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Netlist-Aware Mixed-Cell-Height Legalization

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Abstract

Mixed cell heights pose significant challenges to placement in large-scale integrated circuits. In particular, the heterogeneity of cells remarkably complicates the legalization of mixed-cell-height cells. To preserve the global placement results as much as possible, we propose to directly optimize the total displacement through l_1-norm minimization. For illegal cells that extend beyond the chip boundaries, the netlist information of cells is taken into account and a mixed-integer programming model is employed to determine the optimal placement that minimizes the half-perimeter wirelength (HPWL) variation. This joint optimization strikes a balance between minimizing geometric displacement and HPWL, ensuring that the legalization process remains compact to achieve better placement performance. Experimental evaluations on mixed-cell-height benchmarks demonstrate that the proposed approach achieves smaller total displacement and notably lower HPWL variation within practical time limits compared with state-of-the-art legalization methods. Moreover, this work provides a scalable foundation for future extensions to three-dimensional (3D) IC placement legalization.

Keywords: physical design; placement; legalization; mixed-cell-height cell

1. Introduction

In very-large-scale integration (VLSI) design, physical design is a crucial stage that translates a circuit's logical representation into a manufacturable layout. It involves tasks such as placement [1], routing [2], clock tree synthesis [3], timing closure [4], and power optimization [5]. The main objective of physical design is to ensure that the circuit not only meets its functional requirements but also achieves optimal performance, power efficiency, and area utilization within the constraints of fabrication technology. The critical aspect of physical design is placement, which involves arranging the logical units of a circuit on the chip surface in a way that optimizes performance, power consumption, and area. In standard-cell-based design, placement not only needs to consider the relative positioning of the cells but also must integrate various factors such as timing, routing, and power to ensure the design meets all manufacturing and functional requirements.

In standard-cell design, custom-designed digital cells form the foundation of the standard-cell library [6]. These cells are carefully designed and optimized with specific physical and electrical characteristics, such as area, delay, power, and drive strength. Automated place-and-route tools rely on these standard-cell libraries to perform circuit layout and optimization. The goal is to reduce wire length, optimize timing performance, and meet power constraints within the limited chip area through effective placement. As technology nodes continue to shrink and design complexity increases, achieving efficient placement has become a crucial research area in chip design.

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The placement process is typically divided into several key stages, including global placement, legalization [1], and detailed placement [7]. The primary goal of global placement is to roughly position all cells on the chip, leaving enough space for subsequent routing and optimization. In this stage, overlaps between cells are often overlooked, with the focus on minimizing wire length and overall chip area. Legalization ensures that all logic cells are placed within legal regions, with no overlaps, while adhering to design rules and manufacturing constraints. Detailed placement further refines the positions of the cells to minimize wire length, reduce congestion, and meet timing requirements.

Legalization is an important part of placement design in very-large-scale integration (VLSI) [1,8]. It is known that in the global placement, there are still a lot of undesired overlaps among cells. Hence, the purpose of legalization is to eliminate the overlaps while maintaining the solution of global placement as much as possible. In the previous studies, heuristic [9,10] and analytic algorithms are frequently utilized for legalization. Abacus [11] and Tetris [12] are two representative methods for single-height standard-cell legalization. Abacus is an effective approach based on quadratic programming and dynamic programming. Various variants of Tetris subdivides the layout into regions and perform Tetris in each region to prevent the large displacement observed with the original version.

The linear complementarity problem (LCP) has been considered in [13–16] to put the cells into the chip region by adding linear constraint that each cell's x-coordinate must be greater than zero or within a specified range and use a Tetris-like method to make sure all cells are within the floorplan boundary. In [17], the LCP is transformed to a generalized absolute value equation. In [18], a highly efficient linear-time mixed-cell-height legalization approach that optimizes both the total cell displacement and the maximum cell displacement is presented. In [19], a window-based cell insertion technique and two post-processing network flow-based optimizations are proposed. A reinforcement learning-based method is used for legalization in [20] and power distribution network is utilized in [21]. Complex minimum width constraints and the fragmentation effect are considered in study [22] and vertical cell abutment constraint is considered in study [23]. An optimal region is proposed by S.Goto in [24]. In [25–27], mixed-integer programming (MIP) models have been used for assigning illegal cells to chip regions. Moreover, the minimal displacement has been adopted as a key metric for evaluating the performance of solutions in recent studies [11,16], and the legalization method in 2D placement can be used in 3D placement [28]. Nevertheless, minimizing the total displacement or the max displacement methods still suffer from some shortages. One is the ignorance of the information of the netlist. Another drawback is that minimizing the displacement will not minimize the half-perimeter wirelength (HPWL) variation when some cell displacements are so large that they are outside the fence. In this case, their best sites are occupied by other cells and the new sites have nothing to do with the original location.

In this paper, we address the legalization problem through a multi-stage approach. Firstly, in order to simplify the design, we move each cell to the nearest row to minimize the displacement in the vertical direction (y-coordinate). Then, we split each multi-cell-height cell into multiple single-cell-height cells and carry out an l_1-norm minimization to eliminate the cell overlaps in each row. Next, we examine each row successively to identify the illegal cells that exceed the permitted boundaries and utilize a netlist-aware method to reduce the HPWL by mitigating the impact of illegal cells on the repositioned cells. Finally, the previously split multi-cell-height cells are recombined to complete the legalization process. The legalization framework in this work takes into account the balance between HPWL and the minimum total displacement, achieving better placement results. Experimental results demonstrate the advantages of the proposed method over existing solutions.

