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Research on Formation Rendezvous for Unmanned Surface Vehicles Considering Arbitrary Initial Positions and Headings

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Abstract: The formation of unmanned surface vehicles (USVs) stands as a paramount concern in USV formation control research. In addressing this, this paper introduces a novel formation strategy for USVs, which accounts for their arbitrary initial positions and headings. Firstly, given the random distribution of the USVs' initial positions, to enhance the efficiency of formation assembly, the point set of the USVs is matched with basic formations using the Hausdorff distance to establish the formation. Building upon this foundation and acknowledging the potential hindrance posed by the arbitrary initial headings of the USVs, which could hinder their ability to reach the designated formation target points, a cost matrix is constructed based on the designed distance–azimuth evaluation function. Subsequently, the Hungarian algorithm is utilized to efficiently assign formation target points for the USVs. Ultimately, the effectiveness of the proposed strategy is validated through simulation experiments.

Keywords: USVs; formation determination; evaluation function; assignment of formation target points

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Citation: Zhang, Y.; Peng, P.; Wei, J. Research on Formation Rendezvous for Unmanned Surface Vehicles Considering Arbitrary Initial Positions and Headings. *Electronics* **2024**, *13*, 2867. https://doi.org/10.3390/electronics13142867

Academic Editor: Felipe Jiménez

Received: 27 May 2024
Revised: 9 July 2024
Accepted: 13 July 2024
Published: 20 July 2024

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1. Introduction

As advanced maritime agents, unmanned surface vehicles (USVs) have developed rapidly in recent years. Due to the increasing complexity of the marine environment and the diversity of operational tasks, there is often a need to assemble formations of multiple USVs to achieve higher efficiency and quality in operations. As one of the key technologies of unmanned cooperative systems, unmanned formation control has attracted the attention and research of many scholars.

At present, the most researched methods of USV formation control technology include the leader–follower method [1,2], the behavior-based method [3,4], virtual structures [5,6], the consensus control method [7,8] and others. The authors of [9] proposed a reinforcement learning algorithm based on random braking, which solved the problem of formation transformation and maintenance when some nodes failed while realizing a formation rendezvous of USVs. In [10], the collision avoidance and obstacle avoidance problems of USVs were further considered, and a formation generation algorithm was proposed by combining the virtual structure method and the artificial potential field method. A trajectory planning algorithm based on distributed shell-space decomposition for autonomous aggregation of multiple unmanned underwater vehicles was proposed with the goal of simultaneous aggregation and time optimization in [11]. Aiming at the problem of formation transformation, the authors of [12] proposed a circular trajectory tracking algorithm with adaptive parameters for formation and used the improved differential evolution algorithm to design the formation reconstruction strategy. In order to improve the scalability of formations, a formation control scheme was proposed for USVs based on a distributed deep reinforcement learning algorithm in [13]. This scheme enables the formation to perform

corresponding formation transformation according to the number of USVs. In [14], a cooperative control method based on null space behavior is proposed to realize the formation and maintenance of USVs cluster formation. In [15], the centralized and distributed formation control methods were combined to propose a formation transformation algorithm that avoids collisions between members of the formation.

Research on the assembly of USV formations [16] mainly studies how to determine the formation target point of each USV and reach the point to complete the formation rendezvous, which is the prerequisite for formation control. However, the abovementioned literature did not consider the formation problem when studying formation rendezvous or formation transformation; instead, they adopted the pre-determined rendezvous target points of the USVs. In practice, this method is prone to the situation in which the USVs are far away from their rendezvous points, resulting in low rendezvous efficiency for the USVs. To address the problem of how to select the appropriate target points, the assignment method is generally adopted [17,18]. This method obtains the distance information matrix of the distances between all agents and the aggregated target points and then selects a set of optimal solutions that satisfy the current model through matrix transformation and other operations to minimize the total distance from the agents to the target points. Considering that the method of [17,18] relied on the assumption that each agent knows the state of all other agents, the authors of [19] further proposed a dynamic role assignment algorithm to complete the assembly of formations and realize obstacle avoidance.

It is worth noting that the abovementioned literature generally preset the formation of multiple agents. However, in practice, the initial distribution of USVs at sea is often disorderly and random. A fixed preset formation may result in significant distances between the USVs and their designated points. Furthermore, the main research objects in the above literature on formation research were robots. For USVs, because they cannot move at any angle in practice, the arbitrary initial headings of these vehicles make it difficult to sail along two straight lines to the target points, and the straight-line distance may not be the actual navigation distance.

Based on the above analysis, a formation strategy for USVs considering arbitrary initial positions and headings is proposed in this paper. The contributions of this paper are as follows:

(1) Regarding the problem of formation determination for USVs, different from the conventional approach of pre-determining fixed formation target points as seen in most literature, the formation with the closest resemblance to the initial position distribution of USVs based on the Hausdorff distance is adopted in this paper. To enhance aggregation efficiency, this method allows flexible formation determination based on the initial distribution of USVs.

(2) Regarding the problem of target-point assignment for USVs, different from previous studies [17,18], this paper comprehensively considers the distances between USVs and target points along with the USVs' arbitrary initial headings. A distance–azimuth evaluation function is designed and integrated with the Hungarian algorithm to facilitate the assignment of target points. From a practical point of view, this method effectively avoids the situation in which the USVs cannot reach the target points smoothly due to limitations on the headings of the USVs.

The remainder of this paper is organized as follows: Section 2 discusses the preliminaries, Section 3 presents the research on formation strategy, Section 4 provides a simulation study and Section 5 presents the conclusions.

2. Preliminaries

2.1. Hausdorff Distance

Hausdorff distance is a method used to measure the degree of matching between point sets [20]. This method does not require the establishment of correspondence between point sets; only the maximum distance between sets is required. The Hausdorff distance is used to evaluate the similarity between two finite closed sets.

Assuming that there are two sets, the Hausdorff distance between the two points sets is defined as follows [21].

$$H(X,Y) = \max\{h(X,Y), h(Y,X)\} \tag{1}$$

where $H(X,Y)$ denotes the maximum value of the minimum distance between all points in set X and points in set Y, then denotes the maximum value of the minimum distance between all points in set X and points in set Y, as shown in the following formulae:

$$h(X,Y) = \max_{x \in X}\left\{\min_{y \in Y}\|x - y\|\right\} \tag{2}$$

$$h(Y,X) = \max_{y \in Y}\left\{\min_{x \in X}\|y - x\|\right\} \tag{3}$$

where $\|\cdot\|$ denotes the Euclidean distance between a set and its points.

