## Article

# Accurate Analysis and Design of Integrated Single Input Schmitt Trigger Circuits 

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#### Abstract

Schmitt trigger (ST) circuits are widely used integrated circuit (IC) blocks with hysteretic input/output (I/O) characteristics. Like the I/O characteristics of a living neuron, STs reject noise and provide stability to systems that they are deployed in. Indeed, single-input/single-output (SISO) STs are likely candidates to be the core unit element in artificial neural networks (ANNs) due not only to their similar I/O characteristics but also to their low power consumption and small silicon footprints. This paper presents an accurate and detailed analysis and design of six widely used complementary metal-oxide-semiconductor (CMOS) SISO ST circuits. The hysteresis characteristics of these ST circuits were derived for hand calculations and compared to original design equations and simulation results. Simulations were carried out in a well-established, $0.35 \mu \mathrm{~m} / 3.3 \mathrm{~V}$, analog/mixed-signal CMOS process. Additionally, simulations were performed using a wide range of supplies and process variations, but only 3.3 V supply results are presented. Most of the new design equations provide better accuracy and insights, as broad assumptions of original derivations were avoided.


Keywords: schmitt trigger; artificial neural networks; hysteresis circuits

## 1. Introduction

Artificial neural networks (ANNs) are the core of artificial intelligence (AI) in next generation systems that mimic the parallel processing capabilities of the human brain. One important characteristic of the distributed processing element of the brain, the neuron, is to deal with chaos through its hysteretic I/O response [1]. It is shown that this characteristic of a neuron makes ANNs stable [2] and converge more rapidly [3]. Additionally, the artificial neuron has to be small and consume minimal power to be able to be integrated into mass numbers [4].

The Schmitt trigger (ST) has been used in both analog and digital domains to improve the noise immunity of circuits, thanks to its programmable or hard-wired hysteresis characteristics [5-11]. This characteristic has been utilized in many CMOS circuit blocks including oscillators [12-15], input/output pads of integrated circuits [16,17], image sensors [18-24], triangular carrier-based PWM modulators [25], subthreshold SRAMs [26-29], CMOS transceivers [30-34], impedance-to-frequency converters [35], digital to analog converters (DACs) [36], neuron-based analog to digital converters (ADCs) [37-39], powerline communication systems [40], binary logic circuits (i.e., adders [41] and gates [42]), and sensors [43,44].

CMOSSTs can be categorized based on their mode of operation (voltage or current), inputs (single or differential input), outputs (inverting or noninverting), and hysteresis controls (fixed or programmable). The simplest and most compact STs are the ones with fixed hysteresis, and single voltage input and
single voltage output types. Six well known single input/single output ST topologies are investigated in this paper: Dokic [5] (three types: N, P, and CMOS), Steyaert [6], Pedroni [7], and Al-Sarawi [8]. In this paper, we show how to derive the hysteresis voltages accurately for these STs, and determine their design limitations and sensitivities to process variations. For the analysis and design of an ST circuit, three fundamental input-output (I/O) parameters are considered: high-to-low switching voltage $\left(V_{H L}\right)$, low-to-high switching voltage $\left(V_{L H}\right)$, and hysteresis voltage $\left(\Delta V_{H}=V_{H L}-V_{L H}\right)$, as shown in Figure 1. The hysteresis offset $\left(V_{H O}\right)$ in Figure 1 can be calculated as ( $V_{H O}=V_{L H}+\Delta V_{H} / 2$ ).


Figure 1. I/O characteristics of a voltage mode, inverting ST circuit.
Detailed analysis and the hand calculation equations of each topology are presented in Section 2. Each topology is extensively simulated at different corners of the selected CMOS process. The simulation results are presented in Section 3, as are the comparisons between hand calculations and the simulation results of each topology, in addition to the comparisons between the six topologies. The conclusion is presented in Section 4.

## 2. Analysis of Schmitt Trigger (ST) Circuits

Six well known single input and single output ST topologies and their variants are analyzed in this section, providing transistor level and more accurate and intuitive design equations. They are Dokic [5] (three types), Steyaert [6], Pedroni [7], and Al-Sarawi [8] STs. We used long-channel MOSFET models and high supply voltage process in this section. Equations (1) and (2) are the quadratic MOSFET transistor model equations that were used for the analysis in saturation (SAT) and linear/triode (LIN) regions, respectively [45]. The threshold voltage equation was modified slightly, linearizing bulk-to-source voltage dependency as $V_{t h x}=V_{\text {th } 0}+\psi \cdot V_{S B}$. Here, $\psi$ is defined as $\psi=n \cdot G A M M A \cdot P H I$, where $G A M M A$ is the back-gate effect parameter, $P H I$ is the surface potential, and $n$ is a fitting parameter $(0.3<n<0.5)$ which is determined through the simulation.

$$
\begin{align*}
I_{D S}=\beta\left(V_{G S}-V_{T H}\right)^{2} \quad \text { for } \quad V_{D S} \geq V_{G S}-V_{T H} \quad \text { (Saturation) }  \tag{1}\\
I_{D S}=\beta\left(2\left(V_{G S}-V_{T H}\right) V_{D S}-V_{D S}^{2}\right) \quad \text { for } \quad V_{D S}<V_{G S}-V_{T H} \quad \text { (Linear) } \tag{2}
\end{align*}
$$

where

$$
\begin{equation*}
\beta=\frac{1}{2} K P\left(\frac{W}{L}\right) \tag{3}
\end{equation*}
$$

### 2.1. Dokic Schmitt Trigger Circuits

Dokic proposed three ST topologies in [5]: N-type, P-type, and CMOS-type. These topologies are investigated and detailed, and more accurate design equations for $V_{H L}, V_{L H}$, and $\Delta V_{H}$ are derived.

### 2.1.1. N-Type ST by Dokic

Figure 2a shows the N-type Dokic ST [5]. It is composed of four transistors and its hysteresis is shown in Figure 2b. Depending on how the input signal changes, two I/O characteristics can be observed. If the input goes from low (0) to high $\left(V_{D D}\right)$, the output changes from high to low at $V_{H L}$.

If the input goes from high $\left(V_{D D}\right)$ to low (0), the output changes from low (0) to high ( $V_{D D}$ ) at $V_{L H}$. The $V_{H L}$ and $V_{L H}$ can be found when the input and output voltages are equal to each other at operating points OP1 and OP2, respectively, as marked in Figure 2b.


Figure 2. N-type Dokic ST: (a) circuit diagram, (b) input-output characteristics, and (c) modeled equivalent circuit when M4 is OFF.

