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Study of the Seismoelectric Effect in Saturated Porous Media Using a Bundle of Capillary Tubes Model

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Abstract: The seismoelectric effect is the fundamental basis for seismoelectric logging. Most of the existing theories for the seismoelectric effect are based on the Pride theory, which adopts the assumption of a thin electric double layer and uses the volume-averaging method to derive the seismoelectric coupling equations; hence, the obtained electrokinetic coupling coefficient is not applicable to large-Debye-length cases. In addition, the Pride theory neglects the change in seepage velocity with the radial position of the pore when calculating the streaming current, which leads to an inaccurate reflection of the influence of pore size on the electrokinetic coupling coefficient. In this study, we proposed a flux-averaging method to solve the effective net residual charge density of porous media and further derived the electrokinetic coupling coefficient expressed by the effective net residual charge density. We also investigated the effect of formation parameters and compared the results with those calculated using the Pride theory. Since the proposed method is not limited by the thin electric double layer assumption, it is suitable for both small- and large-Debye-length cases. Moreover, we also carried out flume experiments to investigate the influence of salinity, where both thin and thick electric double layer cases were studied. The comparison between the results of the experiment and simulation verified the correctness of the proposed method. Furthermore, the proposed method took into account the variation in seepage velocity with pore location when solving for the streaming current; therefore, the influence of the pore size on the electrokinetic coefficient can be described more accurately.

Keywords: seismoelectric effect; flux-averaging method; effective net residual charge density; electrokinetic coupling coefficient

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1. Introduction

There is an electric double layer at the solid–liquid interface in a porous medium, and there are excess cations in the pore fluid. When an acoustic field propagates in a porous medium, it will drive the movement of the excess cations in the pore, thus exciting an electromagnetic field; this phenomenon is the seismoelectric effect in a porous medium.

There were many studies on the seismoelectric effect in porous media. In 1944, Frenkel first proposed the coupling equations of the seismoelectric effect in porous media [1], but this theory has some limitations: it ignores the influence of solid skeleton acceleration on the streaming current and suggests that only longitudinal waves can generate a streaming potential. In 1953, Packard proposed the capillary pore model and studied the microscopic mechanism of the seismoelectric effect [2]. However, he regarded the electromagnetic field as a quasi-static field and used the current balance conduction, which is only applicable when the acoustic field is steady. In 1956, Biot proposed the theoretical equations for the acoustic field in porous media [3], which provided a theoretical basis for the study of the acoustic field in porous media. In 1989, Pride and Morgan proposed the capillary model to describe the microscopic mechanism of the seismoelectric effect [4], where their method is the same as that of [2]. In 1991, Pride and Morgan proposed a slit-like pore model to study the "electro-viscous effect" [5] with the same theoretical approach as [2]. In 1994, Pride used the volume-averaging method and combined Biot equations with Maxwell's equations to obtain equations that describe the seismoelectric field, and further derived the electrokinetic coupling coefficient [6]. However, the Pride theory is based on the assumption of a thin electric double layer and is not applicable when the salinity is too small. In 1996, Pride and Haartsen derived the coupling function of a seismoelectric field in isotropic media and the boundary conditions of the seismoelectric field at the interface of porous media [7]. In 1996, Bodapov studied the seismoelectric effect in saturated porous media under the action of an electric field. In 1997, Haasrten and Pride carried out numerical simulations on the seismoelectric effect of a layered medium excited by a point acoustic source based on the equations proposed by Pride [8]. In 1999, Revil and Pezard et al. deduced the acoustoelectric coupling equations and derived the expressions for the electrokinetic coupling coefficient, electrical conductivity, and dynamic permeability [9,10], but their theory is based on the assumption that the electromagnetic field is quasi-static; thus, the result is only suitable for acoustoelectric conversion when the acoustic field is steady. Furthermore, the net residual charge density is obtained by the volume-averaging method, resulting in the obtained streaming current being inconsistent with the Pride theory. In 2005, Zhu et al. decoupled the electric field from the acoustic field and carried out numerical simulations of the seismoelectric field in a wellbore, but they treated the electromagnetic field as a quasi-static field. In 2006, Haasrten and Pride carried out numerical simulations of arbitrary two-dimensional heterogeneous porous media and investigated the resolution of seismoelectric logging [11]. In 2013, Revil and Mahardika considered the influence of water saturation in unsaturated porous media and provided a new expression of the streaming potential coupling coefficient [12]. In recent years, most of the studies on the seismoelectric effect of porous media were focused on the study of the coupling relationship of the seismoelectric field in porous media containing multiple liquid phases [13–15]. The electric double layer theory is also very important for the study of the seismoelectric effect. In 2001, Bohinc and Kralj-Iglič et al. introduced two measures to describe the effective thickness of the electrical double layer [16]. In 2002, Bohinc and Iglič et al. applied a simple statistical mechanical approach to calculate the profile of the density of the number of particles and the profile of the electrostatic potential of an electric double layer formed using a charged cylindrical surface in contact with electrolyte solution [17]. In 2020, Drab and Gongadze et al. derived the modified Langevin Poisson–Boltzmann (LPB) model of EDL by minimizing the corresponding Helmholtz free energy function, which includes the orientational entropy contribution of water dipoles [18].

The seismoelectric effect in porous media can be divided into two cases: (1) when the acoustic field is steady, and therefore, the excited electric field is steady, then the streaming current and conduction current in porous media are equal in amplitude and opposite in direction, and the seismoelectric effect is determined using the streaming potential coupling coefficient; (2) when the acoustic field is time-harmonic, and therefore, the excited electromagnetic field is time-harmonic, then the current balance condition is no longer satisfied, and thus, the seismoelectric effect cannot be described using the streaming potential coupling coefficient. However, the existing theories do not distinguish between the seismoelectric effects in these two cases [2,4,5,9,10] and are mainly based on the Pride theory [7,8,11], which adopts the assumption of a thin electric double layer, and hence, it is not applied in the low-salinity case. In addition, the Pride theory ignores the change in seepage velocity with the radial position of the pore when calculating the streaming current [6], which leads to the obtained results being unable to accurately reflect the effect of the pore size.

In this study, we investigated the governing equation of the seismoelectric effect in porous media, proposed the flux-averaging method to solve for the effective net residual charge density of the porous media, and further derived the streaming potential coupling coefficient in the steady case and the streaming current coupling coefficient in the time-harmonic case. We investigated the influence of formation parameters, such as the porosity, pore fluid viscosity, and salinity, on the seismoelectric effect in the thin electric double layer case and compared the results with those obtained using the Pride theory to verify the effectiveness of the proposed method. The numerical results indicated that since the proposed method was not limited by the condition of the thin electric double layer, the obtained electrokinetic coupling coefficient could also be applied to the low-salinity case, while the Pride theory could not. Moreover, we carried out flume experiments to investigate the influence of salinity on the seismoelectric effect, and the results further demonstrated the effectiveness of our method. In addition, the proposed method fully considered the variation in seepage velocity with pore location when solving for the streaming current, and thus, it can be more accurate for describing the influence of pore size on the coupling coefficient.

2. Methods

In order to study the principle of the seismoelectric effect of porous media, we adopted the parallel capillary bundle model of Ishido [19]. We assumed that all pores had the same radius R, and there were n_0 pores in a unit volume of porous media.

