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Failure-Distribution-Dependent H_∞ Fuzzy Fault-Tolerant Control for Nonlinear Multilateral Teleoperation System with Communication Delays

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Abstract: In practice, the time-varying communication delays and actuator failures are the main inevitable issues in nonlinear multilateral teleoperation systems, which can reduce the performance and stability of the considered systems. This article proposed a novel failure-distribution-dependent H_∞ fuzzy fault-tolerant control scheme to realize position synchronization and force tracking simultaneously for multilateral teleoperation systems. Firstly, the nonlinear multilateral systems were modeled as a kind of T-S fuzzy systems with multiple time-varying delays. Then, based on the distribution characteristic of failures, by introducing a series of tradeoff coefficients, a novel failure-distribution-dependent fault-tolerant control algorithm was provided to ensure force tracking in spite of failures, and the purpose of position synchronization was achieved (not only the master and slave robot position synchronization but also the position synchronization between each slave robot). Finally, a numerical simulation example was given to show the effectiveness of the proposed method.

Keywords: fault-tolerant control; H_∞ performance; T-S fuzzy system; nonlinear multilateral teleoperation system; multiple communication delays



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1. Introduction

Generally, a teleoperation system consists of at least one master manipulator controlled by the operator and a slave manipulator that remotely mimics the maneuvers of the master robot. When there is a teleoperation system with a master robot and a slave robot, a bilateral information flow is established between the two sites, thus forming a bilateral teleoperation system [1]. With the upgrading of communication networks and the progress of technical means, the bilateral teleoperation system is being extended to a multilateral teleoperation system. Compared with bilateral teleoperation, which can only be performed by a single person remotely, multilateral teleoperation has attracted increasing attention in recent years because of its advantage of multi-person cooperation to better accomplish more complex tasks. Similar to bilateral teleoperation, multilateral teleoperation also requires force perceived from the slave site to be fed back to the master site so that there can be actual kinesthetic coupling between the operator and the environment, which is better than traditional ocular or audio information [2,3]. In practice, the multi-master and multi-slave multilateral teleoperation system is more complex than the traditional single-master and single-slave bilateral teleoperation system, and it is more prone to problems such as multi-communication delay and partial failure of actuators, which brings great challenges to research. Odd-come-shortly, some effective research schemes have been expended for multilateral teleoperation systems, such as fuzzy control [4], finite-time control [5,6], adaptive control [7], sliding mode control [8], robust control [9], backstepping control [10], adaptive robust control [11,12], and so on. However, most recent research results have

been obtained without considering any faults, and the problems of partial actuator failure and fault-tolerant control of time-varying delays in multi-degree-of-freedom nonlinear teleoperation systems have not yet been fully investigated. In addition, most studies are limited to the tracking of operating forces while ignoring the telepresence provided by environmental forces to operators. These are the points that motivated the research carried out for this paper.

Among the many methods to solve the failure problem, the fault-tolerant control (FTC) approach is an effective means. In current mainstream research, this method is roughly divided into two branches: active fault-tolerant control (AFTC) [13] and passive fault-tolerant control (PFTC) [14]. The PFTC algorithm is obtained by considering all the fault modes and the normal running state of the system in the design phase [15]. Ref. [16] proposes a novel fault alarm based on a hybrid fault-tolerant control scheme, and the corresponding design system meets the performance requirement of being hybrid passive/ H_∞ . A novel packet-dropout-probability-dependent PFTC approach is presented in [17] for nonlinear networked control systems. In [18], a fresh H_∞ performance index with a summation inequation and S-procedure is proposed for an uncertain discrete-time piecewise-affine (PWA) system with communication delay and actuator failure. For a type of uncertain discrete-time T-S fuzzy systems with actuator saturation and infinite distributed delays, a PFTC algorithm is presented in [19]. But then, the AFTC method provides fault information to the corresponding controller through an adaptive algorithm or fault detection, isolation, and analysis mechanism, so that the controller can be reconfigured [20,21]. For example, ref. [22] contrived an AFTC to solve the attitude control of spacecraft with actuator failures, model uncertainties, and evaluation errors. For the conundrum of bi-directional uncertainty and fault estimation, a new hybrid fault estimation and fault-tolerant control algorithm was proposed in [23]. In [24], a new synthesis method (including a collaborative AFTC and a high-gain observer like-protocol) was provided to overcome multiple types of topology and actuator failures. In fact, the occurrence of system failure often comes from the gradual seriousness of failures, and the characteristic of actuator failure of a nonlinear multilateral teleoperation system is seldom discussed in the existing research, and this is the third point that motivated this research article.

The main contributions of this article are as follows:

- (a) The investigated nonlinear dynamics of teleoperation systems with multiple masters and multiple slaves are modeled as a type of T-S fuzzy system with multiple communication delays.
- (b) Inspired by the fact that the probability of minor failure exists in a way that is often greater than that of serious failure in theory, a novel failure distribution model is established to reflect the distribution characteristic of failures.
- (c) There may be actuator failure in multi-master and multi-slave robots, and a series of failure-distribution-dependent H_∞ fuzzy fault-tolerant controllers are designed, respectively, to ensure force tracking and positional synchronization (master–slave and slave–slave) simultaneously.

The rest of this article is divided into the following parts: the model of a nonlinear multilateral teleoperation system is established in Section 2. In Section 3, a fault-tolerant H_∞ controller is designed for the considered nonlinear multilateral teleoperation system. In Section 4, the effectiveness of the designed controller is verified through a numerical simulation. Finally, the conclusion of this article is summarized.

2. System Modeling

Throughout the article, the following notations are defined: $\mathbb{R}^n$ denotes the n -dimensional Euclidean space, while $\mathbb{R}^{n \times m}$ is the real matrix of all $n \times m$. $\text{diag}\{\dots\}$ represents a block-diagonal matrix, and $'*'$ is a symmetric term in a symmetric matrix. $\mathbb{E}\{\cdot\}$ stands for the mathematical expectation, and A^T denotes the transpose of the matrix A . For a square matrix X and for subsequent simplicity, let us denote $\mathbf{He}(X) = X + X^T$.

The model of a multilateral teleoperation system can be illustrated as [25]

$$\begin{aligned} M_{mi}(q_{mi})\ddot{q}_{mi} + C_{mi}(q_{mi}, \dot{q}_{mi})\dot{q}_{mi} + G_{mi}(q_{mi}) &= F_{mi} + F_{hi} \\ M_{si}(q_{si})\ddot{q}_{si} + C_{si}(q_{si}, \dot{q}_{si})\dot{q}_{si} + G_{si}(q_{si}) &= F_{si} - F_{ei} \end{aligned} \quad (1)$$

where the subscripts m and s stand for the master manipulator and the slave manipulator, respectively. For brevity, one can denote $j = \{m, s\}$ and $i = \{1, \dots, N\}$. q_{ji} , $\dot{q}_{ji}$, and $\ddot{q}_{ji}$ represent joint displacement, joint velocity, and joint acceleration vectors, respectively. $M_{ji}(q_{ji})$ represents mass inertia matrices. $C_{ji}(q_{ji}, \dot{q}_{ji})$ describes coriolis and centripetal matrices. $G_{ji}(q_{ji})$ represents gravitational torques. F_{ji} denotes control torques. F_{hi} and F_{ei} represent the human operator force and environmental force, respectively. The control torques with gravity compensation are designed as $F_{ji} = u_{ji}^F + G_{ji}(q_{ji})$, where u_{ji}^F is the controller output corresponding to the master–slave robot pair. The structure diagram of multilateral teleoperation system is shown in Figure 1.

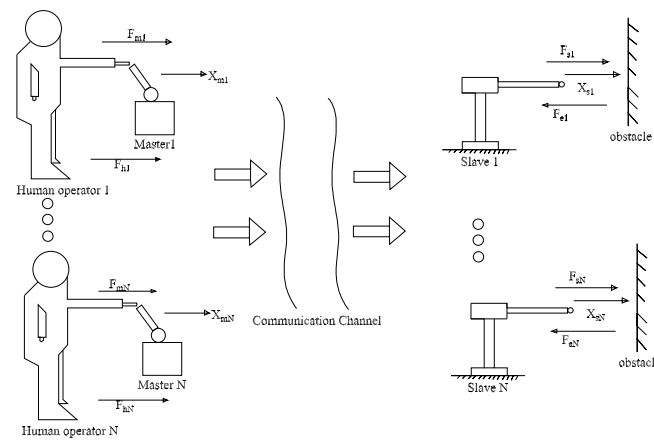


Figure 1. The structure diagram of the multilateral teleoperation system.

Substitute F_{ji} into System (1), so that System (1) can be re-represented as

$$\begin{aligned} M_{mi}(q_{mi})\ddot{q}_{mi} + C_{mi}(q_{mi}, \dot{q}_{mi})\dot{q}_{mi} &= u_{mi}^F + F_{hi} \\ M_{si}(q_{si})\ddot{q}_{si} + C_{si}(q_{si}, \dot{q}_{si})\dot{q}_{si} &= u_{si}^F - F_{ei} \end{aligned} \quad (2)$$

For simplicity, consider that the multilateral teleoperation system consists of N master robots and N slave robots, both of which have two degrees of freedom. The configuration of a two-link robot is displayed in Figure 2, where m_{jia} ($a = 1, 2$) represents the link mass, and $j = \{m, s\}$, $i = \{1, \dots, N\}$. l_{in} ($n = 1, 2$) denotes the link lengths [26].

As in [4,27,28], we define $x_{ji1} = q_{ji1}$, $x_{ji2} = q_{ji2}$, $x_{ji3} = \dot{q}_{ji1}$ and $x_{ji4} = \dot{q}_{ji2}$. Then, System (2) can be described as

$$\begin{aligned} \dot{x}_{mi}(t) &= f(x_{mi}) + \beta(x_{mi})(u_{mi}^F + F_{hi}) \\ \dot{x}_{si}(t) &= f(x_{si}) + \beta(x_{si})(u_{si}^F - F_{ei}) \end{aligned} \quad (3)$$

where

$$f(x_{ji}) = \begin{bmatrix} x_{ji3} \\ x_{ji4} \\ f_{i1}(x_{ji}) \\ f_{i2}(x_{ji}) \end{bmatrix}, \quad \beta(x_{ji}) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ \beta_{i1}(x_{ji}) & \beta_{i2}(x_{ji}) \\ \beta_{i2}(x_{ji}) & \beta_{i3}(x_{ji}) \end{bmatrix},$$

and

$$x_{ji} = [x_{ji1}^T \quad x_{ji2}^T \quad x_{ji3}^T \quad x_{ji4}^T]^T, \quad u_{ji}^F = \begin{bmatrix} u_{ji1}^F \\ u_{ji2}^F \end{bmatrix},$$

$$\begin{aligned}
f_{i1}(x_{ji}) &= \frac{\tilde{\delta}_{i1} [m_{ji2} l_{i1} l_{i2} \tilde{\delta}_{i2} x_{ji3}^2 - m_{ji2} l_{i2}^2 x_{ji4}^2]}{l_{i1} l_{i2} [(m_{ji1} + m_{ji2}) - m_{ji2} \tilde{\delta}_{i2}^2]}, \\
f_{i2}(x_{ji}) &= \frac{\tilde{\delta}_{i1} [-(m_{ji1} + m_{ji2}) l_{i1}^2 x_{ji3}^2 + m_{ji2} l_{i1} l_{i2} \tilde{\delta}_{i2} x_{ji4}^2]}{l_{i1} l_{i2} [(m_{ji1} + m_{ji2}) - m_{ji2} \tilde{\delta}_{i2}^2]}, \\
\beta_{i1}(x_{ji}) &= \frac{m_{ji2} l_{i2}^2}{m_{ji2} l_{i1}^2 l_{i2}^2 [(m_{ji1} + m_{ji2}) - m_{ji2} \tilde{\delta}_{i2}^2]}, \\
\beta_{i2}(x_{ji}) &= \frac{-m_{ji2} l_{i1} l_{i2} \tilde{\delta}_{i2}}{m_{ji2} l_{i1}^2 l_{i2}^2 [(m_{ji1} + m_{ji2}) - m_{ji2} \tilde{\delta}_{i2}^2]}, \\
\beta_{i3}(x_{ji}) &= \frac{(m_{ji1} + m_{ji2}) l_{i1}^2}{m_{ji2} l_{i1}^2 l_{i2}^2 [(m_{ji1} + m_{ji2}) - m_{ji2} \tilde{\delta}_{i2}^2]}, \\
\tilde{\delta}_{i1} &= \sin(x_{ji1}) \cos(x_{ji2}) - \cos(x_{ji1}) \sin(x_{ji2}), \\
\tilde{\delta}_{i2} &= \sin(x_{ji1}) \sin(x_{ji2}) + \cos(x_{ji1}) \cos(x_{ji2}).
\end{aligned}$$

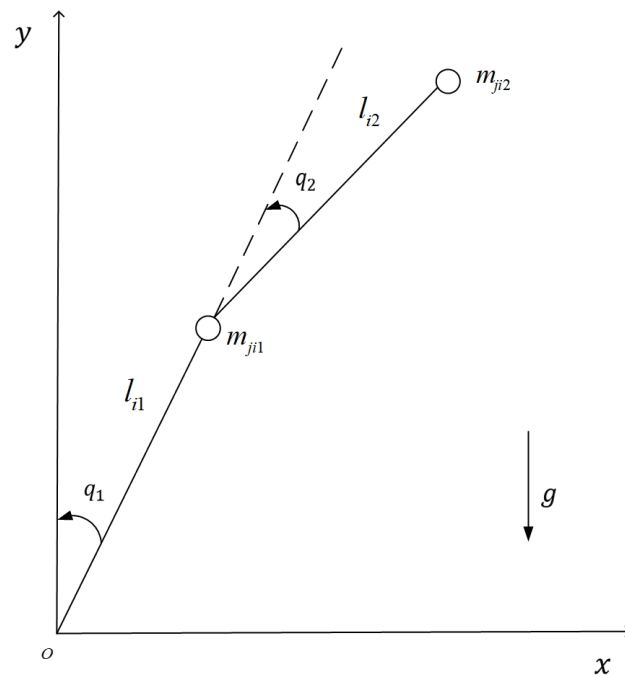


Figure 2. The configuration of a two-link robot.

