electronics

Communication

Chirp Rate Estimation of LFM Signals Based on Second-Order Synchrosqueezing Transform

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Abstract: For the problem of low time-frequency aggregation of the short-time Fourier transform (STFT), which causes the parameter estimation performance degradation of linear frequency modulation (LFM) signals at low signal-to-noise ratio (SNR), second-order synchrosqueezing transform (SSST) is proposed based on the square of STFT amplitude. The time-frequency resolution and energy aggregation are improved by means of squeezing and reassigning the time-frequency spectrum. Meanwhile, in order to decrease the calculation of classical parameter estimation methods, the Hough transform is used for rough estimation, and then the fractional Fourier transform (FRFT) is used for accuracy estimation based on the Renyi entropy. The simulation result shows that higher estimation accuracy is obtained at low SNR, and it has better robustness.

Keywords: LFM signals; parameter estimation; STFT; SSST; Hough transform

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Citation: Zhai, G.; Zhou, J.; Yu, K.; Li, J. Chirp Rate Estimation of LFM Signals Based on Second-Order Synchrosqueezing Transform. *Electronics* **2023**, *12*, 4938. https://doi.org/10.3390/electronics12244938

Academic Editor: Andrea Bonci

Received: 9 October 2023
Revised: 12 November 2023
Accepted: 24 November 2023
Published: 8 December 2023

1. Introduction

An LFM signal is a typical non-stationary signal. The signal is widely used in various radar systems because of its wide bandwidth product, good range resolution, and anti-jamming and anti-interception capability [1,2]. The parameter detection of LFM signals plays an important role in the identification of friend or foe, communication transmission, interference electronic countermeasures [3], and electronic and anti-jamming applications, especially the parameter estimation method of LFM signals based on dewiring modulation [4]. The estimation accuracy of frequency modulation parameters is the prerequisite for the accurate estimation of other parameters [5], so it is of great significance to estimate the frequency modulation of LFM signals with high precision.

Now, the research of LFM signal parameter estimation focuses on the analysis method based on time-frequency characteristics. One of the most classic time-frequency analysis methods, short-time Fourier transform, can describe the local time-frequency characteristics of LFM signals [6]. However, due to the uncertainty criterion, the time resolution and the frequency resolution cannot be obtained with high precision, resulting in non-optimal energy aggregation. The estimation accuracy of the frequency modulation of LFM signals is insufficient at low SNRs. As a kind of nonlinear transform, the Wigner-Ville transform has serious cross interference terms in the condition of multi-component LFM signals. This method will make it impossible to estimate the frequency modulation when the received signals are multi-component LFM signals. By introducing a scale factor into the window function, the wavelet transform (WT) makes the window size of the window function change with the frequency and has a better frequency resolution at the low frequency and a better time resolution at the high frequency. However, because the energy at every moment is always distributed in the frequency band centered on the instantaneous frequency, it cannot have the optimal time resolution and frequency resolution. Therefore, the WT and STFT have similar performance in estimating the frequency modulation of LFM signals.

The S-transform [7] combines the characteristics of the STFT and WT, and introduces the frequency variable into the Gaussian window function of the STFT. Compared with the WT, the result of the transform has a higher time-frequency resolution, but the time-frequency energy is still distributed in the frequency band near the instantaneous frequency, and time-frequency energy aggregation cannot be achieved. The matched Fourier transform (MFT) is used to estimate the frequency modulation of LFM signals. After the MFT, according to the relationship between the peak coordinates of the transform domain and the parameters of LFM signals, the parameters of LFM signals can be estimated [8]. Gabor atoms are used to estimate the parameters of LFM signals, which require a large amount of computation. After determining the most matched atoms, the Hough transform is used to extract the parameters of atoms in linear distribution, which further increases the amount of computation. In addition, Gabor atoms are used to achieve sparse decomposition of LFM signals, and the result shows that their sparsity is not high, which is not conducive to parameter estimation of LFM signals [9]. For the detection of the parameters of LFM signals, a method combining the generalized S transform (GST) and Hough transform was proposed. The GST is the same as the WT, so it cannot have the optimal time resolution and frequency resolution. When the GST is used to estimate the parameters of LFM signals, the detection performance will be greatly reduced at low SNRs [10].

