



Communication

Decentralized Predictor Stabilization for Interconnected Networked Control Systems with Large Uncertain Delays and Event-Triggered Input

Yang Zhu^{1,2}, Meng Zhang³ and Qiang Jiang^{2,*}

- ¹ College of Control Science and Engineering, Zhejiang University, Hangzhou 310027, China; zhuyang88@zju.edu.cn
- ² School of Electronic Information and Artificial Intelligence, Leshan Normal University, Leshan 614000, China
- ³ School of Cyber Science and Engineering, Xi'an Jiaotong University, Xi'an 710049, China; mengzhang2009@xjtu.edu.cn
- * Correspondence: jiangqiang@lsnu.edu.cn

Abstract: In this article, we propose a control scheme with predictors in a decentralized manner for coupled networked control systems (NCSs) under uncertain, large time-delays and event-triggered inputs. The network-induced delays are handled via the prediction; thus, the delay value is allowed to be large, and the burden of the network is relieved by the event-triggered input. Two methods are employed to deal with the large delay issue: the state and output feedback. When the state of each subsystem is measurable, full-state feedback is used, whereas when the plant state cannot be measured, output feedback is employed with the help of an observer, which is more common in practice. Instead of treating the interactive plants like a global system, the exponential stability of the coupled systems, under decentralized predictors with asynchronous sampled-data feedback, is analyzed in a decentralized way. Finally, the proposed methods are verified via an example of three interconnected cart–pendulum systems, while such systems would not be stabilizable by the traditional approach when the network-induced delays are relatively large.

Keywords: decentralized; predictor; delay; interconnected; networked control systems

1. Introduction

Making full use of the burgeoning technologies of digital communication, networked control systems (NCSs) are demonstrated to be a quite effective modern control method. However, the NCSs' development is not without difficulties. One of the important technical challenges in NCSs is the time-delay arising from network transmission, which deteriorates the performance of controlled systems if the delay is ignored in the design. A large body of existing studies on NCSs care about the robustness to delays provided that delay values are not large. In other words, the transmission delays caused by the communication network are not addressed in the control design, and they only explore the maximum delay that the control systems are able to withstand to preserve performance [1,2].

For the purpose of compensating delays that are large, a useful tool is the prediction approach, which has seen popular growth since it was first proposed in 1959 [3]. Nevertheless, a lot of research on the predictor is limited to a centralized controller of a single plant [4–9]. In [10–18], the network-dependent control of interconnected systems under communication time-delay considers predictor-free stabilization where the delays are disregarded when the decentralized controllers are designed; thus, the delay length cannot be "large". Considering two subsystems, a recent paper [19] investigates the continuous-time predictor by state feedback.

On the other hand, as illustrated in [20,21], as networked control systems in either wired or wireless manners have found wide applications in practice, solutions to deal with network



Citation: Zhu, Y.; Zhang, M.; Jiang, Q. Decentralized Predictor Stabilization for Interconnected Networked Control Systems with Large Uncertain Delays and Event-Triggered Input. *Electronics* 2024, *13*, 819. https:// doi.org/10.3390/electronics13050819

Academic Editor: Luca Patanè

Received: 25 April 2023 Revised: 2 February 2024 Accepted: 14 February 2024 Published: 20 February 2024



Copyright: © 2024 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). constraints involving communication and computation have been unavoidable. The effective solution to overcome these constraints is event-based control, which results in reducing the workload of delayed NCSs and has become increasingly popular. In [22], the authors considered decentralized MRAC for interconnected time-delay systems with delays in both the state and in the input via a nested predictor, but they required that the delays were constant and did not use event-triggered schemes to relax the controller workload.

On the basis of the predictor feedback for a single plant [8,9] and the predictor-free feedback for coupled NCSs [16], this paper applies predictor-based stabilization to coupled NCSs with uncertain, large network-induced delays and event-triggered inputs in a decentralized manner, which is a non-trivial problem due to large delays and interactions among subsystems. By "large delays", we refer to those delays that render the control system unstable if we do not introduce any designs to take care of the delay's negative impact [2,23]. In comparison with the literature [24,25], the delays are variable and the event-triggered mechanism is brought in. Two distinct methods are utilized to deal with large delays: full-state and observer-based output feedback. In Section 3, we assume the plant state to be measurable, and use full-state feedback to derive simpler LMI conditions. In Section 4, we take into account a more challenging case of the unmeasurable state, and employ output feedback with the observer, which is important in implementation. Although the global plant is made up of a few of interconnected subsystems, the local control networks are designed in a decentralized manner and do not utilize information from their neighbors. The communication network-induced delays in the subsystems differ from each other, and the sampling instants of the decentralized sensors are asynchronous. An event-triggered strategy is included to decrease the network's workload.

As an alternative to analyze the interactive systems as a whole entity, under decentralized predictors, we construct the Lyapunov–Krasovskii functional of stability analysis in a decentralized way to ensure the interacted systems are exponentially stable.

2. Sampled-Data Control for Coupled NCSs: State Feedback

Consider the interconnected linear systems below:

$$\dot{x}_{i}(t) = A_{i}x_{i}(t) + B_{i}u_{i}(t) + \sum_{j \neq i} F_{ij}x_{j}(t)$$
(1)

$$y_i(t) = C_i x_i(t) \tag{2}$$

where $i = 1, 2, \dots; N$ is the index of the subsystem; $x_i(t) \in \mathbb{R}^{n_i}, y_i(t) \in \mathbb{R}^{q_i}$, and $u_i(t) \in \mathbb{R}^{m_i}$ are the plant state, the plant output, and the control input of the *i*th subsystem, respectively; $x_j(t) \in \mathbb{R}^{n_j}$ is the plant state of the *j*th subsystem; and F_{ij} is the interconnection matrix, which refers to the coupling interaction between the *i*th and *j*th subsystems. We assume that the pair (A_i, B_i) is stabilizable and the pair (A_i, C_i) is detectable.

In this section, we deal with a relatively simple circumstance where full-state feedback is taken into account.

As shown in Figure 1, we apply networked control with sampled data to large-scale systems (1). As revealed in Figure 2, we denote the *i*th subsystem's sampling instants as $\{s_k^i\}, k \in \mathbb{Z}_0^+$, which satisfy

$$0 = s_0^i < s_1^i < \dots < s_k^i < \dots, \quad s_{k+1}^i - s_k^i \le h_i$$
(3)

Signal transmissions suffer from large communication delays in both the sensor-tocontroller channel and the controller-to-actuator channel, which are denoted by $r_i^{sc} + \eta_k^i$ and $r_i^{ca} + \mu_k^i$, respectively, where $r_i^{sc} \ge 0$ and $r_i^{ca} \ge 0$ are known constant delays, whereas η_k^i and μ_k^i are uncertain variable delays such that

$$0 \le \eta_k^i \le \eta_i, \quad 0 \le \mu_k^i \le \mu_i$$

$$\tag{4}$$

Since compensation for large delays is one of the main purposes of this paper, there is no restriction that the delays $r_i^{sc} + \eta_k^i$ and $r_i^{ca} + \mu_k^i$ should be less than the sampling interval h_i .

We assume the controller and actuator to be event-driven; in other words, once they receive new data they update their outputs. As a result, the controller's updating instants and actuation instants are, respectively, the following:

$$\zeta_k^i = s_k^i + r_i^{sc} + \eta_k^i, \quad t_k^i = \zeta_k^i + r_i^{ca} + \mu_{k'}^i$$
(5)

and they satisfy

$$\zeta_k^i \le \zeta_{k+1}^i, \quad t_k^i \le t_{k+1}^i, \quad k \in \mathbb{Z}_0^+ \tag{6}$$



Figure 1. State feedback for interconnected NCSs with predictors and event-triggered controllers.



Figure 2. The timing relationship among the sampling, actuation instants, and transmission delays.

For the event-triggered mechanism, a piece-wise function is defined such that

$$u_i(t) = u_i(\zeta_k^i), \quad t \in [\zeta_k^i, \zeta_{k+1}^i)$$
(7)

In order to relieve the network's burden, the event-triggered scheme is employed. The key idea of the event-triggered strategy is the following: only when the relative change among the sequential inputs is greater than a threshold do we send control signals such that

$$\bar{u}_{i}(\zeta_{k}^{i}) = \begin{cases} \bar{u}_{i}(\zeta_{k-1}^{i}), & \left|u_{i}(\zeta_{k}^{i}) - \bar{u}_{i}(\zeta_{k-1}^{i})\right|^{2} \le \sigma_{i} \left|u_{i}(\zeta_{k}^{i})\right|^{2} \\ u_{i}(\zeta_{k}^{i}), & \left|u_{i}(\zeta_{k}^{i}) - \bar{u}_{i}(\zeta_{k-1}^{i})\right|^{2} > \sigma_{i} \left|u_{i}(\zeta_{k}^{i})\right|^{2} \end{cases}$$
(8)

where $\sigma_i > 0$ and $\bar{u}_i(\zeta_{-1}^i) = 0$.

