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Anti-Disturbance Fault-Tolerant Sliding Mode Control for Systems with Unknown Faults and Disturbances

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Abstract: In this paper, a novel control algorithm with the capacity of fault tolerance and anti-disturbance is discussed for the systems subjected to actuator faults and mismatched disturbances. The fault diagnosis observer (FDO) and the disturbance observer (DO) are successively designed to estimate the dynamics of unknown faults and disturbances. Furthermore, with the help of the observed information, a sliding surface and the corresponding sliding mode controller are proposed to compensate the actuator faults and eliminate the impact of mismatched disturbances simultaneously. Meanwhile, the convex optimization algorithm is discussed to guarantee the stability of the controlled system. The favorable anti-disturbance and fault-tolerant results can also be proved. Finally, the validity of the algorithm is certified by the simulation results for typical unmanned aerial vehicles (UAV) systems.

Keywords: fault-tolerant control; anti-disturbance control; sliding mode control; fault diagnosis observer; disturbance observer

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1. Introduction

It is well known that nearly all control systems are affected by disturbances or uncertainties, which makes it difficult to accurately analyze and control them [1]. Most cases even lead to a prominent drop in the dynamical performance of the system. Therefore, it is very important to discuss effective anti-disturbance control algorithms. In past literature, a large quantity of remarkable control methods, such as H_2/H_∞ control [2,3], active disturbance rejection control (ADRC) [4,5], output regulator theory [6], back-stepping control [7] and robust control [8], have received significant attention in the field of control theory. However, most existing results only relate to the disturbance attenuation problem and cannot directly compensate unknown disturbances [9]. Based on this, disturbance observer-based control (DOBC), as a popular active anti-disturbance method, has been applied in many engineering practices [10–12]. The merit of the DOBC method is that the baseline controller can retain the nominal performance without changing its configuration [13]. The suppression of matching disturbances can be effectively handled by the design of a feedback controller which is based on disturbance observers. However, in practical applications, not every disturbance or uncertainty of the controlled system can strictly meet the matching conditions [14]. For the processing of mismatching disturbances, sliding mode control (SMC) is an effective robust control method. The SMC method is recognized as a powerful methodology, which can manage compensations to disturbances and faults in control systems [15–18]. To date, lots of SMC methods have been studied, such as adaptive SMC methods [19], terminal SMC methods [20] and integral SMC methods [21]. The authors of [22] proposed an SMC method based on a disturbance observer (DO) for uncertain systems with mismatched disturbances. In [23], SMC based on nonlinear DO is designed to suppress mismatched disturbances and reduce chattering. Recently, there have been some novel results related to sliding mode control of second-order systems. In [24], by designing an extended multiple sliding surface, an effective algorithm is proposed

for a system with matched/unmatched uncertainties. For further improving the robustness, [25] addresses a dynamic SMC theory for over-actuated autonomous underwater vehicle (AUV) systems. In [26], a robust SMC method is considered to ensure stability and better performance of a hovering AUV subject to model uncertainties and ocean current disturbance. These proposed SMC methods have potential value in dealing with those uncertain parameters and disturbances that exist in controlled systems.

In the industrial information society, faults of any link in the complex control system will cause immeasurable losses to the entire system [27]. At the same time, it is difficult to monitor the faults in real time, which will increase the operating cost of the control system and reduce the operating efficiency of the system [28]. Therefore, fault diagnosis (FD) and fault-tolerant control (FTC) technologies are proposed to reduce the various impacts caused by faults and maintain the stable operation of the system. Since scholars realized the practical application value of FD and FTC, theoretical research has been developed and fruitful results have been obtained [29–36]. In [29], a spacecraft attitude tracking control method based on a new adaptive integral terminal sliding mode fault-tolerant control strategy was studied. In consideration of actuator failure, external disturbances and actuator saturation, it can ensure the stable tracking performance of the spacecraft's attitude. The authors of [33] analyzed a constrained control scheme based on model-referenced adaptive control. The model considered the longitudinal motion of a commercial aircraft with additive and multiplicative faults and saturated nonlinearity. Many studies assume that the fault information is known. In fact, the fault information is not known in the actual system. Therefore, a fault diagnosis observer (FDO) is proposed to effectively identify faults, which is beneficial to the design of fault-tolerant controllers. The authors of [37] designed a sliding mode observer to estimate faults for nonlinear uncertain systems. The authors of [38] gave an LMI design method for a new type of unknown input observer that can handle noise and uncertainty at the same time. FTC technology can be divided into active FTC and passive FTC [39]. The control method adopted by passive FTC does not depend on the FD system, and the stability of the system is guaranteed by establishing a fixed fault-tolerant controller. Active FTC is a series of control adjustment strategies adopted after the FD system obtains the fault status.

