

Article

Detection-Aided Ordering for LMMSE-ISIC in MIMO Systems

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Abstract: In this paper, the detection-aided ordering schemes are proposed for linear minimum mean-squared-error (LMMSE) iterative soft interference cancellation (ISIC) in multiple-input multiple-output (MIMO) systems. Unlike the conventional LMMSE-ISIC ordering schemes that utilize the channel state information (CSI) only, the proposed ordering schemes utilize the receive signal vector and CSI for the ordering procedure. Then, for each candidate symbol, the sum of the likelihoods except the largest likelihood is calculated to estimate the reliability of the candidate symbol, where the likelihoods are calculated by the LMMSE or LMMSE-ISIC detection-aided ordering procedure. Thus, the proposed ordering schemes can provide a significantly more accurate ordering result than the conventional ordering schemes. As the detection-aided ordering schemes, non-iterative and iterative ordering schemes are proposed, and the constrained iterative ordering scheme is also proposed to resolve the high computational complexity of the original iterative ordering scheme. Numerical simulation results verify that the proposed detection-aided ordering schemes outperform the conventional ordering schemes in terms of convergence speed and error performance.

Keywords: LMMSE-ISIC; MIMO; ordering; iterative detection; likelihoods



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1. Introduction

Multiple-input multiple-output (MIMO) systems have been extensively studied as a key technology of wireless communication systems for recent decades [1–5], and they are adopted in modern communication standards such as 4 G long-term evolution (LTE) [6,7] and 5 G new ratio (NR) [8,9]. MIMO systems are designed to transmit and receive multiple signals without the expansion of frequency and time resources, and this characteristic can be utilized to improve the error performance by diversity techniques (e.g., space-time block codes [1]) or to increase the data rates by spatial multiplexing. In spatially multiplexed MIMO systems, each of multiple signals can include an independent data stream. Thus, the receiver in spatially multiplexed MIMO systems needs to identify the independent data streams from the receive signals [1,2].

The conventional detection approaches for MIMO systems can be classified into linear and non-linear detection schemes [1,2,10]. In general, linear detection schemes such as linear minimum mean-squared-error (LMMSE) and linear zero-forcing (LZF) calculate a filtering matrix, which is a function of the MIMO channel matrix between the transmitter and receiver, and the estimates are obtained by multiplying the filtering matrix to the receive signal vector. Due to their simple structure, linear detection schemes require a relatively smaller computational complexity load compared to non-linear detection schemes. However, the detection performance of linear detection schemes can be significantly degraded from that of non-linear detection schemes. Meanwhile, non-linear detection schemes can outperform linear detection schemes and achieve suboptimum performance at the expense of higher

computational complexity. Specifically, the maximum likelihood (ML) and maximum a posteriori (MAP) detection schemes are known to achieve optimum detection performance in MIMO systems [1]. However, their computational complexity exponentially increases with the number of transmit antennas. This makes optimum MIMO detection schemes impractical for larger MIMO systems, e.g., massive MIMO systems. Thus, suboptimum detection schemes have been widely investigated for MIMO systems [2,10–24].

Among the suboptimum MIMO detection schemes, LMMSE iterative soft interference cancellation (ISIC) [13–17] is known to achieve the near-optimum detection performance approaching the matched filter bound (MFB) of the channel. Basically, the LMMSE-ISIC scheme is designed to iteratively perform the successive interference cancellation (IC) using the soft estimate of each transmit symbol. Thus, similar to successive hard decision IC [1,11,12], the detection order of transmit symbols can significantly impact the performance of LMMSE-ISIC. However, the conventional ordering schemes for LMMSE-ISIC utilize the basic metrics that can be calculated from the channel state information (CSI) only [13,17], such as the combined channel gain or initial signal-to-interference-plus-noise ratio (SINR) of transmit symbols. Thus, by an inaccurate ordering result, the conventional ordering schemes can degrade the performance of LMMSE-ISIC, e.g., convergence speed and error performance.

Therefore, in this paper, new ordering schemes for LMMSE-ISIC are proposed and developed. The new ordering schemes are termed the detection-aided ordering schemes in the sequel, and their ordering procedure is based on the LMMSE or LMMSE-ISIC based detection procedure using the receive signal vector as well as the CSI. Thus, the proposed detection-aided ordering schemes can provide a significantly more accurate ordering result than the conventional ordering schemes, which leads to improved average bit-error rates (BERs) and convergence speed as verified in numerical simulations.

This paper is organized as follows: Section 2 describes the MIMO system model and LMMSE-ISIC procedure considered in this paper. Section 3 describes the conventional and proposed detection-aided ordering schemes, including computational complexity analysis. Section 4 shows numerical simulation results to verify the performance of the proposed detection-aided ordering schemes, and Section 5 makes conclusions.

1.1. Literature Review

Suboptimum iterative MIMO detection schemes have been widely studied, especially for massive MIMO systems in recent years. In [18], the soft-in soft-output (SISO) detection scheme known as subspace marginalization with interference suppression (SUMIS) was developed for suboptimum massive MIMO detection with deterministic complexity. In [19], the channel hardening-exploiting message passing (CHEMP) receiver was studied to exploit the channel hardening phenomenon of massive MIMO systems for suboptimum iterative detection. The large-MIMO approximated message passing (LAMA) algorithm was proposed in [20] for achieving individually-optimal data detection, and the approximated expectation propagation (EP) based detection algorithm was proposed in [21] utilizing the channel-hardening phenomenon for approximation. In [22], the deep learning-aided tabu search detection based on the deep neural network (DNN) architecture for symbol detection was studied. In [23], the unitary approximate message passing (UAMP) based detection was developed for orthogonal-time-frequency-space (OTFS) modulation, and the memory approximate message passing (MAMP) based detection was investigated in [24] for generalized massive MIMO environments.

Meanwhile, LMMSE-ISIC and the related schemes also have been consistently studied. Among the related studies, many approaches have been developed to reduce the computational complexity of the original form of LMMSE-ISIC (e.g., [13]). In [14], an LMMSE-ISIC

scheme to reduce the number of matrix inversions was developed. Further, Neumann series (NS) expansion and diagonal matrix approximation were respectively utilized in [15,16] for matrix inversion. In [17], the LMMSE-ISIC scheme without matrix inversion during the iterative detection procedure was developed, where the matrix inversion is replaced by other operations without any approximation. Nonetheless, there has been little effort to determine the detection order of symbols for iterative sequential detection procedure of LMMSE-ISIC.

1.2. Contributions

As described in Section 1.1, previous research on LMMSE-ISIC mainly focuses on complexity reduction, and the ordering schemes for LMMSE-ISIC have hardly been studied. Thus, this paper proposes detection-aided ordering schemes to improve the convergence speed and error performance of LMMSE-ISIC for MIMO systems.

The contributions of this study can be summarized as follows:

- In the proposed schemes, the detection order of transmit symbols is determined based on the LMMSE or LMMSE-ISIC based detection procedure using the receive signal vector as well as the CSI, unlike the conventional schemes using the CSI only. Further, motivated by the likelihood-based ordering scheme for successive hard decision IC [11,12], among the metrics that can be obtained during the detection-based ordering procedure, the proposed detection-aided ordering schemes calculate the sum of the likelihoods of each transmit symbol except the largest likelihood of the symbol, and the detection order is determined based on the minimum sum of the likelihoods, e.g., a symbol with the smallest sum is selected earlier. Because this sum of likelihoods can accurately represent the reliability of the symbol [11,12], the proposed detection-aided ordering schemes can provide a significantly more accurate ordering result than the conventional ordering schemes based on the combined channel gain or initial SINR.
- As the detection-aided ordering schemes, this paper proposes two basic approaches: non-iterative and iterative ordering schemes. In the non-iterative ordering scheme, the likelihoods of the transmit symbols are obtained by the LMMSE detection principle (i.e., linear detection without soft IC), and the order of the entire transmit symbols is simultaneously determined by the obtained sums of the likelihoods. Meanwhile, in the iterative ordering scheme, a modified LMMSE-ISIC iteration is performed to obtain the likelihoods of the remaining transmit symbols. Then, the symbol with the smallest sum of the likelihoods among the remaining symbols is selected and cancelled by soft IC, and this procedure is repeated until the detection order of all transmit symbols is determined. Thus, the proposed non-iterative ordering scheme performs the LMMSE-detection-aided ordering, and the proposed iterative ordering scheme performs the LMMSE-ISIC-detection-aided ordering. Therefore, while both approaches outperform the conventional ordering schemes, the non-iterative ordering scheme requires a smaller computational complexity than the iterative ordering scheme, while the iterative ordering scheme provides a lower average BER and a faster convergence speed than the non-iterative ordering scheme.
- Finally, to compensate the computational complexity of the iterative ordering scheme, the constrained iterative ordering scheme is also proposed in this paper. In the constrained iterative ordering scheme, the number of the transmit symbols of which the likelihoods are calculated is limited by a given system parameter, while the original iterative ordering scheme is designed to perform the full search (i.e., all the remaining symbols). Therefore, the constrained iterative ordering scheme can maintain practical complexity according to the system environment, at the price of a marginal performance degradation from the original iterative ordering scheme.

1.3. Notations

Matrices and vectors are denoted by upper-case and lower-case boldface letters, respectively. The superscripts T , H , and -1 represent the transpose, conjugate-and-transpose, and inverse operations for a matrix, respectively. $\mathbf{A} = \text{diag}(\mathbf{a})$ is the diagonal matrix $\mathbf{A}$ having $\mathbf{a}$ on its diagonal, and $\mathbf{a} = \text{diag}(\mathbf{A})$ is the vector having the diagonal elements of $\mathbf{A}$. $\mathbf{I}_j$ is the $j \times j$ identity matrix. $[\mathbf{A}]_i$ is the i th row of the matrix $\mathbf{A}$, and $|\mathbb{A}|$ is the number of elements in the set $\mathbb{A}$.