At low SNRs, the time-frequency energy concentration of LFM signals determines the estimation accuracy of frequency modulation. To solve the problem that time-frequency resolution and time-frequency energy concentration of time-frequency analysis methods are not optimal, researchers have done a lot of research, mainly including the synchrosqueezing transform (SST) and time-frequency reassignment (TFR). The SST is a kind of time-frequency analysis method based on the WT proposed by Daubechies et al. [11], which squeezes the energy within a certain frequency range of the wavelet transform spectrum to near the instantaneous frequency of the signal, so as to improve the time-frequency resolution and time-frequency energy aggregation of the signal. By extending the definition of the SST, an improved second-order synchrosqueezing transform (SSST) is proposed by D. Fourer [12,13], which further improves the energy concentration of the signal time-frequency spectrum. By rearranging the time-frequency energy spectrum, the time-frequency reassignment algorithm [14] based on the the STFT squeezes the energy within a certain range of the time-frequency centroid to the centroid position, which significantly improves the time-frequency energy aggregation. However, compared with the SST, it requires more computation.

Through the analysis of several time-frequency analysis algorithms, this paper selects the SSST based on the STFT as the time-frequency analysis tool of LFM signals. First, the SSST is performed on LFM signals to obtain their time-frequency distribution spectrum, and the time-frequency distribution diagram is binarized. Second, combined with the Hough transform, the frequency modulation of LFM signals is roughly estimated. Last, the fractional-order Fourier transform is used to estimate the LFM signals with high precision based on the Renyi entropy.

2. SSST Methods

2.1. The SST Method

For the signal $x(t)$, its short-time Fourier transform is defined as

$$F(t,w) = \int_{-\infty}^{\infty} x(\tau)h(\tau - t)\mathrm{e}^{-\mathrm{i}w\tau}d\tau = A(t,w)\mathrm{e}^{\mathrm{i}\phi(t,w)} \quad (1)$$

where $h(t)$ is the window function, and $A(t,w)$ and $\phi(t,w)$ represent the amplitude and phase of STFT, respectively. The time-frequency energy spectrum of the signal $x(t)$ is defined as follows:

$$|F(t,w)|_2 = F(t,w) \cdot F^*(t,w) \quad (2)$$

where (*) indicates the conjugate operation. Because the time-frequency energy aggregation of signal $x(t)$ is distributed in a certain frequency band centered on the instantaneous frequency, its energy aggregation is poor. In order to improve its time-frequency energy aggregation, the SST squeezes the time-frequency energy distributed in a certain frequency band of the instantaneous frequency to near the instantaneous frequency by estimating its instantaneous frequency. When $h(t)$ is selected as Gaussian window, the steps of the SST are as follows:

(a) Estimate the instantaneous frequency of the signal $x(t)$. According to literature [15], the definition of STFT instantaneous frequency of $x(t)$ is defined as $\hat{w} = \partial_t \phi(t, w)$, where $\hat{w} = \partial_t \phi(t, w)$ represents partial derivative of phase with respect to time. The partial derivative of (1) is defined as

$$\begin{aligned} \frac{\partial F(t,w)}{\partial t} &= \frac{\partial A(t,w)}{\partial t} \mathrm{e}^{\mathrm{i}\phi(t,w)} + \mathrm{i}\frac{\partial \phi(t,w)}{\partial t} A(t,w) \mathrm{e}^{\mathrm{i}\phi(t,w)} \\ &= \frac{\partial_t A(t,w)}{A(t,w)} F(t,w) + \mathrm{i}\frac{\partial \phi(t,w)}{\partial t} F(t,w) \end{aligned} \tag{3}$$

Then, the definition of STFT instantaneous frequency of $x(t)$ is defined as follows [16]:

$$\hat{w}(t,w) = \mathrm{Re}(-\mathrm{i}\frac{\partial_t F(t,w)}{F(t,w)}) \tag{4}$$

where $\partial_t F(t, w)$ is partial derivative to time, and $\mathrm{Re}(\cdot)$ represents the real part of complexes. In practice, to improve efficiency, the following formula can be used to estimate instantaneous frequency:

$$\hat{w}(t,w) = w + \mathrm{Im}(\frac{F^{Dh}(t,w)}{F(t,w)}) \tag{5}$$

where F^{Dh} is defined as the STFT when the window function is the derivative of $h(t)$, and $\mathrm{Im}(\cdot)$ represents the imaginary part of complexes. The formula is to estimate the instantaneous frequency by calculating the energy barycentric coordinates of time-frequency spectrum.