In real applications, it is worthwhile to point out that both disturbances and faults may influence the system performance simultaneously. In fact, this problem has also been studied. Some results only realize the estimation of the faults which have not been compensated effectively [40,41]. Meanwhile, many approaches are used to reject disturbances or uncertainties by feedback control rather than feedforward compensation control [42,43]. More importantly, most results merely relate to the inputs, actuator faults and disturbances which are constrained in the same channel [44,45]. In this case, it is difficult to judge whether the indescribable performance is caused by disturbances or faults. Hence, when the system is simultaneously confronted with faults and disturbances, how to distinguish them is the first problem. In addition, compared with those matched results, the mismatched conditions will also become more challenging in the aspect of controller design and performance analysis.

Motivated by the above analysis, a novel composite observer including FDO and DO is designed to dynamically reject unknown faults and mismatched disturbances. To make up for the mismatched disturbances, a sliding surface and a sliding mode controller are established, which not only compensate the faults and disturbances simultaneously but also guarantee the ultimately uniformly bounded (UUB) performance of a controlled system. A flow chart is shown in Figure 1. It can be seen that the proposed algorithm can realize the unmatched dynamical compensation, which improves matched results and the traditional robust problem. In addition, compared with some single anti-disturbance or fault-tolerance algorithms [46,47], the result also represents a quite significant extension. Finally, as for a typical unmanned aerial vehicle (UAV) system, a simulation example is given to certify the validity of the algorithm.

Figure 1. Anti-disturbance fault-tolerant sliding mode control.

Notation: For a matrix O , $\text{sym}(\cdot)$ is expressed as $\text{sym}(O) = O + O^T$; I and 0 , respectively, denote identity matrix and zero matrix. If there is no description, matrices are supposed to have compatible dimensions.

2. System Description

Consider a system with actuator faults and unknown disturbances, as follows:

$$\begin{cases} \dot{x}(t) = Ax(t) + B_1(u(t) + F(t)) + B_2d(t) \\ y(t) = Cx(t) \end{cases} \quad (1)$$

where $x(t) \in \mathbf{R}^n$, $u(t) \in \mathbf{R}^{m_1}$, $F(t) \in \mathbf{R}^{m_1}$, $d(t) \in \mathbf{R}^{m_2}$ and $y(t) \in \mathbf{R}^p$, respectively, stand for the system state, the control input, the actuator faults, the exogenous disturbances and the control input. A , B_1 , B_2 , C are coefficient matrices.

Obviously, the disturbance and the control input enter the system from different channels, thus forming a mismatched disturbance. It was also mentioned in [14] that the existing DOBC does not work against this disturbance.

Assumption 1. The pair (A, B_1) is controllable, and matrices B_1 and B_2 are full columns.

Assumption 2. The unknown fault $F(t)$ and the mismatched disturbance $d(t)$, respectively, satisfy $\|\dot{F}(t)\| \leq \omega_1$ and $\|d(t)\| \leq \omega_2$, where $\omega_1 > 0$ and $\omega_2 > 0$.

3. Composite Observer Design

For the sake of estimating the actuator faults and the mismatched disturbances, FDO and DO are shown in this section.

3.1. FDO Design

The following FDO is set up to complete the fault estimation:

$$\begin{cases} \hat{F}(t) = v(t) - L_1x(t) \\ \dot{v}(t) = L_1B_1(v(t) - L_1x(t)) + L_1(Ax(t) + B_1u(t) + B_2\hat{d}(t)) \end{cases} \quad (2)$$

where $\hat{F}(t)$ and $\hat{d}(t)$ are estimations of $F(t)$ and $d(t)$, respectively. L_1 is the FDO gain to be solved later. $v(t)$ is a designed auxiliary variable.

