

Article

Flexible Integrated Scheduling Considering Periodic Maintenance

Xueying Ding ¹, Zhiqiang Xie ² , Wei Zhou ^{1,*}, Zhenjiang Tan ¹ and Ming Sun ¹

¹ College of Mathematics and Computer, Jilin Normal University, Siping 136000, China; dingxy@mails.jlnu.edu.cn (X.D.); tanzj@jlnu.edu.cn (Z.T.); jlspsunming@jlnu.edu.cn (M.S.)

² School of Computer Science and Technology, Harbin University of Science and Technology, Harbin 150080, China; xiezhiqiang@hrbust.edu.cn

* Correspondence: jlspsz@jlnu.edu.cn

Abstract: Aiming at the problem of current flexible integrated scheduling, most of the current research shortens the total processing time of products and ignores the loss of the equipment in the processing, which affects the scheduling ability of the equipment; in this context, a flexible integrated scheduling algorithm considering periodic maintenance (FIS-PM) is proposed. The algorithm flexibly mobilizes the processing sequence of the selected processing equipment, and uniformly maintains all equipment at a certain time. First, the ISA-PM algorithm adopts the strategy combination of the layer priority, the short-time, and the equipment priority strategy to schedule the operations. Then, based on the forest firefighting model, the maintenance start point and duration of the equipment are determined by the relationship between the number of the processed operations and the number of the unprocessed operations and the loss of equipment during the maintenance. Finally, the dynamic adjustment strategy is adopted, which not only realizes the maintenance of the equipment, but also reduces the makespan of the products. The experimental results show that the proposed ISA-PM algorithm realizes the optimization effect of reducing the makespan of complex products, completing the maintenance, and improving the overall utilization rate of equipment systems.

Keywords: integrated scheduling; dynamic adjustment; periodic maintenance; loss value



Citation: Ding, X.; Xie, Z.; Zhou, W.; Tan, Z.; Sun, M. Flexible Integrated Scheduling Considering Periodic Maintenance. *Electronics* **2024**, *13*, 3730. <https://doi.org/10.3390/electronics13183730>

Academic Editors: Antonio J. Marques Cardoso and Ping-Feng Pai

Received: 20 July 2024

Revised: 13 September 2024

Accepted: 17 September 2024

Published: 20 September 2024



Copyright: © 2024 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

1. Introduction

Equipment management and maintenance is the basis of modern enterprise production activities. After a long period of work, the wear of equipment will lead to the deterioration of the health of equipment, which will affect the efficiency of production scheduling and production costs [1]. The premise and basis of the use of equipment is the maintenance, which is an important guarantee for the safe operation of equipment. Periodic maintenance can reduce the loss of equipment, reduce the failure rate, and improve the use efficiency of equipment. Therefore, to carry out reasonable periodic maintenance based on the health state of the equipment is an essential link in the production process of enterprises.

Many scholars have carried out in-depth research on the problem of equipment maintenance in scheduling production, and achieved good results [2–13]. For example, Pang Sheng et al. [2] realized the rapid prediction of maintenance equipment demand through simulation, and studied the maintenance support system. Jiang Zhigao et al. [3] proposed the heuristic algorithm LPT-SPT in order to solve the periodic maintenance with variable processing time in single-machine scheduling problem with minimum processing time as the goal. Lou Ping et al. [4] jointly constructed a knowledge graph through a joint model and equipment maintenance, automatically extracted knowledge, and realized providing equipment maintenance schemes according to requirements. Song Wenjia et al. [5] designed a multi-objective hybrid colonial competition algorithm, aiming at reducing maintenance costs, and established an integrated optimization model of equipment preventive maintenance and scheduling. Ali Ashary et al. [6] introduced the modular AMI framework for robot-assisted physical therapy for motion parallax individuals, which can effectively

adjust robot behavior according to the imitation ability of the subject. Ali Ashary et al. [7] proposed the LSTM-RNN and SODTW frameworks for adaptive motion imitation of robot-assisted physical therapy and proved the validity of AMI framework by predicting appropriate robot exercises resembling human subject motions.

With the rapid development of material living standards and technology, the production and manufacturing problems of single-piece and small-batch personalized products with complex constrained processes are becoming increasingly prominent. Therefore, for multi-variety and small-batch products, an integrated scheduling based on collaborative processing and assembly activities is proposed [14] to improve the scheduling efficiency of tree-like complex products [15–28]. In the related research of integrated scheduling, Xie Zhiqiang et al. [11] realized the algorithm of dynamically adjusting the maintenance start time of equipment in general integrated scheduling by calculating the overload of equipment, but this algorithm cannot solve the problem of regular maintenance of equipment in flexible integrated scheduling. Xie Zhiqiang et al. [20] proposed the reverse layer priority strategy, dynamic quasi-long path strategy, equipment selection, and equipment preemption strategies. Finally, based on the completion time flip scheduling scheme, the product completion time in flexible integrated scheduling is shortened. However, only the completion time is considered, and the utilization of equipment and the inability to process due to equipment load operation are not considered. Zhou Wei et al. [28] took the number of processes that can be processed by flexible equipment as the optimization objective, and proposed a strategy to minimize the scheduling scale and dynamically adjust the priority of flexible equipment, so that the utilization rate of flexible equipment can be higher. However, their paper only focuses on the utilization rate of equipment and the total processing time in flexible integrated scheduling, and does not consider that the equipment needs to be maintained after a certain number of processes to ensure more efficient work.

In the flexible scheduling system, the characteristics and processing capabilities of the equipment system are diverse, and the same process can be selected for processing on different equipment. How to use equipment with high flexibility and adaptability to flexibly process, so that the equipment can cope with the regular maintenance activities in the production process within the original plan or even shorter processing time, and improve the overall utilization rate of the equipment system is the main research problem of this paper. In general, integrated scheduling, such as the algorithm in Ref. [11], can not solve the maintenance problem of flexible integrated scheduling. In many research results of flexible integrated scheduling, such as Ref. [20] and Ref. [28], the optimization goal is to shorten the total processing time of complex products, ignoring the need for regular maintenance of equipment in advance due to loss or failure during processing. Therefore, in the current field of integrated dispatching, there is no specific achievement from the perspective of regular maintenance of flexible equipment, so it is necessary to solve the flexible integrated scheduling problem under periodic maintenance.

For the above reasons, this paper presents the flexible integrated scheduling algorithm considering periodic maintenance (FIS-PM) to solve the flexible integrated scheduling problem from the perspective of considering periodic maintenance. The main contributions are:

1. For the flexible integrated scheduling, the scheduling problem under periodic maintenance of equipment was addressed for the first time;
2. For the scheduling of operations, the layer priority strategy and the short-time strategy are preferred when establishing the initial operation set. For the operation adjustment, the flexible equipment priority strategy is used to dynamically adjust the operations.
3. For effective optimization, we have comprehensively considered the horizontal and the vertical directions. For the horizontal direction, the hierarchical relationship and equipment priority in the product structure attribute of the tree are studied, and the intensity of parallel processing is effectively improved. For the vertical direction, based on the dynamic adjustment of the selected flexible equipment, the starting time and the duration of the maintenance are determined, which effectively improves the

intensity of the compact processing of the operation on the equipment and realizes periodic maintenance activities.

