

Article

Analysis on Fairness and Efficiency of the 3-PLP LDM System Using a Normalized Channel Capacity

Ho Jae Kim ¹, Soon-Young Kwon ¹, JaeHwui Bae ², Namho Hur ² and Hyoung-Nam Kim ^{1,*}¹ Department of Electronics Engineering, Pusan National University, Busan 46241, Republic of Korea² Electronics and Telecommunications Research Institute, Daejeon 34129, Republic of Korea

* Correspondence: hnkim@pusan.ac.kr

Abstract: This article presents a novel approach to investigate efficient resource allocation in the layered division multiplexing (LDM) system, a key technology in next-generation broadcasting systems. The LDM system provides diverse services, resulting in different channel capacities for each service, unlike the conventional non-orthogonal multiple access scheme that aims for equal channel capacity among users. To achieve efficient resource allocation among services, we proposed a new method that normalizes the channel capacities allocated to different services while maintaining high fairness. Our proposed method involves normalizing the channel capacities of each service and finding the point where the sum of normalized channel capacities is maximized. This point can be obtained using the Lagrange multiplier method, ensuring a high normalized fairness index. Through simulations, we confirmed that the proposed approach maintains high fairness in the LDM system and achieves the highest normalized channel capacities compared to other transmission techniques. These findings provide valuable insights into resource allocation strategies for the LDM system, enabling a fair comparison of efficiency and performance within a multi-service environment.

Keywords: layered division multiplexing; non-orthogonal multiplexing; Lagrange multiplier; channel capacity; fairness



Citation: Kim, H.J.; Kwon, S.-Y.; Bae, J.; Hur, N.; Kim, H.-N. Analysis on Fairness and Efficiency of the 3-PLP LDM System Using a Normalized Channel Capacity. *Electronics* **2023**, *12*, 4465. <https://doi.org/10.3390/electronics12214465>

Academic Editors: Martin Reisslein and Yung-Fa Huang

Received: 24 August 2023

Revised: 14 October 2023

Accepted: 28 October 2023

Published: 30 October 2023



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1. Introduction

The demand for high-quality broadcasting services has increased rapidly, resulting in significant progress in the digital broadcasting and communication industries. To cater to the evolving needs, the Advanced Television Systems Committee (ATSC) has developed ATSC 3.0 as the standard for next-generation broadcasting systems [1,2]. The physical layer of the ATSC 3.0 system has been designed to enhance flexibility, robustness, and spectral efficiency compared to previous standards, enabling the simultaneous support of a diverse range of services. These include ultra-high definition (UHD) and mobile HD services [3–7].

The physical layer of ATSC 3.0 includes various technologies, such as channel coding and symbol mapping through bit-interleaved coded modulation (BICM) blocks, low-density parity check, bit-interleaver, and a range of constellations. These constellations span from quadrature phase-shift keying (QPSK) to 4096 quadrature amplitude modulation (QAM), and they also include non-uniform constellations [8,9]. The system generates waveforms by performing various interleaving operations based on orthogonal frequency division multiplexing, thus facilitating high-capacity broadcasting services [1–6].

The layered division multiplexing (LDM) system adopted within ATSC 3.0 is a non-orthogonal multiple access (NOMA) scheme which allows the concurrent transmission of various signals through distinct layers [10–17]. The LDM system efficiently utilizes limited time and frequency resources to provide multiple services. Compared to binary time division multiplexing (TDM) and frequency division multiplexing systems, the two-layer LDM system in ATSC 3.0 demonstrates enhanced transmission efficiency [10–17]. Figure 1 illustrates a block diagram of the LDM and TDM systems utilizing two physical layer pipes

(PLPs) [18]. In the LDM system, channel capacity is determined by adjusting the power ratio for each PLP using the injection level parameter α . The parameter α scales the power level of the lower layer relative to the upper layer. Additionally, the power normalizing factor β is multiplied by the combined LDM signal to maintain a constant total transmit power. In contrast, the channel capacity of each PLP within a TDM system is determined by the time ratio [5].

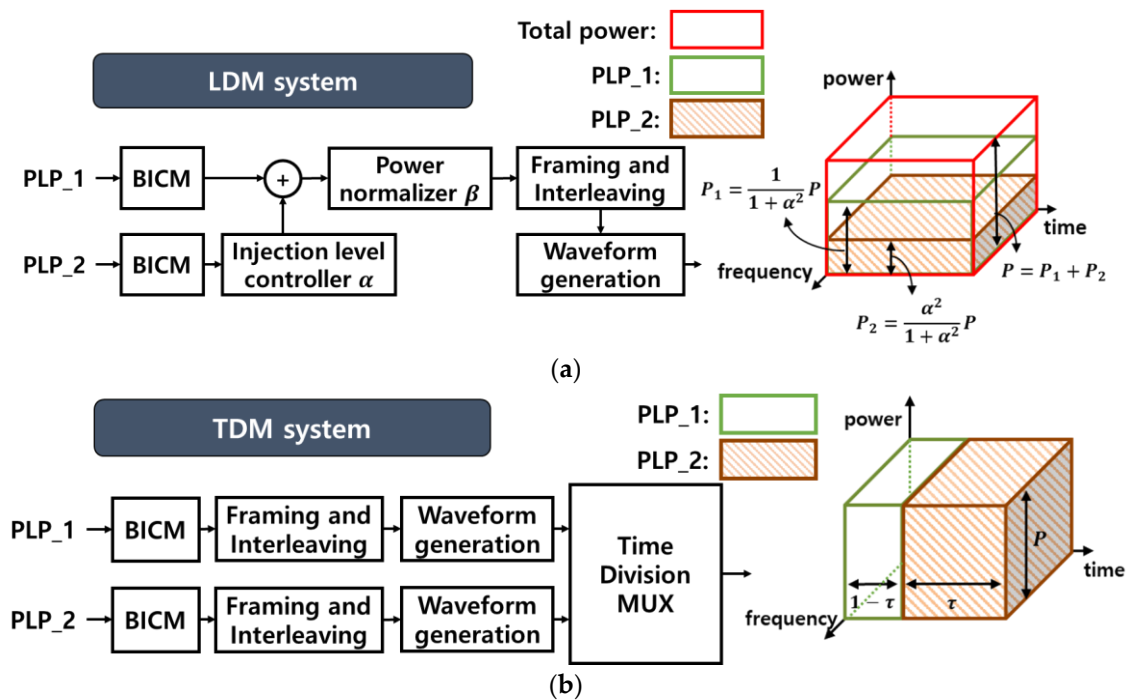


Figure 1. Block diagram of (a) an LDM system with two PLPs, (b) a TDM system with two PLPs.

The LDM system has different objectives as compared to traditional NOMA systems. Traditional NOMA systems aim to maximize spectrum efficiency by allowing multiple users to share the same time and frequency resources. Therefore, the goal is to provide equal channel capacity to all users, considering high fairness among them [19–22]. However, the LDM system in ATSC 3.0 aims to provide various services, such as emergency messaging, mobile communication, and fixed rooftop broadcasting, within limited time and frequency resources [3–7]. The objective is to optimize channel capacity allocation among these services to offer a diverse range of services effectively. In LDM systems, the channel capacity provided for each service is different, so it may be challenging to obtain the fairness and maximum sum rate used in NOMA performance analysis.

In this article, we proposed a method for analyzing the efficiency of the LDM system by normalizing the allocated channel capacities for each service. Through this normalization process, we can evaluate both the normalized fairness and the sum of normalized channel capacities. This approach allows for an equitable comparison of efficiency and resource allocation among various services within the LDM system. In NOMA, maximizing the total channel capacity while upholding fairness is essential. Therefore, we propose a new method for resource allocation in the LDM system that determines where the sum of normalized channel capacities is maximized while preserving normalized fairness. This is achieved by using the Lagrange multiplier method to determine the maximum point. Using this approach, we analyze and compare the channel capacity efficiency and reception performance of implementable LDM, TDM, layered-time division multiplexing (LTDM), and time-layered division multiplexing (TLDM) systems in a 3-PLP configuration.

1.1. Research Background

In previous LDM research, many studies have been conducted to prove the superiority of LDM systems over TDM systems by setting specific injection levels and time ratios [5,23]. Figure 2 compares the LDM system with an injection level α and the TDM system with a time ratio τ [11,24]. The graph shows that the LDM system (represented by the red-line area) outperforms the TDM system (indicated by the red dot) regarding channel capacity for both PLP_1 and PLP_2 at a specific τ . However, determining the most efficient α is not straightforward as the channel capacities of the two PLPs are different. No research has been conducted to determine which LDM system is superior under identical conditions (time duration, bandwidth, etc.), that is, which α value is optimal [5,23]. Therefore, we proposed a new analytical method to determine the optimal value of α and configure the most efficient LDM system. Shannon channel capacities C_1 and C_2 can be defined based on the target received signal-to-noise ratio (SNR) of each PLP [11]. By employing the given SNR, the channel capacity of each PLP in the LDM system can be expressed as a function of α . To determine the optimal value of α , we define the normalized channel capacities of the two PLPs. Normalizing the channel capacity of each PLP allows for a fair comparison of system efficiencies while considering various parameters. The sum of the normalized channel capacities is used to determine the most advantageous combination. Therefore, the most efficient value of α for the LDM system is identified by maximizing the sum of normalized channel capacities.