Defining $e_F(t) = \hat{F}(t) - F(t)$ and $e_d(t) = d(t) - \hat{d}(t)$, the fault estimation error system can be expressed as

$$\begin{aligned}\dot{e}_F(t) &= \dot{\hat{F}}(t) - \dot{F}(t) \\ &= -\dot{F}(t) + L_1 B_1 e_F(t) - L_1 B_2 e_d(t)\end{aligned}\quad (3)$$

In this subsection, an FDO is designed for fault estimation. In the following subsection, we construct a DO so that the mismatched disturbances can be estimated accurately.

3.2. DO Design

The unknown mismatched disturbance $d(t)$ is supposed to be constructed by an exogenous system as

$$\begin{cases} d(t) = Vw(t) \\ \dot{w}(t) = Ww(t) \end{cases}\quad (4)$$

where $w(t)$ is the disturbance vector, W and V are known coefficient matrices.

$w(t)$ should be estimated to identify the mismatched disturbance $d(t)$. According to the state observer programming approach, (4) is able to be re-expressed as

$$\begin{cases} \dot{w}(t) = Ww(t) \\ \sigma(t) = B_2 Vw(t) \end{cases}\quad (5)$$

where $\sigma(t)$ is the equivalent output. Then, error $\tilde{\sigma} = \hat{\sigma} - \sigma$ is defined, and the effect of $\tilde{\sigma}$ is to enhance the estimation precision, where $\hat{\sigma} = B_2 V\hat{w}(t)$. Next, $\hat{w}(t)$ can be set up by the Luenberger observer structure as

$$\begin{aligned}\dot{\hat{w}}(t) &= W\hat{w}(t) + L_2[\hat{\sigma}(t) - \sigma(t)] \\ &= W\hat{w}(t) + L_2[Ax(t) + B_1 u(t) + B_1 \hat{F}(t) + B_2 V\hat{w}(t) - \dot{x}(t)]\end{aligned}\quad (6)$$

In order to remove $\dot{x}(t)$ in (6), by introducing $r(t) = \hat{w}(t) + L_2 x(t)$ into (6), DO can be established as

$$\begin{cases} \dot{\hat{w}}(t) = r(t) - L_2 x(t) \\ \hat{d}(t) = V\hat{w}(t) \end{cases}\quad (7)$$

where $\hat{w}(t)$ is the estimation of $w(t)$, L_2 is DO gain to be solved later and $r(t)$ is an auxiliary variable, which can be described as

$$\dot{r}(t) = (W + L_2 B_2 V)\hat{w}(t) + L_2[Ax(t) + B_1 u(t) + B_1 \hat{F}(t)]\quad (8)$$

Defining $e_w(t) = w(t) - \hat{w}(t)$, the disturbance estimation error system can be obtained from (4)–(8) as

$$\begin{aligned}\dot{e}_w(t) &= \dot{w}(t) - \dot{\hat{w}}(t) \\ &= (W + L_2 B_2 V)e_w(t) - L_2 B_1 e_F(t)\end{aligned}\quad (9)$$

In this subsection, a DO is designed for disturbance estimation. In the next subsection, we determine the FDO gain L_1 and DO gain L_2 to make the fault estimation error $e_F(t)$ and the disturbance estimation error $e_w(t)$ converge to zero.

3.3. Performance Analysis of Error System

By defining $\rho(t) = [e_w^T(t), e_F^T(t)]^T$ and combining estimation error Equations (3) and (9), we have

$$\dot{\rho}(t) = \begin{bmatrix} W + L_2 B_2 V & -L_2 B_1 \\ L_1 B_1 & -L_1 B_2 V \end{bmatrix} \rho(t) + \begin{bmatrix} 0 \\ -I \end{bmatrix} \dot{F}(t)\quad (10)$$

Assumption 3. For the observers of actuator faults (2) and mismatched faults (7), $e_F(t)$ and $e_d(t)$, respectively, satisfy $\|e_F(t)\| \leq \theta_F$ and $\|e_d(t)\| \leq \theta_d$, where θ_F and θ_d are positive scalars.

Next, the following theorems can be obtained by the design of the observers above.

Theorem 1. For known parameters $\lambda_0 > 0$ and $\lambda_1 > 0$, if there exist matrices $P_1 > 0$, R_1 satisfying the following LMI

$$\begin{bmatrix} \text{sym}(R_1 B_1) & P_1 & R_1 B_2 \\ P_1^T & -\lambda_0^2 I & 0 \\ B_2^T R_1^T & 0 & -\lambda_1^2 I \end{bmatrix} < 0 \quad (11)$$

where the fault diagnosis observer gain can be solved by $L_1 = P_1^{-1} R_1$, then the fault estimation error system (3) is UUB.

