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A Novel Bottom-Up/Top-Down Hybrid Strategy-Based Fast Sequential Fault Diagnosis Method

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Abstract: Sequential fault diagnosis is a kind of important fault diagnosis method for large scale complex systems, and generating an excellent fault diagnosis strategy is critical to ensuring the performance of sequential diagnosis. However, with the system complexity increasing, the complexity of fault diagnosis tree increases sharply, which makes it extremely difficult to generate an optimal diagnosis strategy. Especially, because the existing methods need massive redundancy iteration and repeated calculation for the state parameters of nodes, the resulting diagnosis strategy is often inefficient. To address this issue, a novel fast sequential fault diagnosis method is proposed. In this method, we present a new bottom-up search idea based on Karnaugh map, SVM and simulated annealing algorithm. It combines failure sources to generate states and a Karnaugh map is used to judge the logic of every state. Eigenvalues of SVM are obtained quickly through the simulated annealing algorithm, then SVM is used to eliminate the less useful state. At the same time, the bottom-up method and cost heuristic algorithms are combined to generate the optimal decision tree. The experiments show that the calculation time of the method is shorter than the time of previous algorithms, and a smaller test cost can be obtained when the number of samples is sufficient.

Keywords: large-scale fault diagnosis; Karnaugh map; SVM; simulated annealing algorithm; bottom-up; multi-signal model

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1. Introduction

Fault diagnosis is a crucial activity for systems with high safety and mission criticality requirements [1], such as spacecraft [2], military helicopters [3], and aircraft satellites [4]. With the system structure becomes more and more complex [5], how to quickly and low-costly isolate faults [6] becomes a difficult problem. To solve it, Somnath Deb and K.R. Pattipati proposed the multi-signal flow graph model based on AND/OR graph in [7], which make test sequence optimization problem convert to the analysis and processing problem of D-matrix. Because the model is close to the actual physical system [8], and the integration and verification of the model is relatively simple [9], researchers have done a lot on this [10].

Currently, there are mainly two methods of sequential fault diagnosis algorithms based on multi-signal flow graph. One is the top-bottom search strategy, that is, the decision tree is constructed from top to bottom. The initial object is all faults, and test points are selected for fault isolation. There are four kinds of algorithms: (1) Greedy algorithm, whose time complexity is very low, but it is easy to fall into the local optimal solution because it does not consider the whole decision tree. (2) Heuristic search algorithm, which is also known as the AO* algorithm [11]. The heuristic search algorithm based on information theory was applied to generate the sub-tree to handle the imperfect tests existed extensively in the realistic systems [12]. To carry out iterative updating and construct the near-optimal diagnosis strategy, [13] proposed combining a rollout algorithm with information gain heuristic algorithms and [14] makes improvement in different search widths and depths.

Several heuristic algorithms were proposed to find good strategies for large problem instances in [15]. The result of the algorithms is the global optimal solution, but when the heuristic value of each test point is close, it needs to traverse all cases, which takes a lot of time. In addition, with the increase of the number of faults, the time complexity increases exponentially and large storage is required. (3) Intelligent algorithm. In [16], the test sequencing problem was converted to search the minimal complete test sequencing based on the ant algorithm. Based on cuckoo search, [17] proposed an algorithm to generate optimized test sequences, which obtains 100% software coverage. These methods avoid the tedious logic design and seek rules from a large number of permutation and combination data. When the number of faults is large, it needs a lot of iterations and time is uncertain. (4) Hybrid algorithm, the combination of the Algorithms 1, 2 and 3 or the combination of these with other algorithms. Reference [18] proposed a new testing sequence optimization method based on AO* and dynamic programming to improve the efficiency of generating solutions. Reference [19] combined the greedy method with discrete binary particle swarm optimization (DPSO) to construct a test sequential tree. Reference [20] developed iterative algorithm called GLP(τ)S that uses genetic algorithms, LP τ low-discrepancy sequences of points and heuristic rules to find regions of attraction when searching a global minimum of an objective function. Another kind of method is bottom-up search strategy, that is, the fault tree is constructed from bottom to top [21]. The initial state is a single fault, continuous permutation and combination is carried until it is combined into a complete set of faults [22]. This avoids the repeated calculation of the same state, and has advantages in small-scale fault diagnosis. However, in large-scale fault diagnosis, the efficiency of the algorithm is low due to the generation of too many useless states.

To sum up, the main problem is that when the number of faults and test points is large, the time complexity of various algorithms is high [23] and the formation of decision tree is slow [21], that is NP hard problem [24]. Based on the above literature research, this paper proposes a new bottom-up/top-down hybrid strategy-based fast sequential fault diagnosis method. Firstly, a new bottom-up algorithm is established, which uses Karnaugh map, SVM [25] and simulated annealing algorithm [26] to decline the generation of useless state. It is suitable for the situation that the number of test points is more than the number of faults. Then the algorithm is combined with the cost heuristic search algorithm, which is suitable for the situation that the number of test points is less than the number of faults. This method makes the decision tree switch the algorithm according to the situation of nodes in the generation process, so as to improve the computational efficiency. Experimental results show that, compared with heuristic search algorithm and intelligent algorithm, the decision tree of this strategy has shorter generation time and guaranteed accuracy when dealing with large-scale fault dependency matrix.