2. Preliminaries

2.1. System Model

The spatially multiplexed MIMO system with N transmit and M receive antennas is considered throughout this paper, as illustrated in Figure 1. This system model can represent both of the single-user system, e.g., one user with N antennas and one base station (BS) with M antennas, and a multi-user system, e.g., N single-antenna users and one BS with M antennas. In each transmit antenna, a modulated transmit symbol is sent through the MIMO channel environment. For simplicity, the modulation order is fixed to Q for every transmit symbol. Then, each transmit symbol is selected from a Q -ary constellation $\mathbb{S}$ (i.e., $|\mathbb{S}| = Q$), where the elements in $\mathbb{S}$ satisfy $\sum_{s \in \mathbb{S}} s = 0$ and $\sum_{s \in \mathbb{S}} s^2 = Q$, e.g., phase-shift keying (PSK) and quadrature amplitude modulation (QAM). Thus, a transmit signal vector $\mathbf{s} = [s_1, \dots, s_N]^T$ is an $N \times 1$ vector having N modulated symbols of Q -ary, where s_n is the transmit symbol for the $n(1 \leq n \leq N)$ th transmit antenna. Then, an $M \times 1$ receive signal vector $\mathbf{r} = [r_1, \dots, r_M]^T$ can be written as

$$\mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{n}, \quad (1)$$

where $\mathbf{H}$ is the $M \times N$ MIMO channel matrix with $\mathbf{h}_n$ as its $n(1 \leq n \leq N)$ th column and $\mathbf{n}$ is the $M \times 1$ additive white Gaussian noise (AWGN) vector whose elements follow the i.i.d. (independent and identically distributed) Gaussian distributions with zero mean and variance σ^2 . The perfect CSI at the receiver is assumed for simplicity, i.e., $\mathbf{H}$ and σ^2 are perfectly known at the receiver before the LMMSE-ISIC detection procedure. It is worthwhile to mention that $\mathbf{H}$ is the general $M \times N$ MIMO channel model that can include various channel conditions such as the path loss, transmitter or receiver antenna correlation, Rayleigh fading, etc.

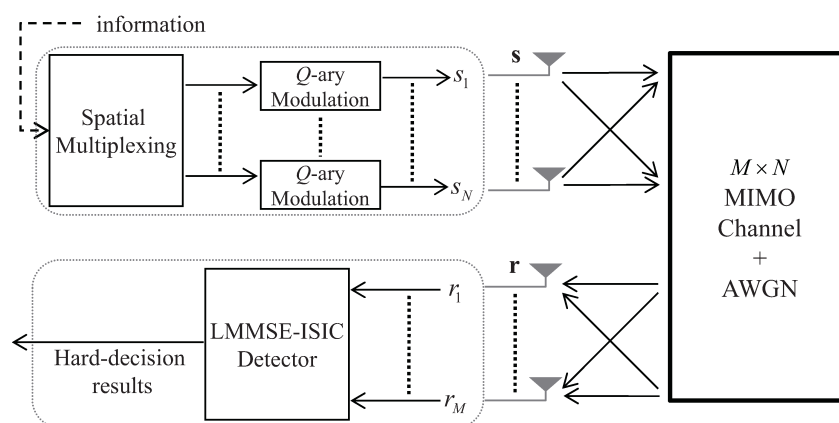


Figure 1. The spatially multiplexed MIMO system model.

2.2. LMMSE-ISIC Detection

In this Section 2.2, the detection procedure of the low-complexity LMMSE-ISIC scheme in [17] is described, which generates the exact same detection estimates to the basic form of LMMSE-ISIC (e.g., [13]) at a significantly lower computational complexity.

Figure 2 illustrates the overall detection procedure of the LMMSE-ISIC scheme in [17]. First, the initialization including the ordering stage is performed prior to the iterative detection procedure. Then, based on the calculated detection order, each s_n for $1 \leq n \leq N$ is sequentially estimated in each LMMSE-ISIC iteration, while the calculated soft estimate of s_n is utilized for the other symbols until it is updated or the detection procedure is over.

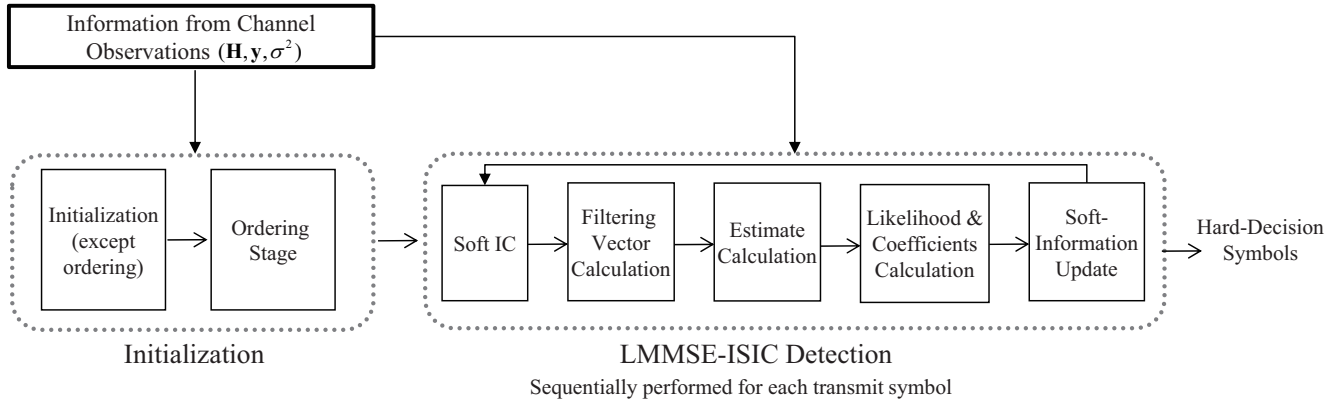


Figure 2. The overall detection procedure of LMMSE-ISIC.

At the initialization, $\bar{s}_n$ and v_n , the soft decision value and residual interference variance for s_n , respectively, are set to 0 and 1 for $1 \leq n \leq N$, because there is no estimation on s_n before the first LMMSE-ISIC iteration, i.e., no a priori information [13,17]. Further, the $N \times N$ matrices $\mathbf{W} = \mathbf{H}^H \mathbf{H}$ and $\mathbf{G} = (\mathbf{W} + \sigma^2 \mathbf{I}_N)^{-1}$ are calculated, where $\mathbf{G}$ corresponds to the inverse matrix part of the LMMSE filtering matrix $(\mathbf{W} + \sigma^2 \mathbf{I}_N)^{-1} \mathbf{H}^H$ when no IC is performed. In addition, the ordering stage is performed to obtain the ordering vector $\mathbf{d} = [d_1, d_2, \dots, d_N]$, where d_n for $1 \leq n \leq N$ is the transmit symbol index that will be estimated in the n th order in each LMMSE-ISIC iteration, i.e., s_{d_n} .

Then, each LMMSE-ISIC iteration is performed to sequentially estimate the transmit symbols from s_{d_1} to s_{d_N} . At the beginning of the detection procedure for s_{d_n} , the $N \times 1$ soft decision vector $\bar{\mathbf{s}}$ and $N \times N$ residual interference variance matrix $\mathbf{V}$ are updated as

$$\bar{\mathbf{s}} = [\bar{s}_1, \dots, \bar{s}_N]^T \quad (2)$$

and

$$\mathbf{V} = \text{diag}(v_1, \dots, v_N). \quad (3)$$

Thus, the soft information updated during the last LMMSE-ISIC detection procedure ($\bar{s}_{d_{n-1}}$ and $v_{d_{n-1}}$ for $n \geq 2$ and $\bar{s}_{d_N}$ and v_{d_N} for $n = 1$) can be utilized for the detection procedure of s_{d_n} .

Using $\bar{\mathbf{s}}$, the soft IC is performed as

$$\tilde{\mathbf{r}} = \mathbf{r} - \mathbf{H}\bar{\mathbf{s}}, \quad (4)$$

where $\tilde{\mathbf{r}}$ is the $M \times 1$ soft interference-cancelled vector. Then, according to $\bar{\mathbf{s}}$, the inverse matrix $\mathbf{G}$ for LMMSE filtering is updated as

$$\mathbf{G} = \mathbf{G} - \frac{(v_k - v'_k) \mathbf{G} ([\mathbf{W}]_k)^H [\mathbf{G}]_k}{(v_k - v'_k) [\mathbf{G}]_k ([\mathbf{W}]_k)^H + 1} \quad (5)$$

where k is the index of the symbol estimated last (i.e., $k = d_{n-1}$ for $n \geq 2$ and $k = d_N$ for $n = 1$) and v'_k is the residual interference variance of s_k before its last update (i.e., the last residual interference variance of s_k used before the calculation of v_k).

Then, considering $\bar{\mathbf{s}}$, the LMMSE filtering vector for s_{d_n} can be obtained using the updated $\mathbf{G}$ as

$$\mathbf{f}_{d_n} = \mathbf{H} [\mathbf{G}]_{d_n}^H. \quad (6)$$

Using $\mathbf{f}_{d_n}$ and $\tilde{\mathbf{r}}$, the LMMSE-ISIC estimate of s_{d_n} , $\hat{s}_{d_n}$ can be calculated as

$$\hat{s}_{d_n} = \beta_{d_n} \mathbf{f}_{d_n}^H \tilde{\mathbf{r}} + \alpha_{d_n} \beta_{d_n} \bar{s}_{d_n}, \quad (7)$$

where the coefficients α_{d_n} and β_{d_n} can be computed by

$$\alpha_{d_n} = \mathbf{f}_{d_n}^H \mathbf{h}_{d_n} \quad (8)$$

and

$$\beta_{d_n} = \frac{1}{(1 - v_{d_n})\alpha_{d_n} + 1}. \quad (9)$$

Because all the information about the transmit symbols including s_{d_n} is cancelled in (4), the weighted conditional symbol mean $\alpha_{d_n} \beta_{d_n} \bar{s}_{d_n}$ is added in the calculation of $\hat{s}_{d_n}$ in (7).