2. Problem Statement

Given a global placement solution with cell overlaps, the aim of legalization is to place the cells into rows and eliminate the overlaps while ensuring the final solution is closed to the solution of global placement as much as possible. Consider n cells, the objective function is given by

$$\min_{x,y} \quad \sum_{i=1}^{n}(|x_i - x_i'| + |y_i - y_i'|), \tag{1}$$

where x_i (y_i) represents the x-coordinate (y-coordinate) of the i-th smallest cell coordinate and x_i' (y_i') is the initial x-coordinate (y-coordinate) obtained by the global placement. The optimization is performed under the constraints: (i) cells are placed in rows; (ii) cells are not overlapped; and (iii) cells are placed inside the chip region.

Note that the mixed-cell-height legalization problem (1) is NP-hard if the relative positions of the cells are not determined. However, by arranging the cells' x-coordinates in an ascending order, then for each row, the legalization constraints (i) and (ii) can be written as

$$x_j - x_k \geq w_k, \tag{2}$$

where $1 \leq k < j \leq n$ and w_k is the width of cell k. By doing so, the mixed-cell-height legalization problem can be formulated as a quadratic programming problem [16]. Instead of a single row as Abacus and Tetris, the mixed-cell-height legalization problem minimizes total displacement across the entire floorplan area. Since the exact row for the cell to be placed is uncertain, directly solving this problem would be time-consuming and challenging. Hence, it is beneficial to place all cells to the nearest row with minimum displacement.

However, after eliminating the overlaps, some cells may exceed the floorplan boundaries. Thus, we need to implement a certain measure to place the illegal cells to the available spaces within the chip region. In this work, the variation of the HPWL will be adopted for such purpose. Let $\Psi = \{\psi_1, \psi_2, \ldots, \psi_m\}$ be the set of all nets and $p_{x,i}$ ($p_{y,i}$) be the x(y)-coordinate of the i-th cell's pin, then the HPWL is determined as

$$\text{HPWL} = \sum_{\psi \in \Psi} (\max_{i,j \in \psi} |p_{x,i} - p_{x,j}| + \max_{i,j \in \psi} |p_{y,i} - p_{y,j}|), \tag{3}$$

and the HPWL variation is given by

$$\Delta\text{HPWL} = \text{HPWL}_{\text{legalized}} - \text{HPWL}, \tag{4}$$

where $\text{HPWL}_{\text{legalized}}$ is the HPWL corresponding to the placement after legalization (replacing the illegal cells).

3. Proposed Algorithm

In this section, we shall firstly introduce the algorithm flow. Next, the method of overlap elimination is described. Finally, the strategy to find suitable sites for illegal cells is developed.

3.1. Introduction of the Legalization Algorithm

The legalization algorithm is initialized by the global placement result with overlaps among the cells, and is implemented by the following steps: (1) row selection; (2) overlapped cells legalization; (3) netlist-aware available region filling; and (4) multi-height cells merging. More specifically, in order to place all the cells into rows and chip region without overlap, they are firstly arranged in a non-decreasing order according to their x-coordinates and placed into neighboring rows. Hence, the relative positions of the cells

can be determined. On this basis, one can eliminate the overlaps among the cells. Due to the factors such as machine precision, boundary overflow beyond the chip region, and solution inaccuracies, some illegal cells may emerge. In such cases, we reposition them within the available chip region using the netlist-aware method which will be detailed in Section 3.3. Finally, multi-cell-height cells are merged.

The proposed method for netlist-aware mixed-cell-height legalization is outlined in Algorithm 1. More concretely, line 1 defines a set of illegal cells, denoted by $\mathcal{O}$, and line 2 generates a non-decreasing order for the placement of cells. Lines 3–5 involve aligning all cells with the nearest row and updating their y-coordinates accordingly. Line 6 eliminates overlaps in each row, in accordance with the established x-coordinate sequence. Lines 7–11 devote to thoroughly examining each cell's legality. For any illegal cell, it is added to the set $\mathcal{O}$. Lines 12–15 address the remaining illegal cells by available region filling which takes the netlist information into account. The procedure will be detailed in Section 3.3. Line 16 merges the multi-cell-height cells, which have been split at the beginning.

Algorithm 1 Netlist-aware mixed-cell-height legalization

Input: Global placement result

1: Define $\mathcal{O}$ as a set of illegal cells (initially empty)
2: Organize the cells in non-decreasing order based on their x-coordinates
3: **for** each cell **do**
4: y $\leftarrow$ y-coordinate of the nearest row
5: **end for**
6: l_1-norm-based overlapped cell legalization
7: **for** each cell **do**
8: **if** cell is illegal **then**
9: put cell
10: **end if**
11: **end for**
12: Remove illegal cells from netlist and row
13: **for** each cell in the set $\mathcal{O}$ **do**
14: netlist-aware available region filling (see Section 3.3)
15: **end for**
16: Multi-cell-height cells merging

Output: Legalization result

In the proposed legalization algorithm, the core steps include the l_1-norm-based overlapped cell legalization and the netlist-aware available region filling. In the following subsections, we will provide a detailed description of these two steps.