The remainder of this paper is organized as follows. Section 2 describes the problem, Section 3 introduces the new proposed approach, Section 4 shows the comparative experiment with other algorithms, and Section 5 provides a summary.

2. Description of the Multi-Signal Flow Graph Combination Problem

In general, the testing sequencing problem is based on the information of failures, including five components:

- $S = \{S_1, S_2, \dots, S_m\}$ is a set of statically independent failure associated with system, where S_i means the i th fault state and m is the number of failure states;
- $P = \{P_1, P_2, \dots, P_m\}$ is denoted as the priori probability vector according to the failure set S ;
- $T = \{T_1, T_2, \dots, T_n\}$ represents a finite set of n reliable binary outcome tests, where T_j checks a subset of S ;
- $C = \{C_1, C_2, \dots, C_n\}$ is a set of tests cost based on time, manpower requirements or other economic factors;

- $D = [d_{ij}]$ represents a binary matrix with $m \times n$, dimension to depict the relationship between the failure set and the test set, where $d_{ij} = 1$ means the test T_j is able to monitor failure state S_i otherwise, $d_{ij} = 0$.

The purpose of the sequential fault algorithm is to generate the optimal binary tree that can incur the least cost of fault separation.

As shown in Table 1, the scale of D1 is $m \times n$, and the matrix is transformed into D2, D2 includes all binary digit combinations. It is shown in Table 2.

Table 1. Fault-test dependency matrix D1.

	T_1	T_2	T_3
S_1	0	1	0
S_2	1	1	0
S_3	0	0	1
S_4	1	0	0

Table 2. Fault-test dependency matrix D2.

	T_1	T_2	T_3
S_0^*	0	0	0
$S_1^* = S_3$	0	0	1
$S_2^* = S_1$	0	1	0
S_3^*	0	1	1
$S_4^* = S_4$	1	0	0
S_5^*	1	0	1
$S_6^* = S_2$	1	1	0
S_7^*	1	1	1

Among them, S_0^* , S_3^* , S_5^* , and S_7^* are fictitious fault modules; if D2 is analyzed, the combination number of test sequence trees is calculated as shown in Equation (1).

$$Num = (n)^{2^0} * (n-1)^{2^1} * \dots * (n - (n-1))^{2^{m-1}} = \prod_{i=0}^{n-1} (n-i)^{2^i} \quad (1)$$

The condition in which the fault in D1 can be completely isolated is shown in Equation (2).

$$m \leq 2^n \quad (2)$$

As shown in Figures 1 and 2, any test sequence tree of D1 is a subset of a test sequence tree of D2.

Figure 1. A test sequence tree of D1.

Figure 2. A test sequence tree of D2.

Different D-matrices of the same scale will generate optimal decision trees with different levels. In the meanwhile, its number of layers will not be larger than $n + 1$. For the convenience of description and calculation, Figure 1 is changed to Figure 3, and the format of Figure 3 is a standard tree with $n + 1$ layers, which is the same to Figure 2.

Figure 3. A standard tree of D1.

As shown in Figure 3, for any node at level K of the binary tree:

$$L_k = \sum_{i=1}^{M_k} L(M_i)_{k_left} * L(M_i)_{k_right} \quad (\text{node size} > 1) \\ L_k = 1 \quad (\text{node size} = 1) \quad (3)$$

$$M_k = \sum_{j=1}^n A_j \quad (4)$$

Node size refers to the number of S contained in the node. If $A_j = 0$, the node can be expanded by test point T_j , and if $A_j = 1$, the node cannot be expanded by the test point T_j ; L_k is the number of combinations that the node can expand, and M_k denotes the number of test points that can be used; $L(M_i)_{k_left}$ and $L(M_i)_{k_right}$ indicate the number of combinations that can be expanded by the two subnodes of the node under condition M_i , and the subnodes are located in the $k + 1$ layer. To calculate the number of combinations of one node in k layer, we should obtain the number of combinations of the subnodes in $k + 1$ layer firstly. Then, we should multiply the combination number of left and right subnodes based on the expansion of different test points. The whole calculation process is from bottom to top.

The combination number of test sequence trees generated by an $n \times m$ D matrix is L_1 ($L_1 < Num$), and the initial value condition is $L_{n+1} = 1$. L_1 is generated by iterative calculation of the formula in the standard tree according to Equations (3) and (4). Additionally, the expected cost J can be calculated as

$$J = P^T B C = \sum_{i=1}^m \sum_{j=1}^n b_{ij} p_i c_j \quad (5)$$

where $B = [b_{ij}]$ is a binary matrix with dimensions of $m \times n$; $b_{ij} = 1$ means that test T_j is selected for the identification of fault state S_j and otherwise, $b_{ij} = 0$.

In summary, the problem of test sequence diagnosis based on a multi-signal flow graph aims to select the test sequence tree with the smallest J among L_1 types of test sequence trees. As m and n increase, the number of combinations L_1 greatly increases, which takes a long time to compare the cost of each tree and select the lowest, that is an NP-hard problem.

3. Proposed Sequential Fault Diagnosis Approach

The flow chart of the hybrid strategy is shown in Figure 4 and has three main parts: generation of parameter set, modification of parameter set and acquisition of switch condition boundary. The first two parts are novel bottom-up algorithms, which represent a great improvement compared with traditional bottom-up algorithms. Part three is used to define the switching conditions of the two algorithms, according to the size of the current node switch using two algorithms.

