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Global Regulation by Integral Feedback for Lower-Triangular Nonlinear Systems with Actuator Failures and Limited Delays

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Abstract: This paper studies the global asymptotic regulation problem for a class of lower-triangular nonlinear systems with actuator failures and limited delays. New integral controllers consisting of an integral dynamic are constructed to make all system states bounded and asymptotically convergent to zero. First, an integral dynamic is constructed and a novel state transformation is introduced, which ensures that the involved systems with actuator failures are converted into a class of auxiliary nonlinear systems without actuator failures. Second, by introducing the static high-gain technique, the problem of designing integral controllers for auxiliary nonlinear systems is converted into that of designing the gain parameter and determining the limit of the actuator delay. At last, with the help of the Lyapunov stability theorem, the gain parameter and the limit of the actuator delay are determined, and the stabilization of the auxiliary nonlinear systems yields the global asymptotic regulation of the involved systems. A physical system example is given to demonstrate the effectiveness of the proposed integral controllers.

Keywords: time-delay systems; actuator failures; feedback regulation; integral control; static high-gain technique

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1. Introduction

In decades, asymptotic control problems for nonlinear systems have attracted continued attention [1,2], and as an important research branch, great achievements have also been reported for lower-triangular nonlinear systems such as stabilization control [3–5], regulation control [6,7], tracking control [8–10], and consensus [11,12].

As we all know, the control signals are applied to practical systems through actuators, and the normal operation of actuators is one of the key factors for guaranteeing system stability. The results mentioned above considered the control problems with the normal operation of actuators. However, in practice, actuators may experience gradual or abrupt failures/faults during system operation, which may lead to a system with bad performance or even instability [13,14]. Therefore, appropriate compensation mechanisms for actuators to ensure their normal operation are of both theoretical and practical importance. To this end, passive methods [15] and many active methods such as fault diagnosis [16], pseudo-inverse method [17], and sliding mode control [18] were proposed to compensate for the actuator failures. Moreover, adaptive failure/fault compensations [19,20] were also proposed recently for the control of lower-triangular nonlinear systems by the backstepping method.

On the other hand, time delay is another primary source of instability and performance degradation, and it makes practical systems hard to control [21–23]. To this end, some important results have been obtained for the control of lower-triangular nonlinear systems with time delays, see [24] and the references therein. Specifically, the stabilization problems were considered for lower-triangular nonlinear systems with state delay [11,25,26] and output delay [27]. Recently, [28] investigated the global stabilization problem for lower-triangular nonlinear systems with input delay by the backsteeping method, and it has been shown that the considered input delay is limited. Moreover, in practice, the actuator may be affected by delays and failures simultaneously, which makes practical systems more difficult to control, and as far as we know, there are no results on the control of such systems with actuator failures and delays.

Partly inspired by [29,30] and in this paper, we will address this challenging problem for a class of lower-triangular nonlinear systems with actuator failures and delays simultaneously. First, we will address a unified control strategy for constructing integral controllers to globally and asymptotically regulate the involved systems for the first time, which is obviously different [28] from solving the stabilization control without actuator failures or [29,30] dealing with the regulation control without time delays. Second, a novel state transformation containing a static high-gain parameter is introduced to convert the involved systems with actuator failures into a class of auxiliary nonlinear systems without actuator failures. Under the transformation, the gain parameter is chosen, the limit of the actuator delay is determined, and an integral controller is constructed with a rigorous stability analysis to achieve the globally asymptotic regulation of the involved systems, which is different from fault diagnosis [16], the pseudo-inverse method [17], the sliding-mode control [18], and the adaptive compensation mechanism [19] for tackling actuator failures.