Proof. A Lyapunov function is selected as

$$\Phi_1(e_F(t), t) = e_F^T(t) P_1 e_F(t) \quad (12)$$

Under the fault estimation error system (3), one has

$$\begin{aligned} \dot{\Phi}_1(e_F(t), t) &= e_F^T(t) (\text{sym}(P_1 L_1 B_1)) e_F(t) - 2e_F^T(t) P_1 \dot{F}(t) - 2e_F^T(t) P_1 L_1 B_2 e_d(t) \\ &\leq e_F^T(t) (\text{sym}(P_1 L_1 B_1)) e_F(t) + \lambda_0^{-2} e_F^T(t) P_1 P_1 e_F(t) + \lambda_0^2 \|\dot{F}(t)\|^2 \\ &\quad + \lambda_1^{-2} e_F^T(t) P_1 L_1 B_2 B_2^T L_1^T P_1 e_F(t) + \lambda_1^2 \|e_d(t)\|^2 \\ &\leq e_F^T(t) (\text{sym}(P_1 L_1 B_1) + \lambda_0^{-2} P_1 P_1 + \lambda_1^{-2} P_1 L_1 B_2 B_2^T L_1^T P_1) e_F(t) + \lambda_0^2 \|\dot{F}(t)\|^2 \\ &\quad + \lambda_1^2 \|e_d(t)\|^2 \end{aligned} \quad (13)$$

Based on a Schur complement formula and LMI (9), we can obtain $\text{sym}(P_1 L_1 B_1) + \lambda_0^{-2} P_1 P_1 + \lambda_1^{-2} P_1 L_1 B_2 B_2^T L_1^T P_1 < 0$. Next, (13) can be rewritten as

$$\dot{\Phi}_1(e_F(t), t) \leq e_F^T(t) \alpha_1 e_F(t) + \lambda_0^2 \omega_1^2 + \lambda_1^2 \theta_d^2 \quad (14)$$

where $\alpha_1 > 0$. It is easy to deduce that $\|e_F(t)\|^2 \leq \alpha_1^{-1} (\lambda_0^2 \omega_1^2 + \lambda_1^2 \theta_d^2)$, which means that $e_F(t)$ is UUB. $\square$

Theorem 2. For a known parameter $\lambda_2 > 0$, if there exist matrices $P_2 > 0$, R_2 satisfying the following LMI

$$\begin{bmatrix} \text{sym}(P_2 W + R_2 B_2 V) & R_2 B_1 \\ B_1^T R_2^T & -\lambda_2^2 I \end{bmatrix} < 0 \quad (15)$$

where the disturbance observer gain can be solved by $L_2 = P_2^{-1} R_2$, then the disturbance estimation error system (9) is UUB.

Proof. A Lyapunov function is selected as

$$\Phi_2(e_w(t), t) = e_w^T(t) P_2 e_w(t) \quad (16)$$

According to the disturbance estimation error system (9), we can obtain

$$\begin{aligned} \dot{\Phi}_2(e_w(t), t) &= e_w^T(t) (\text{sym}(P_2 W + P_2 L_2 B_2 V)) e_w(t) - 2e_w^T(t) P_2 L_2 B_1 e_F(t) \\ &\leq e_w^T(t) (\text{sym}(P_2 W + P_2 L_2 B_2 V)) e_w(t) + \lambda_2^{-2} e_w^T(t) P_2 L_2 B_1 B_1^T L_2^T P_2 e_w(t) \\ &\quad + \lambda_2^2 \|e_F(t)\|^2 \end{aligned} \quad (17)$$

Based on a Schur complement formula and LMI (15), we can obtain $\text{sym}(P_2 W + P_2 L_2 B_2 V) + \lambda_2^{-2} P_2 L_2 B_1 B_1^T L_2^T P_2 < 0$. Next, (17) can be rewritten as

$$\dot{\Phi}_2(e_w(t), t) \leq e_w^T(t) \alpha_2 e_w + \lambda_2^2 \theta_F^2 \quad (18)$$

where $\alpha_2 > 0$. It is easy to deduce that $\|e_w(t)\|^2 \leq \alpha_2^{-1} (\lambda_2^2 \theta_F^2)$, which means that $e_w(t)$ is UUB. $\square$

In the next section, a sliding surface and a sliding mode controller are designed to composite the actuator faults and mismatched disturbances simultaneously to ensure the system stability.