It is well known that the LMMSE estimate $\hat{s}_{d_n}$ can be approximated as a complex Gaussian, i.e.,

$$\hat{s}_{d_n} = \mu_{d_n} s_{d_n} + z_{d_n}, \quad (10)$$

where μ_{d_n} , the filtering bias, is calculated by

$$\mu_{d_n} = \alpha_{d_n} \beta_{d_n} \quad (11)$$

and η_{d_n} , the variance of the residual interference-plus-noise z_n , is calculated by

$$\eta_{d_n} = \mu_{d_n} (1 - \mu_{d_n}). \quad (12)$$

Therefore, $P_{d_n}(s)$, the conditional probability of the symbol given estimate can be updated for each $s \in \mathbb{S}$ as

$$P_{d_n}(s) = \frac{\psi_{d_n}(s)}{\sum_{s' \in \mathbb{S}} \psi_{d_n}(s')}, \quad (13)$$

where the likelihood $\psi_{d_n}(s)$ for each $s \in \mathbb{S}$ is defined as

$$\psi_{d_n}(s) = \exp\left(\frac{-|\hat{s}_{d_n} - \mu_{d_n} s|^2}{\eta_{d_n}}\right) \quad (14)$$

Using the updated conditional probability of the symbol given estimate, the soft symbol mean ($\bar{s}_{d_n}$) and variance (v_{d_n}) can be updated as

$$\bar{s}_{d_n} = \sum_{s \in \mathbb{S}} s P_{d_n}(s) \quad (15)$$

where $\psi_{d_n}(s)$ for each $s \in \mathbb{S}$ is defined as

$$v_{d_n} = \sum_{s \in \mathbb{S}} |s - \bar{s}_{d_n}|^2 P_{d_n}(s), \quad (16)$$

which is the end of the LMMSE-ISIC detection procedure for s_{d_n} at a given LMMSE-ISIC iteration. In each LMMSE-ISIC iteration, the procedures in (2)–(16) are repeated for $\{s_{d_1}, \dots, s_{d_N}\}$, and the procedure to check whether an iteration stopping criterion is satisfied or not can be performed after the end of each iteration. If an iteration stopping criterion is satisfied (e.g., maximum number of iterations), the hard decision symbols are generated as the last procedure.

As described, $\bar{s}_{d_n}$ and v_{d_n} are sequentially updated and the updated $\bar{s}_{d_n}$ and v_{d_n} are utilized from the LMMSE-ISIC detection procedure for the next symbol. That is, in LMMSE-ISIC, the estimate of each transmit symbol is calculated based on the LMMSE principle considering the soft interference-cancelled receive signal vector, where the sequentially updated soft estimates of the other transmit symbols are cancelled in the soft interference-cancelled receive signal vector. By iteratively performing this proce-

ture, the LMMSE-ISIC scheme can achieve a near-optimum detection performance for MIMO systems, especially for massive MIMO systems [13–17].

3. Proposed Detection-Aided Ordering Schemes

The LMMSE-ISIC scheme is basically designed to perform sequential detection based on soft IC operations. Thus, as in the successive hard decision IC schemes [1,11,12], the ordering, i.e., the detection order of transmit symbols, can have a great impact on the detection performance, such as the error performance (e.g., the average BER) and convergence speed (e.g., the number of iterations to achieve a given average BER).

For LMMSE-ISIC, the combined channel gain (e.g., $d_1 = \arg \max \text{diag}(\mathbf{W})$) or initial SINR (e.g., $d_1 = \arg \max \text{diag}(\mathbf{GW})$) of each transmit symbol can be considered as the basic ordering schemes. Further, considering the suboptimum performance of LMMSE-ISIC, the antenna index ordering (i.e., $d_n = n$ for $1 \leq n \leq N$) can be performed as well. These schemes require a low computational complexity to decide the ordering vector $\mathbf{d}$. However, because the ordering is determined without using the receive signal vector or detection estimates, the ordering vector can be inaccurate, e.g., the occurrence of error propagations, which yields the degradation of the error performance and convergence speed for LMMSE-ISIC.

Therefore, this paper proposes the detection-aided ordering schemes for improving the performance of LMMSE-ISIC. The overall procedure of LMMSE-ISIC with the proposed detection-aided ordering schemes is identical to the procedure described in Section 2.2, except the detailed algorithm for the ordering stage during the initialization.

3.1. Non-Iterative Detection-Aided Ordering

In the proposed non-iterative detection-aided ordering scheme, the ordering vector $\mathbf{d}$ is determined by the ordering metric calculated by the LMMSE detection principle, where the sum of the likelihoods of each transmit symbol except the largest likelihood of the symbol is used as the ordering metric. For that, the LMMSE filtering is applied to the receive signal vector to obtain the LMMSE filtering estimates as

$$\hat{\mathbf{s}}^A = [\hat{s}_1^A, \dots, \hat{s}_N^A]^T = \mathbf{G}^A \mathbf{H}^H \mathbf{r}, \quad (17)$$

where $\mathbf{G}^A = (\mathbf{W} + \sigma^2 \mathbf{I}_N)^{-1}$ and $\hat{\mathbf{s}}^A$ is the $N \times 1$ vector including the LMMSE estimates obtained for the proposed non-iterative ordering scheme. Equivalently to (10), each element in $\hat{\mathbf{s}}^A$ can be rewritten as [17,25]

$$\hat{s}_n^A = \mu_n^A s_n + z_n^A, \quad (18)$$

where μ_n^A is calculated for $1 \leq n \leq N$ by [25]

$$\mu_n^A = [\mathbf{G}^A]_n ([\mathbf{W}]_n)^H \quad (19)$$

and η_n^A is calculated for $1 \leq n \leq N$ by

$$\eta_n^A = \mu_n^A (1 - \mu_n^A). \quad (20)$$

Then, based on the Gaussian approximation, the likelihoods of s_n for $1 \leq n \leq N$ for all possible constellation points in $\mathbb{S}$ (i.e., for all $s \in \mathbb{S}$) can be calculated as

$$\psi_n^A(s) = \exp\left(\frac{-|\hat{s}_n^A - \mu_n^A s|^2}{\eta_n^A}\right). \quad (21)$$

For a given n , $s_n^* = \arg \max_s \psi_n^A(s) \forall s \in \mathbb{S}$ indicates the most likely constellation point for s_n based on the likelihoods, and it is shown in [11,12] that the sum of $\psi_n^A(s)$ for all $s \in \mathbb{S}$ except $\psi_n^A(s_n^*)$ represents the reliability of $\hat{s}_n^A$ in inverse proportion, i.e., $\hat{s}_n^A$ accurately

estimates s_n as the sum decreases. Thus, the ordering metric for the non-iterative ordering scheme λ_n^A can be defined as

$$\lambda_n^A = \sum_{s \in \mathbb{S}} \psi_n^A(s) - \max_{s \in \mathbb{S}} \psi_n^A(s) = \sum_{s \in \mathbb{S}} \psi_n^A(s) - \psi_n^A(s_n^*). \quad (22)$$

As λ_n^A decreases, the reliability of $\hat{s}_n^A$ increases, and this indicates that s_n with a smaller λ_n^A needs to be detected earlier during the sequential LMMSE-ISIC detection procedure than the other symbols $s_{n'} (n \neq n')$ with a larger $\lambda_{n'}^A$. Thus, after the calculation of λ_n^A for $1 \leq n \leq N$, the ordering vector $\mathbf{d}$ can be determined as

$$\mathbf{d} = \text{f}^{\text{ascend}}([\lambda_1^A, \dots, \lambda_N^A]), \quad (23)$$

where $\text{f}^{\text{ascend}}(\cdot)$ is the function that sorts the elements of the input vector in an ascending order and generates their indices according to the order.

As described above, based on the LMMSE detection principle, the proposed non-iterative ordering scheme utilizes the CSI and receive signal vector to obtain the likelihoods based on the LMMSE detection principle, and the detection order of transmit symbols is simultaneously determined by the sum of the likelihoods of each symbol except its largest likelihood. In this way, the proposed non-iterative detection-aided ordering scheme can outperform the conventional ordering schemes using the CSI only, while an additional computational complexity load is required to perform (17)–(23).

3.2. Iterative Detection-Aided Ordering

In the proposed iterative detection-aided ordering scheme, each element of the ordering vector $\mathbf{d}$ is sequentially determined by the ordering metric calculated by the LMMSE-ISIC detection principle, where the ordering metric is equivalent to that of the non-iterative ordering scheme in Section 3.1, i.e., the sum of the likelihoods of each transmit symbol except the largest likelihood of the symbol. For that, a modified LMMSE-ISIC iteration is performed to sequentially determine each element of $\mathbf{d}$. Because there is no available pre-determined order at the ordering stage, the LMMSE-ISIC detection procedure in (2)–(16) for the ordering stage is modified to calculate the metrics for all the remaining symbols whose order are not determined.

The modified LMMSE-ISIC iteration for the proposed detection-aided ordering scheme can be described as follows: The modified LMMSE-ISIC iteration sequentially determines the element of $\mathbf{d}$ from d_1 to d_N . Let $\mathbb{N}_n$ denote the set of indices of the remaining transmit symbols not selected in $\{d_1, \dots, d_{n-1}\}$ until the beginning of the modified LMMSE-ISIC iteration to determine d_n , e.g., $\mathbb{N}_1 = \{1, \dots, N\}$.

