

Article

Dual-Random Space Vector Pulse Width Modulation Strategy Based on Optimized Beta Distribution

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Abstract: In the control system of a permanent magnet synchronous motor (PMSM) driven by an inverter, the conventional space vector pulse width modulation (SVPWM) strategy introduces high-frequency current harmonics at the switching frequency and its multiples, resulting in significant high-frequency vibrations during motor operation. To address this issue, a dual-random SVPWM strategy is proposed in this paper, which combines a random switching frequency and random zero-vector to spread the spectrum of high-frequency current harmonics. This approach effectively disperses the high-frequency harmonics concentrated at the switching frequency and its multiples, thereby significantly reducing the motor's high-frequency vibrations. Furthermore, to overcome the limitations of the traditional linear congruential method in generating random numbers, the Beta distribution is introduced and improved in this study. The particle swarm optimization (PSO) algorithm is employed to optimize the shape parameters of the Beta distribution, to achieve the optimal random number performance. Finally, experimental validation is conducted under various speed conditions. Compared with the conventional SVPWM strategy, the results demonstrate that the proposed dual-random SVPWM strategy exhibits superior suppression of both high-frequency harmonics and high-frequency vibrations.

Keywords: permanent magnet synchronous motor; current harmonics; dual-random SVPWM; particle swarm optimization; high-frequency vibration



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1. Introduction

Permanent magnet synchronous motors (PMSMs) are increasingly widely used in fields such as new energy vehicles (EVs), industrial automation, and rail transit, owing to their advantages of high-power density, high efficiency, and low maintenance costs. With the continuous expansion of PMSM application fields, the requirements for its operational performance are constantly increasing, among which the motor vibration level has become one of the key indicators for evaluating its performance [1,2]. The space vector pulse width modulation (SVPWM) strategy is widely adopted in PMSM control systems due to its excellent dynamic performance [3]. When the SVPWM strategy adopts constant switching frequency modulation, the pulse positions remain relatively fixed, and the conduction timing of the power switching devices in the inverter remains basically unchanged. This results in the high-frequency harmonic components generated during inverter operation being mainly concentrated at the switching frequency and its integer multiples. These high-frequency harmonics are the primary sources of vibration in PMSMs [4,5], which can seriously weaken the operational stability and reliability of PMSM control systems.

Therefore, research on vibration suppression in PMSM control systems has significant theoretical value and practical engineering importance.

To address the aforementioned issues, scholars have conducted extensive research from the perspective of optimizing modulation strategies. The primary focus has been on random pulse width modulation (RPWM) strategies [6,7], periodic pulse width modulation (PPWM) strategies [8,9], and chaos pulse width modulation (CPWM) strategies [10,11]. Among these three spread-spectrum modulation approaches, the PPWM strategy employs periodic functions to vary the switching frequency, which can limit the range of high-frequency harmonic sidebands. However, the spectrum within these sidebands remains discrete, resulting in weak suppression of harmonic peaks. In addition, the types of periodic functions are too single, some periodic functions are difficult to implement, and the spreading effect depends on waveform characteristics, all of which limits the development of the PPWM strategy. The CPWM strategy superimposes chaotic perturbation values onto a traditional fixed switching frequency, causing the inverter's switching frequency to vary chaotically within a certain range. The high-frequency harmonic suppression effect is highly dependent on the selected chaotic map and initial conditions. If the selection is improper, it may lead to unsatisfactory results. Furthermore, the broadband white noise characteristics of chaotic signals in the frequency domain cause the sideband harmonics around the switching frequency to spread across the entire frequency range, generating a large number of low-frequency harmonics. Compared to PPWM and CPWM, RPWM has the characteristics of simple implementation and high modulation signal flexibility. It exhibits good continuity in the frequency domain, effectively avoiding spectral discretization and presenting smoother spectral characteristics. With an excellent spread spectrum performance, RPWM has shown significant advantages in suppressing high-frequency harmonics and optimizing system vibration, and has therefore been widely applied in many fields.

The commonly used RPWM strategies include random switching frequency PWM (RSFPWM) [12,13], random pulse position PWM (RPPPWM) [14,15], random zero-vector PWM (RZVPWM) [16], and dual-random PWM (DRPWM) [17–20]. Ref. [21] proposes an SVPWM strategy based on the Gaussian distribution for random pulse positioning, which reduces the amplitude of high-frequency harmonics by randomly altering pulse positions. However, random numbers generated from a Gaussian distribution exhibit poor randomness, often appearing consecutively on the same side of the expected value. Moreover, a standalone random pulse position strategy still contains impulse functions in its power spectral density, resulting in harmonic peaks in the power spectrum. Ref. [22] introduces a strategy that combines a current harmonic spectrum shaping algorithm with a random switching frequency. Compared with the traditional RSFPWM strategy, it can further reduce the high-frequency harmonic amplitude at the switching frequency and its integer multiples. But it does not fully utilize the degree of freedom of the pulse position in the switching function. Ref. [23] proposes a novel variable-sequence PWM strategy to suppress high-frequency harmonics and current ripple. But this approach requires real-time prediction of current ripple trajectories and the root mean square (RMS) value of current harmonics. Ref. [24] presents a novel SVPWM strategy that eliminates harmonics and noise at odd multiples of the switching frequency by redesigning the switching sequence. However, this reconstruction of switching states increases system switching losses. Ref. [25] proposes a new random zero-vector strategy that reduces the amplitude of high-frequency harmonics by dynamically adjusting the allocation time of zero-vectors. Nevertheless, this standalone random zero-vector strategy has limitations, such as the modulation index increasing, and the effective duration of zero-vectors decreasing. Considering the influence of pulse width and dead time, the spread-spectrum effect gradually weakens. Reference [26] proposes a

five-phase dual-random SVPWM strategy for harmonic dispersion at switching frequencies and their harmonics, while generating random numbers by introducing a Beta distribution. However, when optimizing the shape parameters of the Beta distribution, the use of the enumeration method can easily lead to deviations in the optimal shape parameters. Therefore, further optimization can be carried out in the generation of random numbers. The particle swarm optimization (PSO) algorithm, inspired by the foraging behavior of bird flocks [27], is a global optimization algorithm widely used in nonlinear optimization problems due to its simplicity and efficiency. Ref. [2] proposes applying the PSO algorithm to optimize the transition probabilities of Markov chains and random gains, two random parameters in RPWM strategies, to reduce the amplitude of sideband harmonics in motor phase currents, this provides a reason for using the PSO algorithm to optimize random parameters in RPWM strategies. Ref. [28] combines the PSO algorithm for optimizing Beta distribution shape parameters with the long short-term memory (LSTM) model for wind power prediction interval calculation, thereby improving prediction accuracy. However, the method of using the PSO algorithm to optimize Beta distribution shape parameters has not yet been applied to optimizing random PWM strategies.

