

Article

Research on a High-Precision State-of-Charge Estimation Method Based on Forgetting Factor Recursive Least Squares and Adaptive Extended Kalman Filter Applied to LiFePO₄ Battery

Yihui Xia, Zhihao Ye *, Liming Huang, Lucheng Sun and Yunxiang Jiang

School of Electrical Engineering, Naval University of Engineering, Wuhan 430033, China; xiayihui2005@163.com (Y.X.); huangliming1998@163.com (L.H.); d23380807@nue.edu.cn (L.S.); jiangyunxiang@163.com (Y.J.)

* Correspondence: yxyx928@126.com

Abstract: The state-of-charge (SOC) estimation accuracy is closely associated with the estimation method and the battery parameter identification performance. The battery parameter identification method based on forgetting factor recursive least squares (FFRLS) has the advantages of high parameter identification accuracy and fast dynamic response speed. On this basis, the performance of two SOC estimation methods, the extended Kalman filter (EKF) and adaptive extended Kalman filter (AEKF) are compared and studied. The results show that AEKF has better steady-state and dynamic SOC estimation performance, but the estimation accuracy and dynamic response performance are still not objective. To further improve the performance of SOC estimation, a joint SOC estimation method based on FFRLS-AEKF is proposed, and the SOC estimation experimental results with FFRLS-AEKF and AEKF are conducted. The experimental results show that the proposed joint SOC estimation method based on FFRLS-AEKF has a better steady-state and dynamic performance of SOC estimation. The maximum absolute error of the proposed algorithm is 4.97%. As the battery working time increases, the SOC estimation accuracy continues to converge to the true value, and the average absolute error is reduced to 2.5%. The proposed method and theoretical analysis are proven to be correct and feasible.

Keywords: LiFePO₄ battery; high precision; FFRLS; AEKF; SOC estimation



Citation: Xia, Y.; Ye, Z.; Huang, L.; Sun, L.; Jiang, Y. Research on a High-Precision State-of-Charge Estimation Method Based on Forgetting Factor Recursive Least Squares and Adaptive Extended Kalman Filter Applied to LiFePO₄ Battery. *Electronics* **2023**, *12*, 3670. <https://doi.org/10.3390/electronics12173670>

Academic Editors: Luis Hernández-Callejo, Jesús Armando Aguilar Jiménez and Carlos Meza Benavides

Received: 2 July 2023

Revised: 3 August 2023

Accepted: 4 August 2023

Published: 31 August 2023



Copyright: © 2023 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (<https://creativecommons.org/licenses/by/4.0/>).

1. Introduction

The LiFePO₄ battery has advantages such as good safety, long service life, and high power density, and so on [1,2], which is widely used in electric vehicles and energy storage systems. To prevent battery failures such as overcharging and over-discharging, it is necessary to monitor the battery operating status in real time, especially its state of charge (SOC).

The methods of accurate SOC estimation can generally be categorized into the following types: open-circuit voltage, ampere-hour integral-based estimation method, data-driven-based estimation method, and filtering algorithms based on equivalent circuit models [3–5]. There is a monotonic one-to-one correspondence between the open-circuit voltage and the SOC value of the battery. The corresponding table of open-circuit voltage and SOC can be established, and the SOC value can be determined by looking up the table [6]. However, this method requires a long time after the battery is charged and discharged before measuring the open-circuit voltage. Reference [7] proposed a fast open-circuit voltage prediction method. The battery only needs to be placed within 15 min to accurately predict the open-circuit voltage. However, the calculation process is complex, and the requirements for the processor are high. The ampere-hour integral method tracks the SOC by the change in the current integral value. The measurement error will accumulate in the integral, resulting in a large error of SOC. Reference [8] analyzed the error sources

of the ampere-hour analysis method and proposed a method to reduce the cumulative error of time and the proportional error of the SOC. However, it is more dependent on the current accuracy, and the accuracy of the current sensor is higher.

If the data-driven algorithm is trained properly, it can predict the SOC of any type of battery without knowing its inherent characteristics in advance. Reference [9] proposed a method of predicting SOC via artificial neural network machine learning. The root-mean-squared error (RMSE) of the LiFePO₄ battery used in this paper is 0.33% during the test cycle. The author of Reference [10] proposed a recurrent neural network (RNN) with long short-term memory (LSTM) to accurately estimate the SOC of lithium-ion batteries, and it can achieve high accuracy at temperatures from 10 °C to 25 °C. Reference [11] used the support vector machine (SVM) method to estimate the SOC of the battery from the experimental data set, which has high accuracy. In the method of machine learning or deep learning, a data-driven algorithm collects terminal voltage, working current, temperature, and other data in the work of the battery, and it uses neural network and machine learning to train a large amount of data to obtain the current SOC value. This method needs to collect a large amount of data for calculation, which requires a powerful processor and is sensitive to the type of battery. If the internal parameters of the battery change, the data-driven model cannot follow the internal changes well, resulting in a larger SOC estimation error.

The filtering algorithms based on equivalent circuit models mainly use derivative algorithms of Kalman filtering (KF) and particle filtering algorithms for SOC estimation [12–17]. As the most popular model-based method, the KF-based algorithm has enhanced robustness to measurement and process noise and has a high estimation accuracy [18]. Based on the extended Kalman filter (EKF) algorithm, an adaptive dual EKF algorithm is proposed in Ref. [12]; the EKF algorithm is used to improve the nonlinear battery model for SOC estimation in Refs. [17,19] for the LiFePO₄ battery. A strong tracking volume EKF algorithm (STEKF) is proposed in Ref. [20] to provide accurate SOC prediction and a faster computing time. A multi-rate strong tracking EKF algorithm (MRSTEKF) is proposed by introducing multi-rate control strategy and enhancement technology in Ref. [21], and the SOC tracking stability and estimation accuracy is improved from 55.34% and 49.51% with STEKF to 52.66% and 33.88% with MRSTEKF, respectively. An online adaptive EKF algorithm based on the Davidson model is used to estimate SOC [22], and the SOC estimation error can be reduced to 2%. The sigma points Kalman filter (SPKF) is another nonlinear system filtering algorithm that can be used to estimate SOC, which uses numerical approximations instead of analytical approximations of EKF. A joint battery model and SOC estimation algorithm based on SPKF is proposed in Ref. [23], which has the same computational complexity as EKF but has higher accuracy. In Ref. [24], a combination of the volumetric Kalman filtering algorithm and the forgetting factor recursive least squares (FFRLS) algorithm is used to estimate SOC, and the maximum estimation error of SOC under high-rate pulses is reduced to be less than 1%. On the other hand, the method of approximating a probability density function based on a particle filter is used to estimate the SOC. In Ref. [25], a volumetric particle filter for accurate SOC estimation is proposed. In Ref. [26], dual-scale, dual-particle, and dual-scale adaptive particle filters for SOC estimation are proposed. However, compared with SOC estimation algorithms based on the Kalman filter, the SOC estimation algorithms based on the particle filter have a larger computational burden [27]. In Ref. [28], an analytical mathematical formulation of storage and its SOC is presented. In Ref. [29], a power allocation strategy based on cluster switching to relieve the stated problem in two levels is proposed, which is used to eliminate the imbalanced SOC and decrease battery energy loss. To address the power allocation challenges, a novel optimized state-of-charge (SOC) feedback-based energy management strategy in Ref. [30] is proposed for HESS in IPS to restrain the DC bus voltage fluctuation in this paper.