## $V_{H L}$ Voltage

From Figure 2a, in the steady-state and when input is 0 V , the transistors M1 and M2 are OFF, M3 is in the deep LIN region where $V_{d s 3}$ is close to 0 V , and M4 can be considered in the subthreshold region, where $V_{g s 4}<V_{t h n 4}$ while the output is $V_{D D}$. The node voltage $V_{1}$ would be $\left(V_{D D}-V_{t h n 4}\right)$ and rising. When the input is increased and greater than $V_{\text {thn } 0}$, M1 turns ON, discharging $V_{1}$ to a switching point. Since $V_{s b 2}=V_{s b 4}>V_{t h n 0}$, M2 is still OFF and M4 is in SAT. M2 turns ON only when $V_{\text {in }}$ is $V_{t h n 0}$ above the $V_{1}$ voltage. Before M2 turns on, the $V_{1}$ can be found by equating the drain currents of transistors M1 and M4. The general $V_{1}$ equation, including body-effect where $V_{t h n 4} \approx V_{t h n 0}+\psi \cdot V_{1}$, is:

$$
\begin{equation*}
V_{1}=\frac{V_{D D}-V_{\text {thn } 0} \cdot\left(1-\alpha_{1 n}\right)-\alpha_{1 n} V_{i n}}{(1+\Psi)} \tag{4}
\end{equation*}
$$

where

$$
\begin{equation*}
\alpha_{1 n}=\sqrt{\frac{\beta_{1}}{\beta_{4}}} \tag{5}
\end{equation*}
$$

When $V_{\text {in }}$ further increases, $V_{1}$ drops further, and finally, M2 turns ON. Then, the output node starts discharging, first turning M4 OFF and then turning M3 ON during this high to low transition. M2 turns ON when $V_{\text {in(min })}>V_{1}+V_{\text {thn2 }}$. $V_{\text {in (min })}$ could be considered as the threshold voltage of the series combination of M1 and M2, or $V_{\text {thnx }}$. Using (4), it can be found as follows:

$$
\begin{gather*}
V_{\text {in }(\text { min })} \geq V_{1}+V_{\text {thn } 2}=V_{1}+V_{\text {thn } 0}+\Psi \cdot V_{1}=V_{D D}+\alpha_{1 n} V_{\text {thn } 0}-\alpha_{1 n} V_{\text {in }(\text { min })}  \tag{6}\\
V_{\text {thn }}=V_{\text {in }(\text { min })} \geq \frac{V_{D D}+\alpha_{1 n} \cdot V_{\text {thn } 0}}{\left(1+\alpha_{1 n}\right)} \tag{7}
\end{gather*}
$$

When $V_{i n}=V_{H L}$, Dokic [5] assumes that M1 works in LIN, and M2 and M3 are in SAT regions, while M4 is OFF. However, series transistors M1 and M2 (Figure 2c) work in two different operation regions (M1 in LIN, M2 in SAT) that could be simplified into a single NMOS ( $\mathrm{M}_{\mathrm{x}}$ ) transistor working in SAT with an equivalent threshold voltage of $V_{\operatorname{thnx}}$, and $\beta=\beta_{x n}$, as follows:

$$
\begin{gather*}
I_{d s 1}=\beta_{1} \cdot\left(2 \cdot\left(V_{\text {in }}-V_{\text {thn } 0}\right) \cdot V_{1}-V_{1}^{2}\right)  \tag{8}\\
V_{1}=+\left(V_{\text {in }}-V_{\text {thn } 0}\right) \pm \sqrt{\left(V_{\text {in }}-V_{t h n 0}\right)^{2}-\frac{I_{d s 1}}{\beta_{1}}} \tag{9}
\end{gather*}
$$

$$
\begin{equation*}
I_{d s 2}=\frac{1}{2} K P_{n}\left(\frac{W}{L}\right)_{2}\left(V_{i n}-V_{1}-V_{t h n 2}\right)^{2}=\beta_{2}\left(V_{i n}-V_{1}-V_{t h n 0}-\Psi \cdot V_{1}\right)^{2} \tag{10}
\end{equation*}
$$

using (9) in (10):

$$
\begin{equation*}
I_{d s 2}=\beta_{2} \cdot\left(V_{i n}-(1+\Psi) \cdot\left[+\left(V_{\text {in }}-V_{\text {thn } 0}\right) \pm \sqrt{\left(V_{\text {in }}-V_{\text {thn } 0}\right)^{2}-\frac{I_{d s 1}}{\beta_{1}}}\right]-V_{\text {thn } 0}\right)^{2} \tag{11}
\end{equation*}
$$

and assuming that $V_{t h n 1} \cong V_{t h n 2}=V_{t h n 0}$, where $\psi=0$, and $I_{d s 1}=I_{d s 2}=I_{d s x}$, Equation (11) becomes,

$$
\begin{gather*}
I_{d s x}=\beta_{2} \cdot\left[\left(V_{i n}-V_{t h n 0}\right)^{2}-\frac{I_{d s x}}{\beta_{1}}\right]  \tag{12}\\
I_{d s 1}=I_{d s 2}=I_{d s x}=\left[\frac{\beta_{1} \cdot \beta_{2}}{\beta_{1}+\beta_{2}}\right]\left(V_{i n}-V_{t h n 0}\right)^{2}=\beta_{x n} \cdot\left(V_{i n}-V_{t h n 0}\right)^{2} \tag{13}
\end{gather*}
$$

where

$$
\begin{equation*}
\beta_{x n}=\left[\frac{\beta_{1} \cdot \beta_{2}}{\beta_{1}+\beta_{2}}\right] \tag{14}
\end{equation*}
$$

The $V_{H L}$ of the N-type Dokic ST can now be found by equating the drain currents of the transistors M3 and Mx shown in Figure 2c, assuming that both are working in SAT region at OP1:

$$
\begin{gather*}
V_{H L}=\frac{V_{D D}-\left|V_{t h p 0}\right|+\alpha_{2 n} \cdot V_{t h n 0}}{\left(1+\alpha_{2 n}\right)}+\frac{\alpha_{2 n} \cdot\left(V_{D D}-V_{t h n 0}\right)}{\left(1+\alpha_{1 n}\right)\left(1+\alpha_{2 n}\right)}  \tag{15}\\
\text { where, } \alpha_{1 n}=\sqrt{\frac{\beta_{1}}{\beta_{4}}} \text { and } \alpha_{2 n}=\sqrt{\frac{\beta_{x n}}{\beta_{3}}} \tag{16}
\end{gather*}
$$

## $V_{L H}$ Voltage

The $\mathrm{V}_{\mathrm{LH}}$ can be found by considering M4 as being OFF right before the output transition from low to high which occurs at OP2 $\left(V_{\text {in }}=V_{\text {out }}=V_{L H}\right)$, and by equating the drain current of M3 to that of $\mathrm{M}_{\mathrm{x}}$, which both work in SAT. It is important to note here that the current of $\mathrm{M}_{\mathrm{x}}$ is equal to that of M2 (as in (13)) in which $\beta_{x n}, V_{t h n 0}$, and $\alpha_{1 n}$, and $\alpha_{2 n}$ from Equation (16) are used.

$$
\begin{equation*}
V_{L H}=\frac{V_{D D}-\left|V_{t h p 0}\right|+\alpha_{2 n} \cdot V_{t h n 0}}{\left(1+\alpha_{2 n}\right)} \tag{17}
\end{equation*}
$$

Hysteresis Voltage
The hysteresis voltage of the N-type Dokic ST can then be derived by using (15) and (17) as:

$$
\begin{equation*}
\Delta V_{H}=V_{H L}-V_{L H}=\frac{\alpha_{2 n} \cdot\left(V_{D D}-V_{\text {thn0 }}\right)}{\left(1+\alpha_{1 n}\right)\left(1+\alpha_{2 n}\right)} \tag{18}
\end{equation*}
$$

Compared to Dokic's original hysteresis equation (Equation (7) in [5]), (18) provides a transistor-level design strategy, that the hysteresis voltage could be maximized by minimizing $\alpha_{1 n}$ and by maximizing $\alpha_{2 n}$ parameters while ensuring that the individual transistor parameters satisfy $\beta_{4}>\beta_{1}$ and $\beta_{3}<\beta_{x n}$.

