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# Iterative Learning Control for Linear Systems Subject to Random Disturbance: An Impulsive Approach

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**Abstract**

This paper investigates the tracking control problem of linear systems with random disturbances. Distinguishing itself from previous studies, this work models instantaneous external disturbances as impulse effects and employs a Bernoulli random variable to characterize their stochastic occurrence. After this, a random impulsive differential system is established and used for theoretical analysis. Based on this model, two iterative learning control strategies are applied to achieve the robust tracking. The theoretical results demonstrate that the proposed algorithms are successful in reducing the impact caused by random disturbances, enabling the system to perform robust tracking. Finally, a numerical simulation is provided to verify the effectiveness of the proposed approach.

**Keywords:** external disturbance; iterative learning control; impulsive approach; convergence analysis

## 1. Introduction

As is well known, iterative learning control (ILC) has emerged as a powerful control methodology for systems executing repetitive tasks over a fixed finite time horizon, leveraging trial-to-trial error information to iteratively refine control inputs and achieve high-precision trajectory tracking. Due to its simplicity in implementation, strong convergence performance under mild conditions, and robustness to repetitive disturbances, ILC has been widely applied in robotics [1], batch processes [2,3], and network systems [4,5], particularly for linear systems where mathematical modeling is relatively mature. Compared with traditional control approaches, the ILC is less dependent on precise mathematical models. Leveraging this advantage, various ILC-based tracking schemes have been developed, including finite-iteration tracking [6–8], robust learning tracking [9–11], cooperative tracking [12,13], etc. To date, research on ILC continues to attract considerable interest from the science and engineering communities.

Linear systems are the cornerstone of many engineering applications, and their control performance directly determines the reliability and accuracy of the entire system. However, in practical scenarios, linear systems are often subject to external disturbances such as environmental noise [14,15] and mechanical friction [16,17], which degrade tracking performance, induce cumulative errors, and even undermine system stability. This poses a critical challenge to the application of traditional ILC strategies, as most conventional ILC algorithms are designed under the assumption of no external disturbances or only consider repetitive disturbances, lacking effective suppression mechanisms for non-repetitive or random external disturbances. In addition, fluctuations in ambient conditions including temperature and humidity can impair equipment electromagnetic compatibility, which in

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turn gives rise to random transient disturbances. Meanwhile, the coupling effect of environmental complexity, electrical device attributes, and system interaction renders stochastic disturbances inevitable in real-world industrial applications.

To address the external disturbance problem in linear systems, researchers have proposed various improved ILC schemes. For instance, some studies integrated feedback control with ILC to mitigate real-time disturbances [18], but the introduction of feedback loops may increase computational complexity and compromise convergence speed. Others adopted disturbance observers or sliding mode control to compensate for external disturbances [19]. However, these methods rely heavily on accurate system models and can be affected by chattering phenomena or poor adaptability to time-varying disturbances. Furthermore, some studies have analyzed cyber-attacks as a form of external disturbances and subsequently investigated system security with the ILC approach [20,21]. Nevertheless, the cyber-attacks in [20,21] are neither transient nor random, so they cannot properly represent the features of this type of disturbance. Then, a pivotal question arises concerning how to accurately characterize random and transient external disturbances.

The impulsive framework has gained increasing attention in control theory due to its ability to handle systems with instantaneous state changes and discrete control actions [22–25]. This framework adjusts system states by applying transient impulse signals at discrete time points, offering advantages such as low computational cost, rapid response, and strong adaptability to discontinuous dynamical systems. Therefore, a dynamic system incorporating impulsive behavior will be employed to model the transient characteristics of the disturbance. Also, it is meaningful to consider iterative learning control problems within the context of an impulsive dynamic framework. For example, the $P$-type ILC scheme is designed to track the desired discontinuous output trajectory [26]. The uniform tracking problem of multi-agent systems subjected to impulsive input is then addressed by employing the ILC strategy with the initial state of iteration [27]. Additionally, numerous research results are available in references [28–35]. However, the impulses in existing studies are not random. Furthermore, random impulsive phenomena are ubiquitous in physical systems and hold substantial practical significance. Consequently, analyzing the iterative learning control problem under the influence of random impulsive disturbances presents a theoretical challenge.

From the above discussion, one knows that that random and transient disturbances disrupt the stability of systems and give rise to challenges for theoretical analysis. Thus, this paper turns its attention to linear systems that face random external disturbances and examines iterative learning control within an impulsive framework. The main contributions of this study are as follows: (1) Robust tracking problem is discussed under the random disturbance at fixed times. The problem is effectively addressed through the application of the impulsive differential systems approach. (2) A rigorous theoretical analysis was performed on scenarios that involve random instantaneous disturbances. Then, the main results show that tracking errors can converge to a bounded region. (3) The ILC strategies in this paper are concise and easy to implement and verify in practice. In addition, the simulation shows that the randomness of the external disturbance is the primary factor affecting the trajectory tracking. Thus, one should pay attention to the random disturbance in applications.

The remainder of this paper is organized as follows. Section 2 presents the preliminaries, problem formulation, and the proposed ILC schemes. Section 3 analyzes the system and derives the main results. Section 4 provides a numerical example to validate the proposed methods. Finally, Section 5 concludes this paper.

Notations: $x = [x_1, \cdots, x_n]^\top$ denotes real vectors, and symbol $\top$ denotes transpose. Let $\|x\|$ denote the norm of a vector $x$. Assume that $A = (a_{ij})_{n\times m} \in \mathbb{R}^{n\times m}$ is a real matrix, and the matrix norm is expressed as $\|A\|$. Let $\mathbf{O}$ and $I$ denote the zero matrix and the

identity matrix, respectively. $\mathcal{J} = [0, T]$ is the time interval with fixed terminal time $T$. $\mathbb{E}(\cdot)$ is used to describe the mathematical expectation of the stochastic vector.

## 2. Preliminaries and Problem Formulation

### 2.1. Preliminaries

Let $C(\mathcal{J}, \mathbb{R}^n)$ denote the space of continuous functions from $\mathcal{J}$ to $\mathbb{R}^n$, endowed with $\|x\|_C := \sup_{t\in\mathcal{J}}\{\|x(t)\|\}$. Then, the space of piecewise continuous functions is defined as $PC(\mathcal{J}, \mathbb{R}^n) := \{x : \mathcal{J} \to \mathbb{R}^n | x \in C((t_j, t_{j+1}], \mathbb{R}^n),\ j = 0, 1, \cdots, m$, and there exist $x(t_j^-), x(t_j^+)$, and $x(t_j^-) = x(t_j)\}$ endowed with $PC$-norm, i.e., $\|x\|_{PC} := \max_{j\in\{0,1,\cdots,m\}} \sup_{t\in(t_j,t_{j+1}]}\{\|x(t)\|\}$. Obviously, $(C(\mathcal{J}, \mathbb{R}^n), \|\cdot\|_C)$ and $(PC(\mathcal{J}, \mathbb{R}^n), \|\cdot\|_{PC})$ are Banach spaces. Furthermore, the $(PC, \lambda)$-norm is defined as $\|x\|_{PC,\lambda} := \max_{j\in\{0,1,\cdots,m\}} \sup_{t\in(t_j,t_{j+1}]}\{\exp(-\lambda t)\|x(t)\|\}$, $\lambda > 0$.

