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Properties and Analysis of the Guard Interval in Infinite Impulse Response–Orthogonal Frequency Division Multiplexing Systems

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Abstract: Recently, an orthogonal frequency division multiplexing (OFDM) technique for the infinite impulse response (IIR) channel (IIR-OFDM) was proposed, which carries the dedicated guard interval to maintain the circular convolution of the received signal and channel coefficients. Therefore, the loop of an IIR channel can be converted to the frequency domain, and single-tap equalization can still be used to equalize loop interference, like classical OFDM. In this paper, we describe how to build the IIR system based on the channel with loops and derive the properties of the dedicated guard interval for a general multi-order IIR channel, which is different from the classical cyclic prefix (CP) obtained by replicating the samples at the tail end of the signal. In particular, we address two special models for first-order and delay IIR channels. It is demonstrated that the guard interval composition and power characteristics of the two special models are similar. Moreover, the complexity of the guard interval depends not only on the maximum delay of the loop, but also on the number of loops. Finally, we simulate the IIR-OFDM performance under different IIR channels.

Keywords: orthogonal frequency division multiplexing (OFDM); loop interference; single-tap equalization; infinite impulse response (IIR)



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1. Introduction

Orthogonal Frequency Division Multiplexing (OFDM) is a modulation technique commonly used in wireless communication systems. It works by dividing the information signal into multiple sub-channels, transmitting data in an orthogonal manner over each sub-channel, thereby enhancing spectral efficiency and combating multipath interference. The key feature of classical OFDM is the introduction of a cyclic prefix (CP), i.e., a replicated segment of the symbol tail is added before each OFDM symbol. This design effectively eliminates inter-symbol interference (ISI) and simplifies the processing at the receiver, allowing it to better handle time delays caused by multipath transmission [1]. Therefore, OFDM is widely used in modern wireless communication standards such as 4G and 5G, and its excellent resistance to interference and high spectral utilization make it a crucial technology for achieving high-speed data transmission [2].

Generally, a CP of an appropriate length is inserted before each OFDM symbol to prevent multipath interference. However, the channel response in some scenarios is very long, although it has a finite impulse response (FIR), and a CP of a sufficient length can severely degrade the spectral efficiency. If the length of the CP is reduced to improve the spectral efficiency of the OFDM system, the system will inevitably be affected by ISI and inter-carrier interference (ICI). When an OFDM system is subjected to interference and continues to use a single-tap equalization approach, it can lead to a severe bit error rate (BER).

Therefore, researchers have conducted studies on OFDM systems with short or free CPs. In [3], a new OFDM waveform with low CP overhead (SR-OFDM) for narrow-band

Internet of Things (NB-IoT) systems is proposed. Compared to classic OFDM, SR-OFDM employs the concept of symbol repetition across multiple consecutive OFDM symbols, where multiple SR-OFDM symbols occupy only one CP. This significantly reduces CP overhead and improves the spectral and energy efficiency of the NB-IoT system. The precoded vector OFDM scheme is proposed in [4]. By designing a suitable precoding matrix, frequency-valued zeros in the frequency-selective channel band are removed for interference suppression without knowing the channel state information (CSI). Ref. [5] proposes a CP-free OFDM scheme with successive multipath interference cancellation (SMIC) that does not require the CP and uses stored feedback equalization (SFE) to remove the ISI prior to the Fast Fourier Transform (FFT) at the receiver. The SFE operation leaves CP gaps between adjacent OFDM blocks and regenerates the CP using the estimated signal. Ref. [6] proposes a scheme to maximize the channel impulse response (CIR) energy within a given window and shorten the effective CIR, thus reducing the CP length. In addition, the strategy of modeling the channel into infinite impulse response (IIR) with blind estimation is presented in [7]. For channels with long CIR, an IIR filter is applied at the receiver side to eliminate ISI.

However, not all channel responses are FIR, and channels may be IIR in specific electromagnetic environments, such as resonant walls with great reflections or full-duplex communication [8]. Classical OFDM schemes with shortened or free CPs suffer from ISI under IIR channels. Under IIR channels, the signal is affected by previous signals from the loop and the resulting ISI. The results of the study demonstrated the feasibility of communication under IIR channels by mitigating loop interference through a combination of passive suppression and active cancellation of the loop interference [9,10]. The main purpose of passive suppression is to reduce the loop signal power at the receiver, such as antenna isolation, coupling networks, and cross-polarization. Active suppression, on the other hand, utilizes the node's knowledge of its own transmitted signal to eliminate loop interference [11]. Active suppression, which removes loop signal disturbances from the received signal by injecting a suppression waveform, has been extensively studied in the literature. The suppression waveforms can be injected not only over the transmission line, as in [12,13], but also over the air using an additional antenna, as in [14]. Note that neither active nor passive cancellation is able to completely suppress loop interference, which means that IIR channels will always exist.