### 2.1.2. P-Type ST by Dokic

Figure 3 shows the P-type Dokic ST [5]. It is the complementary version of the N-type and has the same hysteresis characteristics. Transistors are numbered the same as the N -type as shown in Figure 2 and the analysis is carried out following the same process presented in the previous section. The $V_{H L}$
and $V_{L H}$ of the P-type Dokic ST can be found when input and output voltages are equal at operating points OP1 and OP2 as shown in Figure 3.


Figure 3. P-type Dokic ST [5]. (a) Circuit diagram and (b) input-output characteristics.
The design equations and parameters for the P-type Dokic ST can be derived as:

$$
\begin{gather*}
V_{L H}=\frac{\alpha_{2 p} \cdot V_{\text {thn } 0}}{\left(1+\alpha_{2 p}\right)}+\frac{\alpha_{1 p} \cdot\left(V_{D D}-\left|V_{\text {thp0 }}\right|\right)}{\left(1+\alpha_{1 p}\right)\left(1+\alpha_{2 p}\right)}  \tag{19}\\
V_{H L}=\frac{V_{D D}-\left|V_{\text {thp0 }}\right|+\alpha_{2 p} \cdot V_{\text {thn } 0}}{\left(1+\alpha_{2 p}\right)} \tag{20}
\end{gather*}
$$

and the hysteresis voltage:

$$
\begin{equation*}
\Delta V_{H}=V_{H L}-V_{L H}=\frac{\left(V_{D D}-\left|V_{t h p 0}\right|\right)}{\left(1+\alpha_{1 p}\right)\left(1+\alpha_{2 p}\right)} \tag{21}
\end{equation*}
$$

where

$$
\begin{equation*}
\alpha_{1 p}=\sqrt{\frac{\beta_{1}}{\beta_{4}}}, \alpha_{2 p}=\sqrt{\frac{\beta_{3}}{\beta_{x p}}}, \beta_{x p}=\left[\frac{\beta_{1} \cdot \beta_{2}}{\beta_{1}+\beta_{2}}\right], \text { and }\left|V_{t h p x}\right|=V_{D D}-\frac{\alpha_{1 p} \cdot\left(V_{D D}-\left|V_{t h p 0}\right|\right)}{\left(1+\alpha_{1 p}\right)} \tag{22}
\end{equation*}
$$

Compared to Dokic's original hysteresis equation (Equation (13) in [5]), (21) provides a transistor-level design strategy, that the hysteresis voltage could be maximized by minimizing both $\alpha_{1 p}$ and $\alpha_{2 p}$ parameters while ensuring the individual transistor parameters satisfies $\beta_{4}>\beta_{1}$ and $\beta_{x p}<\beta_{3}$ conditions.

### 2.1.3. CMOS-Type ST by Dokic

Figure 4 shows the CMOS type Dokic ST circuit [5]. It is composed of six transistors and its hysteresis is shown in Figure 4b. Depending on how the input signal changes, two I/O characteristics can be observed. If the input goes from low (0) to high $\left(V_{D D}\right)$ voltage level, the output state is changed at $V_{H L}$ where the output goes from high $\left(V_{D D}\right)$ to low (0) and vice versa, it switches from low (0) to high $\left(V_{D D}\right)$ at $V_{L H}$. The $V_{H L}$ and $V_{L H}$ can be found when the input and output voltages are equal at operating points OP1 and OP2, as shown in Figure 4b.


Figure 4. CMOS type Schmitt trigger circuit by Dokic [5]. (a) Circuit diagram, (b) input-output characteristics, (c) equivalent circuit for $V_{L H}$ transition, and (d) equivalent circuit for $V_{H L}$ transition.

## $V_{L H}$ Voltage

When the input is $V_{D D}$, the output is close to 0 V and M1, M2, and M5 are OFF, while M6 operates in the subthreshold region. M3 and M4 are in the deep LIN region where $V_{d s 3,4} \approx 0$, and $I_{d s 3,4} \approx 0$. $V_{1}$ approaches but remains below $\left|V_{\text {thp }}\right|$ and is continuously discharged by the subthreshold current of M6. When $V_{\text {in }}$ drops a $\left|V_{\text {thcp }}\right|$ below $V_{D D}$, the series combination of M1 and M2 (MCp) turns ON and works in the SAT region. During this time, M3 still works in SAT, while M5 is OFF and M4 is in LIN, which keeps $V_{2}$ voltage close to 0 V . Thus, the $V_{L H}$ voltage could be determined by equating the SAT current of the transistor MCp to that of MCn formed by the series combination of M3 and M4, while assuming $V_{2}=0$. This means that MCn will have $V_{t h n 0}$ as the threshold voltage, while the threshold voltage of MCp will be $\left|V_{\text {thcp }}\right|$, the same as $\left|V_{\text {thpx }}\right|$ in Equation (22). Thus, the $V_{L H}$ can be derived as:

$$
\begin{equation*}
V_{L H}=\frac{\alpha_{2} \cdot V_{t h n 0}}{\left(1+\alpha_{2}\right)}+\frac{\alpha_{1} \cdot\left(V_{D D}-\left|V_{t h p 0}\right|\right)}{\left(1+\alpha_{1}\right)\left(1+\alpha_{2}\right)} \tag{23}
\end{equation*}
$$

where

$$
\begin{equation*}
\alpha_{1}=\sqrt{\frac{\beta_{1}}{\beta_{6}}}, \alpha_{2}=\sqrt{\frac{\beta_{c n}}{\beta_{c p}}}, \beta_{c p}=\left[\frac{\beta_{1} \cdot \beta_{2}}{\beta_{1}+\beta_{2}}\right] \text {, and } \beta_{c n}=\left[\frac{\beta_{3} \cdot \beta_{4}}{\beta_{3}+\beta_{4}}\right] \tag{24}
\end{equation*}
$$