2.2. Hungarian Allocation Algorithm [22]

The Hungarian method is an algorithm to find the optimal allocation for a given cost matrix. Consider the following cost matrix:

$$C = \begin{bmatrix} c_{(1,1)} & c_{(1,2)} & \cdots & c_{(1,n)} \\ c_{(2,1)} & c_{(2,2)} & \cdots & c_{(2,n)} \\ \vdots & \vdots & & \vdots \\ c_{(n,1)} & c_{(n,2)} & \cdots & c_{(n,n)} \end{bmatrix} \tag{4}$$

For this matrix, the Hungarian algorithm selects multiple c values to minimize the cost function by defining the cost function of Equation (5). The specific principal process can be seen in [22].

$$J = \sum_{i}^{n} c(i, j_i) \tag{5}$$

where j_i is positive integer with $1 \leq j_i \leq n$, $j_1 \neq j_2 \neq \ldots \neq j_n$.

3. Research on Formation Strategy

The purpose of this paper is to propose a formation strategy for USVs considering their arbitrary initial positions and headings. This strategy would enable USVs to complete formation determination and allocation of formation target points. This chapter is divided into two parts: formation determination and allocation of formation target points. A frame diagram of the formation strategy for USV assembly is shown in Figure 1.

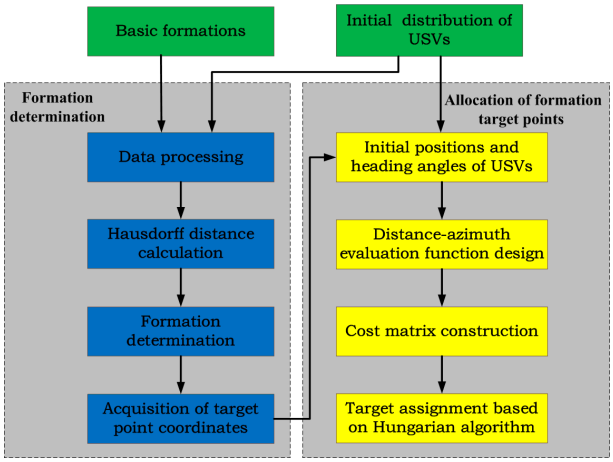


Figure 1. System flow diagram.

3.1. Formation Determination

For the problem of USVs' formation, a formation method based on Hausdorff distance is proposed in this paper. The Hausdorff distance is calculated and used to measure the distribution similarity between the point set of the USVs and the basic formations. The minimum distance is the formation matched by the USVs.

At present, the general research literature mainly focuses on theoretical research on formation control for fewer than 10 USVs. The common basic USV formations are shown in Figure 2.

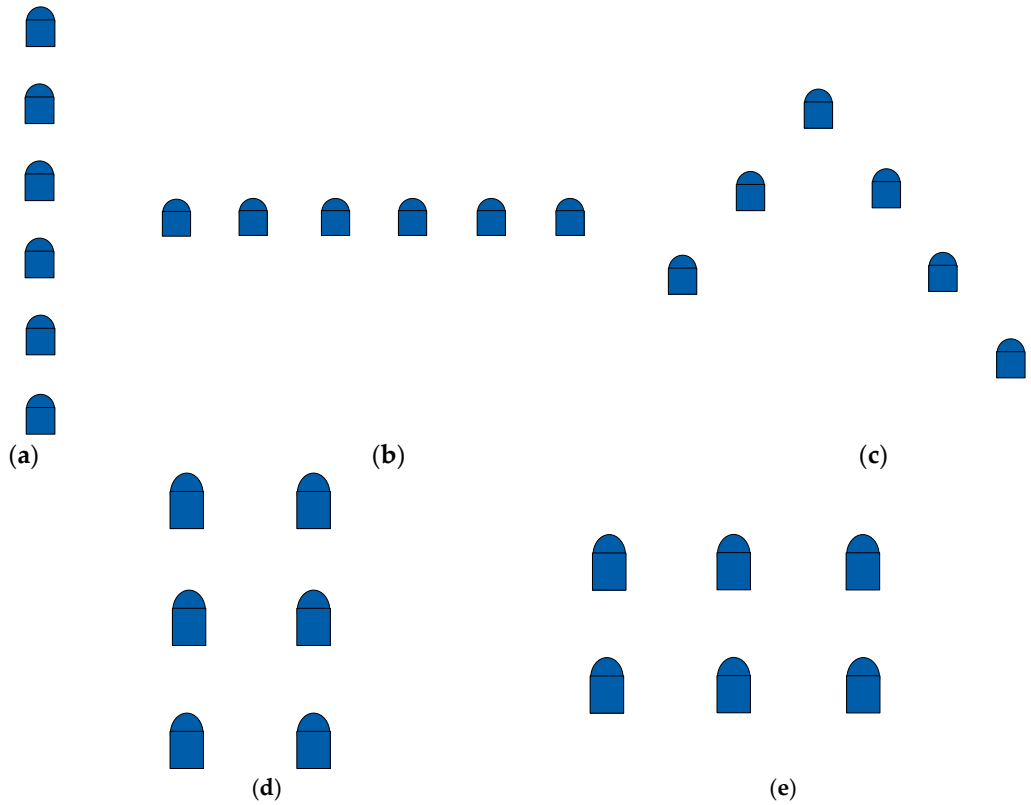


Figure 2. Diagrams of basic USV formations. (**a**) Single longitudinal formation; (**b**) Single horizontal formation; (**c**) Wedge formation; (**d**) Double longitudinal formation; (**e**) Double horizontal formation.

Considering the random distribution of the initial positions of the USVs, in order to reduce the navigation distances of the USVs to their formation points and improve their rendezvous efficiency, it is necessary to select a formation similar to the initial position distributions of the USVs. Therefore, a formation determination method based on Hausdorff distance is proposed in this paper. The specific steps are as follows.

3.1.1. Distance Matrix Construction between the Point Set of the USVs and the Formation

The point set of the USVs is defined as $V = \{v_1, v_2, \ldots, v_n\}$; v_i represents the position of the ith USV, that is, $v_i = [x_{vi}, y_{vi}]^T$; n represents the number of USVs.

In order to make the point set of the USVs and the formation point set match more closely, the two point sets are normalized, respectively. The calculation formulas are as follows.

$$\begin{cases} \overline{x}_{vi} = \dfrac{x_{vi} - \xi_{xv}}{\eta_{xv}}, \overline{y}_{vi} = \dfrac{y_{vi} - \xi_{yv}}{\eta_{yv}} \end{cases} \tag{6}$$

$$\begin{cases} \overline{x}_{gi} = \dfrac{x_{gi} - \xi_{xg}}{\eta_{xg}}, \overline{y}_{gi} = \dfrac{y_{gi} - \xi_{yg}}{\eta_{yg}} \end{cases} \tag{7}$$

where $[\xi_{xv}, \xi_{yv}]$ and $[\xi_{xg}, \xi_{yg}]$ denote the coordinate average value of each respective point set element, that is, $\xi_{yv} = \sum_{i=1}^{n} \frac{y_{vi}}{n}$, $\xi_{xg} = \sum_{i=1}^{n} \frac{x_{gi}}{n}$, $\xi_{yg} = \sum_{i=1}^{n} \frac{y_{gi}}{n}$ and $\xi_{xv} = \sum_{i=1}^{n} \frac{x_{vi}}{n}$, while $[\eta_{xv}, \eta_{yv}]$ and $[\eta_{xg}, \eta_{yg}]$ represent the coordinate variance of each respective point set element, that is, $\eta_{xv} = \sqrt{\frac{\sum_{i=1}^{n}(x_i - \xi_{xv})^2}{n}}$, $\eta_{yv} = \sqrt{\frac{\sum_{i=1}^{n}(y_i - \xi_{yv})^2}{n}}$, $\eta_{xg} = \sqrt{\frac{\sum_{i=1}^{n}(x_{gi} - \xi_{xg})^2}{n}}$ and $\eta_{yg} = \sqrt{\frac{\sum_{i=1}^{n}(y_{gi} - \xi_{yg})^2}{n}}$.