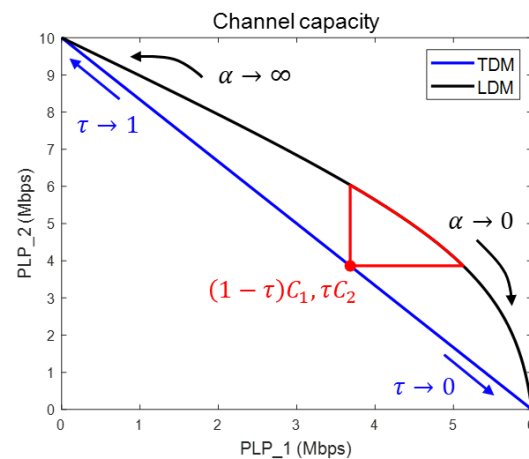


Figure 2. Channel capacity of LDM vs. TDM with two PLPs ($C_1 = 6$ Mbps, $C_2 = 10$ Mbps).

The channel capacity of multi-PLP transmission schemes has been studied in the literature [5,18,25,26]. Efficient channel capacity allocation is necessary for providing various services, such as breaking news in disaster situations, UHD broadcasting services, and mobile HD services [3–7]. Like the LDM system for 2-PLP, the power ratio of each layer in the 3-PLP LDM system is determined by two injection levels. The TDM system transmits using a time ratio divided into three from the total duration [5,18]. The LTDM and TLDM systems combine TDM and LDM principles in a 3-PLP transmission scheme [5,26]. In literature, investigations have shown that performance analyses for particular injection levels and time ratios demonstrate their superiority over TDM, similar to the 2-PLP system [5,26].

Similar to the NOMA system, the LDM system shares the same bandwidth and time resources as all PLPs. In a recent study of the NOMA system, resource allocation has been designed to ensure fairness among multiple users while providing equal channel capacity. This design approach allows NOMA to determine where the sum of the channel capacities of all users is maximized while maintaining fairness [19–22]. However, the primary objective of the LDM system is to provide various services [3–7]. Consequently, it offers a different channel capacity to each PLP. Therefore, when NOMA's fairness analysis is applied to the LDM system, the sum of channel capacities cannot be maximized. Likewise,

maximizing the sum of channel capacities results in an unfair system. To address this, we proposed a new fairness index in LDM to make it comparable. We also introduce a method to analyze whether the sum of normalized channel capacities is maximized while maintaining fairness among PLPs.

Regarding wireless transmission system, the channel capacity is affected by two key factors: bandwidth and signal power, which are considered specifications of a transmission system [27]. These specifications are crucial when comparing different multiplexing methods. We assume the bandwidth and total signal power are equal to ensure a fair comparison. Specifically, in the LDM system, the sum of power of the PLPs is equal to the total power of the TDM system [5,18,26]. By imposing this constraint, the Lagrange multiplier method [28] can be employed to determine the most efficient combination of α , representing the LDM system's power allocation. The goal is to find the value of α that maximizes the sum of normalized channel capacities in the 3-PLP system. The channel capacity efficiency and reception performance of the TDM, LTDM, and TLDM systems can be analyzed and compared by utilizing the Lagrange multiplier method and comparing normalized channel capacities.

1.2. Contribution

The key contributions of this study to the analysis of the LDM system are as follows:

- Proposal of a novel resource allocation method: In conventional NOMA approaches, resource allocation is designed to ensure equal channel capacity among all users, emphasizing fairness [19–22]. However, in the context of the LDM system, where the objective is to offer diverse services, equal channel capacity is not provided for all services. To address this challenge, we propose a novel resource allocation method that normalizes the channel capacities allocated to different services while maintaining high fairness.
- Efficient resource allocation for LDM system: Previous performance analyses of the LDM system relied on arbitrary assignment of resources using injection levels for comparison with other transmission methods [3–7,26]. Therefore, this study introduces a resource allocation method that achieves high normalized fairness and the most efficient transmission strategy for the LDM system. Here, we use the Lagrange multiplier method to identify the optimal point where the sum of normalized channel capacities is maximized.
- Performance evaluation: The proposed LDM system's performance is evaluated through simulations. The results demonstrate that the LDM system maintains a high level of normalized fairness while achieving the highest sum of normalized channel capacities compared to other transmission methods. The performance evaluation shows the superiority of the proposed resource allocation approach in enhancing the overall system efficiency and fairness of the LDM system.

Overall, this study provides valuable insights into the effective resource allocation for the LDM system, enabling a fair comparison of its efficiency and performance with other transmission techniques. The proposed method advances next-generation broadcasting systems by optimizing resource allocation strategies in multi-service environments.

1.3. Organization

The remainder of this article is organized as follows: Section 2 explains the structures of the LDM, TDM, LTDM, and TLDM used in the 3-PLP system. Section 3 discusses the issues related to fairness and performance measures in the LDM system. Section 4 proposes a method to determine the optimal normalized channel capacity using the Lagrange multiplier. Section 5 analyzes the efficiency of various transmission combinations of the 3-PLP system and compares the reception performance of each PLP using the parameters obtained through the Lagrange multiplier. Finally, Section 6 concludes the article.

2. Structure and Channel Capacity of Three PLPs

This section will discuss the structure, parameters, and channel capacity of the four systems: LDM, TDM, LTDM, and TLDM [5,18,26,29].

2.1. TDM System with Three PLPs

Figure 3a,b illustrate the block diagram and signal structure of a TDM system using three PLPs, respectively. In the TDM system, the time ratio $\tau_{TDM,i}$ is applied as the ratio of the allocated time duration within the total time duration for transmission. Each PLP independently generates a signal and provides the final TDM signal at specified time ratios of $\tau_{TDM,1}$, $\tau_{TDM,2}$, and $\tau_{TDM,3}$ set in the time division multiplexer. In an ideal TDM system, the Shannon channel capacity $C_{TDM,i}$ of i -th PLP can be defined as:

$$C_{TDM,i} = \tau_{TDM,i} W \log_2 \left(1 + \frac{P}{N_i} \right), \quad (i = 1, 2, 3), \quad (1)$$

where W is the signal bandwidth, P is the signal power, and N_i is the additive noise of the i -th received PLP signal.

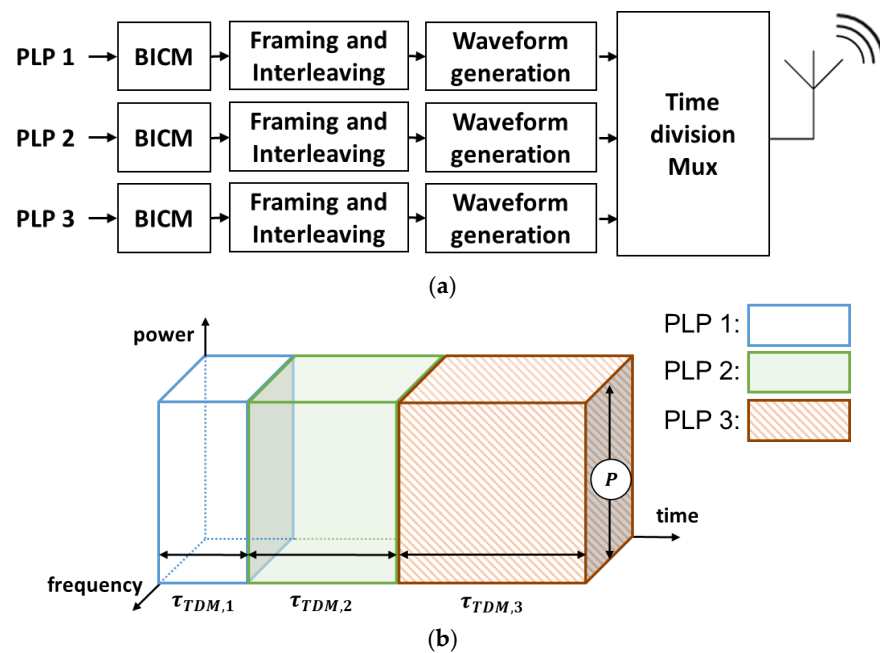


Figure 3. (a) Block diagram, (b) signal configuration of a TDM with three PLPs.

2.2. LDM System with Three PLPs

Figure 4a,b illustrate the block diagram and signal structure of an LDM system using three PLPs, respectively. The LDM system is a non-orthogonal multiplexing method where the power of each PLP signal varies based on the injection level, and all PLPs are simultaneously transmitted.

