid input provides the U-Net with both material-property and spatial-prior information for reconstructing ε_z , ε_x and ε_y .
3. The proposed framework highlights the effectiveness of physically informed input construction without increasing the complexity of the neural-network architecture. Al-

though more sophisticated network architectures may further improve reconstruction accuracy, they generally require higher computational cost, longer training time, and more extensive parameter tuning. In this work, BP-only, DSM-only, and BP-DSM hybrid inputs are evaluated using the same U-Net backbone and identical training settings, enabling a fair assessment of the proposed anisotropic hybrid-input representation.

4. Numerical results demonstrate that the proposed BP-DSM hybrid-input strategy achieves more reliable anisotropic reconstruction than single-input baselines using BP or DSM alone. The reconstructed ε_z , ε_x and ε_y distributions exhibit lower reconstruction errors, improved structural consistency, reduced artifacts, and enhanced robustness under noisy measurement conditions, confirming the effectiveness of the proposed physically informed input design for anisotropic electromagnetic imaging.

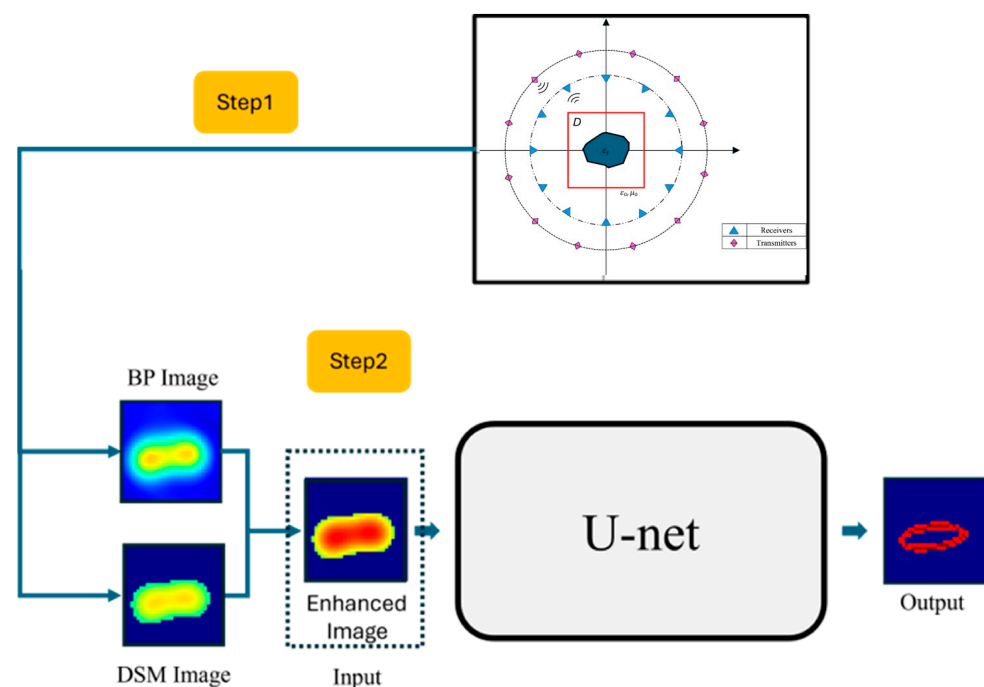


Figure 1. Overall workflow of the proposed hybrid-input U-Net framework.

The remainder of this paper is organized as follows. Section 2 presents the mathematical formulation of the forward and inverse scattering problems. Section 3 introduces the adopted U-Net-based reconstruction framework and the construction of the hybrid input. Section 4 discusses numerical examples and compares the reconstruction performance under different simulation settings. Finally, Section 5 summarizes the major findings and concludes this paper.

2. Problem Formulation

2.1. Direct Problem

Figure 2 illustrates the configuration of an electromagnetic imaging scattering setup designed for anisotropic dielectric inclusions. The target, embedded within the domain of interest D . The surrounding medium is homogeneous and anisotropic, with constant electric permittivity ε_0 and magnetic permeability μ_0 . Here, transmitters (purple diamonds) are positioned uniformly on the outer observation circle, and receivers (blue triangles) are distributed along the inner circle, creating a concentric arrangement that encloses the domain. For an anisotropic dielectric object, the relative permittivity tensor $\bar{\bar{\varepsilon}}_r$ is

characterized by the spatially dependent directional components $\varepsilon_x(x, y)$, $\varepsilon_y(x, y)$ and $\varepsilon_z(x, y)$, which may generally take complex values.

$$\bar{\bar{\varepsilon}}_r = \begin{bmatrix} \varepsilon_x(x, y) & 0 & 0 \\ 0 & \varepsilon_y(x, y) & 0 \\ 0 & 0 & \varepsilon_z(x, y) \end{bmatrix}_{xyz} \quad (1)$$

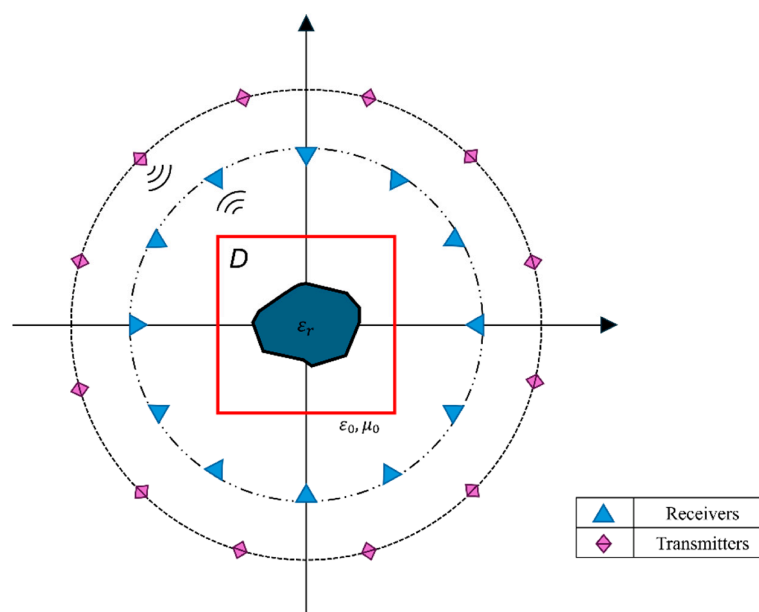


Figure 2. Two-dimensional electromagnetic imaging model with surrounding transmitters and receivers.

Accordingly, the dielectric contrast coefficients are defined as $(\tau_x)_{nn} = \varepsilon_x(\vec{r}) - 1$, $(\tau_y)_{nn} = \varepsilon_y(\vec{r}) - 1$ and $(\tau_z)_{nn} = \varepsilon_z(\vec{r}) - 1$.

In the framework of two-dimensional electromagnetic scattering, the total field can be rigorously formulated through volume integral equations based on the free-space Green's function. For the TM polarization, the incident wave excites only the z -component of the scattered field. Accordingly, the integral representation of the total field inside the domain and the scattered field outside the domain are given by:

$$E_z(\vec{r}) = \int_S G(\vec{r}, \vec{r}') (\varepsilon_z(\vec{r}') - 1) E_z(\vec{r}') ds' + E_z^i(\vec{r}), \quad \vec{r}, \vec{r}' \in S \quad (2)$$

$$E_z^s(\vec{r}) = \int_S G(\vec{r}, \vec{r}') (\varepsilon_z(\vec{r}') - 1) E_z(\vec{r}') ds', \quad \vec{r} \notin S, \vec{r}' \in S \quad (3)$$

where the two-dimensional free-space Green's function is defined as $G(\vec{r}, \vec{r}') = -\frac{jk_0^2}{4} H_0^{(2)}(k_0 |\vec{r} - \vec{r}'|)$, with $H_0^{(2)}$ being the zeroth-order Hankel function of the second kind.