In this paper, a FFRLS-AEKF joint estimation algorithm based on time-varying parameter model is proposed, which considers the characteristics that the FFRLS algorithm can update the battery parameters in real time and the AEKF algorithm can correct the initial value error of SOC and follows the parameters of the battery. Firstly, the working principle

of FFRLS for battery parameter identification is analyzed. Secondly, the second-order equivalent circuit state discretization equation based on the AEKF algorithm is established, and the steps of the battery SOC estimation method based on AEKF are discussed. Thirdly, a joint SOC estimation method based on FFRLS-AEKF is proposed. Fourthly, the SOC estimation performance of three methods as EKF, AEKF, and FFRLS-AEKF is compared by experiments. The experimental results show that the FFRLS-AEKF algorithm can have a higher SOC estimation and faster accuracy convergence speed. Finally, the thesis is summarized.

2. The Basic Principle of Parameter Identification Based on the FFRLS

2.1. Mathematical Modeling of a Second-Order RC Circuit

The second-order RC equivalent circuit model of the LiFePO₄ battery is shown in Figure 1. The two RC circuits, respectively, describe the electrochemical polarization process with a small time constant and the concentration polarization process with a large time constant. Among them, U_{OC} is the open-circuit voltage of the battery, R_0 is the internal resistance, and R_{p1} and C_{p1} are the electrochemical polarization internal resistance and electrochemical polarization capacitance, respectively. R_{p2} and C_{p2} are the concentration difference polarization internal resistance and concentration difference polarization capacitance, respectively. U_L and I_L are load voltage and load current, respectively.

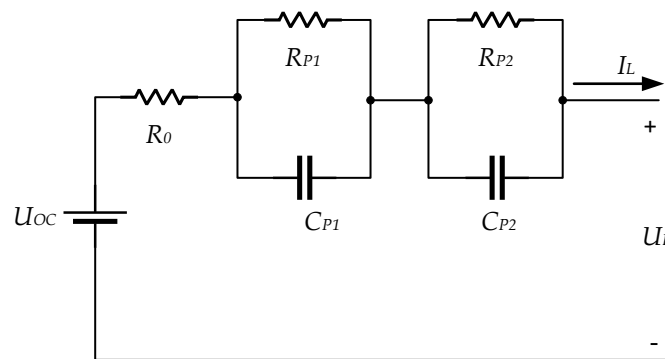


Figure 1. Second-order RC model.

According to Figure 1, the state space equation expression of the second-order RC model circuit can be listed as follows:

$$\begin{pmatrix} \frac{dU_{p1}}{dt} \\ \frac{dU_{p2}}{dt} \end{pmatrix} = \begin{pmatrix} -\frac{1}{R_{p1}C_{p1}} & 0 \\ 0 & -\frac{1}{R_{p2}C_{p2}} \end{pmatrix} \begin{pmatrix} U_{p1} \\ U_{p2} \end{pmatrix} + \begin{pmatrix} \frac{1}{C_{p1}} \\ \frac{1}{C_{p2}} \end{pmatrix} I_L \quad (1)$$

$$U_{OC} = U_{p1} + U_{p2} + R_0 I_L + U_L$$

where $\frac{dU_{p1}}{dt}$ and $\frac{dU_{p2}}{dt}$ represent the electrochemical polarization voltage and the concentration difference polarization voltage, respectively.

2.2. The Basic Principle of the FFRLS

The traditional recursive least squares (RLS) method is widely used in the field of system identification. For linear systems, its mathematical description can be expressed as follows:

$$\begin{aligned} Y(k) &= \varphi(k)\theta(k)^T + e(k) \\ \varphi(k) &= [-Y(k-1) \cdots -Y(k-n) \ u(k) \cdots u(k-n)] \end{aligned} \quad (2)$$

where $Y(k)$ is the output signal of the system at k time, $u(k)$ is the input signal of the system at k time, $\theta(k)$ represents the parameter vector to be estimated, $\varphi(k)$ is the input matrix of

the system at time k , n is the parameter to be estimated, and $e(k)$ is the noise of the system. The recursive formula of the FFRLS method is as follows:

$$\begin{aligned}\hat{\theta}(k+1) &= \hat{\theta}(k) + K(k+1)[Y(k+1) - \varphi^T(k+1)\hat{\theta}(k)] \\ K(k+1) &= P(k+1)\varphi(k+1) \\ P(k+1) &= P(k) - \frac{P(k)\varphi(k+1)\varphi^T(k+1)}{1 + \varphi^T(k+1)P(k)\varphi(k+1)}\end{aligned}\quad (3)$$

In Equation (3), $P(k)$ is the covariance matrix, and $K(k)$ is the gain matrix. During the recursive process, RLS uses the difference between the estimated value and the measured value of the system output, as well as the gain matrix $K(k)$, to make adjustments to the parameter vector $\hat{\theta}(k)$. During initialization, the initial value of $\hat{\theta}(k)$ and $K(k)$ can be any value, $P(0) = \alpha I$, α is as large as possible a constant, and I is the identity matrix.

To reduce the influence of past data and give greater weight to new data, a forgetting factor is introduced by setting it to λ ($0 < \lambda < 1$), leading to the development of FFRLS [31,32]. Equation (3) is modified as follows:

$$\begin{aligned}\hat{\theta}(k+1) &= \hat{\theta}(k) + K(k+1)[y(k+1) - \varphi^T(k+1)\hat{\theta}(k)] \\ K(k+1) &= \frac{P(k)\varphi(k+1)}{\lambda + \varphi^T(k+1)P(k)\varphi(k+1)} \\ P(k+1) &= \frac{1}{\lambda}[1 - K(k+1)\varphi^T(k+1)]P(k)\end{aligned}\quad (4)$$

Equation (4) is a recursive formula based on the forgetting factor recursive least squares method, where the forgetting factor λ is generally taken between 0.95 and 1. The larger the value of λ , the smaller the weight of new data. When $\lambda = 1$, it degenerates into the ordinary RLS method.

2.3. Online Identification of Parameters for a Second-Order RC Circuit

From Figure 1, the KVL (Kirchhoff's voltage law) equation of the circuit can be obtained:

$$U_{OC} = U_{p1} + U_{p2} + R_0 I_L + U_L \quad (5)$$

According to Equation (1), taking the Laplace transform on both sides:

$$U_{OC}(s) = \left(\frac{R_{p1}}{R_{p1}C_{p1}s + 1} + \frac{R_{p2}}{R_{p2}C_{p2}s + 1} + R_0 \right) I(s) + U_L(s) \quad (6)$$

Let the time constants be $\tau_{p1} = R_{p1}C_{p1}$ and $\tau_{p2} = R_{p2}C_{p2}$, and let $a = \tau_{p1}\tau_{p2}$, $b = \tau_{p1} + \tau_{p2}$, $c = R_{p1} + R_{p2} + R_0$; then, Equation (6) can be simplified as:

$$aU_{OC}s^2 + bU_{OC}s + U_{OC} = aR_0I_s^2 + dI_s + cI + aU_s^2 + U_L \quad (7)$$