2. Problem Description and Model Construction

Definition 1. *Layer priority [17]. For the tree-shaped complex product with n -layer structure, the layer priority of the process is defined from the root node. The root node is the first layer, and the layer priority is 1. The layer priority of the second layer process of the tree-shaped complex product from top to bottom is 2, increasing in turn, and the layer priority of the n -layer process is n . The higher the level, the higher the layer priority, and the layer priority of the same layer process is the same.*

Definition 2. *Short-time strategy [18]. For the tree-shaped complex product with n -layer structure, the layer priority of the process is defined from the root node. The root node is the first layer, and the layer priority is 1. The layer priority of the second layer process of the tree-shaped complex product from top to bottom is 2, increasing in turn, and the layer priority of the n -layer process is n . The higher the level, the higher the layer priority, and the layer priority of the same layer process is the same.*

Definition 3. *Equipment priority [19]. The number of the processed operations on each equipment is taken as the equipment priority. The larger the number, the higher the priority.*

Definition 4. *Equipment health value. The equipment health value indicates the current working status of the device. The initial health value of each unprocessed machine is set to 100%. As the number of processes increases, the equipment gradually becomes tired and the health value decreases.*

Definition 5. *Loss coefficient. Dividing the total processing time of complex products without periodic maintenance and dynamic adjustment into 10 parts, 10% of the total processing time is defined as the initial value and increasing step.*

Definition 6. *Loss value. The product of equipment health value and loss coefficient.*

Definition 7. *Starting time of maintenance. The time when the number of the processing operation reaches $2/3$ of the total operation number of the complex product is defined as the starting time of the maintenance, from which all equipment began to carry out maintenance activities.*

Definition 8. *Maintenance time. During the maintenance time, the equipment cannot continue working. The maintenance time is the product of the loss value at the starting time of the maintenance and the starting time.*

Definition 9. *Maintenance period. The interval from the beginning of device processing to the end of the maintenance to ensure the normal operation of the device.*

Definition 10. *Dynamic adjustment strategy. During processing, when the process meets the constraints of the previous process, the total cost of each process on different equipment allowed to be processed is calculated and compared according to the forest firefighting model. According to the comparison results, the processing sequence is dynamically adjusted, and the equipment with the minimum total cost generated by the process is selected for processing.*

In traditional flexible scheduling, different operations have different processing capabilities on the same machine, and the starting time and duration of the maintenance are determined by the number of operations, processing time, and loss value. The specific description is as follows:

- (1) At the beginning of the processing, each machine is idle, the health value is full, and the performance is perfect;

- (2) The sufficient and necessary condition for each operation to start processing is that all the predecessor operations have been completed;
- (3) In order to ensure the long-term stable operation of the equipment, during the maintenance period, the equipment will not be able to perform any processing procedures;
- (4) Operation processing and equipment maintenance are continuous processes, its consistency and stability must be maintained, and any form of the interruption is never allowed;
- (5) The equipment can be idle before starting maintenance.

According to the above conditions, the mathematical model is established as follows:

Let the operation sequence of a single complex product A with n operations be $A = \{A_i\} (1 \leq i \leq n)$, and can be processed on m machines. The machine sequence is represented as $M = \{M_j\} (1 \leq j \leq m)$. The starting time of operation A_i on machine M_j is $T_{A_i M_j}^S$ and the ending time is $T_{A_i M_j}^E$. The processing time of operation A_i on machine M_j is $T_{A_i M_j}$. t_i indicates the working time of the machine in a certain maintenance period. H_i indicates the health status of the machine at the moment $t = i$. E_i represents the ending time of operation i . The mathematical description is as follows:

$$T = \min(\max(E_i) + WT) \quad (1)$$

s.t.

$$T_{A_{i+1} M_j}^S - T_{A_i M_j}^E \geq 0, (1 \leq i \leq n, 1 \leq j \leq m) \quad (2)$$

$$\sum_{i=1}^n \sum_{t=0} A_i / \{A_i\} = 2/3 \quad (3)$$

$$H_i = 100\% - 10\%t_i, (1 \leq i \leq n) \quad (4)$$

$$WT = (1 - H_i)t_i, (1 \leq i \leq n) \quad (5)$$

Formula (1) is the objective function. It is aiming to minimize the makespan of the product, which is the sum of the completion time of the last operation and its maintenance time. Formula (2) indicates the constraints between operations to ensure that the starting time of each operation is later than the completion time of its predecessor. Formula (3) indicates that when the number of the processed operation reaches 2/3 of the number of operations, the maintenance starts. Formula (4) indicates that when $t = n$, the health value of the equipment is equivalent to the initial health value minus the generated loss value. Formula (5) indicates the time spent on maintenance of the equipment.

3. Algorithm Design and Analysis

The proposed algorithm, FIS-PM, adopts a variety of strategies to optimize the scheduling. To begin with, the classic strategies, the layer priority strategy and the short-time strategy, are adopted to improve the parallel processing capability of the equipment. These strategies all take the operation as the optimization object. Next, the equipment priority strategy is introduced, that is, the equipment scheduling with more processing operations is prioritized. It is a strategy that takes the equipment as the optimization object, and can minimize the idle time of the equipment during processing. Moreover, the dynamic adjustment strategy is adopted, which takes the corresponding relationship between the operations and the equipment as the optimization object. It is the core strategy of FIS-PM. It establishes the dynamic adjustment operation sequence and the selected machine sequence through the forest firefighting model, thereby improving the utilization rate of the equipment and completing periodic maintenance activities. Among them, the forest firefighting model refers to the fact that when the forest is on fire, how many members should be sent to fight the fire. The more members are sent, the smaller the forest loss will be, but the more the rescue expenditure will be. The loss cost in the process of fire is usually proportional to the burned area of the forest, and the burned area is related to the fire extinguishing and fire extinguishing time. The fire extinguishing time depends on the number of firefighters, and

the rescue cost is related to the number of firefighters and the length of fire extinguishing time. Therefore, the fire loss and rescue costs should be considered comprehensively, and the number of firefighters dispatched should be determined with the goal of minimizing the total cost.

Therefore, the dynamic adjustment strategy model based on the forest firefighting model is established as follows:

The number of the operations (the firefighters) of a complex product is x . The makespan (the total cost of the firefighting) of the product is $C(x)$, and the total processing time (the loss cost of the firefighting) of the product is $T(x)$. The overall maintenance time (the rescue cost) of the equipment sequence is $F(x)$. The processing capacity of the equipment (the area of the forest burned) is $B(x)$. The maintenance time is regarded as the firefighting time t , and t_1 indicates when the number of the processed operations reaches $2/3$ of x . The processing capacity of the equipment decreases significantly after this moment, and the maintenance time is set as $(t_2 - t_1)$, which is the product of the equipment loss value and time. t_3 is the end time of product processing, and the corresponding relationship between $B(x)$ and t is shown in Figure 1. The time relationship between the optimized equipment processing capacity in this paper is shown in Figure 2, and the optimization objective function in Formula (1) can be rewrite as follows:

$$C(x) = T(x) + F(x) = \beta t_1^2/2 + \beta^2 t_1^2/2(\lambda x - \beta) + \beta t_1 x/(\lambda x - \beta) + x \quad (6)$$

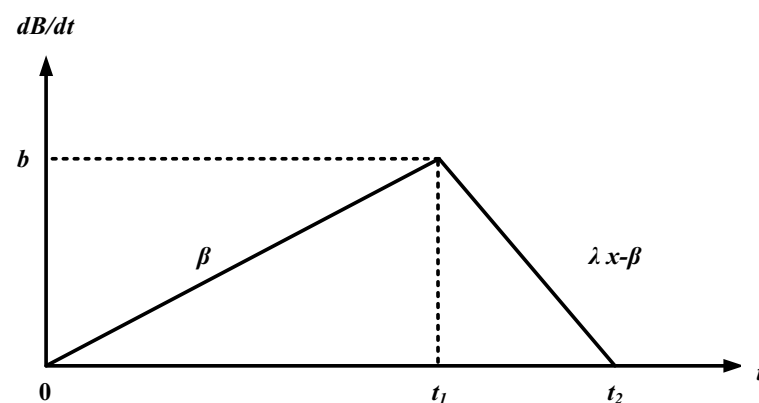


Figure 1. Time relationship diagram of equipment processing capacity.

