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Numerical Model of Current Flow and Thermal Phenomena in Lateral GaN/InGaN LEDs

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Abstract: GaN-based light-emitting diodes (LEDs) became one of the most widely used light sources. One of their key factors is power conversion efficiency; hence, a lot of effort is placed on research to improve this parameter, either experimentally or numerically. Standard approaches involve device-oriented or system-oriented methods. Combining them is possible only with the aid of compact, lumped parameter models. In the paper, we present a new electro-thermal model that covers all the complex opto-electro-thermal phenomena occurring within the operating LED. It is a simple and low computational cost solution that can be integrated with package- or system-oriented numerical analysis. It allows a parametric analysis of the diode structure and properties under steady-state operating conditions. Its usefulness has been proved by conducting simulations of a sample lateral GaN/InGaN LED with the aid of ANSYS software. The results presented illustrate the current density and temperature fields. They allow the identification of 'hot spots' resulting from the current crowding effect and can be used to optimise the structure.

Keywords: LED; electro-thermal model; GaN; energy transfer mechanisms; ANSYS

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1. Introduction

GaN-based light-emitting diodes (LEDs) are recognised as attractive solid-state lighting (SSL) sources due to their advantages compared to traditional light sources, such as lower electric power consumption, higher reliability, very long lifespan, and high colour rendering index. It causes intensive investigation of their design, technology, and features, carried out by both by industry and university research teams. The studies focus on different design concepts such as vertical [1–3], lateral [4,5] or 3D core-shell [6–9] devices, discussing their fabrication technologies [10–12], or aiming at their features employing experimental [13,14] or numerical [15–17] approaches.

SSL LEDs belong to electronic power devices, which means that the conversion efficiency of the delivered electric power into the output optical power is the basic factor for their design. Furthermore, they typically operate with maximum currents. It implies additional constrains of their design, besides those resulting from the one-dimensional physical phenomena taking place in any diodes, e.g., for signal processing applications. The inherent attribute consists in the presence of the lateral component of the current flow, which results from the fact that both the anode contact and the optical window for the light emission are located on the same surface of the diode structure. In general, the radiative recombination below an optical window gives the contribution to the emitted signal. The ohmic contact of the anode above the active region may prevent light escape; therefore, the size, shape, and properties of the contact are of crucial importance for the final efficiency of the diode [18,19]. The two-dimensional current distribution, and more precisely its nonuniformity, is another parameter that must be considered in the case of the power LEDs. It can lead to the disruption of current uniformity in quantum wells, which can result in a decrease in device performance, as well as to the generation of 'hot spots' due to the current crowding effect. The increase in local current density is accompanied by an increase in local Joule heat dissipation, which forces the maximum current reduction to keep the temperature inside the diode at a safe level.

Since both the power LED operating conditions and its reliability are very sensitive to the current distribution, a lot of effort has been devoted both to its evaluation and to its optimisation at the stage of the device design. The conducted studies cover two-dimensional approaches employing models limited to the current flow described by an appropriate resistance network or the Poisson Equation [19–22], as well as more complex models, based on the numerical solution of the physical drift-diffusion model of the semiconductor structure [17,23–25]. The first approach deals with the drift current flow in a 2D or 3D domain covering all the diode design layers and their properties but simplify or even neglect some phenomena taking place in the modelled domain, especially in the junctions. The second approach allows for a large extension omitting the above difficulties, but it leads to more complex, less user-friendly models. In addition, they require many physical parameters that are difficult to get, and very dense computational grids. They are very useful in the case of device-oriented simulations, where the insight in the design of the semiconductor device is necessary, e.g., to improve LED efficiency [24], to test new structure designs [15], or to introduce new quantum effects to the models [23].

On the other hand, package-oriented or system-oriented simulations are focused not mainly on electrical phenomena but also on thermal or fluidic-thermal phenomena that allow analysing the whole system operation under defined environmental conditions. In these cases, more complex finite element or finite volume methods are used to solve a set of partial differential equations [26,27], or alternative compact models [28–30] or their combination [31,32] can be applied. Here, a device is treated as a black box with defined properties, although a new compact model that includes a device-oriented approach has been proposed in the Delphi4LED project [33,34]. It is an interesting concept that is easy to integrate with package-oriented or system-oriented simulations.

In the paper, a new simple electrothermal model allowing investigations of current and heat transfer in the presence of thermal and optical energy exchange phenomena taking place in the structure of lateral GaN/InGaN LEDs is presented. It can be easily integrated with fluid-thermal system-oriented simulations. It allows the conduction of simultaneously package-oriented and device-oriented analysis. The work-out diode model was used in numerical investigations aimed at power management of the selected LED diode.

2. Model Description

The main goal of the model is to allow determination of the distributions of electric current as well as the heat generation and its transfer inside the semiconductor structure in the presence of photon generation and absorption. The LED presented in [5] has been taken as a sample diode for investigation. A model domain has been limited to a 2D cross-section taken along the current flow path marked with the white line (see Figure 1), and it covers both anode and cathode contact.

Although the model is designed to deal with complex opto-electro-thermal phenomena in the LED structure, its computing core is simple and based on the numerical solution of the Poisson Equation. In the research, simulations are conducted with the aid of the Thermal-Electric System available in the commercial package ANSYS Mechanical.

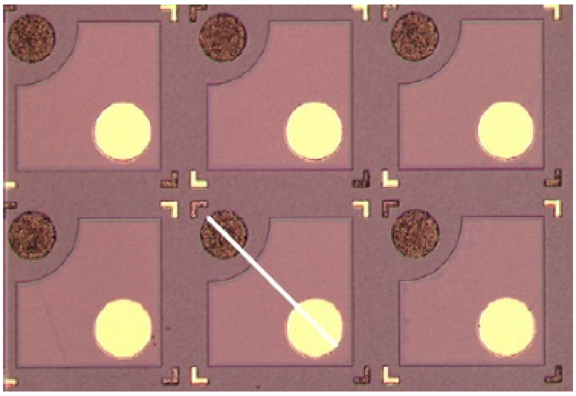


Figure 1. Photograph of several LED chips with a line pointing out the position of the 2D plain of the model (anode—light circle, cathode—dark circle) [5].