Equation (8) can be obtained via discretion using Equation (7) as follows:

$$U(k) = \frac{-bT - 2a}{T^2 + bT + a}U(k-1) + \frac{a}{T^2 + bT + a}U(k-2) + \frac{cT^2 + dT + aR_0}{T^2 + bT + a}I(k) + \frac{-dT - 2aR_0}{T^2 + bT + a}I(k-1) + \frac{aR_0}{T^2 + bT + a}I \quad (8)$$

After simplification, the expression (8) becomes:

$$U(k) = k_1U(k-1) + k_2U(k-2) + k_3I(k) + k_4I(k-1) + k_5I(k-2) \quad (9)$$

Substituting Equation (9) into FFRLS, the value of parameter $\theta(k) = (k_1, k_2, k_3, k_4, k_5)$ can be calculated, and then the circuit model parameter $R_0, R_{p1}, R_{p2}, C_{p1}, C_{p2}$ from the identification results can be deduced as follows:

$$\begin{aligned}\tau_{p1} &= \frac{T}{2(k_1+k_2+1)}[\sqrt{k_1^2 - 4k_2} - k_1 - 2k_2] \\ \tau_{p2} &= -\frac{T}{2(k_1+k_2+1)}[\sqrt{k_1^2 - 4k_2} + k_1 + 2k_2]\end{aligned}\quad (10)$$

Combining Equations (4), (8) and (10), Equation (11) can be obtained as follows:

$$\begin{aligned} R_0 &= \frac{k_5}{k_2} \\ R_{p1} &= \frac{\tau_{p1} \frac{k_3+k_4+k_5}{k_1+k_2+1} + \tau_{p2} R_0 + \frac{T(k_4+2k_5)}{k_1+k_2+1}}{\tau_{p1} - \tau_{p2}} \\ R_{p2} &= \frac{k_3+k_4+k_5}{k_1+k_2+1} - R_{p1} - R_0 \\ C_{p1} &= \frac{\tau_{p1}}{R_{p1}} \\ C_{p2} &= \frac{\tau_{p2}}{R_{p2}} \end{aligned} \quad (11)$$

3. SOC Estimation Method Based on AEKF

3.1. Adaptive Extended Kalman Filter State Equation

The Kalman filter algorithm applies to linear systems. If it is used in a nonlinear system, the Taylor expansion formula needs to be used to locally approximate the nonlinear state equation as a linear equation. The method used for nonlinear systems is called the extended Kalman filter algorithm. Nonlinear systems can be represented by Formula (12):

$$\begin{cases} x_{k+1} = f(x_k, u_k) + w_k \\ y_k = g(x_k, u_k) + v_k \end{cases} \quad (12)$$

where x_k is the state variable, y_k is the observation variable, $f(x_k, u_k)$ is the nonlinear state function, $g(x_k, u_k)$ is the nonlinear observation function, and w_k and v_k are gaussian white noise with zero means and covariances of Q_k and R_k , respectively. The EKF algorithm takes advantage of the local linearity property of nonlinear functions by locally linearizing both nonlinear functions. As described in the previous section, $\hat{x}_{k/k}$ this is the optimal estimated value of the state variable at k time. By performing a first-order Taylor expansion of the nonlinear state function $f(x_k, u_k)$ around $\hat{x}_{k/k}$, Equation (13) can be obtained as follows:

$$f(x_k, u_k) = f(\hat{x}_{k/k}, u_k) + \left. \frac{\partial f}{\partial x_k} \right|_{x_k=\hat{x}_{k/k}} (x_k - \hat{x}_{k/k}) + o(x_k - \hat{x}_{k/k}) \quad (13)$$

In Equation (13), ignoring the high-order term $o(x_k - \hat{x}_{k/k})$ and letting $\left. \frac{\partial f}{\partial x_k} \right|_{x_k=\hat{x}_{k/k}} = F_k$, Equation (2) can be simplified as follows:

$$x_{k+1} = f(\hat{x}_{k/k}, u_k) + F_k(x_k - \hat{x}_{k/k}) + w_k \quad (14)$$

Expanding the nonlinear observation function $g(x_k, u_k)$ around the prior state estimate $\hat{x}_{k/k-1}$ at k time by a first-order Taylor series, Equation (15) can be obtained as follows:

$$g(x_k, u_k) = g(\hat{x}_{k/k-1}, u_k) + \left. \frac{\partial g}{\partial x_k} \right|_{x_k=\hat{x}_{k/k-1}} (x_k - \hat{x}_{k/k-1}) + o(x_k - \hat{x}_{k/k-1}) \quad (15)$$

Ignoring higher-order terms in $o(x_k - \hat{x}_{k/k-1})$ and setting $\left. \frac{\partial g}{\partial x_k} \right|_{x_k=\hat{x}_{k/k-1}} = G_k$, Equation (14) can be simplified as follows:

$$y_k = g(\hat{x}_{k/k-1}, u_k) + G_k(x_k - \hat{x}_{k/k-1}) + v_k \quad (16)$$

If both the nonlinear state equation and the observation equation are linear, then Equation (11) can be rewritten as follows:

$$\begin{cases} x_{k+1} = F_k x_k + f(\hat{x}_{k/k}, u_k) - F_k \hat{x}_{k/k} + w_k \\ y_k = G_k x_k + g(\hat{x}_{k/k-1}, u_k) - G_k \hat{x}_{k/k-1} + v_k \end{cases} \quad (17)$$

Here, matrices F_k and G_k can be obtained by calculating the Jacobian matrices of f and g . If the state variable x is n -dimensional, i.e., $x = [x_1, x_2, \dots, x_n]^T$, then the solution for matrices F_k and G_k is as follows.

$$F_k = \frac{\partial f}{\partial x} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \dots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \dots & \frac{\partial f_2}{\partial x_n} \\ \vdots & \vdots & \dots & \vdots \\ \frac{\partial f_n}{\partial x_1} & \frac{\partial f_n}{\partial x_2} & \dots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \quad (18)$$

$$G_k = \frac{\partial g}{\partial x} = \begin{bmatrix} \frac{\partial g_1}{\partial x_1} & \frac{\partial g_1}{\partial x_2} & \dots & \frac{\partial g_1}{\partial x_n} \\ \frac{\partial g_2}{\partial x_1} & \frac{\partial g_2}{\partial x_2} & \dots & \frac{\partial g_2}{\partial x_n} \\ \vdots & \vdots & \dots & \vdots \\ \frac{\partial g_n}{\partial x_1} & \frac{\partial g_n}{\partial x_2} & \dots & \frac{\partial g_n}{\partial x_n} \end{bmatrix} \quad (19)$$

The EKF algorithm requires advanced calibration of the covariance matrices for observation noise and process noise, which is often calculated by experience, and the covariance matrices Q_k and R_k are fixed values. However, in high-rate conditions of lithium iron phosphate batteries, the noise often changes due to the internal chemical reaction and resulting temperature variation and is no longer a fixed value. To improve the accuracy of SOC estimation, the AEKF algorithm is introduced. Based on the EKF algorithm, the AEKF adds the Sage–Husa adaptive filtering algorithm, enabling the observation noise covariance matrix and the process noise covariance matrix in the EKF algorithm to be adaptively updated, thus improving the accuracy of the SOC estimation. The steps of the AEKF algorithm are listed as follows:

Step 1. Initialization

Set $\hat{x}_0 = x_0, y_0, P_0, Q_0, R_0$ when $k = 0$, where $\hat{x}_0$ is the initial estimate of the state variables, y_0 is the initial observation value, P_0 is the initial value of the error covariance matrix, and Q_0 and R_0 are the initial values of the process covariance matrix and observation noise covariance matrix, respectively.