Under the above mechanism, subsystem (1) becomes

$$\dot{x}_{i}(t) = A_{i}x_{i}(t) + B_{i}\bar{u}_{i}(\zeta_{k}^{i}) + \sum_{j \neq i} F_{ij}x_{j}(t), \quad t \in [t_{k}^{i}, t_{k+1}^{i})$$
(9)

For sake of compensating for the large delay, the predictor is selected as

$$z_i(t) = e^{A_i(r_i^{sc} + r_i^{ca})} x_i(t) + \int_{t - r_i^{ca}}^{t + r_i^{sc}} e^{A_i(t + r_i^{sc} - s)} B_i u_i(s) ds$$
(10)

and the predictor-based state feedback law is designed as

$$u_{i}(\zeta_{k}^{i}) = K_{i}z_{i}(s_{k}^{i})$$

$$= K_{i}\left(e^{A_{i}(r_{i}^{sc} + r_{i}^{ca})}x_{i}(s_{k}^{i}) + \int_{s_{k}^{i} - r_{i}^{ca}}^{s_{k}^{i} + r_{i}^{sc}} e^{A_{i}(s_{k}^{i} + r_{i}^{sc} - s)}B_{i}u_{i}(s)ds\right)$$
(11)
(12)

The clocks for the sensor, controller, and actuator are assumed to be synchronized. Please note the upper limit of integral $s_k^i + r_i^{sc} = \zeta_k^i - \eta_k^i \leq \zeta_k^i$ from (5), which implies that the input $u_i(s)$ over the historical time-interval $[s_k^i - r_i^{ca}, s_k^i + r_i^{sc}]$ is available to the controller at time ζ_k^i . Therefore, the integral term in (12) with a piecewise constant $u_i(s)$ defined by (7) is implementable.

For the stability analysis, we focus on the prediction-based future state governed by

$$\dot{z}_{i}(t) = A_{i}z_{i}(t) + B_{i}u_{i}(t+r_{i}^{sc}) + e^{A_{i}(r_{i}^{sc}+r_{i}^{ca})}\sum_{j\neq i}F_{ij}x_{j}(t) + e^{A_{i}(r_{i}^{sc}+r_{i}^{ca})}B_{i}\Big(\bar{u}_{i}(\zeta_{k}^{i}) - u_{i}(t-r_{i}^{ca})\Big), \quad t \in [t_{k}^{i}, t_{k+1}^{i})$$
(13)

We have the following equalities implied by (7) and (11),

$$u_i(t + r_i^{sc}) = K_i z_i(t - \tau_{i0}(t))$$
(14)

$$u_i(t - r_i^{ca}) = K_i z_i(t - \tau_{i1}(t))$$
(15)

$$\bar{u}_i(\zeta_k^i) = e_i(t) + u_i(\zeta_k^i) = e_i(t) + K_i z_i(t - \tau_{i2}(t))$$
(16)

where

$$\begin{aligned} \tau_{i0}(t) &= t - s_k^i, \quad t \in [\zeta_k^i - r_i^{sc}, \zeta_{k+1}^i - r_i^{sc}) \\ \tau_{i1}(t) &= t - s_k^i, \quad t \in [\zeta_k^i + r_i^{ca}, \zeta_{k+1}^i + r_i^{ca}) \\ \tau_{i2}(t) &= t - s_k^i, \quad t \in [t_k^i, t_{k+1}^i) \\ e_i(t) &= \bar{u}_i(\zeta_k^i) - u_i(\zeta_k^i), \quad t \in [t_k^i, t_{k+1}^i) \end{aligned}$$

From (3), (4), and (8), we have

$$0 \le \tau_{i0}(t) \le h_i + \eta_i = \bar{\tau}_i \tag{17}$$

$$r_i = r_i^{sc} + r_i^{ca} \le \tau_{i1}(t) \le \tau_{i2}(t) \le h_i + r_i^{sc} + r_i^{ca} + \eta_i + \mu_i = \bar{\tau}_i$$
(18)

$$\sigma_i |u_i(\zeta_k^i)|^2 - |e_i(t)|^2 = \sigma_i |K_i z_i(t - \tau_{i2}(t))|^2 - |e_i(t)|^2 \ge 0, \quad t \in [t_0^i, +\infty)$$
(19)

For the *j*th subsystem, the inverse conversion of (10) is given as

$$\begin{aligned} x_j(t) &= e^{-A_j(r_j^{sc} + r_j^{ca})} z_j(t) - \int_{t - r_j^{ca}}^{t + r_j^{sc}} e^{A_j(t - r_j^{ca} - s)} B_j u_j(s) ds \\ &= e^{-A_j(r_j^{sc} + r_j^{ca})} z_j(t) - \xi_j(t) \end{aligned}$$
(20)

where

$$\xi_j(t) = \int_{t-r_j^{ca}}^{t+r_j^{sc}} e^{A_j(t-r_j^{ca}-s)} B_j u_j(s) ds = \int_0^{r_j^{sc}+r_j^{ca}} e^{-A_j\theta} B_j u_j(t+\theta-r_j^{ca}) d\theta$$
(21)

which satisfies the following inequality:

$$\begin{aligned} |\xi_{j}(t)|^{2} &= \left| \int_{0}^{r_{j}^{sc} + r_{j}^{ca}} e^{-A_{j}\theta} B_{j} u_{j}(t+\theta - r_{j}^{ca}) d\theta \right|^{2} \\ &\leq (r_{j}^{sc} + r_{j}^{ca}) \int_{0}^{r_{j}^{sc} + r_{j}^{ca}} \left| e^{-A_{j}\theta} B_{j} u_{j}(t+\theta - r_{j}^{ca}) \right|^{2} d\theta \\ &\leq \underbrace{r_{j} \int_{0}^{r_{j}} \left| e^{-A_{j}\theta} B_{j} \right|^{2} ds |K_{j}|^{2}}_{\delta_{j}} \sup_{\theta \in [-\bar{\tau}_{i},0]} |z_{j}(t+\theta)|^{2} \end{aligned}$$
(22)

Substituting (14)–(16) and (20) into (13), we obtain a closed-loop system for stability analysis as follows:

$$\begin{aligned} \dot{z}_{i}(t) &= A_{i}z_{i}(t) + B_{i}K_{i}z_{i}(t - \tau_{i0}(t)) \\ &+ e^{A_{i}r_{i}}B_{i}K_{i}(z_{i}(t - \tau_{i2}(t)) - z_{i}(t - \tau_{i1}(t))) \\ &+ e^{A_{i}r_{i}}B_{i}e_{i}(t) + e^{A_{i}r_{i}}\sum_{j\neq i}F_{ij}\Big(e^{-A_{j}r_{j}}z_{j}(t) - \xi_{j}(t)\Big), \quad t \in [t_{0}^{i}, +\infty) \end{aligned}$$

$$(23)$$

Theorem 1. Consider a closed-loop system which is made up of the plant (9) and controller (12). Provided positive tuning parameters ϵ_1 , ϵ_2 , and α such that $\epsilon_1 + \epsilon_2 < \alpha$, let matrices $P_i, S_i, R_i, Q_i, U_i, W_i \in \mathbb{R}^{n_i \times n_i} > 0$, $P_{i2}, P_{i3}, G_{i0}, G_{i1}, G_{i2}, G_{i3} \in \mathbb{R}^{n_i \times n_i}$ and $P_j \in \mathbb{R}^{n_j \times n_j} > 0$ and scalar parameters $\sigma_i > 0$; $\lambda_j > 0$, for $j = 1, \dots, M$; and $j \neq i$ satisfy the LMIs:

$$\Phi_{i} < 0, \quad \begin{bmatrix} R_{i} & G_{i0} \\ * & R_{i} \end{bmatrix} > 0, \quad \begin{bmatrix} W_{i} & G_{i1} \\ * & W_{i} \end{bmatrix} > 0,$$