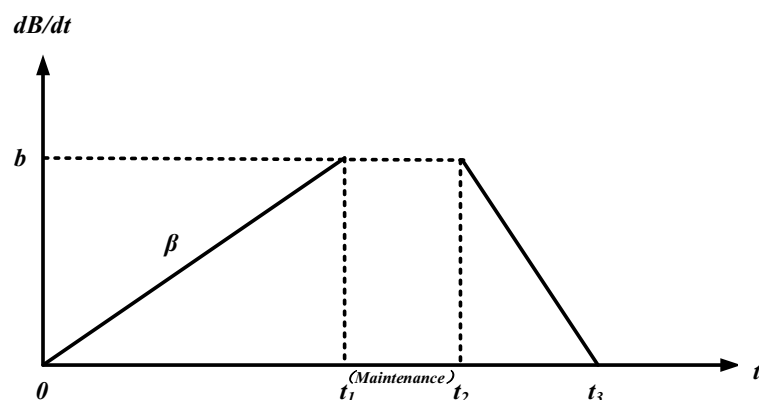


Figure 2. This paper optimizes the time relationship diagram of equipment processing capacity.

Dynamic adjustment strategy: $C_{A_i M_j}$ is defined as the total cost value when operation A_i completes processing on machine M_j . For any complex product, the process scheduling order and the corresponding Gantt chart are obtained by the ‘layer priority + short time + equipment priority’ strategy, as shown in Figure 3a. The next step is to use the dynamic adjustment strategy, $\sum_{i=1}^n \sum_{t=0} A_i / \{A_i\} = 2/3$, which means that when the ratio of the number of completed processing operations to the total number of processes reaches 2/3, the equipment maintenance begins. For the processes A_1 – A_n , the total cost of each process is calculated when the processing is completed on the flexible equipment, and the corresponding processing equipment is assigned to the process to be scheduled from the flexible equipment system to minimize the total cost value. As shown in Figure 3b, the red dotted wireframe represents the original processing equipment and processing time where the process A_i and A_n are located, the green solid wireframe represents the processing equipment and processing time where the process A_i and A_n are adjusted by the dynamic adjustment strategy, and the abscissa t represents the total time spent at the end of the processing of the complex product.

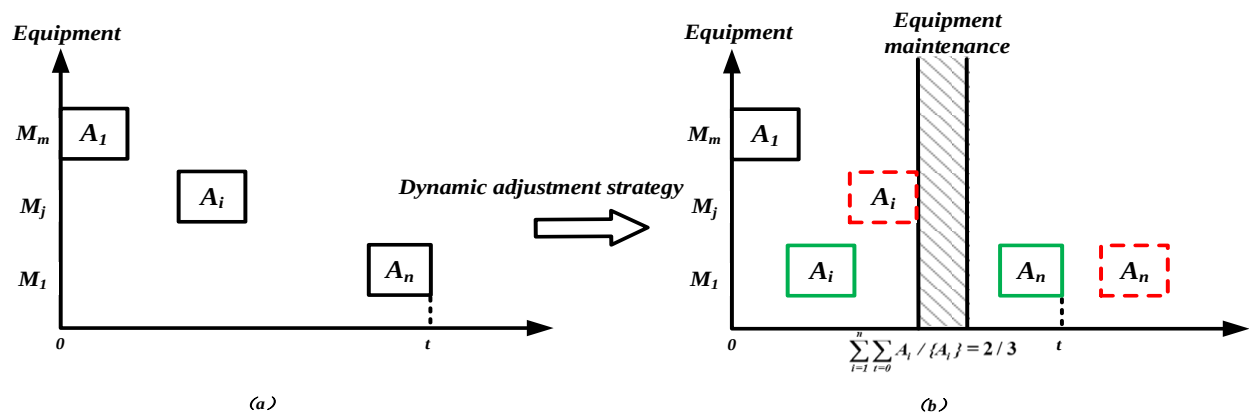


Figure 3. Dynamic adjustment strategy diagram.

3.1. Algorithm Description

The algorithm flow chart of FIS-PM is shown in Figure 4. Specific steps are as follows:

Step 1: Input the processing information;

Step 2: Determine the operation sequence and the corresponding machine sequence without maintenance by the strategies of the layer priority, the short-time, and the equipment priority;

Step 3: Use dynamic adjustment strategy to optimize the operation sequence and select the corresponding machines;

Step 4: Calculate the number of the processed operations, and use Formula (4) to calculate the current health status of each machine according to their working time;

Step 5: If the number of the processed operations reaches 2/3 of the total number of the operations, then turn to Step 6, otherwise, go to Step 7;

Step 6: Start maintenance, and the duration of time of the maintenance is yielded by Formula (5);

Step 7: If the set of the schedulable operation is empty, then go to Step 8, otherwise, remove the completed operations and go to Step 4;

Step 8: Generate and output the Gantt chart of the product. End.