2.1. The Model Domain

The general concept of lateral GaN/InGaN LEDs is well-known, but its fabrication process can be different in the details. Since the model is a general one, it must be created for a defined device structure, whereas gathering the full set of construction data for a specific real structure is almost impossible. Therefore, a representative structure corresponding to a real planar GaN LED design for blue-light emission has been completed based on [2] with additional literature research [21,26–30]. The structure is shown in Figure 2 whereas its basic design data are collected in Table 1. It corresponds to a diode manufactured on a sapphire substrate using MOCVD (metal-organic chemical vapour deposition) technology and consists of a bulk GaN layer, a silicon-doped n-GaN layer, an active layer in a form of multi-quantum-well (MQW) region covering five pairs of 3 nm $In_xGa_{1-x}N$ wells and 10 nm GaN barriers, a magnesium doped p-GaN layer, and a transparent ITO layer. The electron density for the non-intentionally doped GaN layers has been fixed at the level of 5×10^{16}. The ohmic contacts for cathode and anode are formed on n-GaN and ITO layers, respectively.

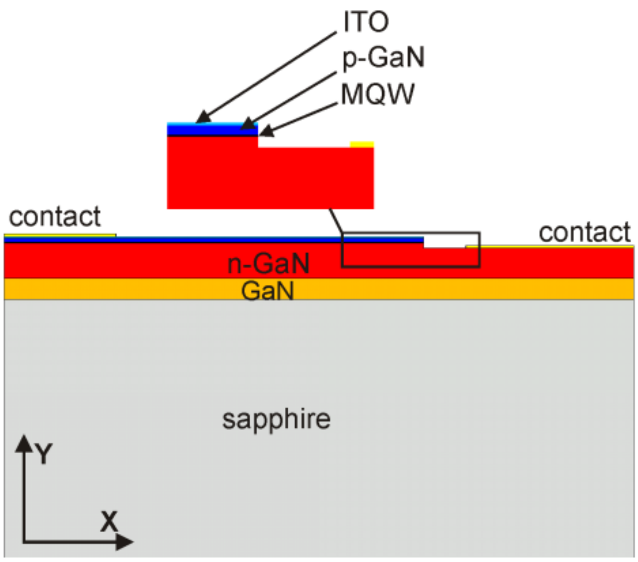


Figure 2. Domain of the model.

Table 1. Parameters and dimensions of the modelled preliminary planar GaN LED structure.

Layer	Thickness [nm]	Width [μm]	Doping/Carrier Concentration [cm^{-3}]	Resistivity [W·cm]	Absorption [cm^{-1}]	Thermal Conductivity [W/mK]
Contact Ni/Au (anode)	200	80	-		-	150
ITO	100	300	-	4×10^{-4}	1.17×10^{4}	11
p-GaN:Mg	300	300	8×10^{17}	2.6	800	120
MQW	65	300	5×10^{16}	17×10^{-2}	400	80
n-GaN:Si	350 2150	300 450	8×10^{18}	3.4×10^{-3}	800	120
GaN	1500	450	5×10^{16}	17×10^{-2}	400	120
Substrate (sapphire)	50,000	450	-		6×10^{-4}	24
Contact Ti/Au (cathode)	200	120	-		-	150

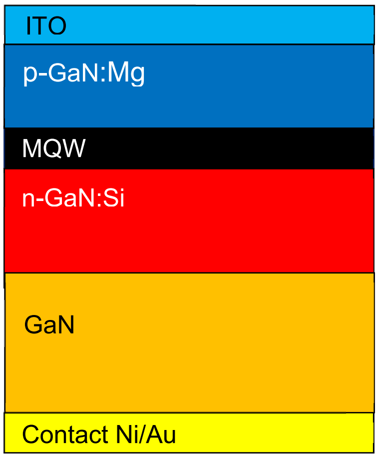
2.2. Current Flow Modelling

Two phenomena are responsible for the current flow in semiconductors; the presence of an electric field leads to the drift current, whereas the difference in carrier density leads to the diffusion current. When the current flows under a homogeneous layer in quasi-neutral conditions, one can assume that it is a pure drift one. Such conditions occur in all layers except the MQW region, where we assume full exchange of electrons and holes as the current carriers, which is acceptable in the considered LED structure. Therefore, it is allowed to determine the drift current distribution in these regions with the aid of Ohm's law described by Equation (1). It determines the current density J as a function of the electric field strength E or the electric potential V with the conductance σ as a material property. In the model, the necessary distribution of the electric potential is obtained by numerical solution of the Poisson Equation (2) with the use of ANSYS software.

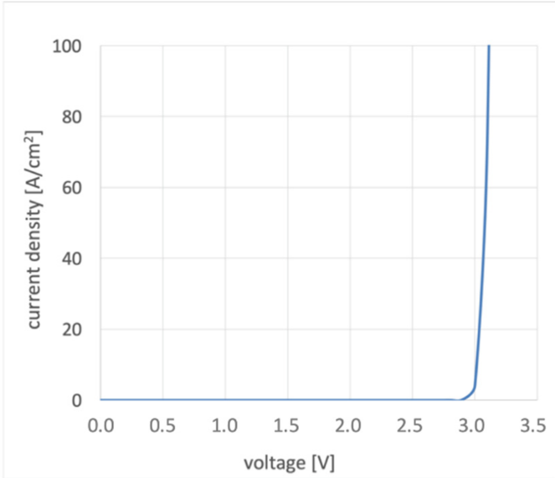
$$J = \sigma E = \sigma \cdot \nabla V \tag{1}$$

$$\nabla(\sigma \cdot \nabla V) = 0 \tag{2}$$

The MQW region requires a distinct approach. It acts as the low doped layer of p-ν-n diode under forward bias, which means that there are injected excess carriers, as well as that almost the entire area is in the junction space charge region (SCR). It implies that although the current parallel to the junction plane can be treated as drift only, the current perpendicular to this plane consists of both drift and diffusion components. It can be solved by introducing the diode equation into the model, as in the case of the resistor network model [23]. However, it is impossible in the case of the model using a numerical solution of the Poisson Equation (2). Therefore, we have employed another approach consisting in introducing an artificial anisotropic conductivity for the MQW region. The components $\sigma_{\|}$ and $\sigma_{\perp}$ represent the conductivity in the direction parallel and perpendicular to the junction plane, respectively. The parallel component represents the real conductivity of the region and allows the calculation of the parallel drift current. The perpendicular one covers the superposition of the drift and diffusion currents. To evaluate it, the I–V characteristic calculated with the aid of a physical drift-diffusion model created in SENTAURUS package has been used. Simulations have been conducted for a simple structure (see Figure 3a) extracted from the test one depicted in Figure 2. The thicknesses of the layers and their properties are consistent with the data gathered in Table 1. For more details, see [25]. The obtained I–V characteristic that allows the determination of the equivalent resistivity $\sigma_{\perp}$ for any current density magnitude is presented in Figure 3b.