$$\begin{bmatrix} W_{i} & G_{i2} \\ * & W_{i} \end{bmatrix} > 0, \quad \begin{bmatrix} W_{i} & G_{i3} \\ * & W_{i} \end{bmatrix} > 0, \quad P_{j} - \lambda_{j} \delta_{j} I_{n_{j}} > 0$$
(24)

where Φ_i is a symmetric matrix consisting of

$$\begin{split} \Phi_{11}^{i} &= A_{i}^{T} P_{i2} + P_{i2}^{T} A_{i} + 2\alpha P_{i} - e^{-2\alpha \bar{\tau}_{i}} R_{i} + S_{i}, \quad \Phi_{12}^{i} = A_{i}^{T} P_{i3} - P_{i2}^{T} + P_{i}, \\ \Phi_{13}^{i} &= e^{-2\alpha \bar{\tau}_{i}} (R_{i} - G_{i0}) + P_{i2}^{T} B_{i} K_{i}, \quad \Phi_{14}^{i} &= e^{-2\alpha \bar{\tau}_{i}} G_{i0}, \\ \Phi_{16}^{i} &= -P_{i2}^{T} e^{A_{i} r_{i}} B_{i} K_{i}, \quad \Phi_{17}^{i} = P_{i2}^{T} e^{A_{i} r_{i}} B_{i} K_{i}, \quad \Phi_{19}^{i} &= P_{i2}^{T} e^{A_{i} r_{i}} B_{i}, \\ \Phi_{1,10}^{i} &= P_{i2}^{T} e^{A_{i} r_{i}} m_{j} = N_{i}^{T} e^{A_{i} r_{i}} B_{i} K_{i}, \quad \Phi_{12}^{i} &= P_{i2}^{T} e^{A_{i} r_{i}} m_{j}, \\ \Phi_{22}^{i} &= -P_{i3}^{T} - P_{i3} + \bar{\tau}_{i}^{2} R_{i} + (\bar{\tau} - r_{i})^{2} W_{i}, \quad \Phi_{23}^{i} &= P_{i3}^{T} e^{A_{i} r_{i}} m_{j}, \\ \Phi_{26}^{i} &= -P_{i3}^{T} e^{A_{i} r_{i}} B_{i} K_{i}, \quad \Phi_{27}^{i} &= P_{i3}^{T} e^{A_{i} r_{i}} B_{i} K_{i}, \quad \Phi_{29}^{i} &= P_{i3}^{T} e^{A_{i} r_{i}} B_{i}, \\ \Phi_{2,10}^{i} &= P_{i3}^{T} e^{A_{i} r_{i}} m_{j} m_{j} = 1, \dots, M \left\{ F_{ij} e^{-A_{j} r_{j}}, j \neq i \right\}, \quad \Phi_{2,11}^{i} &= P_{i3}^{T} e^{A_{i} r_{i}} m_{j}, \\ \Phi_{33}^{i} &= e^{-2\alpha \bar{\tau}_{i}} (G_{i}^{T} + G_{i0} - 2R_{i}), \quad \Phi_{34}^{i} &= e^{-2\alpha \bar{\tau}_{i}} (R_{i} - G_{i0}), \\ \Phi_{44}^{i} &= e^{-2\alpha \bar{\tau}_{i}} (Q_{i} - R_{i} - S_{i}), \quad \Phi_{55}^{i} &= -e^{-2\alpha \bar{\tau}_{i}} W_{i} + e^{-2\alpha r_{i}} (U_{i} - Q_{i}), \\ \Phi_{56}^{i} &= e^{-2\alpha \bar{\tau}_{i}} (G_{i}^{T} + G_{i1} - 2W_{i}), \quad \Phi_{67}^{i} &= e^{-2\alpha \bar{\tau}_{i}} (W_{i} + G_{i2} - G_{i1} - G_{i3}), \\ \Phi_{66}^{i} &= e^{-2\alpha \bar{\tau}_{i}} (G_{i3} - G_{i2}), \\ \Phi_{66}^{i} &= e^{-2\alpha \bar{\tau}_{i}} (G_{i3}^{T} + G_{i3} - 2W_{i}) + K_{i}^{T} K_{i}, \quad \Phi_{78}^{i} &= e^{-2\alpha \bar{\tau}_{i}} (W_{i} - G_{i3}), \\ \Phi_{68}^{i} &= -e^{-2\alpha \bar{\tau}_{i}} (U_{i} + W_{i}), \quad \Phi_{99}^{i} &= -\frac{1}{\sigma_{i}} I_{m_{i}}, \\ \Phi_{10,10}^{i} &= \operatorname{diag}_{j=1, \dots, M} \{ -\frac{2\epsilon_{1}}{M-1} P_{j}, j \neq i \}, \quad \Phi_{11,11}^{i} &= \operatorname{diag}_{j=1, \dots, M} \{ -\frac{2\epsilon_{2}}{M-1} \lambda_{j} I_{n_{j}}, j \neq i \}. \\ \end{array}$$

and I is a unit matrix of appropriate dimensions.

Then, the closed-loop system is exponentially stable with a convergence rate ρ such that $\rho = \alpha - \epsilon_1 - \epsilon_2 e^{2\rho \bar{\tau}}$ with $\bar{\tau} = \max_i \{\bar{\tau}_i\}$.

Proof. Consider the Lyapunov–Krasovskii functional (LKF):

$$V_{i}(t) = V_{P_{i}}(t) + V_{S_{i}}(t) + V_{R_{i}}(t) + V_{Q_{i}}(t) + V_{U_{i}}(t) + V_{W_{i}}(t)$$

where

$$V_{P_i}(t) = z_i^T(t)P_i z_i(t), \quad P_i > 0$$
 (25)

$$V_{S_i}(t) = \int_{t-\bar{\tau}_i}^t e^{2\alpha(s-t)} z_i^T(s) S_i z_i(s) ds, \quad S_i > 0$$
(26)

$$V_{R_i}(t) = \bar{\tau}_i \int_{-\bar{\tau}_i}^0 \int_{t+\theta}^t e^{2\alpha(s-t)} \dot{z}_i^T(s) R_i \dot{z}_i(s) ds d\theta, \quad R_i > 0$$

$$\tag{27}$$

$$V_{Q_i}(t) = \int_{t-r_i}^{t-\bar{\tau}_i} e^{2\alpha(s-t)} z_i^T(s) Q_i z_i(s) ds, \quad Q_i > 0$$
(28)

$$V_{U_i}(t) = \int_{t-\bar{\tau}_i}^{t-r_i} e^{2\alpha(s-t)} z_i^T(s) U_i z_i(s) ds, \quad U_i > 0$$
⁽²⁹⁾

$$V_{W_i}(t) = (\bar{\tau}_i - r_i) \int_{-\bar{\tau}_i}^{-r_i} \int_{t+\theta}^t e^{2\alpha(s-t)} \dot{z}_i^T(s) W_i \dot{z}_i(s) ds d\theta, \quad W_i > 0$$
(30)

Taking the time derivatives of (25), (26), (28), and (29), we obtain

$$\dot{V}_{P_i}(t) + 2\alpha V_{P_i}(t) = 2z_i^T(t)P_i\dot{z}_i(t) + 2\alpha z_i^T(t)P_iz_i(t)$$
(31)

$$\dot{V}_{S_i}(t) + 2\alpha V_{S_i}(t) = z_i^T(t) S_i z_i(t) - e^{-2\alpha \bar{\tau}_i} z_i^T(t - \bar{\tau}_i) S_i z_i(t - \bar{\tau}_i)$$
(32)

$$\dot{V}_{Q_i}(t) + 2\alpha V_{Q_i}(t) = e^{-2\alpha \bar{\tau}_i} z_i^T(t - \bar{\tau}_i) Q_i z_i(t - \bar{\tau}_i) - e^{-2\alpha r_i} z_i^T(t - r_i) Q_i z_i(t - r_i)$$
(33)

$$\dot{V}_{U_i}(t) + 2\alpha V_{U_i}(t) = e^{-2\alpha r_i} z_i^T(t - r_i) U_i z_i(t - r_i) - e^{-2\alpha \bar{\tau}_i} z_i^T(t - \bar{\tau}_i) U_i z_i(t - \bar{\tau}_i)$$
(34)