Figure 4. Flow chart of the bottom-up algorithm.

Firstly, a large number of samples are generated in accordance with the parameters of the test object. Both top-down and bottom-up algorithms are used to generate the optimal decision tree with these samples. Then, the time of the two algorithms is compared to obtain the switching condition. The next follows processing the test object. The size of the initial node is $m \times n$. The algorithm is selected according to the switching condition. After the algorithm is executed, the successor node is generated, and the algorithm is selected according to the situation of the successor node. It is then iterated until no successor nodes are generated. In addition, this flow chart is applicable to any top-down algorithms.

3.1. Generation of Parameter Set Q_k

The following definitions are applied:

- q_k : Any fault subset with size k of the fault complete set S , it contains three parts, k faults, k_1 and k_2 storage location information, and a series of Karnaugh map logic values, the length of which is n ;
- Q_k : The set of all q_k . $q_k(i)$ is used to denote the i th element in Q_k ;
- C_{q_k} : If there are only k failures that belong to q_k in the system, the minimum cost of testing is required;
- P_{q_k} : The sum of the failure rate of k failures in q_k .

$$P_{q_k} = \sum_{i=1}^k P_i, \text{ where } S_i \in q_k \quad (6)$$

The optimal test sequence tree is a full binary tree, and thus any $q_k (k \geq 2)$ can be composed of q_{k1} and q_{k2} , where $k_1 + k_2 = k (k_1 \geq 1, k_2 \geq 1)$. The calculation formula of q_k is shown in Equation (7).

$$C_{q_k} = \frac{P_{q_{k1}} C_{q_{k1}} + P_{q_{k2}} C_{q_{k2}}}{P_{q_{k1}} + P_{q_{k2}}} + \min \{C_{T_j}\} \quad (7)$$

where T_j refers to a test point that can separate q_{k1} and q_{k2} and should satisfy the conditions $\forall S_{i1} \in q_{k1}, \forall S_{i2} \in q_{k2}, \exists d_{S_{i1}T_j} \neq d_{S_{i2}T_j}$; if T_j does not exist, $C_{q_k} = \infty$.

The flow chart of the bottom-up algorithm is shown in Figure 4. There are two main parts: one is the production of Q_k , and the other is the modification of Q_k .

By combining all the elements of Q_{k1} and Q_{k2} ($k_1 + k_2 = k$) in turn, this part calculates and generates q_k , judges the logic of q_k , compares various combinations that can generate q_k , and retains the combination with the least test cost until all combinations are generated, with Q_k calculations completed. When $k = m$, the algorithm is end and decision tree can be obtained from Q_m .

3.1.1. $q_k(i)$ Combination Generation Module

Taking the fault-test dependency matrix in Table 3 as an example, the initial state $Q_1 = \{S_1, S_2, S_3, S_4\}$. Any pairwise combination obtains $Q_2 = \{S_1S_2, S_1S_3, S_1S_4, S_2S_3, S_2S_4, S_3S_4\}$. According to Formula (7), C_{q_2} is $\{1, 1.5, 1.5, 1, 1, 2\}$. Next, we calculate $Q_3 = \{S_1S_2S_3, S_1S_2S_4, S_1S_3S_4, S_2S_3S_4\}$; for example, $S_1S_2S_3$ is constructed in three ways: $S_1S_2 + S_3, S_1S_3 + S_2, S_2S_3 + S_1$. According to Formula (7), the calculation cost is $\{2, 2, \infty\}$, and the minimum value is 2. Similar to the above calculation, $C_{q_3} = \{2, 1.93, 2.75, 2.56\}$, $Q_4 = \{S_1S_2S_3S_4\}$ is constructed in ten ways and compared with each other, and the minimization is $C_{q_4} = \{3.2\}$.

Table 3. Fault-test dependency matrix.

	T_1	T_2	T_3	P
S_1	0	0	1	0.1
S_2	1	0	1	0.2
S_3	0	1	1	0.3
S_4	0	1	0	0.4
C	1	1.5	2	-

If all faults in the fault complete set s can be completely isolated, any subset of the fault complete set q_k can also be completely isolated. According to permutation and combination

$$|Q_k| = C_m^k \quad (8)$$

$|Q_k|$ refers to number of q_k . The computing times of q_k is shown in Equation (9).

$$Times = C_k^1 + C_k^2 + \dots + C_k^{[k/2]} \quad (9)$$

where $[k/2]$ is the largest integer not exceeding $k/2$.

3.1.2. Logic Judgement Module

The output of the previous module calculates the state of redundancy, which should be deleted. For example, the state $S_2S_3S_4$ of Q_3 ; actually, regardless of the type of test sequence, the fault set S is not be separated into fault subset $S_2S_3S_4$. States such as this are non-logical items, and by eliminating the non-logical items to decrease $|Q_k|$, the combination number and calculation time can be greatly reduced. To judge the logic, a Karnaugh map is introduced. Taking the data in Table 3 as an example, the Karnaugh map is constructed with the test points as variables and the faults as logical values in Figure 5.

Figure 5. A complete segmentation method of the Karnaugh map.

A complete segmentation method of the Karnaugh map is defined:

- Each circle conforms to the Karnaugh map circle rule, and its size is 2^y ($0 \leq y \leq n$);
- A large circle containing two smaller circles is treated as one circle;
- S_i ($1 \leq i \leq m$) each have a circle of size one;
- There are only two disjoint circles in a large circle;
- The maximum circle size is 2^n , and it must exist.