During the modified LMMSE-ISIC detection procedure for d_n , the procedures in (2)–(5) are performed as in Section 2.2. Then, instead of the procedures in (6)–(14) for one transmit symbol, the ordering metrics for all the remaining symbols in $\mathbb{N}_n$ need to be obtained. Thus, the LMMSE filtering vectors for the remaining symbols are calculated as

$$\mathbf{f}_{n'}^B = \mathbf{H}([\mathbf{G}^B]_{n'})^H, n' \in \mathbb{N}_n \quad (24)$$

where the initial $\mathbf{G}^B$ at the beginning of the ordering for d_1 is identical to the initial $\mathbf{G} = (\mathbf{W} + \sigma^2 \mathbf{I}_N)^{-1}$ in Section 2.2. Then, the LMMSE filtered estimates are obtained by

$$\hat{s}_{n'}^B = \beta_{n'}^B (\mathbf{f}_{n'}^B)^H \tilde{\mathbf{r}}^B + \alpha_{n'}^B \beta_{n'}^B \bar{s}_{n'}^B, n' \in \mathbb{N}_n, \quad (25)$$

where

$$\alpha_{n'}^B = (\mathbf{f}_{n'}^B)^H \mathbf{h}_{n'}, n' \in \mathbb{N}_n \quad (26)$$

and

$$\beta_{n'}^B = \frac{1}{(1 - v_{n'}^B)\alpha_{n'}^B + 1}, n' \in \mathbb{N}_n. \quad (27)$$

Then, the filtering bias and variance of the residual interference-plus-noise for each $\hat{s}_{n'}^B$ with $n' \in \mathbb{N}_n$ are calculated by

$$\mu_{n'}^B = \alpha_{n'}^B \beta_{n'}^B, n' \in \mathbb{N}_n \quad (28)$$

and

$$\eta_{n'}^B = \mu_{n'}^B (1 - \mu_{n'}^B), n' \in \mathbb{N}_n. \quad (29)$$

Thus, the likelihood $\psi_{n'}^B(s)$ can be calculated for $n' \in \mathbb{N}_n$ and $s \in \mathbb{S}$ as

$$\psi_{n'}^B(s) = \exp\left(-\frac{|\hat{s}_{n'}^B - \mu_{n'}^B s|^2}{\eta_{n'}^B}\right), n' \in \mathbb{N}_n. \quad (30)$$

Then, equivalently to the case of the non-iterative ordering scheme in Section 3.1, the ordering metric $\lambda_{n'}^B$ for the remaining symbol $n' \in \mathbb{N}$ can be obtained by the sum of $\psi_{n'}^B(s)$ except $\max_{s \in \mathbb{S}} \psi_{n'}^B(s)$. This can be written as

$$\lambda_{n'}^B = \sum_{s \in \mathbb{S}} \psi_{n'}^B(s) - \max_{s \in \mathbb{S}} \psi_{n'}^B(s), n' \in \mathbb{N}_n. \quad (31)$$

Thus, d_n can be determined as

$$d_n = \arg \min_{n' \in \mathbb{N}_n} \lambda_{n'}^B. \quad (32)$$

After d_n is decided, the remaining procedure can be performed for one transmit symbol (i.e., s_{d_n}) as in (13), (15), and (16). Then, if $n < N$, the set $\mathbb{N}_{n+1}$ is generated by $\mathbb{N}_{n+1} = \mathbb{N}_n - \{d_n\}$. This is the end of the ordering stage to determine d_n , which is repeated until all $\{d_1, \dots, d_N\}$ is decided. Because $|\mathbb{N}_N| = 1$, the procedure for determining d_N can be omitted by selecting the only remaining element in $\mathbb{N}_N$ as d_N .

As described above, the proposed iterative ordering scheme utilizes the CSI and receive signal vector to obtain the likelihoods based on the LMMSE-ISIC detection principle, and the ordering is sequentially determined per each transmit symbol among all the remaining symbols. In this way, the proposed iterative detection-aided ordering scheme can achieve a near-optimum error performance with a significantly fast convergence speed, which will be verified in Section 4.

Meanwhile, by considering all the remaining symbols during the ordering procedure for each d_n , the iterative ordering scheme in (24)–(32) can have a significantly large computational complexity load. To compensate this issue, the constrained iterative ordering scheme is proposed as well. In the proposed constrained iterative ordering scheme, the maximum number of the remaining symbols of which the likelihoods are calculated is constrained to a given parameter c . This constraint can maintain practical computational complexity according to the system configuration, e.g., the antenna configuration size and load factor, at the price of marginal performance degradation from the original iterative ordering scheme.

Let $\mathbb{N}_n^*$ denote the subset of $\mathbb{N}_n$. In each sequential ordering procedure to determine d_n , the constrained iterative ordering scheme uses $\mathbb{N}_n^*$ instead of $\mathbb{N}_n$ for all procedures in (24)–(32). The number of elements in $\mathbb{N}_n^*$, $|\mathbb{N}_n^*| = \min(c, N + 1 - n)$. That is, $|\mathbb{N}_n^*| = c$ when $c \leq N + 1 - n$ (i.e., $n \leq N + 1 - c$), and $|\mathbb{N}_n^*| = N + 1 - n$ when $c > N + 1 - n$ because $|\mathbb{N}_n| = N + 1 - n$ is less than c .

First, at the beginning of the ordering stage (i.e., before to start the procedure for d_1), the temporary ordering vector $\mathbf{d}^{\text{TEMP}}$ is calculated. $\mathbf{d}^{\text{TEMP}}$ provides a selection rule of $\mathbb{N}_n^*$ from the remaining symbols $\mathbb{N}_n$. To calculate $\mathbf{d}^{\text{TEMP}}$, the conventional ordering schemes (i.e., combined channel gain or initial SINR) can be used, which is based on the assumption that $\mathbf{d}^{\text{TEMP}}$ obtained from the conventional ordering schemes has a certain degree of similarity to $\mathbf{d}$ obtained from the original iterative ordering scheme. Thus, $\mathbf{d}^{\text{TEMP}}$ can be obtained as

$$\mathbf{d}^{\text{TEMP}} = \begin{cases} \mathbf{f}^{\text{descend}}(\text{diag}(\mathbf{W})), & \text{combined channel gain criterion} \\ \mathbf{f}^{\text{descend}}(\text{diag}(\mathbf{GW})), & \text{initial SINR criterion} \end{cases} \quad (33)$$

where $\mathbf{f}^{\text{descend}}(\cdot)$ is the function that sorts the elements of the input vector in an descending order and generates their indices according to the order.

Then, prior to begin the ordering procedure for d_n , $\mathbb{N}_n^*$ is initially set to $\emptyset$, and the following is recursively performed to determine $\mathbb{N}_n^*$ from $\mathbb{N}_n$ by using $\mathbf{d}^{\text{TEMP}} = [d_1^{\text{TEMP}}, \dots, d_N^{\text{TEMP}}]$. Specifically, among the remaining transmit symbols in $\mathbb{N}_n$, the maximum c symbols that have the earlier orders in $\mathbf{d}^{\text{TEMP}}$ than the others are selected as the member of $\mathbb{N}_n^*$. This can be written as

$$\mathbb{N}_n^* = \mathbb{N}_n^* \cup \{ \{d_k^{\text{TEMP}}\} \cap \mathbb{N}_n \}, \quad (34)$$

where k is a variable initially set to 1 and increases by one per recursion. The recursion is performed until $|\mathbb{N}_n^*| = c$ for $c \leq N + 1 - n$ (i.e., $n \leq N + 1 - c$) or $|\mathbb{N}_n^*| = N + 1 - n$ for $c > N + 1 - n$. When $c \geq N + 1 - n$ (i.e., $n \geq N + 1 - c$), $\mathbb{N}_n^* = \mathbb{N}_n$, so the recursion for (34) can be omitted.

After the determination of $\mathbb{N}_n^*$, the rest of the ordering procedure to select d_n for the constrained iterative ordering scheme is identical to the original iterative ordering scheme, e.g., (24)–(32), except that $\mathbb{N}_n$ is replaced by $\mathbb{N}_n^*$.

In this way, the proposed constrained iterative ordering scheme can greatly decrease the computational complexity of the original iterative ordering scheme with full search by setting c properly. By increasing c , more candidates are available to determine each d_n , which can improve the performance of the constrained iterative ordering scheme with an additional complexity. Nonetheless, in Section 4, it will be shown that the constrained iterative ordering scheme with a small c can achieve similar performance to the original iterative ordering scheme with full search.

It is worthwhile to mention that, for all the proposed ordering schemes, including the non-iterative and iterative ones, all the information obtained during the ordering stage (e.g., the filtering estimates, likelihoods, etc.) should not be used during the LMMSE-ISIC detection procedure (2)–(16) executed after the ordering stage, except the ordering vector $\mathbf{d}$. This is to prevent the performance degradation by the utilization of the a priori information not generated from a proper LMMSE-ISIC detection procedure with a sufficiently accurate detection order.

3.3. Computational Complexity

In this Section 3.3, the computational complexities of the ordering schemes are derived and compared by the number of complex multiplications (CMs). The number of complex divisions (CDs) is also included in the number of CMs, and the number of CMs for the inverse of $N \times N$ is calculated as $4N^3/3 - N/3$ assuming the Gaussian elimination.

Table 1 shows the number of CMs for the conventional and proposed ordering schemes. As shown in Table 1, at the expense of an accurate ordering result, the proposed schemes can require a larger number of CMs than the conventional schemes, especially for the iterative ordering schemes. Meanwhile, the selection rule for the proposed constrained iterative

ordering scheme makes little difference for the complexity, i.e., the initial SINR criterion requires N^2 additional CMs than the combined channel gain criterion. It is worthwhile to mention that the results in Table 1 are obtained under the assumption that $\mathbf{W}$ and $\mathbf{G}$ are not pre-calculated before the ordering procedure, e.g., [13]. If $\mathbf{W}$ and $\mathbf{G}$ can be obtained before the ordering, e.g., [17], the numbers of CMs for all the ordering schemes in Table 1 decrease.

Table 1. Computational complexity of ordering.

Ordering	Number of CMs (Including CDs)
Conventional, Channel Gain	MN
Conventional, Initial SINR	$\frac{4}{3}N^3 + N^2 + N(M - \frac{1}{3})$
Proposed, Non-Iterative	$\frac{4}{3}N^3 + 2N^2 + N(2M + 2Q - \frac{1}{3})$
Proposed, Iterative	$N^3(0.5M + \frac{10}{3}) + N^2(2.5M + 1 + Q) + N(Q - \frac{1}{3}) - 2M - 2Q$
Proposed, Constrained - Channel Gain	$\frac{10}{3}N^3 + N^2(M + Mc + 1) + N(M(2.5c - 0.5c^2 - 1) + 2Qc - \frac{1}{3}) + (M + Q)(c - c^2 - 2)$
Proposed, Constrained - Initial SINR	$\frac{10}{3}N^3 + N^2(M + Mc + 2) + N(M(2.5c - 0.5c^2 - 1) + 2Qc - \frac{1}{3}) + (M + Q)(c - c^2 - 2)$

Figure 3 compares the number of CMs for the conventional and proposed ordering schemes when $M = N$. The constrained iterative ordering scheme with the selection rule of the initial SINR is omitted in Figure 3. It is shown that the number of CMs for the proposed non-iterative ordering scheme is similar to that for the conventional ordering scheme with the initial SINR regardless of $M(N)$. Meanwhile, because of employing full search, the proposed iterative ordering scheme requires the greatest number of CMs, and the difference between the iterative ordering scheme and the other ordering schemes increases as $M(N)$ increases. The constrained iterative ordering scheme significantly reduces the number of CMs for the iterative ordering scheme, although more CMs are required than for the non-iterative ordering scheme.