In summary, to effectively suppress vibration noise and enhance electromagnetic compatibility in PMSM drive systems, this paper proposes an optimized Beta distribution-based dual-random SVPWM strategy. Addressing the issue in conventional SVPWM strategies where a constant switching frequency and zero-vector allocation time lead to concentrated high-frequency harmonics at the switching frequency and its integer multiples, the dual-random SVPWM strategy randomizes the switching frequency and zero-vector allocation time. To overcome the limitations of the conventional dual-random SVPWM strategy, which uses the linear congruential generator (LCG) algorithm and suffers from poor randomness, short periodicity, and fixed distribution, this paper proposes using the adjustable shape and high randomness of the Beta distribution to generate random numbers. Additionally, the PSO algorithm is employed to rapidly optimize the shape parameters of the Beta distribution, avoiding the inefficiency and inaccuracy associated with optimal shape parameter selection in conventional enumeration methods. To verify the effectiveness and feasibility of the proposed strategy, comparative experiments were conducted with the conventional SVPWM strategy and the conventional dual-random SVPWM strategy. The results demonstrate that the proposed improved strategy significantly disperses high-frequency harmonic components in the inverter output signal, effectively reducing high-frequency vibrations in the PMSM.

2. Topology of Two-Level Inverter and Principles of Its Modulation Strategy

The topology of a two-level inverter is shown in Figure 1. In the figure, U_{dc} represents the DC-side voltage; C is the DC-side supporting capacitors; i_A , i_B , and i_C denote the three-phase load currents; and each phase leg consists of two switches, S_{x1} and S_{x2} , where $x = A, B, C$. Each phase leg has two valid switching states. Taking phase A as an example, the switching states are presented in Table 1.

Table 1. Switching state of phase A.

Switching State	Output State
S_{A1} ON, S_{A2} OFF	1
S_{A1} OFF, S_{A2} ON	0

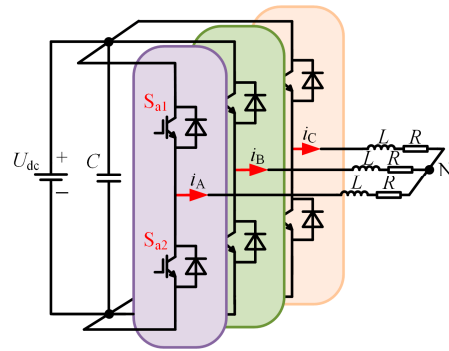


Figure 1. Two-level inverter topology.

From Figure 1 and Table 1, it can be deduced that the inverter has a total of $2^3 = 8$ switching states, corresponding to seven fundamental voltage vectors in the space vector plane, as shown in Figure 2. Among them, the zero-vector V_0 has an amplitude of 0 and corresponds to two switching states; the non-zero vectors V_1 to V_6 have an amplitude of $2U_{dc}/3$ and each corresponds to one switching state.

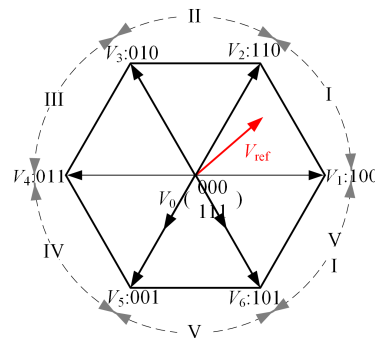


Figure 2. Space vector diagram of a two-level inverter.

The conventional SVPWM strategy is based on the principle of volt second balance, which involves decomposing the product of the reference voltage and the sampling period into the product of the three nearest space vectors and their respective durations. By controlling the switching states of the power devices during the corresponding durations, the three space vectors are output sequentially to achieve modulation. To implement the conventional SVPWM strategy, it is necessary to determine the position of the reference vector and calculate the durations of the fundamental vectors. The durations of each space vector in Figure 2 must satisfy the following conditions:

$$\begin{cases} V_{ref}T_s = V_xT_x + V_yT_y + V_zT_z \\ T_s = T_x + T_y + T_z \end{cases} \quad (1)$$

where V_{ref} represents the reference voltage vector, T_s denotes the sampling period, and V_x , V_y , V_z and T_x , T_y , T_z are the three fundamental vectors closest to the reference voltage vector and their respective durations.

The switching frequency f_s of the conventional SVPWM strategy is a fixed value, and the pulses generated by comparing the modulated wave with the carrier are symmetrical about the carrier vertex, which leads to the concentration of high-frequency harmonics in the output signal at the switching frequency and its multiples. The output signal spectrum of the conventional SVPWM strategy is shown in Figure 3.

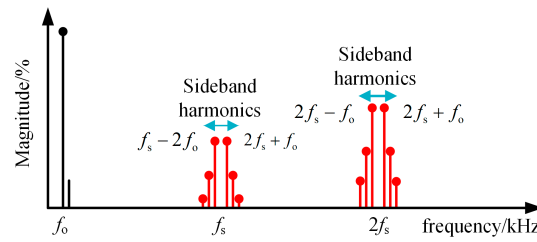


Figure 3. Spectrum diagram of the conventional SVPWM strategy.

3. Conventional Dual-Random SVPWM Strategy

3.1. RZVPWM Strategy

In the conventional SVPWM strategy, the zero-vector duration is uniformly distributed and periodic, which leads to the high-frequency harmonics of the output signal being concentrated at the switching frequency and its integer multiples. However, the duty cycle of the SVPWM strategy is determined solely by the duration of the effective voltage vectors and is independent of the specific allocation method of the zero-vector duration. Therefore, randomizing the allocation of the zero-vector duration can achieve randomization of the pulse conduction positions without affecting the fundamental component of the output voltage. This disrupts the periodic distribution of harmonics in the output signal of the conventional SVPWM strategy, spreading the high-frequency harmonics over a wider frequency range, thereby significantly reducing the harmonic amplitude and improving the high-frequency vibration of the motor. The basic principle of the RZVPWM strategy is illustrated in Figure 4.

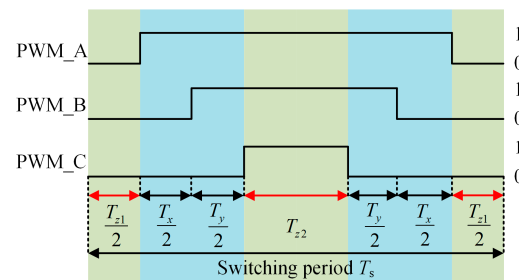


Figure 4. Schematic diagram of the RZVPWM strategy principle.