Figure 5 presents the complete segmentation method of the Karnaugh map. According to the rule of Huffman code [27], a complete segmentation method of Karnaugh maps uniquely is consistent with a decision binary tree. Figure 6 corresponds to the decision tree of Figure 5.

Figure 6. Test sequence tree.

In the Karnaugh map, a circle uniquely corresponds to a series of logical sequences composed of 0, 1 and d (d denotes “don’t care term”), and 0 and 1 denote the judgement results of the corresponding test points. where q_k is composed of q_{k1} and q_{k2} , and q_k is a nonlogical item. The necessary and sufficient condition is described as follows:

1. The circles of the Karnaugh map corresponding to q_{k1} and q_{k2} have no overlap;
2. The circle of the Karnaugh map corresponding to q_k does not contain S_i , $S_i \notin q_{k1}$ and $S_i \notin q_{k2}$.

As shown in Figure 5, S_1 corresponds to the logical value of 001, S_4 corresponds to the logical value of 010 according to the minimum item combination principle [28], and S_1S_4 corresponds to the logical value of 0dd. However, because S_3 is included in 0dd, $q_2 = \{S_1S_4\}$ does not exist. According to the above theory, judging the logic of q_k after it is generated and deleting nonlogic q_k can reduce $|Q_k|$.

3.1.3. Entire Process of Generation of the Parameter Set

In summary, the entire process of generation of the parameter set Q_k is shown in Algorithm 1.

Algorithm 1 Q_k production

- 1: $Q_k = O, k_1 = 1, i = 1$;
 - 2: $k_2 = k - k_1$, take out one element in Q_{k_1} and combine it with one element in Q_{k_2} in turn;
 - 3: Judge the logic of the combination result. If $q_k(i)$ is a non-logic item, return to step 2. Otherwise, go to step 4;
 - 4: Judge whether there is the same $q_k(i)$ in Q_k . If not, it will be included, and $i = i + 1$. Otherwise, compare the C_{q_k} of the two and retain the smaller one;
 - 5: Return to step 2, if all elements in step 2 have been combined, then $k_1 = k_1 + 1$, and if $k_1 > \lfloor k/2 \rfloor$, the calculation is completed; otherwise return to step 2.
-

Furthermore, $|Q_k|$ can be estimated by Equations (10)–(12). D_{xi} is defined as the i th column of the D-matrix with the size of $1 \times m$. The self-generated number SN is the number of q_k that generated directly by D . D_{xi} is equivalent to two fault subsets q_{i_1} and q_{i_2} ($q_{i_1} + q_{i_2} = m$). If $d_{ij} = 0$, $S_i \in q_{i_1}$; otherwise, $S_i \in q_{i_2}$. For example, D_{x1} is equivalent to one q_1 and one q_3 shown in Table 3. Similarly, D presented in Table 3 totally generates two q_1 , two q_2 and two q_3 . In addition, the combination number is defined as the number generated by the combination. The combination of q_{k_3} and q_{k_4} can also produce q_k under

the following conditions: if $S_i \in q_{k_3}$ and $S_i \in q_{k_4}$, then $S_i \in q_k$ ($k < k_3, k < k_4$). For example, the combination of $q_3 = \{S_1 S_3 S_4\}$ in D_{1i} and $q_3 = \{S_1 S_2 S_3\}$ in D_{3i} can produce $q_2 = \{S_1 S_3\}$.

$$CN = \sum_{k_3=k+1}^m \sum_{k_4=k+1}^m \frac{C_{k_3}^k C_{m-k_3}^{k_4-k} |Q_{k_3}| |Q_{k_4}|}{C_m^k} \quad (10)$$

CN refers to combination number. Q_{k_3} and Q_{k_4} have $|Q_{k_3}| |Q_{k_4}|$ kinds of combination and only $\frac{C_{k_3}^k C_{m-k_3}^{k_4-k}}{C_m^k}$ is the logical item. Considering that the sum of different k_3 and k_4 can be k , double summation is done. The combination process may produce repeated q_k (SN and CN both produce it). The repetition rate RR of q_k is

$$RR = \frac{1}{C_m^k} \quad (11)$$

$$|Q_k| = (SN + CN)(1 - RR) \quad (12)$$

3.2. Modification of Parameter Set Q_k

Although the last part can eliminate all nonlogic items and reduce $|Q_k|$, $|Q_k|$ is still extremely large and will generate a lot of combination, which takes a long time. Actually, only a few q_k in Q_k eventually become part of the optimal test sequence binary tree, and the elimination of other q_k does not influence the optimal decision tree. Three points can be used to make the elimination. Firstly, one q_k includes a series of logical number and k faults. The features of q_k is an array but not a number, so each q_k should be analyzed independently. Secondly, the number of eigenvalues of q_k is limited. There is no need to perform principal component analysis to lower the eigenvalues number. Thirdly, the judgement function is generated before the actual test and the sample is sufficient. Therefore, the longer the training time, the better. SVM (support vector machine) is used to distinguish which q_k should be retained and which q_k should be eliminated. The definitions are presented as follows:

- If q_{k_1} and q_{k_2} can be combined into q_k , then q_{k_1} and q_{k_2} are a pair of couples in Q_k ($k > k_1, k > k_2$);
- If q_k can be composed of many pairs of couples, then the least cost pair of couples is called a pair of spouses.