(a)



(b)

Figure 3. Simple vertical structure (**a**) and its I–V characteristic (**b**).

2.3. Energy Transfer Modelling

In steady-state conditions, the energy delivered to a LED as a result of applying external voltage must leave it as the heat transferred from the places of its generation to the surrounding and as the photon flux emitted through the optical window. Both processes are crucially important for diode operation, and their modelling has been included in the developed model.

In solids, heat transfer is mainly connected to the conduction mechanism, and obeys the Fourier law (3a), where T denotes temperature, k is thermal conductivity, and Φ_q is heat flux. After some transformations, we can obtain Formula (3b), which describes the temperature distribution in the presence of the volume density of heat dissipation g(x,y,z). The numerical solution of Equation (3b) with the use of ANSYS software, together with the appropriate boundary conditions and the distribution of heat dissipation g, is the base for the heat transfer model within the considered LED structure (see Section 3).

$$\Phi_q(x,y,z) = -k \cdot \nabla T \tag{3a}$$

$$\nabla(k \cdot \nabla T) = -g(x,y,z) \tag{3b}$$

The proper evaluation of heat dissipation in any semiconductor structure is an intricate problem because of the complexity of energy exchange phenomena in semiconductors and requires the consideration of an energy balance in the structure. To evaluate the energy-transfer balance in a GaN LED, a physical model is necessary to describe the form of energy kept inside the semiconductor structures and the energy exchanged between them. An idea of such a model [32] is presented schematically in Figure 4. It covers a few separated energy stores as well as any possible ways of the energy transfer between them. According to the scheme, the energy transfer from the electric field to the phonons is indirect and consists of at least two steps. At first, the energy has to be taken by free carriers (electrons and holes), and in the next step, the carriers can give it back to phonons in different processes of energy exchange.

This means that to determine heat dissipation in any semiconductor device, the second stage processes such as recombination, relaxation, or contact effects must be taken into account only. The idea has been developed and verified for the p-i-n diode [33]. It was shown that in the considered diode, the power dissipated in recombination can be significantly larger than the delivered electrical power (UI, where U denotes external voltage and I is current). In the structure, both the energy transfer from carries to lattice (heating) and the energy transfer from lattice to carries (cooling) may occur. Similar effects have also been observed in LED structures [34].

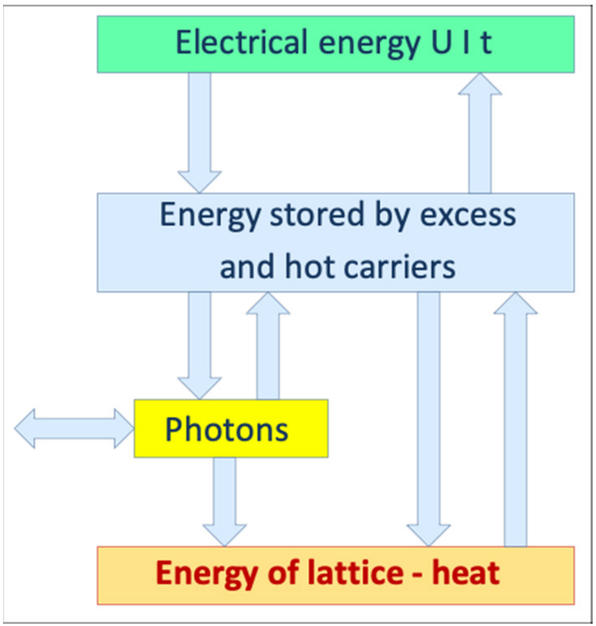


Figure 4. Simplified scheme of energy flow in semiconductor structure.

The simple vertical structure of the GaN LED diode presented in Figure 3a can be considered as some kind of p-ν-n structure, in which the layer ν is represented by the MQW layer. It allows, by analogy, to transfer the formulas describing the energy transfers in the p-ν-n diode to the energy transfers in the model for the GaN LED diode. The developed model takes into account optical phenomena such as photon absorption on electrons and photon generation in radiative recombination. Under the acceptable assumption that the current carrier exchange takes place in the MQW only, the radiative recombination dominates, and its contribution to the total recombination is determined by the internal quantum efficiency IQE. Hence, the model covers the following second-stage processes:

Ohmic losses (Joule losses) are described by the well-known formula:

$$Q_J = J^2 \cdot \rho \tag{4}$$

where J is current density and ρ—electrical resistivity.

Radiative recombination, as shown in Figure 5, takes place with the assumed IQE in the quantum wells where depth decides the colour of emitted light. It is a two-stage process:

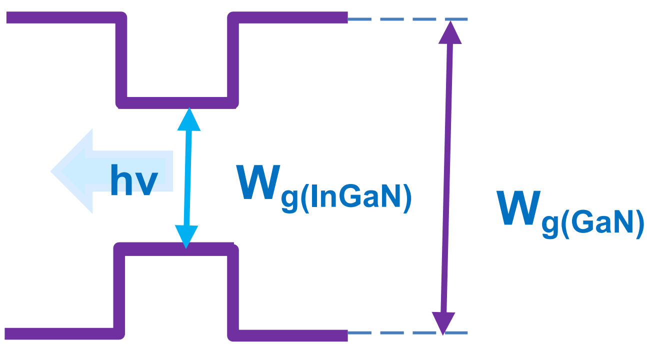


Figure 5. A band diagram of the quantum well structure.

