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Modelling Reliability Characteristics of Technical Equipment of Local Area Computer Networks

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Abstract: Technical systems in the modern global world are rapidly evolving and improving. In most cases, these are large-scale multi-level systems and one of the problems that arises in the design process of such systems is to determine their reliability. Accordingly, in the paper, a mathematical model based on the Weibull distribution has been developed for determining a computer network reliability. In order to simplify calculating the reliability characteristics, the system is considered to be a hierarchical one, ramified to level 2, with bypass through the level. The developed model allows us to define the following parameters: the probability distribution of the count of working output elements, the availability function of the system, the duration of the system's stay in each of its working states, and the duration of the system's stay in the prescribed availability condition. The accuracy of the developed model is high. It can be used to determine the reliability parameters of the large, hierarchical, ramified systems. The research results of modelling a local area computer network are presented. In particular, we obtained the following best option for connecting workstations: 4 of them are connected to the main hub, and the rest (16) are connected to the second level hub, with a time to failure of 4818 h.

Keywords: reliability characteristics; Weibull distribution; models; hierarchical ramified system; computer networks; technical equipment

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1. Introduction

Under the conditions of the market economy, upgrading of quality and economic efficiency of operation of a product, a device, or a system is of particular significance. One of the main indices of quality is reliability [1–4] that, in turn, directly influences economic efficiency [5,6]. A lot of methods, models, and approaches for determination of the reliability parameters of technical systems have already been described in the literature. These approaches are based on the Monte Carlo method [7,8], Markov nets [9], logistic regression [10], stochastic Petri nets [11], and Bayesian networks [12,13], on the use of artificial neural networks [14]. Specialists know that the complexity of systems is much faster and quicker than the development of mathematical methods for investigating their reliability. The existing traditional methods of reliability calculation for devices and simple parallel-sequential systems [15–20] cannot satisfy requirements of reliability investigation for complicated and large systems [21], such as tree-like hierarchical ramified systems [22–24]. Today, a significant part of technical systems are multilevel, hierarchical, and branched systems, namely: computer networks [22], systems of "smart" city [25–27], "smart" house [28,29], smart-grid systems supporting IoT with the use of modern Blockchain technology [30,31], etc. Accordingly, the models developed in this article are intended just for analysis and investigation of complicated hierarchical systems [32–34].

Unlike the existing models, these models possess high accuracy and need small expenses of the resources of personal computers. Therefore, the development of models using the Weibull distribution to determine the reliability characteristics of hierarchical systems is currently an urgent task.

Various authors use the Weibull distribution to determine the reliability parameters, in particular, in the article [35], it was applied to the simple step-stress accelerated life tests for one-shot devices. The reliability of mechanical equipment was evaluated in Reference [36]. The reliability parameters of power and hierarchical systems were investigated in Reference [33], and the reliability and statistical parameters in microelectronics were explored in References [37–39]. The Weibull distribution was also employed to define the reliability of software [3], the reliability of microprograms [40], the reliability indicators in testing tasks [41], etc.

Thus, the purpose of this work is to develop models for determining the reliability parameters of hierarchical systems based on the Weibull distribution. To achieve this goal, it is necessary:

- to develop models for determining reliability parameters based on the Weibull distribution, intended for the study of hierarchical technical systems;
- to apply the developed models for determining the reliability parameters based on the Weibull distribution to the analysis of the local computer network.

This paper is structured as follows. Section 1 (introduction) considers the problem relevancy and a brief review of the related works (the research context). Section 2 describes the research object: local computer network and the model for its representation in the form of a hierarchical ramified system. The main reliability parameters of the elements of the investigated system are shown in Section 3, while Section 4 demonstrates the peculiarities of the minimization of the structure of a hierarchical ramified system. Sections 5–7 contains the models defining the main probability reliability characteristics of the local computer network with the use of the Weibull distribution. The results of calculating the investigated computer network are given in Section 8. The remaining Sections 9 and 10 present conclusions and perspectives for further research of the authors in this area.

2. Representation of a Technical Equipment of a Local Area Computer Network in the Form of a Hierarchical Ramified System

The reliability issues are very important at the development stage and application of systems of different kinds in the industry. The calculation of reliability characteristics is rather difficult due to a great number of factors and general statistical nature of reliability.

As an example, we consider a technical equipment of a computer network [42–45] of the Ethernet 10 Base-T standard, which includes a server, two hubs, and workstations (Figure 1). We can update the network configuration with Ethernet 100 Base-T or Ethernet 1000 Base-T. However, in this case, we analyze the existing network.

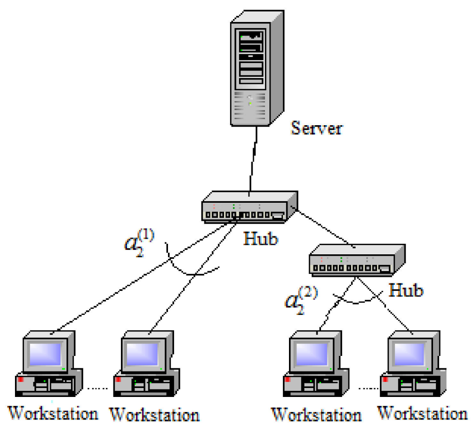

Figure 1. An example of technical equipment of a local area computer network.

A municipal firm has placed 20 workstations, namely seven workstations are connected to the server by the 8-port hub (the 1st hub), and the other 13 workstations are directly connected to the 16-port hub (the 2nd hub). Only the 8-port hub is directly connected to the server, and both hubs are connected with each other. Therefore, the 8-port hub can be considered to be basic. The place, where 13 workstations are placed, is remote from the server at a distance, which does not make it possible to connect them to the basic hub due to a limitation of the Ethernet 10 Base-T standard on the length of a communication line (no more than 100 m) between directly connected nodes for twisted pair cabling. This explains the use of the second hub. The place, where there are workstations connected to the basic hub, is limited in the area. No more than 7 workstations may be put over there. As we can see in standard IEEE 802.3 [46], if a twisted pair is used for the connection, the maximum length of the segment is 100 m (the same in all cases, for Ethernet 10 Base-T, Ethernet 100 Base-T, and Ethernet 1000 Base-T).