Taking the time derivatives of (27) and (30), we obtain

$$\begin{split} \dot{V}_{R_{i}}(t) &+ 2\alpha V_{R_{i}}(t) \\ &= \bar{\tau}_{i} \int_{-\bar{\tau}_{i}}^{0} \dot{z}_{i}^{T}(t) R_{i} \dot{z}_{i}(t) d\theta - \bar{\tau}_{i} \int_{-\bar{\tau}_{i}}^{0} e^{2\alpha\theta} \dot{z}_{i}^{T}(t+\theta) R_{i} \dot{z}_{i}(t+\theta) d\theta \\ &= \bar{\tau}_{i}^{2} \dot{z}_{i}^{T}(t) R_{i} \dot{z}_{i}(t) - \bar{\tau}_{i} \int_{t-\bar{\tau}_{i}}^{t} e^{2\alpha(s-t)} \dot{z}_{i}^{T}(s) R_{i} \dot{z}_{i}(s) ds \\ &\leq \bar{\tau}_{i}^{2} \dot{z}_{i}^{T}(t) R_{i} \dot{z}_{i}(t) - e^{-2\alpha\bar{\tau}_{i}} \begin{bmatrix} z_{i}(t) - z_{i}(t-\tau_{i0}(t)) \\ z_{i}(t-\tau_{i0}(t)) - z_{i}(t-\bar{\tau}_{i}) \end{bmatrix}^{T} \\ &\times \begin{bmatrix} R_{i} & G_{i0} \\ * & R_{i} \end{bmatrix} \begin{bmatrix} z_{i}(t) - z_{i}(t-\tau_{i0}(t)) \\ z_{i}(t-\tau_{i0}(t)) - z_{i}(t-\bar{\tau}_{i}) \end{bmatrix} \end{split}$$
(35)

$$\begin{split} \dot{V}_{W_{i}}(t) &+ 2\alpha V_{W_{i}}(t) \\ &= (\bar{\tau}_{i} - r_{i}) \int_{-\bar{\tau}_{i}}^{-r_{i}} \dot{z}_{i}^{T}(t) W_{i} \dot{z}_{i}(t) d\theta - (\bar{\tau}_{i} - r_{i}) \int_{-\bar{\tau}_{i}}^{-r_{i}} e^{2\alpha\theta} \dot{z}_{i}^{T}(t+\theta) W_{i} \dot{z}_{i}(t+\theta) d\theta \\ &= (\bar{\tau}_{i} - r_{i})^{2} \dot{z}_{i}^{T}(t) W_{i} \dot{z}_{i}(t) - (\bar{\tau}_{i} - r_{i}) \int_{t-\bar{\tau}_{i}}^{t-r_{i}} e^{2\alpha(s-t)} \dot{z}_{i}^{T}(s) W_{i} \dot{z}_{i}(s) ds \\ &\leq (\bar{\tau}_{i} - r_{i})^{2} \dot{z}_{i}^{T}(t) W_{i} \dot{z}_{i}(t) \\ &- e^{-2\alpha\bar{\tau}_{i}} \begin{bmatrix} z_{i}(t-r_{i}) - z_{i}(t-\tau_{i1}(t)) \\ z_{i}(t-\tau_{i2}(t)) - z_{i}(t-\bar{\tau}_{i}) \end{bmatrix}^{T} \\ &\times \begin{bmatrix} W_{i} & G_{i1} & G_{i2} \\ * & W_{i} & G_{i3} \\ * & * & W_{i} \end{bmatrix} \begin{bmatrix} z_{i}(t-r_{i}) - z_{i}(t-\tau_{i1}(t)) \\ z_{i}(t-\tau_{i2}(t)) - z_{i}(t-\bar{\tau}_{i}) \end{bmatrix} \end{split}$$
(36)

Employing the descriptor representation of (23), we have

$$0 = 2 \left[z_i^T(t) P_{i2}^T + \dot{z}_i^T(t) P_{i3}^T \right] \left[-\dot{z}_i(t) + A_i z_i(t) + B_i K_i z_i(t - \tau_{i0}(t)) + e^{A_i r_i} B_i e_i(t) + e^{A_i r_i} B_i K_i(z_i(t - \tau_{i2}(t)) - z_i(t - \tau_{i1}(t))) + e^{A_i r_i} \sum_{j \neq i} F_{ij} \left(e^{-A_j r_j} z_j(t) - \xi_j(t) \right) \right]$$
(37)

Above all, from (19), (22), and (31)–(37), we have

$$\begin{split} \dot{V}_{i}(t) + 2\alpha V_{i}(t) &- \frac{2\epsilon_{1}}{M-1} \sum_{j \neq i} V_{j}(t) - \frac{2\epsilon_{2}}{M-1} \sum_{j \neq i} \sup_{\theta \in [-\bar{\tau}_{j}, 0]} V_{j}(t+\theta) \\ &+ \frac{2\epsilon_{2}}{M-1} \sum_{j \neq i} \lambda_{j} \left(\delta_{j} \sup_{\theta \in [-\bar{\tau}_{j}, 0]} \left| z_{j}(t+\theta) \right|^{2} - \left| \xi_{j}(t) \right|^{2} \right) + |K_{i}z_{i}(t-\tau_{i2}(t))|^{2} - \frac{1}{\sigma_{i}} |e_{i}(t)|^{2} \right) \\ &\leq \eta_{i}^{T}(t) \Phi_{i}\eta_{i}(t) - \frac{2\epsilon_{2}}{M-1} \sum_{j \neq i} \sup_{\theta \in [-\bar{\tau}_{j}, 0]} z_{j}^{T}(t+\theta) \left(P_{j} - \lambda_{j}\delta_{j}I_{n_{j}} \right) \sup_{\theta \in [-\bar{\tau}_{j}, 0]} z_{j}(t+\theta) \leq 0 \end{split}$$
(38)

where $\eta_i(t) = \operatorname{col}\{z_i(t), \dot{z}_i(t), z_i(t - \tau_{i0}(t)), z_i(t - \bar{\tau}_i), z_i(t - r_i), z_i(t - \tau_{i1}(t)), z_i(t - \tau_{i2}(t)), z_i(t - \tau_{i$ $z_i(t - \bar{\tau}_i), e_i(t), \operatorname{col}_{j=1,\dots,M}\{z_j(t), j \neq i\}, \text{ and } \operatorname{col}_{j=1,\dots,M}\{\xi_j(t), j \neq i\}\}.$

Inequality (38) is suggested by LMI (24) and implies

$$\dot{V}(t) + 2(\alpha - \epsilon_1)V(t) - 2\epsilon_2 \sup_{\theta \in [-\bar{\tau}, 0]} V(t + \theta) \le 0$$
(39)

where $V(t) = \sum_{i=1}^{M} V_i(t)$ and $\overline{\overline{\tau}} = \max_i {\{\overline{\overline{\tau}}_i\}}$. Based on Halanay's inequality [2], the closedloop system is exponentially stable by inequality (39). \Box

3. Sampled-Data Control for Coupled NCSs: Output Feedback

This section deals with a more complicated case where the plant state cannot be measured so that the output feedback with the observer is utilized.

As shown in Figure 3, the communication network and the event-triggered scheme in the output feedback are exactly the same as those of the state feedback. The main difference is that the sampled data of the output feedback is the output $C_i x_i(t)$ rather than the state $x_i(t)$. As a result, under the transmission delays and the event-triggered scheme, subsystems (1) and (2) become

$$\dot{x}_i(t) = A_i x_i(t) + B_i \bar{u}_i(\zeta_k^i) + \sum_{j \neq i} F_{ij} x_j(t), \quad t \in [t_k^i, t_{k+1}^i)$$
(40)

$$y_i(t) = y_i(s_k^i) = C_i x_i(s_k^i), \quad t \in [s_k^i, s_{k+1}^i)$$
(41)

where the event-triggered mechanism $\bar{u}_i(\zeta_k^i)$ is defined by (8).



Figure 3. Output feedback for interconnected NCSs with predictors and event-triggered controllers.

We bring in $\hat{x}_i(t)$ to be an estimate of $x_i(t)$ with the estimation error $\tilde{x}_i(t) = x_i(t) - \hat{x}_i(t)$ to overcome the difficulty that the full-state feedback of each sub-plant is not available to be measured. Accordingly, we design the observer as

$$\hat{x}_{i}(t) = A_{i}\hat{x}_{i}(t) + B_{i}u_{i}(t - r_{i}^{ca}) + L_{i}(y_{i}(t) - \hat{y}_{i}(t)),$$

$$t \in [0, \infty)$$
(42)

$$\hat{y}_i(t) = \hat{y}_i(s_k^i) = C_i \hat{x}_i(s_k^i), \quad t \in [s_k^i, s_{k+1}^i)$$
(43)

where $u_i(t)$ is given by (7).