(b) In order to improve the time-frequency resolution of the signal $x(t)$, it is necessary to squeeze the energy in a certain band of the instantaneous frequency to the instantaneous frequency, so the SST is defined as follows:

$$\hat{F}(t,\hat{w}) = \int_R |F(t,w)|^2 \delta(w - \hat{w}(t,w)) dw \tag{6}$$

Compared with the original SST, the square of the STFT amplitude is taken in the formula to avoid energy loss caused by positive and negative cancelling.

2.2. The SSST Method

The definition of instantaneous frequency derived from the SST based on the STFT is shown in Equation (4). From the definition of the equation, it can be seen that the instantaneous frequency estimated by this equation is based on the first partial derivative of time t to estimate the instantaneous frequency of the signal. When the phase of the signal changes linearly with time, the instantaneous frequency obtained from Equation (4) is theoretically the instantaneous frequency of the signal STFT, but when the phase of the signal changes nonlinearly with time, there will be an error between the instantaneous frequency obtained from formula (4) and the real value.

Consider non-stationary signals such as LFM signals

$$x(t) = \mathrm{e}^{\mathrm{i}\pi(2f_0 t + kt^2)} \tag{7}$$

where f_0 is center frequency, and k is the slope of frequency modulation. The phase $\varphi(t)$ is defined as

$$\varphi(t) = 2\pi f_0 t + \pi k t^2 \tag{8}$$

From this equation, it can be seen that the phase is a quadratic function with respect to t, and it has nonlinear properties. According to Lagrange's mean value theorem, the instantaneous frequency can be expressed as

$$\varphi'(t) = \varphi'(\tau) + \varphi''(\tau)(t - \tau) \tag{9}$$

where $\varphi'(t)$ is the first derivative of $\varphi(t)$, and $\varphi''(t)$ is the second derivative of $\varphi(t)$. Compared with the first derivative, the instantaneous frequency calculated by the second derivative is closer to the real value.

Therefore, in order to reduce the deviation when calculating the instantaneous frequency with the SST, Equation (5) is modified by the SSST, and then the modified instantaneous frequency is defined as [17]

$$\hat{w}^{(2)}(t,w) = \begin{Bmatrix} \hat{w}(t,w) + \hat{q}(t,w)(t - \hat{t}(t,w)), & \text{if } \partial_t \hat{t}(t,w) \neq 0 \\ \hat{w}(t,w), & \text{otherwise} \end{Bmatrix} \tag{10}$$

where $\hat{q}(t,w)$ is the modulation operator, and it is defined as follows [18]:

$$\hat{q}(t,w) = \mathrm{Re}\left(\frac{\partial_t(\partial_t F(t,w)/F(t,w))}{2\mathrm{i}\pi - \partial_t(\partial_w F(t,w)/F(t,w))}\right) \tag{11}$$

In this equation, $\hat{w}(t,w)$ is defined as follows:

$$\hat{t}(t,w) = t - \mathrm{Re}\left(\frac{\partial_w F(t,w)}{\mathrm{i}F(t,w)}\right) \tag{12}$$

where $\partial_w F(t,w)$ is the partial derivative to frequency. To further suppress noise, the value of STFT spectrum is set as zero when less than threshold, as follows:

$$F'(t,w) = \begin{Bmatrix} F(t,w), & |F(t,w)| \geq T \\ 0, & |F(t,w)| < T \end{Bmatrix} \tag{13}$$

where T is the threshold. Then, the SSST based on the square of STFT amplitude is defined as

$$\hat{F}(t,\hat{w}^{(2)}) = \int_R \left|F'(t,w)\right|^2 \delta(w - \hat{w}^{(2)}(t,w))dw \tag{14}$$

2.3. Time-Frequency Energy Aggregation Analysis

The STFT of LFM signals describes the linear change of its frequency over time, and its STFT is a straight line. Therefore, the higher the time-frequency energy aggregation of LFM signals, the closer its distribution in the time-frequency domain is to the straight line, the better the anti-noise performance, and the more conducive it is to the subsequent parameter detection.