3.2. l_1-Norm-Based Overlapped Cell Legalization

In order to simplify the design, we assign all cells to nearest row so that the total displacement in the y-direction can be minimized. Consequently, the single-cell-height legalization problem can be written as

$$\begin{aligned} \min_{x} \quad & \sum_{i=1}^{n} |x_i - x_i'| \\ \text{s.t.} \quad & x_j - x_k \geq w_k, \quad \text{if} \quad 1 \leq k < j \leq n. \end{aligned} \tag{5}$$

Instead of relaxing the objective function into a quadratic form, we directly reformulate the problem as an l_1-norm minimization problem as

$$\min_{x} \quad \left\| x - x' \right\|_1, \quad \text{s.t.} \quad Bx \geq b, \tag{6}$$

where B is a matrix with only two nonzero elements, i.e., -1 and 1, in each row. The size of B corresponds to the the number of constraints m and the number of cells n. For example, in Figure 1a, cells c_1 and c_3 are placed in row 1, and cells c_2, c_4, and c_5 in row 2, and we have

$$B = \begin{bmatrix} -1 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix}.$$

Accordingly, we have $b = [w_1, w_2, w_4]^T$.

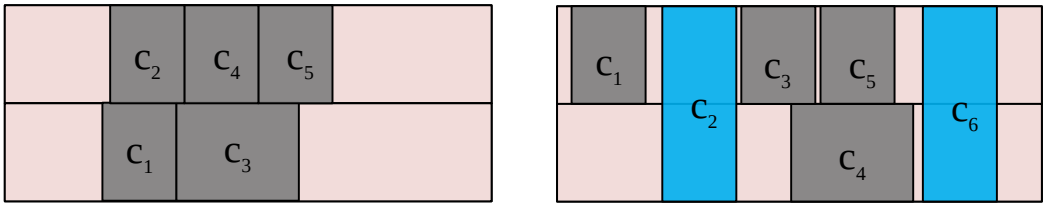


Figure 1. Illustration of the placements with (**a**) single-cell-height and (**b**) double-cell-height cells.

For the case of mixed-cell-height cells, as shown in Figure 1b, the model can be modified as

$$\min_{x} \quad \|x - x'\|_1, \quad \text{s.t.} \quad Bx \geq b, \quad Ex = 0, \tag{7}$$

where the matrix E contains only two non-zero entries -1 and 1 in each row. Suppose that there are r multi-cell-height cells, and their cell heights are h_i, $i = 1, \ldots, r$, then the number of rows in E is $\sum_{i=1}^{r}(h_i - 1)$ and the number of columns is $n + \sum_{i=1}^{r}(h_i - 1)$. For example, in Figure 1b, cells c_2 and c_6 are two-cell-height cells, which must be split into two single-cell-height cells to accommodate the model (7), and we have

$$E = \begin{bmatrix} 0 & -1 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 1 \end{bmatrix}.$$

In this work, the l_1-norm minimization problems (6) and (7) are solved by using the Gurobi optimizer [29] owing to its high performance to tackle large-scale optimization problems.

3.3. Netlist-Aware Available Region Filling

It is worth noting that problems (6) and (7) may not lead to effective solutions, since some cells (e.g., cells c_1 and c_2 in Figure 2) may extend beyond the designated chip region. More specifically, if the left boundary constraints are considered and the right boundary constraints are ignored, or vice versa, it could result in the entire row of cells shifting. This may not only fail to ensure a better HPWL but also increase the number of cells exceeding the chip region in the opposite direction. If both left and right boundary constraints are considered simultaneously, a situation with no feasible solution may arise because the capacity of each row may be insufficient to accommodate the cells whose total area exceeds the available space.

In order to tackle the aforementioned problem and to minimize the displacement, existing designs used the Tetris-like method in [13–16] to place the cells beyond the chip region, like cells c_1 and c_2 in Figure 2, to the available space inside the chip region, as shown in Figure 3a. However, these sites may be outside the desired net. In fact, the expected sites for c_1 and c_2, in terms of the minimum HPWL, should be the positions remarked by blue dashed rectangles. Furthermore, if there is no space available nearby, it becomes necessary

to search for a more distant empty spot, as shown in Figure 3b. In this case, the HPWL will be considerably increased due to large displacements.

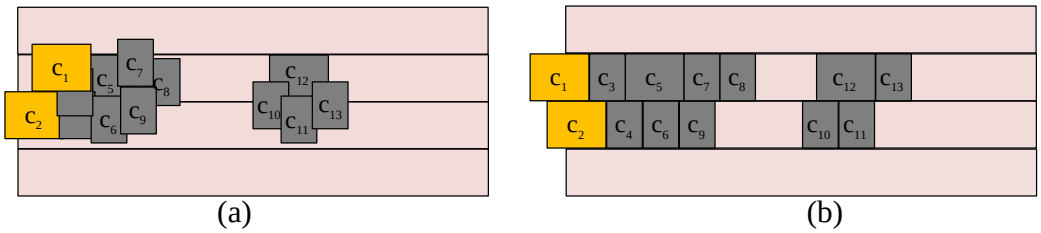


Figure 2. Illustration of placements with cells located outside the chip region. (**a**) Global placement, and (**b**) after overlapped cell legalization.