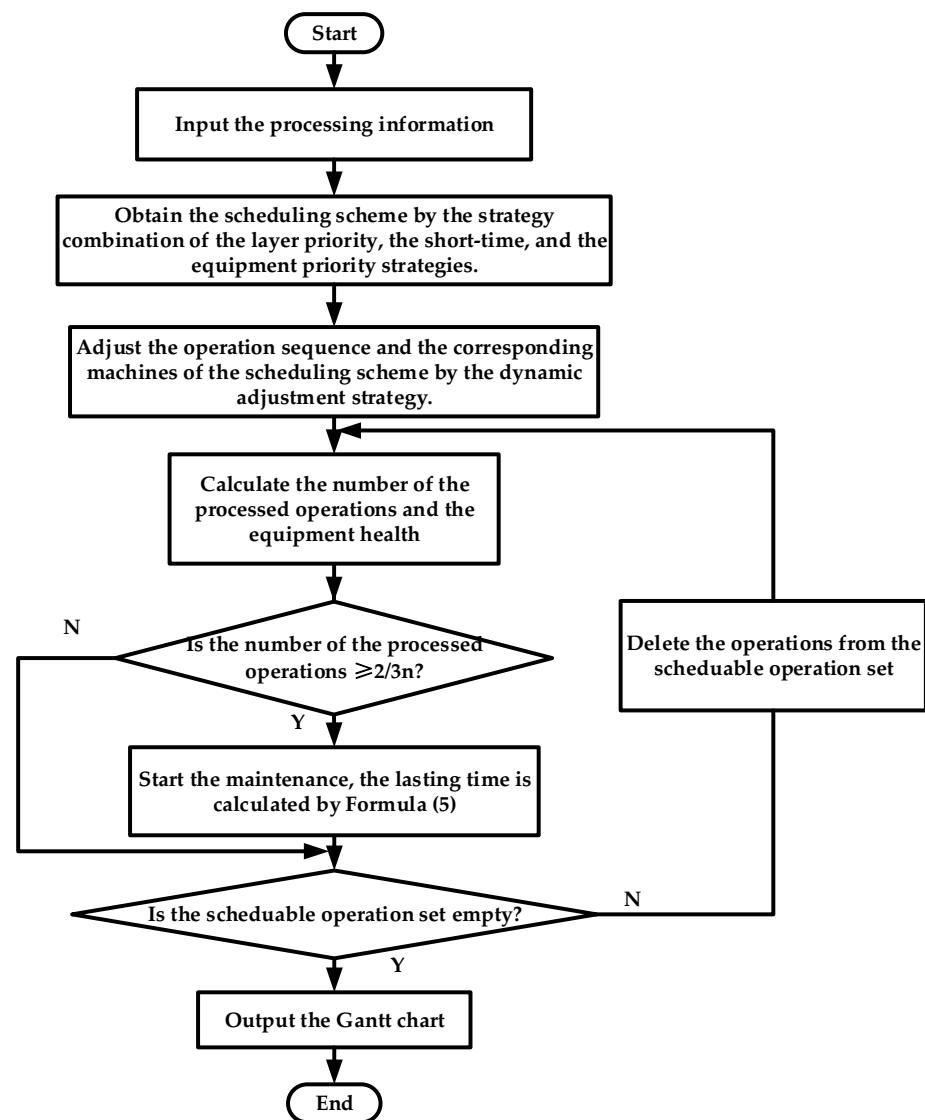


Figure 4. The flow chart of FIS-PM.

3.2. Complexity Analysis

In the proposed algorithm, the following strategies should be adopted in determining the operation sequence and their corresponding machines: the layer priority strategy, the short-time strategy, and the dynamic adjustment strategy. The layer priority strategy is used to select the operation with the highest priority, the short-time strategy is used to select the machine with the shortest processing time for each operation, and the equipment priority strategy is used to compare the number of the processing operations on each machine at different levels. Their time complexities are $O(n)$, $O(mn)$, and $O(n^2)$, respectively. The dynamic adjustment strategy needs to calculate the total processing cost for n nodes, thereby the time complexity is $O(n)$. To sum up the above analysis, the time complexity of the proposed algorithm is $O(n^2)$.

4. Algorithm Example Description

4.1. Petri Net Modeling Analysis

The algorithm in this paper does not take specific examples as the application premise, and has a wide range of versatility. Compared with other complex products with tree structures, it has better scheduling results. Therefore, it is assumed that complex product A is randomly generated, involving 15 processes and 4 flexible devices. Take it as an

example to illustrate the scheduling example, as shown in Figure 5. Each node in the figure includes three elements: the operation, its available machines, and their corresponding processing times. The three elements are separated by “/”. The arrow in the figure indicates the constraint relationship between adjacent operations. The arrow of each operation is pointing to its successor operation.

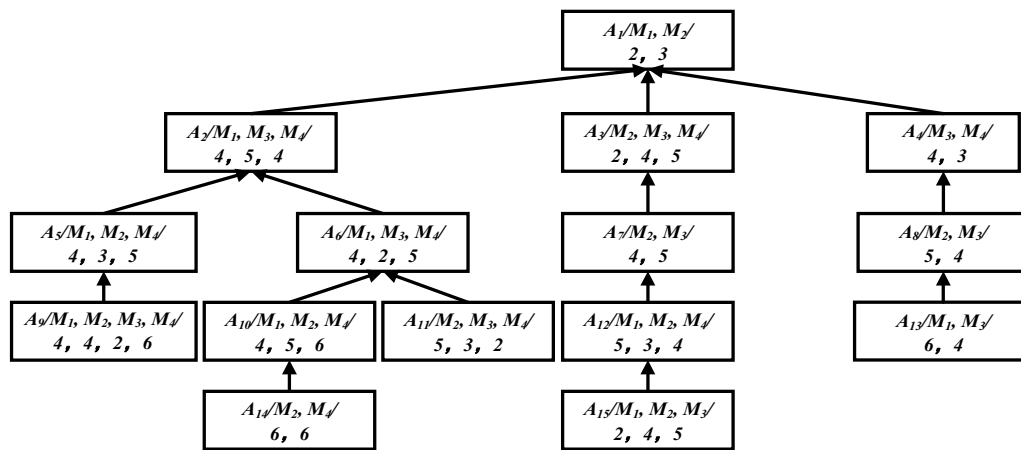


Figure 5. Processing tree.

In this paper, the basic Petri net is used to establish the Petri net model of complex product A on Platform Independent Petri Net Editor V4.3, as shown in Figure 6. Through Petri net modeling, the concurrent behavior can be described more intuitively, which ensures the accuracy and reliability of the model. The place is represented by a circular node, the transition is represented by a short vertical line, and a directed arc is used to represent the relationship between place-directed transitions and the relationship between transitions and places. The token value is calculated by the algorithm.

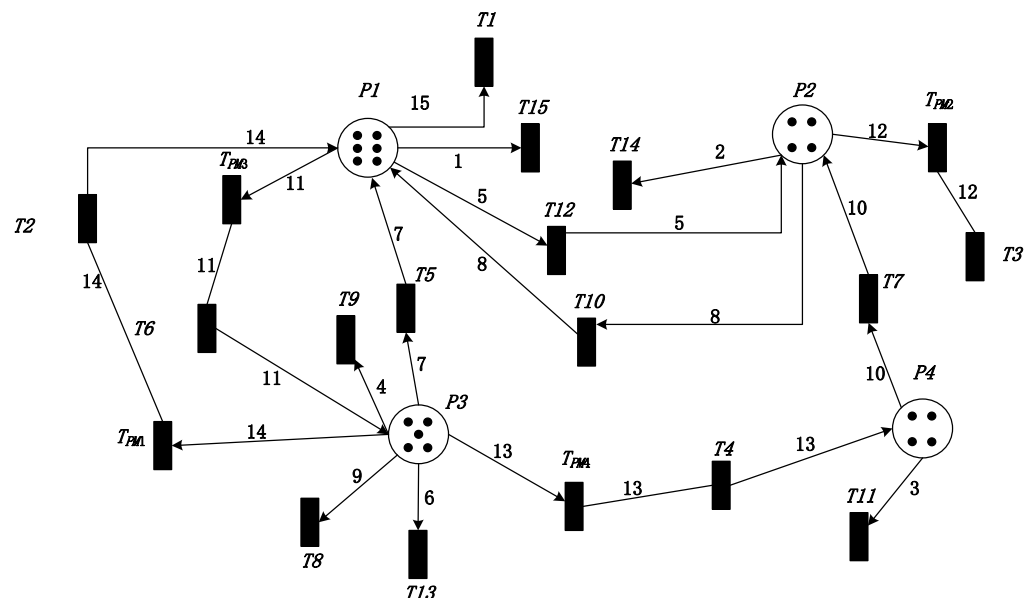


Figure 6. Petri net modeling of product A.

In the Petri net modeling diagram shown in Figure 6, the places P1, P2, P3, and P4 correspond to the equipment M1, M2, M3, and M4 in the complex product A scheduling system, respectively. Transitions T1~T15 correspond to processes A1~A15, and transitions T_{PM1}~T_{PM4} correspond to maintenance activities on equipment M1~M4. The place P1 corresponds to six transitions: T15, T5, T10, T_{PM1}, T2, and T1. The specific role is the input

place of T_{15} , and the output place of the remaining transitions. Place P_2 corresponds to four transitions: the input place of T_{14} , the output place of T_7 , T_{PM2} , and T_3 . The place P_3 corresponds to five transitions: the input place of T_9 , the output place of T_{13} , T_8 , T_{PM3} , and T_6 . P_4 corresponds to four transitions: T_{11} 's input place, T_{12} , T_{PM4} , and T_4 's output place. There are the following states between place and transition:

- (1) Concurrency state: each disjoint transition is preferentially excited in its own place. For example, transitions T_{15} , T_{14} , T_9 , and T_{11} have tokens at the same time at $t = 0$, which are excited at the same time at P_1 , P_2 , P_3 , and P_4 , respectively.
- (2) The sequential state with tight constraints between transitions: only when the tight constraint transition of the transition excites and releases the token, can it have the token in the corresponding place and enter the excitation state.
- (3) The order state of the place constraint relationship between the transitions: in the same place, only after the current transition is triggered, the next transition can have a token.