The predictor is designed in an observer-based manner such that

$$\hat{z}_{i}(t) = e^{A_{i}(r_{i}^{sc} + r_{i}^{ca})} \hat{x}_{i}(t) + \int_{t-r_{i}^{ca}}^{t+r_{i}^{sc}} e^{A_{i}(t+r_{i}^{sc}-s)} B_{i}u_{i}(s)ds$$
(44)

and the predictor-based output feedback law is selected as

$$u_i(\zeta_k^i) = K_i \hat{z}_i(s_k^i) \tag{45}$$

$$=K_{i}\left(e^{A_{i}(r_{i}^{sc}+r_{i}^{ca})}\hat{x}_{i}(s_{k}^{i})+\int_{s_{k}^{i}-r_{i}^{ca}}^{s_{k}^{i}+r_{i}^{sc}}e^{A_{i}(s_{k}^{i}+r_{i}^{sc}-s)}B_{i}u_{i}(s)ds\right)$$
(46)

For stability analysis, the dynamics of $\hat{z}_i(t)$ in (44) and (42) are calculated as

$$\dot{z}_{i}(t) = A_{i}\hat{z}_{i}(t) + B_{i}u_{i}(t + r_{i}^{sc}) + e^{A_{i}(r_{i}^{sc} + r_{i}^{ca})}L_{i}(y_{i}(t) - \hat{y}_{i}(t))
= A_{i}\hat{z}_{i}(t) + B_{i}K_{i}\hat{z}_{i}(t - \tau_{i0}(t)) + e^{A_{i}r_{i}}L_{i}C_{i}\tilde{x}_{i}(t)
+ e^{A_{i}r_{i}}L_{i}C_{i}v_{\tilde{x}_{i}}(t), \quad t \in [s_{k}^{i}, s_{k+1}^{i})$$
(47)

where $v_{\tilde{x}_i}(t) = \tilde{x}_i(s_k^i) - \tilde{x}_i(t)$, and $\tau_{i0}(t)$ is defined underneath (14) and satisfies (17). Subtracting (42) from (40), the estimation error is governed by

$$\begin{split} \dot{\tilde{x}}_{i}(t) &= A_{i}\tilde{x}_{i}(t) - L_{i}(y_{i}(t) - \hat{y}_{i}(t)) + \sum_{j \neq i} F_{ij}x_{j}(t) \\ &+ B_{i}\left(\bar{u}_{i}(\zeta_{k}^{i}) - u_{i}(t - r_{i}^{ca})\right), \quad t \in [t_{k}^{i}, t_{k+1}^{i}) \\ &= (A_{i} - L_{i}C_{i})\tilde{x}_{i}(t) - L_{i}C_{i}v_{\tilde{x}_{i}}(t) \\ &+ B_{i}K_{i}(\hat{z}_{i}(t - \tau_{i2}(t)) - \hat{z}_{i}(t - \tau_{i1}(t))) + B_{i}e_{i}(t) \\ &+ \sum_{j \neq i} F_{ij}\left(\tilde{x}_{j}(t) + e^{-A_{j}r_{j}}\hat{z}_{j}(t) - \xi_{j}(t)\right), \quad t \in [s_{k}^{i}, s_{k+1}^{i}) \cap [t_{0}^{i}, +\infty) \end{split}$$

$$(48)$$

where $\tau_{i1}(t)$, $\tau_{i2}(t)$, and $e_i(t)$ are defined underneath (15) and (16), satisfy (17), and (18) and

$$\sigma_i |u_i(\zeta_k^i)|^2 - |e_i(t)|^2 = \sigma_i |K_i \hat{z}_i(t - \tau_{i2}(t))|^2 - |e_i(t)|^2 \ge 0, \quad t \in [t_0^i, +\infty)$$
(49)

The term $\xi_i(t)$ is given by (21) and meets

$$\left|\xi_{j}(t)\right|^{2} \leq \underbrace{r_{j} \int_{0}^{r_{j}} \left|e^{-A_{j}\theta}B_{j}\right|^{2} ds |K_{j}|^{2}}_{\delta_{j}} \sup_{\theta \in [-\bar{\tau}_{i},0]} \left|\hat{z}_{j}(t+\theta)\right|^{2} \tag{50}$$

Theorem 2. Consider a closed-loop system that consists of plants (40) and (41), observers (42) and (43), and controller (46). Provided tuning parameters $\epsilon_1 > 0$, $\epsilon_2 > 0$ and $\alpha > 0$ such that $0 < \epsilon_1 + \epsilon_2 < \alpha$, let matrices $P_i, O_i, S_i, R_i, Q_i, U_i, W_i, H_i \in \mathbb{R}^{n_i \times n_i} > 0$, $P_{i2}, P_{i3}, P_{i4}, P_{i5}, G_{i0}, G_{i1}, G_{i2}, G_{i3} \in \mathbb{R}^{n_i \times n_i}$,

 $P_j, R_j \in \mathbb{R}^{n_j \times n_j} > 0$, and scalar parameters $\sigma_i > 0$; $\lambda_j > 0$, for $j = 1, \dots, M$; and $j \neq i$ satisfy the LMIs: $\begin{bmatrix} R_i & C_{ij} \end{bmatrix}$

$$\Phi_{i} < 0, \quad \begin{bmatrix} K_{i} & G_{i0} \\ * & R_{i} \end{bmatrix} > 0, \quad \begin{bmatrix} W_{i} & G_{i1} \\ * & W_{i} \end{bmatrix} > 0,$$

$$\begin{bmatrix} W_{i} & G_{i2} \\ * & W_{i} \end{bmatrix} > 0, \quad \begin{bmatrix} W_{i} & G_{i3} \\ * & W_{i} \end{bmatrix} > 0, \quad P_{j} - \lambda_{j} \delta_{j} I_{n_{j}} > 0$$
(51)

where Φ_i is a symmetric matrix made up of

$$\begin{split} & \Phi_{11}^{i} = A_{i}^{T} P_{i2} + P_{i2}^{T} A_{i} + 2\alpha P_{i} - e^{-2\alpha \bar{\eta}} R_{i} + S_{i}, \\ & \Phi_{12}^{i} = A_{i}^{T} P_{i3} - P_{i2}^{T} + P_{i}, \quad \Phi_{13}^{i} = P_{12}^{T} e^{A_{i}r_{i}} L_{i}C_{i}, \quad \Phi_{15}^{i} = P_{12}^{T} e^{A_{i}r_{i}} L_{i}C_{i}, \\ & \Phi_{16}^{i} = e^{-2\alpha \bar{\eta}} (R_{i} - G_{i0}) + P_{i2}^{T} B_{i}K_{i}, \quad \Phi_{17}^{i} = e^{-2\alpha \bar{\eta}} G_{i0}, \\ & \Phi_{22}^{i} = -P_{i3}^{T} - P_{i3} + \bar{\eta}_{i}^{T} R_{i} + (\bar{\tau} - r_{i})^{2} W_{i}, \\ & \Phi_{33}^{i} = (A_{i} - L_{i}C_{i})^{T} P_{i3} + P_{i4}^{T} (A_{i} - L_{i}C_{i}) + 2\alpha O_{i}, \\ & \Phi_{34}^{i} = (A_{i} - L_{i}C_{i})^{T} P_{i5} - P_{i4}^{T} + O_{i}, \quad \Phi_{35}^{i} = -P_{i4}^{T} L_{i}C_{i} \\ & \Phi_{39}^{i} = -P_{i4}^{T} B_{i}K_{i}, \quad \Phi_{3,10}^{i} = P_{i4}^{T} B_{i}K_{i}, \quad \Phi_{3,12}^{i} = P_{i4}^{T} B_{i} \\ & \Phi_{3,14}^{i} = P_{i4}^{T} row_{j=1,\cdots,M} \Big\{ F_{ij}, i \neq i \Big\}, \\ & \Phi_{3,15}^{i} = P_{i7}^{T} P_{i5} + h_{i}^{2} e^{2\alpha h_{i}} H_{i}, \quad \Phi_{45}^{i} = P_{15}^{T} L_{i}C_{i}, \\ & \Phi_{44}^{i} = -P_{15}^{T} - P_{15} + h_{i}^{2} e^{2\alpha h_{i}} H_{i}, \quad \Phi_{45}^{i} = P_{15}^{T} L_{i}C_{i}, \\ & \Phi_{4,14}^{i} = P_{15}^{T} row_{j=1,\cdots,M} \Big\{ F_{ij}, i \neq i \Big\}, \\ & \Phi_{4,14}^{i} = P_{15}^{T} row_{j=1,\cdots,M} \Big\{ F_{ij}, j \neq i \Big\}, \\ & \Phi_{4,15}^{i} = P_{15}^{T} row_{j=1,\cdots,M} \Big\{ F_{ij}, j \neq i \Big\}, \\ & \Phi_{4,16}^{i} = P_{15}^{T} row_{j=1,\cdots,M} \Big\{ F_{ij}, j \neq i \Big\}, \\ & \Phi_{4,16}^{i} = P_{15}^{T} row_{j=1,\cdots,M} \Big\{ F_{ij}, j \neq i \Big\}, \\ & \Phi_{4,16}^{i} = P_{15}^{T} row_{j=1,\cdots,M} \Big\{ F_{ij}, j \neq i \Big\}, \\ & \Phi_{4,16}^{i} = P_{15}^{2} row_{j=1,\cdots,M} \Big\{ F_{ij}, j \neq i \Big\}, \\ & \Phi_{4,16}^{i} = P_{15}^{-2\alpha h_{i}} (W_{i} - G_{i0}), \quad \Phi_{17}^{i} = e^{-2\alpha \bar{\eta}_{i}} (G_{i} - G_{i2}), \\ & \Phi_{55}^{i} = -\frac{\pi^{2}}{4} H_{i}, \quad \Phi_{66}^{i} = e^{-2\alpha \bar{\eta}_{i}} (G_{i}^{T} + G_{i0} - 2R_{i}), \\ & \Phi_{88}^{i} = e^{-2\alpha \bar{\eta}_{i}} (W_{i} - G_{i1}), \quad \Phi_{8,10}^{i} = e^{-2\alpha \bar{\eta}_{i}} (G_{i1} - G_{i2}), \\ & \Phi_{8,11}^{i} = e^{-2\alpha \bar{\eta}_{i}} (W_{i} - G_{i2}), \quad \Phi_{10,11}^{i} = e^{-2\alpha \bar{\eta}_{i}} (G_{i1} - G_{i2}), \\ & \Phi_{9,10}^{i} = e^{-2\alpha \bar{\eta}_{i}} (W_{i} + G_{i2} - G_{i1} - G_{i}), \\ & \Phi_{9,10}^{i} = e^{-2\alpha \bar{\eta}_{i}} (W_{i} + G_{i$$