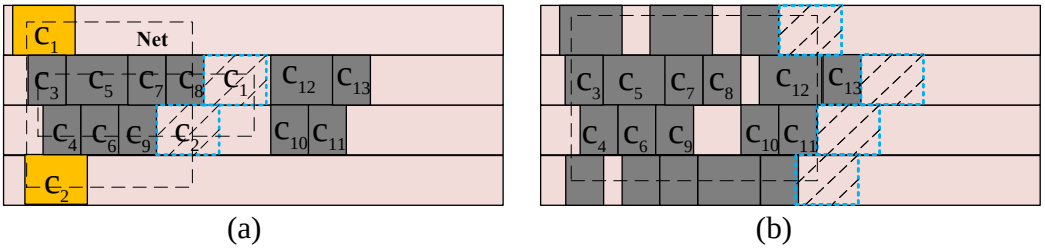


Figure 3. Site candidates for c_1 and c_2. (**a**) The difference between the chosen sites and expected sites. (**b**) More distant sites.

Motivated by these issues, in this work, the following tasks will be considered in our design: (1) Cells must be placed inside the chip region; (2) The HPWL should be minimized; (3) The impact of cell placement order on other nets should be minimized. To handle these tasks, a netlist-aware available region filling method is developed and an MIP model is employed. For task (1), it is easy to find an available space to place. For task (2), we take the netlist information into account to minimize the variation of HPWL. For task (3), the effectiveness of setting a reasonable placement order is notable.

Assuming that there are Q illegal cells beyond the chip region, we legalize them successively. In the q-th ($q = 1, \cdots, Q$) step, let $\mathcal{J}_q$ be the set of placed cells and $\mathcal{I}_q$ be the set of candidate sites for the illegal cells. Moreover, denote by $|\mathcal{J}_q|$ and $|\mathcal{I}_q|$ be the number of placed cells and number of sites, respectively. For $(i, j) \in \mathcal{I}_q \times \mathcal{J}_q$, $1 \leq i \leq |\mathcal{I}_q|$ and $0 \leq j \leq |\mathcal{J}_q|$, the variable $a_j \in \{0, 1\}$ indicates whether the placed cell j is linked (denoted by $a_{ij} = 1$) or not ($a_{ij} = 0$) by an illegal cell re-allocated to site i, according to the netlist information. Since each site can be placed one cell at most, l_i is a binary variable that equals to one if and only if a cell is located at site i, and zero otherwise.

Following the above notations, let ΔHPWL_q be the HPWL variation after legalizing the q-th illegal cell, then the following problem can be formulated as follows:

$$\min_{l,a} \quad \Delta\text{HPWL}_q \tag{8a}$$

$$\text{s.t.} \quad \sum_{i \in \mathcal{I}_q} a_{ij} = 1, \ \forall j \in \mathcal{J}_q, \tag{8b}$$

$$a_{ij} \leq l_i, \ \forall i \in \mathcal{I}_q, j \in \mathcal{J}_q, \tag{8c}$$

$$\sum_{i \in \mathcal{I}_q} l_i = 1, \tag{8d}$$

$$a_{ij} \in \{0, 1\}, \ \forall i \in \mathcal{I}_q, j \in \mathcal{J}_q, \tag{8e}$$

$$l_i \in \{0, 1\}, \ \forall i \in \mathcal{I}_q. \tag{8f}$$

where the constraint (8b) ensures that each cell $j \in \mathcal{J}_q$ is located to a candidate site $i \in \mathcal{I}_q$. Constraint (8c) prohibits allocations to candidate sites that are not chosen as potential sites: when $l_i = 0$ (i.e., $i \notin \mathcal{I}_q$), $a_{ij} = 0 \; \forall j \in \mathcal{J}_q$, i.e., no placed cell can be assigned to the site.

To solve the problem which involves netlist information of the cells, we develop a netlist-aware available region filling method, as outlined in Algorithm 2, where $cost_{hp}$, $cost_{hp1}$, $cost_d$, and $cost_{d1}$ stand for the value of HPWL, the updated HPWL, the value of distance to the initial location, and the updated distance, respectively, while $\mathcal{P}$, $P_{(x,y)}$, and $T_{(x,y)}$ denote the set of available sites, coordinate of each available site, and the coordinate of the chosen site, respectively.

Algorithm 2 Netlist-Aware Available Region Filling

Input: q-th illegal cell

1: Define h as the row height of the q-th illegal cell
2: Define $\mathcal{J}_q$ as a set of cells already inside the chip region
3: Define $\mathcal{P} = \{\mathcal{P}_1, \mathcal{P}_2, \dots\}$ as a set of available sites in each row and $P_{i(x,y)}$ is the coordinate of site in the i-th row.
4: **if** the height of cell is larger than 1 **then**
5: **for** each blank space in each row **do**
6: select the available sites for multi-cell-height illegal cell
7: $P_{i(x,y)} \geq max\{min\{P_{i(x,y)}, \dots, P_{i+h(x,y)}\}\}$
8: $P_{i(x,y)} \leq min\{max\{P_{i(x,y)}, \dots, P_{i+h(x,y)}\}\}$
9: **end for**
10: **end if**
11: $cost_h \leftarrow \infty$ and $cost_d \leftarrow \infty$
12: **if** $\mathcal{J}_q$ is not empty **then**
13: **for** each site $P_{(x,y)}$ in $\mathcal{P}$ **do**
14: **if** $cost_{hp1} < cost_{hp}$ **then**
15: $cost_{hp} \leftarrow cost_{hp1}$ and $T_{(x,y)} \leftarrow P_{(x,y)}$
16: **end if**
17: **end for**
18: Add illegal cell to netlist and row
19: **return** $T_{(x,y)}$
20: **else if** $\mathcal{J}_q$ is empty **then**
21: **for** each site $P_{(x,y)}$ in $\mathcal{P}$ **do**
22: **if** $cost_{d1} < cost_d$ **then**
23: $cost_d \leftarrow cost_{d1}$ and $T_{(x,y)} \leftarrow P_{(x,y)}$
24: **end if**
25: **end for**
26: Add illegal cell to netlist and row
27: **return** $T_{(x,y)}$
28: **end if**