From Figure 4, the duration of the zero-vector can be expressed as follows:

$$\begin{cases} T_{z1} = R_z T_z \\ T_{z2} = (1 - R_z) T_z \end{cases} \quad (2)$$

where T_z represents the total duration of the zero-vector, T_{z1} denotes the zero-vector duration in the switching state (000), T_{z2} denotes the zero-vector duration in the switching state (111), and R_z is a random number varying between 0.15 and 0.85. By randomizing T_{z1} and T_{z2} , the amplitude of high-frequency harmonics can be reduced.

3.2. RSFPWM Strategy

The RSFPWM strategy builds on the conventional SVPWM strategy by randomizing the switching frequency, thus dispersing the harmonics at the switching frequency and its multiples across a wider frequency domain to achieve the suppression of high-frequency electromagnetic vibrations. The expression for the switching frequency after randomization is given by

$$f'_s = f_s \pm \Delta f \cdot R_f \quad (3)$$

where f_s' represents the randomized switching frequency of the inverter. In this paper, f_s is set to 5 kHz, Δf is set to 1.5 kHz, and R_f is a random number varying between $[0, 1]$. After specifying the initial value of Δf , the system's switching frequency is altered by randomly changing the value of R_f . Typically, if the inverter's switching frequency is too low, it can easily lead to significant electromagnetic vibrations; conversely, if the switching frequency is too high, it increases the energy loss in the power devices, making the system operation unstable. Therefore, appropriately selecting the variation range of the switching frequency is crucial for ensuring stable system operation. The principle of the RSFPWM strategy is illustrated in Figure 5.

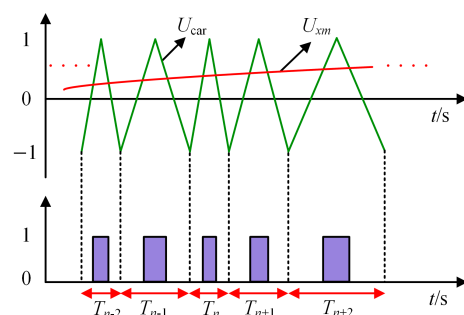


Figure 5. Schematic diagram of the RSFPWM strategy principle.

To address the issue of high-frequency vibration in a PMSM driven by an inverter using the conventional SVPWM strategy, this paper proposes a dual-random SVPWM strategy that combines random zero-vector SVPWM with random switching frequency SVPWM. When applied to the spectral spreading of high-frequency current harmonics, the dual-random SVPWM strategy effectively reduces the motor's high-frequency vibration. The control diagram with the dual-random SVPWM strategy is shown in Figure 6.

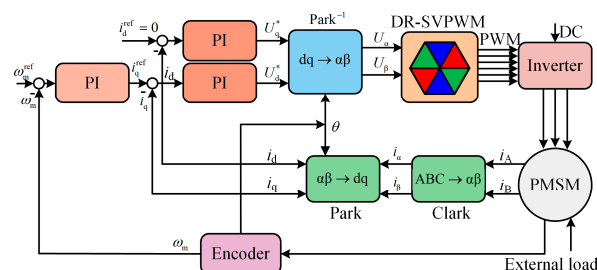


Figure 6. Control diagram with dual-random SVPWM strategy.

When the dual-random SVPWM strategy is applied to motor vibration reduction, the performance of random numbers significantly affects the spread-spectrum effectiveness of the dual-random strategy. The conventional dual-random SVPWM strategy utilizes the LCG algorithm to generate random numbers. The expression for the LCG algorithm is as follows:

$$r_{n+1} = (M_1 \times r_n + M_2) \bmod 2^{N_s} \quad (4)$$

where r_{n+1} and r_n represent the $(n+1)$ th and n th generated random numbers, respectively; M_1 and M_2 are two prime numbers; mod denotes the modulo operation; and N_s is the number of bits of the generated random number.

However, when the LCG algorithm is used to generate random numbers, its randomness is limited by the computer's bit precision and the influence of M_1 and M_2 , resulting in poor randomness and a short periodicity. These factors constrain the spread-spectrum and

vibration reduction effectiveness of the dual-random strategy. Therefore, it is necessary to further optimize the distribution characteristics of the random numbers.

4. Optimization of Random Numbers

4.1. Method Overview

The conventional dual-random SVPWM strategy employs the LCG algorithm to generate random numbers, which exhibit fixed distribution characteristics and lack flexibility, making it impossible to adapt to the specific requirements of harmonic dispersion. This section proposes leveraging the adjustable shape properties of the Beta distribution to generate random numbers in the dual-random SVPWM strategy. Given that the random numbers generated by the Beta distribution are directly influenced by its shape parameters, and to achieve optimal suppression of high-frequency harmonics when these random numbers are applied in the dual-random SVPWM strategy, we propose using the PSO algorithm to rapidly optimize the shape parameters of the Beta distribution, thereby obtaining the best high-frequency harmonic suppression effect.

4.2. The Influence of Different Shape Parameters on the Distribution of Random Numbers

The Beta distribution is a flexible and widely applied continuous probability distribution that can effectively describe the distribution characteristics of variables within the $[0, 1]$ interval. The shape of the Beta distribution is adjustable, making it suitable for generating random numbers that conform to specific distribution shapes. The probability density function of the Beta distribution can be expressed as follows:

$$f(x; a, b) = \frac{x^{a-1}(1-x)^{b-1}}{B(a, b)}, \quad 0 \leq x \leq 1 \quad (5)$$

where a and b are shape parameters, both greater than 0, and $B(a, b)$ is the Beta function. By appropriately selecting the shape parameters a and b , Beta distributions with different shapes can be generated. Depending on the values of the shape parameters a and b , the Beta distribution can primarily be categorized into two distinct cases:

- (1) When $a = b$, the random number distribution exhibits a high degree of symmetry centered around the expected value of 0.5, a characteristic derived from the mathematical properties of the Beta distribution under parameter symmetry.
- (2) When $a \neq b$, the random number distribution displays asymmetry, with its probability density function becoming imbalanced near the expected value of 0.5, skewing toward the side with the larger parameter.

In random PWM strategies, the distribution of random numbers should ideally be symmetrical about the expected value of 0.5. When the distribution of random numbers is asymmetric, it can cause a significant deviation in the overall average switching frequency of the system, manifesting as either an excessively high or low average switching frequency. A higher average switching frequency increases the switching losses of power devices, thereby reducing the system's energy efficiency. Conversely, a lower average switching frequency exacerbates harmonic distortion in the output voltage and current, degrading the waveform quality of the output signal and overall system performance. Both scenarios contradict the original intent of the random strategy, which aims to reduce the amplitude of high-frequency harmonics through randomized switching frequencies. Therefore, this study focuses on the case where the shape parameter $a = b$. Leveraging its symmetric distribution characteristics, the application effects and optimization potential of this case in the dual-random SVPWM strategy are thoroughly investigated to ensure that the system's efficiency and performance meet the expected objectives.