Consider an impulsive differential system as follows:

$$\begin{cases} \dfrac{dx(t)}{dt} = f(t, x(t)), t \in \mathcal{J} \setminus \{t_1, \cdots, t_m\}, \\ x(t_j^+) - x(t_j^-) = h_j(x(t_j^-)), j \in \{1, 2, \cdots, m\}, \\ x(0) = x_0. \end{cases} \tag{1}$$

Based on [25,26], if $f$ and $h_j$ are uniformly Lipschitz, standard arguments guarantee that Equation (1) admits a unique solution $x \in PC(\mathcal{J}, \mathbb{R}^n)$. The solution is given by the following integral equation:

$$x(t) = x_0 + \int_0^t f(s, x(s))ds + \sum_{0<t_j<t} h_j(x(t_j^-)),\ \ t \in \mathcal{J}. \tag{2}$$

In particular, if $f(t, x(t))$ in (1) becomes $Ax(t) + Bu(t)$, then (2) is equal to

$$x(t) = x_0 e^{At} + \int_0^t e^{A(t-s)}Bu(s)ds + \sum_{0<t_j<t} e^{A(t-t_j)}h_j(x(t_j^-)),\ \ t \in \mathcal{J}. \tag{3}$$

### 2.2. Problem Formulation

Consider the dynamics of a linear system as follows:

$$\begin{cases} \dfrac{dx_k(t)}{dt} = Ax_k(t) + Bu_k(t), \\ y_k(t) = Cx_k(t) + Du_k(t), \end{cases} \tag{4}$$

where $t \in \mathcal{J}$, $k$ is the $k$th iteration process, and $x_k(0)$ denotes the $k$th initial state. Here, $x_k(t) \in \mathbb{R}^n$, $u_k(t) \in \mathbb{R}^l$, and $y_k(t) \in \mathbb{R}^m$ are the state, input, and output vectors, respectively. In system (4), $A$, $B$, $C$, and $D$ are matrices of appropriate dimensions. Additionally, two output scenarios are considered in this study: $D = \mathbf{O}$ and $D \neq \mathbf{O}$.

As stated in the Introduction, real-world systems are generally subject to random disturbances. Therefore, for system (4), it is necessary to construct a dynamic model that is subject to randomness. In addition, the system has security mechanisms in place, so disturbances do not always have an effect on it. For the sake of modelling and analysis, this paper supposes that random disturbances occur only at specific moments $\{t_1, t_2, \cdots, t_m\}$. When random and instantaneous disturbances occur, the linear system (4) becomes

$$\begin{cases} \dfrac{dx_k(t)}{dt} = Ax_k(t) + Bu_k(t), \ \ t \in \mathcal{J} \setminus \{t_1, \cdots, t_m\}, \\ x_k(t_j^+) - x_k(t_j^-) = \alpha_{k,t_j}\mu_k(t_j^-), \ \ j \in \{1, 2, \cdots, m\}, \\ y_k(t) = Cx_k(t) + Du_k(t), \end{cases} \tag{5}$$

where $\mu_k(t_j^-)$ denotes the instantaneous external disturbance at time $t_j$. It is bounded, i.e., $\|\mu_k(t_j^-)\| \leq \mu_b$, and $\mu_b$ is a positive constant. The Bernoulli stochastic variable $\alpha_{k,t_j}$ satisfies

$$\text{Prob}\{\alpha_{k,t_j} = 1\} = \mathbb{E}(\alpha_{k,t_j}) = \bar{\alpha}, \ \ 0 \leq \bar{\alpha} < 1, \tag{6}$$

where $\bar{\alpha}$ is a known constant. Furthermore, $\alpha_{k_1,t_1}$ and $\alpha_{k_2,t_2}$ are mutually independent for all $k_1 \neq k_2$ and $t_1, t_2 \in \mathcal{T}$. Thus, $\alpha_{k,t_j}\mu_k(t_j^-)$ indicates the random and instantaneous impulse characteristics of external disturbances. In addition, the meaning of the other parameters is the same as in system (4).

**Remark 1.** *This paper considers the transient and stochastic properties of external disturbances. Thus, dynamic model (5) is more realistic than deterministic impulsive differential systems in [25,26,32]. Moreover, the randomness of transient disturbances also creates some difficulties for theoretical analysis. Accordingly, detailed research is urgently required.*

The main problem with iterative learning control is achieving the tracking of the desired trajectory. As a result, it is necessary to design an appropriate ILC strategy. Let $y_d(t) \in \mathbb{R}^p$ be the desired output. Accordingly, the tracking error is given by $e_k(t) = y_d(t) - y_k(t)$. This paper considers two output modes, i.e., $D = \mathbf{O}$ and $D \neq \mathbf{O}$. To accomplish the tracking task, two different ILC strategies are adopted in the following.

If $D \neq \mathbf{O}$ in (5), the ILC strategy is as follows:

$$u_{k+1}(t) = u_k(t) + \mathcal{K}_1 e_k(t), \tag{7}$$

where $\mathcal{K}_1$ is a learning gain matrix. If $D = \mathbf{O}$ in (5), the ILC strategy can be designed as

$$u_{k+1}(t) = u_k(t) + \mathcal{K}_2 \frac{de_k(t)}{dt}, \tag{8}$$

where $\mathcal{K}_2$ is also a learning gain matrix. For convenience, $\frac{de_k(t)}{dt}$ is represented by $\dot{e}_k(t)$. In general, the initial input $u_0(t)$ in (7) and (8) is chosen arbitrarily. Thereafter, the input control is iteratively improved through repeated training sessions until the tracking objective is ultimately achieved. In fact, control schemes (7) and (8) are classic $P$-type and $D$-type ILC strategies, respectively. They are equipped with a clean design and satisfactory performance, and they are commonly applied in many research studies [26,27,29–31,34]. Motivated by the insights from prior works, this paper continues to investigate the tracking problem by applying these two control algorithms.

**Remark 2.** *In real-world applications, many practical systems exhibit nonlinear characteristics. By utilizing historical experimental data, the ILC strategies in [13,20,21,26,27] achieve the dynamic adjustment of control signals and address the tracking problems in nonlinear systems. Moreover, linearisation techniques can be adopted to analyze and process nonlinear systems. Then, it is feasible to extend ILC schemes (7) and (8) to nonlinear systems.*

**Remark 3.** *While theoretically applicable under noisy conditions, the D-type ILC approach suffers from inherent sensitivity. In low-noise environments, the D-type ILC algorithm is highly effective, with minor noise merely causing slight degradation in steady-state accuracy. While it remains*

*applicable in moderately noisy environments, the incorporation of filtering or robust compensation becomes necessary. However, in high-noise environments, the D-type ILC becomes largely ineffective, necessitating the use of PD-type or robust ILC strategies. This study primarily focuses on the applicable scenarios of the ILC scheme (8).*

This paper focuses on achieving system tracking via ILC algorithms (7) and (8). To achieve this objective, the following assumption and lemma are needed for the theoretical analysis.

**Assumption 1.** *The initial state is invariant across all iterations, i.e.,* $x_k(0) = x_{k+1}(0) \triangleq x_d(0)$.

**Assumption 2.** *Random instantaneous disturbances* $\mu_k(t_j^-)$ *do not occur at the initial moment* $t_0$.

**Lemma 1** (see [36])**.** *Let* $d_k$ *be a sequence of real numbers that converge to the limit* $d_\infty$ *as* $k \to \infty$. *Suppose that* $a_k$ *is a sequence of real numbers such that*

$$pa_k + qa_{k-1} \leq d_k, \quad p > -q \geq 0. \tag{9}$$

*Then, one has*

$$\limsup_{k\to\infty} a_k \leq \frac{d_\infty}{p+q}. \tag{10}$$

## 3. Main Results

Based on ILC schemes (7) and (8), this section investigates the tracking problem in the presence of random impulsive disturbances. The robust convergence of the system trajectories is rigorously analyzed to demonstrate the system's resilience. Furthermore, the proposed strategies are shown to effectively achieve robust tracking.