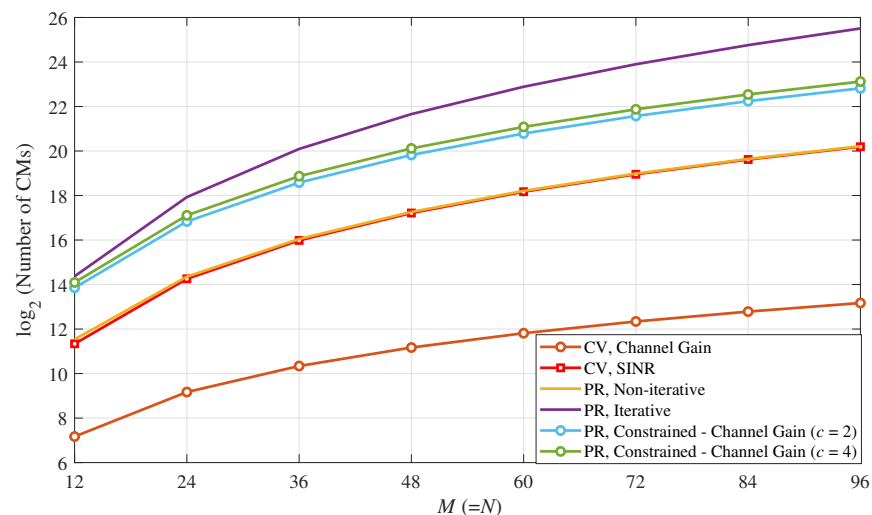


Figure 3. The number of CMs for the ordering schemes when $M = N$ and $Q = 4$.

4. Simulation Results

In this section, the error performance of the proposed ordering schemes in $N \times M$ MIMO systems is evaluated and compared with the conventional ordering schemes by numerical simulations. The average BER of LMSE-ISIC with the ordering schemes is evaluated, and, in addition, the MFB of the channel and the average BER of the LMMSE detection without IC are also evaluated, where the MFB is equivalent to the performance of the maxi-

mal ratio combining in $M \times 1$ single-input multiple-output systems [1]. Unless specified otherwise (e.g., Figures 4–9), (i) Rayleigh MIMO fading channel without antenna correlation is considered, (ii) perfect CSI at the receiver is assumed, (iii) uncoded systems without error correction coding are considered, (iv) the number of LMMSE-ISIC iterations is set to 1 regardless of the ordering scheme, (v) and $c = 4$ for the proposed constrained iterative ordering scheme. When the antenna correlation is assumed (i.e., Figures 10 and 11), the receiver antenna correlation with the exponential correlation model [26] is considered. Further, when the non-perfect CSI is assumed (i.e., Figures 12 and 13), the orthogonal pilot transmission per transmit antenna is considered with least-square (LS) channel estimation. In addition, when the error correction coding is assumed (i.e., Figure 14), a (768,384) block low-density parity-check (LDPC) code is considered, where the min-sum decoding algorithm with 30 decoding iterations is employed for the soft output of the detection schemes. Finally, according to the criterion in (33), the constrained iterative ordering scheme is termed the constrained type-1 ordering for the cases of combined channel gain or the constrained type-2 ordering for the cases of initial SINR in this section.

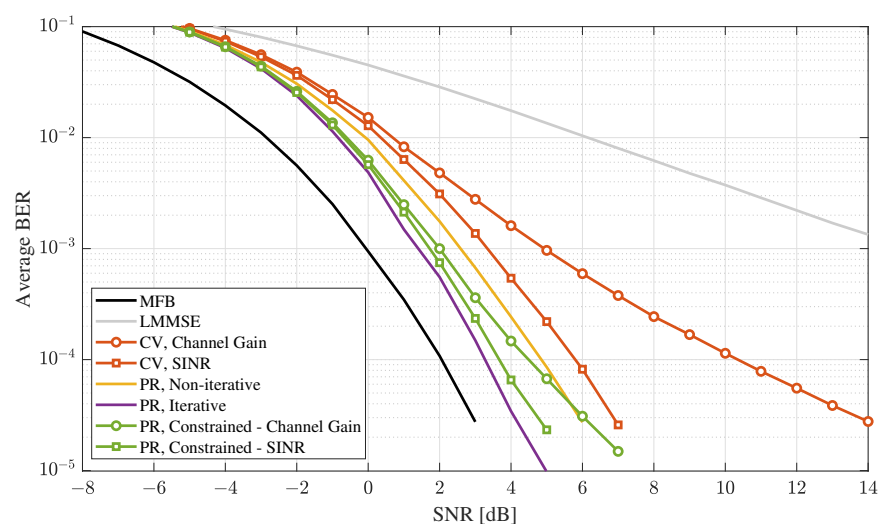


Figure 4. The average BER results in 12×12 MIMO systems for $Q = 4$.

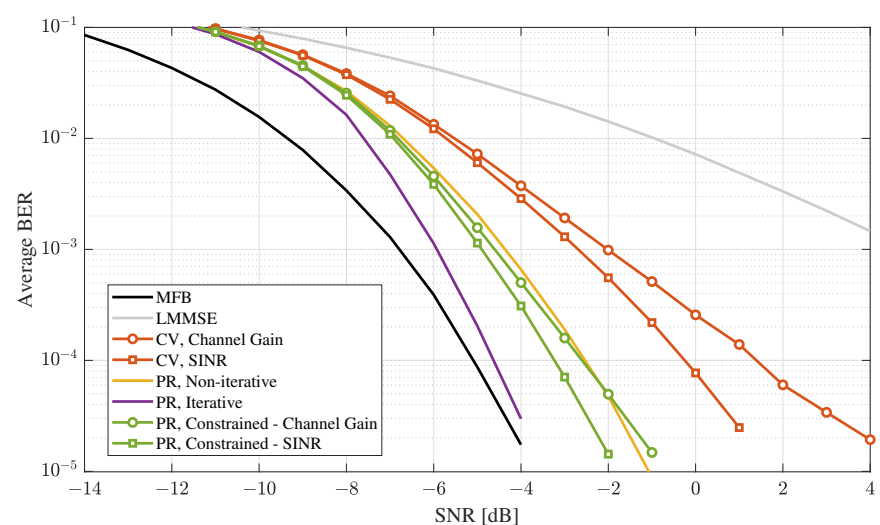


Figure 5. The average BER results in 48×48 MIMO systems for $Q = 4$.

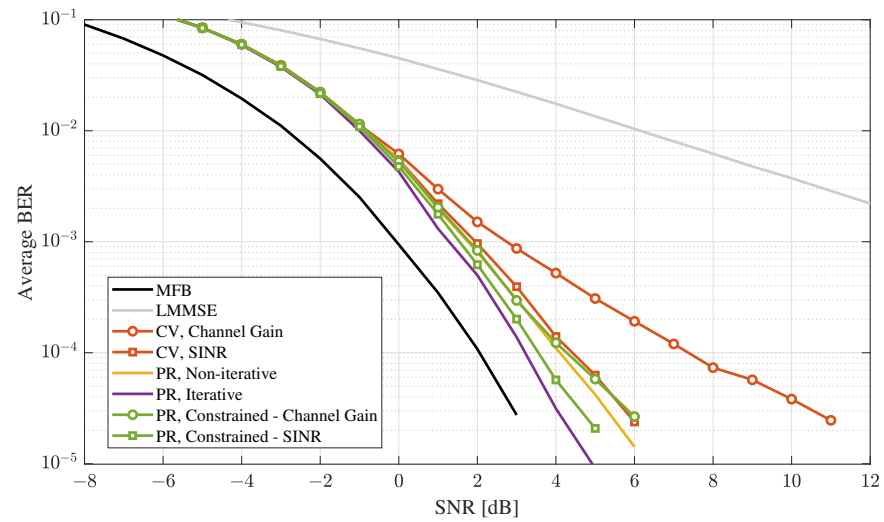


Figure 6. The average BER results in 12×12 MIMO systems for $Q = 4$, where the number of iterations is 3 for LMMSE-ISIC.

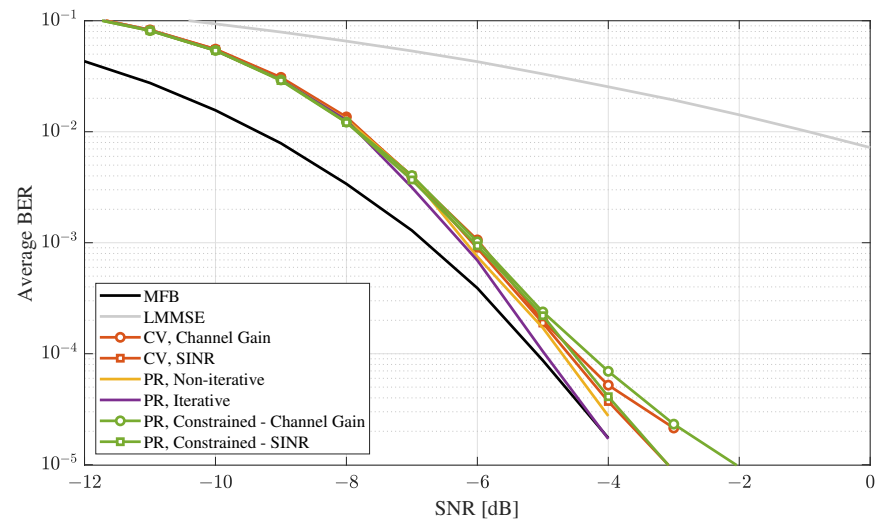


Figure 7. The average BER results in 48×48 MIMO systems for $Q = 4$, where the number of iterations is 3 for LMMSE-ISIC.