RELAXATION (Q_{relax})—Electrons injected into the MQW layer lose part of their energy in the wells, which is dissipated as heat and can be described with the aid of the following formula:

$$Q_{relax} = \frac{1}{q} J \cdot \left(W_{g(GaN)} - W_{g(InGaN)} \right) \tag{5}$$

where $W_{g(GaN)}$ and $W_{g(InGaN)}$ are the energy band-gaps of the GaN barrier and the quantum well, respectively.

LIGHT EMISSION (Q_{rad})—covers any recombination acts when the recombining electron generates a photon; hence:

$$Q_{rad} = IQE \frac{1}{q} J \cdot W_{g(InGaN)} \tag{6}$$

PHONON EMISSION (Q_{phonon})—covers any recombination, when the energy of the recombining electron is finally taken by phonons:

$$Q_{phonon} = (1 - IQE) \frac{1}{q} J \cdot W_{g(InGaN)} \tag{7}$$

Contact losses can be assigned to any interface where carriers are forced to change their energy level. In the case of p-ν-n diode considered in [33], heat absorption/generation was assigned to particular contacts, as shown in Figure 6.

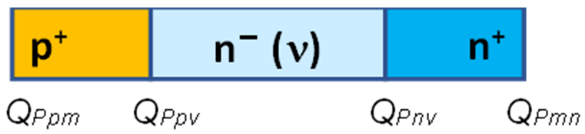


Figure 6. Contact losses in p-ν-n diode.

In the case of GaN LED, when the current carrier exchange takes place in the MQWs only, the currents in p+ and n+ layers are pure majority ones. Therefore, the losses at the metal-semiconductor contacts can be described with the aid of Formula (8):

$$Q_{Pmp} = -\frac{1}{q} J \cdot (W_F - W_V) \qquad Q_{Pmn} = \frac{1}{q} J \cdot (W_C - W_F) \tag{8}$$

where W_C, W_V, and W_F are conduction band edge, valance band edge, and Fermi level, respectively.

Regarding the junction losses, in the case of the p-ν-n diode, the two junctions are so far from each other that separate SCRs are present, as shown in Figure 7a. Therefore, the introduction of two contact heat sources, Q_{Ppv} and Q_{Pnv}, is justified. In LEDs, the width of the MQW area is below 1 µm, which results in an overlap of the SCRs of particular junctions. To check it, additional simulations using SENTAURUS package have been performed. The tested p-i-n diode corresponds to the structure shown in Figure 3a. In the first two cases, it was a simple p-ν-n diode with a 5 µm and 50 nm thick ν-layer, whereas in the third case it was a p-MQW-n diode with a 50 nm MQW layer covering 5 pairs of quantum wells. The results in the form of the electric field and the potential distribution are depicted in Figure 7. In the case of p-ν-n diode with the 5 µm thick ν-layer, two separate potential steps are visible and they determine the contact losses Q_{Ppv} and Q_{Pnv}, respectively. In the case of p-ν-n diode with the 50 nm thick ν-layer and the p-MQW-n diode with the 50 nm thick MQW layer, the SCRs overlap. The presence of wells is only noticeable in the electric field curve, while the potential changes are smooth and the potential drop ΔV over the entire MQW layer occurs. It corresponds to the potential drop in an ordinary p-n junction, which allows us to assume that the heat dissipation in the MQW is described by the formulas:

$$Q_{P(MQW)} = -J \cdot \Delta V \quad (9)$$

Absorption losses are connected with the partial absorbance of the photon flux during its transfer across a semi-transparent solid body. The phenomenon is described by the formula:

$$\Phi_{Ph}(x) = \Phi_{Ph}(0) \cdot exp(-\alpha x) \quad (10)$$

where α is an absorption coefficient of the layer, and $\Phi_{Ph}(x)$ is a photon flux at the distance x from the surface or edge entry to the given layer. Moreover, it is assumed that the light propagates perpendicularly to the MQW layer only, and that there is an ideal mirror at the bottom of the structure, reflecting the photons.

(**a**) (**b**)