### *3.1. Robust Convergence Analysis*

**Theorem 1.** *Under Assumptions 1 and 2, ILC strategy (7) is applied to system (5), where* $D \neq \mathbf{O}$ *in the observation equation. Then,* $e_k(t)$ *is uniformly bounded for* $t \in \mathcal{J}$ *as* $k \to \infty$ *in the sense of expectations and the PC-norm if the following inequality is satisfied:*

$$\rho := \|I - D\mathcal{K}_1\| + \|C\|\|B\mathcal{K}_1\|T\exp\big(\|A\|T\big) < 1. \tag{11}$$

**Proof.** In conjunction with the definition of $e_k(t)$, one knows

$$\begin{aligned} e_{k+1}(t) &= y_d(t) - y_{k+1}(t) \\ &= y_d(t) - y_k(t) + y_k(t) - y_{k+1}(t) \\ &= e_k(t) - \big(y_{k+1}(t) - y_k(t)\big). \end{aligned} \tag{12}$$

From system (5) and ILC strategy (7), $y_{k+1}(t) - y_k(t)$ becomes

$$\begin{aligned} y_{k+1}(t) - y_k(t) &= Cx_{k+1}(t) + Du_{k+1}(t) - \big(Cx_k(t) + Du_k(t)\big) \\ &= C\big(x_{k+1}(t) - x_k(t)\big) + D\big(u_{k+1}(t) - u_k(t)\big) \\ &= C\big(x_{k+1}(t) - x_k(t)\big) + D\mathcal{K}_1 e_k(t). \end{aligned} \tag{13}$$

From (12) and (13), it is not hard to get

$$\begin{aligned} e_{k+1}(t) &= e_k(t) - C\big(x_{k+1}(t) - x_k(t)\big) - D\mathcal{K}_1 e_k(t) \\ &= \big[I - D\mathcal{K}_1\big]e_k(t) - C\big(x_{k+1}(t) - x_k(t)\big). \end{aligned} \tag{14}$$

Let $\Delta x_k(t) := x_{k+1}(t) - x_k(t)$; according to (2) and Assumptions 1 and 2, one can obtain

$$\Delta x_k(t) = \int_0^t A\Delta x_k(s)ds + \int_0^t B\mathcal{K}_1 e_k(s)ds + \sum_{0<t_j<t} \left(\alpha_{k+1,t_j}\mu_{k+1}(t_j^-) - \alpha_{k,t_j}\mu_k(t_j^-)\right). \quad (15)$$

Note that $\forall k$, $\|\mu_k(t_j^-)\| \leq \mu_b$, and taking the norm of both sides of Equation (15), one has

$$\begin{aligned} \|\Delta x_k(t)\| &\leq \int_0^t \|A\|\|\Delta x_k(s)\|ds + \int_0^t \|B\mathcal{K}_1\|\|e_k(s)\|ds \\ &\quad + \sum_{0<t_j<t} \left(\alpha_{k+1,t_j} + \alpha_{k,t_j}\right)\mu_b \\ &\leq a(t) + \|A\| \int_0^t \|\Delta x_k(s)\|ds \end{aligned} \quad (16)$$

where $a(t) = \int_0^t \|B\mathcal{K}_1\|\|e_k(s)\|ds + \sum_{0<t_j<t} \left(\alpha_{k+1,t_j} + \alpha_{k,t_j}\right)\mu_b$; this is a nondecreasing function of $t$.

According to Gronwall inequality, (16) becomes

$$\begin{aligned} \|\Delta x_k(t)\| \leq &\left[\int_0^t \|B\mathcal{K}_1\|\|e_k(s)\|ds + \sum_{0<t_j<t} \left(\alpha_{k+1,t_j} + \alpha_{k,t_j}\right)\mu_b\right] \\ &\cdot \exp\left(\int_0^t \|A\|\,ds\right). \end{aligned} \quad (17)$$

From (14) and (17), it is not hard to get

$$\begin{aligned} \|e_{k+1}(t)\| &\leq \|I - D\mathcal{K}_1\|\|e_k(t)\| + \|C\|\|\Delta x_k(t)\| \\ &\leq \|I - D\mathcal{K}_1\|\|e_k(t)\| \\ &\quad + \|C\|\|B\mathcal{K}_1\| \int_0^t \|e_k(s)\|ds \cdot \exp\left(\|A\|t\right) \\ &\quad + \|C\| \sum_{0<t_j<t} \left(\alpha_{k+1,t_j} + \alpha_{k,t_j}\right)\mu_b \cdot \exp\left(\|A\|t\right). \end{aligned} \quad (18)$$

Taking the $PC$-norm on both side of inequality (18), one has

$$\begin{aligned} \|e_{k+1}\|_{PC} &\leq \|I - D\mathcal{K}_1\|\|e_k\|_{PC} + \|C\|\|B\mathcal{K}_1\|\|e_k\|_{PC} T\exp\left(\|A\|T\right) \\ &\quad + \|C\| \sum_{0<t_j<T} \left(\alpha_{k+1,t_j} + \alpha_{k,t_j}\right)\mu_b \exp\left(\|A\|T\right) \\ &\leq \left[\|I - D\mathcal{K}_1\| + \|C\|\|B\mathcal{K}_1\|T\exp\left(\|A\|T\right)\right]\|e_k\|_{PC} \\ &\quad + \|C\|\exp\left(\|A\|T\right) \sum_{0<t_j<T} \left(\alpha_{k+1,t_j} + \alpha_{k,t_j}\right)\mu_b. \end{aligned} \quad (19)$$

Then, taking the expectation of inequality (19), one knows

$$\mathbb{E}(\|e_{k+1}\|_{PC}) \leq \left[\|I - D\mathcal{K}_1\| + \|C\|\|B\mathcal{K}_1\|T\exp\left(\|A\|T\right)\right]\mathbb{E}(\|e_k\|_{PC}) + 2\|C\|\exp\left(\|A\|T\right)m\bar{\alpha}\mu_b. \quad (20)$$

According to Lemma 1 and the condition (11), one can immediately obtain

$$\limsup_{k\to\infty} \mathbb{E}(\|e_{k+1}\|_{PC}) \leq \frac{M}{1-\rho}. \quad (21)$$

where $M = 2\|C\| \exp\left(\|A\|T\right) m\bar{\alpha}\mu_b$. This means that $e_k(t)$ is uniformly bounded in the sense of expectation and $PC$-norm, i.e., robust tracking is achieved. □

As the convergence condition (11) in Theorem 1 is quite stringent, the next corollary proposes an alternative criterion that is easier to verify in practice.

**Corollary 1.** *Under Assumptions 1 and 2, ILC strategy (7) is applied to system (5), where $D \neq \mathbf{O}$ in the observation equation. Then, $e_k(t)$ is uniformly bounded for $t \in \mathcal{J}$ as $k \to \infty$ in the sense of expectations and the $(PC, \lambda)$-norm if the following inequality is satisfied:*

$$\rho_1 := \|I - D\mathcal{K}_1\| < 1. \tag{22}$$

**Proof.** Taking the $(PC, \lambda)$-norm on both side of inequality (18), it is not hard to get

$$\begin{aligned} \|e_{k+1}\|_{PC,\lambda} &\leq \|I - D\mathcal{K}_1\| \|e_k\|_{PC,\lambda} \\ &\quad + \|C\| \|B\mathcal{K}_1\| \|e_k\|_{PC,\lambda} \frac{\exp\left(\|A\|T\right) - 1}{\lambda} \\ &\quad + \|C\| \sum_{0<t_j<T} \left(\alpha_{k+1,t_j} + \alpha_{k,t_j}\right) \mu_b \\ &\leq \left[\|I - D\mathcal{K}_1\| + \|C\| \|B\mathcal{K}_1\| \frac{\exp\left(\|A\|T\right) - 1}{\lambda}\right] \|e_k\|_{PC,\lambda} \\ &\quad + \|C\| \sum_{0<t_j<T} \left(\alpha_{k+1,t_j} + \alpha_{k,t_j}\right) \mu_b. \end{aligned} \tag{23}$$

where $\lambda$ is sufficient large, i.e., $\lambda >> \|A\|$.