The distance matrix D is constructed as follows:

$$D = \begin{bmatrix} \overline{d}_{11} & \overline{d}_{12} & \cdots & \overline{d}_{1n} \\ \overline{d}_{21} & \overline{d}_{22} & \cdots & \overline{d}_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ \overline{d}_{n1} & \overline{d}_{n2} & \cdots & \overline{d}_{nn} \end{bmatrix} \tag{8}$$

where $\overline{d}_{ij} = \sqrt{\left(\overline{x}_{vi} - \overline{x}_{gj}\right)^2 + \left(\overline{y}_{vi} - \overline{y}_{gi}\right)^2} (i = 1, 2, \ldots, n; j = 1, 2, \ldots, n)$.

3.1.2. Formation Determination Based on Hausdorff Distance

In order to select the formation that is closest to the point set of the USVs, the distance between them is calculated based on the Hausdorff distance; the specific process for formation set D_1 is taken as an example.

$$h_1(D_1) = \max_{i \in n}\left\{\min_{j \in n}(d_{ij})\right\} \tag{9}$$

$$h_2(D_1) = \max_{j \in n}\left\{\min_{i \in n}(d_{ij})\right\} \tag{10}$$

$$H(D_1) = \max\{h_1(D_1), h_2(D_1)\} \tag{11}$$

where $H(D_1)$ is the Hausdorff distance between the point set of the USVs and the formation, which measures the similarity between the two point sets.

The formation point set corresponding to the minimum distance from the unmanned boats' point set is calculated as shown below.

$$H = \min\{H(D_1), H(D_2), H(D_3), H(D_4), H(D_5)\} \tag{12}$$

The point set whose distribution is most similar to that of the point set of the USVs is used as the rendezvous formation of the USVs. The formation point set is expressed as $\begin{cases} \overline{G}_H = \{\overline{g}_{H1}, \overline{g}_{H2}, \ldots, \overline{g}_{Hn}\} \\ \overline{g}_{Hi} = [\overline{x}_{Hi}, \overline{y}_{Hi}]^{\mathrm{T}}, i = 1, 2, \ldots, n \end{cases}$.

In order to obtain the specific coordinates of the target points of the rendezvous formation of the USVs, the formation point set is projected into the coordinate system of the USVs to obtain $\begin{cases} G_H = \{g_{H1}, g_{H2}, \ldots, g_{Hn}\} \\ g_{Hi} = [x_{Hi}, y_{Hi}]^{\mathrm{T}}, i = 1, 2, \ldots, n \end{cases}$, and the calculation formula is as follows.

$$\begin{cases} x_{Hi} = \overline{x}_{Hi}\eta_{Hx} + \xi_{Hx} \\ y_{Hi} = \overline{y}_{Hi}\eta_{Hy} + \xi_{Hy} \end{cases} \tag{13}$$

3.2. Allocation of Formation Target Points

After determining the rendezvous formation of the USVs, it is necessary to consider how the USVs select the formation target points in the corresponding formation. Considering that the arbitrary initial headings of the USVs will limit their maneuvering, in order

to ensure that the USVs could reach the formation target points smoothly, reasonable formation target-point allocation plays a key role. Aiming at this problem, a target allocation method based on distance and heading angle is proposed in this paper.

Target-point allocation is based on the point set of the USVs and the matching formation point set as discussed in the previous section. Usually, the literature generally only considers the distance between the agents and the target points [17,18]. Based on the following distance matrix, the target points are allocated with the minimum distance as the optimization goal.

$$D_d = \begin{bmatrix} d_{11} & d_{12} & \cdots & d_{1n} \\ d_{21} & d_{22} & \cdots & d_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ d_{n1} & d_{n2} & \cdots & d_{nn} \end{bmatrix} \tag{14}$$

where $d_{ij}(i = 1, 2, \ldots, n; j = 1, 2, \ldots, n)$ represent the Euclidean distances between the individual agents and the target points.

However, for USVs with randomly distributed initial headings, the actual navigation distances of the USVs to the formation target points are greater than the ideal linear distances due to the uncertainty of the azimuth angle formed by the target points and the headings of the USVs, which makes it difficult for the USVs to reach the target points smoothly. Therefore, when considering the distances between the USVs and the target points, the influence of the headings needs to be considered. To this end, a distance–azimuth evaluation function for target assignment is proposed in this paper, as shown below.

$$s(\varphi_i, d_{ij}) = d_{ij} \exp\left(k \frac{(\varphi_{ij}/180)^2}{(d_{ij}/\max(D_d))} \right) \tag{15}$$

where φ_i represents the initial azimuth angle formed by the heading angle of the ith USV and the jth target point, and $\varphi_i \in [-180^\circ, 180^\circ]$; $\max(D_d)$ represents the maximum value of all elements in the distance matrix; and $k > 0$ is the parameter to be designed.

Remark 1. The proposed evaluation function with target assignment is based on the following considerations: (a) The original consideration of shortening the navigation distance is retained. (b) The introduction of φ_i into the function $\exp(\cdot)$ means that the evaluation function takes into account the influence of azimuth angle. The larger the absolute value of azimuth angle is, the larger the evaluation function value is. (c) The function $\exp(\cdot)$ contains d_{ij}. When d_{ij} is larger, the value of $\exp(\cdot)$ is smaller. This mainly reflects the consideration that in practice, when the USVs are farther from the target points, the influence of the headings of the USVs on the route is smaller, and the navigation routes are closer to a straight line. (d) $\left(\frac{\varphi_{ij}}{180}\right)^2$ and $\frac{d_{ij}}{\max(D_d)}$ in the function $s(\varphi_i, d_{ij})$ are to normalize the azimuth φ_{ij} and distance d_{ij}.