Renyi entropy can describe the uncertainty of information distribution. The stronger the randomness of the signal, the larger the Renyi entropy value. Conversely, the weaker the randomness of the signal, the smaller the Renyi entropy value. For the time-frequency energy distribution, the more dispersed the time-frequency energy distribution, the larger the Renyi entropy value, and the more concentrated the time-frequency energy, the smaller the Renyi entropy value. Therefore, Renyi entropy can be used to describe the strength of several time-frequency analysis methods. Renyi entropy is defined as

$$H_a(p) = \frac{1}{1-a} \ln \sum_{i=1}^{N} {p_i}^a \tag{15}$$

where a is the order of Renyi entropy, and a is generally an integer. Here, a takes the value of 2.

3. Methodology

The STFT of LFM signals describes the linear relationship of its frequency change with time, because its distribution in the time-frequency domain is a straight line, and the parameters of the line can reflect various modulation parameters of the original LFM signals, such as frequency. Therefore, the detection of the parameters of the line is an important means to detect the parameters of LFM signals. As a method of linear parameter detection, Hough transform has better anti-noise performance and can effectively detect the straight line in the image, and the peak value of the Hough transform domain directly corresponds to the slope and intercept of the line. However, the detection accuracy of parameters is affected by the resolution of coordinate (θ, ρ) in Hough transform domain. In line detection, if a higher angular resolution is adopted, the calculation time and peak search time of Hough transform will be increased. But if a lower angular resolution is adopted, the accuracy of parameter estimation will be lower.

Therefore, in order to obtain higher parameter estimation accuracy of LFM signals and reduce computation and peak search time, this paper first uses Hough transform to perform rough estimation of the slope of frequency modulation with lower angular resolution, and then uses FRFT to perform higher precision estimation of the slope of frequency modulation.

3.1. Rough Estimation of the Frequency Modulation Slope

Hough transform maps a line in an image to a point in Hough transform domain, which uses the line and point correspondence between image space and Hough transform parameter space. According to the principle of Hough transform, if a line is represented by polar coordinates, it can be expressed as

$$\rho = x\cos\theta + y\sin\theta \tag{16}$$

where ρ represents the vertical distance from the origin to the line, θ represents the angle between the vertical line and the horizontal axis of the line, and θ ranges from -90° to 90°. Then, the correspondence between the slope of line and the peak value of the Hough transform domain can be expressed as [19]

$$\hat{k} = -\cot\theta_m \tag{17}$$

where θ_m represents the horizontal coordinate corresponding to the peak point.

When the discrete data space is mapped to the Hough transform domain, the Hough transform domain needs to be discretized, then (θ, ρ) needs to be discretized, θ is evenly divided into L parts, and ρ is evenly divided into M parts. In this case, the Hough transform domain is evenly divided into $L \times M$ parts. After the image space is mapped to the Hough transform domain, the cumulative results of each region are counted, and the coordinates corresponding to the peak can be obtained through peak search.

3.2. Precise Estimation of the Frequency Modulation Slope

After rough estimation of the slope of frequency modulation, accurate estimation can be performed around the rough estimation value. When θ_m is estimated with high precision, the range of search ranges from $[\theta_m - \Delta\theta, \theta_m + \Delta\theta]$, where $\Delta\theta$ represents the angular resolution. In order to obtain accurate estimation of the slope of frequency modulation from the range of search, an accurate estimation method of the slope of frequency modulation based on FRFT is proposed.

The P-order FRFT of signal $x(t)$ is defined as [20]

$$X_P(u) = \int_{-\infty}^{\infty} x(t)K_P(t,u)dt \tag{18}$$

where $K_P(t,u)$ is the kernel function, and it is defined as

$$K_P(t,u) = \begin{cases} A_\alpha e^{i\pi(u^2 \cot\alpha - 2ut\csc\alpha + t^2\cot\alpha)} & \alpha \neq n\pi \\ \delta(t-\tau) & \alpha = 2n\pi \\ \delta(t+\tau) & \alpha = (2n+1)\pi \end{cases} \quad (19)$$

where A_α is equal to $\sqrt{1 - i\cot\alpha}$, and α is equal to $P\pi/2$.

Since the Renyi entropy can describe the strength of the aggregation of energy, the Renyi entropy can be selected to evaluate the aggregation of energy of FRFT result under different rotation order, and the minimum Renyi entropy corresponds to the optimal estimation of the slope of frequency modulation.