Analogous to the subsequent proof of Theorem 1, one has

$$\mathbb{E}(\|e_{k+1}\|_{PC,\lambda}) \leq \left[\|I - D\mathcal{K}_1\| + \|C\| \|B\mathcal{K}_1\| \frac{\exp\left(\|A\|T\right) - 1}{\lambda}\right] \mathbb{E}(\|e_k\|_{PC,\lambda}) + 2m\bar{\alpha}\|C\|\mu_b. \tag{24}$$

According to Lemma 1 and (22), (24) becomes

$$\limsup_{k\to\infty} \mathbb{E}(\|e_{k+1}\|_{PC,\lambda}) \leq \frac{M_1}{1 - \rho_1 - \epsilon}, \tag{25}$$

where $M_1 = 2m\bar{\alpha}\|C\|\mu_b$, and $\epsilon = \|C\| \|B\mathcal{K}_1\| \frac{\exp\left(\|A\|T\right) - 1}{\lambda}$.

Since $\lambda$ is chosen large enough, this means that $\epsilon$ can be small enough. For the inequality $1 - \rho_1 - \epsilon > 0$ to be satisfied, it is sufficient that the condition (22) holds. As a result, $e_k(t)$ is uniformly bounded in the sense of expectations and the $(PC, \lambda)$-norm. □

**Remark 4.** *According to Theorem 1 and Corollary 1, one knows that ILC strategy (7) can effectively achieve robust tracking. Different from [26], the conservatism of convergence results is reduced by applying the $(PC, \lambda)$-norm in the theoretical analysis. In fact, the $(PC, \lambda)$-norm is purely a theoretical analysis tool. It is not used in the implementation process and does not affect control performance. Moreover, Theorem 1 is an extension of the traditional conclusions when $\alpha_{k,t} \equiv 0$, i.e., random disturbances do not occur in systems.*

**Remark 5.** *In practical engineering applications, Assumption 1 often proves to be extremely challenging to strictly satisfy. In the presence of the initial state drift, the control algorithm is merely capable of ensuring the robust convergence of the tracking error [11,19]. Then, the convergence error bound will increase as a result of the combined impact of errors in the initial states and random*

*impulsive disturbances. Accordingly, one can adopt initial-state iterative learning strategies [26,27]. These strategies help prevent initial-state drift.*

### *3.2. Extension to Systems Output Without Control Impact*

In some engineering applications, the system's output is entirely governed by its own state and remains unaffected by input [5,9,10,19,20]. In such cases, ILC strategy (7) can hardly achieve robust tracking of the target trajectory. Thus, ILC strategy (8) is applied to system (5) with $D = \mathbf{O}$ in this subsection.

**Theorem 2.** *Under Assumptions 1 and 2, ILC strategy (8) is applied to system (5), where $D = \mathbf{O}$ in the observation equation. Then, $e_k(t)$ is uniformly bounded for $t \in \mathcal{J}$ as $k \to \infty$ in the sense of expectation and PC-norm if the following inequality is satisfied:*

$$\rho_2 := \|I - CB\mathcal{K}_2\| + \|CA\|\|B\mathcal{K}_2\|T\exp\left(\|A\|T\right) < 1. \tag{26}$$

**Proof.** Note that $e_k(t) = y_d(t) - y_k(t)$, and one obtains

$$e_{k+1}(t) = y_d(t) - y_k(t) + y_k(t) - y_{k+1}(t) = e_k(t) - C\Delta x_k(t),$$

where $\Delta x_k(t) := x_{k+1}(t) - x_k(t)$. Then, it is obtained immediately that

$$\dot{e}_{k+1}(t) = \dot{e}_k(t) - C\Delta\dot{x}_k(t). \tag{27}$$

According to (5) and ILC strategy (8), one has

$$\Delta\dot{x}_k(t) = A\Delta x_k(t) + B\Delta u_k(t) = A\Delta x_k(t) + B\mathcal{K}_2\dot{e}_k(t).$$

Substituting (28) into (27), one obtains

$$\dot{e}_{k+1}(t) = \left[I - CB\mathcal{K}_2\right]\dot{e}_k(t) - CA\Delta x_k(t). \tag{28}$$

Similarly to (15), it is not difficult to get

$$\Delta x_k(t) = \int_0^t A\Delta x_k(s)ds + \int_0^t B\mathcal{K}_2\dot{e}_k(s)ds + \sum_{0<t_j<t}\left(\alpha_{k+1,t_j}\mu_{k+1}(t_j^-) - \alpha_{k,t_j}\mu_k(t_j^-)\right). \tag{29}$$

Note that $\forall k$, $\|\mu_k(t_j^-)\| \le \mu_b$, and taking the norm of both sides of Equation (29),

$$\begin{aligned}\|\Delta x_k(t)\| &\le \int_0^t \|A\|\|\Delta x_k(s)\|ds + \int_0^t \|B\mathcal{K}_2\|\|\dot{e}_k(s)\|ds \\ &\quad + \sum_{0<t_j<t}\left(\alpha_{k+1,t_j} + \alpha_{k,t_j}\right)\mu_b \\ &\le a_1(t) + \|A\|\int_0^t \|\Delta x_k(s)\|ds.\end{aligned} \tag{30}$$

where $a_1(t) = \int_0^t \|B\mathcal{K}_2\|\|\dot{e}_k(s)\|ds + \sum_{0<t_j<t}\left(\alpha_{k+1,t_j} + \alpha_{k,t_j}\right)\mu_b$, and this is also a nondecreasing function of $t$.

Using the Gronwall inequality, (30) becomes

$$\|\Delta x_k(t)\| \le \left[\int_0^t \|B\mathcal{K}_2\|\|\dot{e}_k(s)\|ds + \sum_{0<t_j<t}\left(\alpha_{k+1,t_j} + \alpha_{k,t_j}\right)\mu_b\right]\cdot\exp\left(\int_0^t \|A\|ds\right). \tag{31}$$

Analogous to the proof of process (18)–(20), one can obtain

$$\begin{aligned}\mathbb{E}(\|\dot{e}_{k+1}\|_{PC}) \leq &\Big[\|I - CB\mathcal{K}_2\| + \|CA\|\|B\mathcal{K}_2\|T\exp\big(\|A\|T\big)\Big]\mathbb{E}(\|\dot{e}_k\|_{PC}) \\ &+ 2m\|CA\|\exp\big(\|A\|T\big)\bar{\alpha}\mu_b.\end{aligned} \tag{32}$$

According to Lemma 1 and the condition (26), one can immediately obtain

$$\limsup_{k\to\infty}\mathbb{E}(\|\dot{e}_{k+1}\|_{PC}) \leq \frac{M_1}{1-\rho_2}. \tag{33}$$

where $M_1 = 2m\|CA\|\exp\big(\|A\|T\big)\bar{\alpha}\mu_b$. This implies that $\mathbb{E}(\|\dot{e}_k\|_{PC})$ is uniformly bounded.

Furthermore, note that the following inequality holds,

$$\left\|\int_0^t \dot{e}_k(s)ds\right\|_{PC} \leq \int_0^t \|\dot{e}_k\|_{PC}ds. \tag{34}$$

According to Assumption 1, one has $e_k(0) = C(x_k(0) - x_d(0)) \equiv 0$. As a result, inequality (34) means that

$$\|e_k\|_{PC} \leq \int_0^t \|\dot{e}_k\|_{PC}ds \leq T\|\dot{e}_k\|_{PC}. \tag{35}$$

Then, we have

$$\mathbb{E}(\|e_k\|_{PC}) \leq T\mathbb{E}(\|\dot{e}_k\|_{PC}). \tag{36}$$

Since $\mathbb{E}(\|\dot{e}_k\|_{PC})$ is uniformly bounded, (36) shows that $\mathbb{E}(\|e_k\|_{PC})$ is uniformly bounded, i.e., robust tracking is achieved in the sense of expectations and the $PC$-norm. □

To reduce the conservatism of convergence condition (26), we give the following corollary.