Output: $T_{(x,y)} \leftarrow$ Available site

In Algorithm 2, the input is an illegal cell and the output is the illegal cell's chosen site. Line 1 defines h as the row height of the q-th illegal cell. Line 2 defines a set $\mathcal{J}_q$ of the other cells within the chip region that are listed in the netlist, which initially includes the illegal cell. Line 3 is to search available sites within the chip region. As shown in Figure 4, the red dashed rectangles represent the candidate sites for the illegal cell. To optimize efficiency, the width of the illegal cell is used as the interval for the search sites, rather than the greatest common divisor (GCD) of all cell widths [25], which has a large number of candidate sites and costs much computation time, especially if not performed in a parallel and multi-core environment. For multi-cell-height cell, we first search each row's candidate range using the method of single-cell-height cell candidate sites in line 4. Then, based on the multiple of the illegal cell's row height, we select adjacent rows for

interval merging, ultimately obtaining the candidate interval. Finally, the candidate sites for the illegal multi-cell-height cell can be obtained through interval splitting in lines 5–11, as shown in Figure 4. In each blank in each row, the rightmost site is the smallest site of the rightmost site in successive rows and the leftmost site is the biggest site of the leftmost site. When cells are removed from the netlist, the netlist may contain no cells. Lines 12–22 demonstrate how to find the optimal solution when the remaining netlist is not empty. Once the netlist has been fully processed, the HPWL is inevitably zero, rendering the use of HPWL as an objective function inapplicable. Lines 23–34 illustrate the method for determining the optimal position by minimizing displacement as the objective function, given that the remaining netlist is empty. The operations in lines 21 and 32 add relevant information of the illegal cell to the netlist and row, respectively. Failure to execute these operations may lead to errors during the calculation of the placement of subsequent illegal cells, as the selected site might be occupied by previously placed cells, and the calculation of HPWL might not consider the previously placed cells.

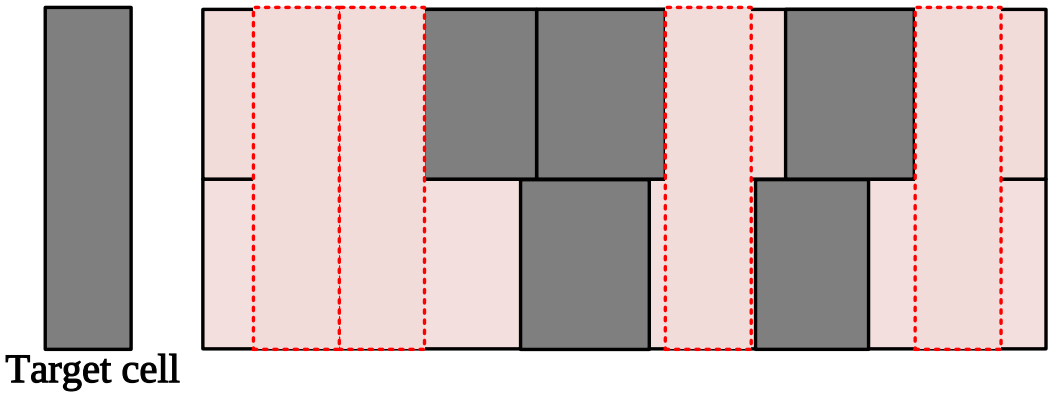


Figure 4. Available sites for double-cell-height cell.

4. Netlist-Aware Available Region Filling

For legalization, the consideration of netlist information pertaining to the cells implies that the number of nets connected to the illegal cell will inevitably have a certain impact. In practice, we can categorize the illegal cells into the following scenarios:

Case 1: In Figure 5a, cell A is included in a single net, and there are already placed cells ($\geq$1) within the chip region.

Case 2: In Figure 5b, cell A is included in multiple nets (>1), and unlike Case 1, the placement of cell A must consider the impact on these nets. Thus, the placement of such cells will have a greater impact on performance compared to the first scenario.

Case 3: In Figure 5c, different from the previous scenarios, the current region lacks placed cells associated with cell A, implying that the placement of cell A does not affect the HPWL performance. In this scenario, the positioning of cell A is not critical, and it can be placed as closely as possible to its original location to minimize displacement.

In what follows, we outline the sequence of the placement of illegal cells. First, we arrange the illegal cells in a descending order of the number of nets they are connected to. Next, if the illegal cells have the same number of connected nets, they are arranged in descending order of size. This method of arrangement sets the priority for placement based on the degree of impact that the illegal cells have on the HPWL. In this way, the placement priority of the illegal cells in Case 3 is the lowest. Given the limited selectable range for multi-cell-height cells, their placement priority is the highest. By setting this order, the impact of the placement sequence of cells on other cells is minimized as much as possible.