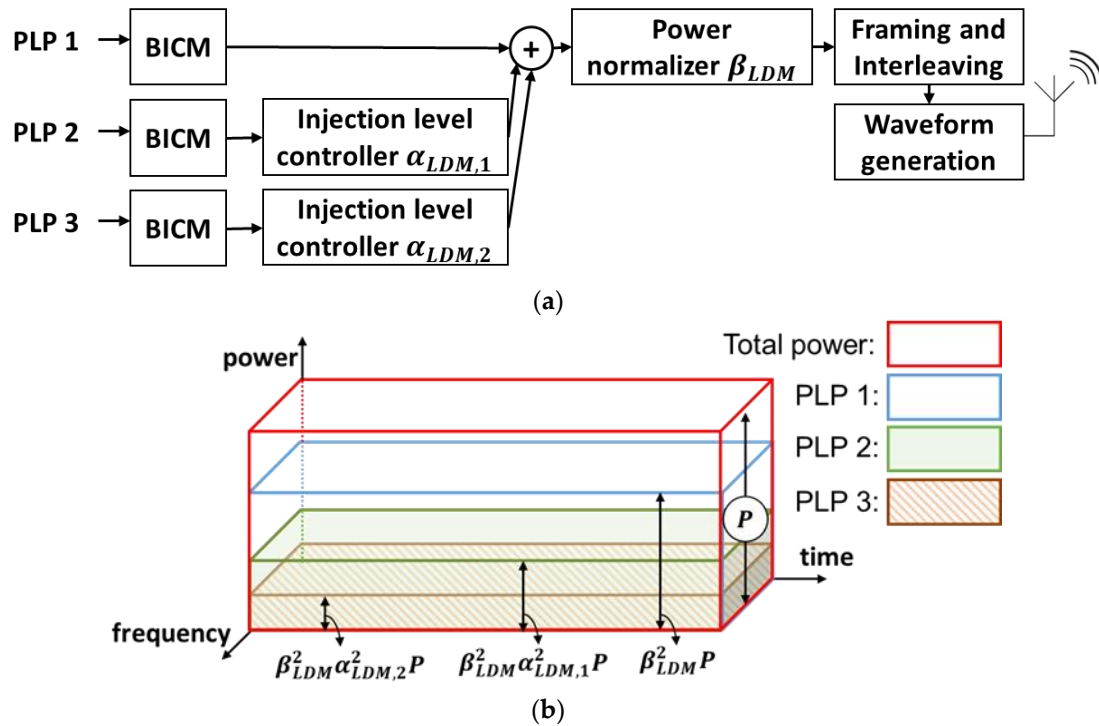


Figure 4. (a) Block diagram, (b) signal configuration of an LDM with three PLPs.

In a 2-PLP LDM system, the upper layer is called the core layer (CL), which aims to provide robust mobile services. The lower layer is the enhanced layer (EL), responsible for delivering UHDTV or full HDTV services [10–15]. To obtain each PLP in a 2-PLP LDM system, the CL signal, which typically requires a low SNR, is first decoded [10–15]. The EL signal is treated as interference during the decoding of the CL signal. However, to obtain the EL signal, it is essential to remove the CL signal [15]. This sequentially removing the upper-layer signals is called successive interference cancellation (SIC) [5,20].

When extending to a 3-PLP LDM system, as shown in Figure 4b, the PLP_2 and PLP_3 signals are considered interference signals when decoding the PLP_1 signal. In this case, the capacity of PLP_1, denoted as $C_{LDM,1}$, is expressed as follows:

$$C_{LDM,1} = W \log_2 \left(1 + \frac{P}{(\alpha_{LDM,1}^2 + \alpha_{LDM,2}^2)P + N_1(1 + \alpha_{LDM,1}^2 + \alpha_{LDM,2}^2)} \right), \quad (2)$$

where $\alpha_{LDM,1}$ and $\alpha_{LDM,2}$ denoted the injection levels of PLP_2 and PLP_3, respectively. Like the 2-PLP LDM system, the PLP_1 signal must be removed to decode the PLP_2 signal. The removal of PLP_1 signal is achieved by the SIC method. After removing the PLP_1 signal, the PLP_2 signal is decoded by considering the PLP_3 signal as an interference signal. The capacity of PLP_2, denoted as $C_{LDM,2}$, can be expressed using the following equation:

$$C_{LDM,2} = W \log_2 \left(1 + \frac{\alpha_{LDM,1}^2 P}{\alpha_{LDM,2}^2 P + N_2(1 + \alpha_{LDM,1}^2 + \alpha_{LDM,2}^2)} \right). \quad (3)$$

Finally, the PLP_3 signal is decoded after regenerating and removing the decoded PLP_2 signal by the same process of SIC. The capacity of PLP_3, denoted as $C_{LDM,3}$, can be expressed using the following equation:

$$C_{LDM,3} = W \log_2 \left(1 + \frac{\alpha_{LDM,2}^2 P}{N_3 (1 + \alpha_{LDM,1}^2 + \alpha_{LDM,2}^2)} \right). \quad (4)$$

2.3. LTDM System with Three PLPs

Figure 5a,b illustrate the block diagram and signal structure of an LTDM system using the three PLPs, respectively. In the LTDM system, either a CL or an EL from the 2-PLP LDM system can be employed as the TDM. Here, we chose to adopt the EL as the TDM, and Figure 5 illustrates this configuration. When transmitting the CL transmitting the PLP_1 signal, the EL can be regarded as an interference signal, and the channel capacity $C_{LTDM,1}$ is computed as follows:

$$C_{LTDM,1} = W \log_2 \left(1 + \frac{P}{\alpha_{LTDM}^2 P + N_1 (1 + \alpha_{LTDM}^2)} \right), \quad (5)$$

where α_{LTDM} denotes is the injection level of an LTDM system. After regenerating and removing the acquired PLP_1 signal (SIC method), the remaining PLP_2 and PLP_3 signals are identical to those of the 2-PLP TDM system. When τ_{LTDM} is the time ratio of the LTDM system, the channel capacities $C_{LTDM,2}$ and $C_{LTDM,3}$ are expressed as follows:

$$C_{LTDM,2} = \tau_{LTDM} W \log_2 \left(1 + \frac{\alpha_{LTDM}^2 P}{N_2 (1 + \alpha_{LTDM}^2)} \right), \quad (6)$$

$$C_{LTDM,3} = (1 - \tau_{LTDM}) W \log_2 \left(1 + \frac{\alpha_{LTDM}^2 P}{N_3 (1 + \alpha_{LTDM}^2)} \right). \quad (7)$$

2.4. TLDM System with Three PLPs

Figure 6a,b illustrate the block diagram and signal structure of a TLDM system delivering three PLPs. In a TLDM system, two TDM signals are allocated to two time-division sections, out of which one is used for the two-layer LDM. Therefore, the channel capacity of PLP_1, denoted as $C_{TLDM,1}$, is defined as follows:

$$C_{TLDM,1} = \tau_{TLDM} W \log_2 \left(1 + \frac{P}{N_1} \right), \quad (8)$$

where τ_{TLDM} denotes the time ratio of the TLDM system. In decoding the PLP_2 signal, the PLP_3 signal is regarded as an interference signal. Thus, the channel capacity of the time-divided CL layer, denoted as $C_{TLDM,2}$, is expressed using the following equation:

$$C_{TLDM,2} = (1 - \tau_{TLDM}) W \log_2 \left(1 + \frac{P}{\alpha_{TLDM}^2 P + N_2 (1 + \alpha_{TLDM}^2)} \right), \quad (9)$$

where α_T is the injection level of the TLDM system. After regenerating and removing the acquired PLP_2 signal, the PLP_3 signal is obtained, and the channel capacity of the timedivided EL layer, denoted as $C_{TLDM,3}$, is computed as follows:

$$C_{TLDM,3} = (1 - \tau_{TLDM}) W \log_2 \left(1 + \frac{\alpha_{TLDM}^2 P}{N_3 (1 + \alpha_{TLDM}^2)} \right). \quad (10)$$