Therefore, an OFDM technique for the IIR channel (IIR-OFDM) was proposed recently [15], which carries the dedicated guard interval to maintain the circular convolution of the received signal and the IIR channel coefficients. Furthermore, in [16], IIR-OFDM is verified to have significantly improved performance in the IIR channel constructed with the full-duplex OFDM amplified-and-forward relay system, compared to existing schemes for suppressing residual loop interference. At the receiver, the loop of an IIR channel can be converted to the frequency domain, and single-tap equalization can still be used to equalize loop interference, like classical OFDM. In this paper, we describe how to build the IIR system based on the channel with loops and derive the properties of the dedicated guard interval in a general multi-order IIR channel, which is different from the classical CP obtained by replicating the samples at the tail end of the signal. In particular, we address two special models for first-order and delay IIR channels. It is demonstrated that the two special models' GI composition and power characteristics are similar. Moreover, the complexity of GI depends not only on the maximum delay of the loop, but also on the number of loops.

The remainder of this paper is composed as follows. In Section 2, we describe the IIR-OFDM system in detail, including how to model the IIR channel, design the guard interval, and demodulate the received signal. Next, we derive the guard interval characteristics of IIR-OFDM under different IIR channel models in Section 3. Finally, the IIR-OFDM is verified by different computer simulations in Section 4.

2. IIR-OFDM

Assume that the number of subcarriers is N and the number of transmitted symbols is V . $d_{n,v}$ is the v th transmitted symbol at the n th subcarrier ($n = 0, \dots, N-1$). Let $\mathbf{d}_v$ represent the transmitted symbol matrix, which can be written as

$$\mathbf{d}_v = [d_{0,v}, \dots, d_{N-1,v}]^T. \quad (1)$$

After the N -point inverse discrete Fourier transform (IDFT), the equivalent baseband transmitted signal can be written as

$$s_v(k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} d_{n,v} e^{j2\pi nk/N}, k \in [0, N-1], \quad (2)$$

where $\frac{1}{\sqrt{N}}$ is the power normalization factor.

Assume that the channel consists of a forward path and several loops, which may be due to reflections or loop interference. The transmitted signal passing through a channel with loops can be indicated as

$$r_v(k) = h(0)[s_v(k) + h(1)r_v(k-1) + \dots + h(l_g)r_v(k-l_g)], \quad (3)$$

where $r_v(k)$ is the received signal corresponding to $s_v(k)$ at the receiver and l_g is the number of loops. Under this channel, only $h(0)$ is the forward path.

Suppose that the transmitted signal passes through a pure IIR channel $H(z) = 1/A(z)$, we have

$$A(z) = \sum_{i=0}^{l_g} a(i)z^{-i}, \quad (4)$$

where $a(i)$ is the coefficient of the IIR channel. $H(z)$ and $A(z)$ are the z -transforms of the channel.

Linking Equation (4) to the channel, we have

$$a(i) = \begin{cases} \frac{1}{h_i}, & i = 0, \\ -h_i, & i = [1, \dots, l_g]. \end{cases} \quad (5)$$

Therefore, the differential Equation (3) for the system of the IIR channel can be expressed as

$$s_v(k) = \sum_{i=0}^{l_g} a(i)r_v(k-i), \quad (6)$$

where received $r_v(k)$ and transmitted signal $s_v(k)$ satisfy linear convolution, like classical OFDM.

We consider $\mathbf{r}_v = [r_v(0), \dots, r_v(N-1)]^T$ as the received signal vector. Then, the transmitted signal vector $\mathbf{s}_v = [s_v(0), \dots, s_v(N-1)]^T$ can be expressed as

$$\mathbf{s}_v = \mathbf{A}\mathbf{r}_v, \quad (7)$$

where $\mathbf{A}$ represents the IIR channel matrix, which can be written as

$$\mathbf{A} = \sum_{i=0}^{l_g} a(i) \Lambda^i = \begin{bmatrix} a(0) & & & & \\ \vdots & \ddots & & & \\ a(l_g) & \cdots & a(0) & & \\ & a(l_g) & \cdots & a(0) & \\ & & \ddots & & \ddots \\ & & & a(l_g) & \cdots & a(0) \end{bmatrix} \quad (8)$$

with

$$\Lambda = \begin{bmatrix} 0 & \cdots & 0 & 0 \\ 1 & \ddots & 0 & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 1 & 0 \end{bmatrix}_{N \times N}, \Gamma = \begin{bmatrix} 0 & \cdots & 0 & 1 \\ 1 & \ddots & 0 & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 1 & 0 \end{bmatrix}_{N \times N}, \quad (9)$$

note that Λ^0 and Γ^0 are the identity matrix.

However, it can be anticipated from (7) that when $k > N - 1$, the received signal will continue to be influenced by the loops of the IIR channel. It can be obtained by

$$a(0)r_v(k) = - \sum_{i=1}^{l_g} a(i)r_v(k-i). \quad (10)$$

The ISI signal can be obtained by

$$\mathbf{s}_v^{\text{ISI}} = \mathbf{A}^{\text{ISI}} \mathbf{r}_v, \quad (11)$$

where $\mathbf{A}^{\text{ISI}}$ represents the ISI matrix of IIR channel, which can be written as

$$\mathbf{A}^{\text{ISI}} = \sum_{i=0}^{l_g} a(i)(\Gamma^i - \Lambda^i) = \begin{bmatrix} 0 & & a(l_g) & \cdots & a(1) \\ \vdots & \ddots & & \ddots & \vdots \\ 0 & \cdots & 0 & & a(l_g) \\ & 0 & \cdots & 0 & \\ & & \ddots & & \ddots \\ & & & 0 & \cdots & 0 \end{bmatrix} \quad (12)$$

where we consider $r_v(k)$ as the transmitted signal and $s_v(k)$ as the received signal; the ISI signal under the IIR channel is similar to the transmitted signal under the FIR channel.