In practice, partial actuator failure is one of the important factors leading to system instability. As in [17,29], the application of fault-tolerant control is an appropriate way to solve partial actuator failures. The controller is designed as $u^F(t) = Mu(t)$, where

$$M = \text{diag}\{m_1, m_2, \dots, m_p\}$$

denotes the partial actuator failure coefficients, which satisfy

$$0 < m_v^{\min} \leq m_v \leq m_v^{\max} \leq 1, \quad v = 1, 2, \dots, p.$$

There are two cases: when $0 < m_v < 1$, it means that the i th actuator has partial failure; when $m_v = 1$, the i th actuator is operating normally. It is worth noting that in the above failure description, the failure distribution is the same. However, in practice, the distribution of failures is different, and minor failures are always more probably to occur than serious failures. In other words, select an appropriate scalar $\tilde{m}_v$ in the interval $[m_v^{\min}, m_v^{\max}]$ to divide the interval into two subintervals as $[m_v^{\min}, \tilde{m}_v)$, $[\tilde{m}_v, m_v^{\max}]$. From

the failure features, majority of the actuator failures fall under the interval $[\tilde{m}_v, m_v^{max}]$ as minor failures. The serious failure occurs at a low probability, which belongs to the interval $[\tilde{m}_v^{min}, \tilde{m}_v]$. Due to the characteristic of partial actuator failures, the failure matrix can be redefined as

$$\hat{M} = \text{diag}\{\theta_1(k)m_1 + (1 - \theta_1(k))\tilde{m}_1, \dots, \theta_p(k)m_p + (1 - \theta_p(k))\tilde{m}_p\}, \quad (4)$$

where

$$\begin{aligned} m_v^{min} \leq m_v \leq \tilde{m}_v, \quad \theta_v(k) = 1 \\ \tilde{m}_v \leq \tilde{m}_v \leq m_v^{max}, \quad \theta_v(k) = 0, \quad v = 1, 2, \dots, p. \end{aligned}$$

$\theta_v(k) (v = 1, \dots, p)$ belongs to the set $[0, 1]$ and is a random variable. In this research, the distribution probability of $\theta_v(k)$ is defined as $\text{Prob}\{\theta_v(k) = 1\} = \tilde{\theta}_v$, and it obeys the Bernoulli distribution. According to the novel failure matrix with failure distributions (4) of the above design, the fault-tolerant controllers can be defined as

$$u_{mi}^F(t) = \hat{M}_{mi}u_{mi}(t), \quad u_{si}^F(t) = \hat{M}_{si}u_{si}(t), \quad (5)$$

where

$$\begin{aligned} \hat{M}_{mi} &= \text{diag}\{\theta_{1mi}(k)m_{1mi} + (1 - \theta_{1mi}(k))\tilde{m}_{1mi}, \\ &\quad \dots, \theta_{pmi}(k)m_{pmi} + (1 - \theta_{pmi}(k))\tilde{m}_{pmi}\}, \\ \hat{M}_{si} &= \text{diag}\{\theta_{1si}(k)m_{1si} + (1 - \theta_{1si}(k))\tilde{m}_{1si}, \\ &\quad \dots, \theta_{psi}(k)m_{psi} + (1 - \theta_{psi}(k))\tilde{m}_{psi}\} \end{aligned}$$

In this paper, $u_{mi}(t)$ and $u_{si}(t)$ are the nominal controllers to be contrived. Obviously, due to System (3) being a kind of nonlinear system with actuator failure, it is difficult to design the corresponding fault-tolerant controller directly for such a system. In order to solve this problem, the nonlinear system (System 3) is modeled as a Takagi–Sugeno (T-S) fuzzy system, and some fuzzy IF-THEN rules can be used to describe it. Among them, the fuzzy rules (a^{th}) regarding the master robot can be written as

$$\begin{aligned} \text{IF } y_{i1}(t) \text{ is } \Psi_{ia1} \text{ and } \dots \text{ and } y_{ip}(t) \text{ is } \Psi_{iap}, \\ \text{THEN } \dot{x}_{mi}(t) = A_{mia}x_{mi}(t) + B_{mia}(u_{mi}^F(t) + F_{hi}(t)), \\ a = 1, 2, 3, \dots, L, \end{aligned}$$

where Ψ_{iab} denotes the fuzzy set, L represents the number of IF-THEN rules, and $y_{i1}(t), \dots, y_{ip}(t)$ are the premise variables. From this, the fuzzy system of the master manipulator can be deduced as follows:

$$\dot{x}_{mi}(t) = \frac{\sum_{a=1}^L \mu_{ia}(y_i(t)) [A_{mia}x_{mi}(t) + B_{mia}(u_{mi}^F(t) + F_{hi}(t))]}{\sum_{a=1}^L \mu_{ia}(y_i(t))}, \quad (6)$$

where

$$\mu_{ia}(y_i(t)) = \prod_{b=1}^p \Psi_{iab}(y_i(t)), \quad y_i(t) = [y_{i1}(t) \ y_{i2}(t) \ \dots \ y_{ip}(t)].$$

We assume that $\mu_{ia}(y_i(t)) \geq 0$ and $\sum_{a=1}^L \mu_{ia}(y_i(t)) \geq 0$. Denoting $h_{ia}(y_i(t)) = \mu_{ia}(y_i(t)) / \sum_{a=1}^L \mu_{ia}(y_i(t))$, it is obvious to infer that $h_{ia}(y_i(t)) \geq 0$ for $a = 1, 2, \dots, L$ and $\sum_{a=1}^L h_{ia}(y_i(t)) = 1$. Thus, the model of the master manipulator can be depicted as

$$\begin{aligned} \dot{x}_{mi}(t) &= \sum_{a=1}^L h_{ia}(y_i(t)) \\ &\quad \times [A_{mia}x_{mi}(t) + B_{mia}(u_{mi}^F(t) + F_{hi}(t))]. \end{aligned} \quad (7)$$

Similarly, the slave manipulator can be modeled as

$$\dot{x}_{si}(t) = \sum_{a=1}^L h_{ia}(\tilde{y}_i(t)) \times [A_{sia}x_{si}(t) + B_{sia}(u_{si}^F(t) - F_{ei}(t))]. \quad (8)$$

For the fuzzy model of the master manipulator (7), select one of the following fuzzy controllers, as follows:

Control Rule k :

IF $y_{i1}(t)$ is Ψ_{ik1} and ... and $y_{ip}(t)$ is Ψ_{ikp} ,
THEN

$$u_{mi}(t) = K_{mik} \left[x_{mi}(t) - \sum_{v=1}^N \beta_{vi} x_{svi}(t - d_{svi}(t)) \right] - F_{ei}(t - d_{sii}(t))$$

where $k = 1, 2, \dots, L$. Then, the nominal fuzzy logic controller for the master manipulator (7) is obtained as

$$u_{mi}(t) = \sum_{k=1}^L h_{ik}(y_i(t)) K_{mik} \times \left[x_{mi}(t) - \sum_{v=1}^N \beta_{vi} x_{svi}(t - d_{svi}(t)) \right] - F_{ei}(t - d_{sii}(t)) \quad (9)$$

Similarly, for slave manipulator (8), the nominal fuzzy controller is given as

$$u_{si}(t) = \sum_{k=1}^L h_{ik}(\tilde{y}_i(t)) K_{sik} (x_{si}(t) - x_{mi}(t - d_{mi}(t))) + F_{hi}(t - d_{mi}(t)) \quad (10)$$

Remark 1. Here, the position tracking errors between the master manipulators and slave manipulators are defined as

$$e_{mi}(t) = x_{mi}(t) - \sum_{v=1}^N \beta_{vi} x_{svi}(t - d_{svi}(t))$$

$$e_{si}(t) = x_{si}(t) - x_{mi}(t - d_{mi}(t))$$

where β_{vi} are the dominance factors having values in $(0, 1)$ (see [30]), and $\sum_{v=1}^N \beta_{vi} = 1$. The position synchronization of the masters and slaves could be achieved, i.e., $\lim_{t \rightarrow 0} \|e_{mi}(t)\| = 0$ and $\lim_{t \rightarrow 0} \|e_{si}(t)\| = 0$.

Assumption 1. We assume that $d_{mi}(t)$ denotes the delay from master i to slave i , and $d_{svi}(t)$ denotes the delay from slave v to master i . Here, it is assumed that the communication delay of each pair of master–slave manipulators is asymmetrically and continuously differentiable functions, satisfying

$$(i) 0 \leq d_{mi}(t) \leq \hat{d}_{mi} \quad (ii) 0 \leq d_{svi}(t) \leq \hat{d}_{si}$$

in which $\dot{d}_{mi}(t) \leq \mu < 1$, $\dot{d}_{svi}(t) \leq \mu < 1$. Additionally, μ , $\hat{d}_{mi}$, and $\hat{d}_{si}$ are positive scalars.

Substituting Equations (5) and (9) into Equation (7), it follows that

$$\begin{aligned} \dot{x}_{mi}(t) = & \sum_{a=1}^L \sum_{k=1}^L h_{ia}(y_i(t)) h_{ik}(y_i(t)) \\ & \times [(A_{mia} + B_{mia} \hat{M}_{mi} K_{mik}) x_{mi}(t) \\ & - \sum_{v=1}^N \beta_{vi} B_{mia} \hat{M}_{mi} K_{mik} x_{svi}(t - d_{svi}(t)) \\ & - B_{mia} \hat{M}_{mi} F_{ei}(t - d_{si}(t)) + B_{mia} F_{hi}(t)]. \end{aligned} \quad (11)$$

Similarly, the following results can be derived from Equations (5), (8), and (10):

$$\begin{aligned} \dot{x}_{si}(t) = & \sum_{a=1}^L \sum_{k=1}^L h_{ia}(\tilde{y}_i(t)) h_{ik}(\tilde{y}_i(t)) \\ & \times [(A_{sia} + B_{sia} \hat{M}_{si} K_{sik}) x_{si}(t) \\ & - B_{sia} \hat{M}_{si} K_{sik} x_{mi}(t - d_{mi}(t)) \\ & - B_{sia} F_{ei}(t) + B_{sia} \hat{M}_{si} F_{hi}(t - d_{mi}(t))]. \end{aligned} \quad (12)$$

Assuming that all the premise variables of the master manipulators and the slave manipulators are the same, one can obtain

$$\begin{aligned} \dot{\hat{x}}_i(t) = & \sum_{a=1}^L \sum_{k=1}^L h_{ia}(y_i(t)) h_{ik}(y_i(t)) \{ \hat{A}_{iak} \hat{x}_i(t) \\ & + \sum_{v=1}^N \hat{A}_{1iakv} \hat{x}_i(t - d_{svi}(t)) + \hat{A}_{2iak} \hat{x}_i(t - d_{mi}(t)) \\ & + \hat{E}_{ia} \hat{\omega}(t) \}, \end{aligned} \quad (13)$$

where

$$\begin{aligned} \hat{A}_{iak} = & \begin{bmatrix} A_{mia} + B_{mia} \hat{M}_{mi} K_{mik} & 0 \\ 0 & A_{sia} + B_{sia} \hat{M}_{si} K_{sik} \end{bmatrix}, \\ \hat{x}_i(t) = & \begin{bmatrix} x_{mi}(t) \\ x_{si}(t) \end{bmatrix}, \hat{A}_{1iakv} = \begin{bmatrix} 0 & 0 \\ 0 & -\beta_{vi} B_{mia} \hat{M}_{mi} K_{mik} \end{bmatrix}, \\ \hat{A}_{2iak} = & \begin{bmatrix} -B_{sia} \hat{M}_{si} K_{sik} & 0 \\ 0 & 0 \end{bmatrix}, \hat{E}_{ia} = \begin{bmatrix} B_{mia} & 0 \\ 0 & B_{sia} \end{bmatrix}, \\ \hat{\omega}(t) = & \begin{bmatrix} F_{hi}(t) - \hat{M}_{mi} F_{ei}(t - d_{si}(t)) \\ \hat{M}_{si} F_{hi}(t - d_{mi}(t)) - F_{ei}(t) \end{bmatrix}. \end{aligned}$$