2. Preliminaries

In this paper, we consider the following time-delay nonlinear system with actuator failures:

$$\begin{cases} \dot{x}_i(t) = x_{i+1}(t-\tau_{i+1}) + f_i(t, x(t), x(t-d), u(t-\tau_1)),\ i = 1, 2, \ldots, n-1, \\ \dot{x}_n(t) = u(t-\tau_1) + f_n(t, x(t), x(t-d), u(t-\tau_1)), \end{cases} \tag{1}$$

where $x(t) = (x_1(t), \ldots, x_n(t))^T \in \mathbb{R}^n$ is the system state and $u(t) \in \mathbb{R}$ is the system input. For $i = 1, \ldots, n$, the real constants $d_i \geq 0$ and $\tau_i \geq 0$ represent the time delays and $x(t-d) = (x_1(t-d_1), \ldots, x_n(t-d_n))^T$ denotes the delayed state. The initial state $x_i(\theta) = \chi_i(\theta) \in \mathcal{C}(-[\max\{d_i, \tau_i\}, 0], \mathbb{R})$, and $f_i(\cdot)$ is a continuous function with $f_i(0,0,0,0) = 0$.

Assumption 1. *For $i = 1, \ldots, n$, there exist known constants c_i such that*

$$|f_i(\cdot)| \leq \sum_{j=1}^{i} c_j(|x_j(t)| + |x_j(t-d_j)|). \tag{2}$$

Assumption 2. *The actuator of interest is modeled as follows:*

$$u(t) = v(t) + \bar{v}, \tag{3}$$

where $v(t)$ is the input of actuator to be designed and $\bar{v}$ is an unknown bias fault.

Remark 1. *It can be seen that system (1) satisfying Assumption 1 is dominated by a lower-triangular model with a known constant growth rate, which was originally studied in [31]. Since then, system (1) satisfying Assumption 1 has been studied widely [32–34]. Moreover, in practice, the actuators may encounter failures, and system (1) satisfying Assumption 2 indicates that the actuator has bias faults. Therefore, it is reasonable to consider the global asymptotic regulation problem for system (1) satisfying Assumptions 1 and 2.*

With the above preliminaries, the problem to be studied is to propose a control strategy to globally asymptotically regulate system (1) under Assumptions 1 and 2.

3. Main Results

In this section, we will solve the global asymptotic regulation of system (1) by the static high-gain technique.

Theorem 1. *Under Assumptions 1 and 2, there are positive constants a_i, $i = 1, \ldots, n+1$ and r such that system (1) can be globally asymptotically regulated via a state feedback integral controller of the form,*

$$\begin{cases} v(t) = -r^{n+1}\Big(a_1 x_0(t) + a_2 \dfrac{x_1(t)}{r} + \cdots + a_{n+1}\dfrac{x_n(t)}{r^n}\Big), \\ \dot{x}_0(t) = x_1(t). \end{cases} \tag{4}$$

Proof. Partly inspired by our previous works [29], we introduce the new state transformation as follows:

$$\begin{cases} \zeta_1(t) = x_0(t) - \dfrac{\bar{v}}{a_1 r^{2n+1}}, \\ \zeta_i(t) = \dfrac{x_{i-1}(t)}{r^{i-1}},\ i = 2, \ldots, n+1 \end{cases} \tag{5}$$

for system (1), where $\zeta(t) = (\zeta_1(t), \ldots, \zeta_{n+1}(t))^T$, and a_1 and $r \geq 1$ will be determined later, then system (1) is converted into

$$\begin{cases} \dot{\zeta}_1(t) = r\zeta_2(t-\tau_0), \\ \dot{\zeta}_i(t) = r\zeta_{i+1}(t-\tau_i) + \dfrac{f_{i-1}(\cdot)}{r^{i-1}},\ i = 2, \ldots, n, \\ \dot{\zeta}_{n+1}(t) = \dfrac{v(t-\tau_1)}{r^n} + \dfrac{\bar{v}}{r^n} + \dfrac{f_n(\cdot)}{r^n(t)}, \end{cases} \tag{6}$$

where $\tau_0 = 0$. It follows from (3) and (5) that the desired controller (4) for system (1) can be rewritten as follows:

$$\begin{aligned} v(t) =& -r^{n+1}\Big(a_1\Big(\zeta_1(t) + \frac{\bar{v}}{r^{2n+1}a_1}\Big) + a_2\zeta_2(t) + \cdots + a_{n+1}\zeta_{n+1}(t)\Big) \\ =& -r^{n+1}(a_1\zeta_1(t) + \cdots + a_{n+1}\zeta_{n+1}(t)) - \frac{\bar{v}}{r^n}, \end{aligned} \tag{7}$$