2.1. IIR-OFDM Transmitter

In order to resist the ISI caused by the loop signals of the IIR channel, IIR-OFDM is based on the guard interval designed for the IIR channel to maintain the circular convolutional structure of the transmitted and received signals. We consider the guard interval for the corresponding transmitted signal $s_v(k)$ to be $\bar{s}_v(k)$; the guard interval vector $\bar{\mathbf{s}}_v = [\bar{s}_v(0), \dots, \bar{s}_v(l_g - 1)]^T$ can be expressed as

$$\bar{\mathbf{s}}_v = \bar{\mathbf{A}}_1 \mathbf{y}_v + \bar{\mathbf{A}}_2 \mathbf{y}_{v-1} \quad (13)$$

where the length of the guard interval is the same as the number of loops, signal vector $\mathbf{y}_v = [y_v(0), \dots, y_v(l_g - 1)]^T$.

$y_v(k)$ can be expressed as

$$y_v(k) = \frac{1}{\sqrt{N}} \sum_{n=0}^{N-1} \frac{d_{n,v}}{A_n} e^{j2\pi(k+N-l_g)n/N} \quad (14)$$

where A_n are the N -point DFT of $a(i)$, which can be expressed as

$$A_n = \sum_{i=0}^{N-1} a(i) e^{-j2\pi i n / N} \quad (15)$$

Then, the guard interval generation matrix $\bar{\mathbf{A}}_1$ and $\bar{\mathbf{A}}_2$ can be expressed as

$$\bar{\mathbf{A}}_1 = \sum_{i=0}^{l_g-1} a(i) \Omega^i = \begin{bmatrix} a(0) & & \\ \vdots & \ddots & \\ a(l_g-1) & \cdots & a(0) \end{bmatrix}, \quad (16)$$

$$\bar{\mathbf{A}}_2 = \sum_{i=0}^{l_g-1} a(l_g-i) \Omega^{-i} = \begin{bmatrix} a(l_g) & \cdots & a(1) \\ & \ddots & \vdots \\ & & a(l_g) \end{bmatrix}, \quad (17)$$

with

$$\Omega = \begin{bmatrix} 0 & \cdots & 0 & 0 \\ 1 & \ddots & 0 & 0 \\ \vdots & \ddots & \ddots & \vdots \\ 0 & \cdots & 1 & 0 \end{bmatrix}_{(l_g-1) \times (l_g-1)}, \quad (18)$$

note that Ω^0 is the identity matrix.

The transmitted signal is inserted into the guard interval to ensure it is free from ISI at the receiver. After inserting the guard interval, the transmitter signal can be expressed as

$$\tilde{\mathbf{s}}_v = [\bar{s}_v(0), \dots, \bar{s}_v(l_g-1), s_v(0) \cdots, s_v(N-1)]^T. \quad (19)$$

2.2. IIR-OFDM Receiver

Suppose $\mathbf{F}$ and $\mathbf{F}^H$ are the DFT and the IDFT matrices, respectively. After inserting the guard interval, the signal can conform to the cyclic convolution, and the signal matrix can be represented as

$$\hat{\mathbf{s}}_v = \mathbf{A}_c \mathbf{r}_v = \mathbf{F}^H \mathbf{d}_v + \mathbf{A}_c \mathbf{w}, \quad (20)$$

where $\mathbf{w}$ represents the additive white Gaussian noise (AWGN) vector whose entries have complex-valued Gaussian distributions of $\mathcal{CN}(0, \sigma_w^2)$. $\mathbf{A}_c$ represents the IIR channel matrix that satisfies the N -point circular convolution, which can be written as

$$\mathbf{A}_c = \sum_{i=0}^{l_g} a(i) \Gamma^i = \begin{bmatrix} a(0) & & & a(l_g) & \cdots & a(1) \\ \vdots & \ddots & & & \ddots & \vdots \\ a(l_g-1) & \cdots & a(0) & & & a(l_g) \\ & a(l_g) & \cdots & a(0) & & \\ & & \ddots & & \ddots & \\ & & & a(l_g) & \cdots & a(0) \end{bmatrix}. \quad (21)$$

The received signal can be recovered by low-order tap equalization, which is similar to classical OFDM. The recovery signal can be expressed as

$$\hat{\mathbf{d}}_v = \mathbf{F} \mathbf{A}_c \mathbf{r}_v = \mathbf{d}_v + \mathbf{F} \mathbf{A}_c \mathbf{w}, \quad (22)$$

where $\hat{\mathbf{d}}_v$ is the matrix of the recovered signal after the equalization, which is only affected by AWGN.