A new variable $z_i(t)$ is defined as the controlled output:

$$z_i(t) = x_{mi}(t) - x_{si}(t) = [I \quad -I] \hat{x}_i(t) = \hat{C} \hat{x}_i(t). \quad (14)$$

Before construing the closed-loop system stability of Equations (13) and (14), basic lemmas are introduced beforehand, as follows:

Lemma 1 ([31]). For any symmetric constant matrix $M > 0$, as well as scalars α and β satisfying $\beta > \alpha$, a vector function $\omega: [\alpha, \beta] \rightarrow \mathbb{R}^n$, such that the integration concerned exists; thus,

$$\begin{aligned} & (\beta - \alpha) \int_{\alpha}^{\beta} \omega^T(s) M \omega(s) ds \\ & \geq \left[\int_{\alpha}^{\beta} \omega(s) ds \right]^T M \left[\int_{\alpha}^{\beta} \omega(s) ds \right]. \end{aligned}$$

Lemma 2 ([32]). Let $L = L^T$, M and N be real symmetric matrices of suitable dimensions with $F(t)$ satisfying $F^T(t)F(t) \leq I$, then the following inequality holds:

$$L + MF(t)N + N^T F(t)^T M^T < 0,$$

if and only if there exists a scalar $\sigma > 0$, such that

$$L + \sigma^{-1}MM^T + \sigma N^TN < 0.$$

Lemma 3 (Schur complement: [33]). *Given a symmetric matrix*

$$S = \begin{bmatrix} S_{11} & S_{12} \\ S_{12}^T & S_{22} \end{bmatrix},$$

the following statements are equivalent:

- (1) $S < 0$;
- (2) $S_{11} < 0, S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$;
- (3) $S_{22} < 0, S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$.

Main Results

In this section, the stability of the closed-loop systems (13), and (14) will be examined. The issue will be depicted in more detail, as follows: on the premise of ensuring the mean square stability of the system (13) with a predefined H_∞ performance requirement, a suitable H_∞ fault-tolerant controller is designed.

1. The system (13) is a mean-square stable system with $\hat{w}(t) = 0$.
2. Under the premise of the initial condition being zero and given a noise attenuation level γ , the system can live up to the following performance requirements.

$$\mathbb{E} \left\{ \int_0^\infty z^T(t)z(t)dt \right\} < \mathbb{E} \left\{ \gamma^2 \int_0^\infty \hat{w}^T(t)\hat{w}(t)dt \right\}.$$

The first result is obtained as follows:

Theorem 1. *For positive scalar $\gamma > 0$, there exist positive definite symmetrical matrices $P_i > 0$, $Q_{svi} > 0$, $Q_{mi} > 0$, $W_{si} > 0$, $W_{mi} > 0$, and matrix N with suitable dimensions. Given the appropriate controller gains K_{mik} and K_{sik} , $k = 1, 2, \dots, L$, as well as matrices $\tilde{M}_{mi}$ and $\tilde{M}_{si}$, the following linear matrix inequality is satisfied, and then the closed-loop system (13) is a mean-square stable system with an H_∞ disturbance attenuation level γ .*

$$\tilde{\Xi} = \begin{bmatrix} \Phi_1 & \Phi_2 & \Phi_3 & \Phi_4 \\ * & \Phi_5 & 0 & 0 \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} < 0, \quad (15)$$

where

$$\begin{aligned} \Phi_1 &= \begin{bmatrix} \Phi_{11} & \Phi_{12} & \Phi_{13} & -P_i & 0 & 0 \\ * & \Phi_{22} & -P_i & \Phi_{23} & 0 & \Phi_{24} \\ * & * & \Phi_{33} & \Phi_{34} & 0 & 0 \\ * & * & * & \Phi_{33} & 0 & \Phi_{24} \\ * & * & * & * & \Phi_{55} & \Phi_{56} \\ * & * & * & * & * & \Phi_{55} \end{bmatrix}, \\ \Phi_2 &= \begin{bmatrix} \Phi_{14} & 0 & W_{si} & -W_{si} & W_{mi} & -W_{mi} \\ 0 & 0 & -W_{si} & W_{si} & -W_{mi} & W_{mi} \\ \Phi_{14} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\ \Phi_3 &= [B_{mia}^T N^T \ 0 \ B_{mia}^T N^T \ 0 \ 0 \ 0]^T, \\ \Phi_4 &= [0 \ B_{sia}^T N^T \ 0 \ B_{sia}^T N^T \ 0 \ 0]^T, \end{aligned}$$

$$\Phi_5 = \begin{bmatrix} \Phi_{57} & \Phi_{58} & 0 & 0 & 0 & 0 \\ * & \Phi_{57} & 0 & 0 & 0 & 0 \\ * & * & -W_{si} & W_{si} & 0 & 0 \\ * & * & * & -W_{si} & 0 & 0 \\ * & * & * & * & -W_{mi} & W_{mi} \\ * & * & * & * & * & -W_{mi} \end{bmatrix},$$

$$\begin{aligned} \Phi_{11} &= \beta_{vi}Q_{svi} + Q_{mi} - W_{si} - W_{mi} + I \\ &\quad + \mathbf{He}(NA_{mia} + NB_{mia}\tilde{M}_{mi}K_{mik}), \\ \Phi_{12} &= -\beta_{vi}Q_{svi} - Q_{mi} + W_{si} + W_{mi} - I, \\ \Phi_{14} &= -NB_{sia}\tilde{M}_{si}K_{sik}, \\ \Phi_{13} &= P_i - N + NA_{mia} + NB_{mia}\tilde{M}_{mi}K_{mik}, \\ \Phi_{22} &= \beta_{vi}Q_{svi} + Q_{mi} - W_{si} - W_{mi} + I \\ &\quad + \mathbf{He}(NA_{sia} + NB_{sia}\tilde{M}_{si}K_{sik}), \\ \Phi_{23} &= P_i - N + NA_{sia} + NB_{sia}\tilde{M}_{si}K_{sik}, \\ \Phi_{24} &= -\beta_{vi}NB_{mia}\tilde{M}_{mi}K_{mik}, \\ \Phi_{33} &= \hat{d}_{si}^2 W_{si} + \hat{d}_{mi}^2 W_{mi} - \mathbf{He}(N), \\ \Phi_{34} &= -\hat{d}_{si}^2 W_{si} - \hat{d}_{mi}^2 W_{mi}, \\ \Phi_{55} &= -(1-\mu)\beta_{vi}Q_{svi}, \quad \Phi_{56} = (1-\mu)\beta_{vi}Q_{svi}, \\ \Phi_{57} &= -(1-\mu)Q_{mi}, \quad \Phi_{58} = (1-\mu)Q_{mi}. \end{aligned}$$

Proof. The following Lyapunov–Krasovskii functions can be considered:

$$\begin{aligned} V(t) &= V_1(t) + V_2(t) + V_3(t), \\ V_1(t) &= \sum_{i=1}^N z_i^T(t) P_i z_i(t), \\ V_2(t) &= \sum_{i=1}^N \left\{ \sum_{v=1}^N \beta_{vi} \int_{t-d_{svi}(t)}^t z_i^T(\phi) Q_{svi} z_i(\phi) d\phi \right. \\ &\quad \left. + \hat{d}_{si} \int_{t-\hat{d}_{si}}^t \int_{t+\sigma}^t z_i^T(\phi) W_{si} z_i(\phi) d\phi d\sigma \right\}, \\ V_3(t) &= \sum_{i=1}^N \left\{ \int_{t-d_{mi}(t)}^t z_i^T(\phi) Q_{mi} z_i(\phi) d\phi \right. \\ &\quad \left. + \hat{d}_{mi} \int_{t-\hat{d}_{mi}}^t \int_{t+\sigma}^t z_i^T(\phi) W_{mi} z_i(\phi) d\phi d\sigma \right\}. \end{aligned} \quad (16)$$

Then, the differentiation of $V(t)$ can be described as

$$\begin{aligned} \mathbb{E}\{\dot{V}_1(t)\} &= \mathbb{E}\left\{ \sum_{i=1}^N 2z_i^T(t) P_i \dot{z}_i(t) \right\}, \\ \mathbb{E}\{\dot{V}_2(t)\} &= \mathbb{E}\left\{ \sum_{i=1}^N \sum_{v=1}^N [\beta_{vi} z_i^T(t) Q_{svi} z_i(t) \right. \\ &\quad \left. - \beta_{vi}(1 - \dot{d}_{svi}(t)) z_i^T(t - d_{svi}(t)) \right. \\ &\quad \left. \times Q_{svi} z_i(t - d_{svi}(t))] \right\} \\ &\quad + \mathbb{E}\left\{ \sum_{i=1}^N [\hat{d}_{si}^2 \dot{z}_i^T(t) W_{si} \dot{z}_i(t) \right. \\ &\quad \left. - \hat{d}_{si} \int_{t-\hat{d}_{si}}^t \dot{z}_i^T(\sigma) W_{si} \dot{z}_i(\sigma) d\sigma] \right\}, \\ \mathbb{E}\{\dot{V}_3(t)\} &= \mathbb{E}\left\{ \sum_{i=1}^N [z_i^T(t) Q_{mi} z_i(t) \right. \\ &\quad \left. - (1 - \dot{d}_{mi}(t)) z_i^T(t - d_{mi}(t)) \right. \\ &\quad \left. \times Q_{mi} z_i(t - d_{mi}(t)) \right. \\ &\quad \left. + \hat{d}_{mi}^2 \dot{z}_i^T(t) W_{mi} \dot{z}_i(t) \right. \\ &\quad \left. - \hat{d}_{mi} \int_{t-\hat{d}_{mi}}^t \dot{z}_i^T(\sigma) W_{mi} \dot{z}_i(\sigma) d\sigma] \right\}. \end{aligned} \quad (17)$$

Under the premise of the time-varying delay assumed above, using Lemma 1 yields

$$\begin{aligned} \mathbb{E}\{\dot{V}(t)\} \leq & \mathbb{E}\left\{\sum_{i=1}^N [2z_i^T(t)P_i\dot{z}_i(t) + z_i^T(t)\sum_{v=1}^N \beta_{vi}Q_{svi}z_i(t) \right. \\ & + \hat{d}_{si}^2\dot{z}_i^T(t)W_{si}\dot{z}_i(t) + z_i^T(t)Q_{mi}z_i(t) \\ & - (1-\mu)z_i^T(t-d_{svi}(t)) \\ & \times \sum_{v=1}^N \beta_{vi}Q_{svi}z_i(t-d_{svi}(t)) \\ & - (z_i(t) - z_i(t-\hat{d}_{si}))^T W_{si}(z_i(t) - z_i(t-\hat{d}_{si})) \\ & + \hat{d}_{mi}^2\dot{z}_i^T(t)W_{mi}\dot{z}_i(t) \\ & - (1-\mu)z_i^T(t-d_{mi}(t))Q_{mi}z_i(t-d_{mi}(t)) \\ & - (z_i(t) - z_i(t-\hat{d}_{mi}))^T \\ & \left. \times W_{mi}(z_i(t) - z_i(t-\hat{d}_{mi}))\right]\}. \end{aligned} \quad (18)$$

By substituting Equation (14) into (18), it follows that

$$\begin{aligned} \mathbb{E}\{\dot{V}(t)\} \leq & \mathbb{E}\left\{\sum_{i=1}^N \sum_{v=1}^N [2\hat{x}_i^T(t) \begin{bmatrix} P_i & -P_i \\ -P_i & P_i \end{bmatrix} \hat{x}_i^T(t) \right. \\ & + \hat{x}_i^T(t) \begin{bmatrix} \beta_{vi}Q_{svi} & -\beta_{vi}Q_{svi} \\ -\beta_{vi}Q_{svi} & \beta_{vi}Q_{svi} \end{bmatrix} \hat{x}_i(t) \\ & + \hat{x}_i^T(t-d_{svi}(t)) \\ & \times \begin{bmatrix} -(1-\mu)\beta_{vi}Q_{svi} & (1-\mu)\beta_{vi}Q_{svi} \\ (1-\mu)\beta_{vi}Q_{svi} & -(1-\mu)\beta_{vi}Q_{svi} \end{bmatrix} \\ & \times \hat{x}_i(t-d_{svi}(t)) \\ & + \hat{x}_i^T(t) \begin{bmatrix} \hat{d}_{si}^2 W_{si} & -\hat{d}_{si}^2 W_{si} \\ -\hat{d}_{si}^2 W_{si} & \hat{d}_{si}^2 W_{si} \end{bmatrix} \hat{x}_i^T(t) \\ & + (\hat{x}_i(t) - \hat{x}_i(t-\hat{d}_{si}))^T \begin{bmatrix} -W_{si} & W_{si} \\ W_{si} & -W_{si} \end{bmatrix} \\ & \times (\hat{x}_i(t) - \hat{x}_i(t-\hat{d}_{si})) \\ & + \hat{x}_i^T(t) \begin{bmatrix} Q_{mi} & -Q_{mi} \\ -Q_{mi} & Q_{mi} \end{bmatrix} \hat{x}_i(t) \\ & + \hat{x}_i^T(t-d_{mi}(t)) \begin{bmatrix} -(1-\mu)Q_{mi} & (1-\mu)Q_{mi} \\ (1-\mu)Q_{mi} & -(1-\mu)Q_{mi} \end{bmatrix} \\ & \times \hat{x}_i(t-d_{mi}(t)) \\ & + (\hat{x}_i(t) - \hat{x}_i(t-\hat{d}_{mi}))^T \begin{bmatrix} -W_{mi} & W_{mi} \\ W_{mi} & -W_{mi} \end{bmatrix} \\ & \times (\hat{x}_i(t) - \hat{x}_i(t-\hat{d}_{mi})) \\ & \left. + \hat{x}_i^T(t) \begin{bmatrix} \hat{d}_{mi}^2 W_{mi} & -\hat{d}_{mi}^2 W_{mi} \\ -\hat{d}_{mi}^2 W_{mi} & \hat{d}_{mi}^2 W_{mi} \end{bmatrix} \hat{x}_i^T(t)\right]\}. \end{aligned} \quad (19)$$