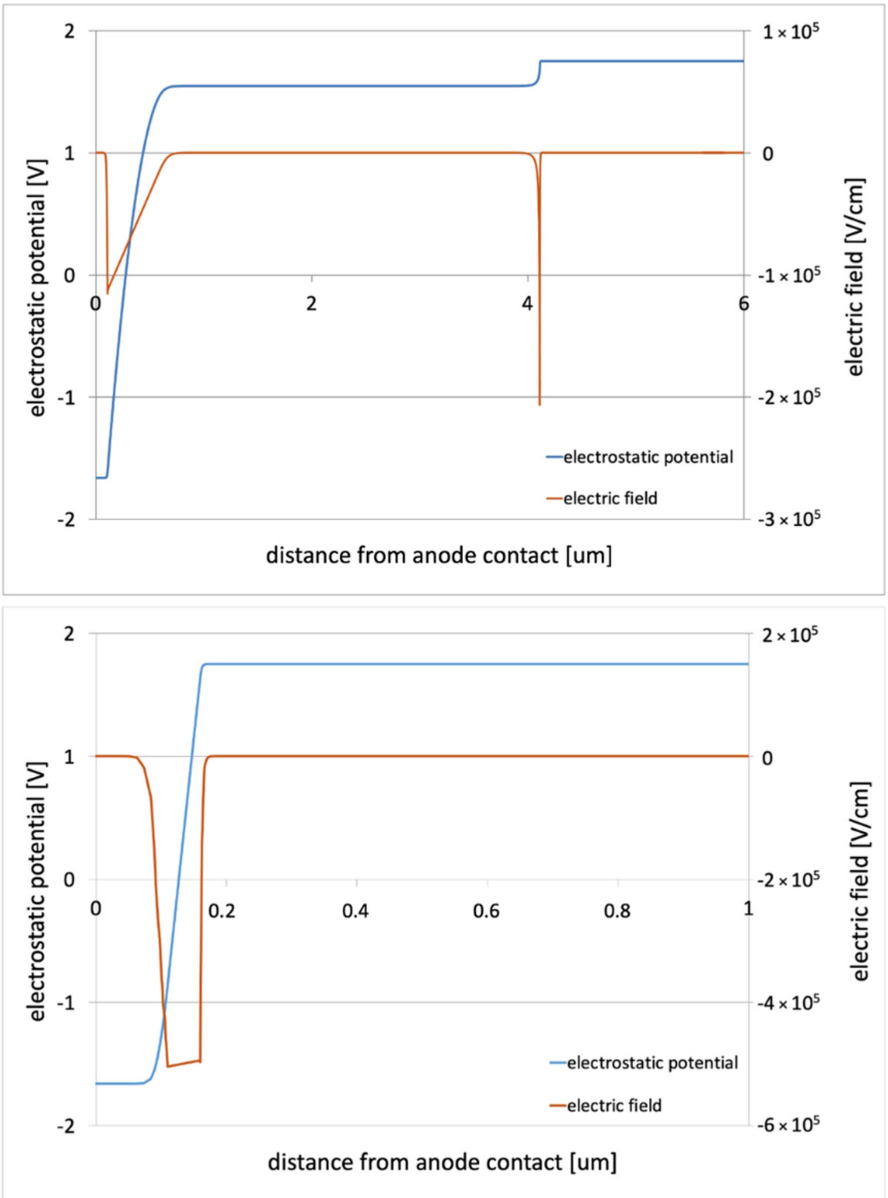


Figure 7. *Cont.*

(c)

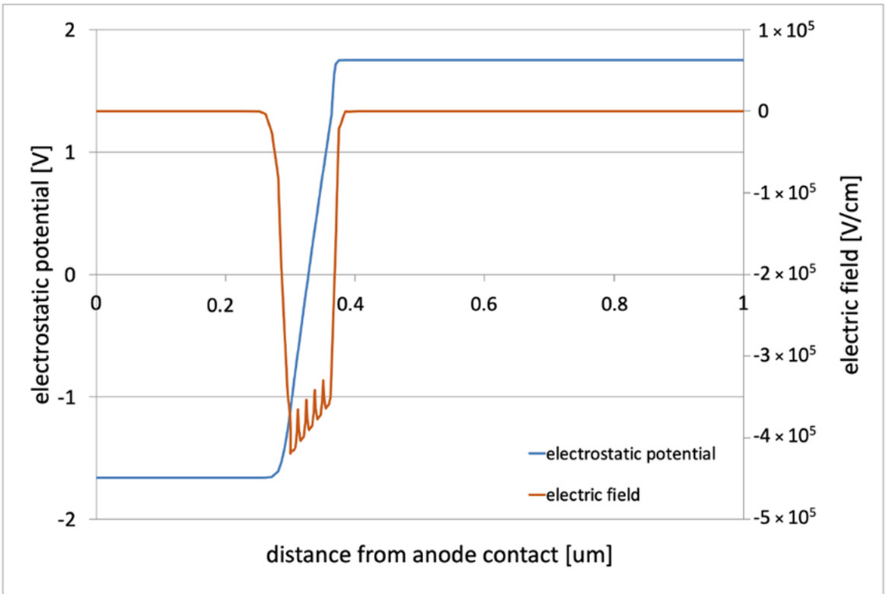


Figure 7. Potential and electric field distribution in the tested diodes: wide and narrow p-i-n diodes and in the LED: (**a**) p-ν-n with 5 µm wide ν-layer, (**b**) p-ν-n with 50 nm wide ν-layer, and (**c**) LED with 50 nm wide MQW-layer.

3. Simulations of the GaN LED Structure

3.1. Preliminary Calculations

To present the usefulness of the developed energy transfer model for a GaN LED, calculations have been performed for the simple vertical structure depicted in Figure 3. The following assumptions have been made:

- photon emission takes place in MQW with IQE = 1 (Wg = 2.79 eV, λ = 428 nm);
- the photons propagate perpendicularly to the MQW layer only as shown in Figure 8;
- the bottom contact is an ideal mirror for the photon flux;
- contact losses are included;
- forward current equals 35 A/cm^2.

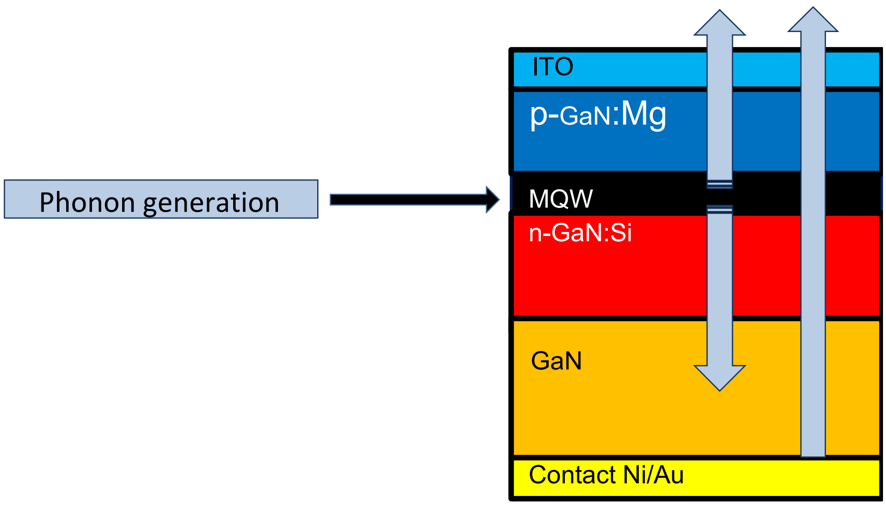


Figure 8. Photon propagation inside the vertical LED structure.

The results of performed calculations are summarised in Table 2. They have been completed for each layer of GaN LED structure and covered the evaluation of each layer's resistivity and absorption coefficients, as well as the resulting energy exchange in the particular processes. The red color represents the energy exchange resulting in an increase of phonon energy (heating), and the blue one represents the energy of photons generated in the MQW layer, which is taken by the photon leaving the structure as the blue radiation. The general conclusions resulting from the data are as follows:

- the ohmic losses are negligible in the semiconductor chip;
- the mirror presence (e.g., metallic layer or distributed Bragg reflector [35]) can enlarge the efficiency by about 59%;
- a large magnitude of emitted energy may result in a negative energy balance inside the semiconductor chip;
- due to high doping level, the contact losses at metal-semiconductor contacts are small or even disappear in the case of degenerate semiconductors;

Table 2. Evaluation of energy exchange at contacts and in processes of radiative recombination, photon absorption, and ohmic losses that take place in particular layers of the considered GaN LED structure.