## $V_{H L}$ Voltage

When the input is 0 V , the output is close to $V_{D D}$ and transistors M3, M4, and M6 are OFF, while M5 operates in the subthreshold region. Transistors M1 and M2 are in the deep LIN region where $V_{s d 1,2} \approx 0$, and $I_{s d 1,2} \approx 0 . V_{2}$ approaches but remains below $V_{D D}$ and is continuously charged by the subthreshold current of M5. When $V_{\text {in }}$ increases a $V_{\text {thcn }}$ above 0 V , the series combination of M3 and M4 (MCn) turns ON and into the SAT region. During this time, M2 still works in SAT, while M6 is OFF and M1 is in the LIN region, which keeps $V_{1}$ voltage close to $V_{D D}$. Thus, the $V_{H L}$ voltage could be determined by equating the SAT current of transistor MCn to that of MCp formed by the series combination of M1 and M2, while assuming $V_{1}=V_{D D}$. This means that MCp will have $\left|V_{t h p 0}\right|$ as threshold voltage, while the threshold voltage of MCn will be $V_{t h c n}$, the same as $V_{t h n x}$ in Equation (7). Thus, $V_{H L}$ can be derived as:

$$
\begin{equation*}
V_{H L}=\frac{V_{D D}-\left|V_{t h p 0}\right|+\alpha_{2} \cdot V_{t h n 0}}{\left(1+\alpha_{2}\right)}+\frac{\alpha_{2} \cdot\left(V_{D D}-V_{\text {thn } 0}\right)}{\left(1+\alpha_{2}\right)\left(1+\alpha_{3}\right)} \tag{25}
\end{equation*}
$$

where

$$
\begin{equation*}
\alpha_{3}=\sqrt{\frac{\beta_{4}}{\beta_{5}}} \text { and } V_{t h c n}=\frac{V_{D D}+\alpha_{3} \cdot V_{t h n 0}}{\left(1+\alpha_{3}\right)} \tag{26}
\end{equation*}
$$

## Hysteresis Voltage

The hysteresis voltage can then be derived by using Equations (23) and (25), as follows:

$$
\begin{equation*}
\Delta V_{H}=\frac{1}{\left(1+\alpha_{2}\right)}\left(\frac{\alpha_{2} \cdot\left(V_{D D}-V_{t h n 0}\right)}{\left(1+\alpha_{3}\right)}+\frac{V_{D D}-\left|V_{t h p 0}\right|}{\left(1+\alpha_{1}\right)}\right) \tag{27}
\end{equation*}
$$

Compared to Dokic's original hysteresis equation (Equation (16) in [5]), (27) provides a transistor-level design strategy, that hysteresis voltage could be adjusted accordingly. The design parameters sensitivity $\left(V_{H L}, V_{L H}\right.$, and $\left.\Delta V_{H}\right)$ in terms of the individual transistor parameters $\left(\alpha_{1}, \alpha_{2}, \alpha_{3}\right)$ can also be determined by using the new design Equations (23), (25) and (27), which provides better insight for the design of CMOS-type Dokic ST.

The new design equations for $\mathrm{N}-$, $\mathrm{P}-$, and CMOS-type Dokic ST presented in this section also provides better hand calculation accuracy in the wide process, supply, and device parameters variation as presented in the next section.

### 2.2. CMOS-Type Schmitt Trigger by Steyaert

Figure 5a shows the CMOS type ST circuit by Steyaert [6]. It is composed of five transistors and has the hysteresis that is shown in Figure 5b. Depending on how the input signal changes, two I/O characteristics can be observed. If the input goes from low (0) to high $\left(V_{D D}\right)$, the output state changes at $V_{L H}$, where the output goes from low $(0)$ to high $\left(V_{D D}\right)$ supply voltage. If the input goes from high $\left(V_{D D}\right)$ to (0) low, in this case, the output changes its value at $V_{H L}$, where the output value goes from high $\left(V_{D D}\right)$ to (0) low. The $V_{H L}$ and $V_{L H}$ can be found when input and output voltages are equal at the operating points OP1 and OP2, as shown in Figure 5b. During the analysis, we will assume that, without M5, the switching voltages of the two inverters are the same. Thus, the device sizes of M1 and M3 and M1 and M3 are the same. We also assume that there is a slight delay between voltages $V_{\text {in }}$ and $V_{\text {out }}$ due to the loading at the $V_{1}$ and output nodes.


Figure 5. CMOS-type ST by Steyaert [6]. (a) Circuit diagram and (b) input-output characteristics.

### 2.2.1. $V_{L H}$ Voltage

When the input is low ( 0 V ), $\mathrm{V}_{1}$ is high $\left(V_{D D}\right)$. Transistors M2 and M3 are ON while M1, M4, and M5 are OFF. At OP1, the $V_{L H}$ voltage is determined by equating the SAT currents of M1 and M2, like a regular CMOS inverter. Thus, the $V_{L H}$ can be derived as:

$$
\begin{equation*}
V_{L H}=\frac{\alpha_{1} \cdot\left(V_{D D}-\left|V_{t h p 0}\right|\right)+V_{t h n 0}}{\left(1+\alpha_{1}\right)} \tag{28}
\end{equation*}
$$

where

$$
\begin{equation*}
\alpha_{1}=\sqrt{\frac{\beta_{2}}{\beta_{1}}} \tag{29}
\end{equation*}
$$

This formulation is the same as the one presented by Steyaert as Equation (1) in [6].

### 2.2.2. $V_{H L}$ Voltage

When the input is high $\left(V_{D D}\right), V_{1}$ is low $(0 \mathrm{~V})$. Transistors M 2 and M3 are OFF while M1, M4, and M5 are ON. At OP2, M2 turns ON, and the $V_{H L}$ voltage could be determined by equating the SAT currents of M1, M2, and M5 while ignoring the channel-length modulation parameter ( $\lambda$ ) of the transistors as follows:

$$
\begin{equation*}
V_{H L}=V_{H L A}\left(1+\sqrt{1-\frac{V_{H L B}}{V_{H L A}}}\right) \tag{30}
\end{equation*}
$$

where

$$
\begin{equation*}
\alpha_{2}=\sqrt{\frac{\beta_{5}}{\beta_{1}}} \tag{31}
\end{equation*}
$$

and

$$
\begin{equation*}
V_{H L A}=\frac{\alpha_{1}^{2} \cdot\left(V_{D D}-\left|V_{\text {thp0 }}\right|\right)-V_{\text {thn } 0}}{\left(\alpha_{1}^{2}-1\right)} \quad V_{H L B}=\frac{\alpha_{1}^{2}\left(V_{D D}-\left|V_{\text {thp0 }}\right|\right)^{2}-V_{\text {thn0 }}^{2}-\alpha_{2}^{2}\left(V_{D D}-V_{\text {thn0 }}\right)^{2}}{\left(\alpha_{1}^{2}-1\right)} \tag{32}
\end{equation*}
$$

The formulation presented by Steyaert (Equation (2) in [6]) is given below, which is much simpler than Equation (30) above, but inaccurate for hand calculations as discussed in the next section.

$$
\begin{equation*}
V_{H L}=V_{L H}-\frac{\alpha_{2} \cdot\left(V_{D D}-V_{t h n 0}\right)}{2 \cdot\left(1+\alpha_{1}\right)} \tag{33}
\end{equation*}
$$

### 2.2.3. Hysteresis Voltage

The hysteresis voltage can then be derived by using Equations (28) and (30) as:

$$
\begin{equation*}
\Delta V_{H}=V_{H L}-V_{L H}=V_{H L A}\left(1+\sqrt{1-\frac{V_{H L B}}{V_{H L A}}}\right)-\frac{\alpha_{1} \cdot\left(V_{D D}-\left|V_{t h p 0}\right|\right)+V_{t h n 0}}{\left(1+\alpha_{1}\right)} \tag{34}
\end{equation*}
$$

Compared with the Steyaert's formula, Equation (34) is more accurate, but less intuitive in designing the ST circuit. More accurate hysteresis voltage could be derived if we include the channel-length modulation mechanism, but the equation becomes more complicated and less intuitive.