Based on the designed evaluation function $s(\varphi_i, d_{ij})$, the cost matrix Q of the point set of the USVs and the selected formation points is constructed.

$$Q = \begin{bmatrix} s_{11} & s_{12} & \cdots & s_{1n} \\ s_{21} & s_{22} & \cdots & s_{2n} \\ \vdots & \vdots & \cdots & \vdots \\ s_{n1} & s_{n2} & \cdots & s_{nn} \end{bmatrix} \tag{16}$$

Based on the constructed cost matrix, the Hungarian algorithm is used to complete the target-point allocation before the formation rendezvous of USVs.

When the formation is determined and the target-point assignment is completed, USVs sail to the assigned formation target points at a constant velocity through the line-of-sight

guidance method. The schematic diagram of the line-of-sight guidance method is shown in Figure 3.

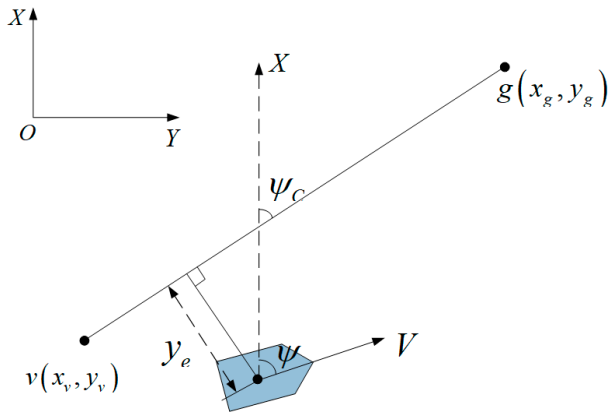


Figure 3. Diagram of line-of-sight guidance method.

In Figure 3, XOY denotes the earth-fixed frame [23]; the starting point $v(x_v, y_v)$ and the end point of $g(x_g, y_g)$, respectively, constitute the expected path of the USV; y_e is the lateral distance between the USV and the desired path; and ψ, ψ_c represents the heading angle and the desired path azimuth of the USV, respectively.

The heading command based on the line-of-sight guidance method is shown below [24]:

$$\psi_d = \psi_c + \arctan\left(-\frac{y_e}{\Delta}\right) \tag{17}$$

where $\Delta > 0$ is a constant to be designated.

4. Simulation Results

In order to verify the effectiveness of the proposed method, a simulation verification is carried out in this paper. The number of USVs is taken as $n = 10$; in order to be closer to reality, the initial position and heading angle of each USV are set to be randomly generated. That is, the position coordinate points of the USVs are randomly generated in the range of $[-200\text{ m}, 200\text{ m}] \times [-200\text{ m}, 200\text{ m}]$, and the headings of the USVs are randomly generated in the range of $[-180°, 180°]$. When the formation is determined and the target-point allocation is completed, the initial velocity of each USV is $V_0 = 0.3$ m/s, and uniform velocity is maintained.

Since the position coordinate points and headings of USVs are randomly distributed, the results of each simulation experiment are different. After many previous simulation experiments, it was determined that, in most cases, the formations were concentrated around the double horizontal formation, the wedge formation and the double longitudinal formation. In order to reflect the effectiveness of the proposed formation strategy, the above three formations are selected as simulation cases for result analysis in multiple experiments. The simulation verification is divided into two parts: simulation of formation determination and simulation of formation target-point allocation.

4.1. Simulation of Formation Determination

This part of the simulation is divided into three formation cases. The initial positions and headings of the USVs in each case are shown in Tables 1–3, and the simulation results are shown in Figures 4–6. It can be seen that by mapping the formation to the coordinate system of the USVs, the formation is distributed near USVs to reduce the distances from the USVs to the formation target points. Then, we determine the formation which represents the minimum Hausdorff distance between the USVs and each formation, as illustrated in Table 4. This formation is identified as the one closest to the distribution of the point set of USV, thereby facilitating the completion of the USV formation matching process.

Table 1. The initial positions and headings of the USVs in Case 1.

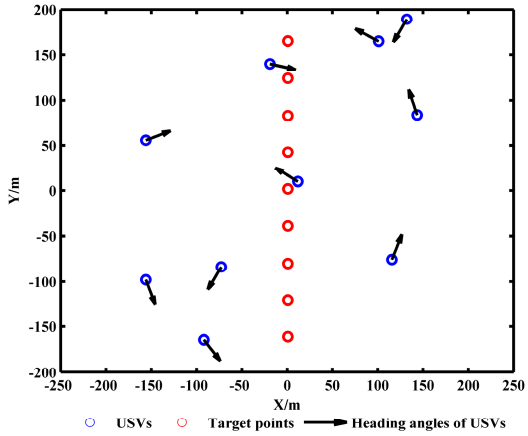
USV Number	Initial Position (m)	Initial Heading (°)	USV Number	Initial Position (m)	Initial Heading (°)
USV1	(11.74, 9.85)	−59.23	USV6	(−19.12, 140.14)	102.64
USV2	(131.99, 189.06)	−149.50	USV7	(100.89, 164.66)	62.02
USV3	(143.50, 84.16)	−18.68	USV8	(−156.06, 55.71)	68.76
USV4	(115.61, −75.26)	22.11	USV9	(−156.10, −97.85)	159.31
USV5	(−72.87, −83.42)	−149.53	USV10	(−92.05, −164.53)	141.63

Table 2. The initial positions and headings of the USVs in Case 2.

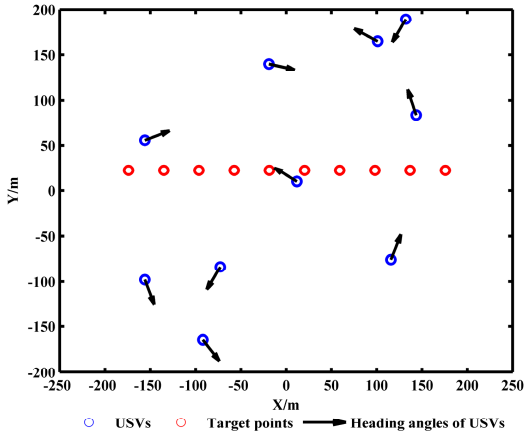
USV Number	Initial Position (m)	Initial Heading (°)	USV Number	Initial Position (m)	Initial Heading (°)
USV1	(103.51, 123.60)	−85.08	USV6	(196.94, −122.49)	−2.07
USV2	(198.09, −57.40)	−159.82	USV7	(120.90, −27.06)	−52.25
USV3	(−125.37, −170.70)	66.18	USV8	(−30.31, 99.66)	−13.53
USV4	(112.46, 36.40)	179.26	USV9	(91.54, −184.33)	−115.58
USV5	(−121.68, 164.08)	−173.58	USV10	(−0.66, 178.53)	145.26

Table 3. The initial positions and headings of the USVs in Case 3.