Figure 7 illustrates the Beta distribution under different shape parameters. As shown in Figure 7, when $a = b = 1.5$, the Beta distribution curve exhibits a convex shape symmetrical about the expected value of 0.5. In this case, as the random number approaches the expected value of 0.5, the probability density of the random number increases, and vice versa. For $a = b > 1$, as the values of a and b increase, the probability density of random numbers near the expected value of 0.5 also increases. When $a = b = 1$, the Beta distribution curve becomes a straight line with a constant probability density of 1, indicating that all random numbers have an equal probability density, and the random numbers follow a uniform distribution. When $a = b < 1$, the Beta distribution curve takes on a concave shape symmetrical about the expected value of 0.5. Here, as the random number approaches the expected value of 0.5, the probability density decreases, and vice versa. For $a = b < 1$, as the values of a and b decrease, the probability density of random numbers near the expected value decreases. It is evident that different shape parameters directly influence the distribution characteristics of random numbers. Therefore, to achieve the optimal spread-spectrum vibration reduction effect when applying the dual-random SVPWM strategy, optimizing the selection of shape parameters is crucial.

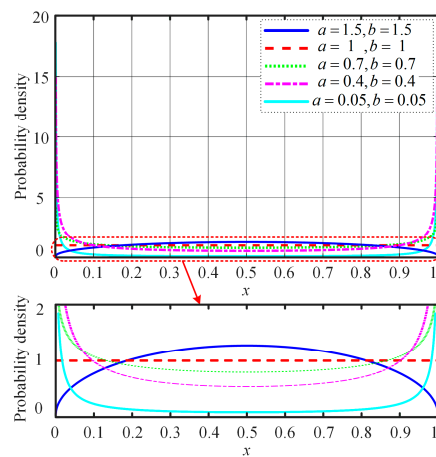


Figure 7. Beta distribution curves under different shape parameters.

4.3. Shape Parameter Optimization Based on PSO Algorithm

To evaluate the effectiveness of the dual-random SVPWM strategy proposed in this paper, the power spectral density (PSD) plot and the HSF are utilized to analyze the impact of different shape parameters on the spread-spectrum effect.

The PSD function of the dual-random SVPWM strategy can be expressed as

$$S_D(\omega, R_T, R_\epsilon) = \frac{1}{E[T_s]} \left\{ E[|F(\omega, R_T, R_\epsilon)|^2] + 2\text{Re} \left[\frac{E[F^*(\omega, R_T, R_\epsilon)] \cdot E[F^*(\omega, R_T, R_\epsilon)e^{j\omega T_s}]}{1 - E[e^{j\omega T_s}]} \right] \right\} \quad (6)$$

where R_T is the reciprocal of R_f , R_ϵ is the pulse delay coefficient corresponding to the random zero vector, F is the Fourier transform of the switching function, F^* represents the complex conjugate of F , T_s is the switching period, and E represents the expectation operator.

The PSD plot, as a critical visualization tool for frequency-domain analysis, can accurately characterize the distribution of signal energy across the frequency axis, thereby providing an intuitive representation of the spectral broadening effect and smoothness achieved by the random strategy during the spread-spectrum process. In contrast to the qualitative analysis provided by the PSD plot, the HSF quantitatively assesses the spread-spectrum effect of the random strategy by calculating the standard deviation of

the harmonic amplitudes in the sample data. A smaller HSF indicates a better harmonic dispersion effect of the employed modulation strategy. The HSF can be expressed as

$$HSF = \sqrt{\frac{1}{N} \sum_{j>1}^N (H_j - H_0)^2} \quad (7)$$

$$H_0 = \frac{1}{N} \sum_{j>1}^N H_j \quad (8)$$

$$H_j = \sqrt{\int_0^1 \int_0^1 f(R_z; a, b) f(R_f; a, b) \left| I(j \cdot f_0, R_z, R_f) \right|^2 dR_z dR_f} \quad (9)$$

where H_0 is the average amplitude of all (N) harmonics excluding the fundamental wave, H_j represents the amplitude of the j -th harmonic, and $I(j \cdot f_0, R_z, R_f)$ represents the Fourier transform of the phase current $i(t)$ at the frequency, with f_0 denoting the spectral resolution. The factors influencing H_j include R_z , R_f , a , b , j , f_0 , and the motor speed. However, when comparing the j -th harmonic amplitudes across different strategies, j , f_0 , and the motor speed should be controlled as identical variables. When random numbers generated by the LCG algorithm and the Beta distribution are applied to R_z and R_f , they will affect H_j .

Since different shape parameters correspond to different Beta distributions, the performance of random numbers generated using the Beta distribution will be affected when applied. To compare the spread-spectrum effects of random numbers under different shape parameters, the PSD plots from simulations are first used for comparison. Taking the shape parameters $a = b = 0.5$, $a = b = 1$, and $a = b = 1.5$ with a speed of 1200 rpm as an example, Figure 8 illustrates the PSD plots of the simulated phase currents for the dual-random SVPWM strategy under these three shape parameter sets.

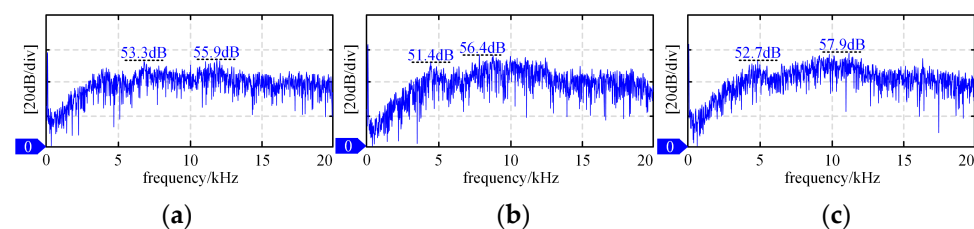


Figure 8. PSD diagram under different shape parameters: (a) $a = b = 0.5$. (b) $a = b = 1$. (c) $a = b = 1.5$.

From the figure above, it can be observed that the harmonic amplitudes at 5 kHz and 10 kHz under the three cases of the dual-random SVPWM strategy exhibit significant differences. This indicates that the performance of random numbers following a Beta distribution largely depends on the choice of shape parameters. The selection of different shape parameters ultimately affects the spread-spectrum effect.

Since the HSF can quantitatively analyze the harmonic spread-spectrum effects of different shape parameters, it is utilized as the evaluation objective function in this study. On this basis, the PSO algorithm is employed to rapidly optimize the shape parameters, aiming to achieve the minimum HSF and thereby obtain the optimal suppression effect on high-frequency vibrations.