In Figure 1, $a_2^{(1)}, a_2^{(2)}$ denote counts of workstations directly connected to the first (basic) hub and to the second hub, correspondingly. The following inequalities are fulfilled.

$$a_2^{(1)} \leq 7, a_2^{(2)} \leq 16, a_2^{(1)} < a_2^{(2)}. \quad (1)$$

The technical equipment of this local area computer network can be represented in the form of a hierarchical ramified system shown in Figure 2.

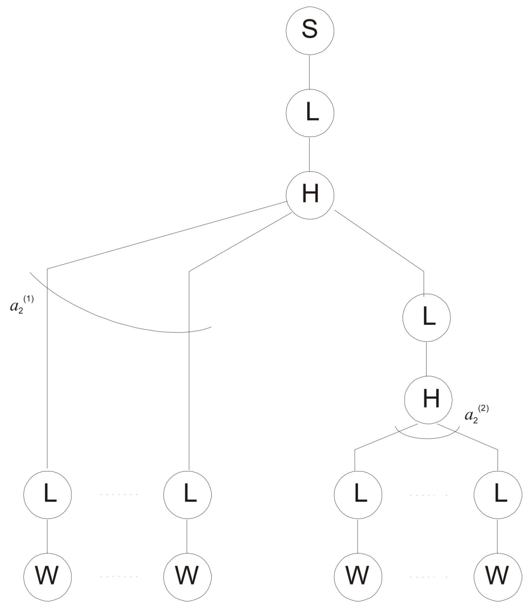

Figure 2. A hierarchical ramified system corresponding to the technical equipment of the local area computer system (where *S* denotes a server, *H*-a hub, *W*-a workstation, *L*-a communication line).

The server, hubs, workstations, and communication lines are the main blocks of the system in terms of reliability. If the server or the basic hub fails, all the systems can be considered inoperable. If the second hub fails, all the workstations, connected to it, can be considered inoperative.

3. Reliability Characteristics of Elements of the System

The investigator's task is to fit a distribution of the probability of a failure-free operation that maintains a model's adequacy for processes of the device's lifetime. The server and workstations include electronic units, which can be considered ageless, as well as mechanical units, that yield exhaustion. In case of aging or exhaustion of elements, their lifetimes are described by the Weibull distribution [47–50]. The lifetimes of electronic units of the server and the lifetimes of workstations are described by the exponential distribution and the lifetimes of mechanical units can be described by the Weibull distribution.

In the process of constructing a mathematical model, we use the notation depicted in Table 1.

Table 1. Nomenclature of the variables and parameters used in the model.

Name	Description	Units
$a_2^{(1)}$	The number of workstations directly connected to the first hub	-
$a_2^{(2)}$	The number of workstations directly connected to the second hub	
t	A moment of time when calculation is conducted	hours
p_s	The probability of failure-free operation of a server	
λ_0	Failure intensity, a parameter of the exponential distribution for the probability of failure-free operation of electronic parts of a server	$\frac{1}{hours}$.
λ_1	Failure intensity, a scale parameter of the Weibull distribution for the probability of failure-free operation of mechanical parts of a server	$\frac{1}{hours}$.
β_1	An aging coefficient (parameter of the Weibull distribution for the probability of failure-free operation of mechanical parts of a server)	-
p_L	The probability of failure-free operation of a communication line as an ageless element	-
λ_2	Failure intensity, a parameter of the exponential distribution for probability of failure-free operation of a communication line	$\frac{1}{hours}$.
p_H	The probability of failure-free operation of a hub	
λ_3	Failure intensity, a parameter of the exponential distribution for probability of failure-free operation of a hub	$\frac{1}{hours}$.
p_W	The probability of failure-free operation of a workstation	
λ_4	Failure intensity, a parameter of the exponential distribution for probability of failure-free operation of electronic parts of a workstation	$\frac{1}{hours}$.
λ_5	Failure intensity, a scale parameter of the Weibull distribution for probability of failure-free operation of mechanical parts of a workstation	$\frac{1}{hours}$.
β_5	An aging coefficient (parameter of the Weibull distribution for probability of failure-free operation of mechanical parts of a workstation)	-
x_2	The number of working output elements (2nd level)	-
$x_2^{(1)}$	The number of working output elements in the first branch (2nd level)	-
$x_2^{(2)}$	The number of working output elements in the second branch (2nd level)	-
p_0	The probability of failure-free operation of the elements at the 0-level (the first hub, server, and communication line)	-
p_1	The probability of failure-free operation of the elements at the first level (the second hub and communication line)	-
p_2	The probability of failure-free operation of the workstations, 2nd level	-
$S_2(z)$	The generating function	
$P_2(x_2)$	A probability distribution of the count of the working output elements of the system	-
$P_2(x_2, t)$	The dependence of probability regarding the count of working output elements of the system upon time	-
k	The number of working output elements (availability condition - no less than k output elements operate)	-
$K_{G2}(k,t)$	The availability function of the system	
$T_2(x_2)$	The duration of the system's stay in the state of x_2 operating output elements	hours
$T_{G2}(k)$	The duration of the system's stay in the prescribed availability condition k	hours
$Q_2(k,t)$	The failure probability in the prescribed availability condition k	-
$a_2(k,t)$	Failure frequency in the prescribed availability condition k	$\frac{1}{hours}$
$\lambda_2(k,t)$	Failure rate in the prescribed availability condition k	$\frac{1}{hours}$

Therefore, the probability of failure-free operation of a server is given by:

$$p_s(t) = e^{-\lambda_0 t} e^{-\lambda_1 t^{\beta_1}}, \quad (2)$$

where λ_0 is a parameter of the exponential distribution for the probability of failure-free operation of electronic parts of a server, λ_1, β_1 are parameters of the Weibull distribution for the probability of failure-free operation of mechanical parts of a server, λ_1 is a scale parameter, and β_1 is an aging coefficient.