For polarization, both the x - and y -components of the electric field are involved. The corresponding integral equations can be expressed as:

$$E_x(\vec{r}) = \left(\frac{\partial^2}{\partial x^2} + k_0^2 \right) \left[\int_S G(\vec{r}, \vec{r}') (\varepsilon_x(\vec{r}') - 1) E_x(\vec{r}') ds' \right] + \frac{\partial^2}{\partial x \partial y} \left[\int_S G(\vec{r}, \vec{r}') (\varepsilon_y(\vec{r}') - 1) E_y(\vec{r}') ds' \right] + \vec{E}_x^i(\vec{r}) \quad (4)$$

$$E_y(\vec{r}) = \frac{\partial^2}{\partial x \partial y} \left[\int_S G(\vec{r}, \vec{r}') (\varepsilon_x(\vec{r}') - 1) E_x(\vec{r}') ds' \right] + \left(\frac{\partial^2}{\partial y^2} + k_0^2 \right) \left[\int_S G(\vec{r}, \vec{r}') (\varepsilon_y(\vec{r}') - 1) E_y(\vec{r}') ds' \right] + \vec{E}_y^i(\vec{r}) \quad (5)$$

In formulating the direct scattering scenario, the incident illumination can be separated into two distinct polarization cases. The first corresponds to a TM wave traveling along the z-direction, denoted as E_z^i , while the second involves a TE wave with field components E_x^i and E_y^i . The scattered fields produced by these excitations can be represented through Hankel functions, which serve as the Green's functions for two-dimensional configurations. For the purpose of numerical simulation, the integral form of the governing equations is discretized using the method of moments, leading to the following matrix representations:

$$(-E_z^i) = ([G_1][\tau_z] - [I])(E_z) \quad (6)$$

$$(E_z^s) = [G_2][\tau_z](E_z) \quad (7)$$

$$\begin{pmatrix} -E_x^i \\ -E_y^i \end{pmatrix} = - \left\{ \begin{bmatrix} G_3 & G_4 \\ G_4 & G_5 \end{bmatrix} \begin{bmatrix} [\tau_x] & 0 \\ 0 & [\tau_y] \end{bmatrix} \begin{bmatrix} [I] & 0 \\ 0 & [I] \end{bmatrix} \right\} \begin{pmatrix} E_x \\ E_y \end{pmatrix} \quad (8)$$

$$\begin{pmatrix} E_x^s \\ E_y^s \end{pmatrix} = \left\{ \begin{bmatrix} G_6 & G_7 \\ G_7 & G_8 \end{bmatrix} \begin{bmatrix} [\tau_x] & 0 \\ 0 & [\tau_y] \end{bmatrix} \right\} \begin{pmatrix} E_x \\ E_y \end{pmatrix} \quad (9)$$

The Green's function matrix can be expressed as:

$$(G_1)_{mn} = \begin{cases} -\frac{j\pi k_0 a_n}{2} J_1(k_0 a_n) H_0^{(2)}(k_0 \rho_{mn}), & m \neq n \\ -\frac{j}{2} [\pi k_0 a_n H_1^{(2)}(k_0 a_n) - 2j], & m = n \end{cases} \quad (10)$$

$$(G_2)_{mn} = -\frac{j\pi k_0 a_n}{2} J_1(k_0 a_n) H_0^{(2)}(k_0 \rho_{mn}) \quad (11)$$

$$(G_3) = \begin{cases} -\frac{j\pi a_n J_1(k_0 a_n)}{2\rho_{mn}^3} \times [k_0 \rho_{mn} (y_m - y_n)^2 H_0^{(2)}(k_0 \rho_{mn}) + \\ ((x_m - x_n)^2 - (y_m - y_n)^2) H_1^{(2)}(k_0 \rho_{mn})], & m \neq n \\ -\frac{j}{4} [\pi k_0 a_n H_1^{(2)}(k_0 a_n) - 4j], & m = n \end{cases} \quad (12)$$

$$(G_4) = \begin{cases} -\frac{j\pi a_n J_1(k_0 a_n)}{2\rho_{mn}^3} (x_m - x_n)(y_m - y_n) \times \\ [2H_1^{(2)}(k_0 \rho_{mn}) - k_0 \rho_{mn} H_0^{(2)}(k_0 \rho_{mn})], & m \neq n \\ 0, & m = n \end{cases} \quad (13)$$

$$(G_5) = \begin{cases} -\frac{j\pi a_n J_1(k_0 a_n)}{2\rho_{mn}^3} \times [k_0 \rho_{mn} (x_m - x_n)^2 H_0^{(2)}(k_0 \rho_{mn}) + \\ ((y_m - y_n)^2 - (x_m - x_n)^2) H_1^{(2)}(k_0 \rho_{mn})], & m \neq n \\ -\frac{j}{4} [\pi k_0 a_n H_1^{(2)}(k_0 a_n) - 4j], & m = n \end{cases} \quad (14)$$

$$(G_6) = -\frac{j\pi a_n J_1(k_0 a_n)}{2\rho_{mn}^3} \times [k_0 \rho_{mn} (y_m - y_n)^2 H_0^{(2)}(k_0 \rho_{mn}) + ((x_m - x_n)^2 - (y_m - y_n)^2) H_1^{(2)}(k_0 \rho_{mn})] \quad (15)$$

$$(G_7)_{mn} = -\frac{j\pi a_n J_1(k_0 a_n)}{2\rho_{mn}^3} (x_m - x_n)(y_m - y_n) [2H_1^{(2)}(k_0 \rho_{mn}) - k_0 \rho_{mn} H_0^{(2)}(k_0 \rho_{mn})] \quad (16)$$

$$(G_8)_{mn} = -\frac{j\pi a_n J_1(k_0 a_n)}{2\rho_{mn}^3} [k_0 \rho_{mn} (x_m - x_n)^2 H_0^{(2)}(k_0 \rho_{mn}) + ((y_m - y_n)^2 - (x_m - x_n)^2) H_1^{(2)}(k_0 \rho_{mn})] \quad (17)$$

where $\rho_{mn} = \sqrt{(x_m - x_n)^2 + (y_m - y_n)^2}$ is the distance between the m-th observation point (x_m, y_m) and the n-th source point (x_n, y_n) . In addition, $H_0^{(2)}$ and $H_1^{(2)}$ denote the zeroth- and first-order Hankel functions of the second kind, respectively, while J_1 denotes the first-order Bessel function.

2.2. Backpropagation (BP)

In this section, the measured scattered-field data are utilized to compute the preliminary dielectric distribution through the BP, which simplifies the learning process of the U-Net. Numerical observations show that BP can effectively reconstruct the scattered field for weak scatterers. Assuming that the back-propagated field is proportional to the induced currents (I_z^b), as following:

$$\begin{pmatrix} I_z^b \end{pmatrix} = Y_m \cdot [G_2]^H (E_z^s) \quad (18)$$

where H denotes the conjugate transpose. Similarly, the induced currents I_x^b and I_y^b can be written as:

$$\begin{pmatrix} I_x^b \\ I_y^b \end{pmatrix} = Y_e \cdot \begin{bmatrix} [G_6] & [G_7] \\ [G_7] & [G_8] \end{bmatrix}^H \begin{pmatrix} E_x^s \\ E_y^s \end{pmatrix} \quad (19)$$

Based on (7), the loss function can be defined as follows:

$$L_m^b(Y) = \left\| (E_z^s) - [G_2] \cdot Y_m \cdot [G_2]^H (E_z^s) \right\|^2 \quad (20)$$

Similarly, based on (9), the loss function L_e^b can be defined as:

$$L_e^b(Y) = \left\| \begin{pmatrix} E_x^s \\ E_y^s \end{pmatrix} - \begin{bmatrix} [G_6] & [G_7] \\ [G_7] & [G_8] \end{bmatrix} \cdot Y_e \cdot \begin{bmatrix} [G_6] & [G_7] \\ [G_7] & [G_8] \end{bmatrix}^H \begin{pmatrix} E_x^s \\ E_y^s \end{pmatrix} \right\|^2 \quad (21)$$

To obtain the minimum of the loss function, the stationary condition is applied. The resulting closed-form expressions for Y_m and Y_e are given as:

$$Y_m = \frac{(E_z^s)^T \cdot ([G_2] ([G_2]^H \cdot (E_z^s)))^*}{\left\| [G_2] ([G_2]^H \cdot (E_z^s)) \right\|^2} \quad (22)$$

$$Y_e = \frac{\begin{pmatrix} E_x^s \\ E_y^s \end{pmatrix}^T \cdot \left(\begin{bmatrix} [G_6] & [G_7] \\ [G_7] & [G_8] \end{bmatrix} \left(\begin{bmatrix} [G_6] & [G_7] \\ [G_7] & [G_8] \end{bmatrix}^H \cdot \begin{pmatrix} E_x^s \\ E_y^s \end{pmatrix} \right) \right)^*}{\left\| \begin{bmatrix} [G_6] & [G_7] \\ [G_7] & [G_8] \end{bmatrix} \left(\begin{bmatrix} [G_6] & [G_7] \\ [G_7] & [G_8] \end{bmatrix}^H \cdot \begin{pmatrix} E_x^s \\ E_y^s \end{pmatrix} \right) \right\|^2} \quad (23)$$

where T denotes transpose and $*$ denotes complex conjugation.