The PSO algorithm is a swarm intelligence optimization algorithm inspired by the collective foraging behavior of bird flocks, classified as a type of metaheuristic optimization method. Unlike traditional algorithms such as Simulated Annealing (SA) and the Genetic Algorithm (GA), PSO does not rely on population crossover or mutation operations. Instead, it iteratively approaches the optimal solution through internal interactions among particles. Owing to its characteristics of global high-precision convergence and robust reliability, PSO

has been widely applied in fields such as multi-objective optimization, adaptive control, nonlinear problems, and multidimensional space optimization.

As a random search algorithm, the core driving mechanism of the PSO algorithm involves the shared iterative updating of the global historical best solution G_{best} and the individual historical best solution P_{best} . This process enables the optimization of both individual extrema and the global optimum of the particle swarm. In this paper, each particle represents a candidate solution vector $P(a_i, b_i)$ for the shape parameters of the Beta distribution, where $a_i = b_i$. The particle's position defines the distribution characteristics of random numbers R_z and R_f . The objective function is the HSF, and optimizing a_i and b_i minimizes the HSF, thereby reducing the high-frequency harmonic peak. The specific workflow of the PSO algorithm is illustrated in Figure 9.

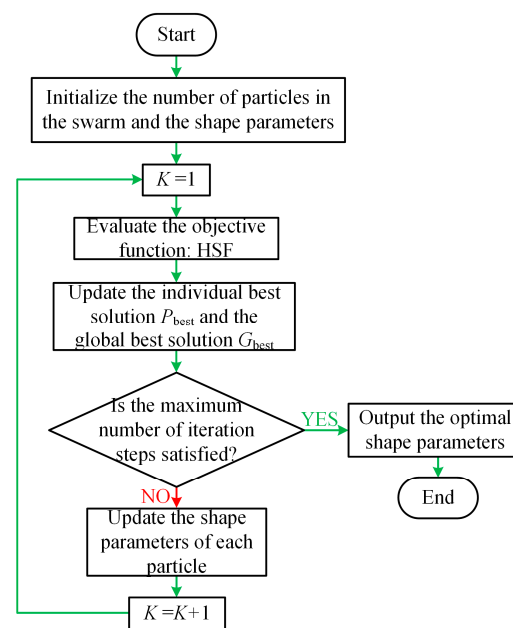


Figure 9. Flowchart of the PSO algorithm.

The screening and solving process for the global optimal solution G_{best} is as follows:

- (1) Calibration of initial system parameters: Based on the phase current data collected from experiments, calibrate parameters such as winding inductance and resistance in the simulation model to ensure that the simulation model aligns with experimental results under various operating conditions.
- (2) Initialize the particle swarm: Restrict the particle range to 0.01~2. Each particle in the swarm contains basic information, namely the shape parameters a and b . During each iteration, the individual best solution P_{best} of each particle is compared and updated with the global best solution G_{best} . Since this study involves an optimization problem with only two parameters, a population of 20 particles is sufficient to cover the solution space, providing adequate diversity to avoid premature convergence to suboptimal solutions. Additionally, the maximum number of iterations is chosen as the convergence criterion, with 60 iterations allowing the algorithm sufficient time to refine the solution. A dynamic inertia weight strategy is adopted, with the weight value linearly decreasing from 0.9 to 0.4 during the iteration process. A larger weight in the early stages ensures a strong global search capability, while a smaller weight in the later stages enhances the local search capability. The learning factors are set to $c1 = c2 = 2.0$, achieving a balance between individual experience and collective collaboration, thus preventing premature convergence to local optima.

- (3) Run the steady-state condition Simulink simulation program, generate the phase current time-domain waveform data in the MATLAB (Version: 9.5.0.944444, R2018b) (MathWorks, Inc., Natick, MA, USA) workspace, and then perform time-frequency conversion on the data using the fast Fourier transform (FFT) program, followed by calculating the HSF, and finally, extract the individual optimal solution P_{best} and the global optimal solution G_{best} .
- (4) Termination condition setting: In the conventional PSO algorithm, termination conditions typically include the number of iteration steps and convergence criteria. To ensure population diversity, this paper uses the number of iteration steps as the termination condition to prevent premature convergence or excessive iteration without convergence of the particle swarm.

The optimization results of the Beta distribution shape parameters based on the PSO algorithm are shown in Figure 10. The HSF stabilizes within 46 iterations, and as the number of iterations continues to increase, the HSF no longer decreases. At this point, the shape parameters a and b are both equal to 0.68, with an HSF of 1.796. Therefore, under these shape parameters, the random numbers generated by the Beta distribution, when applied to the dual-random SVPWM strategy, achieve the optimal spectrum-spreading effect.

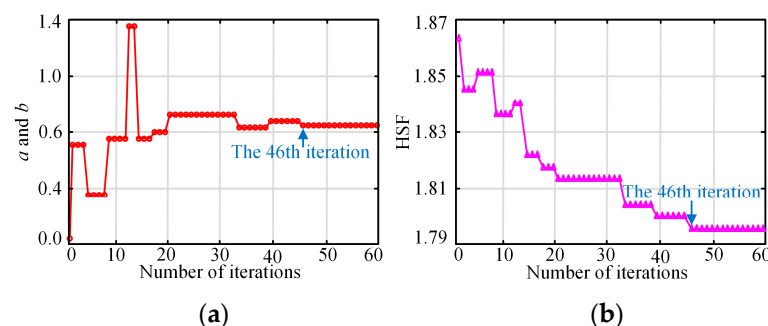


Figure 10. Optimization results of the PSO algorithm: (a) a and b after iteration. (b) HSF after iteration.

Furthermore, Figure 11a,b, respectively, depict an autocorrelation function (ACF) graph and absolute autocorrelation function ($|ACF|$) graph comparing the randomness of random numbers generated by the Beta distribution and the LCG algorithm.

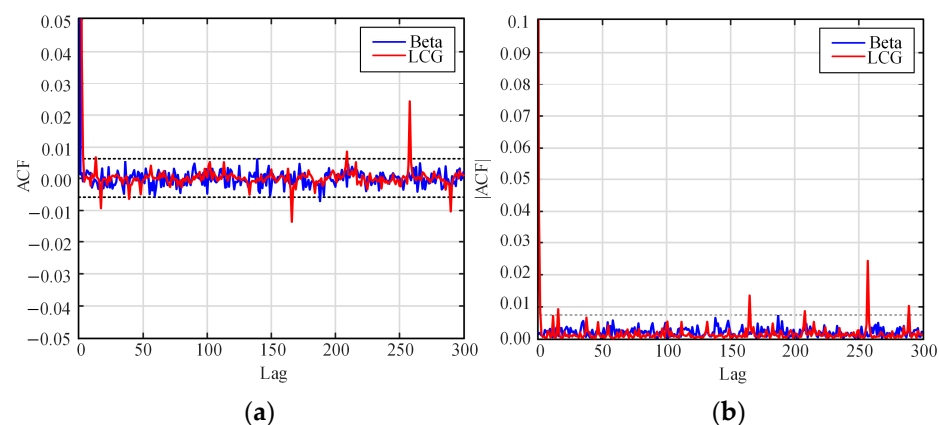


Figure 11. ACF graph and $|ACF|$ graph of different random number algorithms: (a) ACF graph. (b) $|ACF|$ graph.