and then the dynamics of system (6) under controller (7) can be further rewritten as the following compact form:

$$\dot{\zeta}(t) = rA\zeta(t) + \Psi + r\Phi, \tag{8}$$

where $A = \Xi - FK_a$ with Ξ being an $(n+1)\times(n+1)$ matrix of term ϵ_{ij} satisfying $\epsilon_{ij} = 1$ if $i = 1, \ldots, n,\ j = i+1$ and $\epsilon_{ij} = 0$ if $j \neq i+1$, $F = (0\,0\ \cdots\ 0\,1)^T$, $K_a = (a_1, \ldots, a_{n+1})$ with $a_i > 0$ being the coefficients of the Hurwitz polynomial $q(s) = s^{n+1} + a_{n+1}s^n + \cdots + a_2 s + a_1$ [35], $\Psi = (\psi_1, \ldots, \psi_{n+1})^T = (0, f_1(\cdot)/r, \ldots, f_n(\cdot)/r^n)^T$ and $\Phi = (\phi_1, \ldots, \phi_{n+1})^T = (\zeta_2(t-\tau_0) - \zeta_2(t), \zeta_3(t-\tau_2) - \zeta_3(t), \ldots, \zeta_{n+1}(t-\tau_n) - \zeta_{n+1}(t), (v(t-\tau_1) - v(t))/r^{n+1})^T$.

Thus, the problem of designing a state feedback integral controller for system (1) has been converted into that of designing the gain parameter r and determining the limit of the actuator delay such that system (8) is globally asymptotically stable.

Choose the following Lyapunov function candidate for system (8),

$$V_1 = \zeta(t)^T P \zeta(t), \tag{9}$$

where the positive definite matrix P satisfying $PA + A^T P \leq -I$. Taking the derivative of V_1 along system (8), we have

$$\begin{aligned}\dot{V}_1 =& r\zeta(t)^T(A^TP+PA)\zeta(t)+2\zeta(t)^TP\Psi+2r\zeta(t)^TP\Phi\\ \leq& -r\sum_{i=1}^{n+1}\zeta_i^2(t)+2\zeta(t)^TP\Psi+2r\zeta(t)^TP\Phi.\end{aligned} \tag{10}$$

Next, we will give the estimate of the terms $2\zeta(t)^TP\Psi$ and $2r\zeta(t)^TP\Phi$, respectively. When $t \geq \bar{\tau}+\bar{d}$, where

$$\bar{\tau} = \max_{1\leq i\leq n}\{\tau_i\},\ \bar{d} = \max_{1\leq i\leq n}\{d_i\}, \tag{11}$$

it follows from (2) and $r \geq 1$ that $\psi_1 = 0$, and for $i = 2, \ldots, n+1$, we have

$$\begin{aligned}|\psi_i| &= \frac{|f_{i-1}(\cdot)|}{r^{i-1}}\\ &\leq \frac{1}{r^{i-1}}\sum_{j=1}^{i-1}c_j(|x_j(t)|+|x_j(t-d_j)|)\\ &\leq \frac{1}{r^{i-1}}\sum_{j=1}^{i-1}c_j(r^j|\zeta_{j+1}(t)|+r^j|\zeta_{j+1}(t-d_j)|)\\ &\leq \sum_{j=2}^{i}c_{j-1}(|\zeta_j(t)|+|\zeta_j(t-d_j)|),\end{aligned} \tag{12}$$

which indicates

$$2\zeta(t)^TP\Psi \leq \rho_1\sum_{i=1}^{n+1}\zeta_i^2(t)+\rho_2\sum_{i=1}^{n+1}\zeta_i^2(t-d_i), \tag{13}$$

where $\rho_i, i = 1,2$ are known positive constants independent of $\bar{\tau}$ and $\bar{d}$.