Step 2. State prediction

$$\hat{x}_{k/k-1} = f(\hat{x}_{k-1/k-1}, u_{k-1}) \quad (20)$$

Step 3. Prediction of error covariance

$$P_{k/k-1} = F_{k-1}P_{k-1/k-1}F_{k-1}^T + Q_{k-1} \quad (21)$$

Step 4. Calculate the Kalman gain

$$K_k = P_{k/k-1}G_k^T [G_kP_{k/k-1}G_k^T + R_{k-1}]^{-1} \quad (22)$$

Step 5. State estimation

$$\hat{x}_{k/k} = \hat{x}_{k/k-1} + K_k[y_k - g(\hat{x}_{k/k-1}, u_k)] \quad (23)$$

Step 6. Update the noise covariance matrix

$$e_k = y_k - g(\hat{x}_{k/k-1}, u_k) \quad (24)$$

$$\begin{cases} E_k = E(e_k e_k^T) \\ d = e_k^T E_k^{-1} e_k \end{cases} \quad (25)$$

In Equations (24) and (25), e_k is called the innovation matrix, and d is the estimated residual value expressed using the adaptive window factor in the windowing function, which is used to calculate the observation dimension M . According to the principle of the windowing function, if M is small, the computational burden of the adaptive algorithm is reduced, but the accuracy of the algorithm is lower. Conversely, if M is large, the accuracy of the algorithm is significantly improved, but the computational burden is too high, which is not suitable for recursive estimation algorithms. Therefore, M needs to be adjusted according to the convergence time, as shown in Equation (26).

$$\begin{cases} M = 1, d = 1 \\ M = k, d = 0 \\ M = k \times \eta^d, 0 < d < 1 \end{cases} \quad (26)$$

In Equation (26), η is the window convergence rate, which has a range of $0 < \eta < 1$. After calculating M , the noise covariance matrix can be updated as shown in Equations (27) and (28):

$$H_k = \frac{1}{M} \sum_{i=k-M+1}^{i=k} e_k e_k^T \quad (27)$$

$$\begin{cases} R_k = H_k - G_k P_{k/k-1} G_k^T \\ Q_k = K_k H_k K_k^T \end{cases} \quad (28)$$

Step 7. Estimation of error covariance.

$$P_{k/k} = (I - K_k G_k) P_{k/k-1} \quad (29)$$

Subsequently, repeat step 2~step 7 for recursive estimation to obtain the optimal estimate of the state variables.

3.2. Discretization of State Space Equation

Taking the operating current I_L as the input variable, battery SOC, battery electrochemical polarization voltage U_{p1} , and concentration difference polarization voltage U_{p2} as state variables, and U_{OC} as the observation variable, Equation (30) can be obtained by combining Equations (8) and (9):

$$\begin{pmatrix} SOC(t) \\ \frac{dU_{p1}(t)}{dt} \\ \frac{dU_{p2}(t)}{dt} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\frac{1}{R_{p1}(t)C_{p1}(t)} & 0 \\ 0 & 0 & -\frac{1}{R_{p2}(t)C_{p2}(t)} \end{pmatrix} \begin{pmatrix} SOC(t_0) \\ U_{p1}(t) \\ U_{p2}(t) \end{pmatrix} + \begin{pmatrix} -\frac{\int_{t_0}^t \eta dt}{Q_0} \\ \frac{1}{C_{p1}(t)} \\ \frac{1}{C_{p2}(t)} \end{pmatrix} I(t) \quad (30)$$

$$U_L(t) = U_{OC}[SOC(t)] - I(t)R_0(t) - U_{p1}(t) - U_{p2}(t) \quad (31)$$

Equations (30) and (31) are the continuous state equation and continuous observation equation, respectively. Here, $U_{OC}[SOC(t)]$ is the open-circuit voltage at time t obtained using the OCV-SOC relation function.

Setting the Coulomb efficiency η to be 1 and defining the sampling period as T , Equations (30) and (31) are discretized as follows:

$$\begin{pmatrix} SOC(k+1) \\ U_{p1}(k+1) \\ U_{p2}(k+1) \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-\frac{T}{\tau_{p1}(k)}} & 0 \\ 0 & 0 & e^{-\frac{T}{\tau_{p2}(k)}} \end{pmatrix} \begin{pmatrix} SOC(k) \\ U_{p1}(k) \\ U_{p2}(k) \end{pmatrix} + \begin{pmatrix} -\frac{T}{Q_0} \\ R_{p1}(k)(1 - e^{-\frac{T}{\tau_{p1}(k)}}) \\ R_{p2}(k)(1 - e^{-\frac{T}{\tau_{p2}(k)}}) \end{pmatrix} I(k) \quad (32)$$

$$U_L(k) = U_{OC}[SOC(k)] - U_{p1}(k) - U_{p2}(k) - I(k)R_0(k) \quad (33)$$

The vector composed of $SOC(k)$, $U_{p1}(k)$, and $U_{p2}(k)$ in the above equation is the state vector x_k of the system at k time in Equation (11), while $U_L(k)$ is the observation vector y_k of the system at k time. Equation (32) is a linear equation, while the nonlinearity of the battery system state space equation is reflected in the $U_{OC}[SOC(k)]$ part of Equation (33). Therefore, the Kalman filtering algorithm for SOC estimation is carried out using both EKF and AEKF, and the results are compared with the coulomb counting method for SOC estimation.

3.3. SOC Estimation Process for LiFePO4 Battery

Step 1. Data input. Input the test data of the LiFePO4 battery, including the working current sampling data $I_L(k)$, terminal voltage $U_L(k)$, battery capacity of 13 Ah, parameter model identification results, and sampling period $T = 0.1$ s.

Step 2. Algorithm initialization. Initialize the battery state vector $x_0 = [SOC(0), 0, 0]$, estimation error covariance matrix $P_0 = \begin{bmatrix} 10^{-6} & 0 & 0 \\ 0 & 10^{-6} & 0 \\ 0 & 0 & 10^{-6} \end{bmatrix}$, process noise covariance matrix $Q_0 = \begin{bmatrix} 10^{-6} & 0 & 0 \\ 0 & 10^{-6} & 0 \\ 0 & 0 & 10^{-6} \end{bmatrix}$, and observation noise covariance matrix $R_0 = 0.05$.

Step 3. Calculate the prior estimate of the state variable and the predicted covariance matrix. Firstly, according to Equation (10), calculate the state transition matrix $A_k = \text{diag}[1, e^{\frac{-T}{\tau_{p1}(k)}}, e^{\frac{-T}{\tau_{p2}(k)}}]$ and the system input matrix $B_k = [-\frac{T}{Q_0}, R_{p1}(1 - e^{\frac{-T}{\tau_{p1}(k)}}), R_{p2}(1 - e^{\frac{-T}{\tau_{p2}(k)}})]^T$ using the model parameters at time k . Secondly, substitute the state vector $x_k = [SOC(k), U_{p1}(k), U_{p2}(k)]$ and the operating current $I(k)$ at k time into Equation (20) to calculate the prior estimate value $\hat{x}_{k+1/k}$ of the state variable. Finally, calculate the predicted error covariance matrix $P_{k+1/k}$ according to Equation (21).