The probability of failure-free operation of a communication line as an ageless element is described by the exponential distribution.

$$p_L(t) = e^{-\lambda_2 t}, \quad (3)$$

where λ_2 is a parameter of the exponential distribution for the probability of failure-free operation of a communication line.

Hubs are electronic devices that can be considered ageless. The probability of failure-free operation of a hub is described by the exponential distribution.

$$p_H(t) = e^{-\lambda_3 t}, \quad (4)$$

where λ_3 is a parameter of the exponential distribution for the probability of failure-free operation of a hub.

The probability of failure-free operation of a workstation can be written in the form:

$$p_W(t) = e^{-\lambda_4 t} e^{-\lambda_5 t^{\beta_5}}, \quad (5)$$

where λ_4 is a parameter of the exponential distribution for the probability of a failure-free operation of electronic parts of a workstation. λ_5, β_5 are parameters of the Weibull distribution for the probability of a failure-free operation of mechanical parts of a workstation, λ_5 is a scale parameter, and β_5 is an aging coefficient.

4. Contraction of Structure of the Hierarchical Ramified System

For simplification of calculations, it is necessary to reduce (contract) a hierarchical ramified system at the beginning. For this purpose, we merge elements, that do not ramify, into one element. The system will be simplified to a form shown in Figure 3, where elements are designated by integers that are numbers of levels where these elements are located.

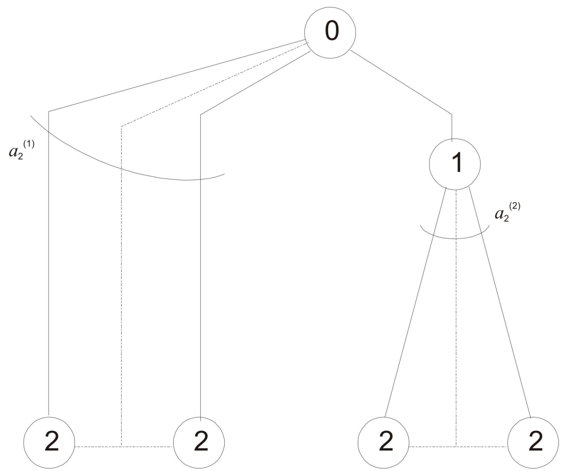

Figure 3. Contracted representation of the hierarchical ramified system.

In this system, the probabilities of failure-free operation of the elements are calculated as follows:

$$p_0 = p_S p_L p_H,\ p_1 = p_L p_H,\ p_2 = p_L p_W, \tag{6}$$

where p_0 is for a hub of the 0-level (basic hub, the first hub), p_1 is for a hub of the 1-level (the second hub), p_2 are for workstations, level 2. Here, we use the same notations $p_S,\ p_L,\ p_H,\ p_W$ as in Equations (2)–(5).

The obtained hierarchical structure is based on the functional purpose of individual elements. Accordingly, the reliability properties of the elements of one level are the same. Therefore, in the presented example, the elements are distributed by levels exactly like that (all the workstations are at one level-number 2 in the Figure 3, and the hubs-at the other two levels, number 0 and 1). Considering the reliability properties of the elements of the hierarchical structure of the network this way, it is more convenient to develop a mathematical model, perform calculations, and automate the calculation process.

5. Construction of the Generation Function and Determination of Probabilistic Reliability Characteristics of the System on the Basis of This Function

According to the binomial theorem, the generating function is written in the form of the following sum of products.

$$\begin{aligned} S_2(z) &= p_0 \sum_{x_1^{(2)}=0}^{1} p_1^{x_1^{(2)}} (p_2 z + q_2)^{a_2^{(2)} x_1^{(2)}} q_1^{1-x_1^{(2)}} \sum_{x_2^{(1)}=0}^{a_2^{(1)}} C_{a_2^{(1)}}^{x_2^{(1)}} p_2^{x_2^{(1)}} q_2^{a_2^{(1)}-x_2^{(1)}} z^{x_2^{(1)}} + q_0 \\ &= \sum_{x_1^{(2)}=0}^{1} \sum_{x_2^{(2)}=0}^{a_2^{(2)} x_1^{(2)}} \sum_{x_2^{(1)}=0}^{a_2^{(1)}} C_{a_2^{(2)} x_1^{(2)}}^{x_2^{(2)}} C_{a_2^{(1)}}^{x_2^{(1)}} p_1^{x_1^{(2)}} q_1^{1-x_1^{(2)}} p_2^{x_2^{(1)}+x_2^{(2)}} q_2^{a_2^{(2)} x_1^{(2)}+a_2^{(1)}-x_2^{(1)}-x_2^{(2)}} z^{x_2^{(1)}+x_2^{(2)}} + q_0 \end{aligned} \tag{7}$$

where $x_1^{(2)}$ is a count of working elements of level 1, $x_2^{(1)}$ is a count of the working elements of level 2 that belong to the first branch, and $x_2^{(2)}$ is a count of the working elements of level 2 that belong to the second branch.

On the basis of the generating function (7), we put down an expression for a probability distribution $P_2(x_2)$ of the count of the working output elements of the system. In addition, we take into account that the general count x_2 of working output elements of the system equals a sum of working elements in two branches. Hence, $x_2^{(2)} = x_2 - x_2^{(1)}$.