Once Y_m is obtained, the induced current can be derived from (18), and the back-propagated total field is computed from (6) as:

$$\begin{pmatrix} E_z^b \end{pmatrix} = \begin{pmatrix} E_z^i \end{pmatrix} + [G_1] \begin{pmatrix} I_z^b \end{pmatrix} \quad (24)$$

Similarly, the induced currents from (19) allow the computation of the back-propagated total field from equate (8):

$$\begin{pmatrix} E_x^b \\ E_y^b \end{pmatrix} = \begin{pmatrix} E_x^i \\ E_y^i \end{pmatrix} + \begin{bmatrix} [G_3] & [G_4] \\ [G_4] & [G_5] \end{bmatrix} \begin{pmatrix} I_x^b \\ I_y^b \end{pmatrix} \quad (25)$$

The relationship between I_z^b and the contrast $[\tau_z^b]$ is:

$$\begin{pmatrix} I_{z,p}^b \end{pmatrix} = \text{diag}([\tau_z^b]) \begin{pmatrix} E_z^b \end{pmatrix} \quad (26)$$

For the TE case, the relations of $I_{x,p}^b$ and $I_{y,p}^b$ and $[\tau_x^b]$ and $[\tau_y^b]$ are:

$$\begin{pmatrix} I_{x,p}^b \\ I_{y,p}^b \end{pmatrix} = \text{diag} \left(\begin{bmatrix} [\tau_x^b] & 0 \\ 0 & [\tau_y^b] \end{bmatrix} \right) \begin{pmatrix} E_x^b \\ E_y^b \end{pmatrix} \quad (27)$$

Here, p denotes the incidence index, while $[\tau_z^b]$, $[\tau_x^b]$ and $[\tau_y^b]$ represent the dielectric coefficient distributions from Backpropagation. By combining all incidences and applying the least squares method, the analytical solution of the n -th element of $[\tau_z^b]$ is:

$$[\tau_z^b] = \frac{\sum_{p=1}^{Ni} I_{z,p}^b(n) \cdot [E_z^b(n)]^*}{\sum_{p=1}^{Ni} \|E_z^b(n)\|^2} \quad (28)$$

Similarly, combining (27) yields the analytical solutions for $[\tau_x^b]$ and $[\tau_y^b]$:

$$\begin{bmatrix} [\tau_x^b] & 0 \\ 0 & [\tau_y^b] \end{bmatrix} = \frac{\sum_{p=1}^{Ni} \begin{pmatrix} I_{x,p}^b \\ I_{y,p}^b \end{pmatrix}(n) \cdot \left[\begin{pmatrix} E_x^b \\ E_y^b \end{pmatrix}(n) \right]^*}{\sum_{p=1}^{Ni} \left\| \begin{pmatrix} E_x^b \\ E_y^b \end{pmatrix}(n) \right\|^2} \quad (29)$$

where p is the number of incident waves.

2.3. Direct Sampling Methods

DSM is a straightforward, rapid, and computationally efficient sampling-based imaging approach. This method uses only the measured scattered-field data to rapidly infer the approximate position and size of the target, without requiring an iterative updating process. In this work, DSM is adopted to estimate the geometry of conducting targets, including their boundary shape and profile information. This property is useful for conductor imaging because the main reconstruction task is to determine the target contour and spatial extent. A conceptual overview of the method is depicted in Figure 3.

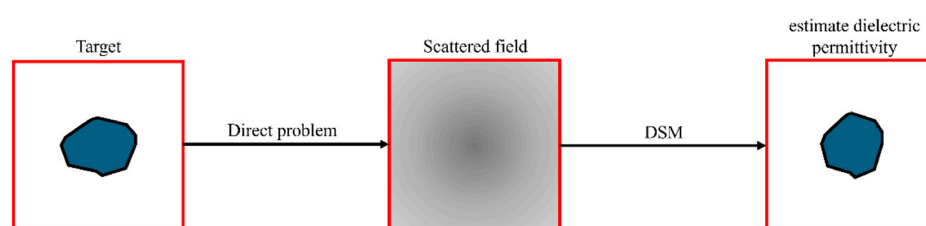


Figure 3. Schematic illustration of the DSM reconstruction procedure.

Initially, DSM was developed for two-dimensional dielectric inverse scattering problems under fixed planar-wave illumination. It was later applied to various inverse scattering scenarios. The advantage of DSM lies in its ability to avoid computationally intensive procedures, including Singular Value Decomposition (SVD), as well as the direct solution of ill-posed integral equations. For TM polarization, the DSM formulation is expressed as:

$$\Psi = \frac{1}{N_i} \sum_{i=1}^{N_i} \frac{\left| \left\langle \begin{pmatrix} E_{z,i}^s \end{pmatrix}, [G_2] \right\rangle \right|}{\left\| \begin{pmatrix} E_{z,i}^s \end{pmatrix} \right\| \left\| [G_2] \right\|} \quad (30)$$

Here, N_i denotes the total number of receiving points. The column vector $(E_z^s)_i$, with dimensions $N_i \times 1$, represents the measured scattered field corresponding to the i -th excitation. The symbol $[G_2]$ is an $N_i \times 1$ Green's-function vector used in the TM-polarized DSM indicator function.

For the x - and y -direction indicator functions, the DSM expression is formulated as:

$$\Psi_{x,y} = \frac{1}{N_i} \sum_{i=1}^{N_i} \frac{\left| \left(\begin{pmatrix} E_{x,i}^s \\ E_{y,i}^s \end{pmatrix} \right) \cdot \begin{bmatrix} G_6 \\ G_7 \end{bmatrix} \right|}{\left\| \left(\begin{pmatrix} E_{x,i}^s \\ E_{y,i}^s \end{pmatrix} \right)_t \right\| \left\| \begin{bmatrix} G_6 \\ G_7 \end{bmatrix} \right\|} \quad (31)$$

When TE waves are employed for reconstruction, the magnitudes of the incident field components E_x^i and E_y^i vary depending on the incident angle. In cases where E_x^i exhibits a large magnitude, the indicator function Ψ_x is utilized for shape reconstruction. Conversely, when E_y^i is dominant, Ψ_y is applied to recover the object's shape. In total, 32 distinct incident wave illuminations are considered.

To align the physics-based preprocessing results with the neural-network reconstruction framework, a hybrid-input U-Net strategy is adopted in this work. Specifically, the BP reconstruction image and the DSM indicator image are first independently generated from the measured scattered-field data and then superimposed at the input-image level to form the hybrid input for the U-Net. In this scheme, BP provides quantitative dielectric-contrast information for distinguishing scatterers with similar geometries but different permittivity values, whereas DSM contributes spatial localization and boundary-related information of the scatterers. The superposition of these two images enables their complementary characteristics to be jointly incorporated into the network input, thereby producing a more informative, spatially accurate, and physically meaningful representation for anisotropic permittivity reconstruction.

3. U-Net

The U-Net structure used in this paper is shown in Figure 4. The architecture follows an encoder–decoder topology for pixel-wise anisotropic permittivity reconstruction. The encoder consists of three resolution levels with 64, 128, and 256 feature channels, respectively. At each level, two consecutive 3×3 convolutional layers are employed, and each convolutional layer is followed by batch normalization and a ReLU activation function. The 3×3 kernels are used to extract local spatial features from the input image, while batch normalization improves training stability and ReLU introduces nonlinear feature mapping. After each encoding block, a 2×2 max-pooling operation is applied for downsampling, reducing the spatial resolution and enlarging the receptive field.

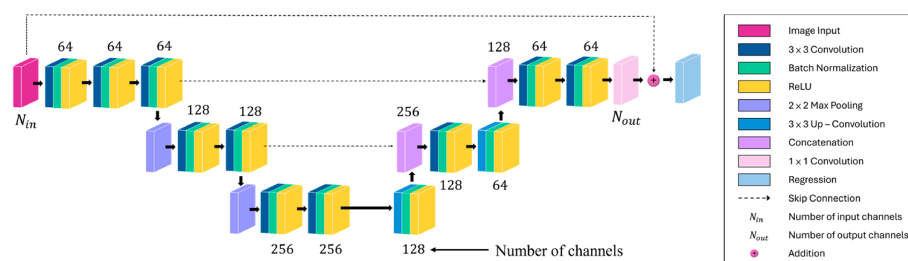


Figure 4. Architecture of the adopted U-Net reconstruction network.

In the decoder path, the spatial resolution is progressively restored through upsampling operations, followed by 3×3 convolutional layers to refine the reconstructed features. Skip connections are introduced between the encoder and decoder at the same spatial scale, where the encoder feature maps are concatenated with the corresponding upsampled decoder feature maps. This design allows low-level spatial details to be preserved while high-level semantic information is incorporated, which is beneficial for maintaining object boundaries and structural consistency in anisotropic electromagnetic image reconstruction.

In the present architecture, N_{in} and N_{out} denote the numbers of input and output channels, respectively. The input channels correspond to the constructed hybrid images, whereas the output channels correspond to the reconstructed anisotropic dielectric components. For multi-component anisotropic reconstruction, the final 1×1 convolutional layer maps the extracted feature representation to the desired output channels and generates the reconstructed permittivity distributions of ε_z , ε_x and ε_y .