The model parameters used in this study are shown in Table 1.

Table 1. Model parameters.

Parameter		Unit
ϕ	Porosity	Dimensionless
k	Static permeability	D
C	Streaming potential coupling coefficient (Pride theory)	V/Pa
C_Q	Streaming potential coupling coefficient (this study)	V/Pa
L_ω	Streaming current coupling coefficient (Pride theory)	A/Pa
L_Q	Streaming current coupling coefficient (this study)	A/Pa
α_∞	Pore tortuosity	Dimensionless
ω_c	Critical frequency	rad/s
ε	Permittivity	F/m
ς	Shear potential	V
σ_0	Conductivity	S/m
d	Debye length	m
Λ	Weighted body surface ratio	m
η_f	Viscosity	$\mathrm{Pa} \cdot \mathrm{m}$
c_0	Salinity	mol/L

Some formation parameters of porous media can be expressed as follows.

Porosity:

$$\phi = n_0 \pi R^2 \tag{1}$$

Static permeability:

$$k = (n_0 \pi R^4)/8 \tag{2}$$

From the above, the pore size has the following relationship with porosity and permeability:

$$R = \sqrt{8k/\phi} \tag{3}$$

According to the Pride theory [6], when the acoustic field is steady, the streaming potential coupling coefficient of the porous medium can be expressed as

$$C = -\frac{\varepsilon\varsigma}{\eta_f \sigma_0}\phi(1 - 2\frac{d}{\Lambda}) \tag{4}$$

where ε denotes the permittivity, ς denotes the shear potential, η_f denotes the pore fluid viscosity, σ_0 denotes the electrical conductivity, d denotes the Debye length, Λ denotes the weighted body surface ratio, and Λ is the pore radius for a cylindrical pore.

On the other hand, when the acoustic field is harmonic, the streaming current coupling coefficient of porous media is correspondingly expressed as

$$L_\omega(\omega) = L_0[1 - i\frac{\omega}{\omega_c}\frac{m}{4}(1-2\frac{d}{\Lambda})^2(1-i^{3/2}\frac{d}{\delta})^2]^{-\frac{1}{2}} \tag{5}$$

$$L_0 = -\frac{\phi}{\alpha_\infty}\frac{\varepsilon\varsigma}{\eta_f}(1-2\frac{d}{\Lambda}) \tag{6}$$

where α_∞ denotes the pore tortuosity, which was taken to be 1 in this study; ω_c denotes the critical frequency:

$$\omega_c = \phi\eta/(\alpha_\infty\kappa_0\rho_f) \tag{7}$$

and m is a dimensionless parameter:

$$m = \phi\Lambda^2/\alpha_\infty\kappa_0 = 8 \tag{8}$$

The pore model is shown in Figure 1.

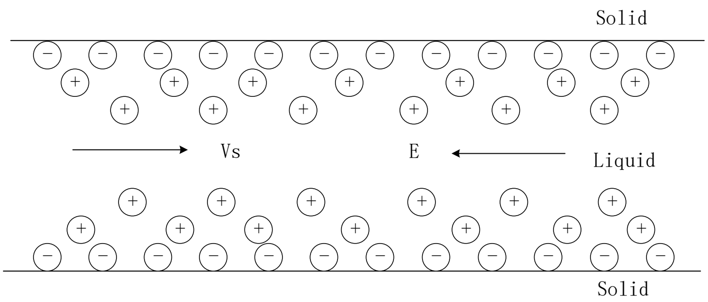


Figure 1. Pore model.

There is an electric double layer at the solid–liquid interface of the pore, the pore wall adsorbs negative ions, and the pore liquid contains residual cations. When there is an acoustic field in the pore, it will drive the pore fluid to move, and the net residual cations will move synchronously, thus stimulating the streaming current in the pore. The streaming current excites an electric field in the opposite direction. When the acoustic field is steady, it excites a steady current, which then excites a steady electric field. In this situation, the conduction current and the streaming current are equal in size and opposite in direction; this is the current balance condition. On the other hand, when the acoustic field is time-harmonic, it excites a time-harmonic current, which then excites a time-harmonic electromagnetic field. In this situation, the conduction current and streaming current are opposite, but their amplitudes are not equal.

The Pride theory adopts the thin electric double layer hypothesis. In this situation, the salinity is not too small, and thus, Debye length is used to estimate the thickness of the electric double layer:

$$d = \sqrt{\varepsilon k_b T/(2e^2N)} \tag{9}$$

where ε denotes the dielectric constant, k_b denotes the Boltzmann constant, T denotes the Kelvin temperature, e denotes the electron charge, and N denotes the ion concentration away from the electric double layer.

When the difference between the Debye length and the pore size is more than two orders of magnitude, the electric double layer can be considered "thin". When the Debye length is comparable to the pore size, we call it a large Debye length.

2.1. Seismoelectric Effect in the Steady Condition

When the acoustic field in the porous medium is a steady field, the electric field excited by the seismoelectric effect is accordingly also a steady electric field. In this case, the streaming current and the conduction current in the porous medium are equal in magnitude and opposite in direction.

The current density in the pore satisfies the Nernst–Planck equation [20]:

$$\mathbf{J}_w = \mathbf{J}_{ws} + \mathbf{J}_{wc} \tag{10}$$

where $\mathbf{J}_{ws}$ denotes the streaming current:

$$\mathbf{J}_{ws} = Q_{ws}\mathbf{v}_{ws} \tag{11}$$

where Q_{ws} denotes the net residual charge density in the pore fluid and $\mathbf{v}_{ws}$ denotes the seepage velocity in the pore. $\mathbf{J}_{wc}$ is the conduction current:

$$\mathbf{J}_{wc} = \sigma_w \mathbf{e}_w \tag{12}$$

where σ_w denotes the pore fluid conductivity and $\mathbf{e}_w$ denotes the electric field in the pore fluid.

According to the volume-averaging theory [21], the volume average of the conduction current can be written as

$$\mathbf{J}_c = \sigma \mathbf{E} \tag{13}$$

where $\mathbf{J}_c$, σ, and $\mathbf{E}$ are the respective physical quantities after volume averaging.

Define the flux average of Q_{ws} relative to $\mathbf{v}_{ws}$ as

$$Q_v^{eff} = \int_{v_f} Q_{ws} v_{ws} dV / \int_{v_f} v_{ws} dV \tag{14}$$

where v_f denotes the volume-averaging reference volume of fluid.

The expression of the streaming current of porous media has the form

$$\mathbf{J}_s = Q_v^{eff} \cdot \mathbf{V}_w \tag{15}$$

where $\mathbf{V}_w$ denotes the macroscopic seepage velocity and Q_v^{eff} denotes the effective net residual charge density of porous media.