and I is the unit matrix of the appropriate dimensions.

Then, the closed-loop system is exponentially stable with a convergence rate ρ such that $\rho = \alpha - \epsilon_1 - \epsilon_2 e^{2\rho \bar{\tau}}$ with $\bar{\tau} = \max_i \{\bar{\tau}_i\}$.

Proof. Consider the Lyapunov–Krasovskii functional (LKF):

$$V_{i}(t) = V_{P_{i}}(t) + V_{O_{i}}(t) + V_{S_{i}}(t) + V_{R_{i}}(t) + V_{Q_{i}}(t) + V_{U_{i}}(t) + V_{W_{i}}(t) + V_{H_{i}}(t)$$

where

$$V_{P_i}(t) = \hat{z}_i^{\mathrm{T}}(t)P_i\hat{z}_i(t), \quad P_i > 0$$
(52)
$$V_{P_i}(t) = \hat{z}_i^{\mathrm{T}}(t)Q_i\hat{z}_i(t), \quad Q_i > 0$$
(52)

$$V_{O_i}(t) = \tilde{x}_i^T(t)O_i\tilde{x}_i(t), \quad O_i > 0$$
(53)

$$V_{S_i}(t) = \int_{t-\bar{\tau}_i} e^{i\mathcal{A}(s-t)} \dot{z}_i^{\ i}(s) S_i \dot{z}_i(s) ds, \quad S_i > 0$$
(54)

$$V_{R_i}(t) = \bar{\tau}_i \int_{-\bar{\tau}_i} \int_{t+\theta} e^{2\alpha(s-t)} \dot{z}_i^T(s) R_i \dot{z}_i(s) ds d\theta, \quad R_i > 0$$

$$(55)$$

$$V_{Q_i}(t) = \int_{t-r_i}^{t-r_i} e^{2\alpha(s-t)} \hat{z}_i^T(s) Q_i \hat{z}_i(s) ds, \quad Q_i > 0$$
(56)

$$V_{U_i}(t) = \int_{t-\bar{\tau}_i} e^{2\alpha(s-t)} \hat{z}_i^{\mathrm{T}}(s) U_i \hat{z}_i(s) ds, \quad U_i > 0$$
(57)

$$V_{W_{i}}(t) = (\bar{\bar{\tau}}_{i} - r_{i}) \int_{-\bar{\tau}_{i}}^{-r_{i}} \int_{t+\theta}^{t} e^{2\alpha(s-t)} \hat{z}_{i}^{T}(s) W_{i} \hat{z}_{i}(s) ds d\theta, \quad W_{i} > 0$$
(58)

$$V_{H_i}(t) = h_i^2 e^{2\alpha h_i} \int_{s_k^i}^t e^{2\alpha(s-t)} \hat{x}_i^T(s) H_i \hat{x}_i(s) ds - \frac{\pi^2}{4} \int_{s_k^i}^t e^{2\alpha(s-t)} [\tilde{x}_i(s_k^i) - \tilde{x}_i(s)]^T H_i [\tilde{x}_i(s_k^i) - \tilde{x}_i(s)] ds, \quad H_i > 0, \quad t \in [s_k^i, s_{k+1}^i)$$
(59)

Taking the time derivatives of (52)–(54), (56), (57), and (59), we have

$$\dot{V}_{P_i}(t) + 2\alpha V_{P_i}(t) = 2\hat{z}_i^T(t) P_i \dot{z}_i(t) + 2\alpha \hat{z}_i^T(t) P_i \hat{z}_i(t)$$
(60)

$$\dot{V}_{O_i}(t) + 2\alpha V_{O_i}(t) = 2\tilde{x}_i^T(t)O_i\dot{\tilde{x}}_i(t) + 2\alpha \tilde{x}_i^T(t)O_i\tilde{x}_i(t)$$
(61)

$$\dot{V}_{S_i}(t) + 2\alpha V_{S_i}(t) = \hat{z}_i^T(t) S_i \hat{z}_i(t) - e^{-2\alpha \bar{\tau}_i} \hat{z}_i^T(t - \bar{\tau}_i) S_i \hat{z}_i(t - \bar{\tau}_i)$$
(62)

$$\dot{V}_{Q_i}(t) + 2\alpha V_{Q_i}(t) = e^{-2\alpha \bar{\tau}_i} \hat{z}_i^T(t - \bar{\tau}_i) Q_i \hat{z}_i(t - \bar{\tau}_i) - e^{-2\alpha r_i} \hat{z}_i^T(t - r_i) Q_i \hat{z}_i(t - r_i)$$
(63)

$$\dot{V}_{U_i}(t) + 2\alpha V_{U_i}(t) = e^{-2\alpha r_i} \hat{z}_i^T(t - r_i) U_i \hat{z}_i(t - r_i) - e^{-2\alpha \bar{\tau}_i} \hat{z}_i^T(t - \bar{\tau}_i) U_i \hat{z}_i(t - \bar{\tau}_i)$$
(64)

$$\dot{V}_{H_i}(t) + 2\alpha V_{H_i}(t) = h_i^2 e^{2\alpha h_i} \dot{\tilde{x}}_i^T(t) H_i \dot{\tilde{x}}_i(t) - \frac{\pi^2}{4} v_{\tilde{x}_i}^T(t) H_i v_{\tilde{x}_i}(t)$$
(65)