Taking the matrix of Table 3 as an example, $S_1 S_3 S_4$ can be composed of $S_1 S_3$ and S_4 , or $S_3 S_4$ and S_1 . Therefore, $S_1 S_3$ and S_4 , $S_3 S_4$ and S_1 are two couples in Q_3 . According to Formula (7), the test costs of the two combinations are 2.75 and 3.25, respectively, and $S_1 S_3$ and S_4 are a pair of spouses in Q_3 . Four parameters are given as follows:

1. $Cnumber_f$: the number of couples q_k generates in $Q_{k+1}, Q_{k+2}, \dots, Q_f$ ($f > k$);
2. $Snumber_f$: the number of spouses q_k generates in $Q_{k+1}, Q_{k+2}, \dots, Q_f$ ($f > k$);
3. $ratio_f$: $ratio_f = Snumber_f / Cnumber_f$. $ratio_f \leq 1$;
4. $Xnumber$: The number of d (irrelevant items) in the logic values of the Karnaugh map corresponding to q_k .

The size of $Cnumber_f$ depends on the logic of q_k , i.e., the distribution of 0 and 1 in the D matrix. The larger $Cnumber_f$, the stronger ability of q_k to combine other fault subsets. On the premise that q_k has been combined with another fault subset to form a pair of couples, if the cost according to Formula (7) is lower than that of other couples, it becomes a pair of spouses. The larger the $ratio_f$, the greater the advantage represented by the cost. According to the analysis in the previous section, the smaller the $Xnumber$ of q_k , the smaller the circle of the Karnaugh map, and the higher the probability of combination with other fault subsets.

If q_k is a part of the optimal test sequence binary tree, then it has the advantage of more times or cost of combining with other fault subsets. Besides, the proportion of "couple"

to “spouse” is higher. As a result, $Cnumber_m$ or $ratio_m$ is higher. However, $Cnumber_m$ and $ratio_m$ are unavailable until Q_m is generated. This method aims to predict $Cnumber_m$, $ratio_m$ through $Cnumber_{k+1} \cdots Cnumber_f$, $ratio_{k+1} \cdots ratio_f$, $Xnumber$ and other parameters, that is, the predication of $Cnumber_m$ and $ratio_m$ through $Q_{k+1} \cdots Q_f (f > k)$. If the prediction result are low, then q_k is eliminated to decrease $|Q_k|$, $f = k + \Delta k$, Δk is a predetermined value. In the current experiment, $f = k + 1$. Each $Q_b (k < b < f + 1)$ can generate a set of eigenvalues to classify q_k . To make the classification more accurate, q_k will be eliminated, if all the classification values according to Δk sets of eigenvalues are 0. The prediction function is obtained by the statistics of sample performance.

3.2.1. Condition Judgement Module *DES*

Not every $|Q_k|$ needs to be reduced by SVM, and Q_k is worth modifying if the time used to statistics q_k eigenvalues and determine the category $\leq$ the time taken of q_k participating in the generation of $Q_{k+1}, Q_{k+2}, \cdots, Q_m$. The Q_k that needs to be modified is denoted as *DES*, and the algorithm enters the modification module only when $k \in DES$; otherwise, $k = k + 1$. Suppose that 0 and 1 in *D* obey a uniform distribution, and the number of *d* in the logic sequence of q_2 obeys the normal distribution with $n/2$ as the mean. Similarly, q_k obeys the normal distribution with $n - \frac{n}{2^{k-1}}$ as the mean value. From the analysis of the last section, if the number of *d* of a q_k is *n*, when $k \neq m$, the corresponding circle of a Karnaugh graph is the whole Karnaugh graph, and thus this q_k must be a nonlogic item. Therefore, the number of *d* of logically valid q_k is at least $n - 1$. Therefore, when $|Q_k|$ satisfies $k - 1 > \log_2^n$, it is wasteful to modify. The largest element in $DES < \log_2^n + 1$. The smaller the value of *k*, the more benefit after Q_k modification because it will participate in a large number of combinations. For example, in the *D* matrix of 30×30 , the values of $|Q_2|$, $|Q_3|$, $|Q_4|$, $|Q_5|$, and $|Q_6|$ are much larger than other $|Q|$. Consider choosing the smaller *k* as much as possible, that is, $DES = \{2, 3, 4\}$.

3.2.2. Prediction Function

The definitions are given as follows:

- r_i : The proportion of 0 and 1 of *D* array, $r_i = \frac{\max\{\text{the number of 0 in } D_{xi}, \text{the number of 1 in } D_{xi}\}}{\min\{\text{the number of 0 in } D_{xi}, \text{the number of 1 in } D_{xi}\}}$
- D_ratio : the average of r_i . $D_ratio = \frac{1}{n} \sum_{i=1}^n r_i$

Samples and test objects should have the same D_ratio , *P* matrix approximation and *C* matrix approximation. In the case of no prior probability distribution, D_ratio is 0.5, P_i obeys the normal distribution with the mean value of $1/m$, and C_i obeys the uniform distribution within a certain range. The prediction function is shown in Algorithm 2. $k \notin DES$.