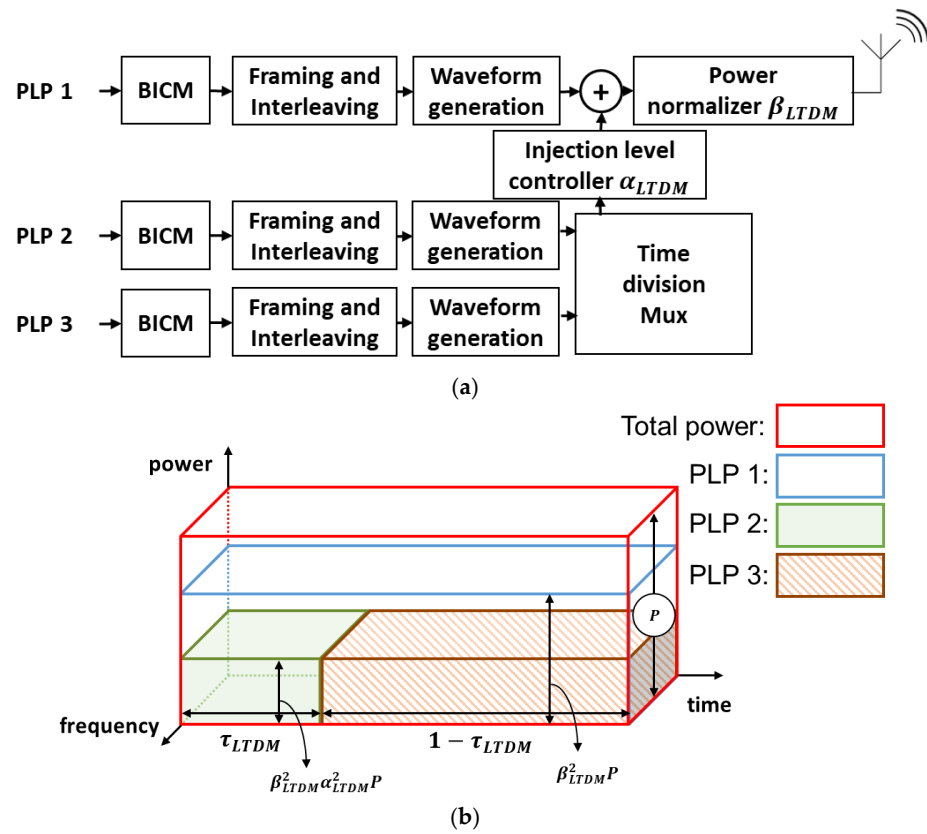


Figure 5. (a) Block diagram, (b) signal configuration of an LTDM with three PLPs.

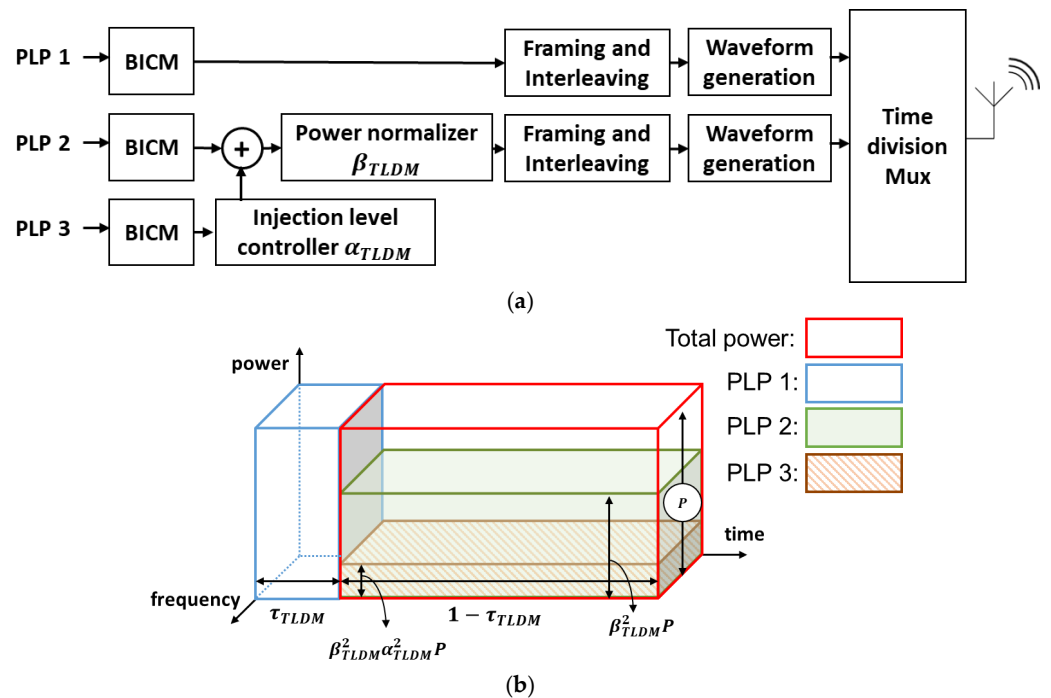


Figure 6. (a) Block diagram, (b) signal configuration of a TLDM with three PLPs.

3. Problem Description

LDM shares time, space, and frequency resources to transmit various services, and its transmission method is similar to NOMA. However, there are differences between the two transmission methods in terms of purpose and application. While NOMA focuses on

maximizing spectral efficiency with the goal of fair channel capacity allocation to all users, the LDM system aims to efficiently transmit multiple services simultaneously [30–33]. In wireless communication systems, Jain's fairness index (JFI) is commonly used to evaluate fairness [19]. The JFI ranges from 0 to 1, and when it is close to 1, it indicates a fair allocation of resources. The JFI index F is:

$$F = \frac{\left(\sum_{i=1}^U D_i\right)^2}{U \sum_{i=1}^U D_i^2}, \quad (11)$$

where U and D_i denote the number of users and i -th user data rate, respectively. In a NOMA system, research is conducted to allocate resources when the JFI is close to 1, ensuring fair distribution of channel capacity among users [20]. However, in the case of LDM, the fairness index tends to decrease as the number of services varies because each service has a different target channel capacity. Since the objective of the LDM system is to transmit multiple services with varying requirements efficiently, the emphasis is on optimizing resource allocation to meet the specific needs of each service rather than achieving perfect fairness among them.

NOMA generally aims to provide equal data rates to all users in a fair scenario, and the maximum sum rate is used as a performance measure [19–22]. However, in the LDM system, each service may have different data rate requirements, and simply using the maximum sum rate as a performance measure may not lead to an efficient allocation of resources among the services. In the given example of a 3-PLP LDM system, the system considers three different types of transmission with their respective minimum data rate requirements:

- Radio transmission ($D_1 \geq 0.5$ Mbps, QPSK, code rate: 3/15)
- Mobile HD transmission ($D_2 \geq 3$ Mbps, QPSK, code rate: 6/15)
- UHD broadcast ($D_3 \geq 20$ Mbps, 64-QAM, code rate: 9/15)

Suppose the maximum sum rate is used as the performance measure in the 3-PLP LDM system. In that case, the resource allocation will prioritize maximizing the data rate of the UHD broadcast service (D_3). In the case of an LDM system, different constellations and code rates are used for each layer based on the specific service requirements. Therefore, if the goal is to achieve the maximum sum rate, the resource allocation strategy is to allocate as many resources as possible to D_3 , having the highest transmission rate per unit sample. Allocating most resources to D_3 , which has the highest transmission rate, can lead to unfairness between services, particularly for D_1 and D_2 , which only meet their minimum performance requirements. In the case of an LDM system where each service has different data rate requirements, simply maximizing the sum rate may not align with achieving fairness among the services.

To address the fairness issue, we propose the utilization of normalized channel capacities as a solution. By normalizing the channel capacities provided to each service, the authors aim to establish a fair comparison measure. The normalized channel capacities can be used to evaluate the fairness among services, similar to the approach used in NOMA. Furthermore, the authors suggest maximizing the sum of normalized channel capacities as a performance measure for the LDM system. By maximizing this value, the system aims to achieve an optimal allocation of resources that balances efficiency and fairness. This approach resembles the objective of maximizing the sum rate in NOMA. It leads to maximizing the overall system performance while ensuring fairness among users.

The maximum channel capacity C_i of PLP $_i$ is the Shannon channel capacity [29] when PLP $_i$ is used alone and can be expressed as follows:

$$C_i = W \log_2 \left(1 + \frac{P}{N_i} \right), \quad (i = 1, 2, 3), \quad (12)$$

Then, the normalized channel capacity of each system, denoted as $R_{M,i}$, is defined as follows:

$$R_{M,i} = \frac{C_{M,i}}{C_i}, \quad (i = 1, 2, 3), \quad (13)$$

where $C_{M,i}$ denotes the channel capacity of PLP_{*i*} in *M* system, which is one of the four multiplexing systems (TDM, LDM, LTDM, and TLDM). The normalized channel capacity measures the efficiency of each PLP in the different multiplexing systems. We present a method that achieves the highest overall efficiency by maximizing the sum of normalized channel capacities for all PLPs. This approach helps to determine the most efficient multiplexing system among the four options (TDM, LDM, LTDM, and TLDM) based on the considered PLPs.

4. Proposed Resource Allocation Method Using the Lagrange Multiplier

In this section, we aim to determine the optimal channel capacity in an LDM system using the Lagrange multiplier. The Lagrange multiplier method is commonly employed to find the optimal solution under constrained conditions [28]. In the example shown in Figure 7, the maximum sum of the channel capacities is 10 Mbps when only PLP₂ is utilized without the PLP₁ service. However, if both services need to be provided simultaneously, some allocation should be made to PLP₁. In the case of LDM, where each PLP may have different channel capacity requirements, it becomes crucial to identify an optimal solution that efficiently utilizes the system's resources. To address this, we propose the use of normalized channel capacity.