Using free-weight matrix technology yields

$$0 = [\hat{x}_i^T(t)\mathcal{N} + \hat{x}_i^T(t)\mathcal{N}]\mathcal{W} + \mathcal{W}^T[\mathcal{N}^T\hat{x}_i(t) + \mathcal{N}^T\hat{x}_i^T(t)], \quad (20)$$

where

$$\begin{aligned} \mathcal{W} = & \sum_{a=1}^L \sum_{k=1}^L h_{ia}(y_i(t))h_{ik}(y_i(t))\{\hat{A}_{iak}\hat{x}_i(t) \\ & + \sum_{v=1}^N \hat{A}_{1iakov}\hat{x}_i(t-d_{svi}(t)) + \hat{A}_{2iak}\hat{x}_i(t-d_{mi}(t)) \\ & + \hat{E}_{ia}\hat{\omega}(t)\} - \hat{x}_i^T(t). \end{aligned}$$

From the above Equations (19) and (20), one can induce that

$$\zeta^T(t)\Xi\zeta(t) = 0, \quad (21)$$

where

$$\zeta(t) = [\hat{x}_i^T(t) \quad \hat{x}_i^T(t) \quad \hat{x}_i^T(t - d_{svi}(t)) \quad \hat{x}_i^T(t - d_{mi}(t)) \\ \hat{x}_i^T(t - \hat{d}_{si}) \quad \hat{x}_i^T(t - \hat{d}_{mi}) \quad \hat{\omega}^T(t)]^T.$$

Next, the system performance requirement can be structured, as follows:

$$J = \sum_{a=1}^L \sum_{k=1}^L h_{ia}(y_i(t)) h_{ik}(y_i(t)) \mathbb{E}\{\dot{V}(t) \\ + z_i^T(t) z_i(t) - \gamma^2 \hat{\omega}^T(t) \hat{\omega}(t)\} \\ \leq \sum_{a=1}^L \sum_{k=1}^L h_{ia}(y_i(t)) h_{ik}(y_i(t)) \mathbb{E}\{\zeta^T(t) \Xi \zeta(t)\}.$$

Define

$$\mathcal{N} = \begin{bmatrix} N & 0 \\ 0 & N \end{bmatrix}, \quad \mathbb{E}\{\hat{M}_{mi}\} = \tilde{M}_{mi}, \\ \mathbb{E}\{\hat{M}_{si}\} = \tilde{M}_{si}, \quad \mathbb{E}\{\Xi\} = \tilde{\Xi},$$

If $J < 0$ holds, $\tilde{\Xi} < 0$ in Equation (15) is naturally satisfied. Then, it follows that

$$\begin{bmatrix} \Phi_1 & \Phi_2 \\ * & \Phi_5 \end{bmatrix} < 0.$$

It is clear that the system (13) is a mean-square stable system with $\hat{\omega}(t) = 0$. In addition, Inequality (15) demonstrates that

$$\mathbb{E}\{\dot{V}(t) + z_i^T(t) z_i(t) - \gamma^2 \hat{\omega}^T(t) \hat{\omega}(t)\} < 0 \quad (22)$$

holds. Furthermore, according to the initial condition of zero, by colligating both sides of Equation (22) from 0 to t and setting $t \rightarrow \infty$, one has

$$\mathbb{E}\left\{\int_0^\infty z_i^T(t) z_i(t) dt\right\} < \mathbb{E}\left\{\int_0^\infty \gamma^2 \hat{\omega}^T(t) \hat{\omega}(t) dt\right\},$$

Thus, the performance requirement is achieved. This means that the system (13) is a mean-square stable system at the level γ of interference attenuation H_∞ . $\square$

Next, the second theorem derived from the first theorem is used to design the controller gains K_{mik} and K_{sik} .

Theorem 2. Given suitable matrices $\tilde{M}_{mi}$ and $\tilde{M}_{si}$, design the controller gains as $K_{mik} = Y_{mik} \hat{N}^{-1}$, $K_{sik} = Y_{sik} \hat{N}^{-1}$. For a positive scalar $\gamma > 0$, there exist positive definite symmetrical matrices $\hat{P}_i > 0$, $\hat{Q}_{svi} > 0$, $\hat{Q}_{mi} > 0$, $\hat{W}_{si} > 0$, $\hat{W}_{mi} > 0$, and matrix $\hat{N}$, Y_{mik} , Y_{sik} with suitable dimensions; thus, the following linear matrix inequalities are satisfied, and then the closed-loop system (13) is a mean-square stable system with an H_∞ disturbance attenuation level γ .

$$\tilde{\Xi}_1 = \begin{bmatrix} \check{\Phi}_1 & \check{\Phi}_2 & \check{\Phi}_3 & \check{\Phi}_4 & \check{\Phi}_6 \\ * & \check{\Phi}_5 & 0 & 0 & 0 \\ * & * & -\gamma^2 I & 0 & 0 \\ * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & -I \end{bmatrix} < 0, \quad (23)$$

where

$$\begin{aligned}\tilde{\Phi}_1 &= \begin{bmatrix} \tilde{\Phi}_{11} & \tilde{\Phi}_{12} & \tilde{\Phi}_{13} & -\hat{P}_i & 0 & 0 \\ * & \tilde{\Phi}_{22} & -\hat{P}_i & \tilde{\Phi}_{23} & 0 & \tilde{\Phi}_{24} \\ * & * & \tilde{\Phi}_{33} & \tilde{\Phi}_{34} & 0 & 0 \\ * & * & * & \tilde{\Phi}_{33} & 0 & \tilde{\Phi}_{24} \\ * & * & * & * & \tilde{\Phi}_{55} & \tilde{\Phi}_{56} \\ * & * & * & * & * & \tilde{\Phi}_{55} \end{bmatrix}, \\ \tilde{\Phi}_3 &= [B_{mia}^T \ 0 \ B_{mia}^T \ 0 \ 0 \ 0]^T, \\ \tilde{\Phi}_4 &= [0 \ B_{sia}^T \ 0 \ B_{sia}^T \ 0 \ 0]^T, \\ \tilde{\Phi}_2 &= \begin{bmatrix} \tilde{\Phi}_{14} & 0 & W_{si} & -W_{si} & W_{mi} & -W_{mi} \\ 0 & 0 & -W_{si} & W_{si} & -W_{mi} & W_{mi} \\ \tilde{\Phi}_{14} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\ \tilde{\Phi}_5 &= \begin{bmatrix} \tilde{\Phi}_{57} & \tilde{\Phi}_{58} & 0 & 0 & 0 & 0 \\ * & \tilde{\Phi}_{57} & 0 & 0 & 0 & 0 \\ * & * & -\hat{W}_{si} & \hat{W}_{si} & 0 & 0 \\ * & * & * & -\hat{W}_{si} & 0 & 0 \\ * & * & * & * & -\hat{W}_{mi} & \hat{W}_{mi} \\ * & * & * & * & * & -\hat{W}_{mi} \end{bmatrix}, \\ \tilde{\Phi}_6 &= [\hat{N} \ -\hat{N} \ 0 \ 0 \ 0 \ 0]^T, \\ \tilde{\Phi}_{14} &= -B_{sia}\hat{M}_{si}Y_{sik}, \\ \tilde{\Phi}_{11} &= \beta_{vi}\hat{Q}_{svi} + \hat{Q}_{mi} - \hat{W}_{si} - \hat{W}_{mi} \\ &\quad + \mathbf{He}(A_{mia}\hat{N} + B_{mia}\hat{M}_{mi}Y_{mik}), \\ \tilde{\Phi}_{12} &= -\beta_{vi}\hat{Q}_{svi} - \hat{Q}_{mi} + \hat{W}_{si} + \hat{W}_{mi}, \\ \tilde{\Phi}_{13} &= \hat{P}_i - \hat{N} + A_{mia}\hat{N} + B_{mia}\hat{M}_{mi}Y_{mik}, \\ \tilde{\Phi}_{22} &= \beta_{vi}\hat{Q}_{svi} + \hat{Q}_{mi} - \hat{W}_{si} - \hat{W}_{mi} \\ &\quad + \mathbf{He}(A_{sia}\hat{N} + B_{sia}\hat{M}_{si}Y_{sik}), \\ \tilde{\Phi}_{23} &= \hat{P}_i - \hat{N} + A_{sia}\hat{N} + B_{sia}\hat{M}_{si}Y_{sik}, \\ \tilde{\Phi}_{24} &= -\beta_{vi}B_{mia}\hat{M}_{mi}Y_{mik}, \\ \tilde{\Phi}_{33} &= \hat{d}_{si}^2\hat{W}_{si} + \hat{d}_{mi}^2\hat{W}_{mi} - \mathbf{He}(\hat{N}), \\ \tilde{\Phi}_{34} &= -\hat{d}_{si}^2\hat{W}_{si} - \hat{d}_{mi}^2\hat{W}_{mi}, \\ \tilde{\Phi}_{55} &= -(1-\mu)\beta_{vi}\hat{Q}_{svi}, \quad \tilde{\Phi}_{56} = (1-\mu)\beta_{vi}\hat{Q}_{svi}, \\ \tilde{\Phi}_{57} &= -(1-\mu)\hat{Q}_{mi}, \quad \tilde{\Phi}_{58} = (1-\mu)\hat{Q}_{mi}.\end{aligned}$$

Proof. We multiply $\text{diag}\{\overbrace{\hat{N}^T, \dots, \hat{N}^T}^{12}, I, I\}$ and $\text{diag}\{\overbrace{\hat{N}, \dots, \hat{N}}^{12}, I, I\}$ on the left and right sides of $\tilde{\Xi}$ (15), respectively. Denoting $\hat{N}^T N = I$, $\hat{P}_i = \hat{N}^T P_i \hat{N}$, $\hat{Q}_{svi} = \hat{N}^T Q_{svi} \hat{N}$, $\hat{Q}_{mi} = \hat{N}^T Q_{mi} \hat{N}$, $\hat{W}_{si} = \hat{N}^T W_{si} \hat{N}$, $\hat{W}_{mi} = \hat{N}^T W_{mi} \hat{N}$, the results listed below are derived:

$$\begin{bmatrix} \tilde{\Phi}_1 & \tilde{\Phi}_2 & \tilde{\Phi}_3 & \tilde{\Phi}_4 \\ * & \tilde{\Phi}_5 & 0 & 0 \\ * & * & -\gamma^2 I & 0 \\ * & * & * & -\gamma^2 I \end{bmatrix} + \begin{bmatrix} \tilde{\Phi}_6 \\ 0_{6 \times 1} \end{bmatrix} [\tilde{\Phi}_6 \ 0_{1 \times 6}]^T < 0.$$

Through the Schur complement of Lemma 3, the corresponding result in Equation (23) is directly obtained. $\square$

However, in Theorems 1 and 2, we assume that $\tilde{M}_{mi}$ and $\tilde{M}_{si}$ are already known, and they are represented as follows:

$$\begin{aligned}\tilde{M}_{mi} &= \text{diag}\{\tilde{\theta}_{1mi}m_{1mi} + (1 - \tilde{\theta}_{1mi})\tilde{m}_{1mi}, \dots, \\ &\quad \tilde{\theta}_{pmi}m_{pmi} + (1 - \tilde{\theta}_{pmi})\tilde{m}_{pmi}\}, \\ \tilde{M}_{si} &= \text{diag}\{\tilde{\theta}_{1si}m_{1si} + (1 - \tilde{\theta}_{1si})\tilde{m}_{1si}, \dots, \\ &\quad \tilde{\theta}_{psi}m_{psi} + (1 - \tilde{\theta}_{psi})\tilde{m}_{psi}\}.\end{aligned}\quad (24)$$