USV Number	Initial Position (m)	Initial Heading (°)	USV Number	Initial Position (m)	Initial Heading (°)
USV1	(103.28, 142.31)	−21.96	USV6	(78.69, −47.36)	93.86
USV2	(154.81, 68.32)	−66.44	USV7	(110.80, 27.07)	8.21
USV3	(−172.48, 9.44)	0.49	USV8	(0.76, 155.14)	152.67
USV4	(−126.59, −80.47)	1.11	USV9	(−29.80, 137.18)	122.78
USV5	(94.83, 81.59)	34.49	USV10	(44.49, 159.52)	−165.11

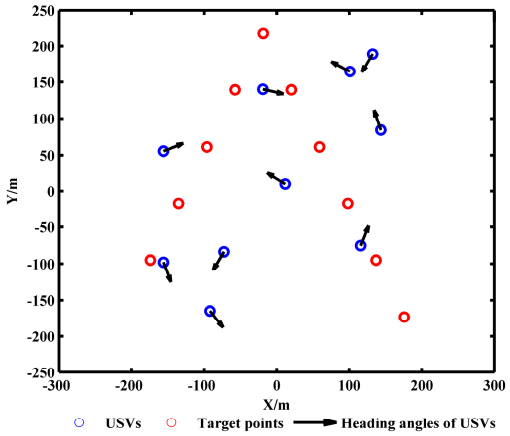


(**a**) Distribution diagram of USVs and single longitudinal formation.

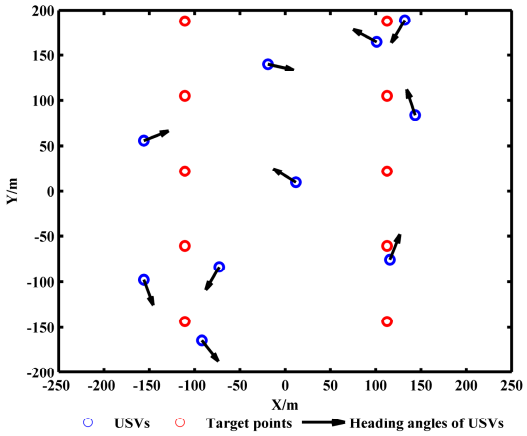


(**b**) Distribution diagram of USVs and single horizontal formation.

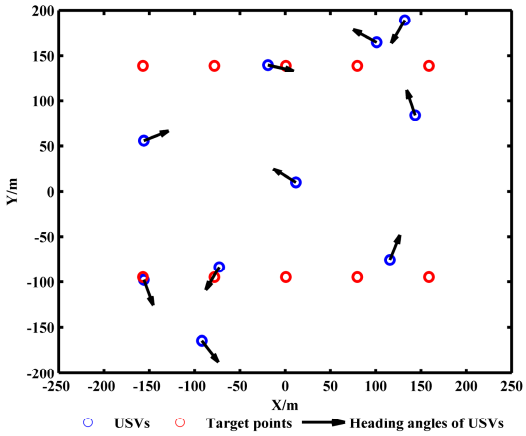
Figure 4. *Cont.*



(**c**) Distribution diagram of USVs and wedge formation.

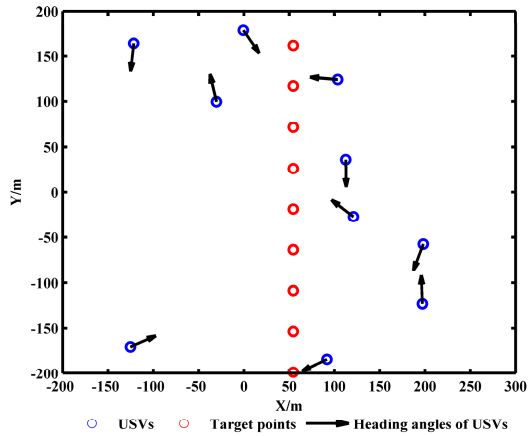


(**d**) Distribution diagram of USVs and double longitudinal formation.

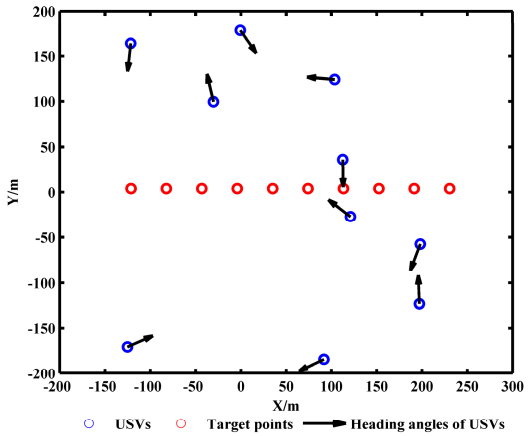


(**e**) Distribution diagram of USVs and double horizontal formation.

Figure 4. Distribution diagram of USVs and formations in Case 1.

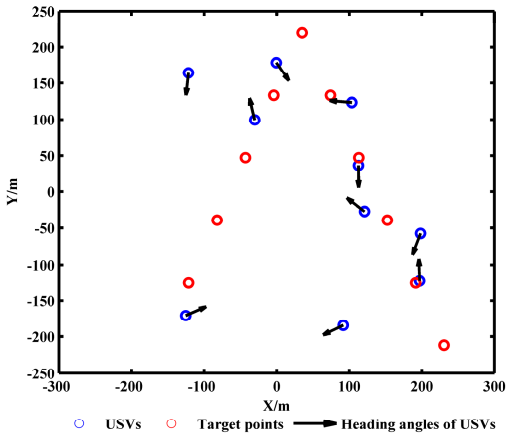


(**a**) Distribution diagram of USVs and single longitudinal formation.

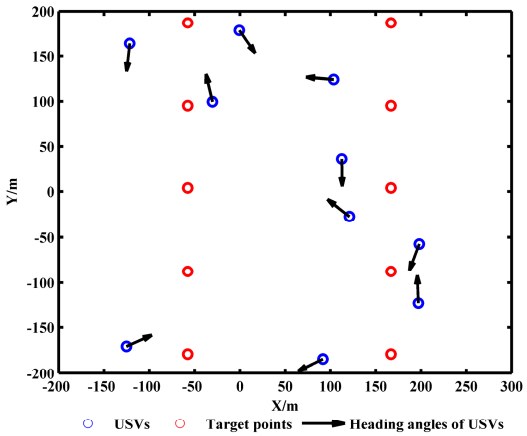


(**b**) Distribution diagram of USVs and single horizontal formation.

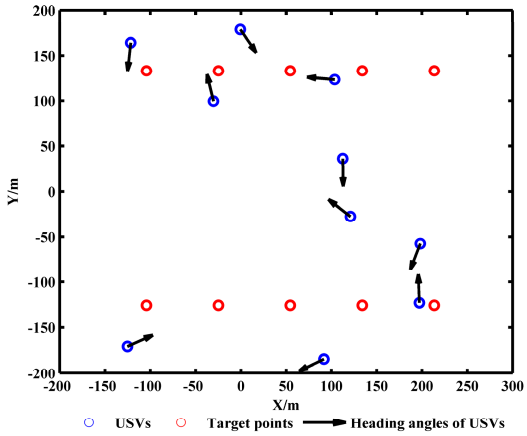
Figure 5. *Cont.*



(**c**) Distribution diagram of USVs and wedge formation.