The optimization target of the U-Net model is formulated in (32), where the network is trained to estimate the three directional permittivity components:

$$\underset{B, i}{\operatorname{argmin}} \sum_{N=1}^{N_t} f \left(B \begin{pmatrix} \varepsilon_x^\alpha \\ \varepsilon_y^\alpha \\ \varepsilon_z^\alpha \end{pmatrix}, \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \end{pmatrix} \right) + \Omega(i) \quad (32)$$

The term f measures the difference between the predicted permittivity components generated by the network B and the corresponding reference values. In this formulation, the three directional components, ε_z , ε_x , and ε_y , are optimized within the same objective function, allowing the model to learn their coupled behavior in a consistent manner. The additional regularization term $\Omega(i)$ is included to improve training stability, suppress overfitting, and enhance the robustness of the reconstruction. The mean-squared error term is adopted because the reconstruction task is formulated as a pixel-wise regression problem for continuous permittivity distributions. It penalizes the difference between the predicted and reference anisotropic dielectric components and encourages accurate recovery of both spatial profiles and dielectric contrast. The regularization term is included to improve training stability and reduce overfitting. This loss formulation enables the network to jointly optimize the reconstructed ε_z , ε_x and ε_y components within a unified reconstruction objective.

4. Numerical Analysis

4.1. Simulation Setup

The simulation domain is modeled using a uniform cell-based discretization. To satisfy the spatial resolution requirement, the cell size is limited by $\frac{0.2\lambda_0}{\sqrt{\varepsilon_r}}$, where λ_0 is taken as the wavelength in free space and ε_r is the largest relative permittivity considered in the anisotropic target. The operating frequency is 3 GHz. A total of 32 transmitter–receiver pairs are placed around the imaging region to collect the scattered-field responses. The numerical tests include both TM and TE incident polarizations. To evaluate noise tolerance, the scattered-field data are contaminated with Gaussian noise at two levels, 5% and 20%. In the TM configuration, the antenna positions are uniformly separated by an angular interval of 11.25° .

For the TE illumination, since the incident field components E_x^i and E_y^i are angle-dependent, only selected incident angles are employed. Specifically, angles between 41° and 48° are used to emphasize E_y^i , 131° to 138° to emphasize E_x^i , 221° to 228° to enhance E_y^i , and 311° to 318° to strengthen E_x^i . Reconstructions of ε_x are carried out whenever E_x^i dominates,

while ε_y is retrieved under conditions where E_y^i is stronger. Altogether, 32 incident angles are incorporated to ensure complete coverage.

The numerical dataset was constructed using anisotropic dielectric objects with three relative-permittivity ranges: 1–1.5, 1.5–2, and 2–2.5. For each range, 500 samples were generated by combining 10 dielectric patterns with 50 randomly selected object locations. For each sample, scattered-field data were simulated under both TM and TE polarized illuminations, and the corresponding BP-only, DSM-only, and hybrid input images were generated using the same preprocessing procedures. The samples were randomly divided into 80% for training and 20% for testing. The testing samples were not used during training and were reserved only for final performance evaluation, allowing the reconstruction performance on unseen samples to be assessed.

The proposed U-Net model was trained using the ADAM optimizer with GPU acceleration. The ADAM parameters were set to $\beta_1 = 0.9$, and $\beta_2 = 0.99$. The initial learning rate was set to 10^{-3} and decreased every 20 epochs until reaching 10^{-4} . The mini-batch size was set to 16, and the training process was conducted for 40 epochs. To reduce overfitting and improve generalization, the training samples were randomly shuffled at the beginning of each epoch. The same training configuration was applied to all compared input strategies, including BP-only, DSM-only, and hybrid-input cases, to ensure a fair comparison.

Reconstruction accuracy is assessed by the Root Mean Square Error (RMSE) and the Structural Similarity Index Measure (SSIM). The RMSE can be defined as following:

$$RMSE = \frac{1}{M_t} \sum_{i=1}^{M_t} \|\varepsilon_r - \varepsilon_r^\alpha\|_F / \|\varepsilon_r\|_F \quad (33)$$

where ε_r denotes the ground truth permittivity, ε_r^α the reconstructed distribution, M_t the total number of testing samples, and F indicates the Frobenius norm. Equation (26) defines the loss function used to train the proposed U-Net model. The SSIM [33], on the other hand, is evaluated as:

$$SSIM = \frac{(2\mu_{\tilde{y}}\mu_y + C_1)(2\sigma_{\tilde{y}y} + C_2)}{(\mu_{\tilde{y}}^2 + \mu_y^2 + C_1)(\sigma_{\tilde{y}}^2 + \sigma_y^2 + C_2)} \quad (34)$$

where $\tilde{y}$ represents the reconstructed distribution and y the reference. Here, μ_y is the mean of y , σ_y^2 the variance of y , and $\sigma_{\tilde{y}y}$ their covariance. Constants $C_1 = (K_1 D)^2$ and $C_2 = (K_2 D)^2$ prevent instability in the denominator, with $K_1 = 0.01$ and $K_2 = 0.03$, and D denotes the intensity range of the image.

For comparison purposes, the free-space permittivity is set as 1.0, and the scatterers are classified into two dielectric-constant ranges:

- (1) 1–1.5, representing low-permittivity dielectric materials, including foams, lightweight polymers, and low-density composite substrates;
- (2) 1.5–2, typical of acrylic, plastics, and epoxy resins;
- (3) 2–2.5, representing low-loss ceramics, glass, and dense polymer composites;

These material categories reflect practical cases commonly encountered in microwave imaging and help ensure the realism of the numerical simulations.

The noisy scattered-field data are defined as:

$$\hat{E}^s = E^s + n \quad (35)$$

where E^s denote the simulated noise-free scattered-field and $\hat{E}^s$ denotes the noisy scattered-field vector and n represents the added Gaussian noise. For a prescribed noise level α , the corresponding signal-to-noise ratio is defined as

$$SNR_{dB} = 10 \log_{10} \left(\frac{1}{\alpha} \right) \quad (36)$$

In this paper, the noise was added based on the measured average power of the scattered-field vector. Therefore, the noise power is given by

$$P_n = \alpha P_s \quad (37)$$

$$P_s = \frac{1}{N} \sum_{m=1}^N |E_s(m)|^2 \quad (38)$$

where P_n denotes the noise power and P_s denotes the measured average power of E^s .

4.2. Reconstruction Results for Permittivity Values of 1–1.5 Under 20% Noise

For this case, the target permittivity is kept in the interval of 1–1.5, but the scattered-field data are further disturbed by 20% Gaussian noise. A total of 500 testing images are prepared by combining 10 permittivity maps with 50 randomly generated object locations. The BP-U-Net and hybrid-U-Net reconstructions are evaluated for both TM and TE incident fields to examine their performance under stronger noise interference.

Figure 5a shows the ground truth of ε_z , while Figure 5b and 5c present the ε_z reconstructions obtained by BP-U-Net and hybrid-U-Net, respectively. Figure 5d–f shows the ground truth, BP-U-Net and hybrid-U-Net results for ε_x , respectively. Figure 5g–i shows the ground truth, BP-U-Net and hybrid-U-Net results for ε_y , respectively.

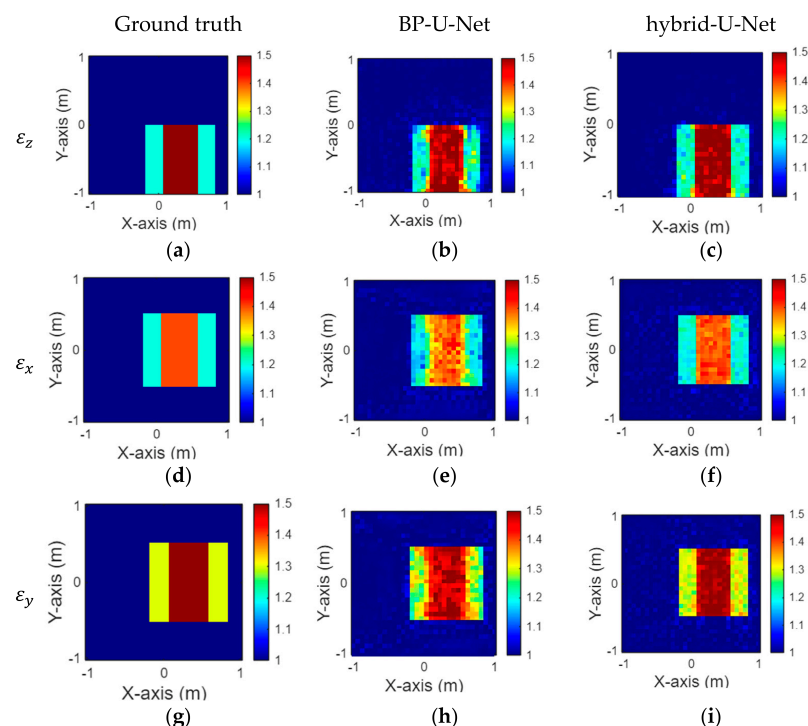


Figure 5. Reconstruction comparison for the 1–1.5 permittivity case under 20% noise. (a) Ground truth of ε_z (b) BP-U-Net reconstruction result of ε_z (c) hybrid-U-Net reconstruction result of ε_z (d) Ground truth of ε_x (e) BP-U-Net reconstruction result of ε_x (f) hybrid-U-Net reconstruction result of ε_x (g) Ground truth of ε_y (h) BP-U-Net reconstruction result of ε_y (i) hybrid-U-Net reconstruction result of ε_y .