4.2. Scheduling Example Analysis

Now we use the proposed algorithm to schedule the product according the processing tree in Figure 5.

First, we use the layer priority strategy, the short-time strategy, and the equipment priority strategy to obtain the operation sequence and their corresponding machine without the maintenance. In layer five, there are two operations: A_{14} and A_{15} . For A_{15} , as $T_{A_{15}M_1} = 2$ is the shortest processing time, A_{15} is assigned to M_1 . For A_{14} , as $T_{A_{14}M_2} = T_{A_{14}M_4} = 6$, we need to compared their equipment priorities. The number of the operations for the ascending order of the machines is 9, 11, 10, and 10, respectively. Thereby the equipment priority of M_2 is greater than that of M_4 , and A_{14} is selected to be scheduled on M_2 . The operation sequence and the corresponding sequence without maintenance at level 5 is $\{A_{15}/M_1/2, A_{14}/M_2/6\}$.

Then, we delete the scheduled operations A_{14} and A_{15} . In layer four, there are five operations: A_9 , A_{10} , A_{11} , A_{12} , and A_{13} . The shortest processing time in this layer is $T_{A_9M_3} = T_{A_{11}M_4} = 2$. The equipment priorities of M_3 and M_4 are both 9. As different machines can be processed in parallel, A_9 and A_{11} are assigned to M_3 and M_4 , respectively. Moreover, A_{12} is assigned to M_2 . For A_{10} and A_{13} , their shortest processing times are the same, but the equipment priority of M_4 is larger than M_1 , so the operation sequence and the corresponding sequence without maintenance at level 5 is $\{A_{11}/M_4/2, A_9/M_3/2, A_{12}/M_2/3, A_{13}/M_3/4, A_{10}/M_1/4\}$.

Delete the scheduled processes A_{11} , A_9 , A_{12} , A_{13} , and A_{10} . There are 4 processes at level 3, which are A_5 , A_6 , A_7 , and A_8 , respectively. The minimum processing time: $T_{A_6M_3} = 2$, the second least processing time is $T_{A_5M_2} = 3$, and then $T_{A_7M_2} = T_{A_8M_3} = 4$, so compare the device priority; the device M_2 priority is 5, the M_3 device priority is 6, so the priority is to schedule the process A_8 on the device M_3 , and the process scheduling order of level 3 is $\{A_6/M_3/2, A_5/M_2/3, A_8/M_3/4, A_7/M_2/4\}$. Similarly, the process of scheduling level 2 can be scheduled in turn according to the short-time strategy. The scheduling order is $\{A_3/M_2/2, A_4/M_4/3, A_2/M_4/4\}$, the total processing time is the processing time of equipment M_1 to complete process A_1 , a total of 25 working hours. Therefore, the corresponding Gantt chart of scheduling complex product A with layer priority, short-time, and equipment priority strategy, and without considering the regular maintenance of equipment, is shown in Figure 7.

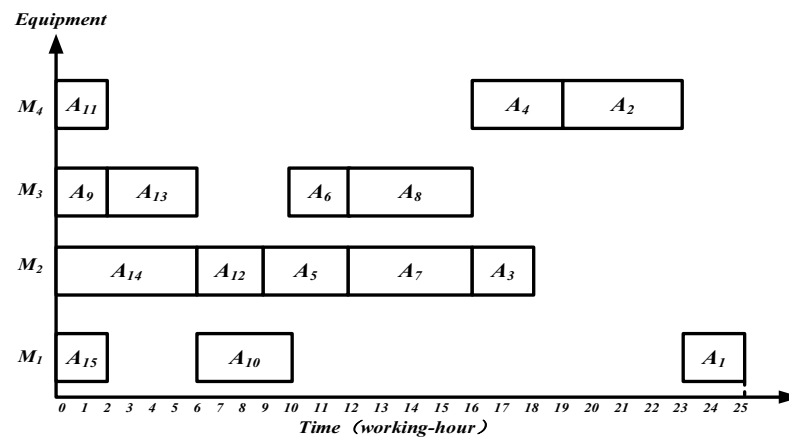


Figure 7. The resulting Gantt chart of complex product A is scheduled by using layer priority, short-time, and device priority strategies.

Furthermore, we introduce the maintenance to the obtained scheduling scheme. According to the definition of the loss value, for every 2.5 working hours, the loss value of the equipment increases by 0.1, and the equipment health decreases by 10%. In the same layer, the operation is dynamically adjusted in the order of increasing the total cost value, as shown in Table 1. For A_{14} and A_{15} in layer five, for calculating the total cost, the number of the processed operation is 1 ($x = 1$). According to Formula (6), $\min(C_{A_{15}M_1}, C_{A_{14}M_1}) = C_{A_{15}M_1} = (2\beta\lambda + 2\beta)/(\lambda - \beta) + 1$ and $C_{A_{14}M_2} = C_{A_{14}M_4} = (18\beta\lambda + 6\beta)/(\lambda - \beta) + 1$, thereby A_{15} is scheduled on M_1 , and A_{14} is scheduled on M_2 according to the equipment priority. In a similar fashion, when scheduling A_{12} at layer four, the number of the processed operations is 3 ($x = 3$), $\min(C_{A_{12}M_1}, C_{A_{12}M_2}, C_{A_{12}M_4}) = C_{A_{12}M_4} = (54\beta\lambda + 18\beta)/(3\lambda - \beta) + 3$, thereby A_{12} is adjusted to M_4 , so that its successor operations $\{A_7, A_3\}$ has the advantage of early processing. Finally, the operation sequences and the corresponding sequences with maintenance at layer five and four are $\{A_{15}/M_1/2, A_{14}/M_2/6\}$ and $\{A_{11}/M_4/2, A_9/M_3/2, A_{12}/M_4/4, A_{13}/M_3/4, A_{10}/M_1/4\}$, respectively.

Table 1. Dynamic adjustment of operation sequence.

Adjustment Operation	Before		After	
	Machine	Completion Time	Machine	Completion Time
A_{12}	M_2	9	M_4	6
A_8	M_3	16	M_3	10
A_5	M_2	12	M_1	6
A_2	M_4	23	M_1	20

For layer three, the number of the processing operations is 8 ($x = 8$), and the least total cost of A_5 is $\min(C_{A_5M_1}, C_{A_5M_2}, C_{A_5M_4}) = C_{A_5M_1} = (144\beta\lambda + 48\beta)/(8\lambda - \beta) + 8$. A_5 is adjusted to M_1 , which is between A_{15} and A_{10} . According to the loss value, M_2 processes A_7 , and M_3 processes A_8 and A_6 . The operation sequence and the corresponding sequence with maintenance at layer three is $\{A_5/M_1/4, A_8/M_3/4, A_7/M_2/4, A_6/M_3/2\}$.