Step 4. Calculating the Kalman gain matrix. Firstly, the nonlinear observation equation is linearized based on Equation (8), and the Jacobian matrix G_k of the observation equation is calculated, as given in equation (33). Secondly, the Jacobian matrix G_k , the predicted error covariance matrix $P_{k+1/k}$, and the observation noise matrix R_k are substituted into Equation (22) to calculate the Kalman gain matrix K_k .

$$G_k = [\frac{\partial U_{OC}[SOC]}{\partial SOC}|_k, \frac{\partial -U_{p1}}{U_{p1}}|_k, \frac{\partial -U_{p2}}{U_{p2}}|_k] = [\frac{\partial U_{OC}[SOC]}{\partial SOC}|_k, -1, -1] \quad (34)$$

Step 5. Calculating the optimal estimated value of the state vector and updating the error covariance matrix. Based on Equation (34), the observation value, i.e., the estimated value $U_L(k)$ of the battery terminal voltage at k time, is calculated. Then, Equation (23) is used to calculate the optimal estimated value $\hat{x}_{k+1/k+1} = [SOC(k+1), U_{p1}(k+1), U_{p2}(k+1)]^T$ of the state vector, where $SOC(k+1)$ is the SOC value estimated by AEKF at $(k+1)$ time. Finally, Equation (13) is used to update the error covariance matrix $P_{k+1/k}$.

Step 6. Updating the noise covariance matrix. The process noise covariance matrix Q_k and the observation noise covariance matrix $R(k)$ are calculated based on Equations (24) and (25). Steps 2 to 6 are repeated recursively to estimate the SOC value at each moment.

3.4. Comparison and Analysis of SOC Estimation Results between EKF and AEKF Algorithms

To compare and analyze the performance of EKF and AEKF algorithms for SOC estimation of lithium iron phosphate batteries, a comparative study is conducted on SOC estimation based on the second-order equivalent circuit parameter model. Through battery charge–discharge testing, the corresponding relationship between battery OCV and SOC can be obtained through fitting. A 13 Ah-rated capacity and 3.2 V-rated voltage LiFePO4 battery is selected for HPPC testing to obtain its SOC and OCV data. The testing procedure involved discharging at 1 C for 10 s, resting for 40 s, and then charging at 0.75 C for 10 s. After resting for 45 min for every 10% drop in SOC, the OCV is measured, followed by another cycle of testing until the SOC of the battery reached 0.06, which is considered as

the end of the discharge. Under the global discharge, the battery terminal voltage U_L , the output current I_L , and the real values of SOC are shown in Figures 2 and 3, respectively.

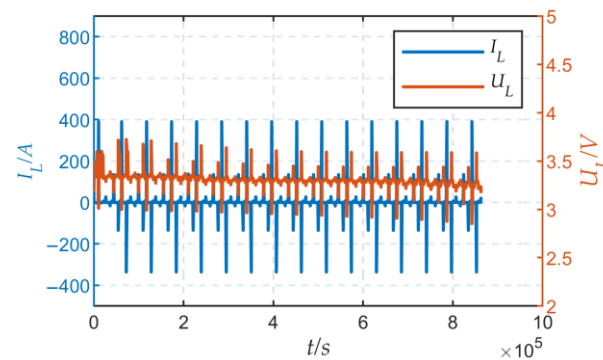


Figure 2. The waveform of terminal voltage U_L and the output current I_L .

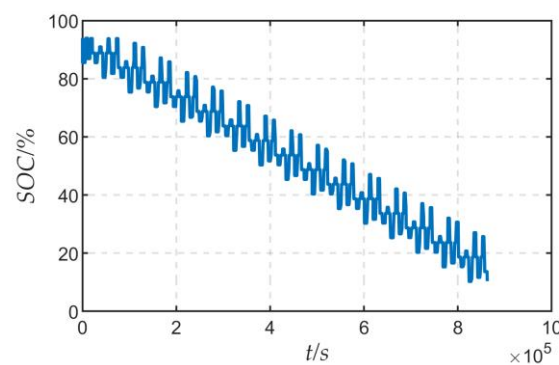


Figure 3. The waveform of real values of SOC.

Figures 4 and 5, respectively, show the SOC estimation results and estimation errors under the EKF and AEKF algorithms. From Figures 4 and 5, it can be seen that compared with the EKF algorithm, the AEKF algorithm can converge to the vicinity of the true value more quickly. The convergence time of the EKF algorithm and the AEKF algorithm is 532.8 s and 124.5 s, respectively. The SOC estimation accuracy of the AEKF algorithm is higher than that of the EKF algorithm. Experimental results show that the AEKF algorithm has a better SOC estimation convergence speed and accuracy.

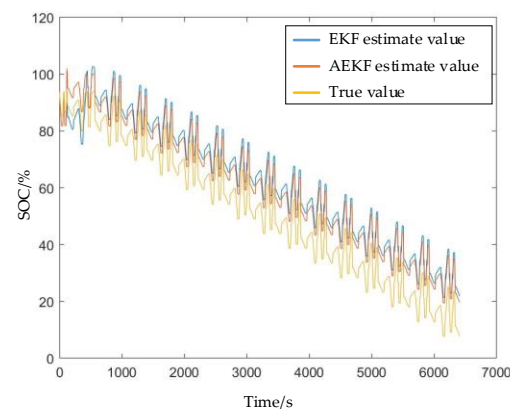


Figure 4. The SOC estimating results with EKF and AEKF.

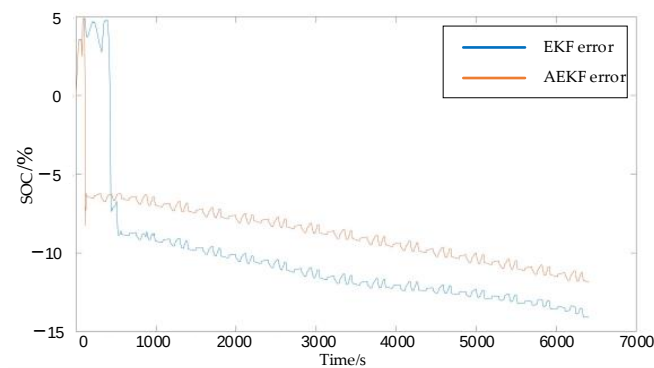


Figure 5. The SOC estimating errors with EKF and AEKF.

The specific statistical characteristics of SOC estimation errors under EKF and AEKF algorithms are shown in Table 1 (absolute error, AE; relative error, RE; mean absolute error, MAE; mean relative error, MRE), and the relevant features are calculated based on the converged results. From Table 1, it can be seen that the SOC estimation accuracy of the AEKF algorithm is better than that of the EKF algorithm, with a maximum absolute error (MaxAE) of 11.9% and a maximum relative error (MaxRE) of 152.6%, which occurs at the end of discharge. And the MAE of the SOC estimation is 8.8%, and the MRE of the SOC estimation is 25.4% with the AEKF algorithm, which is lower than that with the EKF. It can also be seen from Table 1 that the errors of both EKF and AEKF algorithms gradually increase. Therefore, the adaptive noise covariance matrix updating method of the AEKF algorithm can reduce SOC estimation errors. However, the SOC estimation precision is still large, which cannot meet the actual use needs of the battery energy management system. And, further optimization of the SOC estimation algorithm is needed.