From the visual inspection, it is evident that the hybrid U-Net consistently improves the reconstruction quality for all three permittivity components (ϵ_z , ϵ_x , ϵ_y) compared with the standalone BP-U-Net. The hybrid-input framework enhances structural fidelity, suppresses artifacts induced by noise, and produces more accurate dielectric distributions.

Table 1 summarizes the quantitative performance of ϵ_z , ϵ_x and ϵ_y under the condition of 20% Gaussian noise. The results confirm that the hybrid U-net outperforms the conventional BP-U-net in terms of both RMSE and SSIM across all components, demonstrating superior robustness in moderate-contrast but noisy scenarios.

Table 1. Performance of dielectric constant variation of ϵ_z , ϵ_x and ϵ_y from 1 to 1.5 with 20% noise.

Performance	ϵ_z		ϵ_x		ϵ_y	
	RMSE	SSIM	RMSE	SSIM	RMSE	SSIM
BP-U-Net	3.22%	95.32%	2.45%	96.35%	2.48%	97.24%
hybrid-U-Net	1.94%	98.15%	1.76%	96.95%	1.75%	97.48%

4.3. Reconstruction Results for Permittivity Values of 1.5–2 Under 5% Noise

In this case, the relative permittivity of the dielectric target is assigned within the range of 1.5–2, and 5% Gaussian noise is added to the scattered-field data. The dataset contains 500 samples, generated from 10 dielectric patterns combined with 50 randomly selected target positions. The conventional BP method and the proposed hybrid-U-Net method are both evaluated under TM and TE illuminations.

Figure 6a shows the ground truth of ϵ_z , while Figure 6b and 6c present the ϵ_z reconstructions obtained by BP and hybrid-U-Net, respectively. Figure 6d–f shows the ground truth, BP-U-Net and hybrid-U-Net results for ϵ_x , respectively. Figure 6g–i shows the ground truth, BP and hybrid-U-Net results for ϵ_y , respectively.

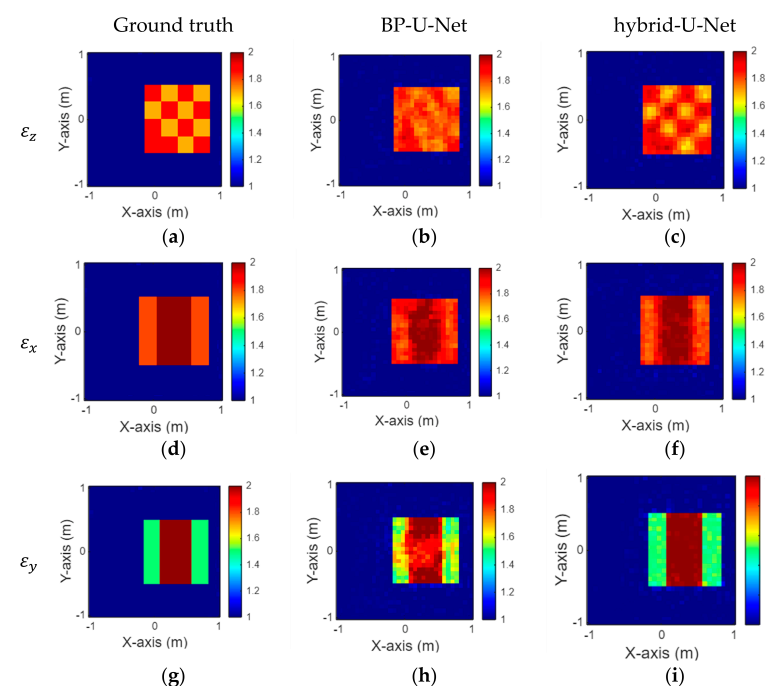


Figure 6. Reconstruction comparison for the 1.5–2 permittivity case under 5% noise. (a) Ground truth of ϵ_z (b) BP-U-Net reconstruction result of ϵ_z (c) hybrid-U-Net reconstruction result of ϵ_z (d) Ground truth of ϵ_x (e) BP-U-Net reconstruction result of ϵ_x (f) hybrid-U-Net reconstruction result of ϵ_x (g) Ground truth of ϵ_y (h) BP-U-Net reconstruction result of ϵ_y (i) hybrid-U-Net reconstruction result of ϵ_y .

By analyzing these results, it is evident that the hybrid-U-Net scheme achieves higher fidelity in reconstructing dielectric properties compared to the standalone BP method. In particular, the hybrid hybrid-U-Net approach preserves the geometric boundaries of scatterers more effectively and yields more accurate values of ε_z , ε_x and ε_y even under noisy conditions. The hybrid-input mechanism therefore demonstrates superior robustness and accuracy for moderate-contrast materials.

Table 2 summarizes the quantitative reconstruction results for ε_z , ε_x , and ε_y under 5% Gaussian noise. The RMSE and SSIM results indicate that the hybrid hybrid-U-Net method provides better reconstruction performance than the BP-U-Net for all three permittivity components, supporting the effectiveness of the proposed framework in practical imaging conditions.

Table 2. Quantitative reconstruction results for ε_z , ε_x , and ε_y with permittivity values of 1.5–2 under 5% noise.

Performance	ε_z		ε_x		ε_y	
	RMSE	SSIM	RMSE	SSIM	RMSE	SSIM
BP-U-Net	5.82%	96.42%	3.17%	98.25%	3.64%	98.13%
hybrid-U-Net	2.52%	98.33%	1.73%	98.76%	1.91%	98.68%

4.4. Reconstruction Results for Permittivity Values of 2–2.5 Under 5% Noise

This case considers a higher-permittivity target, where the relative permittivity varies from 2 to 2.5. The scattered-field data are corrupted by 5% Gaussian noise to evaluate reconstruction performance under mild noise interference. A total of 500 samples are generated by pairing 10 dielectric patterns with 50 randomly selected target locations. The conventional BP method and the proposed hybrid-U-Net method are then tested under both TM and TE incident-field conditions.

Figure 7a shows the ground truth of ε_z , while Figure 7b and 7c present the ε_z reconstructions obtained by BP-U-Net and hybrid-U-Net, respectively. Figure 7d–f shows the ground truth, BP-U-Net and hybrid-U-Net results for ε_x , respectively. Figure 7g–i shows the ground truth, BP-U-Net and hybrid-U-Net results for ε_y , respectively.

The comparison shows that the hybrid hybrid-U-Net framework provides lower reconstruction errors than the BP-U-Net, while maintaining comparable structural similarity for the three permittivity components, ε_z , ε_x , and ε_y . Under moderate noise interference, hybrid-U-Net still preserves the object boundaries, reduces noise-induced distortions, and generates more stable dielectric profiles. These results demonstrate the robustness of the hybrid-input strategy, especially for high-contrast dielectric targets under challenging imaging conditions. Table 3 summarizes the quantitative reconstruction results for ε_z , ε_x , and ε_y in the 2–2.5 permittivity case with 5% Gaussian noise. The obtained RMSE values show that the hybrid hybrid-U-Net framework produces more accurate reconstructions than the BP-U-Net, while the SSIM values remain comparable among different permittivity components.

Table 3. Quantitative reconstruction results for the 2–2.5 permittivity case under 5% noise.