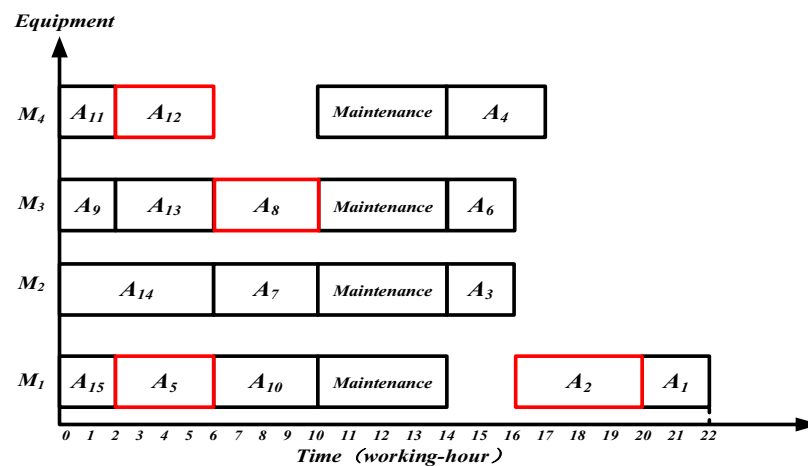
When the number of processing operations reaches 10, which is 2/3 of the total number of operations, the equipment starts the maintenance. At this time $t = 10$, the loss value of the equipment reaches 0.4, as shown in Table 2, so the equipment maintenance time is $0.4 t = 4$, and the processing continues at $t = 14$.

Table 2. Processing information of machines.

Processing Moment	The Number of Processed Operations	The Number of Processing Operations	The State of Equipment Health	Loss Value of Equipment
$t = 2.5$	3	4	90%	0.1
$t = 5$	3	4	80%	0.2
$t = 7.5$	7	3	70%	0.3
$t = 10$	10	0	60%	0.4

For layer two, the number of the processing operations is 12 ($x = 12$), and the least total cost of A_2 is $\min(C_{A_2M_1}, C_{A_2M_3}, C_{A_2M_4}) = C_{A_2M_1} = (2400\beta\lambda + 240\beta) / (12\lambda - \beta) + 12$. A_2 is adjusted to M_1 , and the operation sequence and the corresponding sequence with maintenance at this layer is $\{A_3/M_2/2, A_4/M_4/3, A_2/M_1/4\}$.

The result obtained by the proposed algorithm is 22 working hours, and the Gantt chart is shown in Figure 8.

**Figure 8.** The Gantt chart of the scheduling result by the proposed algorithm.

5. Experimental Analysis

In order to further clarify the effect of the proposed algorithm, two comparative tests are carried out in this section.

5.1. Comparative Analysis of the Dynamic Adjustment Strategy

The algorithm flow chart of the FIS-PM algorithm is shown in Figure 4. The specific steps are as follows:

The scheduling results considering dynamic adjustment and maintenance by the proposed algorithm FIS-PM are compared with the results obtained by the algorithm without the dynamic adjustment strategy and the maintenance (FIS-WDAPM).

As shown in Figure 7, the makespan of product A without considering the maintenance is 25 working hours, and the overall utilization rate of the equipment is 57%. As can be seen from Figure 8, the dynamic adjustment of the operation sequence not only enabled the equipment to realize the period maintenance during the processing and reduce the wear of the equipment, but also shortened the makespan by 3 working hours; the overall utilization rate of the equipment is 92%. As shown in Table 3, compared with the results in Figure 7, the utilization rates of M_1 , M_3 , and M_4 are increased by 40% from $t = 0$ to $t = 10$. Compared with the FIS-WDAPM algorithm, which does not consider the dynamic adjustment and the maintenance, the overall utilization of M_1 , M_3 , and M_4 are increased by 59%, 25%, and 37%, respectively.

Table 3. Equipment utilization.

Machine	Equipment Utilization without the Dynamic Adjustment and the Maintenance		Equipment Utilization by the Proposed Algorithm		Relative Increase Rate of the Overall Equipment Utilization
	$t = 0$ to $t = 10$	Overall Utilization of Equipment	$t = 0$ to $t = 10$	Overall Utilization of Equipment	
M_1	60%	32%	100%	91%	59%
M_2	100%	100%	100%	100%	-
M_3	60%	75%	100%	100%	25%
M_4	20%	39%	60%	76%	37%

5.2. Comparative Analysis of Data Sets

To verify the effect of the FIS-PM algorithm and its adaptability to the products with different tree-like structures, four groups of 40 products of different sizes and process constraint structures were randomly generated under the Matlab R2021b experimental environment. The number of the operations in the four sets of data is [20, 50, 100, 200], and each set has ten instances.

As shown in Figure 9, the optimal solution ratio of the FIS-PM algorithm is always higher than that of FIS-WDAPM, and it always maintains a high ratio. In Figure 10, the variance curve represents the variance of the processing time of the randomly generated processing tree, which reflects the stability of each algorithm. The smaller the variance curve fluctuation amplitude, the more stable the algorithm. The variance curve of the FIS-PM algorithm is always at a lower level and more stable, so the solution obtained by FIS-PM is closer to the optimal solution.

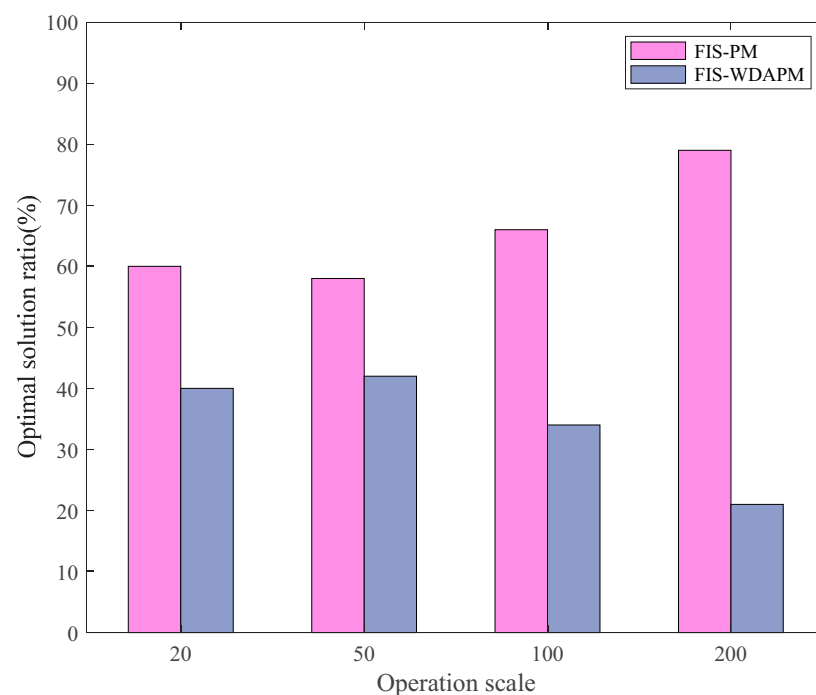


Figure 9. The optimal solution ratio of the two algorithms.