$\mathbf{V}_w$ can be obtained according to Darcy's law:

$$\mathbf{V}_w = -k/\eta_f \cdot \nabla P \tag{16}$$

When the acoustic field is a steady field, the streaming current balances the conduction current:

$$\mathbf{J}_s + \mathbf{J}_c = 0 \tag{17}$$

The electric field can be represented using the streaming potential:

$$\mathbf{E} = -\nabla \phi \tag{18}$$

By substituting the definition of the streaming potential coupling coefficient:

$$C = \nabla \phi / \nabla P \tag{19}$$

we can obtain the streaming potential coefficient:

$$C_Q = kQ_V^{eff} / (\eta_f \sigma) \tag{20}$$

The seepage velocity of the liquid in the pore is

$$v_{ws}(r) = \Delta P/(4\eta)\cdot(R^2 - r^2) \tag{21}$$

The net residual charge density in the pore fluid electric double layer [4,22] can be expressed as

$$Q_{ws} = -2eN\sinh(e\varphi/k_bT) \tag{22}$$

where φ denotes the electrostatic potential in the electric double layer:

$$\varphi = \varsigma I_0(\kappa r)/I_0(\kappa R) \tag{23}$$

$$\kappa = \sqrt{(8\pi e^2N)/(\varepsilon k_bT)} \tag{24}$$

Then, the effective net residual charge density in the cylindrical pores can be obtained using the flux-averaging method according to Equation (14).

By substituting the effective net residual charge density into Equation (20), the streaming potential coupling coefficient can be obtained.

In summary, we finally derived the governing equation of the seismoelectric effect in saturated porous media under the action of a steady acoustic field:

$$\nabla\phi = C_Q\nabla P \tag{25}$$

2.2. Seismoelectric Effect in the Time-Harmonic Condition

When the acoustic field in the porous medium is a time-harmonic field, the electric field excited by the seismoelectric effect is time-harmonic. In this situation, the magnitudes of the streaming current and conduction current in the porous medium are no longer equal.

Same as in the steady case, the local streaming current in the pore is written as

$$\mathbf{J}_{ws} = Q_{ws}\mathbf{v}_{ws} \tag{26}$$

Similar to the seismoelectric effect under the excitation of a steady acoustic field, the flux average of Q_{ws} relative to $\mathbf{v}_{ws}$ is defined as

$$Q_v^{eff} = \int_{v_f} Q_{ws}v_{ws}dV \Big/ \int_{v_f} v_{ws}dV \tag{27}$$

We can obtain the streaming current of porous media:

$$\mathbf{J}_s = Q_v^{eff}\cdot\mathbf{V}_w \tag{28}$$

where $\mathbf{V}_w$ denotes the macroscopic seepage velocity; according to Darcy's law, it can be given as follows:

$$\mathbf{V}_w = \frac{k_w}{\eta_f}(-\nabla\overline{P} + i\omega\rho_f\overline{\mathbf{v}}_s) \tag{29}$$

We can deduce the streaming current of porous media:

$$\mathbf{J}_s = \frac{k(\omega)Q_v^{eff}}{\eta_f}(-\nabla\overline{P} + i\omega\rho_f\overline{\mathbf{v}}_s) \tag{30}$$

According to this definition, the permeability in the pore can be calculated as follows:

$$k(\omega) = 2\eta\phi\int_0^R v_{ws}rdr/R^2 \tag{31}$$

We can further deduce the streaming current coupling coefficient:

$$L_Q = k(\omega)Q_v^{eff}/\eta_f \tag{32}$$

The seepage velocity in the pore satisfies the Navier–Stokes equation [2,23] such that the seepage velocity in the pore is

$$v_{ws} = -\frac{(-\nabla p + i\omega\rho_f v_s)}{i\xi^2\eta}[1 - \frac{I_0(i^{1/2}\xi r)}{I_0(i^{1/2}\xi R)}] \tag{33}$$

The net residual charge density in the pore liquid is the same as in the steady case. The effective net residual charge density in the cylindrical pore can be obtained using the flux-averaging method according to Equation (27). By substituting the effective net residual charge density into Equation (32), we can obtain the streaming current coupling coefficient.

In summary, we finally derived the governing equation of the seismoelectric effect in saturated porous media under the action of a time-harmonic acoustic field:

$$\mathbf{J}_s = L_Q(-\nabla\overline{P} + i\omega\rho_f\overline{\mathbf{v}}_s) \tag{34}$$

2.3. Verifying the Validity of the Flux-Averaging Method

The solution of the streaming current in the Pride theory is obtained using the overall volume average of $\mathbf{J}_{ws}$. In this study, Q_{ws} and $\mathbf{v}_{ws}$ were treated separately, that is, $\mathbf{v}_{ws}$ denotes the volume averaged seepage velocity, Q_{ws} denotes the flux-averaged charge density relative to $\mathbf{v}_{ws}$, and then the two are multiplied to obtain the streaming current. The flux average is similar to the weighted average of Q_{ws} relative to $\mathbf{v}_{ws}$. The equivalence of the flux-averaging method with the volume-averaging method used in the Pride theory is demonstrated below.

The volume average of the seepage velocity is defined as follows:

$$\mathbf{V}_w = \int_{v_f} \mathbf{v}_{ws} dV/v \tag{35}$$

where v denotes the volume-averaged reference volume and v_f denotes the reference fluid volume. The volume average of the streaming current is

$$\mathbf{J}_s = \int_{v_f} Q_{ws} v_w dV/v \tag{36}$$

By multiplying the effective net residual charge density and the seepage velocity, we obtain

$$Q_v^{eff}\mathbf{V}_w = \frac{\int_{v_f} Q_{ws} v_{ws} dV}{\int_{v_f} v_{ws} dV}\frac{\int_{v_f} v_{ws} dV}{v} = \frac{\int_{v_f} Q_{ws} v_w dV}{v} = \mathbf{J}_s \tag{37}$$

It can be proved that the product of the effective net residual charge density and seepage velocity is equal to the streaming current. Therefore, the feasibility of this proposed method can be proved. The method in this study does not need to be restricted to the condition of a thin electric double layer and is more applicable than the Pride theory.

2.4. Experimental Setup

In order to verify that the method used in this study is also applicable to the seismoelectric effect in the low-salinity case, we carried out relevant experiments in a water tank. The experimental model diagram is shown in Figure 2.

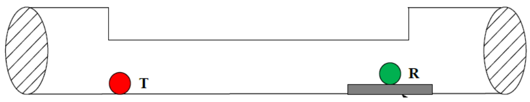


Figure 2. Experimental model diagram.

T refers to the transmitting equipment, i.e., the sound source, which included a signal generator, signal amplification circuit, and piezoelectric transducer. R represents the signal-receiving equipment, which included a receiving antenna and signal amplification circuit.

The tank was a PVC pipe, and a NaCl solution was added to the tank. The sound source was located on the left side of the water tank. The rock sample was a whetstone located on the right side of the water tank. The receiving antenna was closely attached to the rock sample. The excitation source of the acoustic field was a pulse, and the salinity of the solution was changed by adding NaCl to the water tank. An oscilloscope was used to record the electric field waveform under different salinities and extract the peak value of the electric field from it. The experimental results were compared with the simulation results of Equation (32).

3. Simulation and Analysis

3.1. The Streaming Potential Coupling Coefficient

In the simulation discussed in this subsection, the parallel capillary bundle model was used to study the influence of the formation parameters, such as the porosity, pore fluid salinity, and viscosity, on the streaming potential coupling coefficient in the steady case. We compared the results with those calculated using the Pride theory. We mainly studied the difference in the streaming potential coupling coefficient obtained using the two methods in the case of a large Debye length, and we also compared and analyzed the effect of pore size on the streaming potential coupling coefficient calculated using the two methods. In the following figures, C represents the result calculated using Pride theory and C_Q represents the result found using the method developed in this study.