**Corollary 2.** *Under Assumptions 1 and 2, ILC strategy (8) is applied to system (5), where $D = \mathbf{O}$ in the observation equation. Then, $e_k(t)$ is uniformly bounded for $t \in \mathcal{J}$ as $k \to \infty$ in the sense of expectations and the $(PC, \lambda)$-norm if the following inequality is satisfied:*

$$\rho_3 := \|I - CB\mathcal{K}_2\| < 1. \tag{37}$$

**Proof.** The proof of this corollary is similar to that of Corollary 1, so it is omitted here. □

**Remark 6.** *According to Theorem 2 and Corollary 2, ILC strategy (8) is able to achieve robust tracking, even if there is no control impact on the system output. This indicates the effectiveness of the D-type ILC strategy in counteracting random impulsive disturbances.*

**Remark 7.** *Different from earlier studies [26–30], this paper focuses on random impulsive disturbances at fixed times. It is precisely due to the random feature of impulse disturbance that the tracking error can only converge robustly in the expected sense. This shows the main difference between random and deterministic impulsive disturbances. Additionally, accurately determining the prior probabilities in (6) is often challenging in practice. However, this does not compromise the system's performance. Based on (21) and (25), although the tracking error magnitude depends on the prior probability, it remains bounded. Consequently, robust convergence is guaranteed.*

## 4. Illustrative Example

To verify the validity of the theoretical results, a numerical example is provided below.

**Example 1.** *Consider system (4) with a single input and single output and define* $A = \begin{bmatrix} 0 & -0.4 \\ -0.4 & 0 \end{bmatrix}$, $B = \begin{bmatrix} -0.4 \\ 2.0 \end{bmatrix}$, *and* $C = \begin{bmatrix} 0.1 & 0.2 \end{bmatrix}$. *Let* $x_{k+1}(0) = x_k(0) = \begin{bmatrix} 1 \\ -0.5 \end{bmatrix}$. *Moreover, the disturbance times for each iteration are* $t_1 = 0.3$ *and* $t_2 = 0.6$. *Also,* $\bar{\alpha} = 0.5$ *denotes the probability of the external disturbance. Let* $\mu_b = 1$ *be the boundary of disturbance* $\mu_k(t)$, *i.e.,* $\|\mu_k(t)\| \leq \mu_b$. *Further, set the desired trajectory of output as* $y_d(t) = 2\sin(2\pi t)$, $t \in \mathcal{T} = [0, 1]$.

*Case 1. System (5) with* $D \neq \mathbf{O}$.

*In this situation, let* $D = 0.8$ *in system (5). In ILC strategy (7), define* $\mathcal{K}_1 = 0.8$. *The initial input is set as* $u_0(t) = 0$. *By direct computation, we obtain* $\rho = 0.9043 < 1$ *in (11) and* $\rho_1 = \|\mathbf{I} - D\mathcal{K}_1\| = 0.36 < 1$ *in (22). This means the conditions of Theorems 1 and Corollary 1 hold. The tracking performance and errors for Case 1 are depicted in Figure 1.*

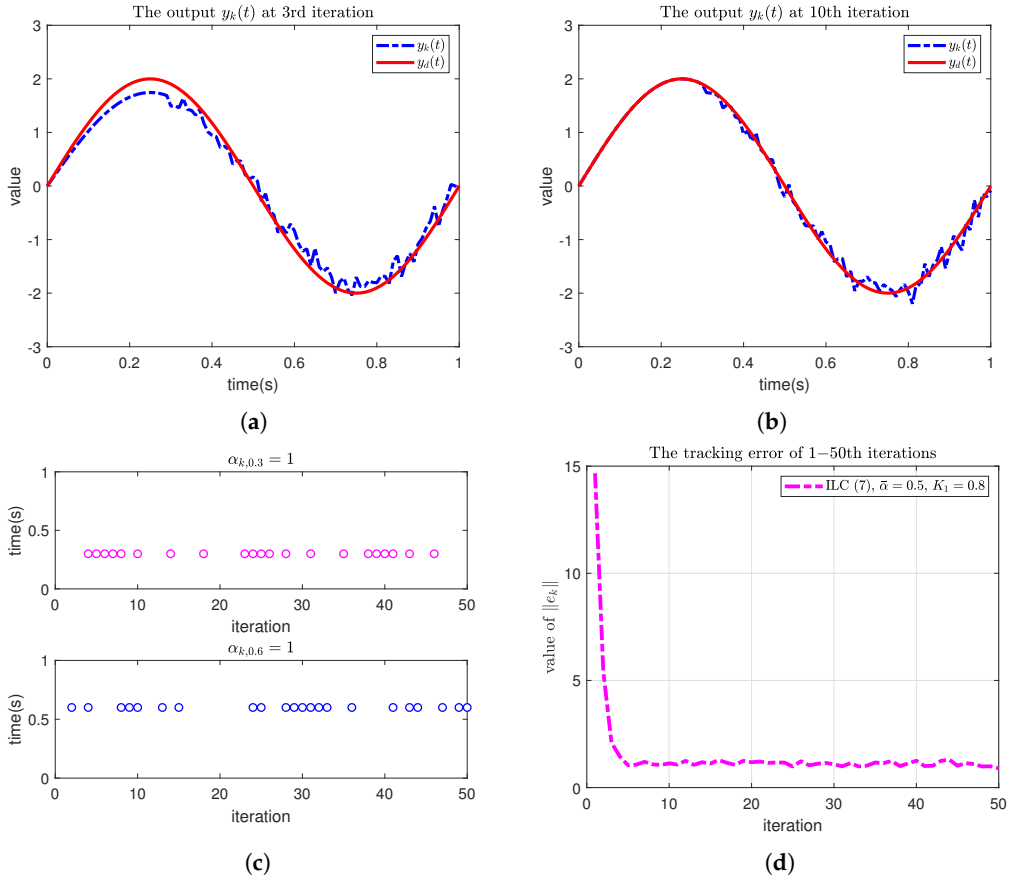



**Figure 1.** Trajectories and the tracking error under random external disturbance in Case 1. (**a**) The tracking trajectory at 3rd iteration. (**b**) The tracking trajectory at 10th iteration. (**c**) Random disturbances occur during the iteration process. (**d**) The tracking error of ILC strategy (7).

*Figure 1a shows the tracking trajectory at the third iteration. Figure 1b shows the tracking trajectory at the* 10*th iteration. According to Figure 1a,b, one knows that Theorem 1 and Corollary 1 in this paper are effective since the tracking error is eventually bounded. Figure 1c shows that random disturbances occur during the iteration process. Also, this implies that the disturbances do not arise at each iterative step. Figure 1d shows the tracking error of ILC strategy (7). According to Figure 1d, the tracking error gradually decreases and eventually fluctuates slightly around a finite value.*

*Case 2. System (5) with* $D \equiv \mathbf{O}$.