Performance	ε_z		ε_x		ε_y	
	RMSE	SSIM	RMSE	SSIM	RMSE	SSIM
BP-U-Net	4.26%	94.85%	3.67%	93.01%	3.23%	93.48%
hybrid-U-Net	2.47%	96.01%	2.96%	95.32%	2.64%	95.65%

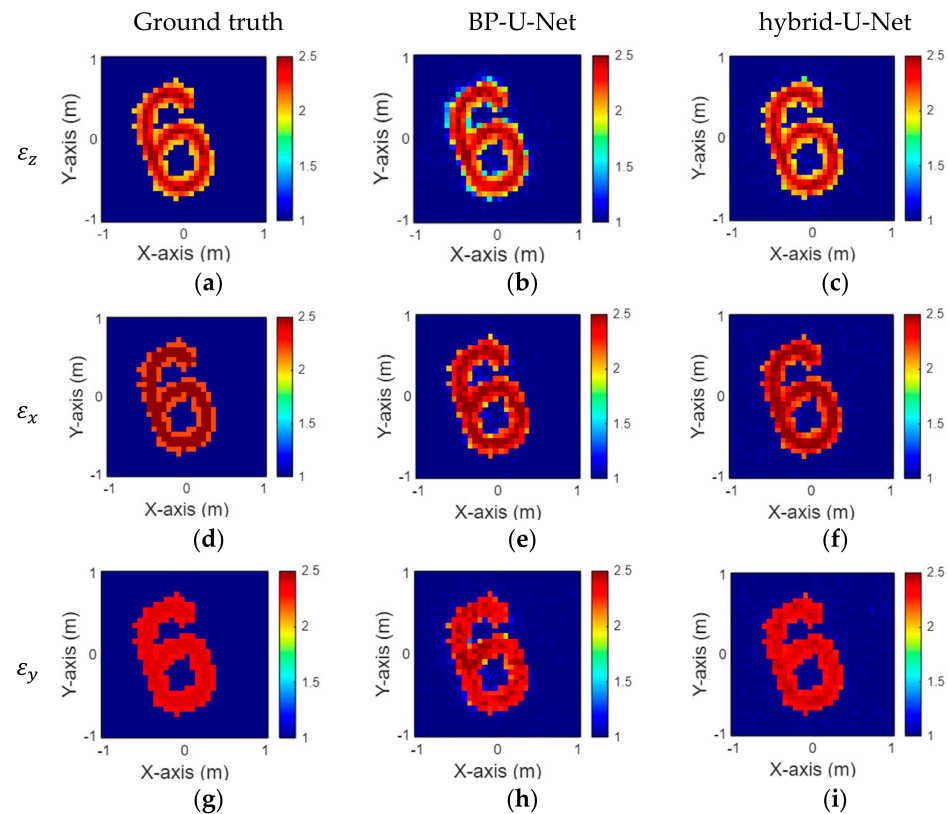


Figure 7. Reconstruction results for the 2–2.5 permittivity case under 5% noise. (a) Ground truth of ε_z (b) BP-U-Net reconstruction result of ε_z (c) hybrid-U-Net reconstruction result of ε_z (d) Ground truth of ε_x (e) BP-U-Net reconstruction result of ε_x (f) hybrid-U-Net reconstruction result of ε_x (g) Ground truth of ε_y (h) BP-U-Net reconstruction result of ε_y (i) hybrid-U-Net reconstruction result of ε_y .

4.5. Generalization Evaluation Using an MNIST-Based Dataset

In this case, 1000 styles were selected for each of the 10 digits, yielding a diverse set of 10,000 object profiles with substantially greater shape variability. Each digit pattern was treated as a dielectric object, and the same electromagnetic simulation procedure was applied to generate the corresponding scattered-field data. CSI, the BP scheme, and DSM were then employed to generate different physics-based preliminary reconstruction images. Based on these inputs, four U-Net-based reconstruction frameworks were compared, namely CSI-10 iteration, BP-U-Net, DSM-U-Net, and the proposed hybrid-input U-Net.

To ensure a fair comparison, all methods used the same U-Net backbone, training configuration, dataset partition, and quantitative evaluation metrics. The difference among the compared frameworks lies only in the input construction strategy. Specifically, CSI is employed as a conventional non-learning-based reconstruction method and is terminated after 10 iterations for comparison. BP-U-Net uses the BP reconstruction image as the network input, whereas DSM-U-Net uses the DSM indicator image.

Figure 8a shows the ground-truth distribution of ε_z , while Figure 8b–e present the corresponding reconstruction results obtained using CSI with 10 iterations, BP-U-Net, DSM-U-Net, and hybrid-U-Net, respectively. Figure 9a shows the ground-truth distribution of ε_x , whereas Figure 9b–e present the corresponding reconstruction results obtained using CSI with 10 iterations, BP-U-Net, DSM-U-Net, and hybrid-U-Net, respectively. Figure 10a shows the ground-truth distribution of ε_y , while Figure 10b–e present the corresponding reconstruction results obtained using CSI with 10 iterations, BP-U-Net, DSM-U-Net, and hybrid-U-Net, respectively.

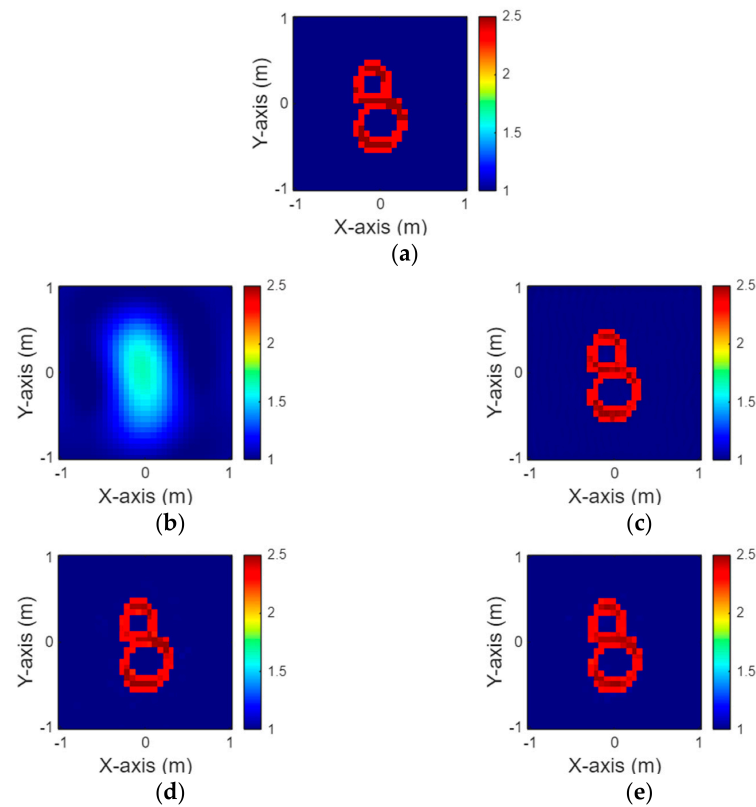


Figure 8. Reconstruction results of ε_z for the MNIST-based dataset under 5% noise. (a) Ground truth (b) CSI reconstruction result (c) DSM-U-Net reconstruction result (d) BP-U-Net reconstruction result (e) hybrid-U-Net reconstruction result.

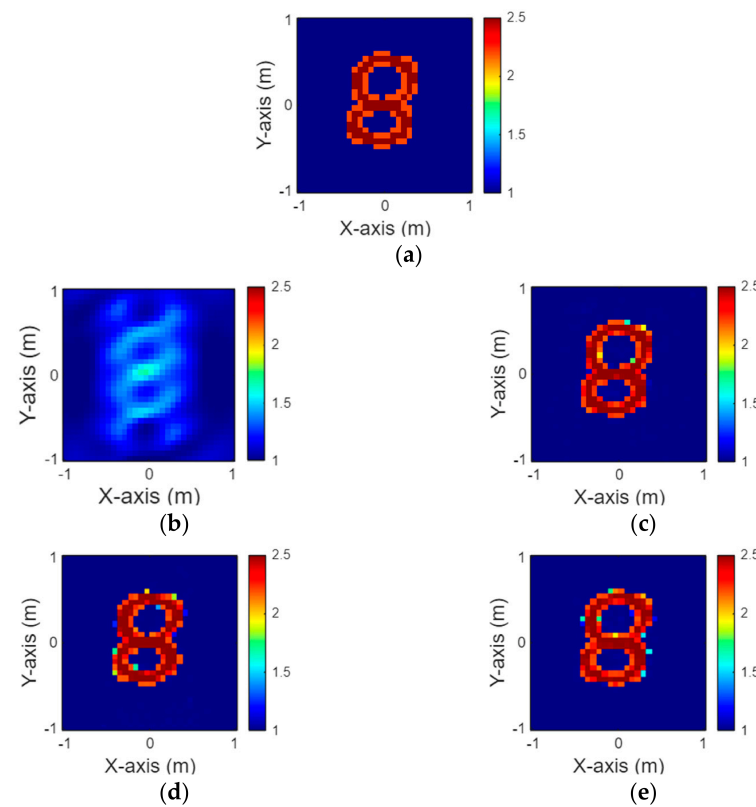


Figure 9. Reconstruction results of ε_x for the MNIST-based dataset under 5% noise. (a) Ground truth (b) CSI reconstruction result (c) DSM-U-Net reconstruction result (d) BP-U-Net reconstruction result (e) hybrid-U-Net reconstruction result.