IIR-OFDM has a more complex guard interval than classical OFDM. Therefore, IIR-OFDM has performance requirements for the transmitter. However, at the receiver, the

equalization process is consistent with classical OFDM. As a result, when deploying IIR-OFDM, people should pay attention to whether the performance of the transmitter can achieve the construction and transmission of complex guard intervals.

3. Guard Interval Analyses

We describe how to build the IIR system based on the channel with loops in Section 2. In this section, we derive the properties of the dedicated guard interval in a general multi-order IIR channel, which is different from the classical CP obtained by replicating the samples at the tail end of the signal. Additionally, we address two special models for 1st-order and delay IIR channels.

3.1. General Multi-Order IIR Channel

The guard interval can be obtained as

$$\begin{aligned}\bar{s}_v(k) &= \sum_{i_0=0}^k a(i_0)y_v(k-i_0) + \sum_{i_1=k+1}^{l_g} a(i_1)y_{v-1}(k-i_1+l_g) \\ &= \frac{1}{\sqrt{N}} \left[\sum_{i_0=0}^k \sum_{n_0=0}^{N-1} a(i_0) \frac{d_{n_0,v}}{A_{i_0}} e^{j2\pi(k-i_0-l_g)n_0/N} \right. \\ &\quad \left. + \sum_{i_1=k+1}^G \sum_{n_1=0}^{N-1} a(i_1) \frac{d_{n_1,v}}{A_{n_1}} e^{j2\pi(k-i_1)n_1/N} \right].\end{aligned}\quad (23)$$

Classical OFDM benefits from the FFT, which reduces its complexity from $\mathcal{O}(N^2)$ to $\mathcal{O}(N \log_2(N))$. The guard interval of IIR-OFDM is different from classical OFDM, which can be the circular convolution of the received signal and channel coefficients. Therefore, the guard interval requires additional complexity $\mathcal{O}((l_g+1)^2 + (l_g+1)) = \mathcal{O}(l_g^2 + 3l_g + 2)$ for design. Note that the complexity of the guard interval is related to the length of loops.

The guard interval consists of a linear superposition of the transmitted symbol and is closely related to the coefficient of the IIR channel. Suppose the expectation of guard interval is $p_v(k) = \mathbb{E}[\bar{s}_v(k)\bar{s}_v(k)^*]$, which can be expressed as

$$\begin{aligned}p_v(k) &= \frac{1}{N} \left[\sum_{i_0=0}^k \sum_{i_1=0}^k a(i_0)a(i_1)^* \sum_{n_0=0}^{N-1} \sum_{n_1=0}^{N-1} \frac{\mathbb{E}[d_{n_0,v-1}d_{n_1,v-1}^*]}{A_{n_0}A_{n_1}^*} e^{j2\pi[(k-l_g)(n_0-n_1)-n_0i_0+n_1i_1]/N} \right. \\ &\quad + \sum_{i_0=0}^k \sum_{i_1=k+1}^{l_g} a(i_0)a(i_1)^* \sum_{n_0=0}^{N-1} \sum_{n_1=0}^{N-1} \frac{\mathbb{E}[d_{n_0,v}d_{n_1,v-1}^*]}{A_{n_0}A_{n_1}^*} e^{j2\pi[k(n_0-n_1)-n_0(i_0+l_g)+n_1i_1]/N} \\ &\quad + \sum_{i_0=k+1}^{l_g} \sum_{i_1=0}^k a(i_0)a(i_1)^* \sum_{n_0=0}^{N-1} \sum_{n_1=0}^{N-1} \frac{\mathbb{E}[d_{n_0,v-1}d_{n_1,v}^*]}{A_{n_0}A_{n_1}^*} e^{j2\pi[k(n_0-n_1)-n_0i_0+n_1(i_1+l_g)]/N} \\ &\quad \left. + \sum_{i_0=k+1}^{l_g} \sum_{i_1=k+1}^{l_g} a(i_0)a(i_1)^* \sum_{n_0=0}^{N-1} \sum_{n_1=0}^{N-1} \frac{\mathbb{E}[d_{n_0,v-1}d_{n_1,v-1}^*]}{A_{n_0}A_{n_1}^*} e^{j2\pi[k(n_0-n_1)-n_0i_0+n_1i_1]/N} \right].\end{aligned}\quad (24)$$

After simplification, we have

$$\begin{aligned}p_v(k) &= \frac{\sigma_d^2}{N} \sum_{n=0}^{N-1} \left[\frac{\sum_{i_0=0}^k \sum_{i_1=0}^k a(i_0)a(i_1)^* e^{j2\pi(i_0-i_1)n/N}}{A_n A_n^*} \right. \\ &\quad \left. + \frac{\sum_{i_0=k+1}^{l_g} \sum_{i_1=k+1}^{l_g} a(i_0)a(i_1)^* e^{j2\pi(i_0-i_1)n/N}}{A_n A_n^*} \right].\end{aligned}\quad (25)$$

The guard interval of IIR-OFDM has a higher complexity than the CP of classical OFDM, and its expectation value increases with the coefficient of the IIR channel. In

addition, a major problem encountered in IIR-OFDM design is to ensure the system stability of the IIR channel model under multiple loops. The unstable IIR channel cannot be demodulated and equalized at the receiver. Generally, loop signals are considered harmful, so increasing the loss of loops and suppressing the amount of multiple paths in loops have been proposed. When in the case of multipath loops, it is required that the loop loss is able to ensure that the IIR channel satisfies the z-transformed poles are all within the unit circle.