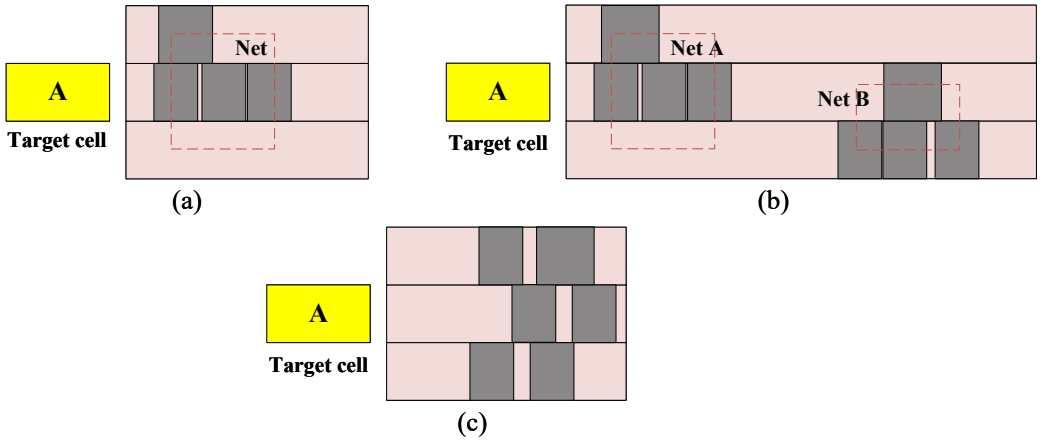


Figure 5. Nets for the target cell. The red dashed rectangle is the net of the legalized cells. (**a**) Target cell is linked to only one net. (**b**) Target cell is linked to multiple nets. (**c**) No net includes the target cell.

After legalization, as shown in Figure 6, we may obtain different placements, where the HPWL remains the same if the illegal cell A is placed at any one site within region R. However, different sites in region R have different effects on other nets. For instance, illegal A in Figure 6b has less effect on other nets than the case in Figure 6a, because illegal A is entirely within region R. Usually, the exact control of candidate site selection during legalization is difficult, and we cannot always achieve a solution as the case in Figure 6b unless some special site (e.g., the site has the same x-coordinate as the pin of the left cell in Figure 6b) can be chosen. Moreover, since all sites within region R have identical HPWL, it is hard to reach the solution as Figure 6b if the site as shown in Figure 6a has been found first.

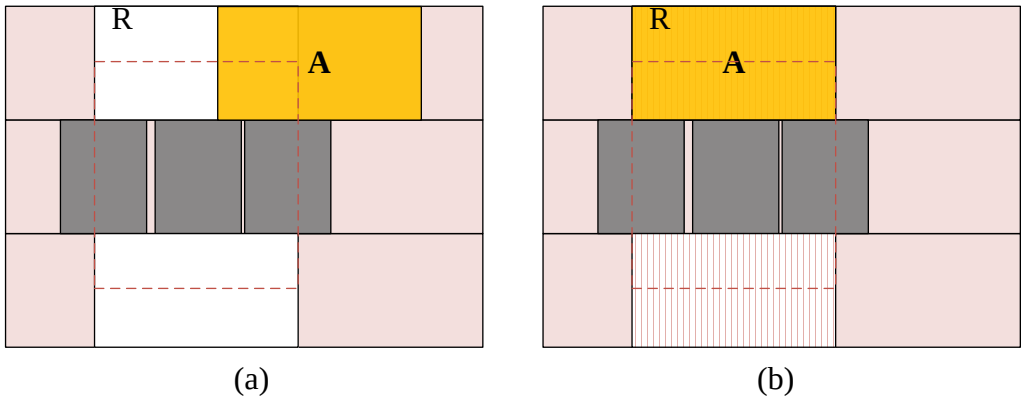


Figure 6. Selected site for cell A after legalization. The red dashed rectangle is the net containing cell in region R. (**a**) Cell A is not entirely within region R. (**b**) Cell A is entirely within region R.

5. The Remarks of l_1-Norm-Based Overlapped Cell Legalization

The goal of legalization is to eliminate the overlap between cells while retaining the result of the global placement as much as possible. Therefore, in the legalization stage, the total displacement volume of the cell will be used as the performance evaluation indicator.

From the expressions of l_1-norm and l_2-norm, it can be seen that l_1-norm allows major displacement to be concentrated in a few more suitable cells, thereby keeping the vast majority of cells almost stationary. The total displacement is smaller, which is more in line with the requirement of "retaining the global placement". l_2-norm will impose a heavy penalty for large displacement and allocate it to more cells, causing more cells to move.

The boundary conditions of the problem have been relaxed. The LCP method for comparison requires setting strict boundary conditions (a left boundary was set). In this

work, no strict boundary conditions were set in this step. The reason is that the existence of the left boundary may make the cells close to the left boundary have stronger consistency in the movement direction (such as moving to the right simultaneously), thereby resulting in the final placement being far from the original placement position. This will lead to a significant increase in HPWL and total dispalcement.

The method for replacing cells that exceed the boundary: the method compared in this paper merely fills this part of the cells using the tetris-like method, without taking into account the netlist information of cells (such as the link between the cells beyond the boundary and the placed cells and the connection between the cells beyond the boundary), which are considered in this work.