Then, for $i = 2, \ldots, n-1$, we have

$$|\phi_i| = |\zeta_{i+1}(t-\tau_i)-\zeta_{i+1}(t)| = \left|-\int_{t-\tau_i}^{t}\dot{\zeta}_{i+1}(s)ds\right|, \tag{14}$$

and it follows from (6), (12) and (14) that we have

$$\begin{aligned}|\phi_i| \leq& r\int_{t-2\bar{\tau}}^{t}|\zeta_{i+2}(s)|ds+\sum_{j=2}^{i+1}c_{j-1}\int_{t-\tau_i}^{t}|\zeta_j(s)|ds\\ &+\sum_{j=2}^{i+1}c_{j-1}\int_{t-\tau_i}^{t}|\zeta_j(s-d_j)|ds\\ \leq& \sqrt{2\bar{\tau}}r\left(\int_{t-2\bar{\tau}}^{t}\zeta_{i+2}^2(s)ds\right)^{\frac{1}{2}}\\ &+\sqrt{\bar{\tau}}\sum_{j=2}^{i+1}c_{j-1}\left(\int_{t-\bar{\tau}}^{t}\zeta_j^2(s)ds\right)^{\frac{1}{2}}\\ &+\sqrt{\bar{\tau}}\sum_{j=2}^{i+1}c_{j-1}\left(\int_{t-\bar{\tau}}^{t}\zeta_j^2(s-d_j)ds\right)^{\frac{1}{2}}\end{aligned} \tag{15}$$

and

$$
\begin{aligned}
|\phi_n| \leq & \sqrt{2\bar{\tau}} r \sum_{j=1}^{n+1} a_j \left(\int_{t-2\bar{\tau}}^{t} \zeta_j^2(s) ds \right)^{\frac{1}{2}} \\
& + \sqrt{\bar{\tau}} \sum_{j=2}^{n+1} c_{j-1} \left(\int_{t-\bar{\tau}}^{t} \zeta_j^2(s) ds \right)^{\frac{1}{2}} \\
& + \sqrt{\bar{\tau}} \sum_{j=2}^{n+1} c_{j-1} \left(\int_{t-\bar{\tau}}^{t} \zeta_j^2(s-d_j) ds \right)^{\frac{1}{2}}.
\end{aligned} \tag{16}
$$

Furthermore, it follows from (6), (7), (12) and (15) that we can easily have

$$
\frac{|v(t-\tau_1) - v(t)|}{r^{n+1}} = \frac{1}{r^{n+1}} \left| \int_{t-\tau_1}^{t} \dot{v}(s) ds \right|, \tag{17}
$$

which indicates

$$
\begin{aligned}
& \frac{|v(t-\tau_1) - v(t)|}{r^{n+1}} \\
& \leq \sqrt{2\bar{\tau}} r \sum_{j=1}^{n+1} \left(a_{n+1} + \frac{a_{j-1}}{a_j} \right) a_j \left(\int_{t-2\bar{\tau}}^{t} \zeta_j^2(s) ds \right)^{\frac{1}{2}} \\
& + \sqrt{\bar{\tau}} \sum_{j=1}^{n+1} a_j \sum_{h=1}^{j} c_{h-1} \left(\int_{t-\bar{\tau}}^{t} \zeta_h^2(s) ds \right)^{\frac{1}{2}} \\
& + \sqrt{\bar{\tau}} \sum_{j=1}^{n+1} a_j \sum_{h=1}^{j} c_{h-1} \left(\int_{t-\bar{\tau}}^{t} \zeta_h^2(s-d_h) ds \right)^{\frac{1}{2}}
\end{aligned} \tag{18}
$$

with $a_0 = 0$. Thus, it follows from (15), (16) and (18) that we have

$$
\begin{aligned}
& 2r\zeta(t)^T P \Phi \\
& \leq \bar{\tau} r^2 \sum_{i=1}^{n+1} \zeta_i^2(t) + \rho_3 \sum_{i=1}^{n+1} \int_{t-2\bar{\tau}}^{t} \zeta_i^2(s) ds \\
& + \rho_4 \sum_{i=1}^{n+1} \int_{t-\bar{\tau}}^{t} \zeta_i^2(s) ds + \rho_5 \sum_{i=1}^{n+1} \int_{t-\bar{\tau}}^{t} \zeta_i^2(s-d_i) ds,
\end{aligned} \tag{19}
$$

where $\rho_i, i = 3, 4, 5$ are known positive constants independent of $\bar{\tau}$ and $\bar{d}$.