### 2.3. Non-Inverting Schmitt Trigger by Pedroni

Figure 6 shows the non-inverting ST by Pedroni [7]. It is composed of six transistors and its hysteresis is shown in Figure 6a. When the input goes from low (0) to high ( $V_{D D}$ ) voltage level, the output state changes at $V_{L H}$ where the output goes from low (0) to high ( $V_{D D}$ ). If the input goes from high $\left(V_{D D}\right)$ to (0) low, the output changes at $V_{H L}$, where the output goes from high ( $V_{D D}$ ) to (0) low. The $V_{H L}$ is defined by the switching point of the inverter composed of M1 and M2 and the $V_{L H}$ is set by the inverter transistors M3 and M4, as shown in Figure 6b. Besides, the hysteresis exists only if $V_{S P 3,4}>V_{S P 1,2}$.


Figure 6. The non-inverting ST by Pedroni [7]. (a) Circuit diagram, (b) input-output characteristics of each input inverter, and (c) overall characteristics of the ST.

## Hysteresis Voltages

When the input is low $(0 \mathrm{~V}), \mathrm{V}_{1}$ and $\mathrm{V}_{2}$ are high $\left(V_{D D}\right)$. M2, M4, and M 5 are ON while $\mathrm{M} 1, \mathrm{M} 3$, and M6 transistors are OFF, and the output is 0 V . At OP1, $V_{1}$ becomes 0 V which turns off M5, and then the output floats maintaining its last state which is 0 V . The output only changes when $V_{2}$ becomes 0 V at OP2 where M6 turns ON and charges the output to $V_{D D}$. Thus, VLH occurs at the switching point $\left(V_{S P}\right)$ of the upper inverter composed of transistors M3 and M4. Thus, $V_{L H}=V_{S P 3,4}$ can be derived as follows:

$$
\begin{equation*}
V_{H L}=\frac{\alpha_{1} \cdot\left(V_{D D}-\left|V_{\text {thp } 0}\right|\right)+V_{t h n 0}}{\left(1+\alpha_{1}\right)} \tag{35}
\end{equation*}
$$

where

$$
\begin{equation*}
\alpha_{1}=\sqrt{\frac{\beta_{4}}{\beta_{3}}} \tag{36}
\end{equation*}
$$

Similarly, the $V_{H L}$ can be found as:

$$
\begin{equation*}
V_{H L}=\frac{\alpha_{2} \cdot\left(V_{D D}-\left|V_{\text {thp0 }}\right|\right)+V_{\text {thn } 0}}{\left(1+\alpha_{2}\right)} \tag{37}
\end{equation*}
$$

where

$$
\begin{equation*}
\alpha_{2}=\sqrt{\frac{\beta_{2}}{\beta_{1}}} \tag{38}
\end{equation*}
$$

The hysteresis voltage could then be derived as:

$$
\begin{equation*}
\Delta V_{H}=V_{H L}-V_{L H}=-\frac{\left(V_{D D}-\left|V_{t h p 0}\right|+V_{t h n 0}\right)\left(\alpha_{1}-\alpha_{2}\right)}{\left(1+\alpha_{1}\right)\left(1+\alpha_{2}\right)} \tag{39}
\end{equation*}
$$

### 2.4. Low-Power CMOS-Type Schmitt Trigger by Al-Sarawi

Figure 7 shows the low-power CMOS-type ST by Al-Sarawi [8]. It is composed of six transistors and its hysteresis is shown in Figure 7d. Depending on how the input signal changes, two I/O characteristics can be observed: If the input goes from low (0) to high ( $V_{D D}$ ) voltage level, the output state changes at $V_{H L}$ where the output goes from high $\left(V_{D D}\right)$ to low (0). If the input goes from high $\left(V_{D D}\right)$ to low (0), in this case, the output changes at $V_{L H}$, where the output goes from low (0) to high $\left(V_{D D}\right)$. The $V_{H L}$ and $V_{L H}$ can be found when the input and the output voltages are equal at the operating points OP1 and OP2, as shown in Figure $7 \mathrm{~b}, \mathrm{c}$. For the analysis, the switching voltage of the two inverters is assumed to be the same. Thus, the device sizes of M1 and M3, and M2 and M4 are the same.


Figure 7. Low-power CMOS-type ST by Al-Sarawi [8]. (a) Circuit diagram, (b) equivalent circuit for $V_{H L}$, (c) equivalent circuit for $V_{L H}$, and (d) input-output characteristics.

### 2.4.1. $V_{H L}$ Voltage

When the input is low $(0 \mathrm{~V}), V_{\text {out }}$ is high $\left(V_{D D}\right)$. Since $V_{\text {out }}$ is a high impedance node and M1 is OFF, $V_{C N}$ drifts towards low (0) potential through the diode-connected M5 that works in the subthreshold region. While M3 works in the deep LIN region that $V_{D S 3}$ is close to 0 . As a result, M6 turns fully ON which pulls the node voltage $V_{P}$ to $V_{D D}$. Thus, when input is low (0) at steady state, transistors M2, M6, and M3 are ON, M5 is in subthreshold and, other transistors are fully OFF. Assuming $V_{H L}$ is larger than the threshold voltage of M1 at OP1, the $V_{H L}$ switching voltage could be determined by equating the SAT currents of M1 and M2, like a regular CMOS inverter with finite $V_{n}$ that keeps M5 on the edge of SAT and OFF regions.

$$
\begin{equation*}
V_{H L}=\frac{V_{n}+\alpha_{1} \cdot\left(V_{D D}-\left|V_{\text {thp0 }}\right|\right)+V_{\text {thn } 0}}{\left(1+\alpha_{1}\right)} \tag{40}
\end{equation*}
$$

where

$$
\begin{equation*}
\alpha_{1}=\sqrt{\frac{\beta_{2}}{\beta_{1}}} \tag{41}
\end{equation*}
$$

$V_{n}$ could be found by using the equivalent circuit shown in Figure 7b. Here, it is assumed that the $V_{\text {out }}$ node voltage is close to $V_{D D}$, and M3 shorts $V_{c n}$ to $V_{n}$ effectively connecting the gate of M5 to its drain, which keeps it in the SAT region. Thus, using SAT currents of transistors M1 and M5, we can determine the $V_{n}$ voltage as follows:

$$
\begin{equation*}
V_{n}=\frac{V_{\text {in }}}{\left(1+\alpha_{2}\right)}+\frac{\left(\alpha_{2}-1\right) \cdot V_{\text {thn } 0}}{\left(1+\alpha_{2}\right)} \tag{42}
\end{equation*}
$$

where

$$
\begin{equation*}
\alpha_{2}=\sqrt{\frac{\beta_{5}}{\beta_{1}}} \tag{43}
\end{equation*}
$$

Using (42) in (40) for $V_{i n}=V_{H L}$, we can find the $V_{H L}$ as follows.

$$
\begin{equation*}
V_{H L}=\frac{2 \cdot \alpha_{2} \cdot V_{\text {thn } 0}+\alpha_{1} \cdot\left(1+\alpha_{2}\right) \cdot\left(V_{D D}-\left|V_{\text {thp } 0}\right|\right)}{\alpha_{1}+\alpha_{2} \cdot\left(1+\alpha_{1}\right)} \tag{44}
\end{equation*}
$$

Compared to Al-Sarawi's equation, Equations (42) and (44) are different than his $V_{n}$ and $V_{H L}$ equations (Equations (2) and (3) in [8]) due to his broad assumption of $V_{\text {thn } 0}=\left|V_{\text {thp0 }}\right|=V_{t h}$.