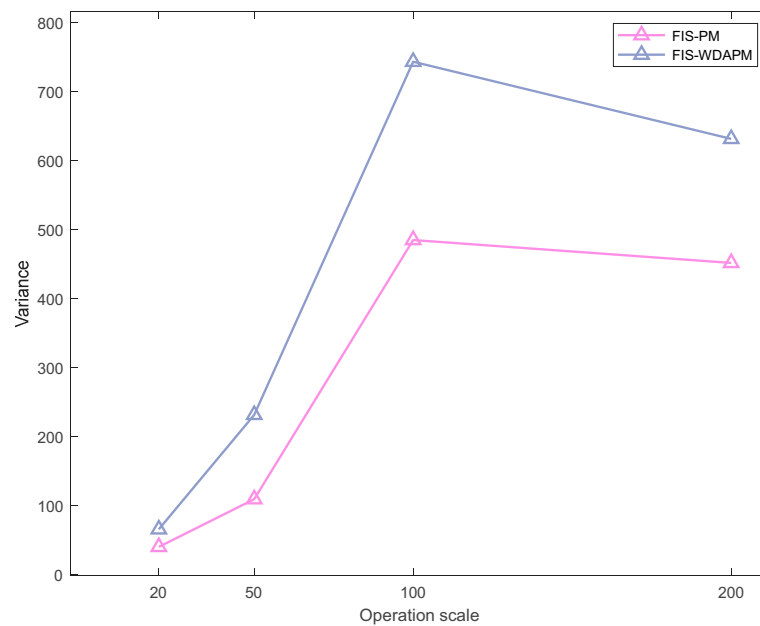


Figure 10. The variance curves of the two algorithms are compared.

5.3. The Advantage Analysis of the Proposed Algorithm

This paper introduces periodic maintenance activities into a flexible integrated scheduling system for the first time. The main advantages are as follows:

- (1) From the perspective of improving the compact scheduling optimization of processes, the classic strategy combination, the layer priority strategy with the short-time strategy, is adopted to effectively shorten the processing time of parallel operations. The dynamic adjustment strategy is proposed according to the total processing cost and significantly optimizes the processing efficiency, which effectively reduces the makespan of the complex product and further shortens the waiting time for operations. With M_1 in Figure 7, A_5 starts to be processed at $t = 2$, which is seven working hours earlier than in Figure 5. This makes the successor operations, A_2 and A_1 , also have the advantage of earlier processing, which is 3 working hours earlier.
- (2) From the perspective of optimizing the utilization rate of the equipment, the proposed algorithm not only improves the overall utilization rate of the equipment and reduces the makespan of the complex product, but also completes the equipment maintenance task. As shown in Figure 7: ① all the operations on M_1 , M_2 , and M_3 achieve seamless high-density scheduling, and the equipment utilization reaches 100% before the maintenance begins; ② the overall utilization rate of M_1 is increased from 32% to 91%, achieving a significant improvement. The machine sequence can process more operations in a short period of time, and achieve the optimization effect of tight machining of the operations on the same machine.

5.4. Analysis of Applicability and Potential Limitations

Based on the forest fire rescue model, this paper proposes a dynamic adjustment strategy to solve the periodic maintenance problem for flexible integrated scheduling. It is suitable for tree-type complex products with tight constraints, reduces production interruptions caused by temporary maintenance, and enables equipment to adapt to diverse production needs and state changes. In modern industrial production, the application of flexible integrated scheduling is increasing, especially in the field of automation and intelligent manufacturing. In process industries, such as iron and steel, chemical, and other heavy industries, the continuous operation of equipment is very important. The algorithm in this paper can balance the needs of production and maintenance, ensure the optimal operation state of equipment in the continuous production process, and avoid

large-scale production stagnation caused by equipment failure. For the electronics industry that requires high-precision and high-speed production, the algorithm can reduce the accuracy degradation caused by excessive operation of the equipment, while ensuring production efficiency. In industries such as multi-variety and small-batch mechanical equipment manufacturing, the algorithm in this paper can adapt to diversified production tasks, optimize the production mode of small-batch and multi-variety processes, and improve the flexibility of production.

In addition, the algorithm also has some potential limitations. For example, in some specific heavy industry applications, the equipment operating environment is complex and the influencing factors are numerous, which may affect the accuracy and practicability of the algorithm.

In short, the algorithm in this paper has high applicability in actual production, which can effectively improve production efficiency and reduce costs. However, its applicability and effectiveness may vary according to different industries and production scales. Therefore, this algorithm can be customized according to its own actual situation when it is used to ensure that the algorithm can maximize its effectiveness.

6. Conclusions

In order to solve the problem of the equipment loss during processing and thus affect the production efficiency, we propose an algorithm considering periodic maintenance of the equipment for the first time in the flexible integrated scheduling of the complex products with multi-varieties and small-batches. By dynamically adjusting the scheduling operation sequence, the equipment utilization rate is improved and periodic maintenance is realized within the original planned makespan. The results show that:

- (1) The proposed dynamic adjustment strategy significantly improves the overall utilization rate of the equipment and realizes the compact operation processing. Compared with non-dynamic adjustment and non-maintenance, the overall equipment utilization rate of the proposed algorithm is improved by 35%.
- (2) In this paper, the loss value and maintenance time are calculated together, which effectively improves the ability of close processing of the operations, reduces the makespan of the product by 12%, and completes the maintenance activities that account for 18% of the total processing time during the processing.
- (3) In this paper, the strategy combination of the layer priority, the short-time, the equipment priority, and the dynamic adjustment strategies is comprehensively applied to realize scheduling optimization of the complex products in vertical and horizontal directions, and periodic maintenance is realized in the flexible integrated scheduling.

Compared with Ref. [11], the algorithm in this paper introduces equipment maintenance activities into flexible integrated scheduling, so that the process can flexibly select equipment during processing. Compared with Refs. [23,27,28], this paper shortens the total processing time of products and improves the utilization rate of equipment. On the basis of completing the regular maintenance activities of equipment in flexible integrated scheduling, equipment failure caused by overload operation is avoided. The proposed algorithm has the ability to optimize production and maintenance plans in the current industrial environment and lays a foundation for periodic maintenance in future studies. In the actual production process, it is also necessary to consider factors such as equipment status and production requirements, ensuring the efficient operation of the algorithm is a challenge in further research. In particular, the integration of the algorithm with the existing industrial automation system may face technical and compatibility challenges. In future research, we can deeply explore more intelligent and dynamic periodic maintenance strategies for flexible integrated scheduling equipment. Combined with the real-time operation status monitoring data of the equipment, the machine learning algorithm is used to predict the potential failure points and the best maintenance time of the equipment, so as to achieve more accurate maintenance arrangements and reduce production interruptions caused by unnecessary maintenance or untimely maintenance. We can also focus on expanding the

scope of application of the algorithm, so that it can adapt to production scheduling systems of different scales and different industries. In the next step, research can be extended to the design and implementation of regular maintenance of complex distributed manufacturing scheduling systems with collaborative computing capabilities, and a more flexible adaptive scheduling mechanism can be developed to adjust according to real-time production conditions and equipment status, or the Internet of Things can be used to collect real-time data to maximize production efficiency and equipment utilization.

Author Contributions: Conceptualization, X.D. and W.Z.; methodology, Z.X.; software, W.Z.; data curation, Z.T.; writing—original draft preparation, X.D.; writing—review and editing, Z.X. and M.S. All authors have read and agreed to the published version of the manuscript.

Funding: This work was partially supported by the National Natural Science Foundation of China (61772160, Author: Z.X.); National Social Science Foundation of China (20BZS058, Author: M.S.); the subject of National College Computer Education Research Association (CERACU2024R13, Author: W.Z.); Jilin Normal University Doctoral Program (2022014, Author: W.Z.); the teaching research topic of Jilin Normal University: (Jilin Normal University Office [2023] 107 (No. 2418 in total), Author: W.Z.); Jilin Normal University Graduate Research Innovation Program Project (201951, Author: Z.T.).

Data Availability Statement: Data are contained within the article.

Conflicts of Interest: The authors declare no conflicts of interest.