Figure 3 shows the effect of porosity on the seismoelectric effect in the steady case. As shown in Figure 3, the streaming potential coupling coefficient was proportional to the porosity, but the effective net residual charge density of the porous media did not change with the porosity. The larger the porosity was, the larger the number of pores, the volume of pore fluid, and the acoustoelectric coupling ability. The coupling coefficient curves calculated using the two methods basically overlapped, and the porosity had the same effect on the coupling coefficient obtained using the two methods.

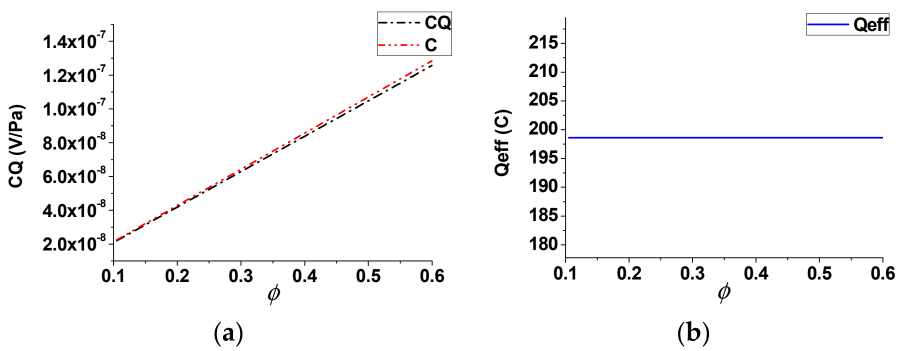


Figure 3. Effect of the porosity (steady case): (**a**) streaming potential coupling coefficient; (**b**) effective net residual charge density.

Figure 4 shows the effect of the pore fluid viscosity on the seismoelectric effect in the steady case. It can be seen from Figure 4 that, with the increase in the pore fluid viscosity, the streaming potential coupling coefficient gradually decreased and the effective net residual charge density in the pore remained unchanged. When the viscosity of the pore fluid became greater, the seepage velocity of the fluid became lower and the acoustoelectric conversion efficiency became lower. The streaming potential coupling coefficient curves calculated using the two methods almost overlapped, i.e., the influence of the pore fluid viscosity on the streaming potential coupling coefficient obtained using the two methods was nearly the same.

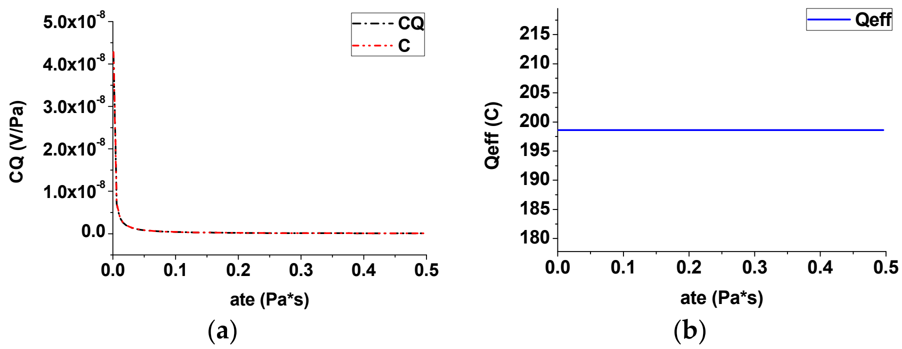


Figure 4. Effect of the viscosity (steady case): (**a**) streaming potential coupling coefficient; (**b**) effective net residual charge density.

Figure 5 shows the effect of the pore fluid salinity on the steady seismoelectric effect in the case of a thin electric double layer. As indicated in Figure 5, with the increase in the pore fluid salinity, the streaming potential coupling coefficient gradually decreased and the effective net residual charge density gradually decreased. In the case of a thin electric double layer, with the increase in the pore fluid salinity, the shear potential of the electric double layer on the pore wall decreases, the electric double layer becomes thinner, the net residual charge density of the electric double layer decreased and the seismoelectric effect weakened. The curves of the streaming potential coupling coefficient calculated using the two methods almost overlap and the influence of salinity on the streaming potential coupling coefficient calculated using the two methods was nearly the same in the thin electric double layer case.

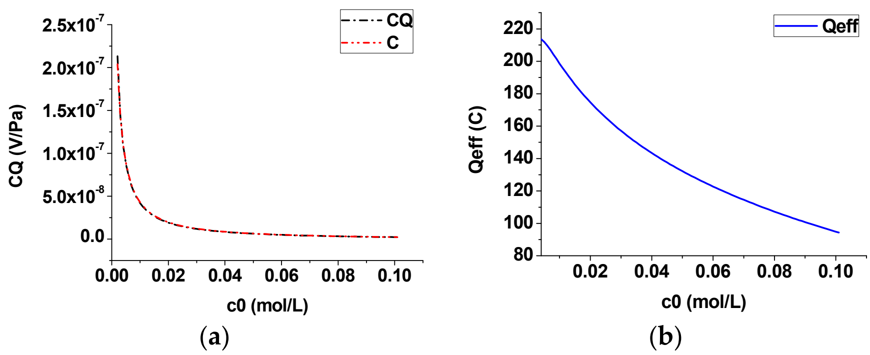
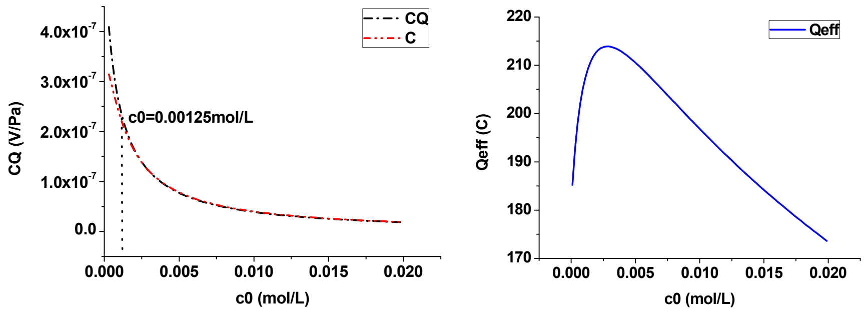


Figure 5. Effect of the salinity (steady case): (**a**) streaming potential coupling coefficient; (**b**) effective net residual charge density.