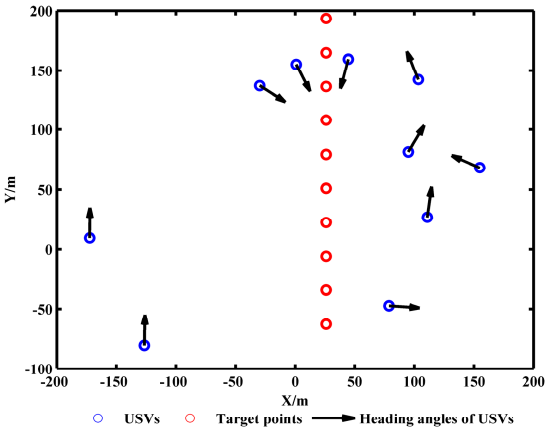


(**d**) Distribution diagram of USVs and double longitudinal formation.

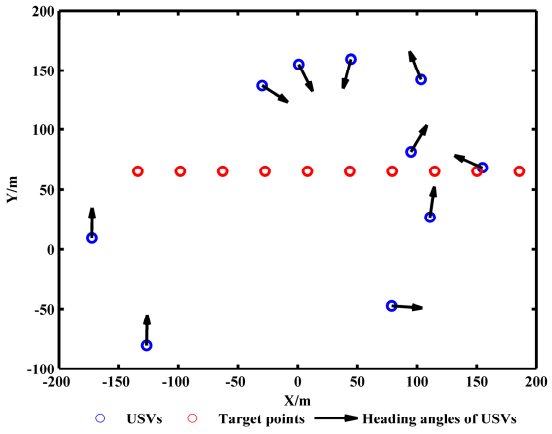


(**e**) Distribution diagram of USVs and double horizontal formation.

Figure 5. Distribution diagram of USVs and formations in Case 2.

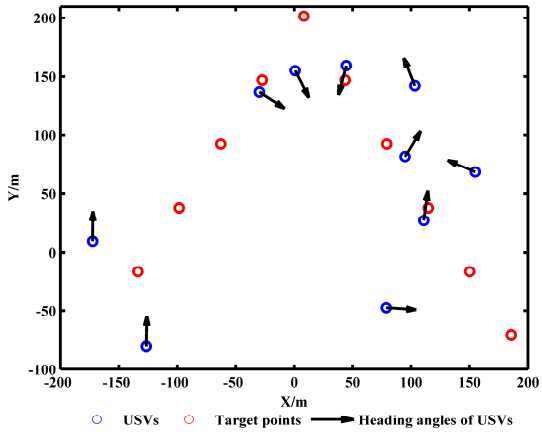


(**a**) Distribution diagram of USVs and single longitudinal formation.

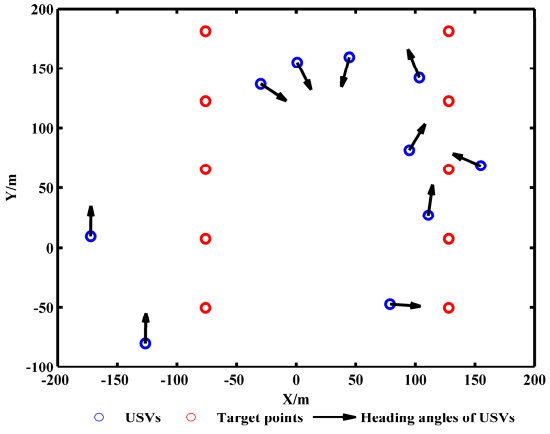


(**b**) Distribution diagram of USVs and single horizontal formation.

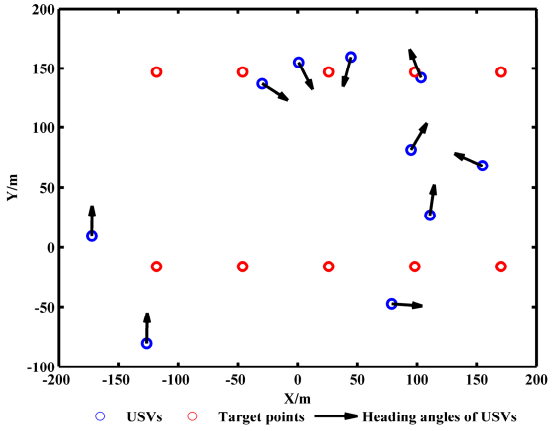
Figure 6. *Cont.*



(**c**) Distribution diagram of USVs and wedge formation.



(**d**) Distribution diagram of USVs and double longitudinal formation.



(**e**) Distribution diagram of USVs and double horizontal formation.

Figure 6. Distribution diagram of USVs and formations in Case 3.

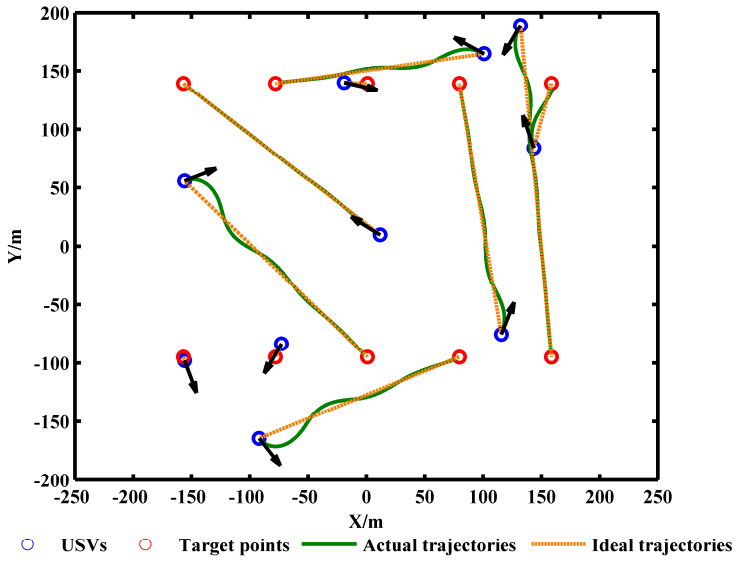
Table 4. The Hausdorff distances between the point sets of the USVs and basic formations.