From the ACF graph, it can be observed that the random numbers generated by the Beta distribution exhibit an ACF with very small fluctuations across all lag points, with values close to 0 and consistently within the confidence interval (indicated by dashed lines),

suggesting that the Beta distribution has almost no autocorrelation; in contrast, the random numbers generated by the LCG algorithm also show an ACF close to 0 at most lags, but they exhibit multiple significant peaks (e.g., around lags 50, 150, and 250), which exceed the confidence interval, indicating clear periodic characteristics.

From the $|ACF|$ graph, it can be observed that the random numbers generated by the Beta distribution exhibit consistently low $|ACF|$ values, fluctuating between 0 and 0.01, indicating extremely low autocorrelation strength. In contrast, the random numbers generated by the LCG algorithm also show low $|ACF|$ values at most lags, but there are several significant peaks (e.g., around lags 50, 150, and 250), corresponding to the ACF peaks observed in the left graph.

Table 2 compares the mean absolute autocorrelation coefficient (MAACF), maximum absolute autocorrelation coefficient (MaxAACF), and the number of significant non-zero lags (NSNLs) of the random numbers generated by these two algorithms.

Table 2. Comparison of randomness metrics for Beta distribution and LCG algorithm.

Algorithm	MAACF	MaxAACF	NSNL
Beta distribution ($a = b = 0.68$)	0.001767	0.007074	2
LCG algorithm	0.002116	0.199811	10

From the table above, it can be observed that the random numbers generated by the Beta distribution have lower values in MAACF = 0.001767, MaxAACF = 0.007074, and NSNL = 2 compared to those generated by the LCG algorithm (MAACF = 0.002116, MaxAACF = 0.199811, NSNL = 10), further indicating that the random numbers generated by the Beta distribution exhibit superior randomness compared to the LCG algorithm.

5. Experimental Results

This paper takes a 4.3 kW PMSM as the research object. To verify the feasibility and superiority of the proposed dual-random SVPWM strategy based on an optimized Beta distribution, experimental validation and analysis were conducted. The proposed strategy was compared with the conventional SVPWM strategy and the conventional dual-random SVPWM strategy in terms of harmonic amplitude and vibration acceleration peak values at different frequencies. The experimental platform is illustrated in Figure 12. This experiment utilized the RT-LAB platform (OPAL-RT Technologies Inc., Montreal, QC, Canada) to implement the system control algorithm, with a two-level inverter serving as the power circuit. Current and voltage data were collected using a Yokogawa high-performance signal analysis oscilloscope, while vibration acceleration signals were acquired using an accelerometer and a Siemens multifunctional data acquisition device. Table 3 classifies different strategies.

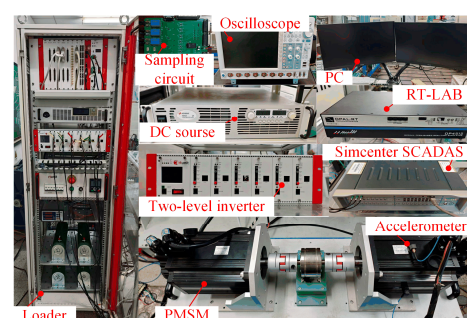


Figure 12. Experimental platform.

Table 3. Classification of different random strategies.

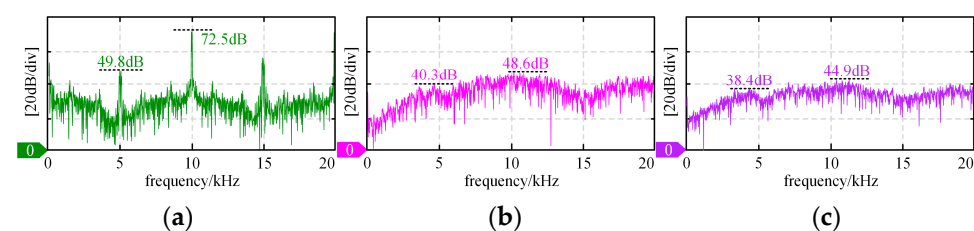
Strategy	Principle
Strategy I	Conventional SVPWM strategy
Strategy II	Dual-random SVPWM strategy based on the LCG algorithm
Strategy III	Dual-random SVPWM strategy based on an optimized Beta distribution

The experimental system parameters are presented in Table 4. The experiments in this study used 300 rpm and 1200 rpm as test speeds based on the following considerations: 300 rpm represents low-speed operation, commonly observed in low-speed cruising for electric vehicles or low-speed servo scenarios in industrial automation; 1200 rpm is close to the rated speed (Table 4 in the paper indicates a rated speed of 1500 rpm), representing medium-to-high-speed operation. Furthermore, as evidenced by the PSD plots from the simulations, Strategy III demonstrates superior harmonic suppression even under other speed conditions. By combining these aspects, the selection of these two speeds effectively reflects the low-speed and high-dynamic-response performance of PMSM applications.

Table 4. Experimental system parameters.

Parameter	Value
Rated voltage/V	220
Rated current/A	20
Rated speed/rpm	1500
V_{DC} bus voltage/V	350
Fixed switching frequency/kHz	5
Random switching frequency variation range/kHz	3.5~6.5

Figures 13 and 14 show the PSD plots of the phase currents for the three strategies at different speeds. To facilitate a comparison between the strategies, the harmonic amplitudes at 5 kHz and 10 kHz frequencies are summarized in Table 5. At a speed of 300 rpm, at the 5 kHz frequency, Strategy II reduces the harmonic amplitude by 9.5 dB compared to Strategy I, while Strategy III further reduces it by 1.9 dB compared to Strategy II; at the 10 kHz frequency, Strategy II reduces the harmonic amplitude by 23.9 dB compared to Strategy I, and Strategy III further reduces it by 3.7 dB compared to Strategy II. At a speed of 1200 rpm, comparing the harmonic amplitudes at 5 kHz, Strategy II achieves a reduction of 15.6 dB compared to Strategy I, with Strategy III further reducing it by 4.5 dB compared to Strategy II. At 10 kHz, Strategy II reduces the harmonic amplitude by 22 dB compared to Strategy I, and Strategy III further reduces it by 1.8 dB compared to Strategy II. Thus, it can be concluded that, across different speeds, Strategy III exhibits the lowest harmonic amplitudes at both 5 kHz and 10 kHz.