Now, we choose the Lyapunov functional

$$
V = V_1 + V_2 + V_3 + V_4 + V_5 + V_6, \tag{20}
$$

where

$$
\begin{cases}
V_2 = (\rho_2 + \bar{\tau}\rho_5) \sum_{i=1}^{n+1} \int_{t-d_i}^{t} \zeta_i^2(s) ds, \\
V_3 = \rho_3 \sum_{i=1}^{n+1} \int_{t-2\bar{\tau}}^{t} \int_{s}^{t} \zeta_i^2(\mu) d\mu ds, \\
V_4 = \rho_4 \sum_{i=1}^{n+1} \int_{t-\bar{\tau}}^{t} \int_{s}^{t} \zeta_i^2(\mu) d\mu ds, \\
V_5 = \rho_5 \sum_{i=1}^{n+1} \int_{t-\bar{\tau}}^{t} \int_{s}^{t} \zeta_i^2(\mu - d_i) d\mu ds,
\end{cases} \tag{21}
$$

and then, taking the derivative of V along system (8) and from (10), (13) and (19), we have

$$
\begin{aligned}
\dot{V} \leq & -r\sum_{i=1}^{n+1}\zeta_i^2(t)+\rho_1\sum_{i=1}^{n+1}\zeta_i^2(t)+\bar{\tau}r^2\sum_{i=1}^{n+1}\zeta_i^2(t)\\
&+(\rho_2+\bar{\tau}\rho_5)\sum_{i=1}^{n+1}\zeta_i^2(t)+2\bar{\tau}\rho_3\sum_{i=1}^{n+1}\zeta_i^2(t)\\
&+\bar{\tau}\rho_4\sum_{i=1}^{n+1}\zeta_i^2(t)\\
\leq & -r\sum_{i=1}^{n+1}\zeta_i^2(t)+(\bar{\tau}r^2+\bar{\tau}\rho_6+\rho_7)\sum_{i=1}^{n+1}\zeta_i^2(t)
\end{aligned}
\tag{22}
$$

with $\rho_6 = \rho_3 + \rho_4 + \rho_5$ and $\rho_7 = \rho_1 + \rho_2$.

Now, choose the gain parameter r and the allowable actuator delay as follows:

$$
r \geq \rho_6 + \rho_7 + 1, \quad \bar{\tau} \leq \frac{1}{2r}, \tag{23}
$$

which makes (22) satisfy

$$
\dot{V} \leq -\sum_{i=1}^{n+1}\zeta_i^2(t). \tag{24}
$$

It follows from (24) and the Lyapunov–Krasovskii theorem that system (8) is globally asymptotically stable, i.e., $\zeta_i(t) \to 0$ as $t \to +\infty$ for $i = 1,\ldots,n+1$. It should be noted that system (8) is equivalent to system (1) with integral controller (4), and it follows from the state transformation (5) and (23) that $x_0(t) \to \frac{\bar{v}}{a_1 r^{2n+1}}$ and $x_i(t) \to 0$ as $t \to +\infty$ for $i = 1,\ldots,n$, which indicates that system (1) with large delays in the state, a limited delay in the input, and actuator failure is globally asymptotically regulated via the integral controller (4) with r determined in (23). □