### 2.4.2. $V_{L H}$ Voltage

When the input is high $\left(V_{D D}\right), V_{\text {out }}$ is low $(0 \mathrm{~V})$. As a result, M 4 is ON shorting drain and gate of M6 that keeps M6 at the edge of SAT and OFF regions. Thus, when the input is high $\left(V_{D D}\right)$, transistors M1,

M5, M4, and M6 are ON and the other transistors are OFF. At OP2, the $V_{L H}$ voltage can be determined by equating the SAT currents of M1 and M2. Here, $V_{p}$ could be considered as finite and less than $V_{D D}$, where M6 is on the edge of the SAT and OFF regions.

$$
\begin{equation*}
V_{L H}=\frac{\alpha_{1} \cdot\left(V_{p}-\left|V_{\text {thp0 }}\right|\right)+V_{\text {thn } 0}}{\left(1+\alpha_{1}\right)} \tag{45}
\end{equation*}
$$

$V_{p}$ could be found by using the equivalent circuit as shown in Figure 7c. Here, it is assumed that the $V_{\text {out }}$ node voltage is approximately equal to 0 V , and M 4 shorts $V_{c n}$ to $V_{p}$ effectively connecting the gate of M6 to its drain, which keeps it in SAT region. Thus, using SAT currents of transistors M2 and M6, we can determine the $V_{p}$ voltage as follows:

$$
\begin{equation*}
V_{p}=\frac{V_{\text {in }}+\alpha_{3} \cdot V_{D D}+\left(1-\alpha_{3}\right)\left|V_{\text {thp0 }}\right|}{\left(1+\alpha_{3}\right)} \tag{46}
\end{equation*}
$$

where

$$
\begin{equation*}
\alpha_{3}=\sqrt{\frac{\beta_{6}}{\beta_{2}}} \tag{47}
\end{equation*}
$$

Using (46) in (45) for $V_{i n}=V_{L H}$, we can derive the $V_{L H}$ as follows:

$$
\begin{equation*}
V_{L H}=\frac{\left(1+\alpha_{3}\right) \cdot V_{t h n 0}+\alpha_{1} \cdot \alpha_{3} \cdot\left(V_{D D}-2 \cdot\left|V_{t h p 0}\right|\right)}{1+\alpha_{3} \cdot\left(1+\alpha_{1}\right)} \tag{48}
\end{equation*}
$$

Al-Sarawi derived the $V_{L H}$ equation (Equation (4) in [8]) rather differently, which assumes $V_{L H}=V_{D D}-V_{H L}$ as well as $V_{t h n 0}=\left|V_{t h p 0}\right|=V_{t h}$. This assumption results in a simple but inaccurate $V_{L H}$ design equation. Additionally, his $V_{L H}$ equation has a typographical error that causes gross calculation error larger than $200 \%$ of simulated values. The corrected equation that gives reasonable hand calculation error ( $<20 \%$ of simulated values) is given below. We used this corrected equation instead of Equation (4) in [8] to compare our new equation.

$$
\begin{equation*}
V_{L H}=V_{D D} \times \frac{(R+1)}{R_{p}(R+1)+1}-V_{t h} \times \frac{R_{p}(2 R-1)-1}{R_{p}(R+1)+1} \tag{49}
\end{equation*}
$$

where

$$
\begin{equation*}
R=1 / \alpha_{1}, \text { and } R_{p}=\alpha_{3} \tag{50}
\end{equation*}
$$

### 2.4.3. Hysteresis Voltages

Formulation provided by Al-Sarawi (Equations (2)-(4) in [8]) assumes that the threshold voltages of the NMOS and PMOS devices are the same and do not cover all design and process variations. Thus, the equations given in (44) and (48) are more detailed and useful for hand calculations and design for the hysteresis voltage $\left(\Delta V_{H}=V_{H L}-V_{L H}\right)$.

## 3. Comparison of Simulation and Hand Calculations

Simulations of all ST circuits were performed using a well established, analog/mixed-signal, $0.35 \mu \mathrm{~m}$ CMOS, 3.3 V , CMOS process with BSIM3v3 Spice models. The process has device characteristics listed in Table 1. This process allows a fair comparison with literature that used long channel MOSFET models and high supply voltages. We used a minimum channel length ( $L_{m i n}$ ) of $0.35 \mu \mathrm{~m}$ and changed widths of transistors to cover a wide range of design spaces. In addition, Monte-Carlo, corner/parameter, or both, sweep simulations were run to find hysteresis voltages ( $V_{L H}$ and $V_{H L}$ ) under various conditions.

Here, we only reported 3.3 V supply results at room temperature ( $T=300 \mathrm{~K}$ ), while similar trends were observed for other supply voltages.

Table 1. $0.35 \mu \mathrm{~m}$ CMOS process parameters for hand calculations.

| Parameter | NMOS | PMOS |
| :---: | :---: | :---: |
| Transconductance $(K P)\left(\mu \mathrm{A} / \mathrm{V}^{2}\right)$ | 170 | 58 |
| Threshold Voltage $\left(V_{\text {th0 }}\right)(\mathrm{V})$ | 0.50 | -0.65 |

### 3.1. Dokic ST Circuits

For the simulations of Dokic's circuits, transistor widths were changed between $2 L_{\text {min }}$ and $20 L_{\text {min }}$ with $2 L_{\text {min }}$ steps such that channel widths of transistors M1, M2, and M4 were set equal to each other and M3 varied separately for P-type and N-type ST circuits. For CMOS type ST, widths of M1, M2, M6 and widths of M3, M4, M5 were set equal, respectively. Since minimum channel length was used for all transistors, setting $\alpha_{1 n}, \alpha_{1 p}, \alpha_{1}$, and $\alpha_{3}$ to 1.0 for all ST types, while $\alpha_{2 n}$ and $\alpha_{2 p}$ were varied between 0.383 and 3.83 for N-type and P-type circuits. For CMOS ST, $\alpha_{2}$ was changed between 0.54 and 5.4. The bulk of all NMOS transistors were connected to ground and the bulk of all PMOS transistors were connected to $V_{D D}$.