4. Controller Design and Dynamical Performance Analysis

A sliding surface is selected as

$$s(t) = B_1^T P_3 x(t) \quad (19)$$

where $P_3 > 0$ can be determined later.

Now, state $x(t)$ can be guaranteed to be UBB and finally reach the sliding surface $s(t)$.

Theorem 3. The system state in (1) is UBB when the matrix P_3 is the solution of matrix inequality

$$P_3 A + A^T P_3 - \gamma^{-1} P_3 B_1 B_1^T P_3 + \eta^{-2} P_3 B_1^\perp B_1^{\perp+} B_2 B_2^T B_1^{\perp+T} B_1^{\perp T} P_3 < 0 \quad (20)$$

where parameters $\lambda > 0$ and $\eta > 0$ can be determined later.

Proof. A Lyapunov function is selected as

$$\Phi_3(x(t), t) = x^T(t) P_3 x(t) \quad (21)$$

According to the system (1), we can obtain

$$\begin{aligned} \dot{\Phi}_3(x(t), t) &= x^T(t) P_3 \left(Ax(t) + B_1(u(t) + F(t)) + (B_1 B_1^\dagger + B_1^\perp B_1^{\perp+}) B_2 d(t) \right) \\ &\quad + \left(Ax(t) + B_1(u(t) + F(t)) + (B_1 B_1^\dagger + B_1^\perp B_1^{\perp+}) B_2 d(t) \right)^T P_3 x(t) \\ &= x^T(t) (P_3 A + A^T P_3) x(t) + 2x^T(t) P_3 B_1 \left(u(t) + F(t) + B_1^\dagger B_2 d(t) \right) \\ &\quad + 2x^T(t) P_3 B_1^\perp B_1^{\perp+} B_2 d(t) \\ &\leq x^T(t) (P_3 A + A^T P_3) x(t) + 2x^T(t) P_3 B_1 \left(u(t) + F(t) + B_1^\dagger B_2 d(t) \right) \\ &\quad + \eta^2 \|d(t)\|^2 + \eta^{-2} x^T(t) P_3 B_1^\perp B_1^{\perp+} B_2 B_2^T B_1^{\perp+T} B_1^{\perp T} P_3 x(t) \\ &\leq x^T(t) \left(P_3 A + A^T P_3 - \gamma^{-1} P_3 B_1 B_1^T P_3 + \eta^{-2} P_3 B_1^\perp B_1^{\perp+} B_2 B_2^T B_1^{\perp+T} B_1^{\perp T} P_3 \right) \\ &\quad x(t) + 2x^T(t) P_3 B_1 \left(u(t) + F(t) + 0.5\gamma^{-1} B_1 P_3 x(t) + B_1^\dagger B_2 d(t) \right) \\ &\quad + \eta^2 \|d(t)\|^2 \\ &\leq x^T(t) \left(P_3 A + A^T P_3 - \gamma^{-1} P_3 B_1 B_1^T P_3 + \eta^{-2} P_3 B_1^\perp B_1^{\perp+} B_2 B_2^T B_1^{\perp+T} B_1^{\perp T} P_3 \right) \\ &\quad x(t) + \eta^2 \omega_2^2 \end{aligned} \quad (22)$$

where $B_1 B_1^\dagger + B_1^\perp B_1^{\perp+} = I$ and $B_1^\perp \in \mathbf{R}^{n \times (n-m)}$. If there is a matrix $P_3 > 0$ that satisfies the inequality (20), it is easy to find that the system state is UUB. $\square$

In this study, it can be guaranteed that the mismatched DO error is finite, but we may not be able to obtain its specific upper limit. Therefore, it is necessary to put forward

adaptive SMC to deal with this issue. Adaptive SMC can meet the reachability condition of (19), as shown below:

$$u(t) = -\left(\alpha s(t) + (B_1^T P_3 B_1) B_1^T P_3 \left(Ax(t) + B_2 \hat{d}(t) + \hat{\theta}_d \|H\| \operatorname{sgn}(s(t))\right)\right) - \hat{F}(t) \quad (23)$$

where $H = (B_1^T P_3 B_1)^{-1} B_1^T P_3 B_2$ and the adaptive law is selected as

$$\dot{\hat{\theta}}_d(t) = h \|H\| \|s(t)\| \quad (24)$$

where h is a positive parameter which can be designed.