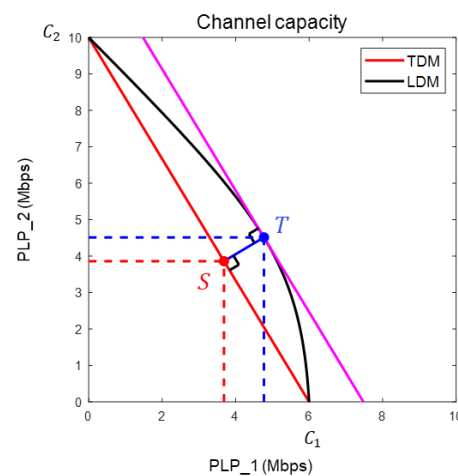


Figure 7. Channel capacity of two PLPs with tangent line (magenta line) for $C_1 = 6$ Mbps and $C_2 = 10$ Mbps.

Figure 8 shows the normalized channel capacity of each PLP, as shown in Figure 7. In the 2-PLP LDM system shown in Figure 8, we can identify the point where the sum of the normalized channel capacities is maximized. Geometrically obtaining the maximum sum involves determining the position where n is maximized in the linear function of $x + y = n$. For the 2-PLP LDM system, the point where n is maximized corresponds to T' , which is a tangent point to the magenta line with a slope of -1 . Point T' represents the normalized position of point T in Figure 7, where point T represents the maximum sum of the normalized channel capacities in the 2-PLP LDM system. The point where the perpendicular to the magenta line passes through point T and intersects the channel capacity of (the red straight line) is called S .

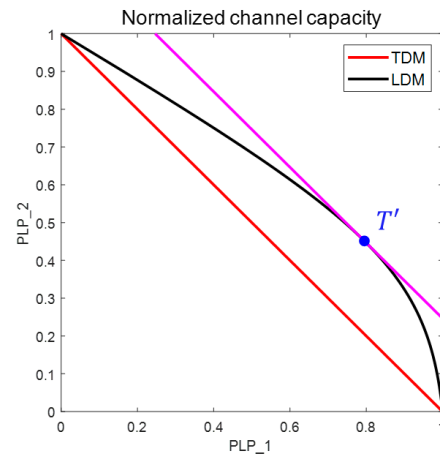


Figure 8. Normalized channel capacity of two PLPs with tangent line (magenta line).

The point T can be analytically determined using the Lagrange multiplier method. Extending to the 3-PLP LDM system, point T in Figure 9 becomes the optimal solution, where the sum of the normalized channel capacities of the three PLPs reaches its maximum. The following subsections describe how the point T can be obtained using the Lagrange multiplier method for the three PLPs.

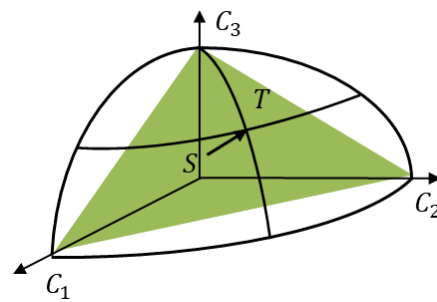


Figure 9. Channel capacity of the LDM (black area) vs TDM (green area) with three PLPs.

4.1. Lagrange Method in the 3-PLP LDM System

The LDM system using the three PLPs is described as follows:

- Initial channel capacities C_i ($i = 1, 2, 3$);
- $\beta_{LDM} = \frac{1}{\sqrt{(1+\alpha_{LDM,1}^2+\alpha_{LDM,2}^2)}}$ (the total powers of TDM and LDM are the same);
- $\alpha_{LDM,1} \neq 0, \alpha_{LDM,2} \neq 0$ (when there is power in all layers);
- PLP_1 has the same additive noise N_i ($i = 1, 2, 3$) in both TDM and LDM;
- Same bandwidth for 3-PLP TDM and 3-PLP LDM systems;
- $N_1 > N_2 > N_3$ (SIC [20])

If the above conditions are satisfied, the channel capacities of the LDM and TDM systems can be configured using three PLPs, as shown in Figure 9. In Figure 9, the channel capacity of the TDM system is represented by the green plane, whereas the black spherical area represents the LDM system. We used the value of $W = 1$ and $P = 1$ to find the optimal point of T . By substituting $\sigma_{LDM,1} = 1 / (1 + \alpha_{LDM,1}^2 + \alpha_{LDM,2}^2)$, $\sigma_{LDM,2} = \alpha_{LDM,1}^2 / (1 + \alpha_{LDM,1}^2 + \alpha_{LDM,2}^2)$, and $\sigma_{LDM,3} = \alpha_{LDM,2}^2 / (1 + \alpha_{LDM,1}^2 + \alpha_{LDM,2}^2)$, Equations (2)–(4) can be rearranged as follows:

$$C_{LDM,1} = \log_2 \left(1 + \frac{\sigma_{LDM,1}}{\sigma_{LDM,2} + \sigma_{LDM,3} + N_1} \right), \quad (14)$$

$$C_{LDM,2} = \log_2 \left(1 + \frac{\sigma_{LDM,2}}{\sigma_{LDM,3} + N_2} \right), \quad (15)$$

$$C_{LDM,3} = \log_2 \left(1 + \frac{\sigma_{LDM,3}}{N_3} \right). \quad (16)$$

By using the above equations, the Lagrangian function can be expressed as follows: [28]

$$L_{LDM}(\sigma_{LDM,1}, \sigma_{LDM,2}, \sigma_{LDM,3}, \lambda) = \frac{C_{LDM,1}}{C_1} + \frac{C_{LDM,2}}{C_2} + \frac{C_{LDM,3}}{C_3} + \lambda(\sigma_{LDM,1} + \sigma_{LDM,2} + \sigma_{LDM,3} - 1) \quad (17)$$

where λ denotes the Lagrange multiplier. The Lagrange multiplier method can obtain a simultaneous equation by assuming that ∇L_{LDM} is 0 [28]. The resulting simultaneous equations are as follows:

$$\frac{\partial L_{LDM}}{\partial \sigma_{LDM,1}} = \frac{1}{C_1(N_1 + 1) \ln 2} + \lambda = 0, \quad (18)$$

$$\frac{\partial L_{LDM}}{\partial \sigma_{LDM,2}} = -\frac{\sigma_{LDM,1}}{C_1(N_1 + 1)(N_1 + \sigma_{LDM,2} + \sigma_{LDM,3}) \ln 2} + \frac{1}{C_2(N_2 + \sigma_{LDM,2} + \sigma_{LDM,3}) \ln 2} + \lambda = 0, \quad (19)$$

$$\frac{\partial L_{LDM}}{\partial \sigma_{LDM,3}} = -\frac{\sigma_{LDM,1}}{C_1(N_1 + 1)(N_1 + \sigma_{LDM,2} + \sigma_{LDM,3}) \ln 2} - \frac{\sigma_{LDM,2}}{C_2(N_2 + \sigma_{LDM,2} + \sigma_{LDM,3})(N_2 + \sigma_{LDM,3}) \ln 2} + \frac{1}{C_3(N_3 + \sigma_{LDM,3}) \ln 2} + \lambda = 0, \quad (20)$$

$$\frac{\partial L_{LDM}}{\partial \lambda} = \sigma_{LDM,1} + \sigma_{LDM,2} + \sigma_{LDM,3} - 1 = 0. \quad (21)$$

Using Equations (18)–(21), we can derive $\sigma_{LDM,1}$, $\sigma_{LDM,2}$ and $\sigma_{LDM,3}$ using the following equations:

$$\sigma_{LDM,1} = \beta_{LDM}^2 = \frac{C_2(N_2 + 1) - C_1(N_1 + 1)}{C_2 - C_1}, \quad (22)$$

$$\sigma_{LDM,2} = \beta_{LDM}^2 \alpha_{LDM,1}^2 = 1 - \sigma_{LDM,1} - \sigma_{LDM,3}, \quad (23)$$

$$\sigma_{LDM,3} = \beta_{LDM}^2 \alpha_{LDM,2}^2 = \frac{C_2 N_2 - C_3 N_3}{(C_3 - C_2)}. \quad (24)$$

By substituting the values of β_{LDM} , $\alpha_{LDM,1}$, and $\alpha_{LDM,2}$ into Equations (2)–(4), the channel capacity of an LDM system with maximum efficiency can be obtained.