Layer	Thickness [nm]	Resistivity [Ω·cm]	Absorption [cm^{-1}]	Losses [W/cm^2]			
				Ohmic	Absorption	Relaxation	Total
p-contact	0	nondegenerate p-layer		$Q_{Pmp} = -3.54\ 10^{-3}$ W/cm^2			
ITO	100	4×10^{-4}	1.17×10^4	4.90×10^{-6}	8.39	-	8.39
p-GaN:Mg	300	2.6	800	9.56×10^{-2}	1.85	-	1.95
MQW	65			-	-	21.00	21.00
				-	−97.65	-	−97.65
MQW-contact				$Q_{P(MQW)} = -0.35$ W/cm^2			
n-GaN:Si	2500	3.4×10^{-3}	400	1.04×10^{-3}	15.28	-	15.28
n-contact	0	degenerate n-layer		$Q_{Pmp} = 0$ W/cm^2			
GaN	1500	17×10^{-2}	400	3.12×10^{-2}	4.52	-	4.55
Contact Ti/Au (cathode)	200	10^{-6} Ω·cm^2	-	1.23×10^{-3}	-	-	1.23×10^{-3}

3.2. Numerical Analysis

The presented above approach makes multi-dimensional investigation of any GaN LED complex structures easy with reduced computational power in comparison to full device-oriented simulations. It can support research aimed at the development and optimisation of new device designs with respect to temperature and current distribution. The usefulness of the developed model is presented below. Numerical analysis has been performed with the aid of ANSYS software for the sample GaN LED structure shown in Figure 2 with the necessary data gathered in Table 1. The mesh contains about 55,000 elements, and it was subjected to sensitivity tests. A series of simulations has been carried out under the following assumption and boundary conditions:

- the temperature at the bottom of the sapphire substrate is constant and equals 29 °C;
- voltage applied across the LED (forward biased) changes in the range 3.1–4.9 V, resulting in the current density (at ITO) changes in the range 28 to 278 A/cm^2 (corresponding current in the range 25 mA to 250 mA);
- absorption, relaxation (IQE = 80%), and contact losses are evaluated for each layer as presented in Section 3.1 and assigned as volumetric heat generation;
- ohmic losses are calculated with the aid of electrical simulations using Equation (2).

Figures 9 and 10 present the 2D distributions of the current density and the corresponding temperature field inside the modelled diode obtained for 3.10 V (28 A/cm^2), 3.62 V (98 A/cm^2), and 4.87 V (278 A/cm^2), respectively. To make the figures more legible, the vertical dimension is scaled 40 times with respect to the horizontal one. Furthermore, almost the entire sapphire layer is not illustrated in figures (only a thin navy blue layer at the bottom of the current density maps). Current crowding is observed in the ITO layer, as well as at the mesa edge, and in the thin n-Gan layer. In this geometry, the temperature field is mainly affected by the phenomenon located near the edge of the anode contact, where the current flows into ITO and spreads. The higher the current, the higher the maximum temperature and the more nonuniform the temperature field are.