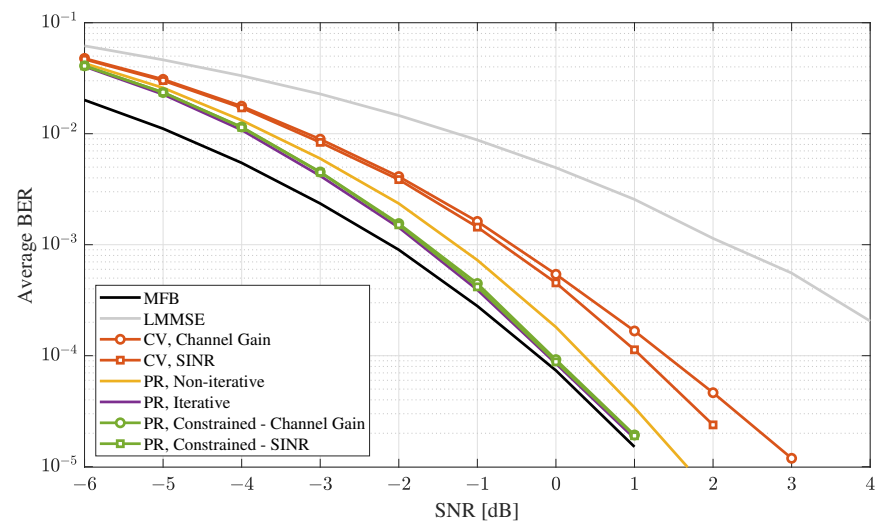


Figure 8. The average BER results in 12×18 MIMO systems for $Q = 4$.

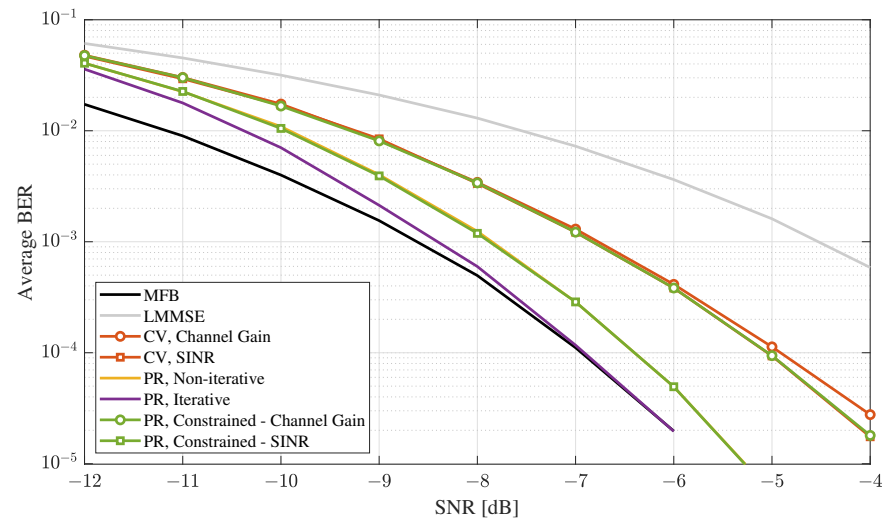


Figure 9. The average BER results in 48×72 MIMO systems for $Q = 4$.

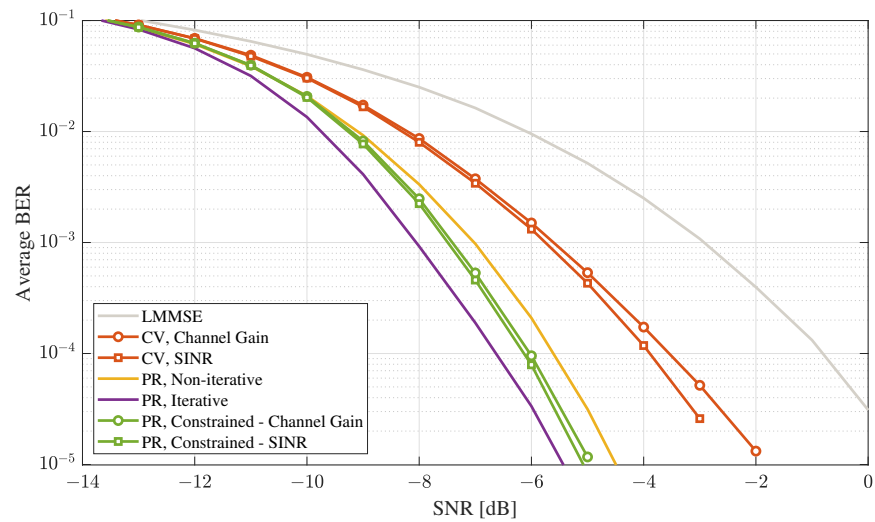


Figure 10. The average BER results in 48×72 MIMO systems for $Q = 4$, where the correlation factor is 0.5.

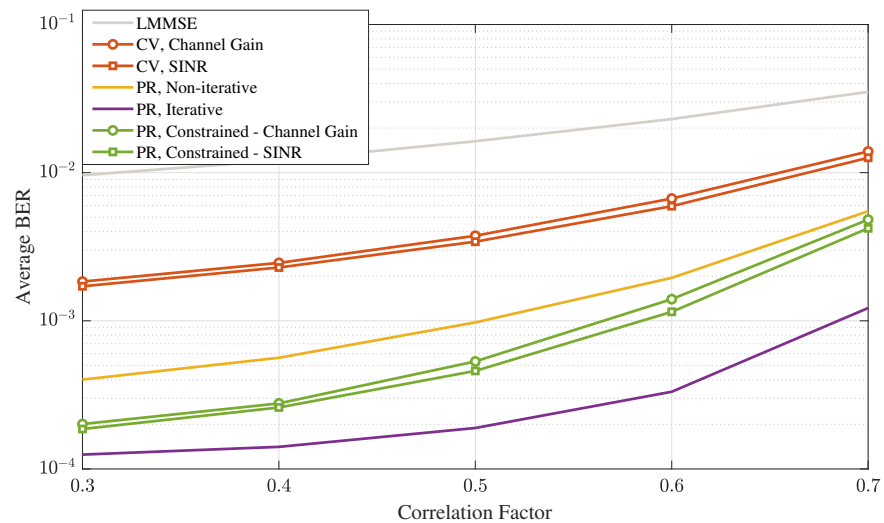


Figure 11. The average BER results in 48×72 MIMO systems for $Q = 4$ according to the correlation factor, where the SNR is -7 dB.

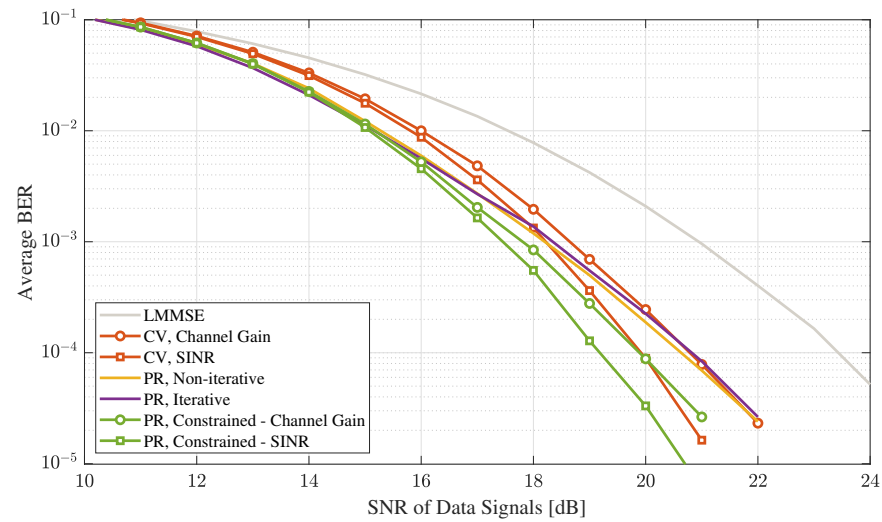


Figure 12. The average BER results in 12×18 MIMO systems for $Q = 4$, where the SNR of pilot signals is identical to the SNR of data signals.

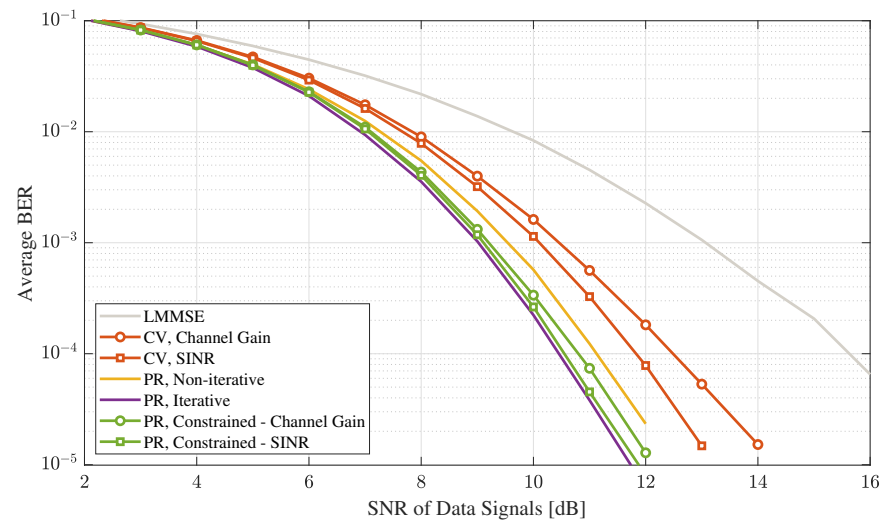


Figure 13. The average BER results in 12×18 MIMO systems for $Q = 4$, where the SNR of pilot signals is 10 dB larger than the SNR of data signals.