Theorem 4. For the designed sliding surface (19), it can be ensured that the state of system can finally reach the designed sliding surface.

Proof. $\tilde{\theta}_d = \theta_d - \hat{\theta}_d$ is defined and the Lyapunov function is selected as

$$V(s(t), (t)) = \frac{1}{2} \left(s^T(t) (B_1^T P_3 B_1)^{-1} s(t) + \frac{1}{h} \tilde{\theta}_d^2 \right) \quad (25)$$

then we take the time derivative of (25), and then it can be found that

$$\begin{aligned} \dot{V}(s(t), (t)) &= s^T(t) (B_1^T P_3 B_1)^{-1} \dot{s}(t) - \frac{1}{h} \tilde{\theta}_d \dot{\hat{\theta}}_d \\ &= s^T(t) (B_1^T P_3 B_1)^{-1} (B_1^T P_3 \dot{x}(t)) - \tilde{\theta}_d \|H\| \|s(t)\| \\ &= s^T(t) (B_1^T P_3 B_1)^{-1} \left[B_1^T P_3 \left(Ax(t) + B_1(u(t) + F(t)) + B_2 d(t) \right) \right] \\ &\quad - \tilde{\theta}_d \|H\| \|s(t)\| \\ &= s^T(t) (B_1^T P_3 B_1)^{-1} \left[B_1^T P_3 \left(Ax(t) + B_1(-\alpha s(t) - (B_1^T P_3 B_1)^{-1} B_1^T P_3 (Ax(t) \right. \right. \\ &\quad \left. \left. + B_2 \hat{d}(t)) - \hat{\theta}_d \|H\| \operatorname{sgn}(s(t)) - B_1 \hat{F}(t) + B_1 F(t)) + B_2 d(t) \right) \right] \\ &\quad - \tilde{\theta}_d \|H\| \|s(t)\| \\ &= s^T(t) H e_d(t) - s^T(t) \alpha s(t) - s^T(t) e_F(t) - s^T(t) \hat{\theta}_d \|H\| - \tilde{\theta}_d \|H\| \|s(t)\| \\ &\leq \|s(t)\| \|H\| \|e_d(t)\| - \alpha \|s(t)\|^2 - \|s(t)\| \|e_F(t)\| - \hat{\theta}_d \|H\| \|s(t)\| \\ &\quad - \tilde{\theta}_d \|H\| \|s(t)\| \\ &\leq \theta_d \|s(t)\| \|H\| - \alpha \|s(t)\|^2 - \|s(t)\| \|e_F\| - \hat{\theta}_d \|H\| \|s(t)\| - \tilde{\theta}_d \|H\| \|s(t)\| \\ &\leq -\alpha \|s(t)\|^2 - \|s(t)\| \|\theta_F\| \\ &\leq 0 \end{aligned} \quad (26)$$

where α is a positive variable. $s^T(t) \dot{s}(t) \leq 0$ can be guaranteed by (26). This means that the reachability condition of SMC can be met. $\square$

5. Numerical Illustrations

In this section, we take the UAV's longitudinal motion equations into consideration. A four-state one-input longitudinal model can be shown as

$$\begin{cases} \dot{V} = \frac{\bar{q}S}{m}(C_{X_1} + C_{X_\alpha}\alpha + C_{X_{\alpha^2}}\alpha^2) + \frac{T\cos\alpha}{m} \\ \dot{\alpha} = \frac{1}{mV}[\bar{q}S(C_{Z_1} + C_{Z_\alpha}\alpha) + \frac{T\cos\alpha}{m}] \\ \dot{q} = \frac{\bar{q}S\bar{c}}{I_{yy}}(C_{M_1} + C_{M_e}\delta_e + C_{M_\alpha}\alpha + C_{M_{\dot{q}}}\dot{q}) \\ \dot{\theta} = q \end{cases} \quad (27)$$

where the state variables, respectively, stand for total velocity V (m/s), angle of attack α (deg), pitch rate q (deg/s) and pitch angle θ (deg). Note that the notion $\bar{q}$ should be the dimensionless number of q , which can be expressed by $\bar{q} = \frac{\bar{c}q}{2V}$. Thrust T (N) and elevator deflection angle δ_e (deg) are control variables. Furthermore, the dynamic pressure $\bar{q}$ can be described as $\bar{q} = \frac{1}{2}\rho V^2$. All related parameters are displayed in Table 1.