The steps of accurate estimation of the slope of frequency modulation based on FRFT are as follows:

(a) For the range of search from $\theta_m - \Delta\theta$ to $\theta_m + \Delta\theta$, the angular resolution is set to $2\Delta\theta/i$, where i is the integer. Then, the rotation angle corresponding to each discrete point is calculated, the FRFT corresponding to each rotation angle and the corresponding Renyi entropy are calculated, and the polar angle θ_1 corresponding to the minimum value is searched;
(b) Determine whether the angular resolution corresponding to step 1 reaches the threshold. If it does, the polar angle corresponding to the minimum Renyi entropy is the required optimal $\hat{\theta}_m$; otherwise, continue the search;
(c) The polar angle value θ_1 and the angular resolution $\Delta\theta'$ constitute the search interval $[\theta_1 - \Delta\theta', \theta_1 + \Delta\theta']$, and continue the search according to step 1.

4. Simulation Results

In this section, the energy aggregation of time-frequency spectrum is analyzed, which corresponds to the STFT, SST, SSST1, and SSST2 of the multi-component LFM signals, where SSST1 represents the SSST based on the square value of STFT, and SSST2 represents the SSST based on the value of STFT. The multi-component LFM signals are defined as

$$x(t) = \exp(i\pi kt^2) + \exp(i2\pi f_0 t - i\pi kt^2) \quad (20)$$

The simulation parameters were set as shown in Table 1.

Table 1. Simulation parameters.

The Name of Simulation Parameters	Value
The form of signal	Chirp
The pulse width of signal (μs)	20
The bandwidth of signal (MHz)	40
The slope of frequency modulation (MHz/μs)	2
Center frequency (MHz)	40
SNR (dB)	−14~0

4.1. The Analysis of the Aggregation of the Time-Frequency Spectrum

The time-frequency distribution of the STFT, SST, SSST1, and SSST2 of multi-component LFM signals under the above simulation parameters is shown in Figure 1, when the SNR is 0 dB. It can be seen that the frequency of LFM signals changes linearly with time from Figure 1. Then, on the basis of the time-frequency transform, the Hough transform can be used for linear detection. On this basis, the parameters of the LFM signals can be estimated. By comparing Figure 1b,c, it can be seen that the time-frequency energy aggregation of the SSST is better than the SST. By comparing Figure 1b–d, it can be seen that the anti-noise performance of the SST based on the square of amplitude of the SSST are better than the SSST based on the amplitude of the STFT.