To compare with the LDM system, we explain the method to obtain point S in the TDM system, which is closest to point T in Figure 9. To determine S , we derived the equation of a straight line passing through point T and parallel to the normal vector of the TDM channel capacity plane (green plane). The equations of straight-line and the green plane are expressed as follows:

$$\frac{x - C_{LDM,1}}{C_2 C_3} = \frac{y - C_{LDM,2}}{C_3 C_1} = \frac{z - C_{LDM,3}}{C_1 C_2}, \quad (25)$$

$$\frac{x}{C_1} + \frac{y}{C_2} + \frac{z}{C_3} = 1, \quad (26)$$

respectively. The solution to the two equations is located at point S . The coordinate of point S is represented as $[\tau_{TDM,1} C_1, \tau_{TDM,2} C_2, \tau_{TDM,3} C_3]$, where values of $\tau_{TDM,i}$ of the TDM system can be derived as a result of the simultaneous Equations (25) and (26). We obtain the following equations:

$$\tau_{TDM,i} = \frac{C_{LDM,i}}{C_i} + A \frac{C_j C_k}{C_i}, (i = 1, 2, 3, i \neq j \neq k), A = \frac{1 - \left(\frac{C_{LDM,1}}{C_1} + \frac{C_{LDM,2}}{C_2} + \frac{C_{LDM,3}}{C_3} \right)}{\frac{C_2 C_3}{C_1} + \frac{C_3 C_1}{C_2} + \frac{C_1 C_2}{C_3}} \quad (27)$$

The method for obtaining the optimal parameters using the Lagrange multiplier method is summarized as follows:

1. Select the initial values of C_1 , C_2 , and C_3 .
2. Calculate the noise corresponding to C_1 , C_2 , and C_3 .
3. Derive $\sigma_{LDM,1}$, $\sigma_{LDM,2}$, and $\sigma_{LDM,3}$ using the Lagrangian function.
4. Calculate the channel capacity of the LDM system using $\sigma_{LDM,1}$, $\sigma_{LDM,2}$, and $\sigma_{LDM,3}$.
5. Find the point S by deriving the equation of the straight line and the equation of the plane and solving these simultaneous equations.
6. Calculate $\tau_{TDM,1}$, $\tau_{TDM,2}$, and $\tau_{TDM,3}$ from the coordinate point S.

4.2. Lagrange Method in the 3-PLP LTDM and TLDM Systems

Figures 10 and 11 illustrate the channel capacities of an LTDM system (blue area) and a TLDM system (red area). The point at which both systems have their optimal solutions when both τ_{LTDM} and τ_{TLDM} are equal to zero, which means they are used as a 2-PLP LDM system. To compare the scenarios using three PLPs, we added a condition that τ_{LTDM} and τ_{TLDM} are equal to $\tau_{LDM,1}$ of the TDM system obtained from Equation (26). The initial conditions for obtaining the optimal α_{LTDM} and α_{TLDM} of the LTDM and the TLDM systems are described as follows:

- Initial channel capacities C_i ($i = 1, 2, 3$).
- $\beta_{LTDM} = \frac{1}{\sqrt{(1+\alpha_{LTDM}^2)}}$, $\beta_{TLDM} = \frac{1}{\sqrt{(1+\alpha_{TLDM}^2)}}$ (the total powers of TDM, LDM, and LTDM are the same).
- $\alpha_{LTDM} \neq 0$, $\alpha_{TLDM} \neq 0$ (when there is power in all layers);
- PLP_{*i*} has the same additive noise N_i ($i = 1, 2, 3$) in TDM, LTDM, and TLDM;
- Same bandwidth of 3-PLP TDM, 3-PLP LTDM, and 3-PLP TLDM systems;
- $\tau_{TDM,1} = \tau_{LTDM} = \tau_{TLDM}$;
- $N_1 > N_2 > N_3$ (SIC [20]);

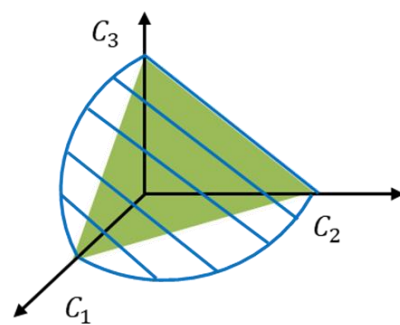


Figure 10. Channel capacity of LTDM (blue area) vs. TDM (green area) with three PLPs.

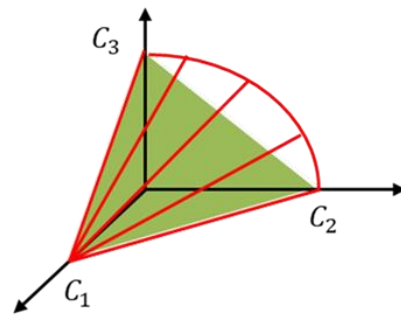


Figure 11. Channel capacity of TLDM (red area) vs. TDM (green area) with three PLPs.

To find α_{TLDM} in the TLDM system, assuming $W = 1$ and $P = 1$, and substituting $\sigma_{TLDM,1} = 1/(1 + \alpha_{TLDM}^2)$ and $\sigma_{TLDM,2} = \alpha_{TLDM}^2/(1 + \alpha_{TLDM}^2)$, Equations (9) and (10) are rearranged as follows:

$$C_{TLDM,2} = (1 - \tau_{TLDM}) \log_2 \left(1 + \frac{\sigma_{TLDM,1}}{\sigma_{TLDM,2} + N_2} \right), \quad (28)$$

$$C_{TLDM,3} = (1 - \tau_{TLDM}) \log_2 \left(1 + \frac{\sigma_{TLDM,2}}{N_3} \right). \quad (29)$$

By using the above equations, the Lagrangian function is given by [28]:

$$L_{TLDM}(\sigma_{TLDM,1}, \sigma_{TLDM,2}, \lambda) = \frac{C_{TLDM,2}}{C_2} + \frac{C_{TLDM,3}}{C_3} + \lambda(\sigma_{TLDM,1} + \sigma_{TLDM,2} - 1). \quad (30)$$

The system of equations using the Lagrange multiplier method is computed as follows [28]:

$$\frac{\partial L_{TLDM}}{\partial \sigma_{TLDM,1}} = \frac{(1 - \tau_{TLDM})}{C_2(N_2 + 1) \ln 2} + \lambda = 0, \quad (31)$$

$$\frac{\partial L_{TLDM}}{\partial \sigma_{TLDM,2}} = -\frac{(1 - \tau_{TLDM})\sigma_{TLDM,1}}{C_2(N_2 + 1)(N_2 + \sigma_{TLDM,2}) \ln 2} + \frac{(1 - \tau_{TLDM})}{C_3(N_3 + \sigma_{TLDM,2}) \ln 2} + \lambda = 0, \quad (32)$$

$$\frac{\partial L_{TLDM}}{\partial \lambda} = \sigma_{TLDM,1} + \sigma_{TLDM,2} - 1 = 0. \quad (33)$$

Using Equations (31)–(33), $\sigma_{TLDM,1}$ and $\sigma_{TLDM,2}$ can be derived as follows:

$$\sigma_{TLDM,1} = \beta_{TLDM}^2 = 1 - \sigma_{TLDM,2}, \quad (34)$$

$$\sigma_{TLDM,2} = \beta_{TLDM}^2 \alpha_{TLDM}^2 = \frac{C_2 N_2 - C_3 N_3}{(C_3 - C_2)}. \quad (35)$$

By substituting the calculated β_{TLDM} and α_{TLDM} into Equations (8)–(10), the channel capacity of the TLDM system with the maximum efficiency can be obtained when $\tau_{TLDM} = \tau_{TDM,1}$.

For the LTDM system, if the Lagrange multiplier method is applied in the same way using Equation (5) (PLP_1) and Equation (7) (PLP_3), the channel capacity of the LTDM system with the maximum efficiency can be obtained when $\tau_{LTDM} = \tau_{TDM,1}$. The LTDM parameters calculated by applying the Lagrange multiplier method using Equations (5) and (7) are as follows:

$$\beta_{LTDM}^2 = 1 - \beta_{LTDM}^2 \alpha_{LTDM}^2, \quad (36)$$

$$\beta_{LTDM}^2 \alpha_{LTDM}^2 = \frac{(1 - \tau_{TDM,1})C_1 N_1 - C_3 N_3}{(C_3 - (1 - \tau_{TDM,1})C_1)} \quad (37)$$

5. Simulation Result

In this section, we present the channel capacity analysis results of the four multiplexing systems using the parameters calculated by the Lagrange multiplier method when operated with three PLPs. The efficiency of each system is compared in terms of the normalized channel capacity $R_{M,i}$. Additionally, we provide the required SNR values to achieve the target bit error rate (BER). The modulation and code rate (ModCod) of the transmitted signal follow the specifications outlined in the physical layer of ATSC 3.0 [2]. It is assumed that perfect channel state information is available without synchronization errors. The received PLP_1 signal, considering channel gain and noise in the frequency domain can be represented as

$$Y_{M,i}[k] = H_i[k]X_M[k] + N_i[k], \quad (38)$$

where $Y_{M,i}[k]$ denotes the received signal of PLP_1 in M system; $X_M[k]$, $H_i[k]$, and $N_i[k]$ represent the transmitted signal in M system; channel gain and noise of PLP_1; and k is a sub-carrier index. Since we assume perfect channel state information, the estimated transmission signal $\hat{X}_M[k]$ can be given by

$$\hat{X}_M[k] = X_M[k] + \frac{N_i[k]}{H_i[k]}. \quad (39)$$

The transmitter signal of PLP_1 in the M system, denoted as $X_{M,i}[k]$, needs to be obtained. This signal can be obtained through DEMUX in the case of TDM system or by using the SIC technique in the case of LDM system [5,18].