Algorithm 2 Prediction function

- 1: According to the test objects *D*, *P* and *C*, generate several similar samples;
 - 2: Take a sample, and calculate $Q_1, Q_2, \cdots, Q_f$ in turn;
 - 3: Take an element *k* in the *DES*. Statistics $Cnumber_{k+1} \cdots Cnumber_f$, $ratio_{k+1} \cdots ratio_f$, $Xnumber$. Take $Cnumber_{k+1}$, $ratio_{k+1}$, $Xnumber, k + 1, m, n$ as the first set of eigenvalues. Take $Cnumber_{k+2}$, $ratio_{k+2}$, $Xnumber, k + 2, m, n$ as the second set of eigenvalues. $\cdots$ Take $Cnumber_{k+\Delta k}$, $ratio_{k+\Delta k}$, $Xnumber, k + \Delta k, m, n$ as the Δk th set of eigenvalues. Statistics each q_k $Cnumber_m$ and $ratio_m$, and calculate the average value: *average 1*, *average 2*;
 - 4: If $Cnumber_m$ and $ratio_m$ are lower than *mod1-average 1* and *mod2-average 2*, the q_k is marked as 0; otherwise, it is marked as 1 (*mod1* and *mod2* are coefficient parameters, which are taken as 1 in this experiment);
 - 5: Delete this element *k* in *DES*, and return to step 3 until $DES = \emptyset$;
 - 6: The eigenvalues obtained in step 2 and the tag values obtained in step 4 are used as the training matrix. Return to step 2 until all samples are calculated, and use all of the training matrix to train the prediction function.
-

3.2.3. Statistical Eigenvalue Module

Each q_k has j groups of eigenvalues. Only if all the classification value calculated by the j groups of eigenvalues is 0, q_k will be deleted. However, after calculating Q_{k+1} , some q_k satisfies the first set of classification values of 1, which can no longer attract the attention. The next focus is those q_k with a classification value of 0. If we continue to calculate Q_{k+2} directly, q_k , which has been determined to be 1, will participate, causing useless calculation. Therefore, the simulated annealing algorithm is used to obtain all the eigenvalue without calculating complete $Q_{k+\Delta k}$. The statistical eigenvalue module is displayed in Algorithm 3.

Algorithm 3 Statistical eigenvalue

- 1: Through Q_{k+1} , the eigenvalues $Cnumber_{k+1}$, $ratio_{k+1}$, $Xnumber_{k+1}$ are obtained, and the $q_k(i)$ with classification value 0 is included in the set E , $j = 1$;
 - 2: If $j = \Delta k$, then go to step 5. Create an array: *EIGEN* to record the eigenvalues. Randomly take the subsets of $Q_1, Q_2, \dots, Q_{k-1}, E, Q_{k+1}, \dots, Q_{k+j}$ as $Q_1^*, Q_2^*, \dots, Q_{k-1}^*, E^*, Q_{k+1}^*, \dots, Q_{k+j}^*$ (The size of the subset is $1/mod_stay$ of the total set; in this paper, mod_stay is 10), use $Q_1^*, Q_2^*, \dots, Q_{k-1}^*, E^*, Q_{k+1}^*, \dots, Q_{k+j}^*$ to calculate Q'_{k+j+1} ;
 - 3: Record all the couples and spouses in each E , and add the couples and spouses not included in the *EIGEN* into it;
 - 4: Return to step 2; if the eigenvalue is not updated in multiple circles, it shows that the statistics of couples and spouses are completed, the eigenvalues $Cnumber_{k+1+j}$ and $ratio_{k+1+j}$ are obtained, and the elements in E take $Cnumber_{k+1+j}$, $ratio_{k+1+j}$, $Xnumber_{k+1+j}$ as the eigenvalue to obtain the classification values. Delete the element in E with classification value of 1, $j = j + 1$, empty the *EIGEN*, and return to step 2;
 - 5: Finally, the remaining elements in E are the elements to be eliminated.
-

3.2.4. Entire Process of Modification of the Parameter Set

In summary, the entire process of parameter set modification is shown in Algorithm 4.

Algorithm 4 Q_k Modification

- 1: Judge whether $k \in DES$; if yes, go to step 2, and otherwise, go to step 4;
 - 2: Calculate Q_{k+1} in the same way as the Q_k production part;
 - 3: Enter the statistical eigenvalue module, use the prediction function to obtain the classification value of Q_k , and eliminate the q_k with the classification value of 0;
 - 4: $k = k + 1, i = 1$.
-

The size of the modified $|Q_k|$ depends on four parameters: $mod1$, $mod2$, Δk and the number of samples. The larger $mod1$ and $mod2$, the smaller $|Q_k|$ will be. However, the accuracy of the results will be reduced because the components of the optimal test sequence tree may be removed. The larger the value of Δk , the more q_k is retained, and the larger the value of $|Q_k|$, the higher the accuracy will be, but with a longer calculation. When the number of samples is sufficiently large, even if $mod1$ and $mod2$ are large and Δk is small, the accuracy of the results is guaranteed. The prediction function is calculated in advance and is not included in the test time.