Taking the time derivatives of (55) and (58), we obtain

$$\begin{split} \vec{V}_{R_{i}}(t) &+ 2\alpha V_{R_{i}}(t) \\ &= \vec{\tau}_{i} \int_{-\vec{\tau}_{i}}^{0} \dot{z}_{i}^{T}(t) R_{i} \dot{z}_{i}(t) d\theta \vec{\tau}_{i} \int_{-\vec{\tau}_{i}}^{0} e^{2\alpha \theta} \dot{z}_{i}^{T}(t+\theta) R_{i} \dot{z}_{i}(t+\theta) d\theta \\ &= \vec{\tau}_{i}^{2} \dot{z}_{i}^{T}(t) R_{i} \dot{z}_{i}(t) - \vec{\tau}_{i} \int_{t-\vec{\tau}_{i}}^{t} e^{2\alpha(s-t)} \dot{z}_{i}^{T}(s) R_{i} \dot{z}_{i}(s) ds \\ &\leq \vec{\tau}_{i}^{2} \dot{z}_{i}^{T}(t) R_{i} \dot{z}_{i}(t) - e^{-2\alpha \vec{\tau}_{i}} \left[\dot{z}_{i}(t) - \dot{z}_{i}(t-\tau_{i0}(t)) \\ \dot{z}_{i}(t-\tau_{i0}(t)) - \dot{z}_{i}(t-\vec{\tau}_{i}) \right]^{T} \\ &\times \begin{bmatrix} R_{i} & G_{i0} \\ * & R_{i} \end{bmatrix} \begin{bmatrix} \dot{z}_{i}(t) - \dot{z}_{i}(t-\tau_{i0}(t)) \\ \dot{z}_{i}(t-\tau_{i0}(t)) - \dot{z}_{i}(t-\vec{\tau}_{i}) \end{bmatrix} \end{split}$$
(66)

$$\begin{split} \dot{V}_{W_{i}}(t) &+ 2\alpha V_{W_{i}}(t) \\ &= (\bar{\tau}_{i} - r_{i}) \int_{-\bar{\tau}_{i}}^{-r_{i}} \dot{z}_{i}^{T}(t) W_{i} \dot{z}_{i}(t) d\theta - (\bar{\tau}_{i} - r_{i}) \int_{-\bar{\tau}_{i}}^{-r_{i}} e^{2\alpha\theta} \dot{z}_{i}^{T}(t + \theta) W_{i} \dot{z}_{i}(t + \theta) d\theta \\ &= (\bar{\tau}_{i} - r_{i})^{2} \dot{z}_{i}^{T}(t) W_{i} \dot{z}_{i}(t) - (\bar{\tau}_{i} - r_{i}) \int_{t - \bar{\tau}_{i}}^{t - r_{i}} e^{2\alpha(s - t)} \dot{z}_{i}^{T}(s) W_{i} \dot{z}_{i}(s) ds \\ &\leq (\bar{\tau}_{i} - r_{i})^{2} \dot{z}_{i}^{T}(t) W_{i} \dot{z}_{i}(t) \\ &- e^{-2\alpha \bar{\tau}_{i}} \begin{bmatrix} \hat{z}_{i}(t - r_{i}) - \hat{z}_{i}(t - \tau_{i1}(t)) \\ \hat{z}_{i}(t - \tau_{i1}(t)) - \hat{z}_{i}(t - \tau_{i2}(t)) \\ \hat{z}_{i}(t - \tau_{i2}(t)) - \hat{z}_{i}(t - \tau_{i1}(t)) \end{bmatrix}^{T} \\ &\times \begin{bmatrix} W_{i} & G_{i1} & G_{i2} \\ * & W_{i} & G_{i3} \\ * & * & W_{i} \end{bmatrix} \begin{bmatrix} \hat{z}_{i}(t - r_{i2}(t)) - \hat{z}_{i}(t - \tau_{i2}(t)) \\ \hat{z}_{i}(t - \tau_{i2}(t)) - \hat{z}_{i}(t - \bar{\tau}_{i}) \end{bmatrix} \end{split}$$
(67)

Employing the descriptor representations of (47) and (48), we have

$$0 = 2 \left[\hat{z}_{i}^{T}(t) P_{i2}^{T} + \dot{z}_{i}^{T}(t) P_{i3}^{T} \right] \left[-\dot{z}_{i}(t) + A_{i} \hat{z}_{i}(t) + B_{i} K_{i} \hat{z}_{i}(t - \tau_{i0}(t)) + e^{A_{i} r_{i}} L_{i} C_{i} \tilde{x}_{i}(t) + e^{A_{i} r_{i}} L_{i} C_{i} v_{\tilde{x}_{i}}(t) \right]$$
(68)

$$0 = 2 \left[\tilde{x}_{i}^{T}(t) P_{i4}^{T} + \dot{x}_{i}^{T}(t) P_{i5}^{T} \right] \left[-\dot{x}_{i}(t) + (A_{i} - L_{i}C_{i})\tilde{x}_{i}(t) - L_{i}C_{i}v_{\tilde{x}_{i}}(t) + B_{i}K_{i}(\hat{z}_{i}(t - \tau_{i2}(t)) - \hat{z}_{i}(t - \tau_{i1}(t))) + B_{i}e_{i}(t) + \sum_{j \neq i}F_{ij}\left(\tilde{x}_{j}(t) + e^{-A_{j}r_{j}}\hat{z}_{j}(t) - \xi_{j}(t)\right) \right]$$
(69)

Above all, from (49), (50), and (60)–(69), we have

$$\dot{V}_{i}(t) + 2\alpha V_{i}(t) - \frac{2\epsilon_{1}}{M-1} \sum_{j \neq i} V_{j}(t) - \frac{2\epsilon_{2}}{M-1} \sum_{j \neq i} \sup_{\theta \in [-\bar{\tau}_{j,0}]} V_{j}(t+\theta) + \frac{2\epsilon_{2}}{M-1} \sum_{j \neq i} \lambda_{j} \left(\delta_{j} \sup_{\theta \in [-\bar{\tau}_{j,0}]} \left| \hat{z}_{j}(t+\theta) \right|^{2} - \left| \xi_{j}(t) \right|^{2} \right) + |K_{i}\hat{z}_{i}(t-\tau_{i2}(t))|^{2} - \frac{1}{\sigma_{i}} |e_{i}(t)|^{2} \leq \eta_{i}^{T}(t) \Phi_{i}\eta_{i}(t) - \frac{2\epsilon_{2}}{M-1} \sum_{j \neq i} \sup_{\theta \in [-\bar{\tau}_{j,0}]} \hat{z}_{j}^{T}(t+\theta) \left(P_{j} - \lambda_{j}\delta_{j}I_{n_{j}} \right) \sup_{\theta \in [-\bar{\tau}_{j,0}]} \hat{z}_{j}(t+\theta) \leq 0$$

$$(70)$$

where $\eta_i(t) = \operatorname{col}\{\hat{z}_i(t), \hat{z}_i(t), \tilde{x}_i(t), \tilde{x}_i(t), z_i(t - \tau_{i0}(t)), \hat{z}_i(t - \bar{\tau}_i), \hat{z}_i(t - r_i), \hat{z}_i(t - \tau_i), \hat{z}_i(t - \tau_i),$

Inequality (3) is suggested by LMI (51) and implies

$$\dot{V}(t) + 2(\alpha - \epsilon_1)V(t) - 2\epsilon_2 \sup_{\theta \in [-\bar{\tau}, 0]} V(t + \theta) \le 0$$
(71)

where $V(t) = \sum_{i=1}^{M} V_i(t)$ and $\overline{\tau} = \max_i {\{\overline{\tau}_i\}}$. Based on Halanay's inequality [2], the closed-loop system is exponentially stable via inequality (39). \Box

4. Applicable Example of Physical Systems

In this section, we take into account an application of three coupled inverted pendulums on three carts, which is borrowed from [16,26–28] (as revealed in Figure 4). We utilize the control mechanism proposed in previous sections.

the control mechanism proposed in previous sections. The system matrices are $A_1 = A_2 = A_3 = \begin{bmatrix} 2.9156 & 0 & -0.005 & 0 \\ 2.9156 & 0 & -0.0002 & 0 \\ -1.6663 & 0 & 0.002 & 0 \end{bmatrix}$, $B_1 = B_2 = B_3 = \begin{bmatrix} -0.0042 \\ 0.0012 \\ 0.0167 \end{bmatrix}$, $C_1 = C_2 = C_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$, and $F_{12} = F_{21} = F_{13} = F_{31} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -0.0003 & 0 & -0.0002 & 0 \end{bmatrix}$. The control gains are selected as $K_1 = \begin{bmatrix} 11396 & 7196.2 & 573.96 & 1199.0 \end{bmatrix}$ and $K_2 = K_3 = \begin{bmatrix} 29241 & 18135 & 2875.3 & 3693.9 \end{bmatrix}$. The observer gains are selected as $L_1 = L_2 = L_3 = \begin{bmatrix} 11.7 & -1.2 \\ 37 & -8.9 \\ -7.9 & 36 \end{bmatrix}$. The initial states are set as $x_1 = \begin{bmatrix} 0, 0, -0.1, 0 \end{bmatrix}$, $x_2 = \begin{bmatrix} 0, 0, 0.1, 0 \end{bmatrix}$, and $x_3 = \begin{bmatrix} 0, 0, 0.2, 0 \end{bmatrix}$.