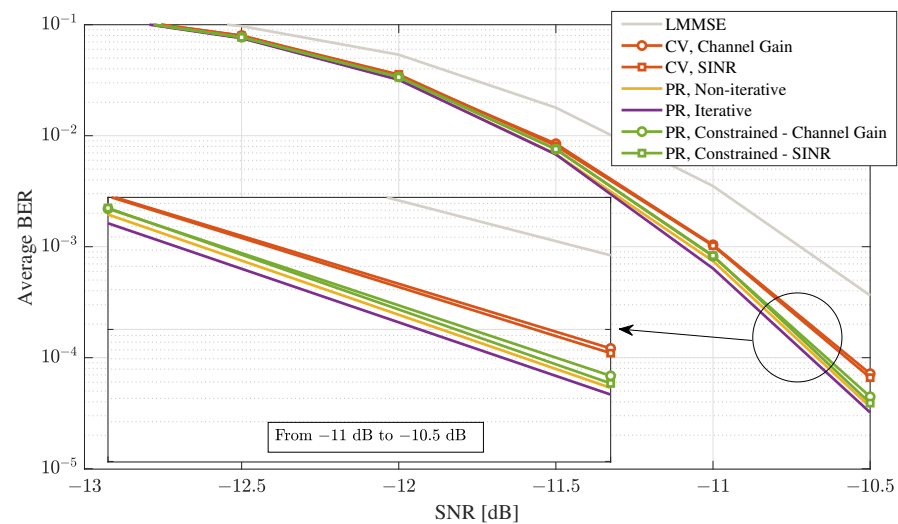


Figure 14. The average BER results in 24×36 MIMO systems for $Q = 4$, where the LDPC code of rate 0.5 is considered.

Figures 4 and 5 show the average BER results in MIMO systems with the load factor 1 for 4-QAM modulation, where Figures 4 and 5 consider 12×12 and 48×48 antenna configurations, respectively. In all the considered antenna configurations, the proposed iterative ordering scheme outperforms the other schemes and achieves near-optimum performance by approaching the MFB of the channel in one LMMSE-ISIC iteration, which verifies the accuracy of the ordering results by the proposed iterative ordering scheme. Further, the proposed constrained type-1 (combined channel gain criterion) and type-2 (initial SINR criterion) ordering schemes show a better error performance than the proposed non-iterative ordering scheme at the low SNR region. In the high SNR region, the performance of the constrained type-1 and non-iterative ordering schemes becomes similar. Further, in the high SNR region, the constrained type-2 ordering scheme outperforms the constrained type-1 and non-iterative ordering schemes. This is because, as shown in the results of the conventional ordering scheme with combined channel gain criterion and initial SINR criterion, the selection rule of initial SINR can be more accurate than the selection rule of combined channel gain. Meanwhile, all the proposed ordering schemes obtain a significant SNR gain from the conventional ordering schemes, and the SNR gain increases for a larger antenna configuration size.

Figures 6 and 7 show the average BER results in MIMO systems with the load factor 1 for 4-QAM modulation when the number of LMMSE-ISIC iterations is set to 3, where Figures 6 and 7 consider 12×12 and 48×48 antenna configurations, respectively. Comparing the results in Figures 4 and 5, it is observed that the proposed ordering schemes still outperform the conventional ordering schemes in a small MIMO system (e.g., 12×12 for Figures 4 and 6), while the performance of the conventional ordering schemes approaches that of the proposed ordering schemes in a large MIMO system (e.g., 48×48 for Figures 5 and 7). This implies that the performance improvement by an additional LMMSE-ISIC iteration is more clearly shown for the conventional ordering schemes as the antenna configuration size increases. Considering the results in Figures 4 and 5 that the SNR gain of the proposed schemes over the conventional schemes increases with the antenna configuration size, it can be concluded that the convergence speed of the proposed schemes is more accelerated as the antenna configuration size increases. Although the conventional schemes can have a similar performance to the proposed scheme in a large MIMO system, this requires more ISIC iterations, incurring additional computational complexity load and latency. Further, in addition to the better convergence speed, the proposed schemes can outperform the conventional schemes in terms of error performance at the high SNR region in a small MIMO system regardless of the number of LMMSE-ISIC iterations. Meanwhile, it is shown in Figures 6 and 7 that the proposed iterative ordering scheme still outperforms the other ordering schemes regardless of the antenna configuration.

Next, Figures 8 and 9 show the average BER results in MIMO systems with the load factor $2/3$ for 4-QAM modulation, where Figures 8 and 9 consider 12×18 and 48×72 antenna configurations, respectively. Compared with the results in Figures 4 and 5, because of the decreased load factor (i.e., increased M), the performance of the ISIC schemes in Figures 8 and 9 approaches closely to the MFB and the performance gap between the ordering schemes are decreased for a given N . The proposed constrained type-1 ordering scheme outperforms the conventional ordering schemes when $N = 12$, and it achieves a near-identical performance to the conventional ordering schemes when $N = 48$. This is because, in addition to the use of the combined channel gain criterion inferior to the initial SINR criterion, a fixed number of $c (= 4)$ increases inaccuracy for ordering during the selection by (34) as the number of possible candidate symbols (e.g., N) increases. A similar phenomenon is observed for the proposed constrained type-2 ordering scheme,

as it outperforms the proposed non-iterative ordering scheme when $N = 12$ but achieves a near-identical performance when $N = 48$.

In Figures 10 and 11, the average BER results in 48×72 MIMO systems are shown for 4-QAM modulation when the receiver antenna correlation is considered. From Figure 10, it is shown that the proposed schemes outperform the conventional schemes when the correlation factor is 0.5. Because of the receiver antenna correlation, the information obtained from the receive antennas can be decreased, and thereby the performance characteristics shown in Figure 10 for the 48×72 correlated MIMO system becomes similar to those shown in Figure 5 for 48×48 the uncorrelated MIMO system instead of Figure 9 for the uncorrelated 48×72 MIMO system. To check the effect of the correlation factor more clearly, Figure 11 shows the average BER results according to the correlation factor when SNR is -7 dB. It is shown that, although the average BER performance of the detection schemes becomes worse for a larger correlation factor, the proposed schemes still outperform the conventional schemes and the performance characteristics still remain similar to those observed in Figure 10.

Figures 12 and 13 show the average BER results in 12×18 MIMO systems for 4-QAM modulation with the LS channel estimation, where the SNR of pilot signals is identical to the SNR of data signals in Figure 12 and 10 dB larger than the SNR of data signals in Figure 13. It is shown in Figure 12 that the proposed iterative ordering scheme shows the degraded performance than the other ordering schemes as the SNR increases, while the other proposed schemes outperform the conventional schemes. This implies that the non-ignorable channel estimation error can yield non-ignorable error propagations during the ordering procedure of the iterative ordering scheme. This can be overcome by using the proposed constraint iterative ordering scheme utilizing the selection rule as the baseline. Meanwhile, as shown in Figure 13, with a more accurate channel estimation, the proposed iterative ordering schemes can show a better error performance than the other schemes and the performance characteristics become similar to those observed in previous figures with perfect CSI.

Next, Figure 14 shows the average BER results in 24×36 MIMO systems for 4-QAM modulation when the LDPC code of rate 0.5 is employed, i.e., coded MIMO systems. It is shown that the performance gap between the ordering schemes is significantly reduced compared with the results in uncoded systems. This is because the high SNR region (i.e., the SNR region with low average BERs) of coded systems corresponds to the low SNR region (i.e., the SNR region with high average BERs) of uncoded systems, where the performance gap between the ordering schemes in the low SNR region of uncoded systems is not significant, as shown in the previous figures. Nonetheless, the proposed ordering schemes show a better average BER than the conventional schemes.

Figures 15 and 16 show the average BER results in 24×36 MIMO systems, where Figures 15 and 16 consider 16-QAM ($Q = 16$) modulation and 256-QAM ($Q = 256$) modulation, respectively. The performance characteristics observed in Figures 15 and 16 for an increased Q are similar to those in the previous results. Thus, the proposed ordering schemes are effective in terms of performance for a high order modulation as well. Meanwhile, the performance gap between the proposed ordering schemes is slightly larger when $Q = 256$ than when $Q = 16$, and the SNR gain of the conventional ordering scheme with initial SINR over the conventional ordering scheme with combined channel gain is similar when both $Q = 16$ and $Q = 256$. This implies that a more accurate criterion for the detection-aided ordering procedure can be required to prevent error propagations as the modulation order increases.

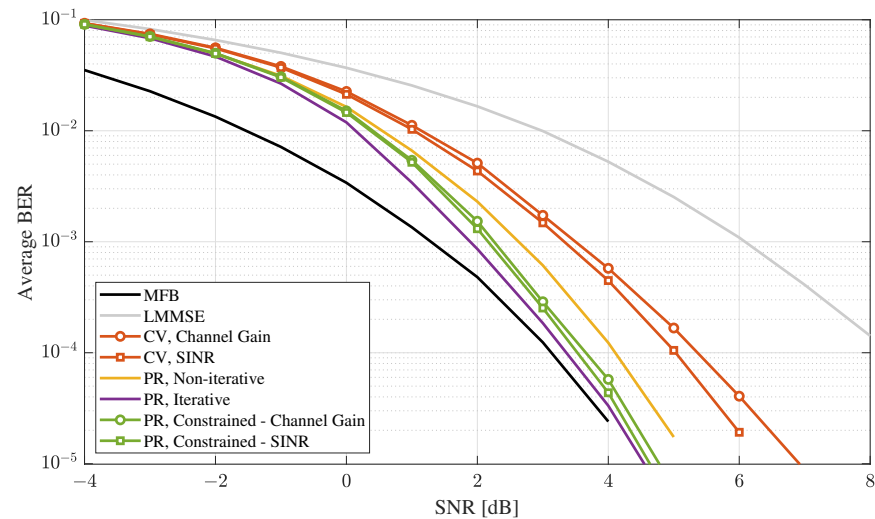


Figure 15. The average BER results in 24×36 MIMO systems for $Q = 16$.

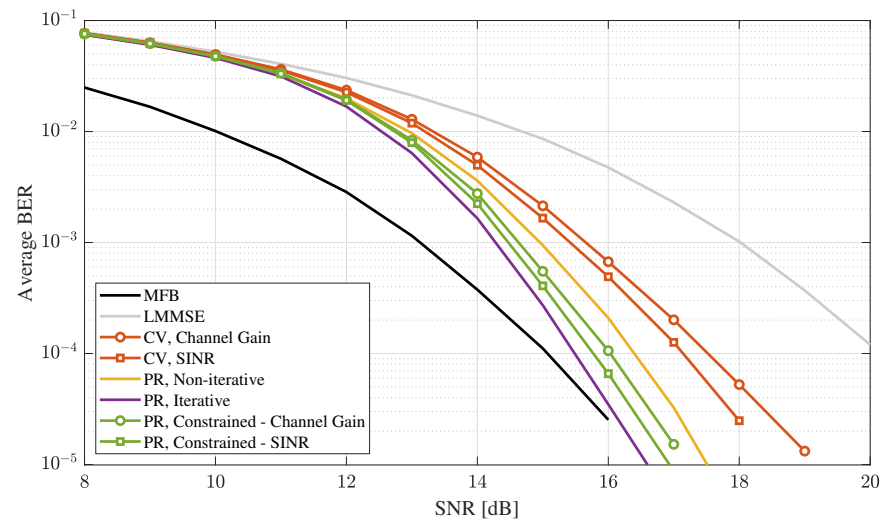


Figure 16. The average BER results in 24×36 MIMO systems for $Q = 256$.