Case Number	Single Horizontal Formation	Single Longitudinal Formation	Wedge Formation	Double Horizontal Formation	Double Longitudinal Formation
Case 1	178.85 m	158.57 m	122.19 m	101.15 m	102.07 m
Case 2	218.98 m	235.07 m	175.11 m	141.43 m	129.97 m
Case 3	207.86 m	224.91 m	107.08 m	125.27 m	122.27 m

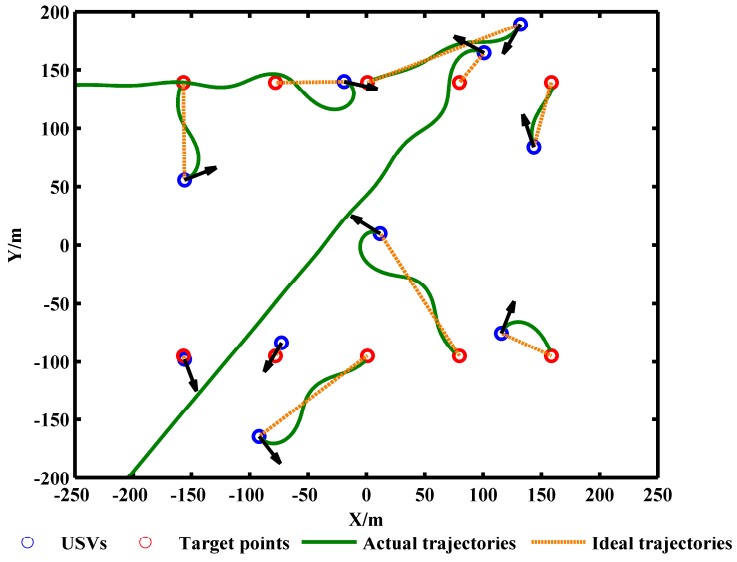
4.2. Simulation of Formation Target-Point Allocation

In order to verify the superiority of the proposed formation target-point allocation method, based on the conventional Hungarian allocation algorithm, the proposed formation target-point allocation strategy (denoted as scheme 'A') is compared with the allocation strategy of minimizing the navigation distance (denoted as scheme 'B'). The simulation results are shown in Figures 7–9. The black arrows indicate the initial headings of USVs in Figures 7–9. It can be seen that scheme B aims to minimize the distance, and the formation target points are allocated to the nearest USVs as far as possible, but the headings of

the USVs are ignored. Due to the limitation of the actual turning radius, some USVs in Figures 7b, 8b and 9b cannot successfully reach the formation target point, and it is difficult to meet the formation rendezvous requirements of USVs in reality. The allocation of target points in scheme A comprehensively considers the azimuths and distances between the USVs and the target points. It can be seen that in the simulation results shown in Figures 7b, 8b and 9b (corresponding to scheme A), for the case of a small azimuth, the distance between the USV and the target point is close, and the efficiency of aggregation is improved while the USV successfully reaches the target point. For the case of a large azimuth, the distance between the USV and the target point is often far away, which can meet the space requirements for the USV to adjust its course and reach the assigned target point smoothly. Therefore, by comparison, the target-point allocation strategy of scheme A is more effective and more in line with the actual needs.

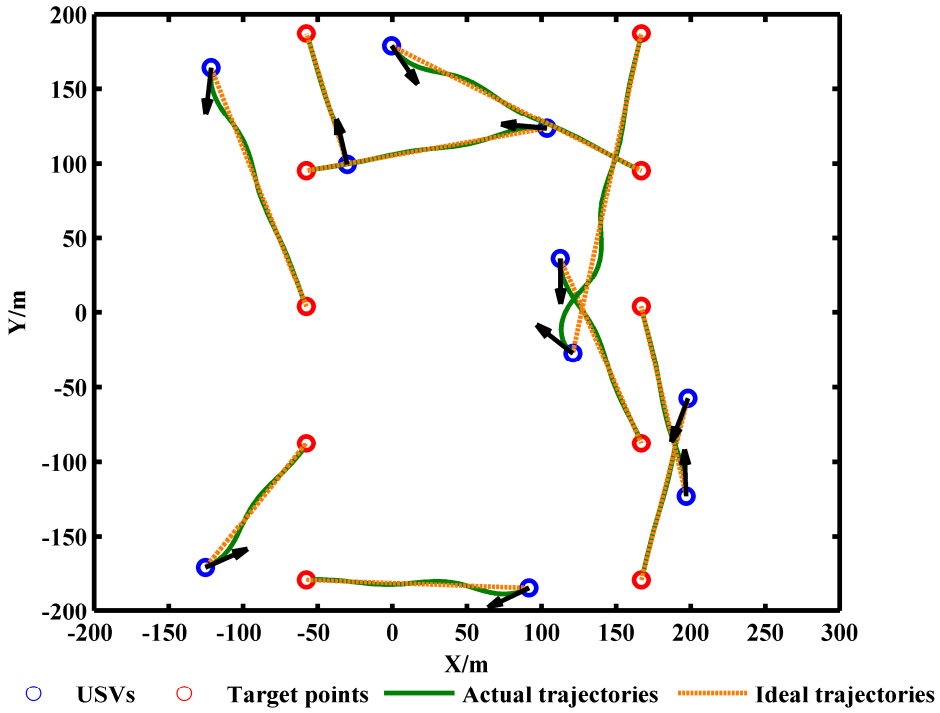


(**a**) Formation target-point assignment based on the designed evaluation function.

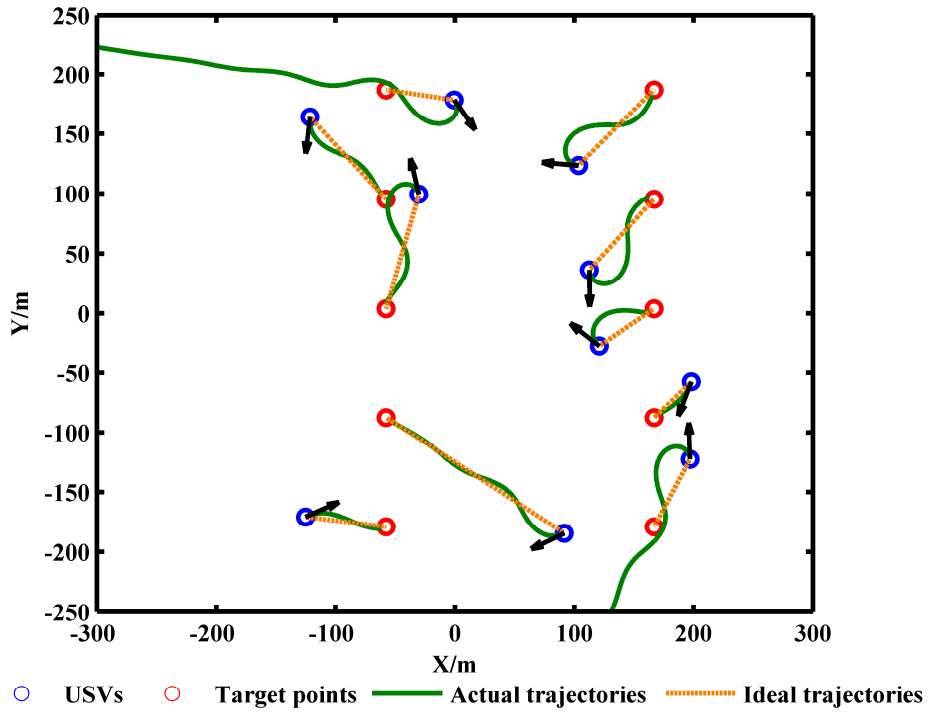


(**b**) Formation target-point assignment based on distance minimization.