Table 1. Parameters in (25).

Parameters	Values
Acceleration of gravity (g)	9.81 m/s ²
Air density (ρ)	1.205 kg/m ³
Mass of the UAV (m)	28 kg
Wing planform area (S)	1.8 m ²
Mean aerodynamic chord ($\bar{c}$)	0.58 m
Pitch moment-of-inertia (I_{yy})	10.9(±0.05) kg·m ²
Lift force coefficient (C_{Z_1})	1.29×10^{-2} (±0.10)
Lift force coefficient (C_{Z_α})	-3.25 (±0.20)
Drag force coefficient (C_{X_1})	-2.12×10^{-2} (±0.10)
Drag force coefficient (C_{X_α})	-2.66×10^{-2} (±0.20)
Drag force coefficient ($C_{X_{\alpha^2}}$)	-1.55 (±0.20)
Pitch force coefficient (C_{M_1})	2.08×10^{-2} (±0.10)
Pitch force coefficient (C_{M_e})	5.45×10^{-1} (±0.20)
Pitch force coefficient (C_{M_α})	-9.03×10^{-2} (±0.20)
Pitch force coefficient ($C_{M_{\dot{q}}}$)	-9.83 (±0.20)

The linearization UAV systems which are subject to actuator faults and mismatched disturbances can be described by

$$\begin{cases} \dot{x}(t) = Ax(t) + B_1(u(t) + F(t)) + B_2d(t) \\ y(t) = Cx(t) \end{cases} \quad (28)$$

where

$$A = \begin{bmatrix} -0.0088 & -0.0105 & 0 & -0.0409 \\ -0.0915 & -0.4917 & 1 & 0 \\ -0.0294 & -2.5464 & -0.8966 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0 \\ -0.1011 \\ -7.7307 \\ 0 \end{bmatrix}, \quad B_2 = \begin{bmatrix} 0.1 \\ 0 \\ -3 \\ 0.1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}^T.$$

The initial conditions of the state are defined as $V_0 = 100\text{m/s}$, $\alpha_0 = 1\text{deg}$, $q = 1\text{deg}$ and $\theta = 0.1$.

In the following section, we consider FDO and DO for different conditions.

Model 1: The actuator fault is supposed to occur at the 15th second as

$$F(t) = \begin{cases} 0, & \text{if } t < 10 \\ 1, & \text{if } t \geq 10 \end{cases} \quad (29)$$

and the mismatched disturbances can be described as (9) with

$$W = \begin{bmatrix} 0 & -4 \\ 4 & -0.3 \end{bmatrix}, \quad V = \begin{bmatrix} 0.5 \\ -0.1 \end{bmatrix}^T,$$

Meanwhile, by defining the parameters $\lambda_1 = \lambda_2 = 1$, $\gamma = 1$, $\eta = 0.1$, $h = 1$ and solving inequations (6), (15), (20), the control gains P_1 , P_2 , the FDO gain L_1 and the DO gain L_2 are found to be

$$P_1 = [0.5897], \quad P_2 = 1 \times 10^8 \begin{bmatrix} 4.7891 & 0.1073 \\ 0.1073 & 4.6246 \end{bmatrix},$$

$$L_1 = [0 \quad 9.9370 \quad 0.0641 \quad 0], \quad L_2 = \begin{bmatrix} 0 & -0.9665 & 0.0126 & 0 \\ 0 & 0.1826 & -0.0024 & 0 \end{bmatrix},$$

where the sliding surface gain P_3 is calculated as

$$P_3 = 1 \times 10^{-6} \begin{bmatrix} 0.0040 & 0.0142 & -0.0175 & 0.0002 \\ 0.0142 & 0.5985 & 0.1107 & -0.0014 \\ -0.0175 & 0.1107 & 0.2947 & -0.0011 \\ 0.0002 & -0.0014 & -0.0011 & 0.0012 \end{bmatrix}$$

The initial values of states are assumed as $x_0 = [100, 0, -2, -1]^T$. Figures 2 and 3, respectively, display the dynamic trajectories of constant fault and attenuated harmonic disturbance, as well as their accurate estimates. It can be concluded that the designed FO and DO show effective estimation performances. The dynamic trajectory of the sliding surface is shown in Figure 4, and it can be proved that the sliding surface is reachable. Figures 5 and 6, respectively, show the thrust and the elevator deflection angle of a UAV system that vary with time, which also imply that the UAV system is eventually stable.