Define

$$\begin{aligned}\tilde{\Theta}_{mi} &= \text{diag}\{\tilde{\theta}_{1mi}, \dots, \tilde{\theta}_{pmi}\}, \quad \tilde{\Theta}_{si} = \text{diag}\{\tilde{\theta}_{1si}, \dots, \tilde{\theta}_{psi}\}, \\ \Gamma_{mi} &= \text{diag}\{m_{1mi}, \dots, m_{pmi}\}, \quad \bar{\Gamma}_{mi} = \text{diag}\{\tilde{m}_{1mi}, \dots, \tilde{m}_{pmi}\}, \\ \Gamma_{si} &= \text{diag}\{m_{1si}, \dots, m_{psi}\}, \quad \bar{\Gamma}_{si} = \text{diag}\{\tilde{m}_{1si}, \dots, \tilde{m}_{psi}\},\end{aligned}$$

and Equation (24) can be rewritten as

$$\begin{aligned}\tilde{M}_{mi} &= \tilde{\Theta}_{mi}\Gamma_{mi} + (1 - \tilde{\Theta}_{mi})\bar{\Gamma}_{mi}, \\ \tilde{M}_{si} &= \tilde{\Theta}_{si}\Gamma_{si} + (1 - \tilde{\Theta}_{si})\bar{\Gamma}_{si}.\end{aligned}\quad (25)$$

Nevertheless, in practice, the accurate values of Γ_{mi} , Γ_{si} , $\bar{\Gamma}_{mi}$, and $\bar{\Gamma}_{si}$ are unknown beforehand. In order to address this problem, we introduce the characteristics of m_{ami} , m_{asi} , $\tilde{m}_{ami}$, and $\tilde{m}_{asi}$. For the purpose of being concise, j is used to represent m and s , that is, $j = m$ indicates the master robot and $j = s$ indicates the slave robot; thus, the following results can be derived:

$$\begin{aligned}m_{ajio} &= \frac{m_{aji}^{\min} + \tilde{m}_{aji}}{2}, \quad \tilde{m}_{ajio} = \frac{\tilde{m}_{aji} + m_{aji}^{\max}}{2}, \\ r_{aji} &= \frac{\tilde{m}_{aji} - m_{aji}^{\min}}{\tilde{m}_{aji} + m_{aji}^{\min}}, \quad \bar{r}_{aji} = \frac{m_{aji}^{\max} - \tilde{m}_{aji}}{\tilde{m}_{aji} + m_{aji}^{\max}}, \\ \bar{g}_{aji} &= \frac{\tilde{m}_{aji} - \tilde{m}_{ajio}}{\tilde{m}_{ajio}}, \quad g_{aji} = \frac{m_{aji} - m_{ajio}}{m_{ajio}}.\end{aligned}\quad (26)$$

Then, for $a = 1, 2, \dots, p$, it follows that

$$\begin{aligned}m_{aji} &= m_{ajio}(1 + g_{aji}), \quad |g_{aji}| \leq r_{aji} \leq 1, \\ \tilde{m}_{aji} &= \tilde{m}_{ajio}(1 + \bar{g}_{aji}), \quad |\bar{g}_{aji}| \leq \bar{r}_{aji} \leq 1.\end{aligned}$$

Define

$$\begin{aligned}\Gamma_{jio} &= \text{diag}\{m_{1jio}, m_{2jio}, \dots, m_{pjio}\}, \\ \bar{\Gamma}_{jio} &= \text{diag}\{\tilde{m}_{1jio}, \tilde{m}_{2jio}, \dots, \tilde{m}_{pjio}\}, \\ G_{ji} &= \text{diag}\{g_{1ji}, g_{2ji}, \dots, g_{pji}\}, \\ |G_{ji}| &= \text{diag}\{|g_{1ji}|, |g_{2ji}|, \dots, |g_{pji}|\}, \\ R_{ji} &= \text{diag}\{r_{1ji}, r_{2ji}, \dots, r_{pji}\}, \\ \bar{R}_{ji} &= \text{diag}\{\bar{r}_{1ji}, \bar{r}_{2ji}, \dots, \bar{r}_{pji}\}, \\ \bar{G}_{ji} &= \text{diag}\{\bar{g}_{1ji}, \bar{g}_{2ji}, \dots, \bar{g}_{pji}\}, \\ |\bar{G}_{ji}| &= \text{diag}\{|\bar{g}_{1ji}|, |\bar{g}_{2ji}|, \dots, |\bar{g}_{pji}|\}.\end{aligned}$$

Γ_{ji} and $\bar{\Gamma}_{ji}$ can be rewritten as

$$\begin{aligned}\Gamma_{ji} &= \Gamma_{jio}(I + G_{ji}), \quad |G_{ji}| \leq R_{ji} \leq I, \\ \bar{\Gamma}_{ji} &= \bar{\Gamma}_{jio}(I + \bar{G}_{ji}), \quad |\bar{G}_{ji}| \leq \bar{R}_{ji} \leq I.\end{aligned}\quad (27)$$

Replacing Equation (27) by Equation (25), $\tilde{M}_{mi}$ and $\tilde{M}_{si}$ can be depicted as

$$\begin{aligned}\tilde{M}_{mi} &= \tilde{\Theta}_{mi}\Gamma_{mio} + (1 - \tilde{\Theta}_{mi})\bar{\Gamma}_{mio} + \tilde{\Theta}_{mi}\Gamma_{mio}G_{mi} \\ &\quad + (1 - \tilde{\Theta}_{mi})\bar{\Gamma}_{mio}\bar{G}_{mi}, \\ (|G_{mi}| \leq R_{mi} \leq I, \quad |\bar{G}_{mi}| \leq \bar{R}_{mi} \leq I) \\ \tilde{M}_{si} &= \tilde{\Theta}_{si}\Gamma_{sio} + (1 - \tilde{\Theta}_{si})\bar{\Gamma}_{sio} + \tilde{\Theta}_{si}\Gamma_{sio}G_{si} \\ &\quad + (1 - \tilde{\Theta}_{si})\bar{\Gamma}_{sio}\bar{G}_{si}, \\ (|G_{si}| \leq R_{si} \leq I, \quad |\bar{G}_{si}| \leq \bar{R}_{si} \leq I)\end{aligned}\quad (28)$$

From Equation (28), one can obtain the following theorem:

Theorem 3. For positive scalars $\gamma > 0$, $\sigma_a > 0$, $a = 1, 2, 3, \dots, 12$, there exist positive definite matrices $\hat{P}_i > 0$, $\hat{Q}_{svi} > 0$, $\hat{Q}_{mi} > 0$, $\hat{W}_{si} > 0$, $\hat{W}_{mi} > 0$, and matrices $\hat{N}$, Y_{mik} and Y_{sik} with appropriate dimensions, the following linear matrices inequality holds, and the closed-loop system (13) is a mean-square stable system with an H_∞ disturbance attenuation level γ .

$$\Omega = \begin{bmatrix} \bar{\Omega}_1 & \bar{\Omega}_2 & \bar{\Omega}_3 \\ * & \bar{\Omega}_4 & 0 \\ * & * & \bar{\Omega}_5 \end{bmatrix} < 0, \quad (29)$$

where

$$\begin{aligned} \bar{\Omega}_1 &= \begin{bmatrix} \Omega_1 & \Omega_2 & \Omega_3 & \Omega_4 & \Omega_6 \\ * & \Omega_5 & 0 & 0 & 0 \\ * & * & -\gamma^2 I & 0 & 0 \\ * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & -I \end{bmatrix}, \\ \Omega_1 &= \begin{bmatrix} \Omega_{11} & \bar{\Phi}_{12} & \Omega_{13} & -\hat{P}_i & 0 & 0 \\ * & \Omega_{22} & -\hat{P}_i & \Omega_{24} & 0 & \Omega_{26} \\ * & * & \Omega_{33} & \bar{\Phi}_{34} & 0 & 0 \\ * & * & * & \Omega_{44} & 0 & \Omega_{26} \\ * & * & * & * & \bar{\Phi}_{55} & \bar{\Phi}_{56} \\ * & * & * & * & * & \bar{\Phi}_{55} \end{bmatrix}, \\ \bar{\Omega}_2 &= \begin{bmatrix} \hat{\Omega}_1 & \hat{\Omega}_1 & \hat{\Omega}_2 & \hat{\Omega}_2 & \hat{\Omega}_3 & \hat{\Omega}_3 \end{bmatrix}, \\ \hat{\Omega}_1 &= \begin{bmatrix} Y_{mik} & 0_{1 \times 14} \end{bmatrix}, \hat{\Omega}_2 = \begin{bmatrix} 0_{1 \times 6} & -Y_{sik} & 0_{1 \times 8} \end{bmatrix}, \\ \bar{\Omega}_3 &= \begin{bmatrix} \hat{\Omega}_4 & \hat{\Omega}_4 & \hat{\Omega}_2 & \hat{\Omega}_2 & \hat{\Omega}_4 & \hat{\Omega}_4 \end{bmatrix}, \\ \bar{\Omega}_4 &= \text{diag}\{-\sigma_1 I, -\sigma_2 I, -\sigma_3 I, -\sigma_4 I, -\sigma_5 I, -\sigma_6 I\}, \\ \bar{\Omega}_5 &= \text{diag}\{-\sigma_7 I, -\sigma_8 I, -\sigma_9 I, -\sigma_{10} I, -\sigma_{11} I, -\sigma_{12} I\}, \\ \hat{\Omega}_2 &= \begin{bmatrix} 0_{1 \times 6} & -Y_{sik} & 0_{1 \times 8} \end{bmatrix}, \hat{\Omega}_3 = \begin{bmatrix} 0 & Y_{sik} & 0_{1 \times 13} \end{bmatrix}, \\ \hat{\Omega}_4 &= \begin{bmatrix} 0_{1 \times 5} & -Y_{mik} & 0_{1 \times 9} \end{bmatrix}, \\ \Omega_2 &= \begin{bmatrix} \Omega_{17} & 0 & \hat{W}_{si} & -\hat{W}_{si} & \hat{W}_{mi} & -\hat{W}_{mi} \\ 0 & 0 & -\hat{W}_{si} & \hat{W}_{si} & -\hat{W}_{mi} & \hat{W}_{mi} \\ \Omega_{17} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \\ \Omega_3 &= \begin{bmatrix} B_{mia}^T & 0 & B_{mia}^T & 0 & 0 & 0 \end{bmatrix}^T, \\ \Omega_4 &= \begin{bmatrix} 0 & B_{sia}^T & 0 & B_{sia}^T & 0 & 0 \end{bmatrix}^T, \\ \Omega_6 &= \begin{bmatrix} \hat{N} & -\hat{N} & 0 & 0 & 0 & 0 \end{bmatrix}^T, \end{aligned}$$

$$\Omega_5 = \begin{bmatrix} \tilde{\Phi}_{57} & \tilde{\Phi}_{58} & 0 & 0 & 0 & 0 \\ * & \tilde{\Phi}_{57} & 0 & 0 & 0 & 0 \\ * & * & -\hat{W}_{si} & \hat{W}_{si} & 0 & 0 \\ * & * & * & -\hat{W}_{si} & 0 & 0 \\ * & * & * & * & -\hat{W}_{mi} & \hat{W}_{mi} \\ * & * & * & * & * & -\hat{W}_{mi} \end{bmatrix},$$

$$\Omega_{11} = \beta_{vi}\hat{Q}_{svi} + \hat{Q}_{mi} - \hat{W}_{si} - \hat{W}_{mi} + \mathbf{He}(A_{mia}\hat{N} + B_{mia}\tilde{\Theta}_{mi}\Gamma_{mio}Y_{mik} + B_{mia}(1 - \tilde{\Theta}_{mi})\bar{\Gamma}_{mio}Y_{mik}) + \sigma_1 B_{mia}\tilde{\Theta}_{mi}\Gamma_{mio}R_{mi}R_{mi}^T\Gamma_{mio}^T\tilde{\Theta}_{mi}^TB_{mia}^T + \sigma_3 B_{sia}\tilde{\Theta}_{si}\Gamma_{sio}R_{si}R_{si}^T\Gamma_{sio}^T\tilde{\Theta}_{si}^TB_{sia}^T + \sigma_2 B_{mia}(1 - \tilde{\Theta}_{mi})\bar{\Gamma}_{mio}R_{mi}\bar{R}_{mi}^T\bar{\Gamma}_{mio}^T \times (1 - \tilde{\Theta}_{mi})^TB_{mia}^T + \sigma_4 B_{sia}(1 - \tilde{\Theta}_{si})\bar{\Gamma}_{sio}\bar{R}_{si}\bar{R}_{si}^T\bar{\Gamma}_{sio}^T \times (1 - \tilde{\Theta}_{si})^TB_{sia}^T,$$