3.3. Acquisition of the Switch Condition Boundary

Top-down and bottom-up algorithms have their own advantages. They have the following characteristics:

- In the top-down algorithm. $|Q_1|, |Q_2|, \dots, |Q_m|$ is not affected by P and C when $|DES| = 0$; i.e., the calculation time is unchanged. After adding the prediction function, $|Q_1|, |Q_2|, \dots, |Q_m|$ is less affected by P and C , and the calculation time is stable. When the cost of each test point and the probability of each fault are similar, the time of top-down search is short. When the cost of each test point and the probability of each fault are much different, the time of top-down search is long;

- The bottom-up algorithm has an advantage when the number of test points $|T|$ is much larger than the number of faults $|S|$ because the bottom-up algorithm takes faults as the core and calculates the combination of its subsets. The upper limit of calculation times is

$$|Q_1| + |Q_2| + \dots + |Q_m| \leq 2^m - 1 \quad (13)$$

- The core of bottom-up algorithm is comparing the characteristic of each test point. When the number of test points $|T|$ is large, the calculation time is long. In contrast, when the number of test points $|T|$ is much less than the number of faults $|S|$, top-down search has more advantages.

In large-scale tests such as $m = n = 100$, the top-down search method is adopted. For the third level of the test sequence binary tree, as shown in Figure 7, the fault matrix problem of 100×100 is transformed into solving four 25×98 . The top-down algorithm has more advantages for the transformation and the bottom-up algorithm has more advantages for the 25×98 problem. The boundary is that the computation time of the bottom-up algorithm is the same as that of bottom-up algorithm. This boundary is obtained by sample simulation. Under the conditions of the same D , P , and C , we compare the computational efficiency of the two algorithms with different scales.

Figure 7. Part of a test sequence tree.

4. Experiment

The experiment is divided into two parts. One part compares the efficiency of the hybrid algorithm (the combination of bottom up and cost heuristic algorithm in this paper) and the algorithm [14] in dealing with a large-scale fault-test dependency matrix with simulation data. The other part compares the time spent and expected cost of the hybrid algorithm and algorithm [14,29] in dealing with the superheterodyne receiver example [30].

4.1. Large-Scale Fault-Test Dependency Matrix

The experimental data are based on different scales of the D matrix, the D_ratio and the distribution of C_j . According to the parameter settings, 8 groups of samples were randomly generated, and the average value was calculated. This experiment runs on MATLAB 2018b on a 3.60 GHz, 16.0 GB RAM desktop.

D-matrix scale change: $D_ratio = 1$, C_j obeys the uniform distribution of (1,2), and P_i^* obeys the uniform distribution of (0,1), $P_i = \frac{P_i^*}{\sum_{k=1}^m P_i^*}$, the size of D-matrix changes from 60×60 to 80×80 , and the interval is 5. The result is shown in Figure 8.

Figure 8. Average calculation time of different D-matrix scale.

D_ratio change: The size of D-matrix is 60×60 , C_j obeys the uniform distribution of (1,2), P_i^* obeys the uniform distribution of (0,1), and $P_i = \frac{P_i^*}{\sum_{k=1}^m P_i^*}$, $D_ratio = 1, 21/19, 22/18, 23/17, 24/16, 12/8$. The result is shown in Figure 9.

Figure 9. Average calculation time of different D_ratio .

Distribution of C_j change: The size of the D-matrix is 60×60 , $D_ratio = 1$, P_i^* obeys the uniform distribution of (0,1), and $P_i = \frac{P_i^*}{\sum_{k=1}^m P_i^*}$, and C_j obeys the uniform distribution of (1,2), (1,10), (1,20), (1,50) and (1,100). The result is shown in Figure 10.

Figure 10. Average calculation time of different C_j .

The boundary is determined by comparing the time of two algorithms under different settings. For example, in the D-matrix scale change experiment, the scale of 80×80 is taken as an example. The experiment is shown in Table 4. The rows represent the number of failures $|S|$, and the columns represent the number of test points $|T|$, where 1 means that the efficiency of the bottom-up algorithm is higher, and 0 means that it is lower. It can be observed from the figure that when $65 < |T| < 80$, $|S| < 10$, the bottom-up algorithm has more advantages—i.e., when the heuristic search node size is less than 10—the bottom-up algorithm is used.

Table 4. Boundary of 80×80 D.

$ T \backslash S $	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
4	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
5	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
7	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
8	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1
9	1	1	1	1	1	1	1	1	1	1	0	1	0	1	0	0
10	1	1	1	1	1	1	0	1	0	1	1	1	0	0	1	1
11	0	1	0	0	1	1	0	0	0	0	0	0	0	0	0	0
12	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
13	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

The experimental results show that when the C gap is smaller, the D_ratio is smaller, the calculation time is longer, and the combination algorithm is better than the rollout and information heuristic algorithm because the heuristic values of each measurement point are similar, which all need to be expanded. The more times the extended nodes switch to the bottom-up algorithm, the higher the efficiency.

4.2. Superheterodyne Receiver Example

In this experiment, the example is the five-tube superheterodyne receiver originally designed for the U.S. Navy [30,31], the system consists of 36 different tests and 22 failure states, and all the test costs are 1, as shown in Table 5. This experiment runs on MATLAB 2018b on a 3.60 GHz, 16.0 GB RAM desktop.