It is worth noting that IIR-OFDM signals occupy the same bandwidth and transmitter bits as classical OFDM. Therefore, IIR-OFDM has the same spectral efficiency as classical OFDM. For power consumption, IIR-OFDM has the same transmission power as classical OFDM when the guard interval and CP are ignored. However, from (24), the guard interval expectation of IIR-OFDM is higher than OFDM symbols. Therefore, the total transmission power is the same, and the number of transmitted symbols of IIR-OFDM will be less than classical OFDM. IIR-OFDM has a higher delay required to process data at the transmitter than CP due to the complexity of the guard interval. At the receiver, the IIR-OFDM equalization process is like OFDM without additional delay. However, it is worth noting that IIR-OFDM is specifically proposed for under IIR channels.

3.2. The 1st-Order IIR Channel

Owing to passive and active cancellation techniques, the multipath effect of the loop can be significantly suppressed [17]. Suppose that the loop's multipath effect is suppressed; the loop coefficient is only $a(1)$. The guard interval under the 1st-order IIR channel can be expressed as

$$\bar{s}_v(0) = a(0)y_v(0) + a(1)y_{v-1}(0). \quad (26)$$

Under the 1st-order IIR channel, the guard interval complexity is related to the maximum delay of the loop as $\mathcal{O}(N \log_2(N) + l_g^2 + 3l_g + 2)$, i.e., $\mathcal{O}(N \log_2(N) + 6)$.

The expectation of the guard interval is

$$\begin{aligned} p_v(0) &= \frac{\sigma_d^2}{N} \sum_{n=0}^{N-1} \left[\frac{a(0)a(0)^* + a(1)a(1)^*}{A_n A_n^*} \right] \\ &= \sigma_d^2 \frac{a(0)a(0)^* + a(1)a(1)^*}{a(0)a(0)^* - a(1)a(1)^*}, \end{aligned} \quad (27)$$

where the guard interval is only connected to the forward path $a(0)$ and the loop $a(1)$.

Similarly, to ensure system stability, it is necessary to satisfy $a(0)a(0)^* - a(1)a(1)^* > 0$. In addition, the IIR channel coefficients also have to satisfy $a(0)a(0)^* - a(1)a(1)^* \gg 0$ for the guard interval expectation to be in a reasonable range. It is demonstrated that for IIR-OFDM, suppressing loops maintains the system stability and reduces the guard interval expectation. However, loops cannot be completely suppressed and the performance of IIR-OFDM under IIR channels will outperform that of classical OFDM.

3.3. Delay IIR Channel

Assume that the loop's multipath effect is suppressed and the IIR channel order still is l_g . However, the IIR channel coefficients have $a(i) = 0, i = [1, \dots, l_g - 1]$ under this special delay IIR channel, which can be regarded as a single-loop IIR channel with a large loop delay. The expression for the guard interval under a special delay IIR channel is the same as (23), which can be expressed as

$$\begin{aligned} \bar{s}_v(k) &= \sum_{i_0=0}^k a(i_0)y_v(k-i_0) + \sum_{i_1=k+1}^{l_g} a(i_1)y_{v-1}(k-i_1+l_g) \\ &= a(0)y_v(k) + a(1)y_{v-1}(k-1+l_g). \end{aligned} \quad (28)$$

Under the delay IIR channel, the complexity of GI depends not only on the maximum delay of the loop, but also on the number of loops. Therefore, the complexity of the guard interval is $\mathcal{O}(N \log_2(N) + (l_g + 1)^2 + l_g)$, i.e., $\mathcal{O}(N \log_2(N) + 5)$.

The expectation of guard interval under a special delay IIR channel is

$$\begin{aligned}
 p_v(k) &= \frac{\sigma_d^2}{N} \sum_{n=0}^{N-1} \left[\frac{\sum_{i_0=0}^k \sum_{i_1=0}^k a(i_0) a(i_1)^* e^{j2\pi(i_0-i_1)n/N}}{A_n A_n^*} \right. \\
 &\quad \left. + \frac{\sum_{i_0=k+1}^{l_g} \sum_{i_1=k+1}^{l_g} a(i_0) a(i_1)^* e^{j2\pi(i_0-i_1)n/N}}{A_n A_n^*} \right] \\
 &= \frac{\sigma_d^2}{N} \sum_{n=0}^{N-1} \left[\frac{a(0) a(0)^* + a(l_g) a(l_g)^*}{A_n A_n^*} \right] \\
 &= \sigma_d^2 \frac{a(0) a(0)^* + a(l_g) a(l_g)^*}{a(0) a(0)^* - a(l_g) a(l_g)^*}.
 \end{aligned} \tag{29}$$

Note that the special delay IIR channel has the same expectation although the amplitudes of the protection intervals are different, and the expression is the same as (27). Therefore, this IIR channel should satisfy $a(0) a(0)^* - a(1) a(1)^* \gg 0$ for a reasonable guard interval expectation and system stability. It proves that when the loop delay is large, the number of GIs must be greater than or equal to the maximum number of loop delays to maintain the circular convolution of the received signal and the loop channel coefficients. However, the complexity of the guard interval does not increase with the maximum delay of the loop and depends only on the number of loops.