$$\Omega_{13} = \hat{P}_i - \hat{N} + A_{mia}\hat{N} + B_{mia}\tilde{\Theta}_{mi}\Gamma_{mio}Y_{mik} + B_{mia}(1 - \tilde{\Theta}_{mi})\bar{\Gamma}_{mio}Y_{mik} + \sigma_1 B_{mia}\tilde{\Theta}_{mi}\Gamma_{mio}R_{mi}R_{mi}^T\Gamma_{mio}^T\tilde{\Theta}_{mi}^TB_{mia}^T + \sigma_2 B_{mia}(1 - \tilde{\Theta}_{mi})\bar{\Gamma}_{mio}\bar{R}_{mi}\bar{R}_{mi}^T\bar{\Gamma}_{mio}^T \times (1 - \tilde{\Theta}_{mi})^TB_{mia}^T,$$

$$\Omega_{33} = \hat{d}_{si}^2\hat{W}_{si} + \hat{d}_{mi}^2\hat{W}_{mi} - \mathbf{He}(\hat{N}) + \sigma_1 B_{mia}\tilde{\Theta}_{mi}\Gamma_{mio}R_{mi}R_{mi}^T\Gamma_{mio}^T\tilde{\Theta}_{mi}^TB_{mia}^T + \sigma_2 B_{mia}(1 - \tilde{\Theta}_{mi})\bar{\Gamma}_{mio}\bar{R}_{mi}\bar{R}_{mi}^T\bar{\Gamma}_{mio}^T \times (1 - \tilde{\Theta}_{mi})^TB_{mia}^T + \sigma_9 B_{sia}\tilde{\Theta}_{si}\Gamma_{sio}R_{si}R_{si}^T\Gamma_{sio}^T\tilde{\Theta}_{si}^TB_{sia}^T + \sigma_{10} B_{sia}(1 - \tilde{\Theta}_{si})\bar{\Gamma}_{sio}\bar{R}_{si}\bar{R}_{si}^T\bar{\Gamma}_{sio}^T \times (1 - \tilde{\Theta}_{si})^TB_{sia}^T,$$

$$\Omega_{24} = -\beta_{vi}B_{mia}\tilde{\Theta}_{mi}\Gamma_{mio}Y_{mik} - \beta_{vi}B_{mia}(1 - \tilde{\Theta}_{mi})\bar{\Gamma}_{mio} \times Y_{mik} + \sigma_5 B_{sia}\tilde{\Theta}_{si}\Gamma_{sio}R_{si}R_{si}^T\Gamma_{sio}^T\tilde{\Theta}_{si}^TB_{sia}^T + \sigma_6 B_{sia}(1 - \tilde{\Theta}_{si})\bar{\Gamma}_{sio}\bar{R}_{si}\bar{R}_{si}^T\bar{\Gamma}_{sio}^T \times (1 - \tilde{\Theta}_{si})^TB_{sia}^T,$$

$$\Omega_{26} = -\beta_{vi}B_{mia}\tilde{\Theta}_{mi}\Gamma_{mio}Y_{mik} - \beta_{vi}B_{mia} \times (1 - \tilde{\Theta}_{mi})\bar{\Gamma}_{mio}Y_{mik},$$

$$\Omega_{17} = -B_{sia}\tilde{\Theta}_{si}\Gamma_{sio}Y_{sik} - B_{sia}(1 - \tilde{\Theta}_{si})\bar{\Gamma}_{sio}Y_{sik},$$

$$\Omega_{22} = \beta_{vi}\hat{Q}_{svi} + \hat{Q}_{mi} - \hat{W}_{si} - \hat{W}_{mi} + \mathbf{He}(A_{sia}\hat{N} + B_{sia}\tilde{\Theta}_{si}\Gamma_{sio}Y_{sik} + B_{sia}(1 - \tilde{\Theta}_{si})\bar{\Gamma}_{sio}Y_{sik}) + \sigma_5 B_{sia}\tilde{\Theta}_{si}\Gamma_{sio}R_{si}R_{si}^T\Gamma_{sio}^T\tilde{\Theta}_{si}^TB_{sia}^T + \sigma_7 \beta_{vi}^2 B_{mia}\tilde{\Theta}_{mi}\Gamma_{mio}R_{mi}R_{mi}^T\Gamma_{mio}^T\tilde{\Theta}_{mi}^TB_{mia}^T + \sigma_6 B_{sia}(1 - \tilde{\Theta}_{si})\bar{\Gamma}_{sio}\bar{R}_{si}\bar{R}_{si}^T\bar{\Gamma}_{sio}^T(1 - \tilde{\Theta}_{si})^TB_{sia}^T + \sigma_8 \beta_{vi}^2 B_{mia}(1 - \tilde{\Theta}_{mi})\bar{\Gamma}_{mio}\bar{R}_{mi} \times \bar{R}_{mi}^T\bar{\Gamma}_{mio}^T(1 - \tilde{\Theta}_{mi})^TB_{mia}^T,$$

$$\Omega_{44} = \hat{d}_{si}^2\hat{W}_{si} + \hat{d}_{mi}^2\hat{W}_{mi} - \mathbf{He}(\hat{N}) + \sigma_5 B_{sia}\tilde{\Theta}_{si}\Gamma_{sio}R_{si}R_{si}^T\Gamma_{sio}^T\tilde{\Theta}_{si}^TB_{sia}^T + \sigma_6 B_{sia}(1 - \tilde{\Theta}_{si})\bar{\Gamma}_{sio}\bar{R}_{si}\bar{R}_{si}^T\bar{\Gamma}_{sio}^T(1 - \tilde{\Theta}_{si})^TB_{sia}^T + \sigma_{11} \beta_{vi}^2 B_{mia}\tilde{\Theta}_{mi}\Gamma_{mio}R_{mi}R_{mi}^T\Gamma_{mio}^T\tilde{\Theta}_{mi}^TB_{mia}^T + \sigma_{12} \beta_{vi}^2 B_{mia}(1 - \tilde{\Theta}_{mi})\bar{\Gamma}_{mio}\bar{R}_{mi} \times \bar{R}_{mi}^T\bar{\Gamma}_{mio}^T(1 - \tilde{\Theta}_{mi})^TB_{mia}^T,$$

$$\Phi_{12} = -\beta_{vi}\hat{Q}_{svi} - \hat{Q}_{mi} + \hat{W}_{si} + \hat{W}_{mi},$$

$$\tilde{\Phi}_{55} = -(1 - \mu)\beta_{vi}\hat{Q}_{svi}, \tilde{\Phi}_{56} = (1 - \mu)\beta_{vi}\hat{Q}_{svi},$$

$$\tilde{\Phi}_{57} = -(1 - \mu)\hat{Q}_{mi}, \tilde{\Phi}_{58} = (1 - \mu)\hat{Q}_{mi}.$$

Proof. Replace $\tilde{M}_{mi}$ and $\tilde{M}_{si}$ of Equation (28) into $\tilde{\Xi}_1$ of Equation (23); then, $\tilde{\Xi}_1$ can be separated into two subparts, as $\tilde{\Xi}_1 = \Lambda + \Delta\Lambda$, where Λ indicates the constant part and is expressed as follows:

$$\Lambda = \begin{bmatrix} \Lambda_1 & \Lambda_2 & \tilde{\Phi}_3 & \tilde{\Phi}_4 & \tilde{\Phi}_6 \\ * & \tilde{\Phi}_5 & 0 & 0 & 0 \\ * & * & -\gamma^2 I & 0 & 0 \\ * & * & * & -\gamma^2 I & 0 \\ * & * & * & * & -I \end{bmatrix}, \quad (30)$$

where

$$\Lambda_1 = \begin{bmatrix} \Lambda_{11} & \tilde{\Phi}_{12} & \Lambda_{13} & -\hat{P}_i & 0 & 0 \\ * & \Lambda_{22} & -\hat{P}_i & \Lambda_{23} & 0 & \Lambda_{24} \\ * & * & \tilde{\Phi}_{33} & \tilde{\Phi}_{34} & 0 & 0 \\ * & * & * & \tilde{\Phi}_{33} & 0 & \Lambda_{24} \\ * & * & * & * & \tilde{\Phi}_{55} & \tilde{\Phi}_{56} \\ * & * & * & * & * & \tilde{\Phi}_{55} \end{bmatrix},$$

$$\Lambda_{11} = \mathbf{He}(A_{mia}\hat{N} + B_{mia}\tilde{\Theta}_{mi}\Gamma_{mio}Y_{mik} + B_{mia}(1 - \tilde{\Theta}_{mi})\bar{\Gamma}_{mio}Y_{mik}) + \beta_{vi}\hat{Q}_{svi} + \hat{Q}_{mi} - \hat{W}_{si} - \hat{W}_{mi},$$

$$\Lambda_{13} = \hat{P}_i - \hat{N} + A_{mia}\hat{N} + B_{mia}\tilde{\Theta}_{mi}\Gamma_{mio}Y_{mik} + B_{mia}(1 - \tilde{\Theta}_{mi})\bar{\Gamma}_{mio}Y_{mik},$$

$$\Lambda_{22} = \mathbf{He}(A_{sia}\hat{N} + B_{sia}\tilde{\Theta}_{si}\Gamma_{sio}Y_{sik} + B_{sia}(1 - \tilde{\Theta}_{si})\bar{\Gamma}_{sio}Y_{sik}) + \beta_{vi}\hat{Q}_{svi} + \hat{Q}_{mi} - \hat{W}_{si} - \hat{W}_{mi},$$

$$\Lambda_{23} = \hat{P}_i - \hat{N} + A_{sia}\hat{N} + B_{sia}\tilde{\Theta}_{si}\Gamma_{sio}Y_{sik} + B_{sia}(1 - \tilde{\Theta}_{si})\bar{\Gamma}_{sio}Y_{sik},$$

$$\Lambda_{24} = -\beta_{vi}B_{mia}\tilde{\Theta}_{mi}\Gamma_{mio}Y_{mik} - \beta_{vi}B_{mia}(1 - \tilde{\Theta}_{mi})\bar{\Gamma}_{mio}Y_{mik}$$

$\Delta\Lambda$ denotes the uncertain part and it is depicted as

$$\Delta\Lambda = \mathbf{He}\left\{ \begin{bmatrix} B_{mia}\tilde{\Theta}_{mi}\Gamma_{mio} \\ 0 \\ B_{mia}\tilde{\Theta}_{mi}\Gamma_{mio} \\ 0_{12 \times 1} \end{bmatrix} G_{mi} \begin{bmatrix} Y_{mik} & 0_{1 \times 14} \end{bmatrix} \right. \\ + \begin{bmatrix} B_{mia}(1 - \tilde{\Theta}_{mi})\bar{\Gamma}_{mio} \\ 0 \\ B_{mia}(1 - \tilde{\Theta}_{mi})\bar{\Gamma}_{mio} \\ 0_{12 \times 1} \end{bmatrix} \bar{G}_{mi} \\ \times \begin{bmatrix} Y_{mik} & 0_{1 \times 14} \end{bmatrix} \\ + \begin{bmatrix} B_{sia}\tilde{\Theta}_{si}\Gamma_{sio} \\ 0_{14 \times 1} \end{bmatrix} G_{si} \begin{bmatrix} 0_{1 \times 6} & -Y_{sik} & 0_{1 \times 8} \end{bmatrix} \\ + \begin{bmatrix} B_{sia}(1 - \tilde{\Theta}_{si})\bar{\Gamma}_{sio} \\ 0_{13 \times 1} \end{bmatrix} \bar{G}_{si} \\ \left. \times \begin{bmatrix} 0_{1 \times 6} & -Y_{sik} & 0_{1 \times 8} \end{bmatrix} \right\}$$

$$\begin{aligned}
& + \begin{bmatrix} 0 \\ B_{sia} \tilde{\Theta}_{si} \Gamma_{sio} \\ 0 \\ B_{sia} \tilde{\Theta}_{si} \Gamma_{sio} \\ 0_{11 \times 1} \end{bmatrix} G_{si} \begin{bmatrix} 0 & Y_{sik} & 0_{1 \times 13} \end{bmatrix} \\
& + \begin{bmatrix} 0 \\ B_{sia} (1 - \tilde{\Theta}_{si}) \bar{\Gamma}_{sio} \\ 0 \\ B_{sia} (1 - \tilde{\Theta}_{si}) \bar{\Gamma}_{sio} \\ 0_{11 \times 1} \end{bmatrix} \bar{G}_{si} \begin{bmatrix} 0 & Y_{sik} & 0_{1 \times 13} \end{bmatrix} \\
& + \begin{bmatrix} 0 \\ \beta_{vi} B_{mia} \tilde{\Theta}_{mi} \Gamma_{mio} \\ 0_{13 \times 1} \end{bmatrix} G_{mi} \\
& \times \begin{bmatrix} 0_{1 \times 5} & -Y_{mik} & 0_{1 \times 9} \end{bmatrix} \\
& + \begin{bmatrix} 0 \\ \beta_{vi} B_{mia} (1 - \tilde{\Theta}_{mi}) \bar{\Gamma}_{mio} \\ 0_{13 \times 1} \end{bmatrix} \bar{G}_{mi} \\
& \times \begin{bmatrix} 0_{1 \times 5} & -Y_{mik} & 0_{1 \times 9} \end{bmatrix} \\
& + \begin{bmatrix} 0_{2 \times 1} \\ B_{sia} \tilde{\Theta}_{si} \Gamma_{sio} \\ 0_{12 \times 1} \end{bmatrix} G_{si} \begin{bmatrix} 0_{1 \times 6} & -Y_{sik} & 0_{1 \times 8} \end{bmatrix} \\
& + \begin{bmatrix} 0_{2 \times 1} \\ B_{sia} (1 - \tilde{\Theta}_{si}) \bar{\Gamma}_{sio} \\ 0_{12 \times 1} \end{bmatrix} \bar{G}_{si} \\
& \times \begin{bmatrix} 0_{1 \times 6} & -Y_{sik} & 0_{1 \times 8} \end{bmatrix} \\
& + \begin{bmatrix} 0_{3 \times 1} \\ \beta_{vi} B_{mia} \tilde{\Theta}_{mi} \Gamma_{mio} \\ 0_{11 \times 1} \end{bmatrix} G_{mi} \\
& \times \begin{bmatrix} 0_{1 \times 5} & -Y_{mik} & 0_{1 \times 9} \end{bmatrix} \\
& + \begin{bmatrix} 0_{3 \times 1} \\ \beta_{vi} B_{mia} (1 - \tilde{\Theta}_{mi}) \bar{\Gamma}_{mio} \\ 0_{11 \times 1} \end{bmatrix} \bar{G}_{mi} \\
& \times \begin{bmatrix} 0_{1 \times 5} & -Y_{mik} & 0_{1 \times 9} \end{bmatrix} \}.
\end{aligned}$$