Calculation time: This information is shown in Table 6. The search time of rollout and information heuristic algorithm [14] is more than 15 min. Because the test cost is 1 and the failure probability is similar, the heuristic function value of each test point is similar, the number of test points is greater and the number of failures is less, which leads to the long operation time for the heuristic search in this type of problem. The algorithm runs from bottom to top with $DES = \emptyset$, takes 88.45 s, and the optimal test cost is 3.3473. The test sequence is shown in Table 7. For example, S_1 , and after 7 test points, $T_{34}, T_{30}, T_{26}, T_{31}, T_{25}, T_{12}, T_{35}$ are finally isolated. In combination with SVM, $DES = \{2, 3, 4\}$ takes 60.97 s, but the computational cost depends on the number of samples generated. When the number of samples exceeds 30, it is easy to obtain the minimum value.

Table 5. Test matrix and fault probabilities for the superheterodyne receiver example.

[illegible]

Table 6. Comparison of calculation time.

Algorithm	Rollout and Information Heuristic Algorithm	Hybrid Algorithm ($DES=\emptyset$)	Hybrid Algorithm ($DES=\{2,3,4\}$)
Time	More than 15 min	88.45 s	60.97 s

Table 7. Test sequence.

Sequence S	1	2	3	4	5	6	7	8
1	T_{34}	T_{30}	T_{26}	T_{31}	T_{25}	T_{12}	T_{35}	-
2	T_{34}	T_8	T_{19}	T_{26}	-	-	-	-
3	T_{34}	T_{30}	T_{26}	-	-	-	-	-
4	T_{34}	T_{30}	T_{26}	T_{31}	T_{25}	T_{21}	T_{22}	T_{13}
5	T_{34}	T_{30}	T_{26}	T_{31}	T_{25}	T_{12}	T_{35}	-
6	T_{34}	T_8	T_{19}	T_{35}	-	-	-	-
7	T_{34}	T_{30}	T_{28}	T_{29}	T_{35}	T_{25}	-	-
8	T_{34}	T_{30}	T_{28}	T_{29}	-	-	-	-
9	T_{34}	T_{30}	T_{28}	-	-	-	-	-
10	T_{34}	T_{30}	T_{26}	T_{31}	-	-	-	-
11	T_{34}	T_8	T_{35}	-	-	-	-	-
12	T_{34}	T_{30}	T_{26}	T_{31}	T_{25}	T_{21}	T_{22}	-
13	T_{34}	T_{30}	T_{26}	T_{31}	T_{25}	T_{12}	-	-
14	T_{34}	T_8	T_{35}	-	-	-	-	-
15	T_{34}	T_{30}	T_{26}	T_{31}	T_{25}	T_{21}	-	-
16	T_{34}	T_{30}	T_{26}	T_{31}	T_{25}	T_{21}	T_{22}	T_{13}
17	T_{34}	T_8	T_{19}	T_{35}	-	-	-	-
18	T_{34}	T_{30}	T_{28}	T_{29}	T_{35}	T_{31}	-	-
19	T_{34}	T_8	T_{19}	T_{26}	T_{21}	-	-	-
20	T_{34}	T_{30}	T_{28}	T_{29}	T_{35}	T_{31}	-	-
21	T_{34}	T_{30}	T_{28}	T_{29}	T_{35}	T_{25}	-	-
22	T_{34}	T_8	T_{19}	T_{26}	T_{21}	-	-	-

Test cost: This information is shown in Table 8. According to reference [29], the optimal cost of the genetic algorithm is 3.9526 and that of the information gain algorithm is 6.9124. The cost of the hybrid algorithm is 3.3473, which is minimal.

Table 8. Comparison of cost.

Algorithm	Genetic Algorithm	Information Gain Algorithm	Rollout and Information Heuristic Algorithm	Hybrid Algorithm
Cost	3.9526	6.9124	3.3473	3.3473

Because of the uncertainty of the algorithms, the worst case time complexity is compared. In the worst case, only one fault can be isolated for each test, so the decision tree has m layer and $m - 1$ test points. Each test point has $n, n - 1, \dots, n - m + 2$ choices respectively. n^{m-1} times calculation is required and the worst case time complexity of Information gain algorithm and rollout and information heuristic algorithm is $O(n^{m-1})$. The worst complexity of genetic algorithm is $O(Max_gen \cdot \mu)$, Max_gen refers to the maximum number of iterations and μ refers to the population size. For the bottom-up algorithm, according to the analysis in Section 3.3, the calculation time is $2^m - 1$ and each time has n loops. Therefore, the worst-case time complexity is $O(n \cdot 2^m)$. The worst-case time complexity of the hybrid algorithm is $O(n^{m-k} + n \cdot 2^k)$, k refers to the number of fault sources of the boundary.

The above experiments show that the bottom-up algorithm has advantages in cost and computing time when dealing with this type of problem.

5. Conclusions

In this paper, the problem of the test sequencing for large-scale systems is considered. A novel bottom-up algorithm based on the Karnaugh map, SVM, and the simulated annealing algorithm is proposed. When the heuristic values of each test point are similar or the number of faults is far less than the number of test points, this algorithm is suitable. Furthermore, decision tree processing strategy for large-scale D-matrix is proposed, which can combine the advantages of bottom-up and top-down algorithms. It can switch the algorithm based on the relationship between the number of test points and the number of faults of the node. Moreover, the strategy can be applied to sequential fault diagnosis of large complex equipment, and can be used in testability optimization of equipment. Experiments under different parameter settings are carried and demonstrate that this method has a short operation time and high efficiency when dealing with a large-scale D matrix. More research should be carried out to determine prior knowledge of C, P, and *D_ratio*, which can significantly affect the efficiency.

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