To evaluate the fairness of the system, we can calculate the normalized fairness index (NFI) using the normalized channel capacity. The NFI measures fairness in resource allocation among the different services in an LDM system. The formula to calculate the NFI in 3-PLPs is as follows:

$$NF_M = \frac{\left(\sum_{i=1}^3 R_{M,i}\right)^2}{3 \times \sum_{i=1}^3 (R_{M,i})^2}. \quad (40)$$

We derived the parameters through the Lagrange multiplier method using the values C_i presented in Table 1 (provided in Ref. [5]) to determine the sum of normalized channel capacities. The parameters provided in [5] and the calculated using the proposed method are listed in Table 2. $C_{M,i}$ and $R_{M,i}$ can be computed using the parameters given in Table 2, and the sum of the normalized channel capacities, denoted as R_M is defined as follows:

$$R_M = \sum_{i=1}^3 R_{M,i}. \quad (41)$$

Table 1. Channel capacity C_i presented in paper [5].

Parameters	Value
maximum channel capacity of PLP_1 (C_1)	1 bps/Hz (SNR: 0 dB)
maximum channel capacity of PLP_2 (C_2)	3.16 bps/Hz (SNR: 9 dB)
maximum channel capacity of PLP_3 (C_3)	7.65 bps/Hz (SNR: 23 dB)

Table 2. Simulation parameters derived by the proposed method for comparison with the parameters presented in the article [5].

Parameters	Value	
	Ref. [5]	Proposed Method
$\alpha_{LDM,1}$ of LDM	0 dB	5.6 dB
$\alpha_{LDM,1}$ of LDM	4 dB	9.54 dB
$\tau_{TDM,1}$ of TDM	0.25 (25%)	0.209 (20.9%)
$\tau_{TDM,2}$ of TDM	0.25 (25%)	0.264 (26.4%)
$\tau_{TDM,3}$ of TDM	0.50 (50%)	0.527 (52.7%)
τ_{LTDM} of LTDM	0.4 (40%)	0.209 (20.9%)
α_{LTDM} of LTDM	0 dB	9.04 dB
τ_{TLDM} of TLDM	0.30 (30%)	0.209 (20.9%)
α_{TLDM} of TLDM	4 dB	10.6 dB

Tables 3 and 4 show the calculated results for $C_{M,i}$, $R_{M,i}$, R_M , and NF_M , which are used to evaluate the performance of different multiplexing systems. The results demonstrate that the sum of the normalized channel capacities obtained using the Lagrange multiplier method is higher than the comparison parameters provided in reference [5]. For the TDM system, since the channel capacity $C_{TDM,i}$ changes linearly according to τ_i , the normalized channel capacity $R_{TDM,i}$ is equal to τ_i . In other words, $R_{TDM} = 1$ for any τ_i . Therefore, the result is the same if R_{TDM} is calculated using the Lagrange multiplier method or the comparison parameters provided in [5]. However, the channel capacities do not change linearly with the injection levels for other multiplexing systems, unlike in the TDM system. Therefore, the sum of the normalized channel capacities differed depending on the injection level. For an LDM system, the sum of the normalized channel capacities obtained using the Lagrange multiplier method is 0.08 higher than that mentioned in [5]. This means that the channel efficiency has improved by 8% when considering the TDM system's channel efficiency as 100%. Furthermore, it has been confirmed that the NFI is also higher at 0.9256. Similarly, the Lagrange multiplier method can achieve 15% and 8% better performances in the LTDM and TLDM systems, respectively. However, it should be noted that the normalized fairness indices are lower than those presented in [5].

Table 3. The channel capacity $C_{M,i}$ and normalized channel capacity $R_{M,i}$ at each PLP using parameters in Tables 1 and 2.

System	Parameters	Value					
		PLP_1		PLP_2		PLP_3	
		Ref. [5]	Proposed	Ref. [5]	Proposed	Ref. [5]	Proposed
LDM	$C_{LDM,i}$ (bps/Hz)	0.34	0.65	1.28	0.97	5.09	4.09
	$R_{LDM,i}$	0.34	0.65	0.41	0.31	0.67	0.53
TDM	$C_{TDM,i}$ (bps/Hz)	0.25	0.21	0.79	0.84	3.82	4.03
	$R_{TDM,i}$	0.25	0.21	0.25	0.26	0.5	0.53
LTDM	$C_{LTDM,i}$ (bps/Hz)	0.42	0.85	0.93	0.19	3.99	3.59
	$R_{LTDM,i}$	0.42	0.85	0.29	0.06	0.52	0.47
TLDM	$C_{TLDM,i}$ (bps/Hz)	0.3	0.21	1.02	1.94	4.1	3.23
	$R_{TLDM,i}$	0.3	0.21	0.32	0.61	0.54	0.42

Table 4. The sum of normalized channel capacities R_M and the normalized fairness index NF_M at each PLP using parameters in Table 3.

System	Parameters	Value	
		Ref. [5]	Proposed
LDM	R_M	1.41	1.49
	NF_M	0.9175	0.9256
TDM	R_M	1	1
	NF_M	0.8889	0.849
LTDM	R_M	1.23	1.38
	NF_M	0.9499	0.6703
TLDM	R_M	1.16	1.24
	NF_M	0.9267	0.8649

Here, we compared the four multiplexing systems; we found that the LDM system is the most efficient technique. In terms of the sum of normalized channel capacities derived using the Lagrange multiplier method, the LDM system is 49% more efficient than the TDM system. The LTDM and TLDM systems were 38% and 24% more efficient than the TDM system, respectively. Furthermore, when resource allocation is carried out using the Lagrange multiplier method, it can be observed that the NFI of the LDM system is 0.9256, achieving the highest NFI compared to other transmission methods. Through this analysis, it can be confirmed that the LDM system utilizing the Lagrange multiplier method achieves the highest sum of normalized channel capacities even under fair conditions.

Table 5 represents the ModCod supported by the physical layer of ATSC 3.0, along with the parameters C_i obtained using the Lagrange multiplier method. C_i is calculated by substituting the required SNR at $\text{BER} = 10^{-4}$ obtained from the simulations into Equation (11). Using the parameters listed in Table 5, we calculated the $C_{M,i}$, $R_{M,i}$, R_M , and NF_M values as listed in Tables 6 and 7. From the simulation results in Table 7, we can see the sum of the normalized channel capacities is 1.4342. It shows that the LDM system is more efficient and outperformed other transmission systems.

Table 5. Parameter values obtained using the Lagrange multipliers.

Parameters	Value
FFT size	8K FFT
Guard interval	1/16
Bandwidth (W)	6 MHz
C_1 (QPSK, code rate: 6/15)	5.64 Mbps (required SNR at $\text{BER} = 10^{-4}$: −0.2 dB)
C_2 (16-QAM, code rate: 6/15)	11.56 Mbps (required SNR at $\text{BER} = 10^{-4}$: 4.7 dB)
C_3 (256-QAM, code rate: 12/15)	40.17 Mbps (required SNR at $\text{BER} = 10^{-4}$: 20.7 dB)
$\alpha_{LDM,1}$ of LDM	4.98 dB
$\alpha_{LDM,1}$ of LDM	7.25 dB
$\tau_{TDM,1}$ of TDM	0.239 (23.9%)
$\tau_{TDM,2}$ of TDM	0.191 (19.1%)
$\tau_{TDM,3}$ of TDM	0.57 (57%)
τ_{LTDM} of LTDM	0.239 (23.9%)
α_{LTDM} of LTDM	8.74 dB
τ_{TLDM} of TLDM	0.239 (23.9%)
α_{TLDM} of TLDM	8.45 dB

Table 6. The channel capacity $C_{M,i}$ and normalized channel capacity $R_{M,i}$ at each PLP using parameters in Table 5.