By applying Lemmas 2 and 3, the corresponding result in Equation (29) can be obtained directly. $\square$

3. Numerical Example

To validate the effectiveness of the closed-loop system (13), four identical two-DOF manipulators are set as the master and slave parts, respectively. Slaves 1 and 2 are controlled with the control signals of Masters 1 and 2, respectively. At the same time, the signals from Slaves 1 and 2 are fed back to both the masters, so that Masters 1 and 2 can synchronously control Slaves 1 and 2. Let $m_{j1} = 1\text{kg}$, $m_{j2} = 1\text{kg}$, $l_{j1} = 1\text{m}$, and $l_{j2} = 1\text{m}$, with angular positions q_{mi1} , q_{mi2} , q_{si1} , and q_{si2} being constrained with $[-\pi/2 \ \pi/2]$. We have utilized a

nine-rule fuzzy model for the master manipulators and the slave manipulators, respectively. As in [4,28], the specific fuzzy rules for the first master manipulator are listed as follows:

Rule 1 If x_{m11} is about $-\frac{\pi}{2}$ and x_{m12} is about $-\frac{\pi}{2}$,

then $\dot{x} = A_{m11}x + B_{m11}(u_{m1}^F + F_{h1})$.

Rule 2 If x_{m11} is about $-\frac{\pi}{2}$ and x_{m12} is about 0,

then $\dot{x} = A_{m12}x + B_{m12}(u_{m1}^F + F_{h1})$.

Rule 3 If x_{m11} is about $-\frac{\pi}{2}$ and x_{m12} is about $\frac{\pi}{2}$,

then $\dot{x} = A_{m13}x + B_{m13}(u_{m1}^F + F_{h1})$.

Rule 4 If x_{m11} is about 0 and x_{m12} is about $-\frac{\pi}{2}$,

then $\dot{x} = A_{m14}x + B_{m14}(u_{m1}^F + F_{h1})$.

Rule 5 If x_{m11} is about 0 and x_{m12} is about 0,

then $\dot{x} = A_{m15}x + B_{m15}(u_{m1}^F + F_{h1})$.

Rule 6 If x_{m11} is about 0 and x_{m12} is about $\frac{\pi}{2}$,

then $\dot{x} = A_{m16}x + B_{m16}(u_{m1}^F + F_{h1})$.

Rule 7 If x_{m11} is about $\frac{\pi}{2}$ and x_{m12} is about $\frac{\pi}{2}$,

then $\dot{x} = A_{m17}x + B_{m17}(u_{m1}^F + F_{h1})$.

Rule 8 If x_{m11} is about $\frac{\pi}{2}$ and x_{m12} is about 0,

then $\dot{x} = A_{m18}x + B_{m18}(u_{m1}^F + F_{h1})$.

Rule 9 If x_{m11} is about $\frac{\pi}{2}$ and x_{m12} is about $-\frac{\pi}{2}$,

then $\dot{x} = A_{m19}x + B_{m19}(u_{m1}^F + F_{h1})$.

where

$$A_{m11} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 5.927 & -0.315 & -0.001 & -8.4 \times 10^{-6} \\ -6.859 & 3.155 & 0.002 & 6.2 \times 10^{-6} \end{bmatrix},$$

$$A_{m12} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 3.0428 & 0.1791 & -0.0011 & -0.0002 \\ 3.5436 & 2.5611 & 0.0313 & 1.14 \times 10^{-5} \end{bmatrix},$$

$$\begin{aligned}
A_{m13} &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 6.2728 & 0.4339 & 0.0030 & -0.0001 \\ 9.1041 & -1.0574 & 0.0158 & -3.2 \times 10^{-5} \end{bmatrix}, \\
A_{m14} &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 6.4535 & 1.2427 & 0.0017 & 0.0002 \\ -3.1873 & 5.1911 & -0.0306 & -1.8 \times 10^{-5} \end{bmatrix}, \\
A_{m15} &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 11.1336 & -1.8145 & 0.0000 & 0.0000 \\ -9.0918 & 9.1638 & 0.0000 & 0.0000 \end{bmatrix}, \\
A_{m16} &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 6.1702 & 1.6870 & -0.0010 & -0.0002 \\ -2.3559 & 4.5298 & 0.0314 & 1.1 \times 10^{-5} \end{bmatrix}, \\
A_{m17} &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 6.1206 & 0.6205 & -0.0041 & 0.0001 \\ 8.8794 & -1.0119 & -0.0193 & 4.4 \times 10^{-5} \end{bmatrix}, \\
A_{m18} &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 3.6421 & 0.0721 & 0.0018 & 0.0002 \\ 2.4290 & 2.9832 & -0.0305 & -1.9 \times 10^{-5} \end{bmatrix}, \\
A_{m19} &= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 6.2933 & -0.2188 & -0.0009 & -1.2 \times 10^{-5} \\ -7.4649 & 3.2693 & 0.0024 & 9.2 \times 10^{-6} \end{bmatrix}, \\
B_{m11} &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & -1 \\ -1 & 2 \end{bmatrix}, \quad B_{m12} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.5 & 0 \\ 0 & 1 \end{bmatrix}, \\
B_{m13} &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}, \quad B_{m14} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.5 & 0 \\ 0 & 1 \end{bmatrix}, \\
B_{m15} &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & -1 \\ -1 & 2 \end{bmatrix}, \quad B_{m16} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.5 & 0 \\ 0 & 1 \end{bmatrix}, \\
B_{m17} &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}, \quad B_{m18} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.5 & 0 \\ 0 & 1 \end{bmatrix}, \\
B_{m19} &= \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & -1 \\ -1 & 2 \end{bmatrix}
\end{aligned}$$

The nine fuzzy rules for the master manipulators in System 2 and the slave manipulators are similar to the rules for the master manipulators in System 1. For simplicity, some membership functions (Figure 3) are used for Rule 1 through Rule 9.

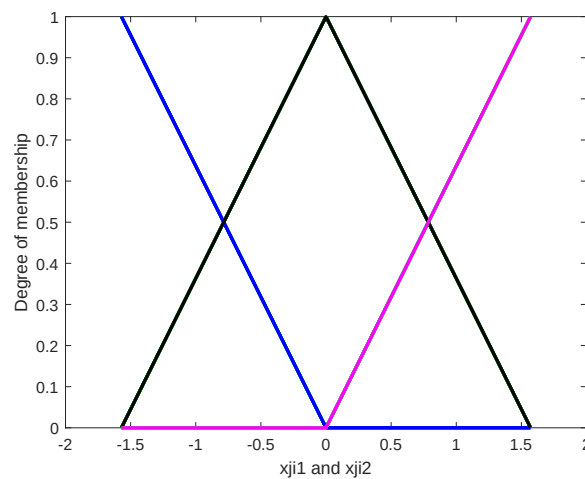


Figure 3. Fuzzy sets.

The communication delays are set as

$$\begin{aligned} d_{m1} &= \sin\left(\frac{1}{2}t\right) + 1, & d_{m2} &= \sin\left(\frac{1}{5}t\right) + 1 \\ d_{s1} &= \sin\left(\frac{1}{4}t\right) + 1, & d_{s2} &= \sin\left(\frac{1}{3}t\right) + 1, & \mu &= 0.7 \end{aligned}$$

The parameters of failures are selected as $\tilde{m}_{mib} = 0.85$, $\tilde{m}_{sib} = 0.85 (i = 1, 2) (b = 1, 2)$, $\mathbb{E}\{\theta_{nmi}(k) = 1\} = 0.35$, $\mathbb{E}\{\theta_{nsi}(k) = 1\} = 0.45$, $n = 1, 2, \dots, p$, where

$$\begin{cases} 0.75 \leq m_{nmi} \leq 0.85, & \theta_{nmi} = 1 \\ 0.85 \leq m_{nmi} \leq 0.95, & \theta_{nmi} = 0 \end{cases}, \begin{cases} 0.75 \leq m_{nsi} \leq 0.85, & \theta_{nsi} = 1 \\ 0.85 \leq m_{nsi} \leq 0.95, & \theta_{nsi} = 0 \end{cases}$$

Through Theorem 3, the following results are obtained:

$$\begin{aligned} K_{m11} &= \begin{bmatrix} 0.0206 & -0.0181 & 0.0010 & -0.0027 \\ -0.0339 & 0.0368 & 0.0002 & 0.0062 \end{bmatrix}, \\ K_{m21} &= \begin{bmatrix} 0.0280 & -0.0173 & -0.0004 & -0.0006 \\ -0.0480 & 0.0389 & 0.0016 & 0.0037 \end{bmatrix}, \\ K_{s11} &= \begin{bmatrix} 0.0344 & -0.0285 & 0.0020 & -0.0043 \\ -0.0544 & 0.0560 & -0.0004 & 0.0094 \end{bmatrix}, \\ K_{s21} &= \begin{bmatrix} 0.0337 & -0.0309 & 0.0027 & -0.0055 \\ -0.0540 & 0.0603 & -0.0013 & 0.0110 \end{bmatrix}, \\ K_{m12} &= \begin{bmatrix} 0.0012 & -0.0009 & 0.0007 & 0.0000 \\ 0.0001 & 0.0043 & -0.0008 & 0.0002 \end{bmatrix}, \\ K_{m22} &= \begin{bmatrix} 0.0017 & -0.0013 & 0.0001 & -0.0006 \\ 0.0028 & 0.0057 & -0.0005 & -0.0007 \end{bmatrix}, \end{aligned}$$