Figure 7. Comparison of formation target-point allocation methods in Case 1.

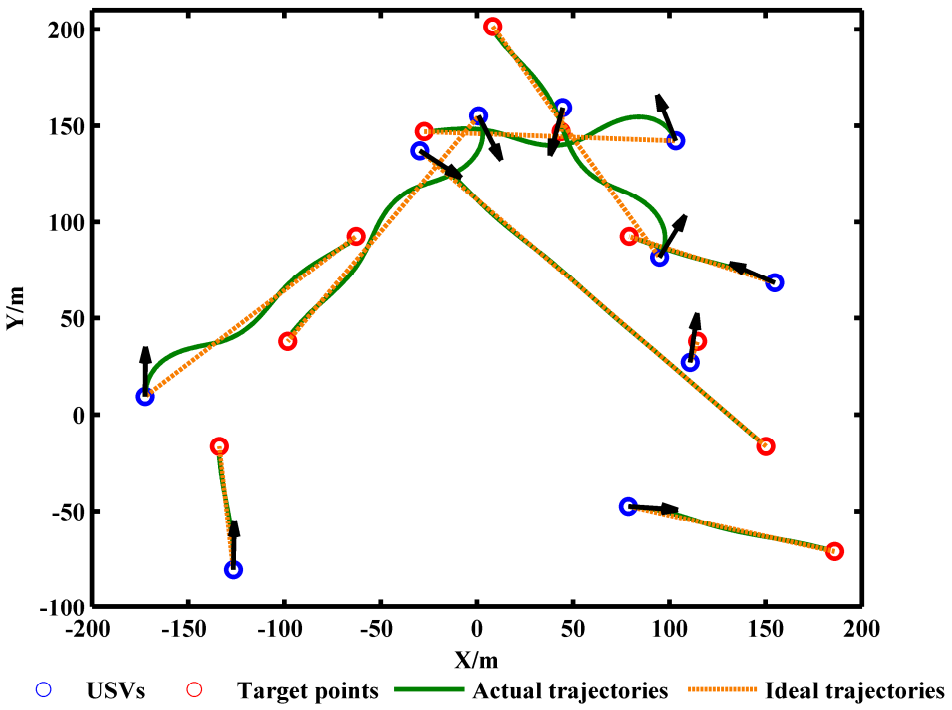


(**a**) Formation target-point assignment based on the designed evaluation function.

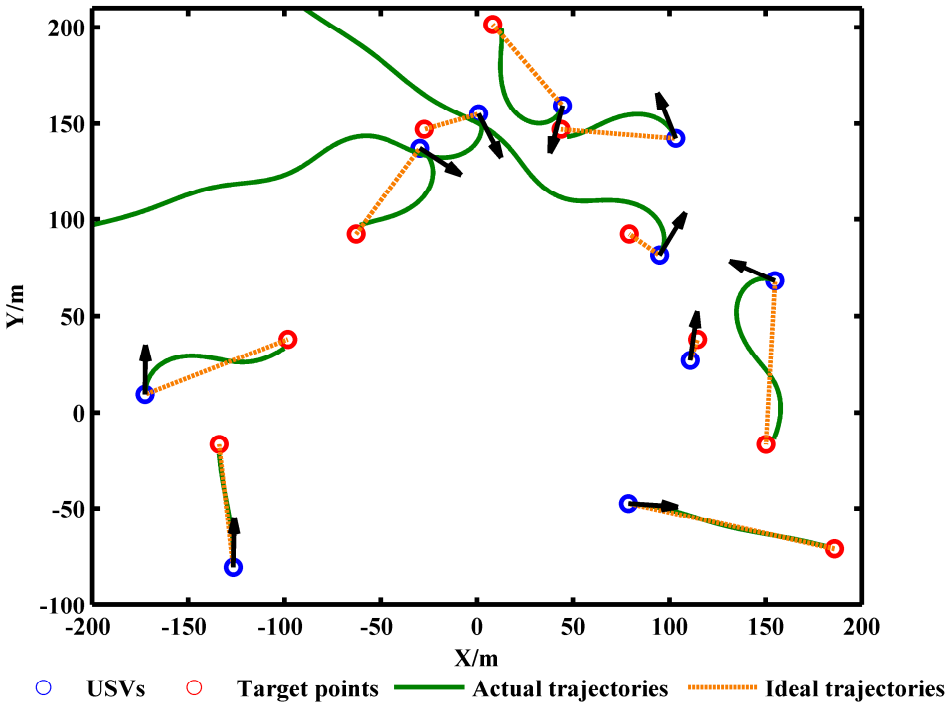


(**b**) Formation target-point assignment based on distance minimization.

Figure 8. Comparison of formation target-point allocation methods in Case 2.



(**a**) Formation target-point assignment based on the designed evaluation function.



(**b**) Formation target-point assignment based on distance minimization.

Figure 9. Comparison of formation target-point allocation methods in Case 3.

5. Conclusions

In this paper, a USV formation strategy considering initial positions and headings is proposed. Through theoretical elaboration and simulation verification, the following conclusions are obtained:

(1) The proposed method for formation determination, based on Hausdorff distance, takes into account the random distribution of the initial positions of the USVs. Consequently, the formation that best matches this distribution is chosen as the rendezvous formation and positioned at the center of the USVs' distribution to enhance rendezvous efficiency. The simulation effectively illustrates the process of formation determination according to this method.

(2) The proposed method for allocating formation target points, based on the distance–azimuth evaluation function, takes into account the arbitrary initial headings of the USVs. In practice, distances between USVs and target points, along with the azimuth angles formed, serve as evaluation metrics. A dedicated distance–azimuth evaluation function is designed, guiding the application of the Hungarian allocation algorithm for assigning target points among the USVs. The effectiveness and practicality of this method are validated through comparative simulations.

Author Contributions: Conceptualization: Y.Z.; Methodology: Y.Z. and P.P.; Validation: Y.Z.; software: Y.Z.; formal analysis: P.P.; investigation: P.P.; resources: J.W.; writing—review and editing: J.W. All authors have read and agreed to the published version of the manuscript.

Funding: This research received no external funding.

Data Availability Statement: Data are contained within the article.

Acknowledgments: The authors of this paper would like to express their sincere gratitude to the editors and reviewers for their hard work.

Conflicts of Interest: The authors declare no conflict of interest.

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