### 3.1.1. N-Type ST by Dokic

Figure 8 shows the simulation results for N-type Dokic ST. Hysteresis voltage $\left(\Delta V_{H}\right)$ as large as 1 V could be achieved for $\alpha_{2 n}=3.83$. Moreover, smaller hysteresis voltages close to 0.1 V are also possible with N-type Dokic ST. Although they are not the same, Equations (15) and (17) result in the same $V_{H L}$ and $V_{L H}$ hand calculation values as the original Dokic equations (Equations (5) and (6) in [5]). Hand calculation accuracy compared to the simulation results are shown in Figure 9. Hand calculations result in lower than -13 and $+5 \%$ errors for $V_{H L}$ and $V_{L H}$, respectively. Wide hysteresis voltage can be achieved by choosing $(W / L)_{1,2,4}=20\left(\alpha_{1 n}=1.0\right)$, and $(W / L)_{3}=2\left(\alpha_{2 n}=3.83\right)$.


Figure 8. Simulated hysteresis voltages ( $V_{H L}, V_{L H}, \Delta V_{H}$ ) of N-type Dokic ST for different device sizes $\left(0.383<\alpha_{2 n}<3.83\right)$ and $V_{D D}=3.3 \mathrm{~V}$.


Figure 9. Hand calculation errors of hysteresis voltages using (15) and (17) for N-Type ST (or with Equations (5) and (6) in [5]) for $V_{D D}=3.3 \mathrm{~V}$ and different device sizes ( $0.383<\alpha_{2 n}<3.83$ ).

### 3.1.2. P-Type ST by Dokic

For P-type Dokic ST, the channel length of M3 is set to $4 L_{\text {min }}$ and the bulk of M4 is connected to the node $V_{1}$ while the dimensions of other transistors and bulk connections are kept the same as N-type Dokic ST. Figure 10 shows the simulation results of P-type Dokic ST for 3.3 V supply voltage. Hysteresis voltage $\left(\Delta V_{H}\right)$ as large as 0.9 V could be achieved for $\alpha_{2 p}=0.383$. Moreover, hysteresis voltages close to 0.1 V are also possible with P-type Dokic ST for larger $\alpha_{2 p}$ values. The simulation results for the hysteresis voltages for different device sizes and supply voltages show that, typically, the $V_{H L}$ voltage is widely controlled by the design parameters. Figure 11 shows the hand calculation errors of the hysteresis voltages for the design parameter $\alpha_{2 p}$ using Equations (19) and (20) (or with Equations (8) and (12) in [5]). The error could be less than $\pm 13 \%$.


Figure 10. Simulated hysteresis voltages ( $V_{H L}, V_{L H}, \Delta V_{H}$ ) of P-type Dokic ST for different device sizes $\left(0.383<\alpha_{2 p}<3.83\right)$ and $V_{D D}=3.3 \mathrm{~V}$.


Figure 11. Hand calculation errors of hysteresis voltages using (19) and (20) for P-Type ST (or with Equations (8) and (12) in [5]) for different device sizes ( $0.383<\alpha_{2 p}<3.83$ ) and $V_{D D}=3.3 \mathrm{~V}$.

### 3.1.3. CMOS-Type ST by Dokic

For CMOS-type Dokic ST circuit simulations, the bulk of M6 is connected to the node $\mathrm{V}_{1}$. Figure 12 shows the simulation results for the 3.3 V supply voltage. Hysteresis voltage $\left(\Delta V_{H}\right)$ as large as 1.2 V could be achieved for $\alpha_{2}=0.54$. The simulation results show that a smaller hysteresis voltage is not possible. Additionally, hand calculation equations (Equations (23) and (25)) and original Dokic equations (Equations (14) and (15) in [5]) do not provide good approximations that result in up to $\pm 50 \%$ error for $V_{H L}$ and between $+10 \%$ and $-25 \%$ error for $V_{L H}$, as shown in Figure 13 .


Figure 12. Simulated hysteresis voltages $\left(V_{H L}, V_{L H}, \Delta V_{H}\right)$ of CMOS-type Dokic ST for different device sizes $\left(0.54<\alpha_{2}<5.4\right)$ at $V_{D D}=3.3 \mathrm{~V}$.


Figure 13. Hand calculation errors of hysteresis voltages using (23) and (25) for CMOS-type ST (or with Equations (14) and (15) in [5]) for different device sizes $\left(0.54<\alpha_{2}<5.4\right)$ at $V_{D D}=3.3 \mathrm{~V}$.

### 3.2. CMOS-Type ST by Steyaert

Transistor widths were changed between $2 L_{\min }$ and $10 L_{\min }$ with $L_{\min }$ steps for transistors M1, M3, and M5, and between $7 L_{\text {min }}$ and $14 L_{\min }$ with $4 L_{\min }$ steps for transistors M2 and M4 during the simulation. The channel lengths of NMOS and PMOS transistors were set to $10 L_{\text {min }}$ and $L_{\text {min }}$, respectively. As a result, a wide design space was covered for simulations and calculations. A new design parameter, the transconductance factor ratio ( $\kappa$ ) which is defined as the ratio between $\beta_{2}$ (M2) and $\beta_{1}+\beta_{5}$ (M1 and M5 combination) represents the design space. $\kappa$ was set between 1.0 and 12 . This results in the $V_{H L}$ voltage being between 0.55 and 2.04 V , the $V_{L H}$ between 1.7 and 2.30 V , and the hysteresis voltage between 0.24 and 1.15 V for 3.3 V supply voltage, as shown in Figure 14a.


Figure 14. (a) Simulated hysteresis voltages of Steyaert ST , (b) hand calculation accuracy of $V_{H L}$ using Equation (30) and Equation (1) in [6], and (c) hand calculation error of $\Delta V_{H}$ using Equation (34) and Equations (1) and (2) in [6].

Derived $V_{L H}$ equation in this work, Equation (28), and the original Steyaert equation ((1) in [6]) are the same, resulting in a maximum $-4 \%$ hand calculation error. Steyaert's $V_{H L}$ equation ((2) in [6]), on the other hand, results in gross hand calculation errors up to $-120 \%$ for smaller $\mathcal{K}$ values, while Equation (30) presented in this work results in less than $-25 \%$ as shown in Figure 14b. As a result, overall hand calculation error for the hysteresis voltage $\Delta V_{H}$ by Equation (34) is lower than that of the original Steyaert equation, (Equations (1) and (2) in [6]), as shown in Figure 14c.

### 3.3. Non-Inverting ST by Pedroni

The hysteresis voltages of Pedroni ST can be set by modifying the sizes of NMOS (M1, M3, M5) or PMOS (M2, M4, M6) transistors. For simulation, NMOS transistor widths were changed between $2 L_{\text {min }}$ and $10 L_{\text {min }}$ with $2 L_{\text {min }}$ steps. The multiplication factor $(M)$ of M1 is set to $\mathrm{M}_{\mathrm{x}}$ and M3 and M5 is to unity, respectively. Similar widths and steps used for the PMOS transistors while setting the multiplication factor of M 4 to be Mx , and keeping the multiplication factor of M 2 and M 6 to be unity. The channel length of NMOS and PMOS transistors were set to $L_{\text {min }}$ or $0.35 \mu \mathrm{~m}$. Multiplication factor of transistor M1, M4, the $M_{x}$, varied from 2 to 6 while other Ms were kept constant at unity so that a wide range of $V_{H L}$ and $V_{L H}$ voltages were achieved.