$$\begin{aligned}
K_{s12} &= \begin{bmatrix} 0.0022 & -0.0013 & 0.0011 & 0.0000 \\ 0.0003 & 0.0072 & -0.0012 & 0.0004 \end{bmatrix}, \\
K_{s22} &= \begin{bmatrix} 0.0018 & -0.0016 & 0.0017 & 0.0004 \\ -0.0002 & 0.0073 & -0.0018 & -0.0002 \end{bmatrix}, \\
K_{m13} &= \begin{bmatrix} 19.7263 & -16.8186 & 3.4577 & -2.9499 \\ -100.8789 & 87.0535 & -16.7270 & 14.3567 \end{bmatrix}, \\
K_{m23} &= \begin{bmatrix} 19.6649 & -16.7280 & 3.4105 & -2.9245 \\ -101.0030 & 87.2124 & -16.8201 & 14.4125 \end{bmatrix}, \\
K_{s13} &= \begin{bmatrix} 19.6173 & -16.6109 & 3.4252 & -2.9185 \\ -100.9905 & 87.2515 & -16.7678 & 14.3959 \end{bmatrix}, \\
K_{s23} &= \begin{bmatrix} 19.6623 & -16.6773 & 3.4472 & -2.9360 \\ -100.8945 & 87.1448 & -16.8763 & 14.4926 \end{bmatrix}, \\
K_{m14} &= \begin{bmatrix} 0.0052 & -0.0012 & 0.0007 & -0.0001 \\ -0.0052 & 0.0093 & 0.0001 & 0.0015 \end{bmatrix}, \\
K_{m24} &= \begin{bmatrix} 0.0076 & -0.0007 & -0.0015 & 0.0005 \\ -0.0099 & 0.0136 & -0.0002 & -0.0028 \end{bmatrix}, \\
K_{s14} &= \begin{bmatrix} 0.0098 & -0.0022 & 0.0011 & -0.0001 \\ -0.0094 & 0.0172 & 0.0003 & 0.0023 \end{bmatrix}, \\
K_{s24} &= \begin{bmatrix} 0.0081 & -0.0024 & 0.0020 & -0.0002 \\ -0.0077 & 0.0144 & 0.0006 & 0.0039 \end{bmatrix}, \\
K_{m15} &= \begin{bmatrix} 0.1468 & -0.1382 & 0.0042 & -0.0081 \\ -0.2253 & 0.2248 & -0.0059 & 0.0150 \end{bmatrix}, \\
K_{m25} &= \begin{bmatrix} 0.1831 & -0.1605 & -0.0027 & 0.0004 \\ -0.2954 & 0.2747 & 0.0018 & -0.0008 \end{bmatrix}, \\
K_{s15} &= \begin{bmatrix} 0.2434 & -0.2275 & 0.0051 & -0.0104 \\ -0.3675 & 0.3670 & -0.0065 & 0.0191 \end{bmatrix}, \\
K_{s25} &= \begin{bmatrix} 0.2262 & -0.2155 & 0.0074 & -0.0107 \\ -0.3421 & 0.3448 & -0.0085 & 0.0193 \end{bmatrix}, \\
K_{m16} &= \begin{bmatrix} 0.0022 & -0.0005 & 0.0000 & 0.0000 \\ -0.0013 & 0.0027 & 0.0001 & 0.0001 \end{bmatrix}, \\
K_{m26} &= \begin{bmatrix} 0.0040 & 0.0001 & -0.0007 & 0.0004 \\ -0.0036 & 0.0053 & -0.0007 & -0.0011 \end{bmatrix}, \\
K_{s16} &= \begin{bmatrix} 0.0041 & -0.0009 & 0.0000 & 0.0000 \\ -0.0023 & 0.0050 & 0.0001 & 0.0002 \end{bmatrix}, \\
K_{s26} &= \begin{bmatrix} 0.0032 & -0.0010 & 0.0001 & -0.0000 \\ -0.0017 & 0.0039 & 0.0008 & 0.0002 \end{bmatrix}, \\
K_{m17} &= \begin{bmatrix} 19.6178 & -16.9980 & -0.1422 & 0.1019 \\ -126.1310 & 110.4224 & -1.6427 & 1.7055 \end{bmatrix}, \\
K_{m27} &= \begin{bmatrix} 19.4410 & -16.8080 & -0.1425 & 0.0881 \\ -126.4699 & 110.7685 & -1.5951 & 1.6406 \end{bmatrix},
\end{aligned}$$

$$\begin{aligned}
K_{s17} &= \begin{bmatrix} 19.6332 & -16.9116 & -0.1323 & 0.0967 \\ -126.1837 & 110.5798 & -1.6380 & 1.7060 \end{bmatrix}, \\
K_{s27} &= \begin{bmatrix} 19.7837 & -17.0679 & -0.1058 & 0.0749 \\ -125.9301 & 110.3306 & -1.5890 & 1.6657 \end{bmatrix}, \\
K_{m18} &= \begin{bmatrix} 0.0034 & -0.0017 & 0.0017 & -0.0002 \\ -0.0050 & 0.0071 & -0.0001 & 0.0032 \end{bmatrix}, \\
K_{m28} &= \begin{bmatrix} 0.0102 & -0.0024 & 0.0009 & -0.0002 \\ 0.0063 & 0.0183 & -0.0002 & 0.0016 \end{bmatrix}, \\
K_{s18} &= \begin{bmatrix} 0.0074 & -0.0038 & 0.0037 & -0.0005 \\ -0.0108 & 0.0155 & -0.0003 & 0.0070 \end{bmatrix}, \\
K_{s28} &= \begin{bmatrix} 0.0029 & -0.0048 & 0.0039 & -0.0004 \\ -0.0166 & 0.0094 & 0.0000 & 0.0074 \end{bmatrix}, \\
K_{m19} &= \begin{bmatrix} 0.0503 & -0.0391 & 0.0028 & -0.0074 \\ -0.0825 & 0.0779 & -0.0008 & 0.0160 \end{bmatrix}, \\
K_{m29} &= \begin{bmatrix} 0.0586 & -0.0354 & -0.0008 & -0.0015 \\ -0.1020 & 0.0773 & 0.0031 & 0.0064 \end{bmatrix}, \\
K_{s19} &= \begin{bmatrix} 0.0853 & -0.0632 & 0.0050 & -0.0119 \\ -0.1352 & 0.1225 & -0.0024 & 0.0246 \end{bmatrix}, \\
K_{s29} &= \begin{bmatrix} 0.0828 & -0.0670 & 0.0073 & -0.0148 \\ -0.1327 & 0.1292 & -0.0049 & 0.0281 \end{bmatrix}.
\end{aligned}$$

In this article, we consider the interactive torque of human operators and environments as the passive spring–damper model, which can be described as

$$\begin{aligned}
F_{hi} &= f_{hi} + D_{hi}\dot{q}_{mi} + S_{hi}q_{mi} \\
F_{ei} &= f_{ei} + D_{ei}\dot{q}_{si} + S_{ei}q_{si}
\end{aligned}$$

where $D_{hi}, D_{ei} \in \mathcal{R}^{n \times n}$ denote the damping matrices of the operator and environment. $S_{hi}, S_{ei} \in \mathcal{R}^{n \times n}$ express the spring matrices of the operator and environment. f_{hi} and f_{ei} are the extra operator and environment torque, and they are assumed to be bounded. $D_{hi}, D_{ei}, S_{hi}, S_{ei}$ are all positive definite and diagonal constant matrices, and $D_{h1} = D_{h2} = \text{diag}(0.05, 0.05)$, $S_{h1} = \text{diag}(3, 3)$, $S_{h2} = \text{diag}(2.95, 2.95)$, $D_{e1} = D_{e2} = \text{diag}(0.008, 0.008)$, $S_{e1} = \text{diag}(0.08, 0.08)$, and $S_{e2} = \text{diag}(0.06, 0.06)$.

The position tracking performance of the corresponding master–slave robots were achieved and are shown in Figure 2. The synchronous motion performance of the slave robots is shown in Figure 3. Obviously, Figures 2 and 3 reflect the good displacement tracking performance of the designed control scheme, and we introduced partial actuator failure fault in the simulation period of 2 to 3 s, with the simulation results showing strong system robustness. The corresponding force feedback tracking performance of environmental force and human force is shown in Figure 4 and Figure 5, respectively. This reflects the good fitting and tracking performance of the controller's environmental force and operating force. From the above results, it is obvious that the designed controller is effective, even in the case of partial actuator failure. By using the algorithm in reference [4], a comparison example is given in Figures 6–8, and the closed loop is uncontrollable when actuator failure exists.

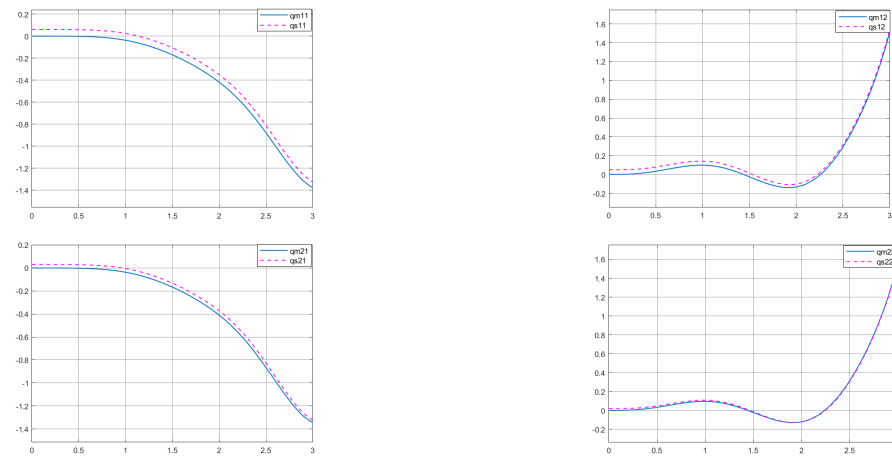


Figure 4. Synchronous motion of master and slave robots by using the proposed fault-tolerant control scheme.

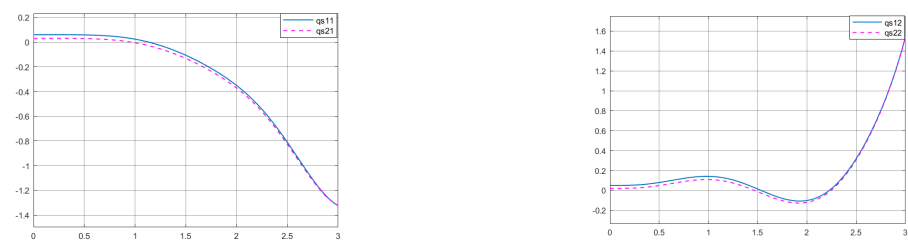


Figure 5. Synchronous motion of Slave 1 and Slave 2 robots.

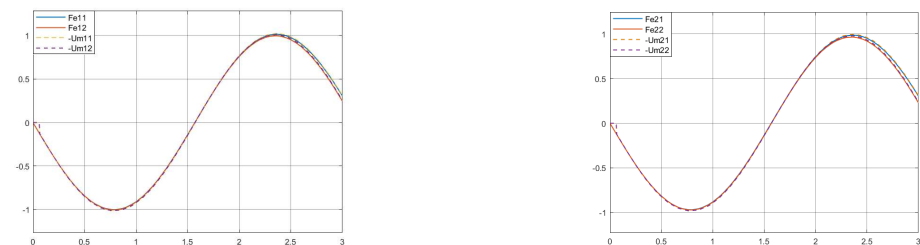


Figure 6. Fitting of environmental force.

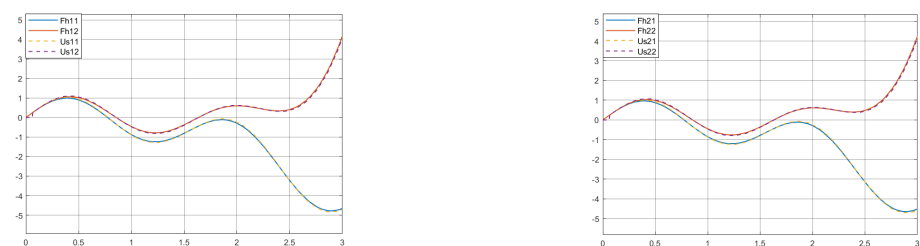


Figure 7. Fitting of human operating force.

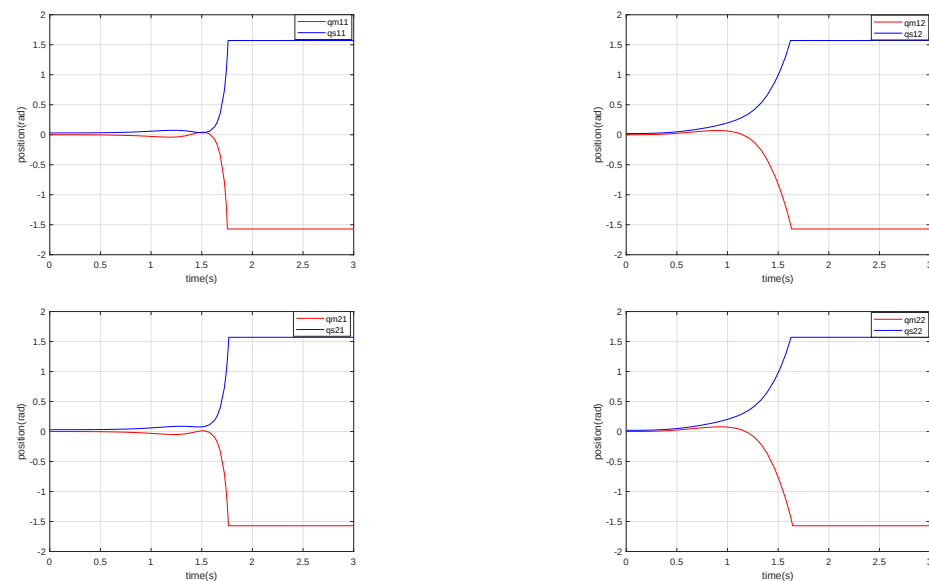


Figure 8. Synchronous motion of master and slave robots by using the algorithm in reference [4].

4. Conclusions

For the considered multilateral teleoperation systems that may have partial actuator failure, the issues of position synchronization and force tracking were achieved simultaneously through a novel failure-distribution-dependent H_∞ fuzzy fault-tolerant control scheme. In view of the fact that the multilateral teleoperation systems are nonlinear and have communication delays, the nonlinear multilateral systems are modeled as a kind of T-S fuzzy system with multiple time-varying delays. Then, based on the distribution characteristic of failures, by introducing a series of tradeoff coefficients, a novel failure-distribution-dependent fault-tolerant control algorithm is provided to ensure force tracking in spite of failures, and the purpose of position synchronization is achieved (not only the master and slave robots' position synchronization but also the position synchronization between each slave robot). After this, the sufficient conditions to ensure the mean square stability and interference suppression level γ of the closed-loop system are given. In the end, a numerical example is given to show the effectiveness of the proposed method.

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