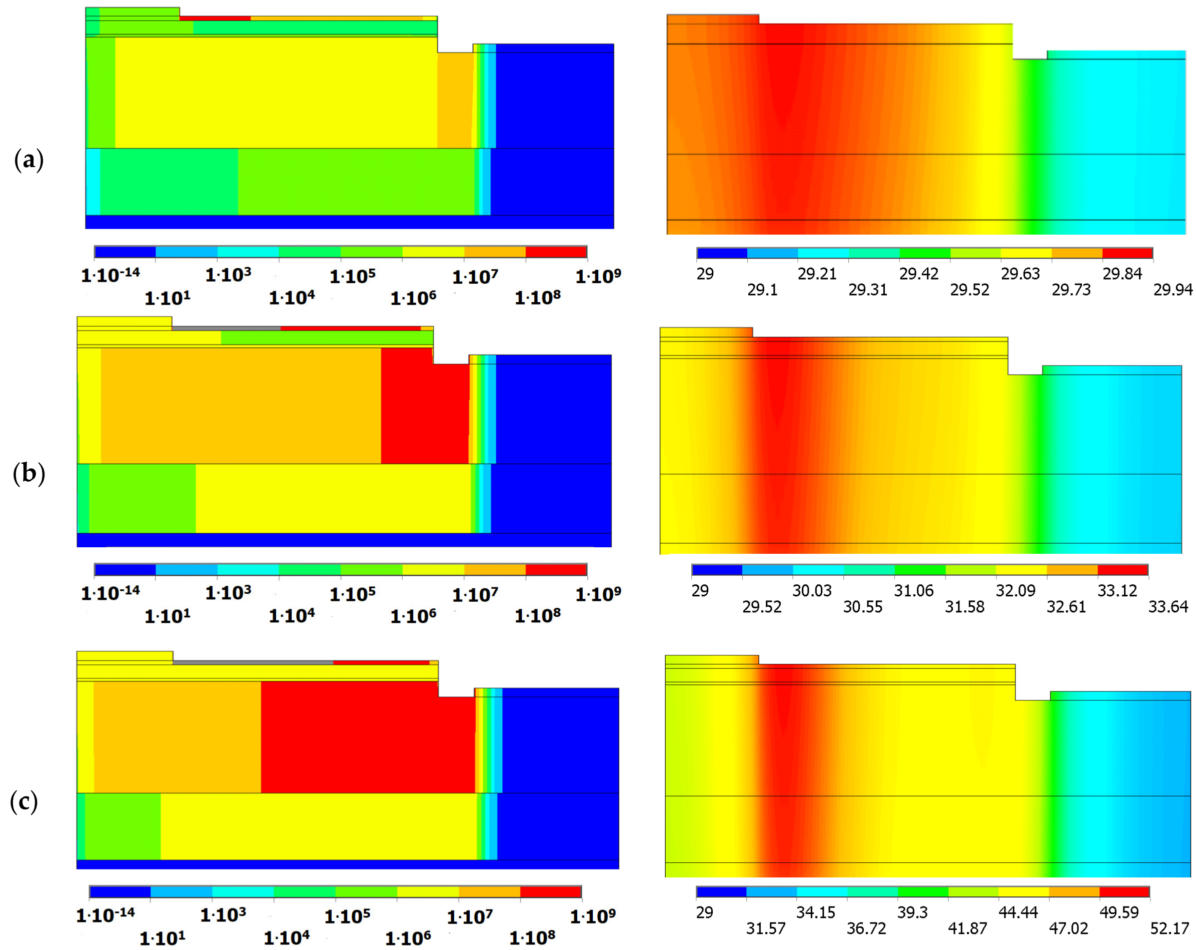


Figure 9. Current density (**left**) and temperature (**right**) distributions within the modelled preliminary structure for a forward bias of (**a**) 3.10 V, (**b**) 3.62 V, and (**c**) 4.87 V.

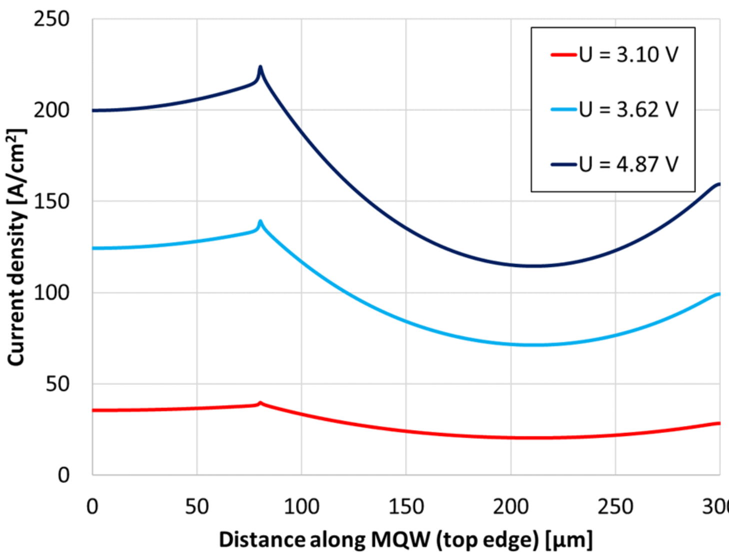


Figure 10. Current density distribution along the top edge of the MQW layer for three values of forward bias.

Better insight in the operation of the considered diode structure can be obtained from the current density distributions along the top edge of the MQW layer shown in Figure 10. The current crowding corresponding to the contact edge is visible at a distance of approximately 80 µm. Furthermore, the current density under the anode contact, which is opaque for generated photons, is larger than that under the optical window. The phenomenon is highly dependent on the applied voltage and the resulting current values.

The results presented above illustrate the spatial variability of the phenomena in the representative structure described by the data in Table 1. It gives insight into device-oriented simulations during package- or system-oriented simulations, which is the basic advantage of the model. It can also be used to optimise the structure design by the analysing of phenomena runs resulting from the changes of the chosen parameters. An example of such simulations is shown in Figure 11, which illustrates the influence of the thickness of the n-GaN:Si layer on the current and temperature distributions within the analysed domain. The series of simulations has been conducted for a similar set of assumptions and boundary conditions, except the change in n-GaN: Si thickness (*d*) and the applied forward bias voltage of 3.62 V and 3.78 V and 4.13 V for *d* of 2.15 µm, 1.5 µm, and 1.0 µm, respectively. The current density at the ITO layer has been kept at a constant value of 98 A/cm^2. The influence of considered changes inside the diode structure is easily noticeable. The current crowding and at the mesa edge and its growing influence on the temperature field are depicted on the colour maps in Figure 11 and in the characteristics taken along the top edge of the MQW layer for the current density and temperature shown in Figure 12. The maximum temperature is shifted from the point closer to the anode contact edge towards the mesa edge.