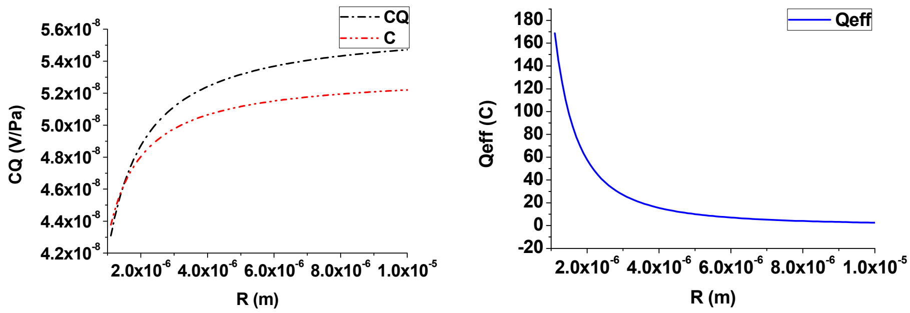
Figure 6 shows the effect of salinity (including the large-Debye-length case) on the streaming potential coupling coefficient calculated using the two methods. The Debye length was used to describe the thickness of the electric double layer. The Debye length increased with the decrease in the salinity. As shown in Figure 6, the coupling coefficient of the streaming potential increased with the decrease in the salinity. The effective net residual charge density first increased and then decreased with the decrease in the salinity, but the conductivity decreased with the decrease in the salinity, resulting in the streaming potential coupling coefficient increasing with the decrease in the salinity. When the salinity was large, the Debye length was much smaller than the size of the pore; in this case, the curves of the streaming potential coupling coefficient calculated using the two methods overlapped. However, as the salinity decreased, the Debye length gradually increased; when the salinity was 0.00125 mol/L, the Debye length reached the order of 10^{-7} m. In this situation, the Debye length was comparable to the pore size (2×10^{-6} m), and thus, the results calculated by Pride theory differed from those found using our method.



(**a**) (**b**)

Figure 6. Effect of the salinity (steady condition, including the large-Debye-length case): (**a**) streaming potential coupling coefficient; (**b**) effective net residual charge density.

Figure 7 shows the effect of pore size on the seismoelectric effect in the steady case. As shown in Figure 7, with the increase in the pore radius, the streaming potential coupling coefficient increased monotonically while the effective net residual charge density decreased monotonically. The larger the pore radius, the smaller the effective net residual charge density in the pore; however, as the corresponding permeability increased (quadratic relationship with pore size), the streaming potential coupling coefficient increased. The streaming potential coupling coefficient calculated using the flux averaging method was more affected by the pore size. The Pride theory adopts the assumption of a thin electric double layer and ignores the variation in the seepage velocity with the radial position of the pore [6]; therefore, its results are less affected by the pore radius and the coupling coefficient of streaming potential obtained by the Pride theory cannot truly reflect the influence of the pore size. The method developed in this study does not have this defect, and thus, the results it produces are more reliable.



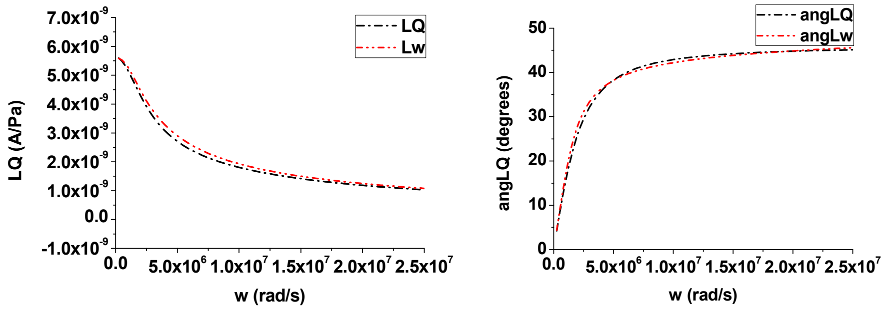
(**a**) (**b**)

Figure 7. Effect of the pore size (steady case): (**a**) streaming potential coupling coefficient; (**b**) effective net residual charge density.

3.2. The Streaming Current Coupling Coefficient

We also carried out numerical simulations of the streaming current coupling coefficient in the time-harmonic case. The effect of the formation parameters was analyzed and compared with the results of the Pride theory. We focused on the difference in the streaming current coupling coefficient calculated using the two methods in the case of a thick electric double layer, and the influence of pore size on the result of the two methods was compared and analyzed. We also investigated the variation in the amplitude and phase of the coupling coefficient with the frequency of the acoustic field. In the following figures, L_w represents the result calculated using the Pride theory and L_Q represents the result calculated using the method developed in this study.

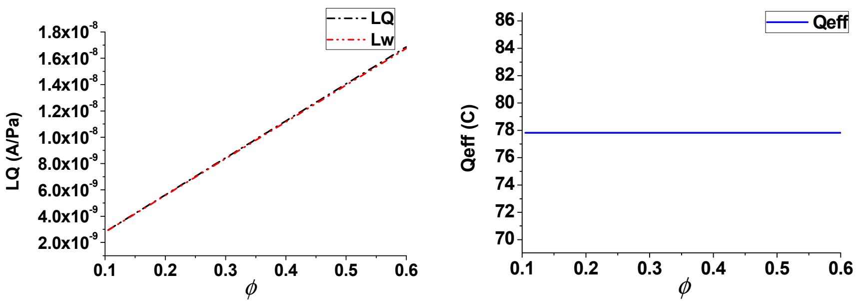
Figure 8 shows the variation in the amplitude and phase of the streaming current coupling coefficient with frequency in the time-harmonic case. It can be seen from Figure 8 that the amplitude and phase curves of the coupling coefficient calculated using the two methods almost overlapped. As the frequency increased, the amplitude of the coupling coefficient decreased and the phase increased and approached a constant value ($\pi/4$).



(**a**) (**b**)

Figure 8. Variation with the frequency (time-harmonic case): (**a**) amplitude; (**b**) phase.

Figure 9 shows the effect of porosity on the seismoelectric effect in the time-harmonic case. It can be seen from Figure 9 that the streaming current coupling coefficient was proportional to the porosity, and the effective net residual charge density did not change with the porosity. When the porosity became larger, the number of pores became larger, the volume of pore fluid became larger, and the conversion ability between the acoustic field and the electric field became stronger. The coupling coefficient curves calculated using the two methods almost overlapped, and the porosity had the same effect on the streaming current coupling coefficient calculated using the two methods.



(**a**) (**b**)

Figure 9. Effect of the porosity (time-harmonic case): (**a**) streaming current coupling coefficient; (**b**) effective net residual charge density.

Figure 10 shows the effect of pore fluid viscosity on the seismoelectric effect in the time-harmonic case. As shown in Figure 10, with the increase in pore fluid viscosity, the streaming current coupling coefficient gradually decreased and the effective net residual charge density in the pore remained unchanged. When the viscosity of the pore fluid became greater, the seepage velocity of the fluid became smaller, and therefore, the acousto-electric coupling ability became weaker. The coupling coefficient curves calculated using the two methods almost overlapped and the influence of the viscosity on the streaming potential coupling coefficient obtained using the two methods was basically the same.

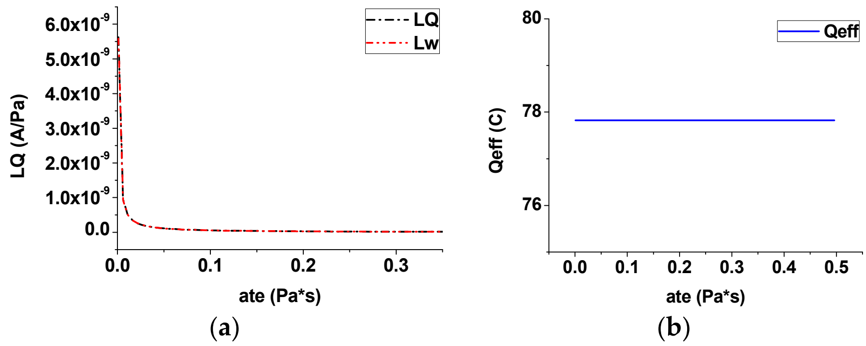


Figure 10. Effect of the viscosity (time-harmonic case): (**a**) streaming current coupling coefficient; (**b**) effective net residual charge density.