4. Simulation Results

In this section, we validate IIR-OFDM from the bit error rate (BER). Suppose the signal-to-noise ratio (SNR) in the simulation is σ_d^2 / σ_w^2 . BER indicates the probability of a bit being incorrectly demodulated, while SNR represents the power ratio of a useful symbol to the noise it carries. On the transmitter, the number of subcarriers is 128, transmitting one frame containing two symbols at a time, i.e., $v = 2$. In addition, the simulation is based on IIR-OFDM symbols modulated with quadrature phase shift keying (QPSK) in different IIR channels. The loop response of the IIR channel follows a complex zero-mean Gaussian Random Variable. IIR-OFDM is compared to classical OFDM, where both schemes carry the guard interval of length l_g . Note that the guard interval for classical OFDM is replicated from the tail sample of the signal. The details of the simulation parameters are as follows.

- Modulation: QPSK.
- Number of subcarriers: $N = 128$.
- Number of transmitted symbols: $V = 2$.
- The loop response of the IIR channel: complex zero-mean Gaussian Random Variable.

The loss of loops is assumed to be β dB, which is to maintain the stability of the IIR channel system under multiple loops. In addition, residual loops remain due to the difficulty of complete loop suppression in practice. Figure 1 shows the amplitude of the IIR-OFDM signal at values of different β ; the red and black lines are the amplitudes of the guard interval and OFDM signal, respectively. The number of loops is $l_g = 8$, so the length of the guard interval is 8. When $\beta = 10$ dB, the poles of the system appear outside the unit circle, the system is unstable, and the amplitude of the guard interval is larger than the OFDM signal. Too large a guard interval will result in the Radio Frequency side of the transmitter being unable to amplify linearly, and an unstable system will result in the receiver being unable to demodulate. However, when $\beta = 20$ dB, the guard interval amplitude is similar to the OFDM signal since the system is stable.

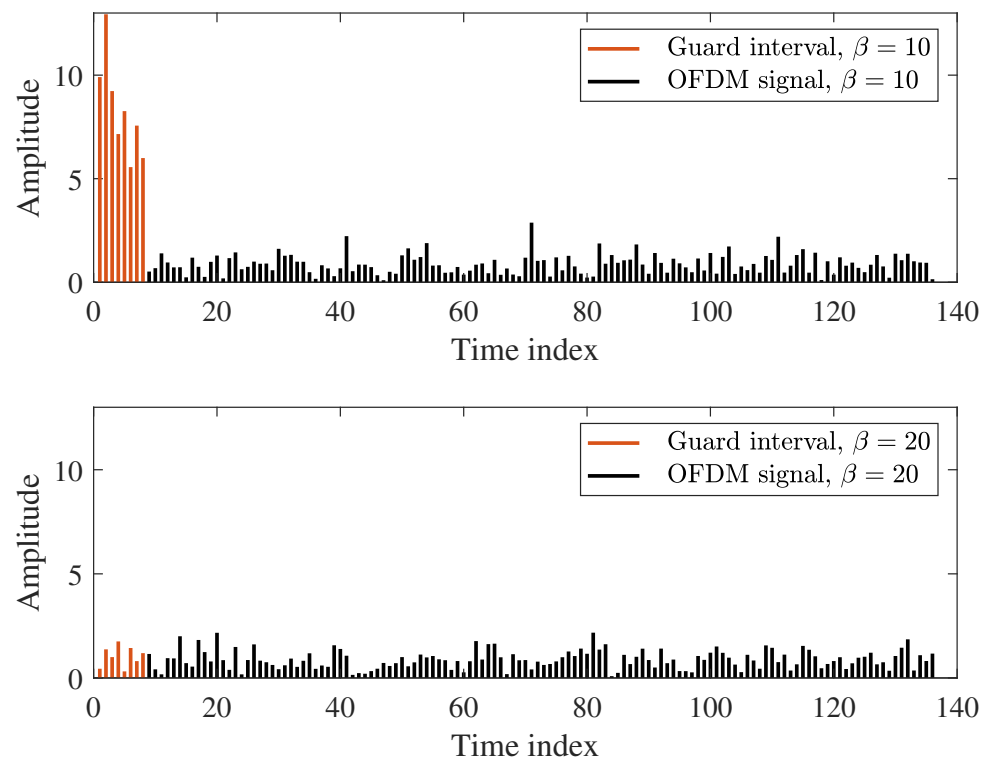


Figure 1. The amplitude of the IIR-OFDM signal at different β values.