We also take into the account that count $x_1^{(2)}$ of working elements of level 1 cannot be less than $\frac{x_2^{(2)}}{a_2^{(2)}}$, the count $x_2^{(1)}$ of working elements of level 2 that belong to the first branch cannot be more than both $a_2^{(1)}$ and x_2, and cannot be less than $x_2 - a_2^{(2)}$ provided that $x_2 > a_2^{(2)}$.

$$P_2(x_2) = p_0 \sum_{x_2^{(1)}=\max\{0, x_2 - a_2^{(2)}\}}^{\min\{x_2, a_2^{(1)}\}} \sum_{x_1^{(2)}=ceil(\frac{x_2 - x_2^{(1)}}{a_2^{(2)}})}^{1} C_{a_2^{(2)} x_1^{(2)}}^{x_2 - x_2^{(1)}} C_{a_2^{(1)}}^{x_2^{(1)}} p_1^{x_1^{(2)}} q_1^{1-x_1^{(2)})} p_2^{x_2} q_2^{a_2^{(2)} x_1^{(2)} + a_2^{(1)} - .x_2} \tag{8}$$

Under the conditions $0 < x_2 \le a_2^{(1)} + a_2^{(2)}$, $a_2^{(1)} < a_2^{(2)}$, we obtain:

At $x_2 = 0$ the right side of Equation (8) should increase by q_0.

We pass to dependence of probability of the count of working output elements of the system upon time. Substituting Equations (3)–(5) into (6), we obtain:

$$p_0 = e^{-(\lambda_0+\lambda_2+\lambda_3)t-\lambda_1 t^{\beta_1}},\ p_1 = e^{-(\lambda_2+\lambda_3)t},\ p_2 = e^{-(\lambda_2+\lambda_4)t-\lambda_5 t^{\beta_5}}. \tag{9}$$

Substituting Equation (9) into Equation (8), we obtain under the conditions $0 < x_2 \leq a_2^{(1)} + a_2^{(2)}, a_2^{(1)} < a_2^{(2)}$:

$$P_2(x_2,t) = e^{-(\lambda_0+\lambda_2+\lambda_3)t-\lambda_1 t^{\beta_1}} \sum_{x_2^{(1)}=\max\{0,x_2-a_2^{(2)}\}}^{\min\{x_2,a_2^{(1)}\}} \sum_{x_1^{(2)}=ceil\left(\frac{x_2-x_2^{(1)}}{a_2^{(2)}}\right)}^{1} C_{a_2^{(2)}x_1^{(2)}}^{x_2-x_2^{(1)}} \times C_{a_2^{(1)}}^{x_2^{(1)}} e^{-(\lambda_2+\lambda_3)x_1^{(2)}t} \left(1-e^{-(\lambda_2+\lambda_3)t}\right)^{1-x_1^{(2)}} \times e^{-x_2((\lambda_2+\lambda_4)t+\lambda_5 t^{\beta_5})} \left(1-e^{-(\lambda_2+\lambda_4)t-\lambda_5 t^{\beta_5}}\right)^{a_2^{(2)}x_1^{(2)}+a_2^{(1)}-x_2} \quad (10)$$

At $x_2 = 0$, the right side of Equation (10) should increase by $1 - e^{-(\lambda_0+\lambda_2+\lambda_3)t-\lambda_1 t^{\beta_1}}$. The probability that there are no less than k working output elements when $0 < k \leq \left(a_2^{(1)} + a_2^{(2)}\right)$ is the availability function of the system. Under the condition $a_2^{(1)} < a_2^{(2)}$, this function is written in the following way.

$$K_{G2}(k,t) = \sum_{x_2=k}^{a_2^{(1)}+a_2^{(2)}} P_2(x_2,t) = \sum_{x_2=k}^{a_2^{(1)}+a_2^{(2)}} e^{-(\lambda_0+\lambda_2+\lambda_3)t-\lambda_1 t^{\beta_1}} \times \sum_{x_2^{(1)}=\max\{0,x_2-a_2^{(2)}\}}^{\min\{x_2,a_2^{(1)}\}} \sum_{x_1^{(2)}=ceil\left(\frac{x_2-x_2^{(1)}}{a_2^{(2)}}\right)}^{1} C_{a_2^{(2)}x_1^{(2)}}^{x_2-x_2^{(1)}} C_{a_2^{(1)}}^{x_2^{(1)}} e^{-(\lambda_2+\lambda_3)x_1^{(2)}t} \times \left(1-e^{-(\lambda_2+\lambda_3)t}\right)^{1-x_1^{(2)}} e^{-x_2((\lambda_2+\lambda_4)t+\lambda_5 t^{\beta_5})} \left(1-e^{-(\lambda_2+\lambda_4)t-\lambda_5 t^{\beta_5}}\right)^{a_2^{(2)}x_1^{(2)}+a_2^{(1)}-x_2}. \quad (11)$$

It should be noted that, if new structural elements/levels are added to the system (e.g., WiFi routers, BT devices, LTE, etc.), then the structure of the mathematical model will be analogous, but it will contain more parameters and variables. Calculations will be more complicated and time-consuming.

6. Calculations of Time Reliability Characteristics of the System

We pass to two main time reliability characteristics of the system: the average duration of the system's stay in each of its states and the average duration of the system's stay in the prescribed availability condition.

The duration of the system's stay in the state of x_2 operating output elements is obtained by integrating the expression for the probability distribution of the count of output working elements with respect to t from 0 to ∞.

Under the conditions $0 <\leq a_2^{(1)} + a_2^{(2)}, a_2^{(1)} < a_2^{(2)}$, we obtain:

$$T_2(x_2) = \int_0^\infty P_2(x_2,t)dt, \quad (12)$$

where $P_2(x_2,t)$ is calculated by Equation (10).

The duration of the system's stay in the prescribed availability condition k, which means that no less than k output elements operate, equals a sum of durations of the system's stay in states over the count of output elements from k to $a_2^{(1)} + a_2^{(2)}$. Under the conditions $0 < k \leq a_2^{(1)} + a_2^{(2)}$, we obtain:

$$T_{G2}(k) = \sum_{x_2=k}^{a_2^{(1)}+a_2^{(2)}} T_2(x_2), \quad (13)$$

where $T_2(x_2)$ is calculated by Equation (12).

7. Calculations of Conventional Reliability Characteristics for Unrestorable Ramified Systems

Under the prescribed availability condition, on the basis of the expression for the availability function of the system, it is possible to estimate the system's reliability by means of three conventional characteristics specified by standards for unrestorable systems: the failure probability, the failure frequency, and the failure rate.