Figure 11 shows the effect of the pore fluid salinity on the time-harmonic seismoelectric effect in the case of a thin electric double layer. It can be seen from Figure 11 that with the increase in pore fluid salinity, the streaming current coupling coefficient gradually decreased and the effective net residual charge density gradually decreased. In the case of a thin electric double layer, as the salinity of the pore liquid increased, the shear potential of the electric double layer at the pore wall decreased, the electric double layer became thinner, the net residual charge density of the electric double layer decreased, and the acoustoelectric coupling ability weakened. The coupling coefficient curves calculated using the two methods almost overlapped. The effect of salinity on the streaming current coupling coefficient obtained using the two methods was basically the same in the thin electric double layer case.

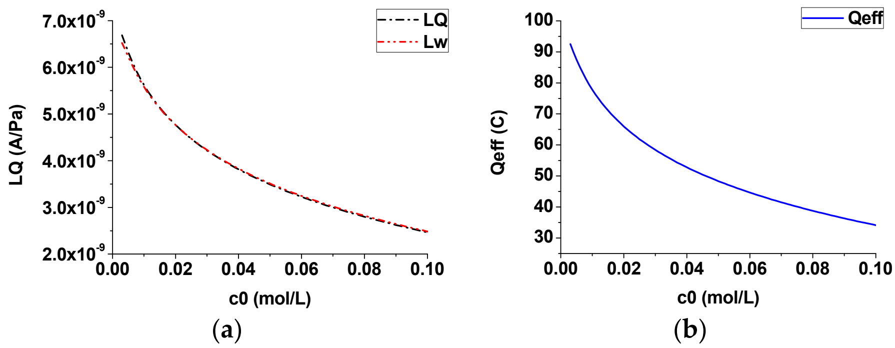


Figure 11. Effect of the salinity (time-harmonic case): (**a**) streaming current coupling coefficient; (**b**) effective net residual charge density.

Figure 12 shows the effect of the salinity (including the large-Debye-length case) on the streaming current coupling coefficient calculated using the two methods. The Debye length increased with the decrease in salinity. As shown in Figure 12, with the increase in salinity, the coupling coefficient of the streaming current increased first and then decreased. The effective net residual charge density first increased and then decreased with the increase in salinity, leading to the streaming current coupling coefficient first increasing and then decreasing with the decrease in salinity. When the salinity was large, the Debye length was much smaller than the pore size and the curves of the streaming current coupling coefficient calculated using the two methods overlapped. As the salinity decreased, the Debye length gradually increased; when the salinity was 0.005 mol/L, the Debye length reached the order of 10^{-7} m. In this situation, the Debye length was comparable to the pore size (1.7×10^{-6} m), and thus, the results calculated using the Pride theory differed from those found using our method. When the Debye length was greater than the pore radius, the streaming current coupling coefficient calculated using the Pride theory became negative, which is obviously inconsistent with the physical facts and verified that the Pride theory is not suitable when the salinity is too low.

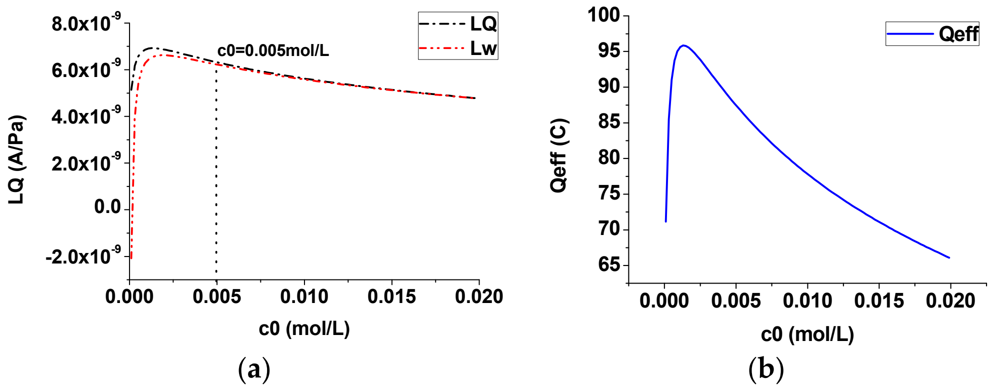


Figure 12. Effect of the salinity (time-harmonic condition, including the large-Debye-length case): (**a**) streaming current coupling coefficient; (**b**) effective net residual charge density.

Figure 13 shows the effect of the pore size on the seismoelectric effect in the time-harmonic case. It can be seen from Figure 13 that as the pore radius increased, the streaming current coupling coefficient increased monotonically and the effective net residual charge density decreased monotonically. When the pore radius became larger, the effective net residual charge density became smaller, but the corresponding permeability increased (quadratic relationship with pore size), resulting in a larger streaming current coupling coefficient. The streaming current coupling coefficient calculated using the flux-averaging method was more affected by the pore size. Because the Pride theory adopts the assumption of a thin electric double layer and ignores the variation in the seepage velocity with the radial position of the pore [6], its results are insensitive to the change in the pore radius, and thus, the streaming current coupling coefficient obtained using the Pride theory cannot truly reflect the influence of pore size on the acoustoelectric effect. In contrast, the method developed in this study has no such defect, and thus, its results are more reliable.

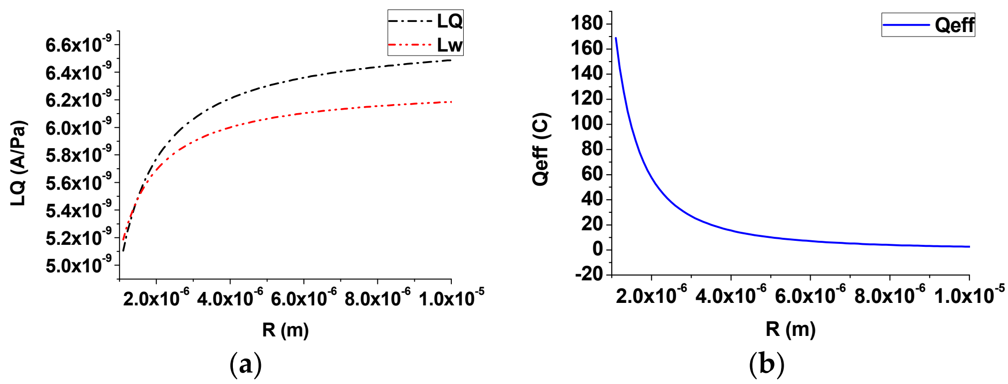


Figure 13. Effect of the pore size (time-harmonic case): (**a**) streaming current coupling coefficient; (**b**) effective net residual charge density.

4. Experimental Results and Analysis

We carried out the experiment in the tank, where the experimental device is shown in Figure 14.

Figure 14. Experimental device.