6. Experimental Results

We use the benchmarks provided by the ICCAD-2015 contest [30] and the 2018 Initial-Detailed-Routing contest [31]. These benchmarks do not need to consider fence regions in the original benchmarks and are under consideration and exclusively consist of cells with a single row height. These benchmarks have different orders of magnitude in the number of cells, different floorplan sizes, different cell characteristics, and different densities. Density is an important indicator for measuring the degree to which a local area of the chip is occupied by standard cells, reflecting the relationship between area utilization and space congestion. These benchmarks also have different netlist information and placement characteristics. Selecting these benchmarks can well test the applicability of the method. The global placement results were generated using OpenROAD [32,33]. To evaluate the algorithm's applicability with mixed-cell-height cells, we make the same modifications as [16] to create test benchmarks comprising both single-cell-height and multi-cell-height cells, thereby ensuring that there is sufficient space within the chip region to accommodate all cells. The benchmarks ended with "_md" in the tables are such modified versions. All experiments are conducted on a Windows workstation with Intel(R) Core(TM) i7-7700 3.60 GHz CPU and 16 GB memory.

Table 1 shows the overlapped cells legalization experiment results. In this table, "Density" refers to the density of the design, "GP HPWL" the wirelength of global placement result, "#S. Cell" the number of standard single-cell-height cells, and "#M. Cell" the number of multi-cell-height cells.

Table 1. The information of benchmarks.

Benchmark	Density	GP HPWL (m)	#S. Cell	#M. Cell
fft_1	0.84	0.825	32,281	0
fft_2	0.5	0.382	32,281	0
perf_1	0.91	1.103	112,644	0
mult_1	0.81	3.410	155,325	0
ispd18_test1	0.86	0.101	8879	0
fft_1_md	0.84	0.825	30,362	1919
fft_2_md	0.5	0.382	30,362	1919
perf_1_md	0.91	1.103	104,159	8485
mult_1_md	0.81	3.410	152,632	2693

Tables 2 and 3 give the experimental results of the entire legalization flow. In these tables, "%ΔHPWL(solving problem (7))" denotes the percentage change in HPWL relative to "GP HPWL" after solving (7), "Total Disp" represents the total displacement of all cells, and "Max Disp" gives the maximum displacement across all cells. "LG HPWL" represents the HPWL after legalization and "%ΔHPWL" indicates the increase in "LG HPWL" from the results of global placement.

Table 2. Overlapped cells legalization experimental results. Bolded values highlight the better method.

Benchmark	%ΔHPWL (Solving Problem (7))		Max Disp (nm)		Total Disp (cm)	
	LCP	Ours	Tetris-like	Ours	Tetris-like	Ours
fft_1	6.102	**5.406**	29,197	**17,472**	5.577	**4.815**
fft_2	**5.93**	6.09	**8249**	8609	2.03	**1.94**
perf_1	12.54	**10.93**	23,736	**20,475**	17.34	**14.76**
mult_1	7.23	**7.13**	24,894	**24,780**	28.28	**27.35**
ispd18_test1	51.260	**37.572**	346,809	**203,892**	16.24	**10.92**
fft_1_md	14.20	**9.09**	49,786	**28,485**	47.86	**19.61**
fft_2_md	**5.778**	5.947	7639	**7550**	2.12	**2.01**
perf_1_md	28.77	**25.90**	112,644	**57,684**	490.9	**209.0**
mult_1_md	13.782	**10.785**	76,346	**45,282**	108.6	**61.18**
Average	16.177	**13.206**	75,478	**46,025**	79.88	**39.06**

Table 3. Final legalization experimental results. Bolded values highlight the better method.

Benchmark	%ΔHPWL		LG HPWL (dm)		RT (s)
	Tetris-like	Ours	Tetris-like	Ours	
fft_1	6.23	**6.03**	8.78	**8.74**	2.34
fft_2	**5.93**	6.086	**4.050**	4.056	3.38
perf_1	12.77	**11.35**	12.44	**12.28**	3.18
mult_1	7.24	**7.13**	36.57	**36.53**	4.55
ispd18_test1	48.68	**33.76**	1.63	**1.47**	3.3
fft_1_md	27.21	**13.99**	10.49	**9.40**	9.09
fft_2_md	**5.78**	5.94	4.04	**4.04**	2.44
perf_1_md	171.96	**58.64**	30.01	**17.59**	17.28
mult_1_md	15.59	**12.18**	39.42	**38.25**	81.67
Average	33.52	**17.23**	16.38	**14.70**	14.13

The experimental results show that our method can achieve improved results on multiple benchmarks, fully demonstrating the applicability and effectiveness of this method for different placement characteristics, and also reflecting that this method can achieve a better balance in the optimization between HPWL and the total displacement of the cells. Our work achieves 18.4% smaller smaller HPWL increase rate after solving (7) compared to the LCP method and 51.1% smaller total displacement and 39% smaller maximum displacement compared to the Tetris-like method from Table 2. It is seen from Table 3 that, when all cells have been legalized, our work achieves 48.59% smaller HPWL increase rate and 10.23% smaller HPWL increase in "LG HPWL". Since the double-cell-height cells in mixed-cell-height benchmarks are randomly chosen, their unpredictable positions maybe lead to substantial variations in HPWL post-legalization. The results of some benchmarks have a relatively large gap, which may be related to the density, placement size, and number of cells of the benchmarks. These factors lead to the fact that some rows may have a large number of cells, and the strict boundary conditions in the LCP method cause these cells to be far from their original positions. Figure 7 shows the global placement result of the benchmark fft_2_md, and Figure 8 provides a comparison of the legalization results that only include standard cells between our method and the LCP method. Figure 9a illustrates the legalization result of the benchmark fft_2_md, and the partial zoom-in result from Figure 9a is presented in Figure 9b.