Design parameter $\mathrm{M}_{\mathrm{x}}, \alpha_{1}$, and $\alpha_{2}$ can be used for setting hysteresis voltages ( $V_{H L}, V_{L H}$, and $\Delta V_{H}$ ) as shown in Figure 15. Hysteresis voltage as large as 1 V can be achieved by increasing all design parameters, however, this will result in a large silicon footprint. Equations (35) and (37) predict the hysteresis voltages, $V_{H L}$, and $V_{L H}$ with $+14 \%$ to $-8 \%$ calculation errors as shown in Figure 16.


Figure 15. Simulated hysteresis voltages of Pedroni ST for different device sizes $\alpha_{1}, \alpha_{2}$, and Mx at $V_{D D}$ $=3.3 \mathrm{~V}(\mathbf{a}) V_{L H}$ vs. $\alpha_{2},(\mathbf{b}) V_{H L}$ vs. $\alpha_{1}$, and (c) $\Delta V_{H}$ vs. $\alpha_{1}$.


Figure 16. Hand calculation accuracy of hysteresis voltages $V_{H L}$ and $V_{L H}$ using Equations (35) and (37).

### 3.4. CMOS-Type ST by Al-Sarawi

The sensitivity of the $V_{H L}$ to $\alpha_{1}$ parameter, which can be derived from Equation (44), is much higher than the $\alpha_{2}$ and $\alpha_{3}$ parameters. Thus, for simulation and evaluation of the design equation accuracies, $\alpha_{1}$ parameter varied from 0.15 to 1.3 while $\alpha_{2}$ and $\alpha_{3}$ were set to 1.41 and 2 , respectively. We proposed new hysteresis design equations for the Al-Sarawi ST. Figure 17 shows the simulated and calculated values by using Al-Sarawi's original design equation and the proposed design Equation (44)
of the hysteresis voltages $V_{H L}, V_{L H}$, and $\Delta V_{H}$ versus $\alpha_{1}$ parameter. The simulation results show that the $\Delta V_{H}$ reaches 0.98 V (Figure 17c), which is larger than Al-Sarawi's original design equation. However, it saturates around these values even if $\alpha_{1}$ parameter is increased.


Figure 17. Comparison of design equations and simulated hysteresis voltages of Al-Sarawi's ST for 3.3 V supply voltage (a) $V_{H L}$ vs. $\alpha_{1}$, (b) $V_{L H}$ vs. $\alpha_{1}$, and (c) $\Delta V_{H}$ vs. $\alpha_{1}$.

Figure 18 illustrates the hand calculation errors of proposed design equations in Section 2.4 as well as Al-Sarawi's original design equations related to the simulated values of $V_{H L}, V_{L H}$, and $\Delta V_{H}$. It can be noticed from Figure 18b that $V_{L H}$ hand calculation error of the proposed design Equation (48) is less than $2 \%$ while it could be as large as $17 \%$ for Al-Sarawi's original design equation. Additionally, hand calculation errors of $V_{H L}$ and $\Delta V_{H}$ by the proposed design equations are always inversely proportional to $V_{H L}$ and $\Delta V_{H}$, respectively, and are lower than the calculation errors of Al-Sarawi's original design equations.


Figure 18. Comparison of hand calculation errors of hysteresis voltages of Al-Sarawi's ST for 3.3 V supply voltage. (a) error vs. $V_{H L}$, (b) error vs. $V_{L H}$, and (c) error vs. $\Delta V_{H}$.

### 3.5. Comparison of All Circuits

Table 2 summarizes a comparison between Dokic [5], Steyaert [6], Pedroni [7], and Al-Sarawi [8] ST circuits in terms of total area, simulated power consumption, transition delays, and maximum hysteresis voltage based on the variation of device dimensions. There are tradeoffs among these parameters that the larger $\Delta V_{H}$ design may have the higher-power consumption or the larger footprint or the lower speed. By design, it is desirable to have larger $\Delta V_{H}$, and smaller delay, area, and power
consumption. Thus, we propose a figure of merit (FoM) based on these parameters. FoM for ST circuits can be calculated using the following equation:

$$
\begin{equation*}
\text { FoM }=\frac{10000 \times \Delta V_{H}(V)}{\operatorname{Area}\left(\mu m^{2}\right) \times \text { Power consumtion }(\mu W) \times \operatorname{Delays}(n s)} \tag{51}
\end{equation*}
$$

Table 2. Performance comparison of the six ST circuits.

| ST Circuit | $\mathrm{T}_{\text {rise }}(\mathbf{n s})$ | $\mathrm{T}_{\text {fall }}(\mathbf{n s})$ | Area $(\mu \mathbf{m})$ | Power $(\mu \mathbf{W})$ | $\Delta \mathbf{V}_{\mathbf{H}}(\mathbf{V})$ | FoM |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Dokic (N) [5] | 1.636 | 0.109 | 7.60 | 58.51 | 1.110 | 1.43 |
| Dokic (P) [5] | 0.412 | 1.507 | 6.86 | 61.58 | 0.855 | 1.05 |
| Dokic (CMOS) [5] | 1.239 | 0.372 | 7.80 | 75.31 | 1.015 | 1.07 |
| Steyaert [6] | 1.308 | 3.043 | 38.20 | 186.48 | 1.150 | 0.04 |
| Pedroni [7] | 0.425 | 0.338 | 13.70 | 63.20 | 0.952 | 1.44 |
| Al-Sarawi [8] | 1.320 | 1.185 | 5.40 | 43.23 | 0.983 | 1.68 |

This equation is scaled by a multiplying factor of 10,000 for better number representation. Delay was measured at $50 \%$ of the supply voltage level when a 100 fF loading is added at the output of each ST design. The sum of rising $\left(t_{\text {rise }}\right)$ and falling times $\left(t_{\text {fall }}\right)$ was calculated as the delay.

From Table 2, if the large hysteresis voltage is the main requirement, the Steyaert ST circuit is the best, yet it has the lowest FoM due to large power consumption and area. Overall, Al-Sarawi's ST circuit offers the best FoM. However, it is slower than Pedroni's ST, which is the second-best choice among the investigated topologies.

## 4. Conclusions

In this paper, detailed reviews of Dokic [5], Steyaert [6], Pedroni [7], and Al-Sarawi [8] SISO ST circuits are presented. The paper starts with the detailed derivation of the hysteresis voltages ( $V_{H L}$, $V_{L H}$, and $\Delta V_{H}$ ) for each topology. Then, we propose some new design equations which result in a more intuitive, and accurate design through hand calculations. Simulations were run to verify that the derived and the original design equations are accurate. The simulations were carried out in a well-established $0.35 \mu \mathrm{~m} / 3.3 \mathrm{~V}$ analog/mixed-signal CMOS process. For each ST circuit, hysteresis voltages ( $V_{H L}, V_{L H}$, and $\Delta V_{H}$ ) were calculated with respect to different device sizes, which cover wide design space at a process supply voltage of 3.3 V . The hand calculation results derived from both the original and the new design equations were presented for each ST circuit and are compared with simulation results. The comparisons show that the new design equations are better than the original ones in terms of accuracy and intuition. Finally, the proposed FoM in Equation (51) can work as a criterion to compare different ST circuits. It is found that Al-Sarawi's ST circuit offers the best FoM of the investigated topologies.

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