Figure 15 shows the comparison between the experimental and simulation results. Both the simulation and experiment investigated the case of a large Debye length. The red curve is the simulation result showing that the coupling coefficient of the streaming current changed with the salinity calculated using the method developed in this study. The blue dots are the peak values of the electric field signal measured under different salinities. It can be seen from the figure that with the increase in the salinity of pore fluid, the electrokinetic coupling coefficient obtained via the simulation increased first and then decreased; meanwhile, the peak value of the electric field caused by the seismoelectric effect of the rock samples increased first and then decreased, and the changing trend of the two with the salinity was basically consistent. It can be seen from Equation (32) that the streaming current coupling coefficient was proportional to the effective net residual charge density, and the effective net residual charge density first increased and then decreased with the increase in salinity (shown in Figure 12b). When the salinity was low, the Debye length was large. At this time, the effective net residual charge density increased with the increase in salinity. When the salinity increased to a certain extent, the electric double layer became thinner. In this situation, the shear potential of the electric double layer decreased and the effective net residual charge density decreased with the increase in salinity, leading to a decrease in the coupling coefficient. Therefore, the coupling coefficient of the streaming current first increased and then decreased with the increase in salinity.

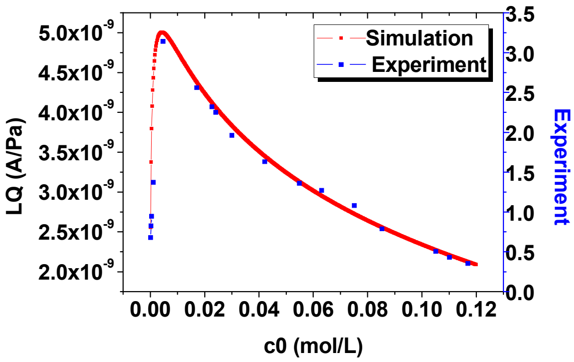


Figure 15. Comparison between the experiment and simulation.

Figure 16 shows the comparison of simulation results between the Pride theory and the proposed theory in the experimental condition; when the salinity was 0.003 mol/L, the Debye length reached the order of 10^{-7} m. In this situation, the Debye length was comparable to the pore size (10^{-6} m), and thus, the results calculated by Pride theory differed from those found using our method. When the Debye length was greater than the pore radius (c_0 = 0.0003 mol/L), the streaming current coupling coefficient calculated using the Pride theory became negative, which is obviously inconsistent with the experimental results, indicating that the Pride theory is only valid in the case of a thin electric double layer. In contrast, the theory developed in this study does not have this limitation and is applicable to the cases of both small and large Debye lengths.

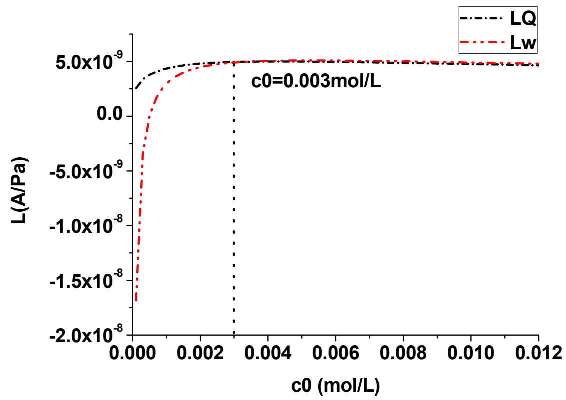


Figure 16. Comparison of the simulation results between the Pride theory and the proposed theory (experimental condition, including the large-Debye-length case).

5. Discussion

In Section 3.1, we discussed the variation in the streaming potential coupling coefficient and the effective net residual charge density with the porosity, pore fluid salinity, viscosity, and pore size; then, we compared the results with those calculated using the Pride theory and analyzed the reasons.

It can be seen from Figures 3–5 that in the case of a thin electric double layer, the variation trend of streaming potential coupling coefficient with the porosity, salinity, and viscosity obtained using the method developed in this study and the Pride theory was consistent. This also verified the correctness of the theory developed in this study to a certain extent.

Figure 6 shows that when the salinity was low and the Debye length was large, the coupling coefficient of the streaming potential calculated using the Pride theory was different from that found using our method because the Pride theory was derived based on the assumption of a thin double electric layer. Figure 7 shows that the influence of the pore size on the streaming potential coupling coefficient calculated using the method developed in this study was greater than that calculated using the Pride theory. This was consistent with our analysis, which showed that the Pride theory ignored the change in the seepage velocity with the radial position when solving the streaming current, and reduced the influence of the pore size on the streaming potential coupling coefficient.

In Section 3.2, we discussed the variation in the streaming current coupling coefficient and the effective net residual charge density with the porosity, pore fluid salinity, viscosity, and pore size; then, we compared the results with those calculated using the Pride theory and analyzed the reasons.

Figure 8 shows that the influence of frequency on the streaming current coupling coefficient obtained using the two methods was the same. It can be seen from Figures 9–11 that in the case of a thin electric double layer, the variation trends of the streaming potential coupling coefficient with the porosity, salinity, and viscosity obtained using the method developed in this study and the Pride theory were the same. This also verified the correctness of the theory developed in this study to a certain extent.

Figure 12 shows that when the salinity was low and the Debye length was large, the streaming current coupling coefficient calculated using the Pride theory was different from that found using our method because the Pride theory was derived based on the assumption of a thin double electric layer. Figure 15 shows that the streaming current coupling coefficient calculated using our method was consistent with the experimental results, while the results of the Pride theory were not consistent (when the salinity is very small, it will become negative). This showed that the method developed in this study is applicable to both small- and large-Debye-length cases. Figure 13 shows that the influence of the pore size on the streaming current coupling coefficient calculated using the method developed in this study was greater than that calculated using the Pride theory. This was consistent with our analysis that the Pride theory ignores the change in the seepage velocity with the radial position when solving for the streaming current, and thus, will reduce the influence of the pore size on the streaming current coupling coefficient.

It can be seen from Equations (4) and (6) that the Pride theory will use the thickness of the electric double layer when calculating the electrokinetic coupling coefficient. The Pride theory adopts the thin electric double layer hypothesis and uses the Debye length to describe the thickness of the electric double layer. According to [16], when the salinity is very low, the Debye length has a deviation when describing the thickness of the electric double layer. Therefore, the results of the Pride theory will be biased in the low-salinity case. This study also used the Debye length to describe the thickness of an electric double layer to verify that the Pride theory is not applicable in the case of a large Debye length. It can be seen from Equations (20) and (32) that the method developed in this study does not use the thickness of electric double layer when calculating the electrokinetic coupling coefficient, and thus, the results in this study are also applicable to the case of low salinity without the limitation of the Pride theory.

In this study, we only investigated the cylindrical pore model; other pore models, such as a slit-like pore, have not been studied, which is what we need to study in the next step.

6. Conclusions

The previous studies on the seismoelectric effect are mainly based on the Pride theory [7,8,11], which cannot be applied to the case of a thick electric double layer, nor can it accurately reflect the influence of pore size on the acoustoelectric effect. Besides, when the acoustic field in porous media is respectively steady and time-harmonic, the governing equations of seismoelectric effect will be different accordingly, while the existing theories don't distinguish the two cases [2,4,5,9,10]. In this study, we proposed the flux-averaging method to investigate the seismoelectric effect of a saturated porous media under the action of a steady acoustic field and a time-harmonic acoustic field, then deduced the coupling coefficient and compared the results with those calculated using the Pride theory. Through the simulation and analysis, the following conclusions were drawn.