*In strategy (8), define* $\mathcal{K}_2 = 1.8$. *Let the initial input be* $u_0(t) = 0$. *By computation, one obtains* $\rho_2 = 0.8419 < 1$ *and* $\rho_3 = \|\mathbf{I} - CB\mathcal{K}_2\| = 0.3520 < 1$; *then, the conditions of Theorems 2 and Corollary 2 hold. The tracking trajectories and errors of Case 2 are shown in Figure 2. Figure 2a*

*shows the tracking trajectory at the* 120*th iteration. Figure* 2*b shows the tracking trajectory at the* 200*th iteration. Figure* 2*c shows that random disturbances occur during* 100*th–*200*th iteration process. From Figure* 2*c, one knows the disturbances do not arise at each iterative step. Figure* 2*d shows that the tracking error gradually decreases and eventually fluctuates slightly around a finite value. Therefore, Figure* 2 *indicates that ILC scheme (*8*) is effective.*

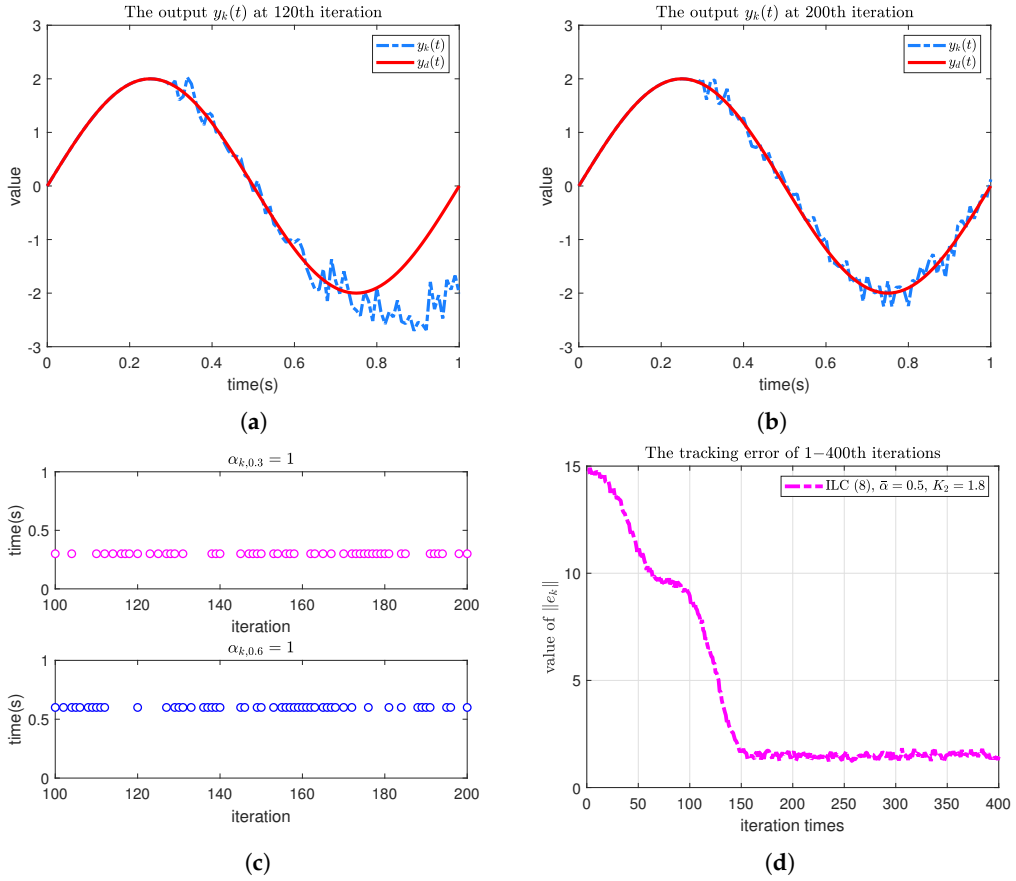



**Figure 2.** Trajectories and the tracking error under random external disturbance in Case 2. (**a**) The tracking trajectory at 120th iteration. (**b**) The tracking trajectory at 200th iteration. (**c**) Random disturbances occur during 100th-200th iteration process. (**d**) The tracking error of ILC strategy (8).

*Figure* 3 *shows the differences between two control algorithms by presenting the tracking errors for various learning parameters and disturbance probabilities. According to Figure* 3*a,c, the results indicate that the ILC strategy (*7*) yields a faster convergence rate. This can be attributed to the control term in the observer, which is designed to mitigate the effects of external disturbances. Moreover, different learning rates result in different rates of error convergence for the same probability of disturbance. A faster convergence rate tends to result in a larger convergence error bound. Nevertheless, navigating the trade-off between the convergence rate and steady-state error bound remains a primary challenge for ILC in practice. The crux of this challenge lies in the presence of non-repetitive disturbances, such as random noise. To achieve such a balance, it is insufficient to rely merely on a single theoretical formula. Instead, parameter tuning must fully consider the system's frequency-domain characteristics, noise intensity, and practical engineering constraints.*

*Figure* 3*b,d illustrate the tracking error of the two ILC strategies when faced with different probabilities of disturbance. As is apparent from this, the magnitude of the disturbance probability has an effect on the convergence error bound. A greater disturbance probability corresponds to more active uncertainties and thus a larger convergence error bound.*

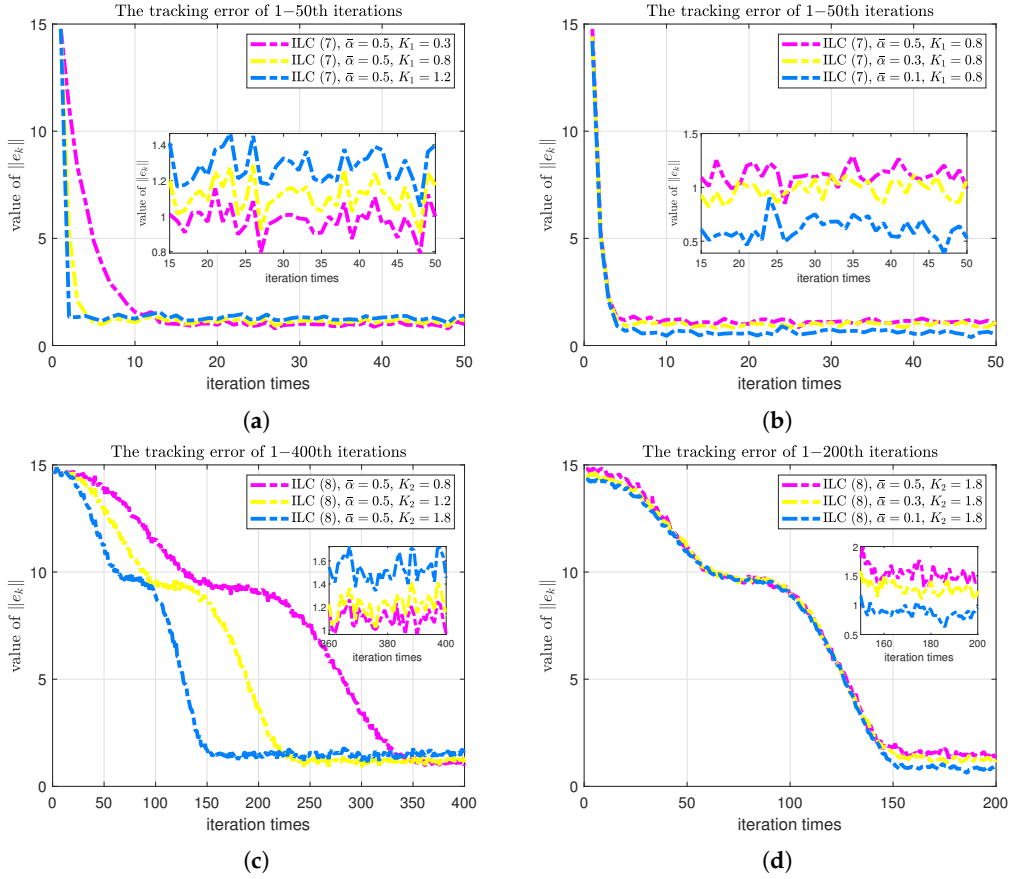



**Figure 3.** The tracking errors under different learning parameters and probabilities of disturbance. (**a**) The tracking error under different $K_1$ in ILC (7). (**b**) The tracking error under different $P\{\alpha_{k,t_j} = 1\}$ in ILC (7). (**c**) The tracking error under different $K_2$ in ILC (8). (**d**) The tracking error under different $P\{\alpha_{k,t_j} = 1\}$ in ILC (8).