Table 1. SOC estimation error under the EKF algorithm and AEKF algorithm.

Estimation Algorithm	MaxAE	MaxRE	MAE	MRE
EKF	14.1%	180.3%	10.9%	31.2%
AEKF	11.9%	152.6%	8.8%	25.4%

4. SOC Estimation Method Based on FFRLS-AFKF

The FFRLS-AEKF joint estimation algorithm uses the SOC value estimated using the AEKF algorithm to replace the estimated value of the ampere-hour integration method in the FFRLS algorithm to improve the battery equivalent circuit parameters identification accuracy. The identified model parameters are substituted into the AEKF algorithm to recursively estimate the SOC and improve the accuracy of the AEKF algorithm. Then, the estimated SOC value is fed back to FFRLS, and through positive feedback, the SOC estimation accuracy is finally improved. The specific calculation process is shown in Figure 6.

Step 1. Initialize the FFRLS algorithm and the AEKF algorithm, where $SOC(0)$ is obtained by the open-circuit voltage method.

Step 2. Obtain the model parameters at k time through the FFRLS algorithm, substitute them into the AEKF algorithm, construct the corresponding transfer matrix and input matrix, and obtain the SOC at $(k + 1)$ time.

Step 3. Substitute the SOC at $(k + 1)$ time into the OCV-SOC relationship formula, and then use the FFRLS algorithm to obtain the model parameters at $(k + 1)$ time.

Step 4. Update the transfer matrix and input matrix in the AEKF algorithm to obtain the SOC of the next time step; repeat steps 2 to step 4 and recursively obtain the SOC at each time step. The specific calculation process is shown in the diagram.

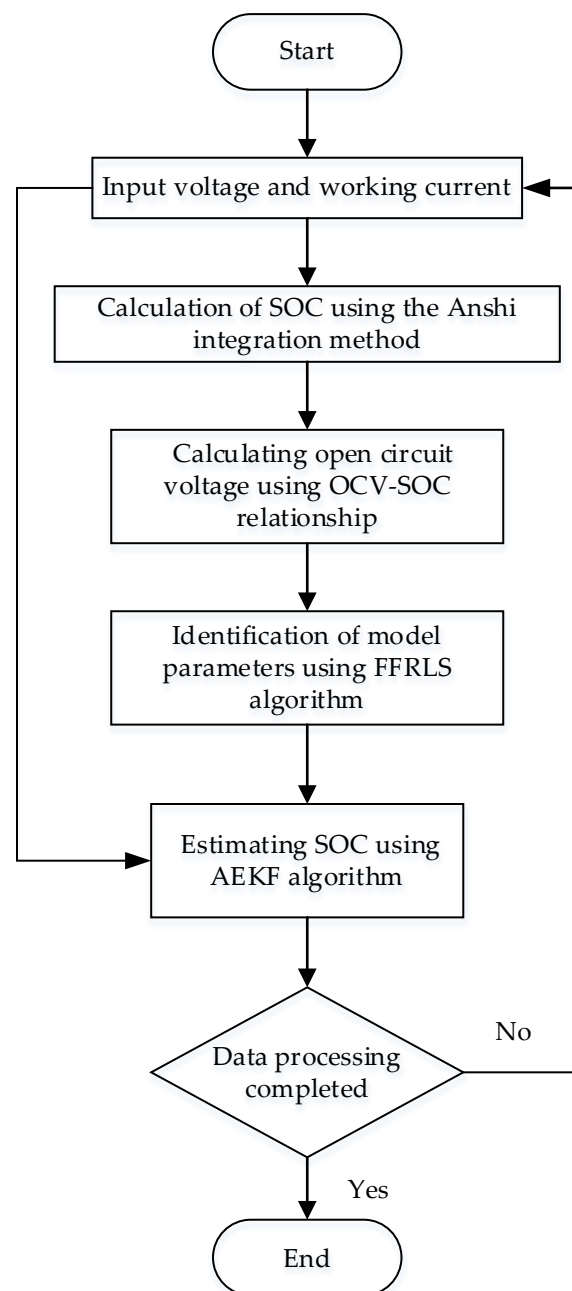


Figure 6. FFRLS + AEKF processes.

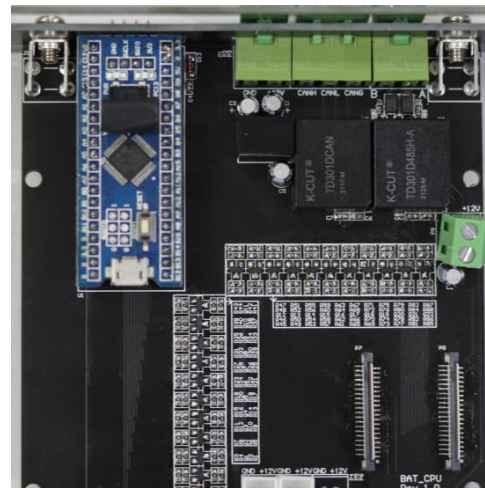
5. Experimental Results

The experimental parameters setting is the same as in Section 3.2. The structure of the battery pack used in the experiment is first parallel and then series. The batteries used are all new, and the working temperature is 25 °C. According to the method in Section 2.3, the battery parameters R_0 , R_{p1} , C_{p1} , R_{p2} , and C_{p2} can be identified at 25 °C, and the identification results are in Table 2. Four single cells form a parallel unit, and ten parallel units, a total of forty battery cells form a series battery pack. The parallel unit is composed of ten parallel units, and a total of forty battery cells are composed of series battery packs. The model of the battery is IFP9380, the rated capacity is 15 Ah, the nominal voltage is 3.2 V, the operating voltage range is 2.0–3.65 V, and the maximum discharge current is 2.00 A. The main control chip of the switching circuit selects STM32F103VB8T6 battery voltage measurement using the battery management chip LTC6811. The measurement error is less than 1 mV, and the sampling frequency is greater than 3 kHz. The sampling of the circuit

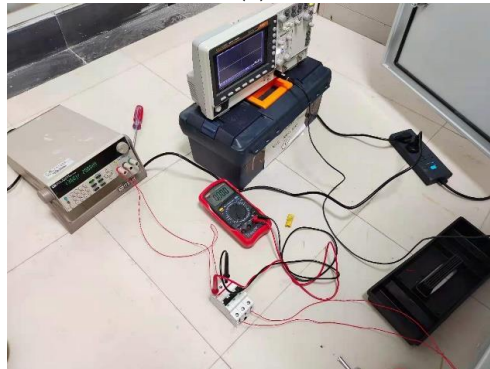
current uses a shunt, and the sampling error is less than 0.1%. The experimental platform is shown in Figure 7.

Table 2. The identification results.

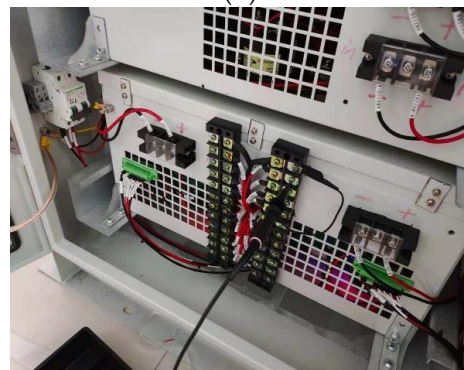
R_0/Ω	R_{p1}/Ω	C_{p1}/F	R_{p2}/Ω	C_{p2}/F
0.0005206917	0.0001007158	1256.806552	0.0024315296	26355.45136



(a)



(b)



(c)

Figure 7. Experimental platform of battery charge and discharge. (a) Control board physical diagram. (b) DC power supply charging. (c) Battery pack interface diagram.