For the seismoelectric effect in the steady condition:

(1) In the case of a thin electric double layer, the porosity, viscosity, and salinity had the same influence on the streaming potential coupling coefficient obtained using the two methods. It was feasible to calculate the streaming potential coupling coefficient in the steady case through the flux-averaging method.
(2) The effect of the pore size on the streaming potential coupling coefficient obtained using the flux-averaging method was stronger. The flux-averaging method could better reflect the influence of the pore size on the seismoelectric effect in the steady case.
(3) The streaming potential coupling coefficient obtained using the Pride theory was not suitable for the large-Debye-length case, but the method developed in this study does not have this limitation.

For the seismoelectric effect in the time-harmonic condition:

(1) The amplitude and phase of the streaming current coupling coefficient obtained using the two methods had the same variation trend with frequency.
(2) In the case of a thin electric double layer, the porosity, viscosity, and salinity had the same influence on the streaming current coupling coefficient obtained using the two methods. It was feasible to calculate the streaming current coupling coefficient in a time-harmonic case through the flux-averaging method.
(3) The influence of the pore size on the streaming current coupling coefficient obtained using the flux-averaging method was stronger. The coupling coefficient obtained using the flux-averaging method could better reflect the influence of the pore size on the acoustoelectric effect in the time-harmonic condition.

(4) The streaming current coupling coefficient obtained using the Pride theory was not suitable for the case of a large Debye length, but the method proposed in this study does not have this limitation.

Through the flume experiments, the influence of the salinity on the seismoelectric effect was investigated, and both cases of a thin and thick electric double layer were studied. The conclusions can be summarized as follows:

(1) As the salinity of the pore fluid increased gradually, the streaming current coupling coefficient of the porous medium first increased and then decreased, and the coupling coefficient had a peak value.
(2) The theory developed in this study had no limit on the thickness of the electric double layer and was suitable for both the small- and large-Debye-length cases; therefore, its application fields are more extensive than the Pride theory.

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References

1. Frenkel, J. On the theory of seismic and seismoelectric phenomena in a moist soil. *J. Eng. Mech.* **2005**, *131*, 879–887. [CrossRef]
2. Packard, R.G. Streaming potentials across glass capillaries for sinusoidal pressure. *J. Chem. Phys.* **1953**, *21*, 303–307. [CrossRef]
3. Biot, M.A. Theory of propagation of elastic waves in a fluid-saturated porous solid. II. Higher frequency range. *J. Acoust. Soc. Am.* **1956**, *28*, 179–191. [CrossRef]
4. Pride, S.; Morgan, F.D. On the importance of electrokinetic forces in the acoustics of porous media. In *SEG Technical Program Expanded Abstracts*; Society of Exploration Geophysicists: Tulsa, OK, USA, 1989; pp. 579–581.
5. Pride, S.; Morgan, F.D. Electrokinetic dissipation induced by seismic waves. *Geophysics* **1991**, *56*, 914–925. [CrossRef]
6. Pride, S. Governing equations for the coupled electromagnetics and acoustics of porous media. *Phys. Rev. B* **1994**, *50*, 15678–15696. [CrossRef] [PubMed]
7. Pride, S.R.; Haartsen, M.W. Electroseismic wave properties. *J. Acoust. Soc. Am.* **1996**, *100*, 1301–1315. [CrossRef]
8. Haartsen, M.W.; Pride, S.R. Electroseismic waves from point sources in layered media. *J. Geophys. Res. Solid Earth* **1997**, *102*, 24745–24769. [CrossRef]
9. Revil, A.; Schwaeger, H.; Cathles, L.M., III; Manhardt, P.D. Streaming potential in porous media: 2. Theory and application to geothermal systems. *J. Geophys. Res. Solid Earth* **1999**, *104*, 20033–20048. [CrossRef]
10. Revil, A.; Pezard, P.A.; Glover, P. Streaming potential in porous media: 1. Theory of the zeta potential. *J. Geophys. Res. Solid Earth* **1999**, *104*, 20021–20031. [CrossRef]
11. Haartsen, M.W.; Pride, S.R. Seismoelectric numerical modeling on a grid. *Geophysics* **2006**, *71*, N57–N65.
12. Revil, A.; Mahardika, H. Coupled hydromechanical and electromagnetic disturbances in unsaturated porous materials. *Water Resour. Res.* **2013**, *49*, 744–766. [CrossRef] [PubMed]
13. Zyserman, I.; Monachesi, L.B.; Jouniaux, L. Dependence of shear wave seismoelectrics on soil textures: A numerical study in the vadose zone. *Geophys. J. Int.* **2017**, *208*, 918–935. [CrossRef]
14. Pesavento, F.; Schrefler, B.A.; Sciumè, G. Multiphase flow in deforming porous media: A review. *Arch. Comput. Methods Eng.* **2017**, *24*, 423–448. [CrossRef]
15. Revil, A.; Barnier, G.; Karaoulis, M.; Sava, P.; Jardani, A.; Kulessa, B. Seismoelectric coupling in unsaturated porous media: Theory, petrophysics, and saturation front localization using an electroacoustic approach. *Poult. Sci.* **2013**, *94*, 955–964. [CrossRef]
16. Bohinc, K.; Kralj-Iglič, V.; Iglič, A. Thickness of electrical double layer. effect of ion size. *Electrochim. Acta* **2001**, *46*, 3033–3040. [CrossRef]

17. Bohinc, K.; Iglič, A.; Slivnik, T.; Kralj-Iglič, V. Charged cylindrical surfaces: Effect of finite ion size. *Bioelectrochemistry* **2002**, *57*, 73–81. [CrossRef]
18. Drab, M.; Gongadze, E.; Kralj-Iglič, V.; Iglič, A. Electric double layer and orientational ordering of water dipoles in narrow channels within a modified Langevin Poisson-Boltzmann model. *Entropy* **2020**, *22*, 1054. [CrossRef]
19. Ishido, T.; Mizutani, H. Experimental and theoretical basis of electrokinetic phenomena in rock-water systems and its applications to geophysics. *J. Geophys. Res. Solid Earth* **1981**, *86*, 1763–1775. [CrossRef]
20. Revil, A.; Woodruff, W.F.; Lu, N. Constitutive equations for coupled flows in clay materials. *Water Resour. Res.* **2011**, *47*, W05548. [CrossRef]
21. Howes, F.A.; Whitaker, S. The spatial averaging theorem revisited. *Chem. Eng. Sci.* **1985**, *40*, 1387–1392. [CrossRef]
22. Keh, H.J.; Liu, Y.C. Electrokinetic flow in a circular capillary with a surface charge layer. *J. Colloid Interface Sci.* **1995**, *172*, 222–229. [CrossRef]
23. Yang, J.; Grundke, K.; Bellmann, C.; Michel, S.; Kostiuk, L.W.; Kwok, D.Y. Oscillating streaming potential and electro-osmosis of multilayer membranes. *J. Phys. Chem. B* **2004**, *108*, 2103–2110. [CrossRef]