Figure 2. Fault $F(t)$ and its estimation value in model 1.

Figure 3. Disturbance $d(t)$ and its estimation value in model 1.

Figure 4. The sliding mode surface in model 1.

Figure 5. Thrust of UAV system in model 1.

Figure 6. Elevator deflection angle of UAV system in model 1.

Model 2: The second kind of the actuator fault is supposed to occur at the 15th second as

$$F(t) = \begin{cases} 0, & \text{if } t < 10 \\ 2 + 0.5\sin t, & \text{if } t \geq 10 \end{cases} \quad (30)$$

and the mismatched disturbances can be described as in (9) with

$$W = \begin{bmatrix} 0 & 5 \\ -5 & 0 \end{bmatrix}, \quad V = \begin{bmatrix} 1 \\ 0 \end{bmatrix}^T,$$

At the same time, by defining the parameters $\lambda_1 = \lambda_2 = 0.5$, $\gamma = 1$, $\eta = 0.5$, $h = 1 \times 10^6$ and solving inequations (6), (15), (20), the control gains P_1 , P_2 , the FDO gain L_1 and the DO gain L_2 are found to be

$$P_1 = [0.3606], \quad P_2 = 1 \times 10^8 \begin{bmatrix} 4.1638 & -0.5562 \\ -0.5562 & 4.1638 \end{bmatrix},$$

$$L_1 = [0 \quad 13.4033 \quad 0.0187 \quad 0], \quad L_2 = \begin{bmatrix} 0 & -1.3990 & 0.0183 & 0 \\ 0 & -0.1869 & 0.0024 & 0 \end{bmatrix},$$

where the sliding surface gain P_3 is calculated as

$$P_3 = 1 \times 10^{-6} \begin{bmatrix} 0.0040 & 0.0142 & -0.0175 & 0.0002 \\ 0.0142 & 0.5985 & 0.1107 & -0.0014 \\ -0.0175 & 0.1107 & 0.2947 & -0.0011 \\ 0.0002 & -0.0014 & -0.0011 & 0.0012 \end{bmatrix}$$

The initial values of states are $x_0 = [100, 0, -2, -1]^T$. A favorable fault identification effect is displayed in Figure 7. Meanwhile, from Figure 8, we can find that the mismatched disturbance can be effectively estimated. In addition, the process of system states is shown in Figure 9, which shows that the system states can eventually reach the sliding surface. Figures 10 and 11 show the trajectories of thrust and the elevator deflection angle of the UAV system. The states of the UAV model are designed as in Figures 12–15, respectively, which reflect the satisfactory stability performance. It can be seen from the simulation results that the designed anti-disturbance fault-tolerant sliding mode control algorithm is effective.

Figure 7. Fault $F(t)$ and its estimation value in model 2.

Figure 8. Disturbance $d(t)$ and its estimation value in model 2.

Figure 9. The sliding mode surface in model 2.

Figure 10. Thrust of UAV system in model 2.

Figure 11. Elevator deflection angle of UAV system in mode 2.

Figure 12. The velocity of UAV system.

Figure 13. The angle of attack of UAV system.

Figure 14. The pitch rate of UAV system.

Figure 15. The pitch angle of UAV system.

6. Conclusions

In this paper, a novel control algorithm with the capacity of fault tolerance and anti-disturbance is put forward for systems subject to unknown faults and mismatched disturbances. In order to distinguish the effects of the faults and disturbances, FDO and DO are designed to estimate the dynamics of unknown faults and disturbances, respectively. With the help of the dynamical estimation results, both a sliding surface and a corresponding sliding mode controller are successively proposed to realize the rejection of the faults and mismatched disturbances. It is noted that the proposed algorithm not only solves the difficulty of unmatched dynamical compensation but also extends single anti-disturbance or fault-tolerance results.

In order to achieve faster convergence, a finite time control problem will be carefully considered as one of our future research topics. Moreover, experimental verification relying on practical equipment will also be discussed in future related work.

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