The BER comparison under the IIR channel of the multipath loop is shown in Figure 2. The number of loops is $l_g = 8$ and follows a complex zero-mean Gaussian Random Variable with $\sigma_w^2 = 1$. Suppose that $h_0 = 1$ to avoid the effects of the forward path, i.e., $a(0) = 1$. The IIR channel system with $l_g = 8$ is stable when $\beta > 20$ dB. When the guard interval is ignored, the OFDM signal transmission power is the same for both schemes. Note that the BER performance of both schemes increases with increasing loss of loops. However, the BER of IIR-OFDM outperforms that of classical OFDM regardless of the loop loss. It is demonstrated that IIR-OFDM signals can resist ISI from multipath loops.

Figure 3 and Figure 4 show the IIR-OFDM under the first-order IIR channel and the special delay IIR channel, respectively. Suppose that $h_0 = 1$ for both IIR channels to avoid the effects of the forward path, i.e., $a(0) = 1$. In addition, the delay channel is a two-order IIR channel with $a(1) = 0$. The loops for both IIR channels follow a complex zero-mean Gaussian Random Variable with $\sigma_w^2 = 1$. The loop loss is increased by active and passive suppression, so both IIR channel systems are stable when $\beta > 15$ dB. IIR-OFDM outperforms classical OFDM in terms of BER performance in both channels. When $\beta = 15$ dB, IIR-OFDM has a BER gain of about 0.8 dB compared to classical OFDM at a BER of 10^{-3} over the first-order IIR channel and the special delay IIR channel. However, when $\beta = 20$ dB, the BER gain of IIR-OFDM is about 0.2 dB at a BER of 10^{-3} . Note that the performance gap between IIR-OFDM and classical OFDM decreases as the loop loss increases. When loops are completely eliminated, both performances become identical. It is well known that the loop signal causes ISI to the received signal at the receiver. Therefore, many active and passive suppression techniques for loops have been proposed. Generally, passive isolation of the loop should be maximized to minimize the number of more complex active pair cancellation techniques. This means that the loop signal power leaking to the receiver is small, and does not require much active cancellation to attenuate it to a tolerable level. However, completely canceling loops is a challenging problem. For active cancellation, even if the transmitted signal is known in the digital baseband, it cannot be completely canceled in the receiver due to radio frequency (RF) impairments and the large power difference between the transmitted and received signals [18]. In addition, RF

components have many non-idealities compared to an ideal demonstration setup, and low-cost, small-sized full-duplex transceivers are more suitable for mass-market products [19]. Although the loop cannot be completely eliminated, the ISI caused by loop signals can be greatly mitigated by the cancellation. Note that the channel model after loop cancellation is still an IIR channel due to the presence of the residual loops.

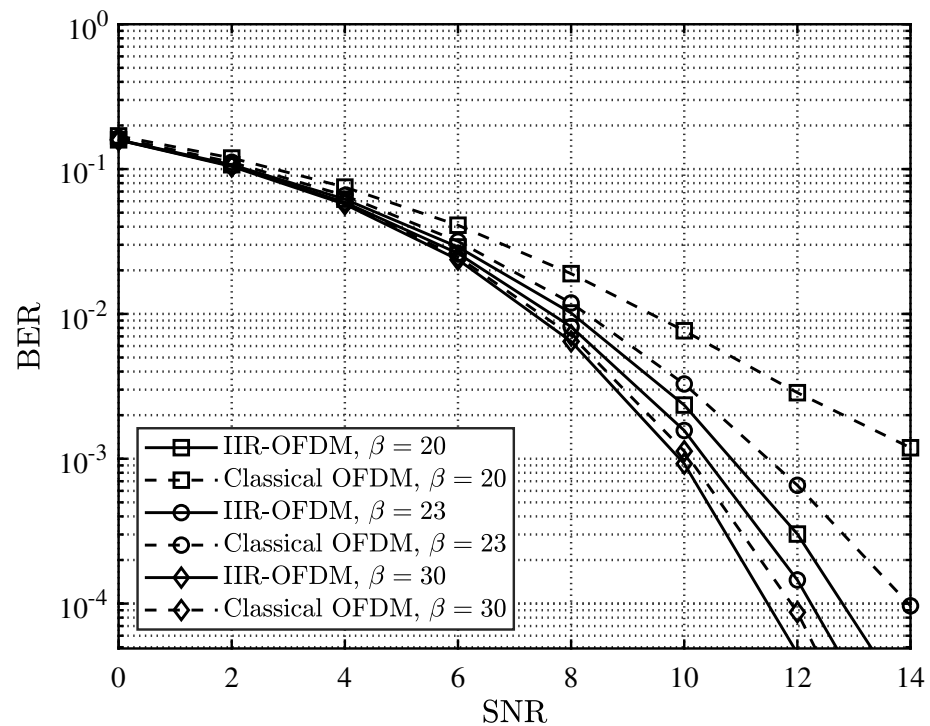


Figure 2. BER comparison under the IIR channel of the multipath loop.