*To validate the effectiveness of the proposed ILC strategies, a comparison is conducted with the PD-type ILC algorithm (2) in [33]. These methods are applied to the system described in Example 1, with the resulting tracking errors depicted in Figure 4.*

*In Case 1, as shown in Figure 4a,b, the tracking error of the PD-type ILC is highly sensitive to the error derivative learning gain. If the magnitude of the error derivative becomes excessive, the tracking error is prone to diverge (see Table 1). These results demonstrate that the proposed ILC strategy (7) outperforms the traditional PD-type algorithm in this scenario.*

*In Case 2, Figure 4c reveals that both the PD-type ILC and the proposed method (8) exhibit virtually identical convergence under appropriate learning gains. However, Figure 4d highlights a critical trade-off: While a larger proportional learning gain $K_p$ accelerates convergence for the PD-type ILC algorithm, an excessively large value significantly inflates the error bound and risks divergence (see Table 2). This suggests that to ensure robustness, the PD-type ILC tuning must remain conservative. Furthermore, when subjected to actuator saturation and physical hardware constraints, the proportional error correction term in PD-type ILC is strictly bounded, rendering the proportional gain largely ineffective. Consequently, the derivative term assumes a dominant role, making the dynamic convergence process and steady-state tracking performance of the PD-type ILC practically indistinguishable from those of the proposed ILC strategy (8).*

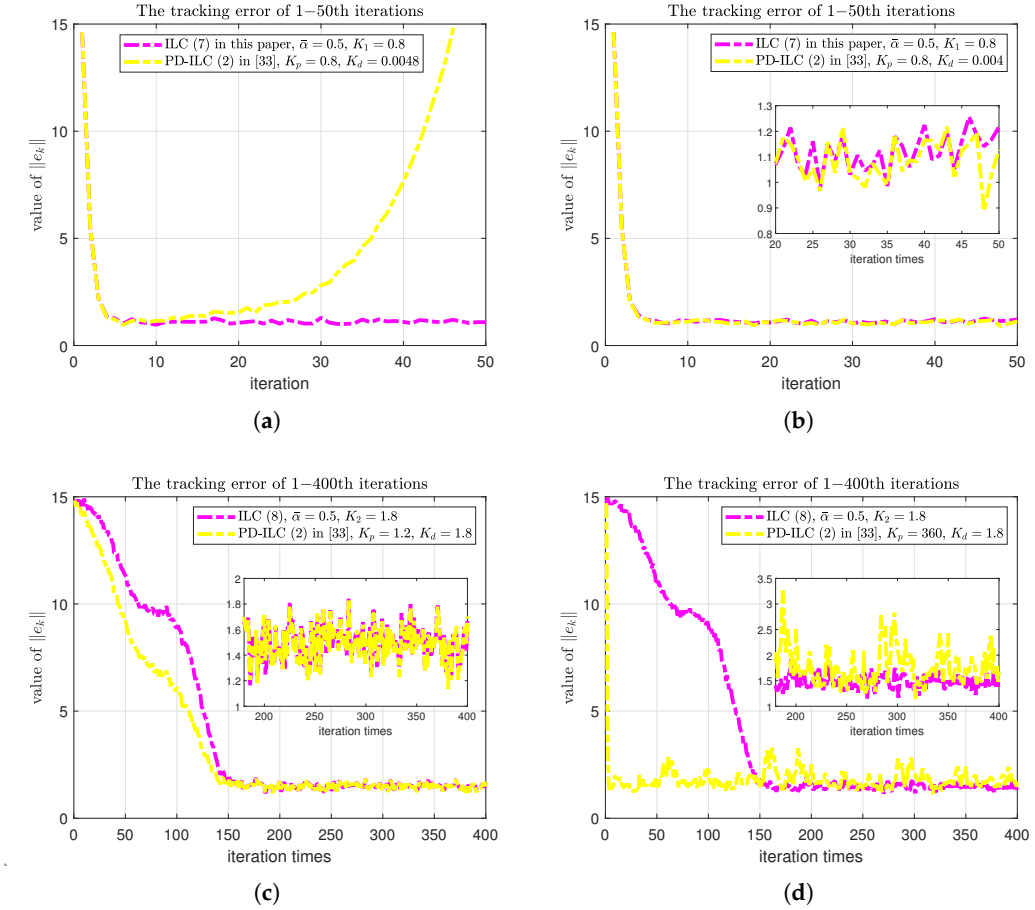



**Figure 4.** The tracking errors under different ILC strategies. (**a**) Case 1: ILC (7) ($K_1 = 0.8$), *PD*-ILC ($K_p = 0.8, K_d = 0.0048$). (**b**) Case 1: ILC (7) ($K_1 = 0.8$), *PD*-ILC ($K_p = 0.8, K_d = 0.0040$). (**c**) Case 2: ILC (8) ($K_2 = 1.8$), *PD*-ILC ($K_p = 1.2, K_d = 1.8$). (**d**) Case 2: ILC (8) ($K_2 = 1.8$), *PD*-ILC ($K_p = 360, K_d = 1.8$).

**Table 1.** Tracking errors of Figure 4a.

| Iteration | 10 | 20 | 30 | 40 |
|---|---|---|---|---|
| ILC (7), $K_1 = 0.8$ | 1.1589 | 1.0227 | 1.2208 | 1.0409 |
| *PD*-ILC (2) in [33], $K_p = 0.8$, $K_d = 0.0048$ | 1.2397 | 1.6136 | 3.2793 | 7.2750 |

**Table 2.** Tracking errors of Figure 4d.

| Iteration | 100 | 200 | 300 | 400 |
|---|---|---|---|---|
| ILC (8), $K_2 = 1.8$ | 8.9473 | 1.5935 | 1.4977 | 1.4448 |
| *PD*-ILC (2) in [33], $K_p = 360$, $K_d = 1.8$ | 2.1757 | 2.3272 | 1.8930 | 1.7133 |

*The above analysis demonstrates that random external disturbances ultimately impair trajectory tracking, rendering the system unable to realize ideal tracking. Fortunately, the ILC schemes proposed in this paper can effectively achieve robust trajectory tracking. This implies that the ILC algorithm can, to some extent, mitigate the impact of uncertain disturbances. Additionally, convergence conditions* (22) *and* (37) *are concise and easy to implement and verify in practice. According to [10], the convergence conditions* (22) *and* (37) *can be formulated as linear matrix inequalities, thereby facilitating the determination of the learning parameters. In additions, given the detrimental impacts induced by random instantaneous disturbances, they should be adequately considered in real-world applications.*

## 5. Conclusions

This paper addressed the tracking control of continuous-time linear systems experiencing random instantaneous disturbances. Treating the system within an impulsive framework, two ILC strategies have been applied to achieve the robust tracking. Based on norm estimation and inequality analyses, sufficient conditions were established to guarantee precise tracking. The derived criteria are tractable and readily applicable to practical implementations. A numerical example has been ultimately presented to corroborate the theoretical results.