Next, Figures 17 and 18 compare the average BER results of the proposed ordering schemes in MIMO systems for 16-QAM modulation, where Figures 17 and 18 consider 12×12 and 48×48 antenna configurations, respectively. As shown in Figures 17 and 18, the proposed constrained type-2 ordering scheme always outperforms the proposed constrained type-1 ordering scheme for a given c , although the performance gap is slightly reduced as c or N increases. As expected, by considering more candidates during the selection in (34), the performance of the proposed constrained iterative ordering schemes improves for a larger c , and the degree of the improvement becomes larger as N increases. Meanwhile, the performance gap between the proposed iterative ordering scheme with full search and the proposed constrained type-2 ordering scheme with $c = 6$ increases for a larger antenna configuration, and the required c for the proposed constrained iterative ordering schemes to achieve a similar error performance to the proposed non-iterative ordering scheme is also increased for a larger antenna configuration. This implies that the required number of c for an accurate ordering in the constrained iterative ordering scheme increases with the total number of candidates (i.e., the number of transmit symbols N) for the ordering.

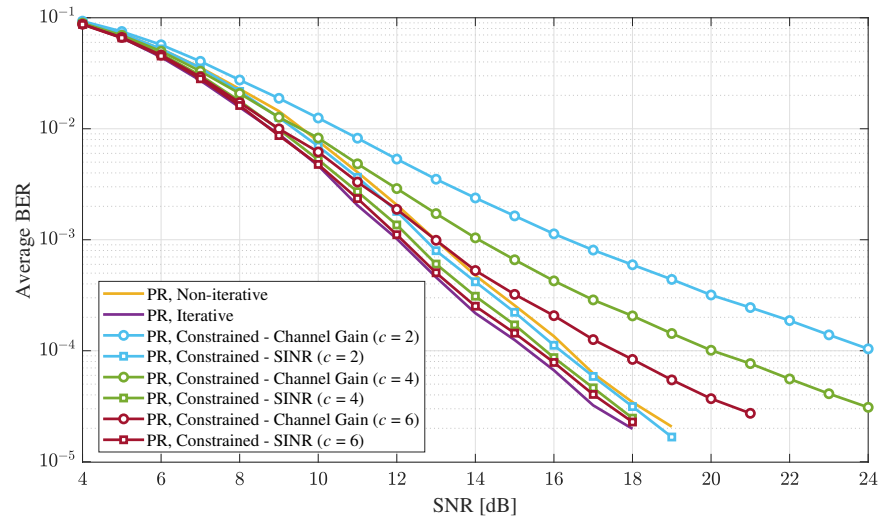


Figure 17. The average BER results of LMMSE-ISIC with the proposed ordering schemes in 12×12 MIMO systems for $Q = 16$.

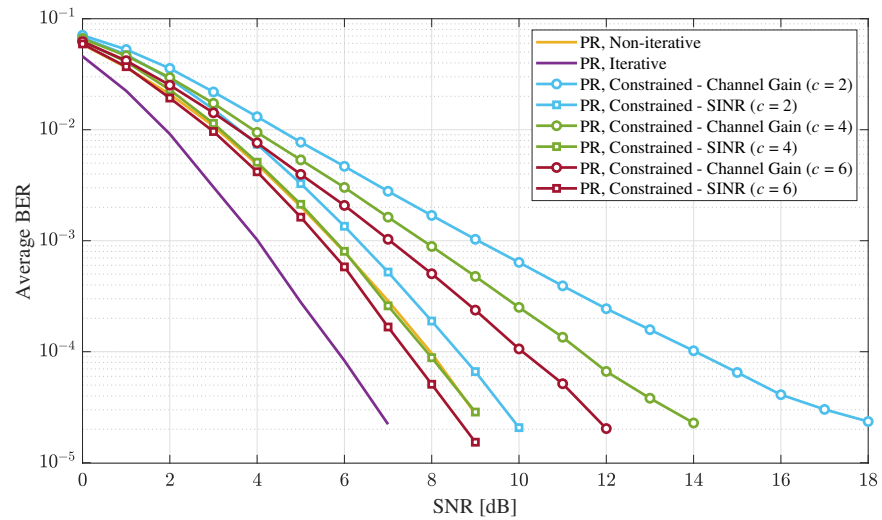


Figure 18. The average BER results of LMMSE-ISIC with the proposed ordering schemes in 48×48 MIMO systems for $Q = 16$.

Finally, in Figure 19, the average BER results of LMMSE-ISIC according to the detection order are compared in 24×24 MIMO systems for 16-QAM modulation at the SNR of 9 dB, where the average BERs of the symbol detected first (i.e., s_{d_1}) and the symbol detected last (i.e., s_{d_N}) are shown with the average BERs of all symbols. The proposed ordering schemes outperform the conventional ordering schemes for both s_{d_1} and s_{d_N} . Further, the proposed iterative ordering scheme and non-iterative ordering scheme achieve the lowest average BER for s_{d_1} , which can prevent error propagations to remaining symbols. It is worthwhile to mention that the proposed iterative and non-iterative ordering schemes in Figure 19 achieve the identical average BER for s_{d_1} because s_{d_1} in both schemes are always equal, as discussed in Section 3. In addition, for both s_{d_1} and s_{d_N} , the average BER of the proposed constraint iterative ordering schemes improve with c and the proposed constrained type-2 ordering scheme always outperforms the proposed constrained type-1 ordering scheme for a given c . Thus, it is verified that the proposed ordering schemes provide a more accurate ordering result than the conventional ordering scheme to prevent error propagations.

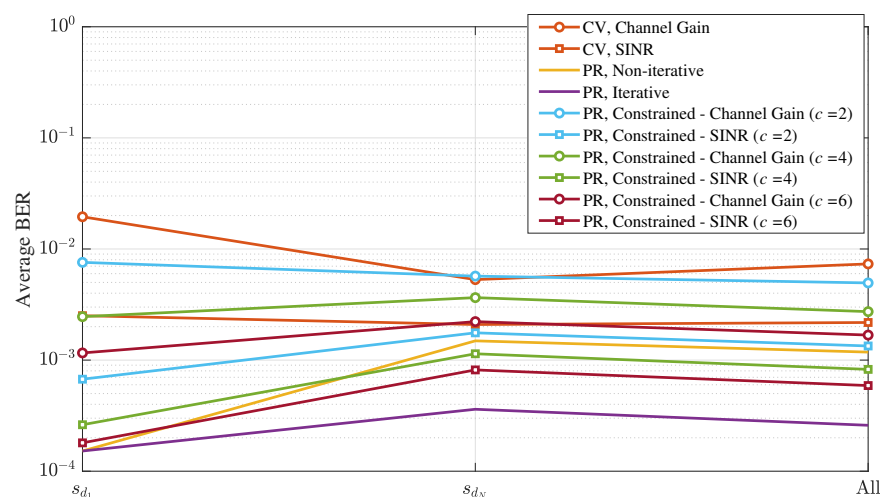


Figure 19. The average BER results of LMMSE-ISIC according to the detection order in 24×24 MIMO systems for $Q = 16$, where the SNR is 9 dB.

5. Conclusions

This paper proposes detection-aided ordering schemes for LMMSE-ISIC. By employing the detection principle for ordering, the proposed ordering schemes can significantly improve the convergence speed of LMMSE-ISIC from the conventional ordering schemes. Although the proposed ordering schemes yield additional computational complexity, it is verified by numerical simulations that the additional complexity load for the non-iterative ordering scheme and constrained iterative ordering scheme can be marginal compared to the decreased complexity load resulting from fewer ISIC iterations needed to achieve similar error performance. In addition, it is shown that the proposed ordering schemes outperform the conventional ordering schemes and significantly accelerate the convergence speed.

One disadvantage of the proposed iterative ordering schemes, including the constraint ordering schemes, is the high complexity load compared with the other schemes. Thus, it is necessary to decrease the complexity of the proposed iterative ordering schemes without sacrificing performance. Further, for the proposed non-iterative ordering scheme, a better ordering criterion instead of the current likelihood-based criterion can be investigated to achieve an improved error performance approaching that of the proposed iterative ordering scheme. In addition, considering the effects of error propagations, a more detailed analysis of the iterative detection behavior of LMMSE-ISIC based on the ordering scheme is required, e.g., extrinsic information transfer (EXIT) chart analysis, as in [27]. These will be addressed in future works.

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Abbreviations

The following abbreviations are used in this manuscript:

BER	Bit-error rate
BS	Base station

CD	Complex divisions
CHEMP	Channel hardening-exploiting message passing
CM	Complex multiplications
CSI	Channel state information
DNN	Deep neural network
EP	Expectation propagation
EXIT	Extrinsic information transfer
IC	Interference cancellation
ISIC	Iterative soft interference cancellation
LAMA	Large-MIMO approximated message passing
LDPC	Low-density parity-check
LMMSE	Linear minimum mean-squared-error
LS	Least-square
LTE	Long-term evolution
LZF	Linear zero-forcing
MAMP	Memory approximate message passing
MAP	Maximum a posteriori
MFB	Matched filter bound
MIMO	Multiple-input multiple-output
ML	Maximum likelihood
NR	New radio
OTFS	Orthogonal-time-frequency-space
PSK	Phase-shift keying
QAM	Quadrature amplitude modulation
SINR	Signal-to-interference-plus noise ratio
SISO	Soft-in soft-output
SNR	Signal-to-noise ratio
SUMIS	Subspace marginalization with interference suppression
UAMP	Unitary approximate message passing

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