Remark 2. *It should be noted that the proposed integral-control method has the limitation for the considered fault sizes. Firstly, the proposed integral-control method can deal with the actuator failures satisfying Assumption 2, which indicates that the actuator has bias faults. However, the proposed integral-control method cannot deal with the partial loss of effectiveness of the actuators. Secondly, in Assumption 2, the actuator of interest is modeled as follows $u(t) = v(t) + \bar{v}$ $t \in [0, +\infty)$, where the unknown bias fault $\bar{v}$ must be an unknown constant. Moreover, the proposed integral-control method can also deal with the case that $u_j(t) = v(t) + \bar{v}_j$, $t \in [t_{j-1}, t_j]$, $j = 1, 2, \ldots$, where $\bar{v}_j$ is the unknown bias fault and $t_0 = 0, t_1, \ldots, t_j, \ldots$ are unknown time series. In this case, we can introduce the following state transformation as follows:*

$$
\begin{cases}
\zeta_1(t) = x_0(t) - \dfrac{\bar{v}_j}{a_1 r^{2n+1}},\\
\zeta_i(t) = \dfrac{x_{i-1}(t)}{r^{i-1}},\ i = 2,\ldots,n+1,\ t \in [t_{j-1}, t_j],\ j = 1, 2, \ldots,
\end{cases}
\tag{25}
$$

for system (1). Then, it follows from (25) and the proof of Theorem 1 that system (1) can be globally asymptotically regulated via the state feedback integral controller (4).

Remark 3. *As we all know, many practical control systems are designed to be asymptotically stable. However, due to the influence of external disturbances, actuator failures, time delays, etc., the asymptotically stable control is hard to achieve. Therefore, the regulation control, as one of the basic problems of control systems, can make system states bounded and asymptotically convergent to zero via feedback controllers, which has attracted the continuous attention of scholars. In this paper, we aim to design new integral controllers to achieve the global asymptotic regulation problem for a class of lower-triangular nonlinear systems with actuator failures and limited delays.*

4. A Simulation Example

In this section, the state feedback integral controller is applied to a controlled pendulum model [36] when considering delays in the state and input, and actuator failures, whose model can be described as follows:

$$ml\ddot{\theta}(t) + kl\dot{\theta}(t) + mg\sin\theta(t - d_1) = u(t - \tau_1), \tag{26}$$

where $\theta(t)$ is the acute angle between the rod and the vertical axis; $\dot{\theta}(t)$ is the angular velocity of the rod; $u(t)$ is the actuator control voltage with a bias fault $\bar{v}$; d_1 and τ_1 are the system time delays; m and l are the mass of the bob and the length of the rod, respectively; g is the acceleration of gravity; and k is an constant representing the frictional coefficient.

It follows the transformation $x_1(t) = ml\theta(t)$ and $x_2(t) = ml\dot{\theta}(t)$ that system (26) can be converted as follows:

$$\begin{cases} \dot{x}_1(t) = x_2(t), \\ \dot{x}_2(t) = u(t - \tau_1) - mg\sin\left(\dfrac{x_1(t - d_1)}{ml}\right) - \dfrac{k}{m}x_2(t). \end{cases} \tag{27}$$

It is obvious that system (27) satisfies Assumption 1. By Theorem 1, there is a state feedback integral controller

$$\begin{cases} v(t) = -r^3\left(a_1x_0(t) + a_2\dfrac{x_1(t)}{r} + a_3\dfrac{x_2(t)}{r^2}\right), \\ \dot{x}_0(t) = x_1(t) \end{cases} \tag{28}$$

that globally asymptotically regulates system (27) if the actuator delay is limited and has a bias fault.

In the simulation, we firstly consider the case where the time delays are $d_1 = 0.5$ and $\tau_1 = 0.1$; the bias fault is $\bar{v} = 20$; and the parameters are $m = 0.25$, $l = 4$, $k = 0.25$, $g = 10$. Therefore, the Hurwitz polynomial coefficients are chosen as $a_1 = 2$, $a_2 = 5$ and $a_3 = 4$ [35], and the control gain is chosen as $r = 2$. Figures 1 and 2 show the effectiveness of the integral controller (28) when the initial state $(x_0(s), x_1(s), x_2(s)) = (0, -2, 1)$ for $s \in [-0.5, 0]$. It can be clearly observed from Figures 1 and 2 that the states $x_1(t)$ and $x_2(t)$ are bounded and asymptotically convergent to zero, the integral state $x_0(t)$ is bounded and asymptotically convergent to $\frac{\bar{v}}{a_1r^5}$, and the state of controller (28) is bounded and asymptotically convergent to $-\frac{\bar{v}}{r^2}$. Similar results can also be obtained from Figures 3 and 4, where the parameters d_1, τ_1, and $\bar{v}$ are reselected as $d_1 = 0.4$, $\tau_1 = 0.1$, and $\bar{v} = 100$ with the same values of other parameters.