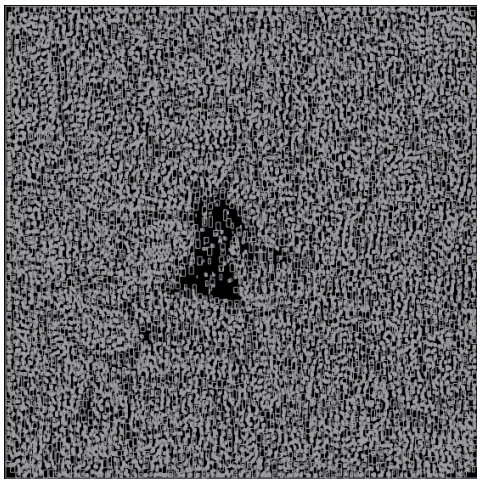
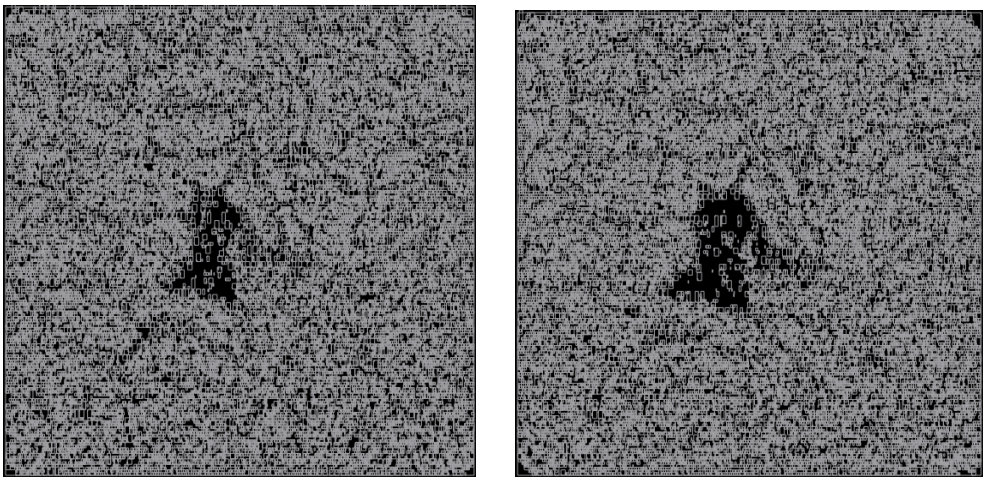
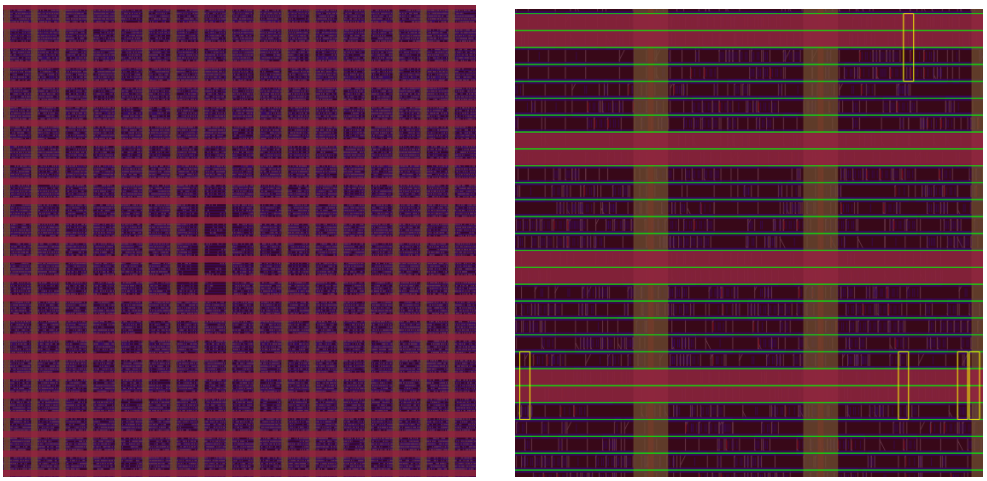


Figure 7. The result of global placement.



(a) Proposed method (b) LCP and Tetris method

Figure 8. Legalization comparion on fft_2_md benchmark.



(a) Legalization results (b) Details results

Figure 9. Legalization and details of proposed method on fft_2_md benchmark.

7. Conclusions

In this work, we have presented a mixed-cell-height standard-cells legalization algorithm. To maximize the preservation of the results from the global placement, we proposed

to directly optimize the total displacement. Moreover, to ensure that all the cells are placed inside the chip region, we proposed an algorithm to minimize the HPWL increase based on the existing placement and obtained improved results. Experimental results have shown the effectiveness and superiority of the proposed approach, achieving at least a 4% improvement in the final legalized placement performance in multiple benchmarks. Optimization of HPWL directly affects routing complexity and timing performance, while reducing total cell displacement helps lower power consumption and enhance layout stability. Furthermore, in the context of 3D chip design, the scalability and portability of the legalization method become increasingly important. The portability of legalization method across different 3D chip configurations and technologies will be key to ensuring the continued scalability and practical application of VLSI design principles in the future of 3D ICs.

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