Figures 8 and 9 show the SOC estimation results and errors using the AEKF and FFRLS-AEKF algorithms, respectively. From Figures 6 and 7, it can be seen that the estimation results with the FFRLS-AEKF algorithm are closer to the true values, and the SOC estimation

errors with the FFRLS-AEKF algorithm are smaller than with the AEKF algorithm. The estimation errors of the AEKF and FFRLS-AEKF algorithms are shown in Table 3. It can be seen from Table 3 that the maximum absolute error of the joint estimation algorithm is 4.97%, and the error is controlled within 5%. As the battery operating time increases, the SOC estimation accuracy continuously converges to the vicinity of the true value, with the average absolute error decreasing to 2.5%. The experimental results show that the FFRLS-AEKF joint estimation algorithm has good convergence performance and high estimation accuracy, verifying that the proposed method is correct and feasible. Therefore, the SOC estimation performances with EKF, AEKF, and FFRLS-AEKF are shown in Table 4.

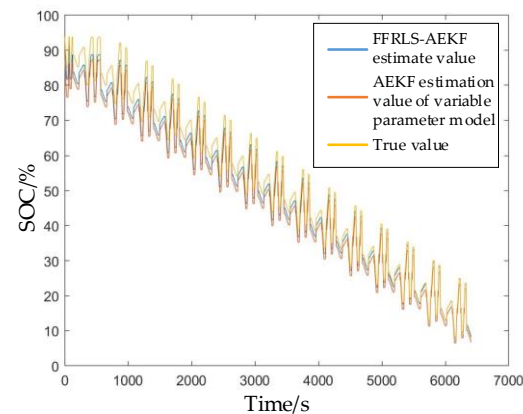


Figure 8. The SOC estimating results of the FFRLS-AEKF and AEKF.

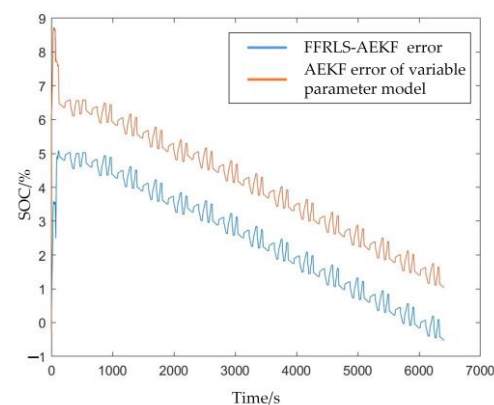


Figure 9. The SOC estimating errors of the FFRLS-AEKF and AEKF.

Table 3. Compared FFRLS-AEKF and AEKF algorithm of SOC errors.

Estimation Algorithm	MaxAE	MaxRE	MAE	MRE
AEKF	6.6%	15.7%	4.0%	8.1%
FFRLS-AEKF	4.97%	6.8%	2.5%	4.3%

Table 4. The SOC estimation performances with EKF, AEKF, and FFRLS-AEKF.

Estimation Algorithm	EKF	AEKF	FFRLS-AEKF
Steady-precision	low	lower	lowest
Convergence speed	fast	faster	fastest

6. Conclusions

The working principle of FFRLS for battery parameter identification is analyzed. The second-order equivalent circuit state discretization equation based on the AEKF algorithm

is established, and the steps of the battery SOC estimation method based on AEKF are discussed and simulated. The simulation results show that it has the disadvantages of low steady-state accuracy and slow convergence rate. To improve SOC estimation precision, combining the advantages of high precision and adaptability of FFRLS and AEKF, a joint SOC estimation method based on FFRLS-AEKF is proposed and experimented with. The experimental results show that the FFRLS-AEKF algorithm can have higher SOC estimation and faster accuracy convergence speed, verifying that the proposed method is correct and feasible.

In the actual use of the battery pack, it is often necessary to combine the battery cells in series and parallel to provide sufficient capacity and voltage level. However, in the process of series and parallel grouping of the battery pack, there will be problems such as more complex model parameters and difficult data sampling. Therefore, the next step in this research field needs to consider the influence of physical parameters such as the small number of sampling points, incomplete sampling data, and contact resistance on the accuracy of the battery model. The establishment of a perfect equivalent model of the battery pack is the focus of the next step in this research.

Author Contributions: Conceptualization, Y.X.; methodology, Z.Y.; software, L.H.; validation, Y.X., Z.Y. and L.S.; formal analysis, L.H.; investigation, L.H.; resources, Z.Y.; data curation, Y.X.; writing—original draft preparation, L.H.; writing—review and editing, Y.X.; visualization, Z.Y.; supervision, L.H.; project administration, L.S.; funding acquisition, Y.J. All authors have read and agreed to the published version of the manuscript.

Funding: This research was funded by the National Natural Science Foundation of China (Grant No. 51507183 and 51877212).

Data Availability Statement: The data that support the findings of this study are available from the corresponding author upon reasonable request.

Conflicts of Interest: The authors declare no conflict of interest.

References

- Marc, D.; Thomas, F.F.; John, N. Modeling of Galvanostatic Charge and Discharge of the Lithium/Polymer/Insertion Cell. *J. Electrochem. Soc.* **2019**, *140*, 13–21.
- Hu, X.S.; Tang, X.L. Review of modeling techniques for lithium-ion traction batteries in electric vehicles. *J. Mech. Eng.* **2017**, *16*, 20–31. [\[CrossRef\]](#)
- Qays, M.O.; Buswig, Y.; Hossain, M.L.; Abu-Siada, A. Recent progress and future trends on the state of charge estimation methods to improve battery-storage efficiency: A review. *CSEE J. Power Energy Syst.* **2022**, *8*, 105–114.
- Xiong, R.; Cao, J.; Yu, Q.; He, H.; Sun, F. Critical Review on the Battery State of Charge Estimation Methods for Electric Vehicles. *IEEE Access* **2017**, *6*, 1832–1843. [\[CrossRef\]](#)
- Naguib, M.; Kollmeyer, P.; Emadi, A. Lithium-Ion Battery Pack Robust State of Charge Estimation, Cell Inconsistency, and Balancing: Review. *IEEE Access* **2021**, *9*, 50570–50582. [\[CrossRef\]](#)
- He, H.; Zhang, X.; Xiong, R.; Xu, Y.; Guo, H. Online model-based estimation of state-of-charge and open-circuit voltage of lithium-ion batteries in electric vehicles. *Energy* **2012**, *39*, 310–318. [\[CrossRef\]](#)
- Meng, J.; Stroe, D.-I.; Ricco, M.; Luo, G.; Swierczynski, M.; Teodorescu, R. A Novel Multiple Correction Approach for Fast Open Circuit Voltage Prediction of Lithium-Ion Battery. *IEEE Trans. Energy Convers.* **2018**, *34*, 1115–1123. [\[CrossRef\]](#)
- Movassagh, K.; Raihan, A.; Balasingam, B.; Pattipati, K. A Critical Look at Coulomb Counting Approach for State of Charge Estimation in Batteries. *Energies* **2021**, *14*, 4074. [\[CrossRef\]](#)
- Vidal, C.; Hausmann, M.; Barroso, D.; Shamsabadi, P.M.; Biswas, A.; Chemali, E.; Ahmed, R.; Emadi, A. Hybrid Energy Storage System State-Of-Charge Estimation Using Artificial Neural Network for Micro-Hybrid Applications. In Proceedings of the 2018 IEEE Transportation Electrification Conference and Expo (ITEC), Long Beach, CA, USA, 13–15 June 2018.
- Chemali, E.; Kollmeyer, P.J.; Preindl, M.; Ahmed, R.; Emadi, A.; Kollmeyer, P. Long Short-Term Memory Networks for Accurate State-of-Charge Estimation of Li-ion Batteries. *IEEE Trans. Ind. Electron.* **2018**, *65*, 6730–6739. [\[CrossRef\]](#)
- Antón, J.C.; Nieto, P.J.G.; Viejo, C.B.; Vilán, J.A.V. Support Vector Machines Used to Estimate the Battery State of Charge. *IEEE Trans. Power Electron.* **2013**, *28*, 5919–5926. [\[CrossRef\]](#)
- Hou, J.; Yang, Y.; He, H.; Gao, T. Adaptive Dual Extended Kalman Filter Based on Variational Bayesian Approximation for Joint Estimation of Lithium-Ion Battery State of Charge and Model Parameters. *Appl. Sci.* **2019**, *9*, 1726. [\[CrossRef\]](#)
- Hu, X.; Li, S.; Peng, H. A comparative study of equivalent circuit models for Li-ion batteries. *J. Power Sources* **2012**, *198*, 359–367. [\[CrossRef\]](#)