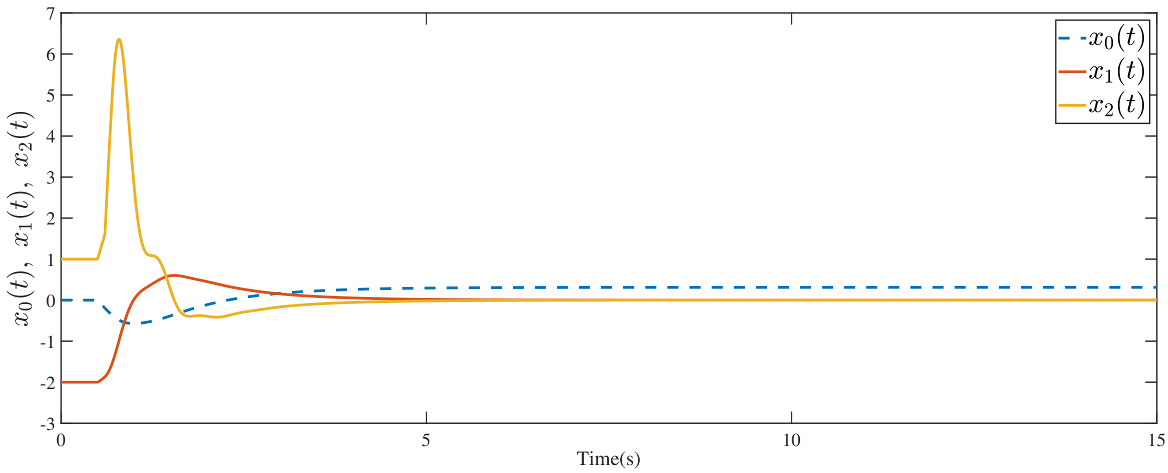

Figure 1. Trajectories of the states of system (27).

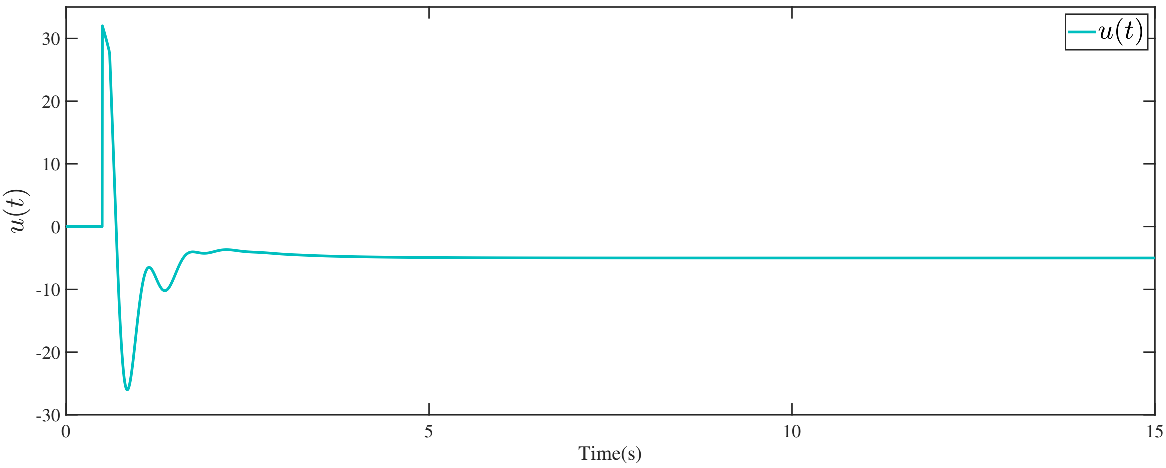

Figure 2. Trajectories of the state of controller (28).