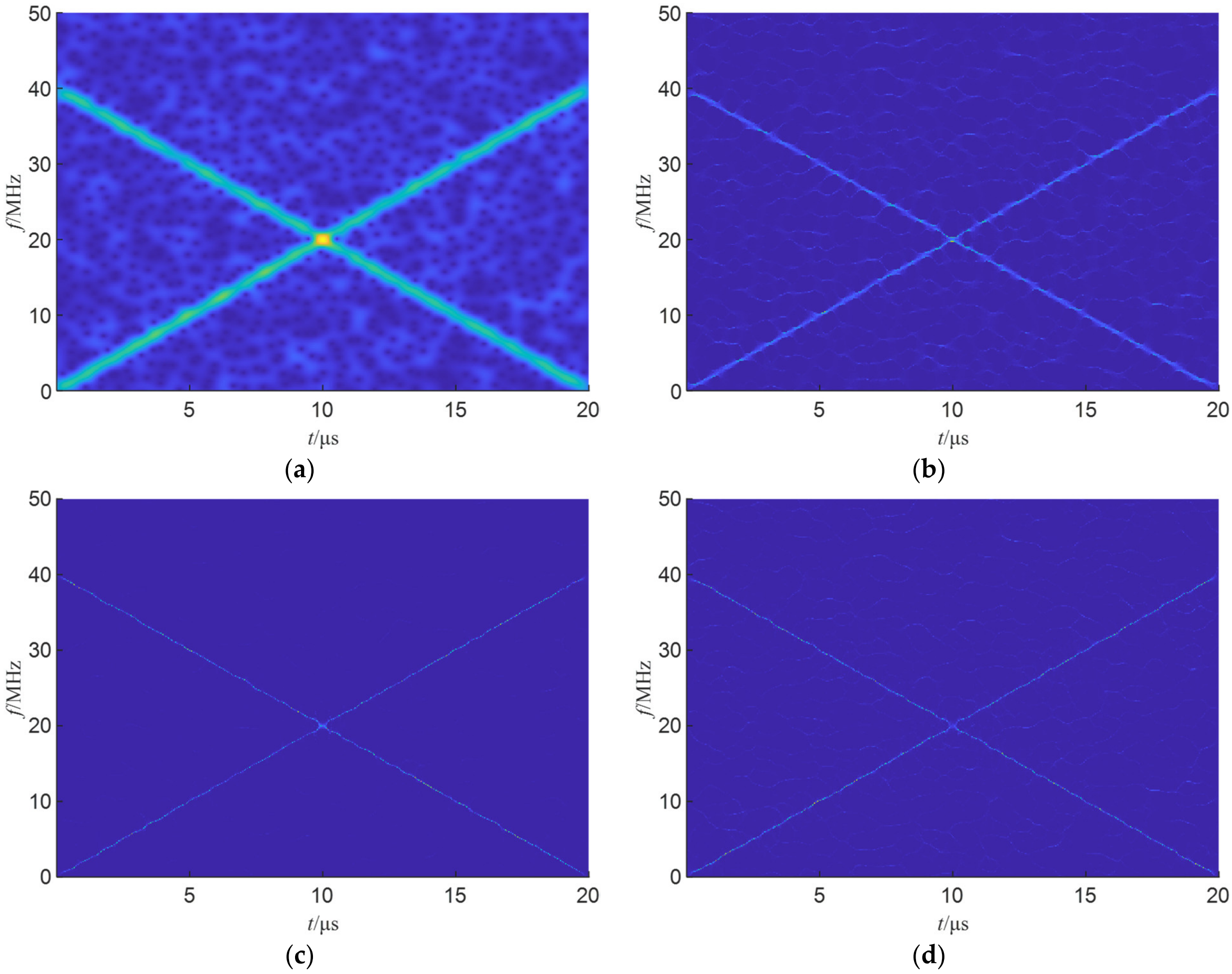

Figure 1. Time-frequency spectrum of LFM signals based on (**a**) STFT, (**b**) SST, (**c**) SSST1, (**d**) SSST2.

The Renyi entropy change under different SNRs of the SST, SSST1, and SSST2 is shown in Figure 2, when the Renyi entropy is used as an evaluation index of the time-frequency energy aggregation. It can be seen that the time-frequency energy aggregation of the SSST based on the square amplitude of the STFT is superior to the SST and the SSST based on the amplitude of the STFT. With the increase of SNRs, the time-frequency energy concentration of the SSST is better than that of the SST and STFT.

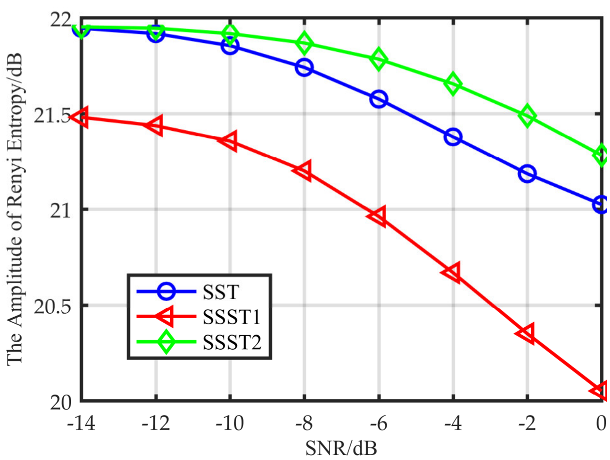

Figure 2. Aggregation of time-frequency energy based on SST, SSST1, and SSST2.