System	Parameters	Value		
		PLP_1	PLP_2	PLP_3
LDM	$C_{LDM,i}$ (Mbps)	3.3	3.16	23.16
	$R_{LDM,i}$	0.5849	0.2729	0.5764
TDM	$C_{TDM,i}$ (Mbps)	1.35	2.2	22.88
	$R_{TDM,i}$	0.2396	0.1907	0.5696
LTDM	$C_{LTDM,i}$ (Mbps)	4.74	0.6	17.26
	$R_{LTDM,i}$	0.8408	0.0521	0.4297
TLDM	$C_{TLDM,i}$ (Mbps)	1.35	6.78	17.61
	$R_{TLDM,i}$	0.2396	0.5867	0.4383

Table 7. The sum of normalized channel capacities R_M and the normalized fairness index NF_M at each PLP using parameters in Table 6.

System	Parameters	Value
LDM	R_M	1.4342
	NF_M	0.9156
TDM	R_M	1
	NF_M	0.7969
LTDM	R_M	1.3326
	NF_M	0.652
TLDM	R_M	1.2646
	NF_M	0.8978

Unfortunately, it is difficult to find a ModCod supported by ATSC 3.0 that matches channel capacity as $C_{M,i}$ obtained using the Lagrange multiplier method. Therefore, we set the ModCod to perform similarly to $C_{M,i}$ and analyzed the required SNR accordingly. Figure 12 shows the BER performance of PLP_3 under an additive white Gaussian noise (AWGN) channel using the ModCod, with the closest value to $C_{M,i}$ listed in Table 6. For example, in the LDM system, the ModCod used for PLP_3 is 64-QAM with a code rate of 9/15, and it is evident that a required SNR of approximately 21.1 dB is needed, which is close to the $C_3 = 20.7$ dB listed in Table 5. In this manner, we identified ModCods closest to $C_{M,i}$ for all cases, as listed in Table 8.

Table 9 lists the required SNR at $\text{BER} = 10^{-4}$ obtained from simulations conducted in both AWGN and fading channels, using the ModCods listed in Table 8. It also includes the corresponding data rate. The data rate $r_{M,i}$ is calculated using the following equation:

$$r_{M,i} = \left(\frac{1}{GI + 1} \right) \tau_{M,i} W q_{M,i} \times \text{Coderate}. \quad (42)$$

where GI represents the guard interval time, $\tau_{M,i}$ is the time ratio of i -th PLP of each system, and $q_{M,i}$ is the number of bits per symbol used for modulation. As an example of $q_{M,i}$, when using the ModCod for PLP_3 in the LDM system, 64-QAM is employed, which uses 6 bits. Therefore, $q_{LDM,3} = 6$. Both PLP_1 and PLP_2 used TU-6 channel [5,34], which is a Rayleigh fading channel in a mobile environment, considering mobile reception at 100 km/h. PLP_3 used RL 20 channel [5,35,36], which is a fixed Rayleigh fading channel, considering the fixed UHD service.

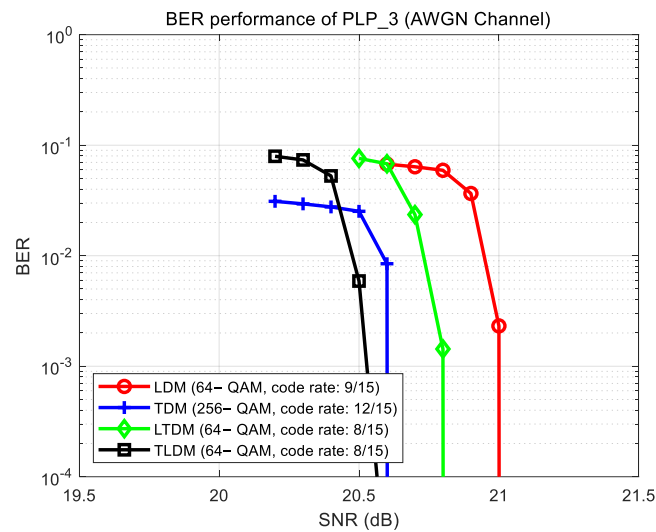


Figure 12. BER performance of PLP_3 under AWGN channel.

Table 8. ModCod configuration.

System	Parameters	Value		
		PLP_1	PLP_2	PLP_3
LDM	Modulation	QPSK	QPSK	64-QAM
	Code rate	3/15	3/15	9/15
TDM	Modulation	QPSK	16-QAM	256-QAM
	Code rate	6/15	6/15	12/15
LTDM	Modulation	QPSK	QPSK	64-QAM
	Code rate	3/15	2/15	8/15
TLDM	Modulation	QPSK	QPSK	64-QAM
	Code rate	6/15	7/15	8/15

Table 9. Required SNR at $\text{BER} = 10^{-4}$ and data rate.

System		Value		
		PLP_1	PLP_2	PLP_3
AWGN channel	LDM	−0.5 dB	4.6 dB	21.1 dB
	TDM	−0.2 dB	4.7 dB	20.7 dB
	LTDM	−0.8 dB	4.9 dB	20.9 dB
	TLDM	−0.2 dB	4.8 dB	20.6 dB
Fading channel	LDM	1.6 dB	6.6 dB	23.3 dB
	TDM	2.2 dB	6.3 dB	24.3 dB
	LTDM	0.9 dB	6.2 dB	23.2 dB
	TLDM	2.2 dB	7.2 dB	23 dB
Data rate $r_{M,i}$	LDM	2.26 Mbps	2.26 Mbps	20.33 Mbps
	TDM	0.54 Mbps	1.72 Mbps	20.59 Mbps
	LTDM	2.26 Mbps	0.36 Mbps	13.75 Mbps
	TLDM	0.54 Mbps	4.01 Mbps	13.75 Mbps

Table 9 shows a comparative analysis of the reception performance and data rates for each PLP category. For PLP_1, both the LTDM and LDM systems had the same data rate. However, the LTDM system exhibited approximately 0.3 dB better reception performance under the AWGN channel and about 0.7 dB better SNR performance under the fading channel. Moving on to PLP_2, the TLDM system achieved the highest data rate, approximately twice as much as that of the LDM system. However, the reception performance deteriorated the most among the four transmission methods to 7.2 dB in the fading channel. As for PLP_3, although the data rates of the LDM and TDM systems are similar, the LDM system outperformed by 1 dB in the fading channel. By using these simulations, we can analyze the performance of received signals by allocating resources in each transmission system and understand the focus of operations for different services.

6. Conclusions

We proposed a novel approach for efficient resource allocation in the LDM system. To achieve efficient resource allocation while maintaining a high level of NFI, a new method is introduced that normalizes the channel capacities allocated to different services using the Lagrange multiplier.

The simulation results demonstrated the superiority of the proposed approach over other transmission techniques. The LDM system exhibited the highest NFI of over 0.9 and achieved the highest sum of normalized channel capacities at over 1.4, indicating its efficiency in serving multiple services simultaneously. By comparing the LDM system with other multiplexing systems, it is evident that the LDM system outperformed in fairness and efficiency. Furthermore, our study explored the practical implementation of ModCods and analyzed their performance under AWGN and fading channels. Here, we can analyze the performance of received signals by allocating resources in each transmission system and gain an understanding of the operational focus on different services. In our simulation experiments, we used the ATSC 3.0 system with three PLPs as an example, this method can be applied to future transmission systems that offer various services and multiple PLPs. This approach is expected to be valuable for performance analysis in terms of system efficiency.

Overall, the research provides valuable insights into resource allocation strategies for the LDM system, paving the way for fair comparisons of efficiency and performance within multi-service environments. The proposed approach contributes to the advancement of next-generation broadcasting systems by enhancing resource allocation efficiency and overall system fairness.

Author Contributions: H.J.K., S.-Y.K. and H.-N.K. took the lead in writing papers, developing control algorithms, and conducting experiments. J.B. and N.H. reviewed the overall contents and supervised the control development. All authors have read and agreed to the published version of the manuscript.

Funding: This work was supported by BK21FOUR, Creative Human Resource Education and Research Programs for ICT Convergence in the 4th Industrial Revolution and Institute of Information and Communications Technology Planning and Evaluation (IITP) grant funded by the Korea government (MSIT) (No. 2017-0-00081, Development of Transmission Technology for Ultra High Quality UHD).

Data Availability Statement: Not applicable.

Conflicts of Interest: The authors declare no conflict of interest.

Abbreviations

The following abbreviations are used in this manuscript:

LDM	Layered division multiplexing
ATSC	Advanced Television Systems Committee
UHD	Ultra-high definition
HD	High definition
BICM	Bit-interleaved coded modulation
QPSK	Quadrature phase-shift keying
QAM	Quadrature amplitude modulation
NOMA	Non-orthogonal multiple access
TDM	Time division multiplexing
PLP	Physical layer pipe
LTDM	Layered-time division multiplexing
TLDM	Time-layered division multiplexing
SNR	Signal-to-noise ratio
CL	Core layer
EL	Enhanced layer
JFI	Jain's fairness index
SIC	Successive interference cancellation
BER	Bit error rate
ModCod	Modulation and code rate
NFI	Normalized fairness index
AWGN	Additive white Gaussian noise

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