**Figure 13.** PSD graphs of phase currents for three strategies at 300 rpm: (a) Strategy I. (b) Strategy II. (c) Strategy III.

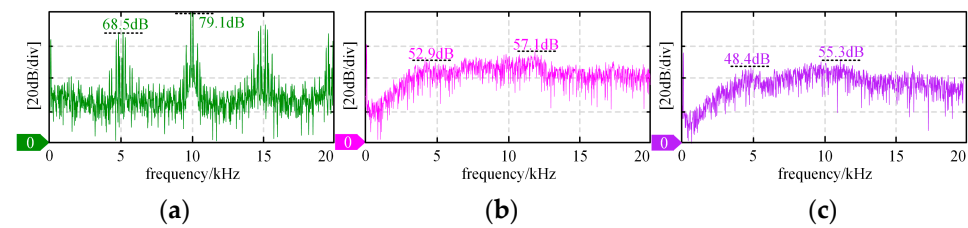


Figure 14. PSD graphs of phase currents for three strategies at 1200 rpm: (a) Strategy I. (b) Strategy II. (c) Strategy III.

Table 5. Comparison of harmonic amplitudes at different speeds.

Strategy	300 rpm		1200 rpm	
	5 kHz	10 kHz	5 kHz	10 kHz
Strategy I	49.8 dB	72.5 dB	68.5 dB	79.1 dB
Strategy II	40.3 dB	48.6 dB	52.9 dB	57.1 dB
Strategy III	38.4 dB	44.9 dB	48.4 dB	55.3 dB

Figures 15 and 16 illustrate the vibration acceleration plots for the three strategies at different speeds. To facilitate comparison between the strategies, the peak vibration acceleration values at 5 kHz and 10 kHz frequencies are summarized in Table 6. At a speed of 300 rpm, at the 5 kHz frequency, Strategy II reduces the peak vibration acceleration by 0.0174 g compared to Strategy I, while Strategy III further reduces it by 0.0004 g compared to Strategy II; at the 10 kHz frequency, Strategy II reduces the peak vibration acceleration by 0.0691 g compared to Strategy I, and Strategy III further reduces it by 0.0063 g compared to Strategy II. At a speed of 1200 rpm, at the 5 kHz frequency, Strategy II reduces the peak vibration acceleration by 0.0386 g compared to Strategy I, with Strategy III further reducing it by 0.0008 g compared to Strategy II; at the 10 kHz frequency, Strategy II reduces the peak vibration acceleration by 0.1732 g compared to Strategy I, and Strategy III further reduces it by 0.0085 g compared to Strategy II. Therefore, it can be concluded that, across different speeds, Strategy III consistently exhibits the lowest peak vibration acceleration at both 5 kHz and 10 kHz.

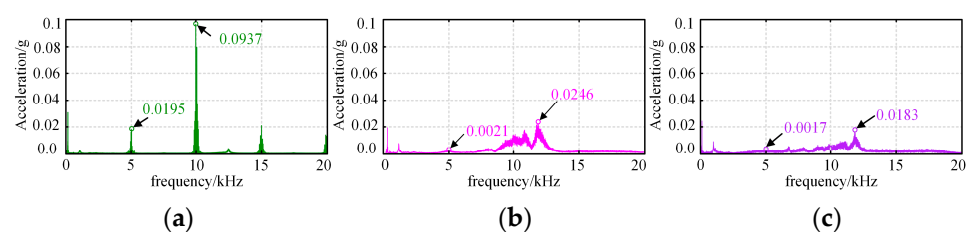


Figure 15. High-frequency vibration acceleration graphs for three strategies at 300 rpm: (a) Strategy I. (b) Strategy II. (c) Strategy III.

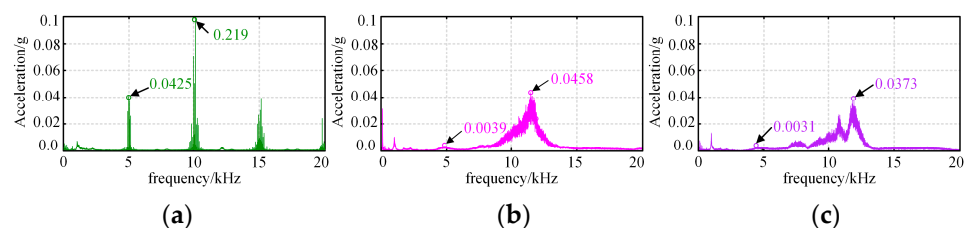


Figure 16. High-frequency vibration acceleration graphs for three strategies at 1200 rpm: (a) Strategy I. (b) Strategy II. (c) Strategy III.

Table 6. Comparison of vibration acceleration peaks at different speeds.

Strategy	300 rpm		1200 rpm	
	5 kHz	10 kHz	5 kHz	10 kHz
Strategy I	0.0195 g	0.0937 g	0.0425 g	0.2190 g
Strategy II	0.0021 g	0.0246 g	0.0039 g	0.0458 g
Strategy III	0.0017 g	0.0183 g	0.0031 g	0.0373 g

As Strategy I employs a constant switching frequency and fixed zero-vector allocation modulation, pulse positions are relatively fixed. This causes the high-frequency harmonic components generated during inverter operation to form clustered harmonic spikes at the switching frequency and its multiples. Strategy II uses the LCG algorithm to generate random numbers. However, the random numbers generated by the LCG algorithm exhibit poor randomness and short periodicity, and they cannot be adjusted based on specific harmonic dispersion requirements. Consequently, although Strategy II introduces a degree of randomness, its harmonic dispersion effect remains limited, achieving only a moderate reduction in harmonic peaks, while harmonic spikes persist.

Compared to Strategy I, this paper proposes randomizing the switching frequency and zero-vector allocation time to avoid high-frequency harmonics caused by a fixed switching frequency and fixed zero-vector allocation time. Compared to Strategy II, this paper proposes using the Beta distribution to generate random numbers, addressing the issues of poor randomness and short periodicity in random numbers generated by the LCG algorithm. Additionally, it combines the PSO algorithm to rapidly optimize the shape parameters of the Beta distribution, improving the quality and generation efficiency of random numbers. Consequently, compared to Strategies I and II, Strategy III is optimal, achieving the lowest harmonic amplitudes at the switching frequency and its multiples, and minimizing the peak vibration acceleration of the permanent magnet synchronous motor.