References

1. Han, J. Influencing Factors of Mechanical Wear and Routine Maintenance. *Mod. Manuf. Technol. Equip.* **2019**, *12*, 159–161. [\[CrossRef\]](#)
2. Pang, S.; Jia, Y.X.; Liu, X.; Deng, Y.C. Study on simulation modeling and evaluation of equipment maintenance. *J. Shanghai Jiaotong Univ. (Sci.)* **2016**, *21*, 594–599. [\[CrossRef\]](#)
3. Jiang, Z.G.; Dong, M. Study on Single-machine Problem with Maintenance and Variable Processing Time. *Ind. Eng. Manag.* **2011**, *16*, 68–74. [\[CrossRef\]](#)
4. Lou, P.; Yu, D.; Jiang, X.M.; Hu, J.M.; Zeng, Y.H. Knowledge Graph Construction Based on a Joint Model for Equipment Maintenance. *Mathematics* **2023**, *11*, 3748. [\[CrossRef\]](#)
5. Song, W.J.; Zhang, C.Y.; Yin, Y.; Shao, X.Y. Integrated Optimization of Equipment Maintenance and Shop Scheduling Problem Based on Multi-objective Hybrid Imperialist Competitive Algorithm. *China Mech. Eng.* **2015**, *26*, 1478–1487. [\[CrossRef\]](#)
6. Ashary, A.; Rayguru, M.M.; SharafianArdakani, P.; Kondaurava, I.; Popa, D.O. Multi-Joint Adaptive Motion Imitation in Robot-Assisted Physiotherapy with Dynamic Time Warping and Recurrent Neural Networks. In Proceedings of the SoutheastCon 2024, Atlanta, GA, USA, 15–24 March 2024; IEEE: New York, NY, USA, 2024; pp. 1388–1394. [\[CrossRef\]](#)
7. Ashary, A.; Rayguru, M.M.; Dowdy, J.; Taghavi, N.; Popa, D.O. Adaptive Motion Imitation for Robot Assisted Physiotherapy Using Dynamic Time Warping and Recurrent Neural Network. In Proceedings of the 17th International Conference on Pervasive Technologies Related to Assistive Environments, Crete, Greece, 26–28 June 2024; pp. 55–62. [\[CrossRef\]](#)
8. Sheen, G.; Liao, L.; Lin, C. Optimal parallel machines scheduling with machine availability and eligibility constraints. *Int. J. Adv. Manuf. Technol.* **2008**, *36*, 132–139. [\[CrossRef\]](#)
9. Li, D.W.; Zhang, Z.H.; Liu, T.H.; Zhong, Q.H. Product Maintenance Strategy Research Based on Scheduled Servicing. *Acta Aeronaut. Astr.-Onautica Sin.* **2013**, *34*, 1101–1107. [\[CrossRef\]](#)
10. Bingchan, L.; Mao, B.; Cao, J. Maintenance and Management of Marine Communication and Navigation Equipment Based on Virtual Reality. *Procedia Comput. Sci.* **2018**, *139*, 221–226. [\[CrossRef\]](#)
11. Xie, Z.Q.; Zhou, W.; YU, Z.R. Integrated Scheduling Algorithm for Dynamic Adjustment of Equipment Maintenance Start Time. *J. Mech. Eng.* **2021**, *57*, 240–246.
12. Zhang, G.P. Research on predictive maintenance system of mechanical equipment based on mega data. *Mach. China* **2023**, *36*, 61–64+69.
13. Sun, K.; Li, H. Scheduling problems with multiple maintenance activities and non-preemptive jobs on two identical parallel machines. *Int. J. Prod. Econ.* **2010**, *124*, 151–158. [\[CrossRef\]](#)
14. Xie, Z.Q. Study on Operation Scheduling of Complex Product with Constraint among Jobs. Ph.D. Thesis, Harbin University of Science and Technology, Harbin, China, 2009.
15. Xie, Z.Q.; Zhou, W.; Yang, J. Resource Cooperative Integrated Scheduling Algorithm Based on Sub-tree Cycle Decomposition of Process Tree. *J. Mech. Eng.* **2022**, *58*, 228–239. [\[CrossRef\]](#)
16. Xie, Z.Q.; Yang, D.; Ma, M.R.; Yu, X. An improved artificial bee colony algorithm for the flexible integrated scheduling problem using networked devices collaboration. *Int. J. Coop. Inf. Syst.* **2020**, *29*, 2040003. [\[CrossRef\]](#)

17. Xie, Z.Q.; Yang, G.; Tan, G.Y. An algorithm of JSSP with dynamic collection of job with priority. In Proceedings of the International Technology and Innovation Conference 2006-Advanced Manufacturing Technologies, Hangzhou, China, 6–7 November 2006; pp. 106–111.
18. Xie, Z.Q.; Hao, S.Z.; Ye, G.J.; Tan, G.Y. A new algorithm for complex product flexible scheduling with constraint between jobs. *Comput. Ind. Eng.* **2009**, *57*, 766–772. [[CrossRef](#)]
19. Zhou, W.; Xie, Z.Q. Resource Collaborative Integrated Scheduling Algorithm Considering Multi-process Equipment Weight. *J. Electron. Inf. Technol.* **2022**, *44*, 1625–1635. [[CrossRef](#)]
20. Xie, Z.Q.; Wang, Q. Flexible Integrated Scheduling Algorithm Based on Reverse Order Layer Priority. *J. Electron. Inf. Technol.* **2022**, *44*, 1554–1562. [[CrossRef](#)]
21. Teng, K.; Xie, Z.Q.; Yue, X.G. A Flexible Integrated Scheduling Algorithm for Dynamic Determination Processing Equipment Considering the Root-Subtree Vertical and Horizontal Pre-Scheduling. *IEEE Access* **2022**, *10*, 132298–132314. [[CrossRef](#)]
22. Zhou, W.; Zhou, P.W.; Zheng, Y.; Xie, Z.Q. A Heuristic Integrated Scheduling Algorithm via Processing Characteristics of Various Machines. *Symmetry* **2022**, *14*, 2150–2170. [[CrossRef](#)]
23. Gui, Z.Y.; Yang, J.; Xie, Z.Q. Scheduling algorithm for flexible job shop based on pruning and layering. *Control Decis.* **2017**, *32*, 1921–1932. [[CrossRef](#)]
24. Xie, Z.Q.; Zhou, W.; Yang, J. Resource cooperative integrated scheduling algorithm considering hierarchical scheduling order. *Comput. Integr. Manuf. Syst.* **2022**, *28*, 3391–3402. [[CrossRef](#)]
25. Zhou, P.W.; Xie, Z.Q.; Zhou, W.; Tan, Z.J. A Heuristic Integrated Scheduling Algorithm Based on Improved Dijkstra Algorithm. *Electronics* **2023**, *12*, 4189. [[CrossRef](#)]
26. Zhou, W.; Zhou, P.W.; Yang, D.; Cao, W.C.; Tan, Z.J. Symmetric Two-Workshop Heuristic Integrated Scheduling Algorithm Based on Process Tree Cyclic Decomposition. *Electronics* **2023**, *12*, 1553. [[CrossRef](#)]
27. Xie, Z.Q.; Zhou, H.X.; Yu, J.; Gui, Z.Y. Conflict Mediation Algorithm of the Integrated Flexible Scheduling Based on Device Driver. *Trans. Beijing Inst. Technol.* **2014**, *34*, 1150–1156. [[CrossRef](#)]
28. Zhou, W.; Ding, X.Y.; Xie, Z.Q. An Integrated Scheduling Algorithm Considering the Processing Capacity of Flexible Equipment. *J. South China Norm. Univ. (Nat. Sci. Ed.)* **2024**, *56*, 110–118. [[CrossRef](#)]

Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.