The failure probability in the prescribed availability condition k, where $0 < k \leq a_2^{(1)} + a_2^{(2)}, a_2^{(1)} < a_2^{(2)}$, is written in the following way:

$$Q_2(k,t) = 1 - K_{G2}(k,t) = 1 - \sum_{x_2=k}^{a_2^{(1)}+a_2^{(2)}} e^{-(\lambda_0+\lambda_2+\lambda_3)t-\lambda_1 t^{\beta_1}} \times \sum_{x_2^{(1)}=\max\{0,x_2-a_2^{(2)}\}}^{\min\{x_2,a_2^{(1)}\}} \sum_{x_1^{(2)}=ceil\left(\frac{x_2-x_2^{(1)}}{a_2^{(2)}}\right)}^{1} C_{a_2^{(2)}x_1^{(2)}}^{x_2-x_2^{(1)}} C_{a_2^{(1)}}^{x_2^{(1)}} e^{-(\lambda_2+\lambda_3)x_1^{(2)}t} \times \left(1-e^{-(\lambda_2+\lambda_3)t}\right)^{1-x_1^{(2)}} e^{-x_2((\lambda_2+\lambda_4)t+\lambda_5 t^{\beta_5})} \left(1-e^{-(\lambda_2+\lambda_4)t-\lambda_5 t^{\beta_5}}\right)^{a_2^{(2)}x_1^{(2)}+a_2^{(1)}-x_2}. \quad (14)$$

For calculation of the failure frequency in the prescribed availability condition k, where $0 < k \leq a_2^{(1)} + a_2^{(2)}$, $a_2^{(1)} < a_2^{(2)}$, we obtain:

$$a_2(k,t) = -\frac{dK_{G2}(k,t)}{dt}, \quad (15)$$

where $K_{G2}(k,t)$ is calculated by Equation (11).

The main characteristic for reliability evaluation of unrestorable systems, according to standards, is the failure rate [51,52]. Under the prescribed availability condition k and under the conditions $0 < k \leq a_2^{(1)} + a_2^{(2)}$, $a_2^{(1)} < a_2^{(2)}$, we obtain:

$$\lambda_2(k,t) = \frac{a_2(k,t)}{K_{G2}(k,t)}, \quad (16)$$

where $a_2(k,t)$ and $K_{G2}(k,t)$ are calculated by Equations (15) and (11), respectively.

8. An Example of Calculations

The input data for calculating the reliability characteristics of the system shown in Figure 1 are the parameters of the server, the hubs, the workstations, and communication line, namely: λ_0, λ_1, β_1, λ_2, λ_3, λ_4, λ_5, β_5, $a_2^{(1)}, a_2^{(2)}$, x_2 (they are described in detail in Table 1) and t is a moment of time when calculation is conducted.

For the server, when the average operating time to failure is 172,199 h [53], we obtain:

$$\lambda_0 = 5,807 * 10^{-6} \frac{1}{hours}. \quad (17)$$

For mechanical parts of the server (e.g., cooler with the average operating time to failure at about 100,000 h [54]), an aging coefficient should be chosen between 1.1 and 1.3. At $\beta_1 = 1,2$ for the given average operating time to failure, we obtain:

$$\lambda_1 = 9,2927 * 10^{-7} \frac{1}{hours}. \quad (18)$$

Analogously for the workstations, proceeding from the average operating time to failure at 8766 h, we obtain:

$$\lambda_4 = 1.14 * 10^{-4} \frac{1}{hours}, \ \lambda_5 = 2,80651 * 10^{-5} \frac{1}{hours}, \ \beta_5 = 1,2. \quad (19)$$

For the communication lines, granting stationary service conditions, it is necessary to assign:

$$\lambda_2 = 6,12105 * 10^{-8} \frac{1}{hours}. \quad (20)$$

Proceeding from the average operating time to failure of the hub at 492,096 h (we consider 16-Port Fast Ethernet unmanaged switch DES-1016D [55]) and taking into account that the 8-port hub has almost the same operating time to failure (for example, D-Link DGS 1008D 8-port switch [56]), we obtain:

$$\lambda_3 = 2,03 * 10^{-6} \frac{1}{hours}. \quad (21)$$

As a result of calculations, it is possible to obtain the following main output data [48,49]: the probability distribution of a count of operating output elements, the availability function, the duration of the system's stay in each of its working states, the duration of the system's stay in the prescribed availability condition, the failure frequency in the prescribed availability condition, and the failure rate in the prescribed availability condition.

The results of calculations of the availability function, the failure frequency, and the failure rate of the system in the prescribed availability condition and under the condition $0 \le t \le 5000$ h are shown in Figures 4–6.

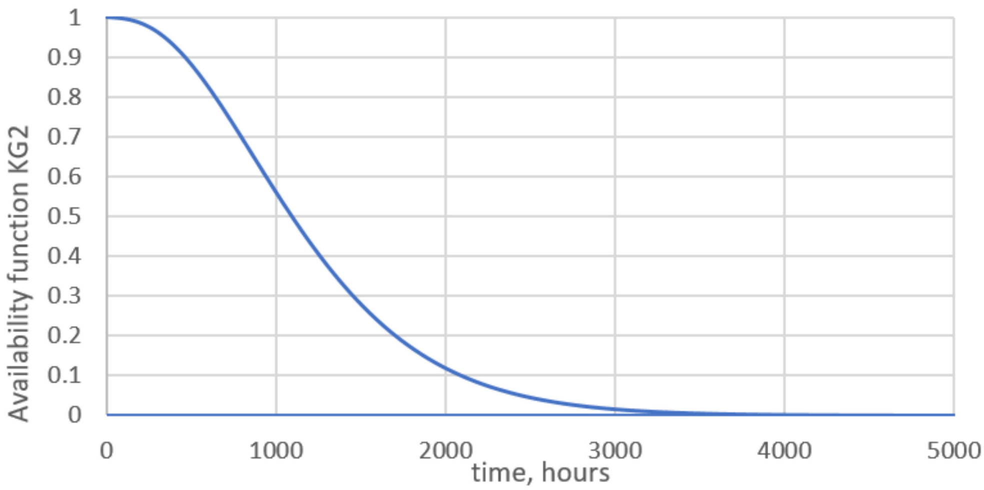

Figure 4. The availability function of the hierarchical ramified system in the availability condition k = 18.