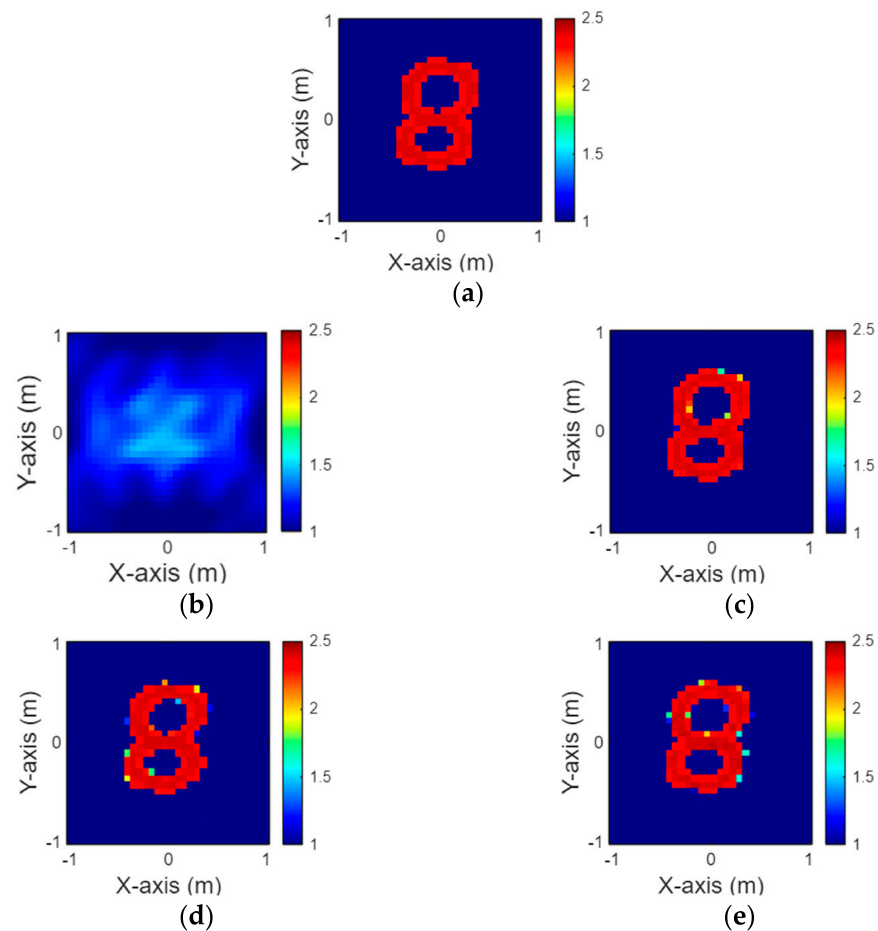


Figure 10. Reconstruction results of ε_y for the MNIST-based dataset under 5% noise. (a) Ground truth (b) CSI reconstruction result (c) DSM-U-Net reconstruction result (d) BP-U-Net reconstruction result (e) hybrid-U-Net reconstruction result.

As observed from Figures 8–10, the reconstruction obtained by CSI after 10 iterations still exhibits noticeable errors, including blurred boundaries, inaccurate permittivity distributions, and residual artifacts. By contrast, the U-Net-based frameworks use the preliminary images generated by the corresponding imaging methods as network inputs and further refine their spatial structures and dielectric distributions. The results demonstrate that neural-network-based refinement can effectively improve the reconstruction quality of the preliminary images obtained from the BP scheme, DSM, and their hybrid representation. Among the compared frameworks, hybrid-U-Net consistently produces clearer object boundaries, fewer reconstruction artifacts, and more accurate permittivity distributions.

Table 4 summarizes the quantitative results of the four methods on the large-scale MNIST-based dataset. The U-Net-based frameworks exhibit improved reconstruction performance compared with CSI using 10 iterations, indicating that the preliminary images generated by physics-based imaging methods can be effectively refined through neural-network reconstruction. In particular, hybrid-U-Net achieves the best overall performance across the three anisotropic permittivity components, demonstrating the effectiveness of combining the complementary dielectric-contrast and spatial-localization information provided by the BP scheme and DSM. These results further support the generalization capability of the proposed framework for large-scale datasets containing diverse and irregular object profiles.

Table 4. Quantitative reconstruction results for the MNIST-based dataset under 5% noise.

Performance	ε_z		ε_x		ε_y	
	RMSE	SSIM	RMSE	SSIM	RMSE	SSIM
CSI-10 iteration	26.06%	48.86%	30.55%	38.66%	32.66	27.73%
BP-U-Net	3.49%	97.83%	2.84%	98.81%	2.21%	98.63%
DSM-U-Net	3.56%	97.73%	2.86%	98.35%	2.23%	98.57%
hybrid-U-Net	3.39%	98.08%	2.79%	98.82%	2.19%	98.74%

5. Conclusions

This paper developed a learning-based electromagnetic reconstruction method for anisotropic objects by combining BP quantitative information with DSM-based spatial indicators. These complementary features were incorporated into a U-Net model to improve the reconstruction of directional dielectric properties. The BP component provides useful permittivity-related information, whereas DSM offers localization cues for the scatterers. By integrating these two types of physical information, the proposed method enhances reconstruction stability and accuracy while maintaining a reasonable computational cost.

A distinctive advantage of the proposed hybrid-U-Net framework is its ability to address the difficulties associated with TE-wave imaging, where nonlinear effects and field coupling typically degrade reconstruction accuracy. By employing precise adjustments of incident angles, these limitations are mitigated, resulting in more consistent and interpretable outcomes. Furthermore, the proposed method can reconstruct high-permittivity targets with real-time efficiency, indicating its applicability to practical electromagnetic imaging scenarios. Although the proposed framework demonstrates improved reconstruction performance, several limitations should be noted. Since the hybrid input is constructed from BP- and DSM-derived preliminary images, the reconstruction quality depends on the reliability of the dielectric and spatial information provided by these preprocessing methods. When the measurement aperture is severely limited, the DSM indicator may become less accurate, leading to degraded spatial localization and blurred reconstructed boundaries. This issue is particularly relevant to half-space buried-object imaging, where the available observation angles are restricted and the scattered-field information is incomplete. Therefore, the present framework is more suitable for free-space or multilayer imaging scenarios with sufficient measurement aperture. Future work will extend the method to more challenging half-space buried-object reconstruction, complex multilayer anisotropic targets, lossy dielectric objects, complex permittivity reconstruction, and practical uncertainty models such as phase noise, calibration errors, correlated channel noise, antenna-position errors, and forward-model mismatch. In particular, the reconstruction of lossy media will require estimating both the real and imaginary parts of the complex permittivity, so that dielectric storage and conductive loss characteristics can be simultaneously characterized. Moreover, advanced generative learning frameworks, including GAN-based reconstruction models and diffusion-model-based inverse solvers, will be explored to further improve reconstruction fidelity, artifact suppression, and generalization capability for complex anisotropic imaging scenarios. These developments may further support practical applications in non-destructive testing of anisotropic composite materials, where accurate reconstruction of direction-dependent dielectric properties and loss-related material parameters is important for defect detection and material characterization.