14. Wan, E.A.; Van Der Merwe, R. The unscented Kalman filter for nonlinear estimation. In Proceedings of the IEEE 2000 Adaptive Systems for Signal Processing, Communications, and Control Symposium (Cat. No.00EX373), Lake Louise, AB, Canada, 4 October 2000.
15. Plett, G.L. Extended Kalman filtering for battery management systems of LiPB-based HEV battery packs—Part 3. State and parameter estimation. *Power Sources* **2004**, *134*, 277–292. [\[CrossRef\]](#)
16. Bhangu, B.; Bentley, P.; Stone, D.; Bingham, C. Nonlinear Observers for Predicting State-of-Charge and State-of-Health of Lead-Acid Batteries for Hybrid-Electric Vehicles. *IEEE Trans. Veh. Technol.* **2005**, *54*, 783–794. [\[CrossRef\]](#)
17. Pavkovic, D.; Smetko, V.; Hrgetic, M.; Komljenovic, A. Dual Kalman filter-based SOC/SoH estimator for an ultracapacitor module. In Proceedings of the 2014 IEEE Conference on Control Applications (CCA), Juan Les Antibes, France, 8–10 October 2014.
18. Lai, X.; Zheng, Y.; Sun, T. A comparative study of different equivalent circuit models for estimating state-of-charge of lithium-ion batteries. *Electrochim. Acta* **2018**, *259*, 566–577. [\[CrossRef\]](#)
19. Zhou, D.; Yin, H.; Xie, W.; Fu, P.; Lu, W. Research on Online Capacity Estimation of Power Battery Based on EKF-GPR Model. *J. Chem.* **2019**, *2019*, 1–9. [\[CrossRef\]](#)
20. Gao, Z.C.; Chin, C.S.; Toh, W.D.; Chiew, J.; Jia, J. State-of-Charge Estimation and Active Cell Pack Balancing Design of Lithium Battery Power System for Smart Electric Vehicle. *J. Adv. Transp.* **2017**, *2017*, 1–14. [\[CrossRef\]](#)
21. Jia, J.; Lin, P.; Chin, C.S.; Toh, W.D.; Gao, Z.; Lyu, H.; Cham, Y.T.; Mesbahi, E. Multirate strong tracking extended Kalman filter and its implementation on lithium iron phosphate (LiFePO₄) battery system. In Proceedings of the 11th IEEE International Conference on Power Electronics and Drive Systems, Sydney, NSW, Australia, 9–12 June 2015.
22. Ali, M.U.; Khan, H.F.; Masood, H.; Kallu, K.D.; Ibrahim, M.M.; Zafar, A.; Oh, S.; Kim, S. An adaptive state of charge estimator for lithium-ion batteries. *Energy Sci. Eng.* **2022**, *10*, 2333–2347. [\[CrossRef\]](#)
23. Ge, D.; Zhang, Z.; Kong, X.; Wan, Z. Online SoC Estimation of Lithium-Ion Batteries Using a New Sigma Points Kalman Filter. *Appl. Sci.* **2021**, *11*, 11797. [\[CrossRef\]](#)
24. Zhang, Z.Y.; Wang, G.S.; Nie, S.X.; Xing, P.X. State of Charge Estimation of LiFePO₄ battery under Pulse High Rate Discharge. *J. Electr. Eng. Technol.* **2019**, *8*, 1769–1779.
25. Xia, B.; Sun, Z.; Zhang, R.; Lao, Z. A Cubature Particle Filter Algorithm to Estimate the State of the Charge of Lithium-Ion Batteries Based on a Second-Order Equivalent Circuit Model. *Energies* **2017**, *10*, 457. [\[CrossRef\]](#)
26. Ye, M.; Guo, H.; Xiong, R.; Yu, Q. A double-scale and adaptive particle filter-based online parameter and state of charge estimation method for lithium-ion batteries. *Energy* **2017**, *144*, 789–799. [\[CrossRef\]](#)
27. Schwunk, S.; Armbruster, N.; Straub, S.; Kehl, J.; Vetter, M. Particle filter for state of charge and state of health estimation for lithium-iron phosphate batteries. *J. Power Sources* **2013**, *239*, 705–710. [\[CrossRef\]](#)
28. Giannelos, S.; Borozan, S.; Aunedi, M.; Zhang, X.; Ameli, H.; Pudjianto, D.; Konstantelos, I.; Strbac, G. Modelling Smart Grid Technologies in Optimisation Problems for Electricity Grids. *Energies* **2023**, *16*, 5088. [\[CrossRef\]](#)
29. Li, X.; Lyu, L.; Geng, G.; Jiang, Q.; Zhao, Y.; Ma, F.; Jin, M. Power Allocation Strategy for Battery Energy Storage System Based on Cluster Switching. *IEEE Trans. Ind. Electron.* **2021**, *69*, 3700–3710. [\[CrossRef\]](#)
30. Gao, X.; Fu, L. SOC Optimization Based Energy Management Strategy for Hybrid Energy Storage System in Vessel Integrated Power System. *IEEE Access* **2020**, *8*, 54611–54619. [\[CrossRef\]](#)
31. Song, Q.; Mi, Y.; Lai, W. A Novel Variable Forgetting Factor Recursive Least Square Algorithm to Improve the Anti-Interference Ability of Battery Model Parameters Identification. *IEEE Access* **2019**, *7*, 61548–61557. [\[CrossRef\]](#)
32. Badoni, M.; Singh, A.; Singh, B. Variable Forgetting Factor Recursive Least Square Control Algorithm for DSTATCOM. *IEEE Trans. Power Deliv.* **2015**, *30*, 2353–2361. [\[CrossRef\]](#)

Disclaimer/Publisher’s Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.