4.2. Error Analysis of Estimation Error of Frequency Modulation

The estimation error of the slope of frequency modulation is analyzed below, and the simulation steps are as follows:

(a) For a certain SNR, the LFM signal model is used to generate the data sample, and the generated two-dimensional time-frequency spectrum is binarized after the SSST;
(b) The slope of frequency modulation is a rough estimation by the Hough transform. The initial polar resolution is set to 1°, and the Hough transform is performed on the image generated in step 1 to detect the coordinate of the peak. In this case, the value of θ_m corresponds to the horizontal coordinate, and θ_m is the rough estimation of the polar angle;
(c) The slope of frequency modulation is accurately estimated based on the FRFT. The initial search interval ranges from $(\theta_m - 1^\circ)$ degree to $(\theta_m + 1^\circ)$ degree, i is set to 20, and the threshold is set to 0.001°. $\hat{\theta}_m$ is obtained according to the corresponding steps, and $\hat{\theta}_m$ is the accurate estimation of the slope of frequency modulation;
(d) Repeat steps (a), (b), and (c). Perform 100 Monte Carlo experiments for each SNR, and calculate the relative estimation error of the slope of frequency modulation. The relative estimation error is defined as follows:

$$\mathrm{RMSE} = \frac{\sqrt{\frac{1}{N}\sum_{i=1}^{N}(\hat{k}_i - k)^2}}{k} \tag{21}$$

The relative estimation error of the slope of frequency modulation at different SNRs is shown in Figure 3. Compared with two estimation methods based on the STFT and SSST1, the estimation error of the slope of frequency modulation based on the SSST1 is smaller than that based on the STFT at low SNR. At different SNRs, the estimation error of the slope of frequency modulation decreases with the increase of SNR. This is because the higher the SNR, the closer the time-frequency distribution is to a straight line. The simulation result showed that the parameter estimation method based on the SSST1 had better robustness and higher precision than that based on the STFT.

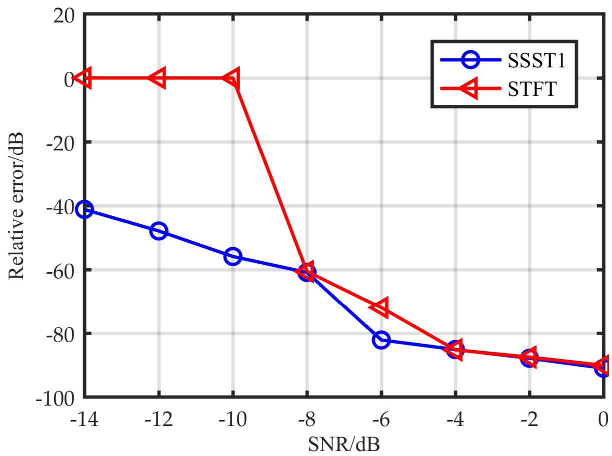

Figure 3. Relative estimation error of the slope of frequency modulation.

The computational analysis of the SSST mainly focuses on the STFT and Hough transform. Assuming that the matrix size of LFM signals after time-frequency conversion is $N \times P$, the STFT needs $NP\log_2 P$ operations, and the Hough transform needs $N \times P \times L$ operations. Therefore, the computation required for the SSST is $NP(\log_2 P + L)$ operations.

5. Discussion

The advantages of the method presented in this paper are as follows:

(a) In order to improve the time-frequency energy aggregation of the STFT, the SSST based on the square of the STFT amplitude is proposed, which has better anti-noise effect than the direct addition of the result of the STFT;
(b) In order to reduce the amount of computation of the Hough transform, combined with the linear detection ability of the Hough transform for time-frequency spectrum with low SNR, the LFM signal is coarsely estimated with low angular resolution, and then the FRFT is used for parameter precision estimation based on the Renyi, entropy;
(c) Using the Renyi entropy as the evaluation criterion of FRFT energy concentration, the traditional two-dimensional peak search method is transformed into a one-dimensional minimum search, which reduces the search time and has better anti-noise performance;
(d) The center frequency of LFM signals can be further estimated on the basis of accurate estimation of the slope of frequency modulation.

According to the analysis of the SSST above, the SSST is used to rearrange the time-frequency energy based on the STFT. Compared with the STFT, the calculation amount of the SSST is greatly increased. Therefore, in the future, we can combine the SSST and the sparse Fourier transform to reduce the calculation amount.

Author Contributions: Validation, J.L.; Writing—original draft, G.Z.; Writing—review & editing, J.Z. and K.Y. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by Young scientist fund of NSFC (China) grant number 61801212.

Data Availability Statement: The data presented in this study are available on request from the corresponding author. The data are not publicly available due to ongoing research.

Conflicts of Interest: The authors declare no conflict of interest.

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