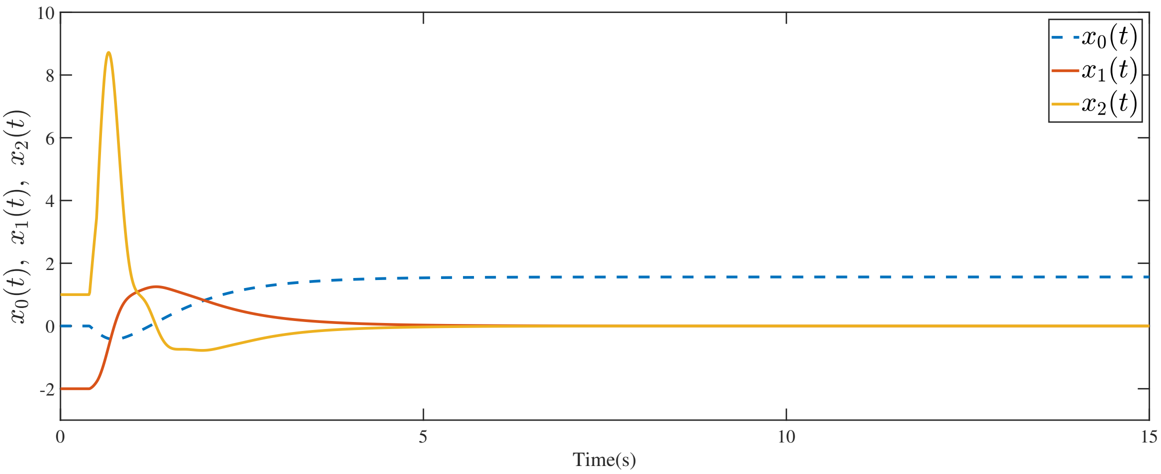

Figure 3. Trajectories of the states of system (27).

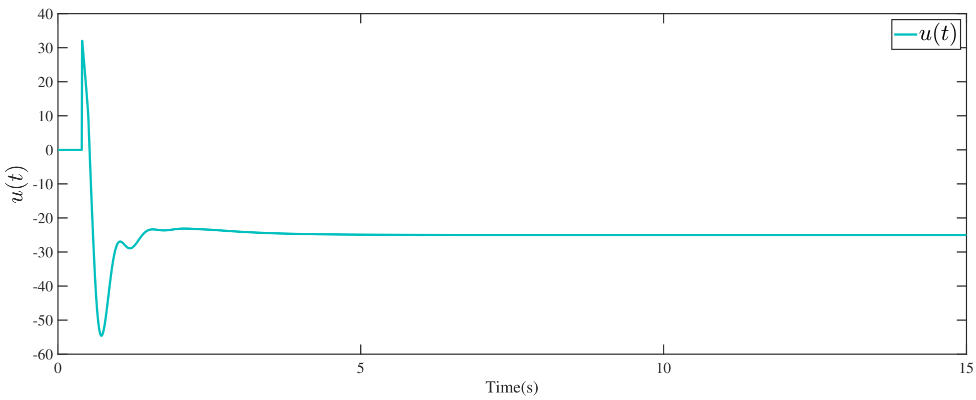

Figure 4. Trajectories of the state of controller (28).

5. Conclusions and Future Works

This paper has constructed integral controllers to globally and asymptotically regulate a class of time-delay lower-triangular nonlinear systems with actuator failures. First, by introducing an integral dynamic and a new state transformation, the involved systems with actuator failures are transformed into a class of auxiliary nonlinear systems without actuator failures. Second, the static high-gain technique is applied to achieve the global stabilization of the auxiliary nonlinear systems, which indicates the global regulation of the involved systems. Future potential works aim to design output feedback controllers for the involved systems.

Author Contributions: Conceptualization, J.L. and X.C.; methodology, X.C.; software, D.W. and G.Z.; formal analysis, G.Z.; investigation, X.C. and D.W.; writing—original draft preparation, X.C. and J.L.; visualization, X.C. and G.Z.; supervision, J.L.; and project administration, J.L. All authors have read and agreed to the published version of the manuscript.

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