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References

1. Menshov, V.I.; Okhmatovski, V.I. Superlens Enhanced 2-D Microwave Tomography with Contrast Source Inversion Method. *IEEE Open J. Antennas Propag.* **2021**, *2*, 453–463. [\[CrossRef\]](#)
2. Yang, C.; Zhang, J.; Tong, M.S. An FFT-Accelerated Particle Swarm Optimization Method for Solving Far-Field Inverse Scattering Problems. *IEEE Trans. Antennas Propag.* **2021**, *69*, 1078–1093. [\[CrossRef\]](#)
3. Yang, C.X.; Zhang, J.; Tong, M.S. A Hybrid Quantum-Behaved Particle Swarm Optimization Algorithm for Solving Inverse Scattering Problems. *IEEE Trans. Antennas Propag.* **2021**, *69*, 5861–5869. [\[CrossRef\]](#)
4. Huang, X.; Hu, Y. Accelerating Uncertainty Quantification for Nonlinear Inverse Scattering Problems with High Contrast Media by Direct Envelope Methods and Krylov Subspace Iterative Integral Equation Solvers. *IEEE Trans. Geosci. Remote Sens.* **2021**, *59*, 8895–8902. [\[CrossRef\]](#)
5. Narendra, C.; Mojabi, P. A Combined Inverse Source and Scattering Technique for Dielectric Profile Design to Tailor Electromagnetic Fields. *IEEE Trans. Antennas Propag.* **2022**, *70*, 2149–2160. [\[CrossRef\]](#)
6. Pavone, S.C.; Sorbello, G.; Di Donato, L. Forward and Inverse Scattering of Metallic Objects Through Focused Bessel-Shaped Fields. *IEEE Open J. Antennas Propag.* **2022**, *3*, 911–916. [\[CrossRef\]](#)
7. Sun, S.; Dai, D.; Wang, X. A Fast Algorithm of Cross-Correlated Contrast Source Inversion in Homogeneous Background Media. *IEEE Trans. Antennas Propag.* **2023**, *71*, 4380–4393. [\[CrossRef\]](#)
8. Xie, L.; Huang, Y.; Huo, J.; Zhao, Q. Born Iterative Method Based on Convolutional Dictionary Learning for Inverse Scattering Problems. *IEEE Antennas Wirel. Propag. Lett.* **2023**, *22*, 54–58. [\[CrossRef\]](#)
9. Baharian, M.; Mohassel, J.A.R. Inverse Scattering for Coupled Nonuniform Lossless Transmission Lines Based on a State-Space Method and Nonuniform Frequency Sampling. *IEEE Trans. Antennas Propag.* **2024**, *72*, 915–925. [\[CrossRef\]](#)
10. Li, J.; Li, Z.; Guan, Z.; Han, F. 2-D Electromagnetic Scattering and Inverse Scattering from Anisotropic Objects Under TE Illumination Solved by the Hybrid SIM/SEM. *IEEE Trans. Antennas Propag.* **2024**, *72*, 3517–3528. [\[CrossRef\]](#)
11. Batista, C.; Adriano, R.; Batista, L.S. EISPY2D: An Open-Source Python Library for the Development and Comparison of Algorithms in Two-Dimensional Electromagnetic Inverse Scattering Problems. *IEEE Access* **2025**, *13*, 92134–92154. [\[CrossRef\]](#)
12. Zhang, X.; Ye, X. Solving Strongly Nonlinear Inverse Scattering Problems for Mixture of Dielectric and PEC with Novel Contraction T-Matrix Equation. *IEEE Trans. Microw. Theory Tech.* **2025**, *73*, 4791–4805. [\[CrossRef\]](#)
13. Wen, L.; Zhao, S.; Yin, T.; Guo, Y.; Chen, X. Inverse Scattering Approach to Integration of Communication and Sensing. *IEEE J. Sel. Top. Electron. Antennas Propag.* **2025**, *1*, 20–36. [\[CrossRef\]](#)
14. Zhou, Y.; Zhong, Y.; Wei, Z.; Yin, T.; Chen, X. An Improved Deep Learning Scheme for Solving 2-D and 3-D Inverse Scattering Problems. *IEEE Trans. Antennas Propag.* **2021**, *69*, 2853–2863. [\[CrossRef\]](#)
15. Wei, Z.; Chen, X. Uncertainty Quantification in Inverse Scattering Problems with Bayesian Convolutional Neural Networks. *IEEE Trans. Antennas Propag.* **2021**, *69*, 3409–3418. [\[CrossRef\]](#)
16. Liu, J.; Zhou, H.; Ouyang, T.; Liu, Q.; Wang, Y. Physical Model-Inspired Deep Unrolling Network for Solving Nonlinear Inverse Scattering Problems. *IEEE Trans. Antennas Propag.* **2022**, *70*, 1236–1249. [\[CrossRef\]](#)
17. Yago Ruiz, Á.; Cavagnaro, M.; Crocco, L. A Physics-Assisted Deep-Learning Microwave Imaging Framework for Real-Time Shape Reconstruction of Unknown Targets. *IEEE Trans. Antennas Propag.* **2022**, *70*, 6184–6194. [\[CrossRef\]](#)
18. Luo, F.; Wang, J.; Zeng, J.; Zhang, L.; Zhang, B.; Xu, K.; Luo, X. Cascaded Complex U-Net Model to Solve Inverse Scattering Problems with Phaseless-Data in the Complex Domain. *IEEE Trans. Antennas Propag.* **2022**, *70*, 6160–6170. [\[CrossRef\]](#)
19. Yao, H.M.; Guo, R.; Li, M.; Jiang, L.; Ng, M.K.P. Enhanced Supervised Descent Learning Technique for Electromagnetic Inverse Scattering Problems by the Deep Convolutional Neural Networks. *IEEE Trans. Antennas Propag.* **2022**, *70*, 6195–6206. [\[CrossRef\]](#)
20. Wang, Y.; Zhao, Y.; Wu, L.; Yin, X.; Zhou, H.; Hu, J.; Nie, Z. An Early Fusion Deep Learning Framework for Solving Electromagnetic Inverse Scattering Problems. *IEEE Trans. Geosci. Remote Sens.* **2023**, *61*, 1–14. [\[CrossRef\]](#)

21. Shan, T.; Lin, Z.; Song, X.; Li, M.; Yang, F.; Xu, S. Neural Born Iterative Method for Solving Inverse Scattering Problems: 2D Cases. *IEEE Trans. Antennas Propag.* **2023**, *71*, 818–829. [[CrossRef](#)]
22. Costanzo, S.; Flores, A. Deep-Learning Enhancement of Born Iterative-Based Inverse Scattering Solution for Breast Cancer Detection. In Proceedings of the 2023 IEEE Conference on Antenna Measurements and Applications (CAMA), Genoa, Italy, 15–17 November 2023. [[CrossRef](#)]
23. Zhou, H.; Xu, C.; Cheng, Y.; Liu, Q.; Hu, S.; Wang, Y. Self-Supervised Learning of Physics-Guided Deep Unfolding Parallel Networks for Solving Nonlinear Inverse Scattering Problems. *IEEE Trans. Microw. Theory Tech.* **2024**, *72*, 5204–5217. [[CrossRef](#)]
24. Hu, Y.-D.; Wang, X.-H.; Zhou, H.; Wang, L. A Priori Knowledge-Based Physics-Informed Neural Networks for Electromagnetic Inverse Scattering. *IEEE Trans. Geosci. Remote Sens.* **2024**, *62*, 1–9. [[CrossRef](#)]
25. Yao, H.M.; Jiang, L.; Ng, M. Enhanced Deep-Learning Approach Based on the Conditional Generative Adversarial Network for Electromagnetic Inverse Scattering Problems. *IEEE Trans. Antennas Propag.* **2024**, *72*, 6133–6138. [[CrossRef](#)]
26. Xing, Z.; Zhang, Z.; Chen, Z.; Wang, Y.; Ma, H.; Wei, Z. VBIM-Net: Variational Born Iterative Network for Inverse Scattering Problems. *IEEE Trans. Geosci. Remote Sens.* **2025**, *63*, 1–16. [[CrossRef](#)]
27. Farook Maricar, M.; Zakaria, A.; Qaddoumi, N. Attention-Unet for Electromagnetic Inverse Scattering Problems in Microwave Imaging. *IEEE Trans. Microw. Theory Tech.* **2025**, *73*, 1290–1304. [[CrossRef](#)]
28. Huang, Q.; Li, C.; Ye, X.; Xu, K.; Song, R. Meta-Learning-Assisted Untrained Neural Network for Electromagnetic Inverse Scattering Problems. *IEEE Trans. Antennas Propag.* **2025**, *73*, 2548–2560. [[CrossRef](#)]
29. Wang, M.; Sun, S.; Zhang, Y.; Dai, D.; Wu, H.; Su, Y. PUP-Net: A Twofold Physical Model Embedded 3-D U-Net With Polarization Fusion for Solving Inverse Scattering Problems With a Sparse Planar Array. *IEEE Trans. Microw. Theory Tech.* **2025**, *73*, 2123–2136. [[CrossRef](#)]
30. Du, C.; Pan, J.; Yang, D.; Hu, J.; Nie, Z.; Chen, Y. A Physics-Assisted Deep-Learning Scheme Based on Globally Perceptive Modules for Electromagnetic Inverse Scattering Problems. *IEEE Trans. Geosci. Remote Sens.* **2025**, *63*, 1–14. [[CrossRef](#)]
31. Xu, Z.; Zheng, Y.; Zeng, Y.; Yang, Z.; Zhang, Y.; Wu, Q.; Ai, X. Measurement and Calibration Methods for Full-Polarization Characteristics of Radar Targets at Subterahertz Frequencies. *IEEE Trans. Antennas Propag.* **2026**, *74*, 3471–3479. [[CrossRef](#)]
32. Zhu, Y.; Xu, Z.; Ai, X.; Chen, Z.; Yang, Z.; Xiao, S. Sub-Terahertz Full-Polarimetric Measurement and ISAR Imaging of Scaled Space Targets. *IEEE Trans. Antennas Propag.* **2026**, *in press*. [[CrossRef](#)]
33. Huang, Y.; Song, R.; Xu, K.; Ye, X.; Li, C.; Chen, X. Deep Learning-Based Inverse Scattering with Structural Similarity Loss Functions. *IEEE Sens. J.* **2021**, *21*, 4900–4907. [[CrossRef](#)]

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