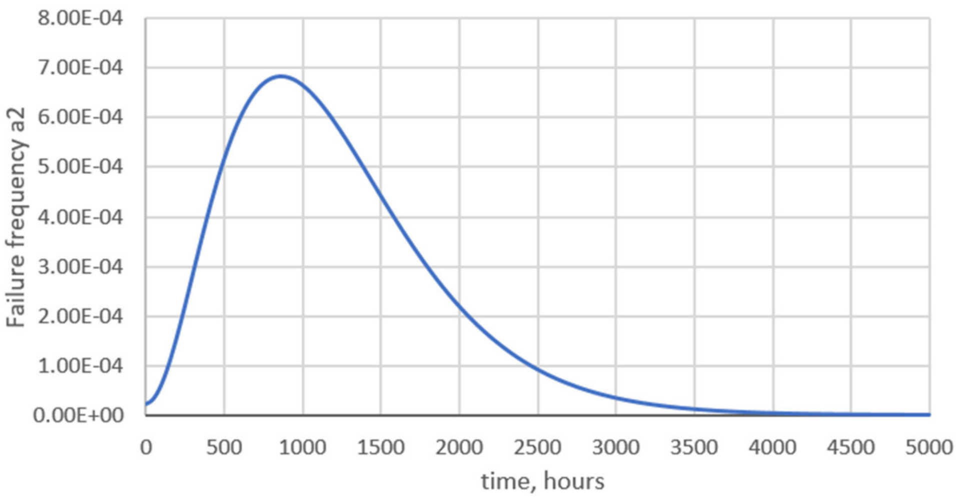

Figure 5. The failure frequency of the hierarchical ramified system in the availability condition k = 18.

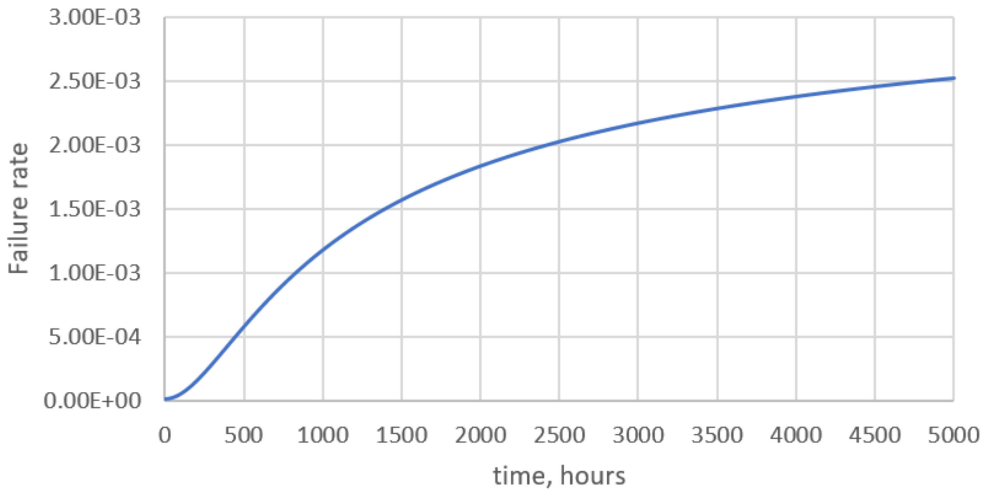

Figure 6. The failure rate of the hierarchical ramified system in the availability condition k = 18.

For validation of the results, we take into consideration conditions (1) and the conditions below.

$$a_2^{(1)} + a_2^{(2)} = 20. \tag{22}$$

The choice of the ramification coefficients $a_2^{(1)}$, $a_2^{(2)}$, which ensure the highest reliability of the system under the conditions (1) and (22).

The decision about an optimal variant of placement of 20 workstations under the conditions (1) and (22) should be made on the basis of the following reliability characteristics of the system: the availability function, the failure frequency in the prescribed availability condition, the failure rate in the prescribed availability condition, and the duration of the system's stay in the prescribed availability condition. As a result of the calculations, it appears that, under the condition $0 \leq t \leq 5000$ h, the availability function, the failure frequency, and the failure rate in the prescribed availability condition do not depend on the variant of placement that responds to the conditions (1) and (22).

The average duration of the system's stay in the state of x_2 working output elements of the system under the condition $8 \leq x_2 \leq 20$ does not depend on the variant of placement responding to the conditions (1) and (22).

All the 20 output elements of the system will operate on the average for 242.28 h from the beginning of the system's operation. Afterward, one of the elements will fail and 19 output elements will operate on an average of 475.27 h. After failure of the second output element, 18 output elements will operate, on average, for 708.79 h.

In Figures 7–10, the fragments of histograms of the system's operation in the course of time under different variants of placement that satisfy the conditions (1) and (22) are presented. From the initial moment of time (t = 0) and up to t = 3000 h—the value is constant, that is why it is not shown in Figures 7–10 (for better visualization). These figures demonstrate that, under the condition $1 \leq x_2 \leq 7$, a maximum value of a moment of time, until which no less than x_2 output elements operate, is reached when $a_2^{(1)} = 4$, $a_2^{(2)} = 16$. Thus, the optimal variant of placement of output elements of the system is connecting four workstations to the basic hub and 16 workstations to the second hub.