Please note that the maximum delays allowed by predictor-free controllers in [16,26–28] are less than 0.03s.

As shown in Figure 5, when the predictor is not employed in the feedback, the three sub-plants become unstable if the delay lengths are 0.1 s. In contrast, as revealed in Figures 6 and 7, when the predictor is utilized in the feedback, the three sub-plants are still stable even if the delay lengths are as large as 0.2 s. It is seen that, relative to the delay length promised by the predictor-free controller, the predictor-based controller allows for a larger delay.



Figure 4. Three coupled cart–pendulum systems.



Figure 5. Predictor-free feedback under small delay of 0.1 s.



Figure 6. Predictor-based feedback under large delay of 0.2 s.



Figure 7. Predictor event-triggered control input under large delay of 0.2 s.

5. Conclusions

This paper develops predictor-dependent stabilization for coupled networked control systems under large, uncertain transmission delays and event-triggered strategies in a decentralized manner. The network-induced delays in the communication network in our paper are addressed by a prediction method; thus, the delay length is promised to be large. The local control laws of the subsystems that are coupled work independently using no information from their neighbors and operate asynchronously under respective sampling instants. Given the controller and observer gains that stabilize the systems in the case of no delay, a couple of prediction-dependent controllers are proposed to compensate the large delays: prediction using a state feedback method and prediction using an output feedback method. The first kind of method leads to simpler conditions, whereas the second is easily implementable in practice. The stability analysis via the Lyapunov–Krasovskii method is carried out in a decentralized way. The practical implementation of three coupled cart–pendulum systems is taken into account to validate the given method in a case where the transmission delays are so large that the closed-loop control system cannot be stabilized if we do not use a predictor.

Author Contributions: Conceptualization, Y.Z. and M.Z.; Methodology, Y.Z. and Q.J.; Simulation, Y.Z. All authors have read and agreed to the published version of the manuscript.

Funding: This work is supported by National Key R&D Program of China (Grant No. 2021YFB1714100).

Data Availability Statement: All data underlying the results are available as part of the article and no additional source data are required.

Conflicts of Interest: The authors declare no conflicts of interest.

References

- 1. Shi, P.; Li, F.; Wu, L.; Lim, C.C. Neural network-based passive filtering for delayed neutral-type semi-Markovian jump systems. *IEEE Trans Neural Netw. Learn. Syst.* 2017, 28, 2101–2114. [CrossRef]
- 2. Fridman, E. Introduction to Time-Delay Systems: Analysis and Control; Birkhauser: Basel, Switzerland, 2014.
- 3. Smith, O.J.M. A controller to overcome dead time. *ISA* **1959**, *6*, 28–33.
- 4. Novella-Rodriguez, D.F.; Tudon-Martinez, J.C.; Vazquez-Guerra, R.J.; Marquez-Rubio, J.F. Feedback control based on a sequential observer-predictor for systems with unknown actuator delay. *Int. J. Control. Autom. Syst.* 2022, 20, 2779–2791. [CrossRef]
- 5. Wu, F.; Lian, J. Truncated predictor feedback control for switched linear systems subject to input delay and saturation. *Int. J. Control. Autom. Syst.* 2022, 20, 2849–2856. [CrossRef]

- 6. Rajkumar, S.M.; Chakraborty, S.; Dey, R.; Deb, D. Online delay estimation and adaptive compensation in wireless networked system: An embedded control design. *Int. J. Control. Autom. Syst.* **2020**, *18*, 856–866. [CrossRef]
- 7. Artstein, Z. Linear systems with delayed controls: A reduction. IEEE Trans. Autom. Control 1982, 27, 869–879. [CrossRef]
- Selivanov, A.; Fridman, E. Predictor-based networked control under uncertain transmission delays. *Automatica* 2016, 70, 101–108. [CrossRef]
- 9. Selivanov, A.; Fridman, E. Observer-based input-to-state stabilization of networked control systems with large uncertain delays. *Automatica* **2016**, *74*, 63–70. [CrossRef]
- 10. Michiels, W.; Niculescu, S.I. Stability and Stabilization of Time-Delay Systems: An Eigenvalue-Based Approach; SIAM: Philadelphia, PA, USA, 2007.
- 11. Bakule, L. Decentralized control: An overview. Annu. Rev. Control. 2008, 32, 87–98. [CrossRef]
- 12. Mahmoud, M.S. Decentralized stabilization of interconnected systems with time-varying delays. *IEEE Trans. Autom. Control* 2009, 54, 2663–2668. [CrossRef]
- 13. Erol, H.E.; Iftar, A. Stabilization of decentralized descriptor-type neutral time-delay systems by time-delay controllers. *Automatica* **2016**, *64*, 262–269. [CrossRef]
- Zhang, M.; Shen, C.; Wu, Z.; Zhang, D. Dissipative filtering for switched fuzzy systems with missing measurements. *IEEE Trans. Cybern.* 2020, 20, 1931–1940. [CrossRef]
- 15. Zhang, M.; Shi, P.; Ma, L.; Cai, J.; Su, H. Quantized feedback control of fuzzy markov jump systems. *IEEE Trans. Cybern.* **2019**, 49, 3357–3384. [CrossRef]
- 16. Freirich, D.; Fridman, E. Decentralized networked control of systems with local networks: A time-delay approach. *Automatica* **2016**, 69, 201–209. [CrossRef]
- 17. Zhang, M.; Dong, S.; Wu, Z.-G.; Chen, G.; Guan, X. Reliable Event-Triggered Load Frequency Control of Uncertain Multi-Area Power Systems with Actuator Failures. *IEEE Trans. Autom. Sci. Eng.* **2023**, *20*, 1545–1595.
- Zhang, M.; Dong, S.; Shi, P.; Chen, G.; Guan, X. Distributed Observer-based Event-Triggered Load Frequency Control of Multi-Area Power Systems under Cyber Attacks. *IEEE Trans. Autom. Sci. Eng.* 2023, 20, 2435–2444. [CrossRef]
- 19. Liu, K.; Sun, X.; Krstic, M. Distributed predictor-based stabilization of continuous interconnected systems with input delays. *Automatica* **2018**, *91*, 69–78. [CrossRef]
- 20. Tabuada, P. Event-triggered real-time scheduling of stabilizing control tasks. *IEEE Trans. Autom. Control* 2007, 52, 1680–1685. [CrossRef]
- Heemels, W.P.M.H.; Johansson, K.H.; Tabuad, P. An introduction to event-triggered and self-triggered control. In Proceedings of the 51st IEEE Conference on Decision and Control, Maui, HI, USA, 10–13 December 2012; pp. 3270–3285.
- Hashemipour, S.H.; Vasegh, N.; Sedigh, A.K. Decentralized model reference adaptive control for interconnected time delay systems with delay in state and compensation of long delay in input by nested prediction. *Int. J. Control. Autom. Syst.* 2017, 5, 1198–1206. [CrossRef]
- 23. Krstic, M. Delay Compensation for Nonlinear, Adaptive, and PDE Systems; Birkhauser: Berlin, Germany, 2009.
- 24. Zhu, Y.; Fridman, E. Predictor methods for decentralized control of large-scale systems with input delays. *Automatica* 2020, 116, 108903. [CrossRef]
- 25. Zhu, Y.; Fridman, E. Observer-based decentralized predictor control for large-scale interconnected systems with large delays. *IEEE Trans. Autom. Control* **2020**, *66*, 2897–2904. [CrossRef]
- Borgers, D.; Heemels, W. Stability analysis of large-scale networked control systems with local networks: A hybrid small-gain approach. In Proceedings of the 17th International Conference on Hybrid Systems: Computation and Control, Berlin, Germany, 15–17 April 2014; pp. 103–112.
- Heemels, W.; Borgers, D.; Wouw, N.v.; Nesic, D.; Teel, A. Stability analysis of nonlinear networked control systems with asynchronous communication: A small-gain approach. In Proceedings of the 52th IEEE Conference on Decision and Control, Florence, Italy, 10–13 December 2013; pp. 4631–4637.
- 28. Dolk, V.-S.; Borgers, D.-P.; Heemels, W.-P.-M.-H. Output-based and decentralized dynamic event-triggered control with guaranteed *L*_p-gain performance and zeno-freeness. *IEEE Trans. Autom. Control* **2017**, *62*, 34–49. [CrossRef]

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.