Theoretically, increasing Δf enhances high-frequency harmonic dispersion but tends to introduce low-frequency harmonics, leading to distortion in the inverter's output line voltage, increased current fluctuations, and potential EMC compliance issues. Conversely, reducing Δf weakens the dispersion effect but improves control precision. Variations in R_z also affect harmonic dispersion: a smaller R_z range reduces dispersion capability, while excessive randomness in R_z increases waveform fluctuations, impacting precision in industrial automation applications.

From the above analysis, it is evident that, compared to the conventional dual-random SVPWM strategy, the use of the dual-random SVPWM control based on an optimized Beta distribution in a PMSM can effectively achieve the effects of spectrum spreading and vibration reduction.

6. Discussion

In this study, the PSO algorithm is executed offline. Once the optimal parameters are determined, they are fixed and hard-coded into the control system. Consequently, there is no need to run the PSO algorithm in real-time during motor operation. For random number sampling from the Beta distribution, this study employs the inverse transform sampling method, which can be efficiently implemented in real-time systems and is computationally feasible for embedded systems. By pre-computing a lookup table for the cumulative distribution function (CDF), random numbers can be rapidly generated during real-time operations. Furthermore, the FFT and HSF calculations in this study are primarily used for offline analysis and evaluation and are not part of the real-time control loop. During real-time operation, the control algorithm relies solely on pre-optimized shape parameters and random number generation, without requiring FFT or HSF computations.

Although this study employs the PSO algorithm to optimize the shape parameters of the Beta distribution, with the optimized parameters subsequently hard-coded into the system—a process that does not directly impact real-time response time—the offline optimization process itself demands significant computational resources and high-performance computing equipment, thereby increasing development costs. The PSO algorithm involves multiple iterations (60 iterations with 20 particles in this study), each requiring Simulink simulations, FFT computations, and HSF calculations, which may pose challenges for resource-constrained research environments or small development teams.

The offline PSO algorithm for optimizing Beta distribution shape parameters involves 60 iterations with 20 particles, each requiring Simulink simulations, FFT, and HSF calculations, necessitating high-performance computing resources but not affecting real-time operations. Real-time computations encompass Beta distribution random number generation, utilizing the inverse transform sampling method to compile a pre-computed cumulative distribution function lookup table for generating random parameters R_z and R_f , which consumes control loop time and may introduce delays. Additionally, dynamic PWM adjustments for the switching frequency and zero-vector allocation time increase the computational burden of the control loop, potentially interfering with other tasks. These factors impact the response time of the PMSM control system through computational delays and timing conflicts. For field-oriented control (FOC) and direct torque control (DTC), the delays are typically acceptable. However, for model predictive control (MPC), delays may lead to accumulated prediction errors, requiring compensation mechanisms. Furthermore, timing conflicts can cause sampling window offsets, compromising the accuracy of feedback signals.

The concentration of high-frequency harmonics is a primary source of electromagnetic interference (EMI). The strategy proposed in this study randomizes the switching frequency (3.5–6.5 kHz) and zero-vector allocation time (based on random numbers R_z and R_f), dispersing the high-frequency harmonics, which are typically concentrated at the switching frequency and its multiples (e.g., 5 kHz, 10 kHz) in conventional SVPWM, across a broader spectrum. This dispersion reduces the energy at individual frequency points, thereby mitigating electromagnetic radiation and conducted interference, and enhancing the system's electromagnetic compatibility (EMC). Compared to the random numbers generated by the conventional LCG algorithm, which suffer from poor randomness and short periodicity, the high-quality random numbers produced by the Beta distribution (with optimized parameters $a = b = 0.68$) through the PSO algorithm ensure more uniform spectrum spreading. The superior randomness of the Beta distribution is validated in this study through the autocorrelation function (ACF) and absolute autocorrelation coefficients (MAACF = 0.001767, MaxAACF = 0.007074), which reduce periodic harmonic peaks and further lower EMI risks. However, while high-frequency harmonics are effectively dispersed, the randomization process may introduce low-frequency harmonics. In this study, the proposed dual-random SVPWM strategy exhibits high stability under light load conditions, attributed to low current demands and effective high-frequency harmonic dispersion, making it suitable for applications such as small servo motors and fan drives. However, stability may be compromised under high load conditions due to increased current harmonics and response lag induced by computational delays, necessitating self-adaptive parameter correction and overload protection to enhance fault tolerance. By implementing control optimization, fault detection algorithms, and parameter adaptation, issues related to low-frequency harmonics, delays, and fault diagnosis can be mitigated, ensuring the strategy's robustness and reliability across industrial applications ranging from low-power to high-performance scenarios.

The proposed solution in this study primarily achieves high-frequency harmonic dispersion by modifying the PWM signal generation module, which can be integrated as an independent module into existing PMSM control systems. The PSO algorithm optimizes the Beta distribution shape parameters offline and hard-codes them, with real-time operations limited to random number generation and PWM adjustments, decoupled from upper-level control logic (such as FOC, DTC, and MPC), thereby reducing integration complexity. Modern PMSM control systems, typically based on high-performance DSPs, FPGAs, or MCUs, possess the capability to support dynamic PWM adjustments, store CDF lookup tables, and handle additional computational loads. However, PWM modules in low-end systems may lack support for dynamic frequency adjustments or sufficient clock precision, resulting in modulation errors. The CDF lookup table requires 10–100 KB of storage space, which may exceed the memory capacity of low-end MCUs, necessitating external Flash storage or table compression. Hardware incompatibility in low-end MCUs may lead to signal distortion, thereby reducing the effectiveness of harmonic dispersion.

7. Conclusions

To address the high-frequency vibration issue of PMSM, this paper proposes an optimized Beta distribution-based dual-random SVPWM strategy to suppress high-frequency harmonics and reduce motor high-frequency vibrations. The advantages of the proposed method are summarized as follows:

- (1) To address the high-frequency vibration issue in conventional SVPWM strategies caused by a constant switching frequency and fixed zero-vector allocation time, randomization of the switching frequency and zero-vector allocation time is proposed to reduce high-frequency vibrations in PMSM.
- (2) To address the issues of poor randomness and short periodicity of random numbers generated by the LCG algorithm in conventional dual-random SVPWM strategies, the use of the Beta distribution for random number generation is proposed to improve the quality of random numbers.
- (3) To address the inefficiency of using enumeration methods to find optimal shape parameters in traditional Beta distribution-based random number generation, a PSO algorithm is proposed to rapidly optimize shape parameters, thereby improving efficiency.

In summary, the proposed method not only improves the efficiency and quality of random number generation but also further suppresses high-frequency harmonics in phase currents, thereby reducing high-frequency vibrations in PMSM.

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