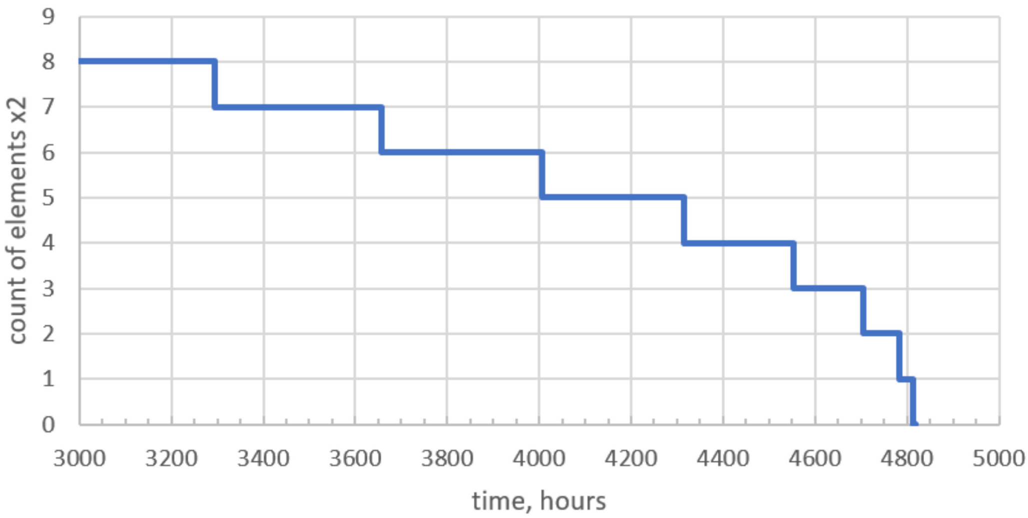

Figure 7. A fragment of a histogram of the hierarchical ramified system's operation in the course of time for $a_2^{(1)} = 4$, $a_2^{(2)} = 16$.

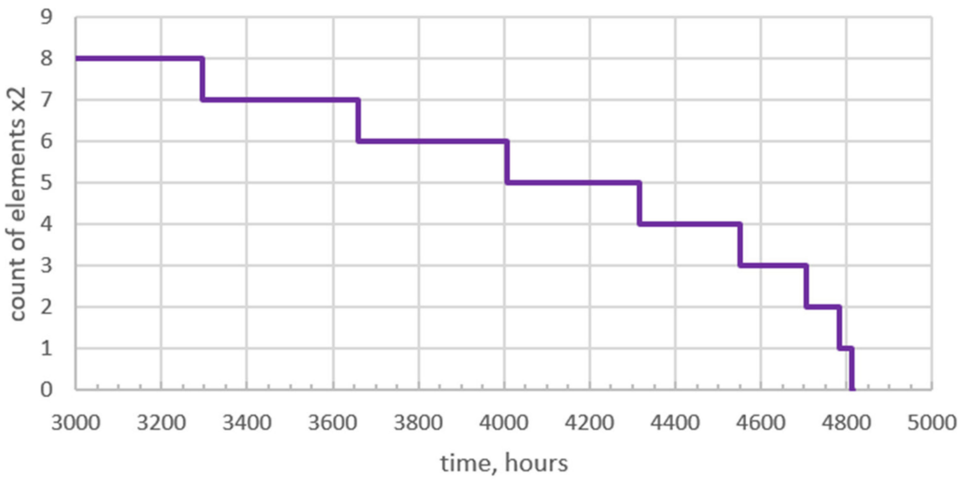

Figure 8. A fragment of a histogram of the hierarchical ramified system's operation in the course of time for $a_2^{(1)} = 5$, $a_2^{(2)} = 15$.

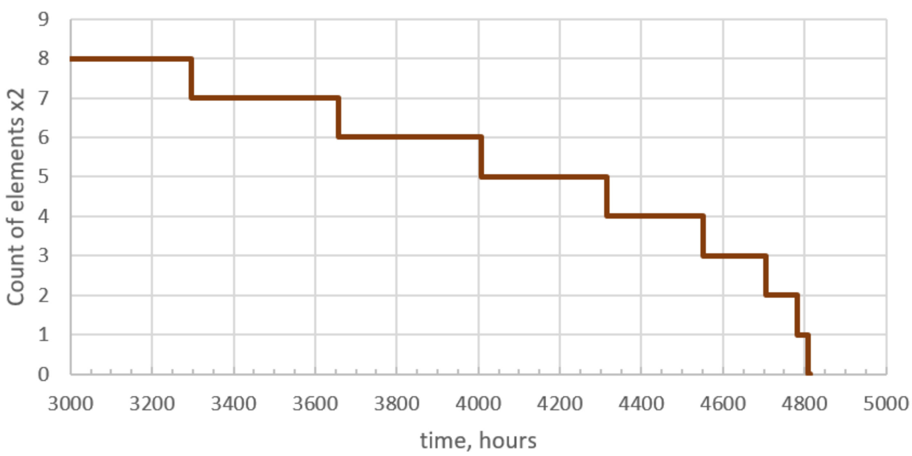

Figure 9. A fragment of a histogram of the hierarchical ramified system's operation in the course of time for $a_2^{(1)} = 6$, $a_2^{(2)} = 14$.

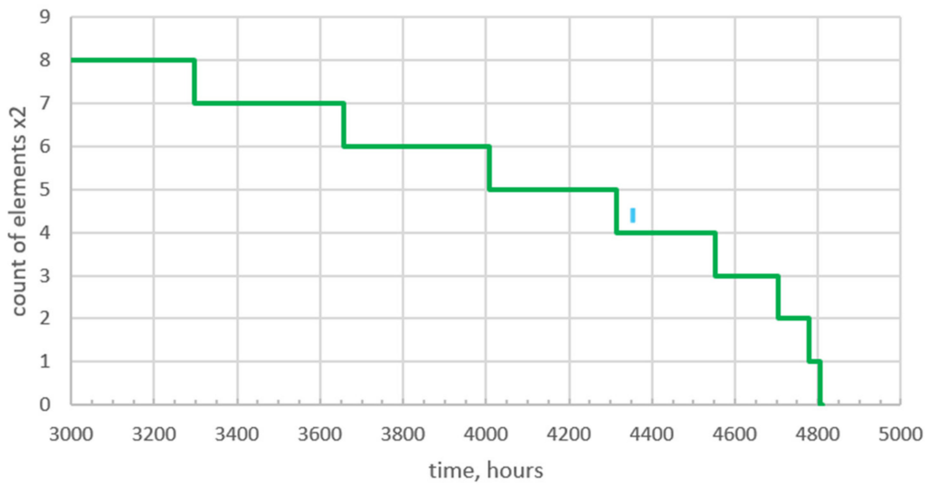

Figure 10. A fragment of a histogram of the hierarchical ramified system's operation in the course of time for $a_2^{(1)} = 7$, $a_2^{(2)} = 13$.