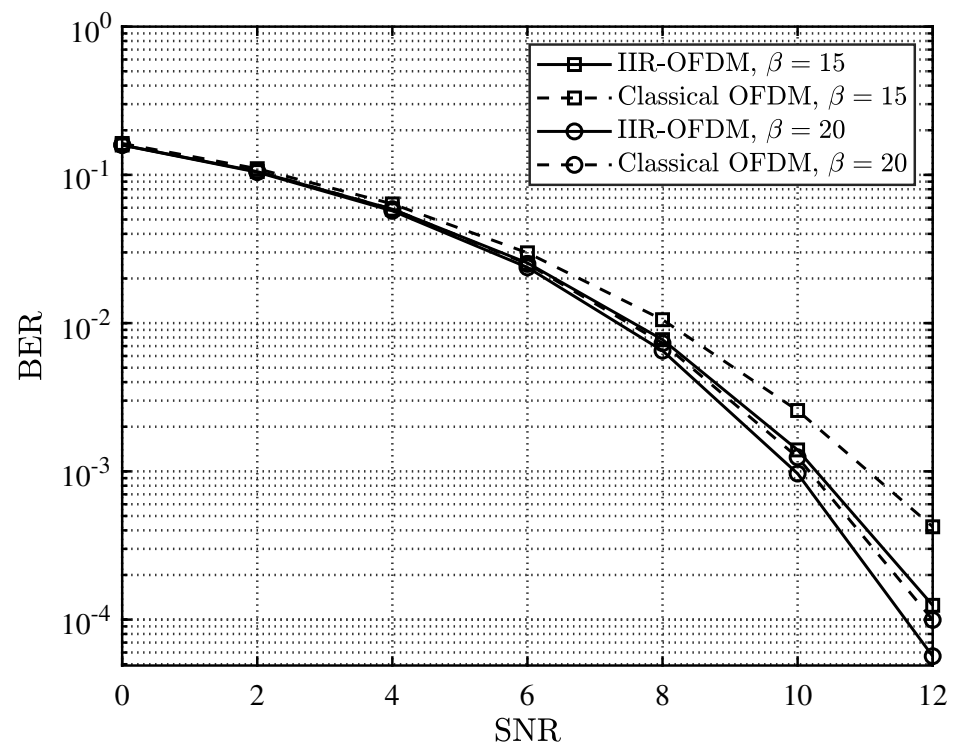


Figure 3. BER comparison under the 1st-order IIR channel.

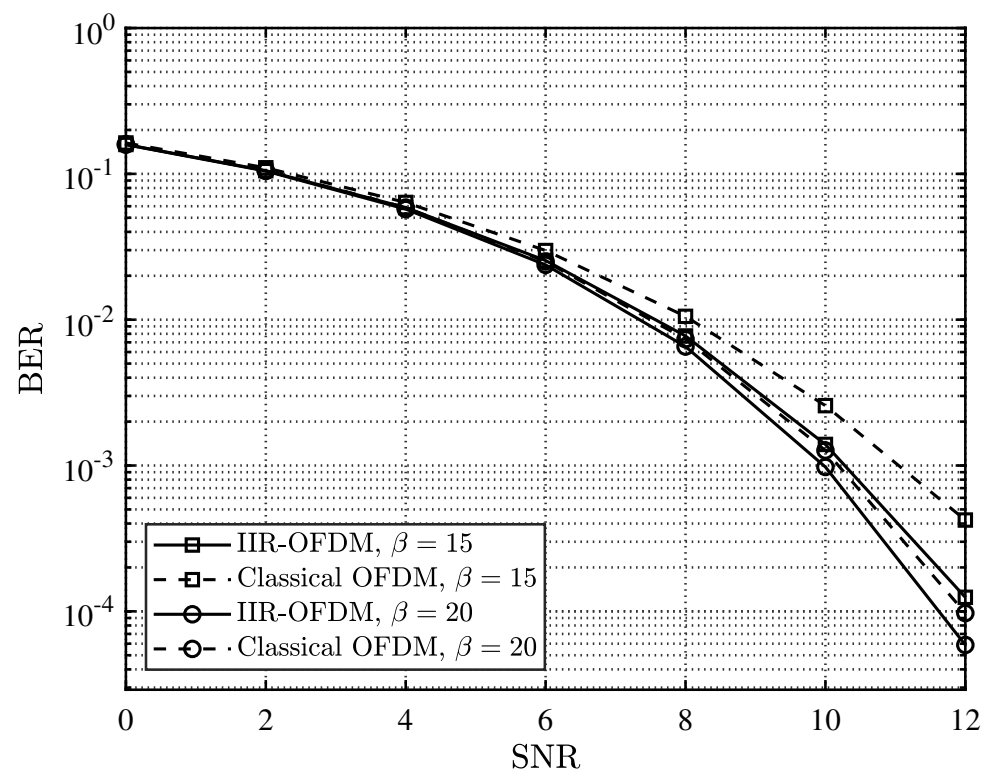


Figure 4. BER comparison under the special delay IIR channel.

5. Conclusions

In this paper, we summarize and derive the guard interval characteristics of IIR-OFDM under different IIR channel models. By modeling IIR systems with different channels, the IIR-OFDM carrying the guard interval is resistant to interference from loop signals. Moreover, the complexity of GI depends not only on the maximum delay of the loop, but also on the number of loops. It is shown that IIR-OFDM performs better than classical OFDM based on a theoretical derivation and simulation. As a remark, IIR-OFDM can only be transmitted in stable IIR systems.

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