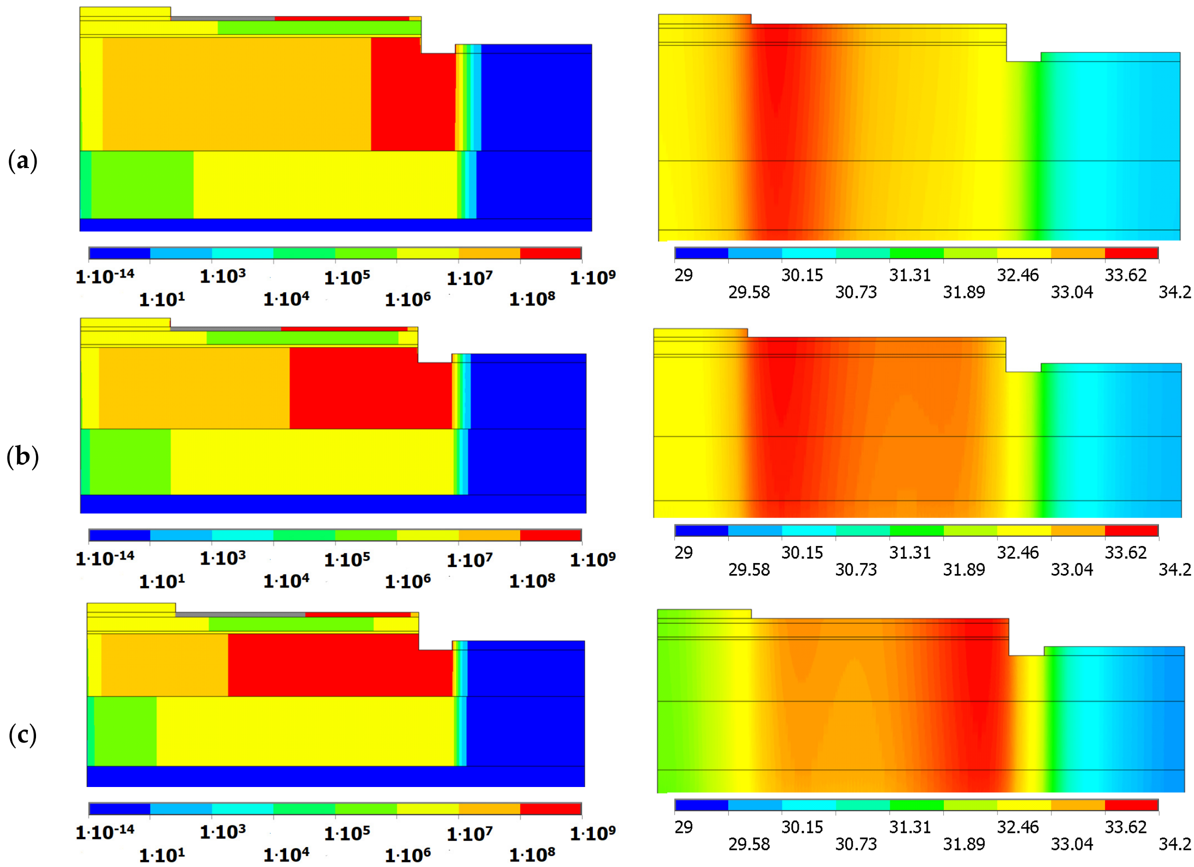


Figure 11. Results illustrating the influence of the thickness of the n-GaN:Si layer. Current density (**left**) and temperature (**right**) distributions within the modified modelled structure for a forward bias of: (**a**) 3.62 V (d = 2.15 µm), (**b**) 3.78 V (d = 1.5 µm), and (**c**) 4.13 V (d = 1.0 µm).

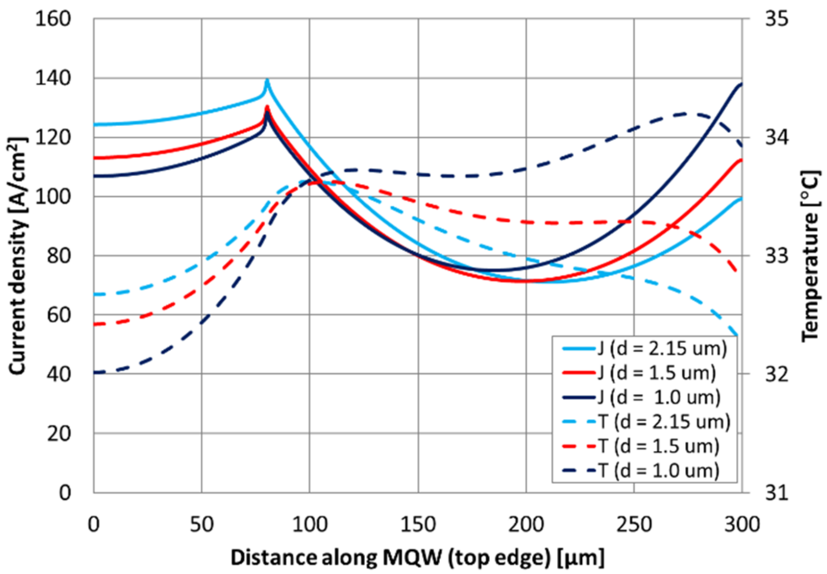


Figure 12. Current density distribution (continuous lines) and temperature distribution (dashed lines) along the top edge of the MQW layer for three thickness values of the n-GaN:Si layer.

4. Discussion

One of the most widely investigated problems in the design of LEDs is current crowding and its influence on diode efficiency, reliability, and lifespan. On the one hand, research [3,18–20,36] focuses on the top contact shapes that ensure a more uniform current spread. On the other hand, studies concentrate on reducing the current crowding effect by improving the geometry [11,17] or properties [15,21] of transparent conductive layers. In [36,37], additional graphene interlayers are considered to improve lateral current flow for better distribution, while entering the active region. The dimensions of n-GaN layers are considered to be more critical in the case of 3D structures [6,17]. However, they might be interesting in lateral diode designs as presented in the paper. The developed electro-thermal model of lateral LED considers all the phenomena taking place in the devices that influence current density and temperature fields. It can be used to perform parametric analysis of the diode structure and to investigate the influence of different material properties on the operation of the tested device.

The results obtained from the conducted analysis agree well with the data in the literature. Although preliminary calculations for the simple vertical diode shown in Figure 3 suggested that ohmic losses might be of negligible importance, the simulations carried out for the lateral LED proved that the situation is the opposite. Joule losses can increase from several percent of total heat dissipation for low current densities of 25 A/cm^2 to over 50% for current densities of 250 A/cm^2. The current crowding effect is observed not only in the transparent conductive layer but also in GaN areas, where local contractions are present. Furthermore, cooling of the local junction is possible and in agreement with other studies [38].

5. Conclusions

The LED solid-state lighting sources belong to the power electronic devices that operate at maximum currents, and the conversion efficiency of electrical power into optical power is one of the basic factors of their performance. The developed electro-thermal model of lateral LED takes into account all the complex opto-electro-thermal phenomena occurring in the operating devices, which influence current density and temperature fields in the structure. It does not use a physical drift-diffusion model but only the Poisson Equation for the drift component, accompanied by a simplified version of the law of energy conservation, hence it requires coarser mesh and is characterised by lower computational power. It allows us to conduct a precise multidimensional investigation of any GaN LED complex structure.

Its huge benefit, available mainly for compact models, is an easy integration of the model as the device-oriented approach with the package-oriented simulations.

As presented in the paper, it can be used to perform a parametric analysis of the diode structure as well as to investigate the influence of different material properties on the operation of the tested device. It allows one to identify 'hot spots' resulting from the current crowding effect. As presented here, the phenomena may appear within different areas of the tested structure, and they are highly dependent on the total current values. It can support research aimed at the development and optimisation of new device designs focusing on steady-state operating conditions. The usefulness of the model has been proved by its application to numerical investigations of the representative GaN LED structure. They covered both the investigations of special current distribution and temperature distribution for typical steady-state diode operation as well as the influence of one of the geometrical parameters on current density and temperature fields.

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