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## References

1. Wang, M.; Zhang, Y.; Dong, H.; Yu, J. Trajectory tracking control of a bionic robotic fish based on iterative learning. *Sci. China Inform. Sci.* **2020**, *63*, 170202. [CrossRef]
2. Tao, H.; Zheng, J.; Wei, J.; Paszke, W.; Rogers, E.; Stojanovic, V. Repetitive process based indirect-type iterative learning control for batch processes with model uncertainty and input delay. *J. Process Control* **2023**, *132*, 103112. [CrossRef]
3. Geng, Y.; Ruan, X.; Yang, X.; Zhou, Q. Data-based iterative learning control for multiphase batch processes. *Asian J. Control* **2023**, *25*, 1392–1406. [CrossRef]
4. Zhang, T.; Li, J. Quantized iterative learning control for nonlinear multi-agent systems with limited information communication and input saturation. *J. Frankl. Inst.* **2024**, *361*, 1620–1630. [CrossRef]
5. Chen, B.; Chu, B. Distributed norm optimal iterative learning control for high performance consensus tracking. *IEEE Trans. Auto. Control* **2026**, *71*, 753–768. [CrossRef]
6. Xiong, W.; Xu, L.; Huang, T.; Yu, X.; Liu, Y. Finite-iteration tracking of singular coupled systems based on learning control with packet losses. *IEEE Trans. Syst. Man Cyber.-Syst.* **2020**, *50*, 245–255. [CrossRef]
7. Xiong, W.; Ho, D.W.; Wen, S. A periodic iterative learning scheme for finite-iteration tracking of discrete networks based on FlexRay communication protocol. *Inform. Sci.* **2021**, *548*, 344–356. [CrossRef]
8. Chi, R.; Liu, Z.; Lin, N.; Hou, Z.; Huang, B. Data-driven finite-iteration learning control. *IEEE Trans. Syst. Man Cybern. Syst.* **2024**, *54*, 5296–5306. [CrossRef]
9. Meng, D.; Moore, K.L. Robust iterative learning control for nonrepetitive uncertain systems. *IEEE Trans. Auto. Control* **2017**, *62*, 907–913. [CrossRef]
10. Meng, D.; Zhang, J. Convergence analysis of robust iterative learning control against nonrepetitive uncertainties: System equivalence transformation. *IEEE Trans. Neur. Net. Lear. Syst.* **2021**, *32*, 3867–3879. [CrossRef]
11. Ruan, X.; Hussain, I.; Liu, C.; Liu, Y.; Li, B. An algebraic approach to robust iterative learning control for linear discrete-time singular system with initial state shifting. *IEEE Trans. Syst. Man Cybern. Syst.* **2025**, *55*, 4485–4497. [CrossRef]
12. Liang, J.; Bu, X.; Hou, Z. Finite-time asymmetric bipartite consensus for multi-agent systems using data-driven iterative learning control. *IEEE Trans. Signal Inform. Process. Netw.* **2024**, *10*, 227–238. [CrossRef]
13. Zhang, J.; Meng, D. Cooperative iterative learning for uncertain nonlinear agents in leaderless switching networks. *Automatica* **2021**, *129*, 109692. [CrossRef]
14. Liu, W.; Shi, P.; Pan, J.-S. State estimation for discrete-time markov jump linear systems with time-correlated and mode-dependent measurement noise. *Automatica* **2017**, *85*, 9–21. [CrossRef]
15. Chen, K.; Liu, M.; Zhu, Z.; Wang, J. Distributed observers for linear systems with communication noises over Markovian switching topologies. *Inter. J. Syst. Sci.* **2025**, *56*, 1973–1985. [CrossRef]

16. Cho, S.-I.; Ha, I.-J. A learning approach to tracking in mechanical systems with friction. *IEEE Trans. Auto. Control* **2000**, *45*, 111–116.
17. Riaz, S.; Qi, R.; Tutsoy, O.; Iqbal, J. A novel adaptive PD-type iterative learning control of the PMSM servo system with the friction uncertainty in low speeds. *PLoS ONE* **2023**, *8*, e0279253. [CrossRef] [PubMed]
18. Chin, I.; Qin, S.J.; Lee, K.S.; Cho, M. A two-stage iterative learning control technique combined with real-time feedback for independent disturbance rejection. *Automatica* **2004**, *40*, 1913–1922. [CrossRef]
19. Yang, R.; Gong, Y.; Paszke, W. ILC-based tracking control for linear systems with external disturbances via an SMC scheme. *IEEE Trans. Auto. Sci. Eng.* **2024**, *22*, 9698–9707. [CrossRef]
20. Xiong, W.; Gong, K.; Wen, G.; Lü, J. Security analysis of discrete nonlinear systems with injection attacks under iterative learning schemes. *IEEE Trans. Syst. Man Cybern. Syst.* **2022**, *52*, 927–935. [CrossRef]
21. Yu, W.; Bu, X.; Hou, Z. Security data-driven control for nonlinear systems subject to deception and false data injection attacks. *IEEE Trans. Net. Sci. Eng.* **2022**, *9*, 2910–2921. [CrossRef]
22. Yang, T. *Impulsive Control Theory*; Springer: Berlin/Heidelberg, Germany, 2002.
23. Li, X.; Peng, D.; Cao, J. Lyapunov stability for impulsive systems via event-triggered impulsive control. *IEEE Trans. Auto. Control* **2020**, *65*, 4908–4913. [CrossRef]
24. Liu, B.; Xu, B.; Zhang, G.; Tong, L. Review of some control theory results on uniform stability of impulsive systems. *Mathematics* **2019**, *7*, 1186. [CrossRef]
25. Lakshmikantham, V.; Bainov, D.D.; Simeonov, P. *Theory of Impulsive Differential Equations*; World Scientific: Singapore, 1989.
26. Liu, S.; Wang, J.; Wei, W. A study on iterative learning control for impulsive differential equations. *Commun. Nonlinear Sci. Numer. Simul.* **2015**, *24*, 4–10. [CrossRef]
27. Cao, X.; Feckan, M.; Shen, D.; Wang, J. Iterative learning control for multi-agent systems with impulsive consensus tracking. *Nonlinear Anal. Model. Control* **2021**, *26*, 130–150. [CrossRef]
28. Jin, X.; Wang, J.; Shen, D. Convergence analysis for iterative learning control of impulsive linear discrete delay systems. *J. Diff. Equ. Appl.* **2021**, *27*, 739–762. [CrossRef]
29. Aydin, M.; Mahmudov, N.I. Iterative learning control for impulsive fractional order time-delay systems with nonpermutable constant coefficient matrices. *Intern. J. Adapt. Control Sign. Process.* **2022**, *36*, 1419–1438. [CrossRef]
30. Liu, Q.; Tian, S.; Gu, P. P-type iterative learning control algorithm for a class of linear singular impulsive systems. *J. Frankl. Inst.* **2018**, *355*, 3926–3937. [CrossRef]
31. Raghavan, D.; Nagarajan, S. Convergence on iterative learning control of Hilfer fractional impulsive evolution equations. *Asian J. Control* **2023**, *25*, 2183–2193. [CrossRef]
32. Gao, Z.; Liu, S.; Wang, J. PID-type iterative learning control for impulsive ordinary differential equations. *J. Appl. Math. Comput.* **2017**, *54*, 41–55. [CrossRef]
33. Wang, J.; Fečkan, M.; Liu, S. Convergence characteristics of PD-type and PD$D^{\alpha}$-type iterative learning control for impulsive differential systems with unknown initial states. *J. Vib. Control* **2018**, *24*, 3726–3743. [CrossRef]
34. Vivek, D.; Sunmitha, S.; Elsayed, E.M. P-type iterative learning control for impulsive pantograph equations with Hilfer fractional derivative. *Math. Found. Comput.* **2026**, *10*, 17–34. [CrossRef]
35. Qiu, W.; Wang, J.; Shen, D. Quantized iterative learning control for impulsive differential inclusion systems with data dropouts. *ISA Trans.* **2024**, *145*, 285–297. [CrossRef]
36. Sun, M. Robust convergence analysis of iterative learning control systems. *Control Theory Appl.* **1998**, *15*, 320–326.


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