The analysis of the obtained results shows that, on average, they will remain in the system:

- x_2 = 6 operating output elements at t = 3658 h (if $a_2^{(1)} = 4$, $a_2^{(2)} = 16$), t = 3658 h ($a_2^{(1)} = 5$, $a_2^{(2)} = 15$), t = 3658 h ($a_2^{(1)} = 6$, $a_2^{(2)} = 14$), t = 3658 h ($a_2^{(1)} = 7$, $a_2^{(2)} = 13$);
- x_2 = 5 operating output elements at t = 4315 h ($a_2^{(1)} = 4$, $a_2^{(2)} = 16$), t = 4315 h ($a_2^{(1)} = 5$, $a_2^{(2)} = 15$), t = 4315 h ($a_2^{(1)} = 6$, $a_2^{(2)} = 14$), t = 4315 h ($a_2^{(1)} = 7$, $a_2^{(2)} = 13$);
- x_2 = 4 operating output elements at t = 4552 h ($a_2^{(1)} = 4$, $a_2^{(2)} = 16$), t = 4552 h ($a_2^{(1)} = 5$, $a_2^{(2)} = 15$), t = 4552 h ($a_2^{(1)} = 6$, $a_2^{(2)} = 14$), t = 4552 h ($a_2^{(1)} = 7$, $a_2^{(2)} = 13$);
- x_2 = 3 operating output elements at t = 4705 h ($a_2^{(1)} = 4$, $a_2^{(2)} = 16$), t = 4705 h ($a_2^{(1)} = 5$, $a_2^{(2)} = 15$), t = 4705 h ($a_2^{(1)} = 6$, $a_2^{(2)} = 14$), t = 4704 h ($a_2^{(1)} = 7$, $a_2^{(2)} = 13$);
- x_2 = 2 operating output elements at t = 4783 h ($a_2^{(1)} = 4$, $a_2^{(2)} = 16$), t = 4783 h ($a_2^{(1)} = 5$, $a_2^{(2)} = 15$), t = 4782 h ($a_2^{(1)} = 6$, $a_2^{(2)} = 14$), t = 4780 h ($a_2^{(1)} = 7$, $a_2^{(2)} = 13$);
- x_2 = 1 operating output element at t = 4812 h ($a_2^{(1)} = 4$, $a_2^{(2)} = 16$), t = 4811 h ($a_2^{(1)} = 5$, $a_2^{(2)} = 15$), t = 4809 h ($a_2^{(1)} = 6$, $a_2^{(2)} = 14$), t = 4806 h ($a_2^{(1)} = 7$, $a_2^{(2)} = 13$);
- x_2 = 0 operating output elements at t = 4818 h ($a_2^{(1)} = 4$, $a_2^{(2)} = 16$), t = 4816 h ($a_2^{(1)} = 5$, $a_2^{(2)} = 15$), t = 4814 h ($a_2^{(1)} = 6$, $a_2^{(2)} = 14$), t = 4811 h ($a_2^{(1)} = 7$, $a_2^{(2)} = 13$).

Thus, no later than in 4818 h, on average, the system will fail at $a_2^{(1)} = 4$, $a_2^{(2)} = 16$.

9. Conclusions

A technical equipment of a local area computer network, whose main blocks are a server, hubs, workstations, and communication lines, is considered as a hierarchical ramified system with aging elements. In order to simplify the calculations of reliability characteristics, this system is represented in contracted form as a hierarchical system with the bypath through one level, ramified to level 2.

Models in the form of expressions are constructed for calculations of the following:

- probabilistic reliability characteristics of the system (the probability distribution of a count of operating output elements, the availability function);
- time reliability characteristics of the system (the duration of the system's stay in each of its working states, the duration of the system's stay in the prescribed availability condition);
- conventional reliability characteristics specified by standards for unrestorable systems (the failure probability in the prescribed availability condition, the failure frequency

in the prescribed availability condition, and the failure rate in the prescribed availability condition).

The results of calculations of reliability characteristics are presented for the case of a technical equipment of a local area computer network.

Without the use of reliability characteristics, it is impossible to settle a number of problems of systems' design and operation, such as a selection of structure and rational redundancy, an organization of inspection monitoring, and preventive maintenance. It is necessary to work out the methods of reliability prognostication with regard to systems' specific features, such as the possibility of structure rearrangement, and preservation of serviceability in case of partial failures at the expense of structural redundancy.

The analytical models, determining reliability parameters with high accuracy, have been developed for modern hierarchical systems widely used in technology. These models are based on the use of the Weibull distribution. The accuracy of the developed analytical models is determined by the errors of the input data. In addition, the proposed models, due to the regularity of such hierarchical systems, can help automate the process of their synthesis and can be used to explore the parameters of large systems. The study presented in the article can be generalized for hierarchical systems with more levels and elements at each level.

Thus, the model for determining the reliability parameters of hierarchical, multilevel, branched systems has been developed and is presented in this paper. The proposed model uses a system of analytical expressions, which allows us to determine the reliability parameters of the studied object with high accuracy and can be used for large, multi-level, hierarchical, ramified systems.

10. Prospects for Future Research

The future research of this theme will be focused on the following. First, we will work on the development of the generalized algorithm for automatic forming of the optimal/quasi-optimal placement based on the input data. Second, we plan to introduce several parameters into the model, which allow us to take into account different types of cabling (not only twisted pair, but an optical fiber and coaxial cable) as well as different reliability indicators of hubs and workstation connections.

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