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Article

Beyond an Input/Output Paradigm for Systems: Design Systems by Intrinsic Geometry

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Abstract: Given a stress-free system as a perfect crystal with points or atoms ordered in a three dimensional lattice in the Euclidean reference space, any defect, external force or heterogeneous temperature change in the material connection that induces stress on a previously stress-free configuration changes the equilibrium configuration. A material has stress in a reference which does not agree with the intrinsic geometry of the material in the stress-free state. By stress we mean forces between parts when we separate one part from another (tailing the system), the stress collapses to zero for any part which assumes new configurations. Now the problem is that all the new configurations of the parts are incompatible with each other. This means that close loop in the earlier configuration now is not closed and that the two paths previously joining the same two points now join different points from the same initial point so the final point is path dependent. This phenomenon is formally described by the commutators of derivatives in the new connection of the stress-free parts of the system under the control of external currents. This means that we lose the integrability property of the system and the possibility to generate global coordinates. The incompatible system can be represented by many different local references or Cartan moving Euclidean reference, one for any part of the system that is stress-free. The material under stress when is free assumes an equilibrium configuration or manifold that describes the intrinsic "shape" or geometry of the natural stress—the free state of the

material. Therefore, we outline a design system by geometric compensation as a prototypical constructive operation.

Keywords: intrinsic geometry; holonomic constraints; nonholonomic systems; dissipative systems; free stress material; Cartan moving reference; Maxwell-like Gauge approach; generalized Gauge as compensation; non-conservative gravity; gravity with torsion; physical theory as system; crystal defects; memristors

1. Introduction

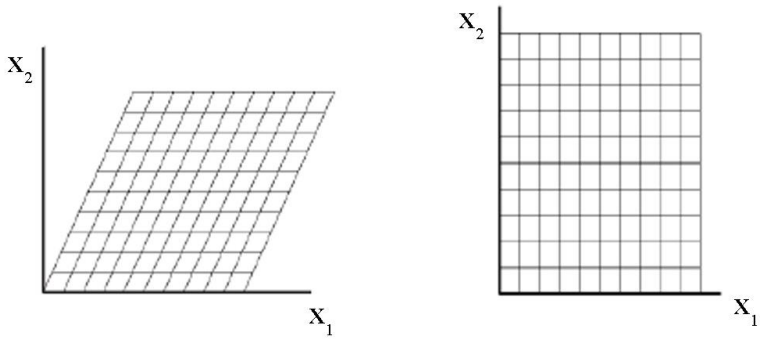
Given the stress-free system in the Euclidean reference space, any field of forces between particles due to gravity, electromagnetic, heterogeneous temperature, dissipation, or crystal defects will be called stress field. The defects or the physical fluxes change the material connection that induce stress on a previously stress-free configuration as in the holonomic system and as the equilibrium configuration or geometry change. Now the problem is that all the new configurations of the parts are incompatible with each other, with a geometry that differs from the intrinsic geometry of the system. This incompatibility creates defects in the reference. The coordinates of non-intrinsic geometry are not commutative and any loop cannot return to the initial value. This means that the integration operator is not unique and the system is not conservative. A simple example of incompatible geometry is given by rotation movement in the flat geometry. The geometry without curvature is not the intrinsic geometry of the rotation so stress forces appear as centripetal and centrifugal forces to compute the movement. When we use the intrinsic geometry for rotation as curvilinear coordinates, the reference is stress-free. The incompatible system for the defects (singularity) cannot be represented by a global reference but can be represented by many different references or Cartan moving references, one for any part of the system that is stress-free or locally compatible. The material under stress when is free assumes an equilibrium configuration or manifold that describes the intrinsic "shape" or geometry of the natural stress-free state of the material. The article underlines that the appearance of non-conservative facets in systems is a universal aspect which may be explained analyzing the structural links between quantum mechanics and Maxwell's equations, and also between gravitation and Maxwell's equations, thus outlining a general theory of open and nonholonomic systems. All that generalizes "input" and "output" concepts in Systems Theory (every "law" is a systemic connection among a series of input/output(s), under specific boundary conditions) has already been overcome by Einstein geometry that radically changes the old Newtonian concept of input (force) and output (acceleration).

The main examples of the intrinsic geometry for gravity force as a stress are the Einstein general relativity with curvature (defects in rotations) without torsion and the example of Cartan moving reference in gravity is the "Teleparallel" with torsion (defects in translation) without curvature. In this paper, we use a Maxwellian-like generalized gauge approach to get the intrinsic geometry in different systems. Here, we follow Caianello's idea [1] that any description of a physical theory or model represents a "system"—in formal and conceptual senses—and new possibilities of description emerge when we introduce new hypotheses to modify the logical closeness of this system.

2. Local Intrinsic Geometry Used to Map Global Intrinsic Geometry

With a moving local reference it is possible to detect the geometric nature of the system. Historically we remember the Galileo principle for which systems with constant velocity are all equal by local reference that moves with the system (inertial movement). Any local reference cannot be detected if the system moves and the velocity itself, too, cannot be detected. In this Galilean situation, any local reference has the same geometry of the global reference, the topology of the system is always the same (conservative system). In Figure 1, we show a transformation of the system that cannot change the connection elements or geometry between one point and another point. The local geometry is the intrinsic geometry of all the system.

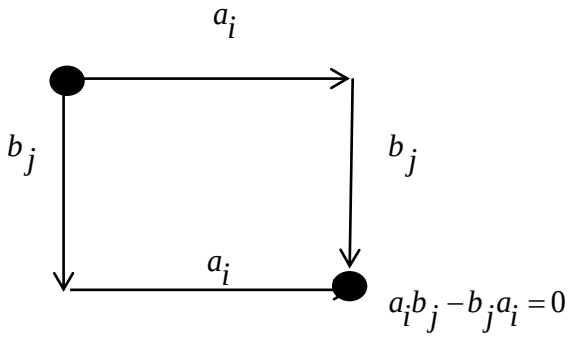
Figure 1. The Cartesian reference has no defects and local geometry is the same of the global geometry. After the transformation, we have another reference that has the same properties of the original Cartesian coordinates (Definition: A system is compatible if the local geometry is the same as the global geometry).



To know if a system has no defects or is compatible we take a local reference that we move to form a loop. If, after the loop, we return to the same point and to the same states, the system is compatible and conservative. We know that the Euclidean geometry in the Cartesian reference is a compatible geometry without defect for which any derivative commutes one with the other in this way

$$\left[\frac{\partial}{\partial x_i},\frac{\partial}{\partial y_j}\right]\psi=\left(\frac{\partial}{\partial x_i}\frac{\partial}{\partial y_j}-\frac{\partial}{\partial y_j}\frac{\partial}{\partial x_i}\right)\psi=\frac{\partial^2\psi}{\partial x_i\partial y_j}-\frac{\partial^2\psi}{\partial y_j\partial x_i}=0 \quad (1)$$

We can see the compatible property by this categorical diagram



where

$$a_i = \frac{\partial}{\partial x_i}, b_j = \frac{\partial}{\partial y_j} \tag{2}$$

Given the rotation system, we know that the tangent vector in any point in the Cartesian reference is given by the tangent vector

$$v = \begin{bmatrix} -y \\ x \end{bmatrix} \tag{3}$$

The directional derivative is given by the scalar product of the vector

$$\nabla \psi = \begin{bmatrix} \frac{\partial \psi}{\partial x} \\ \frac{\partial \psi}{\partial y} \end{bmatrix} = \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{bmatrix} \psi \tag{4}$$

With v. So we have

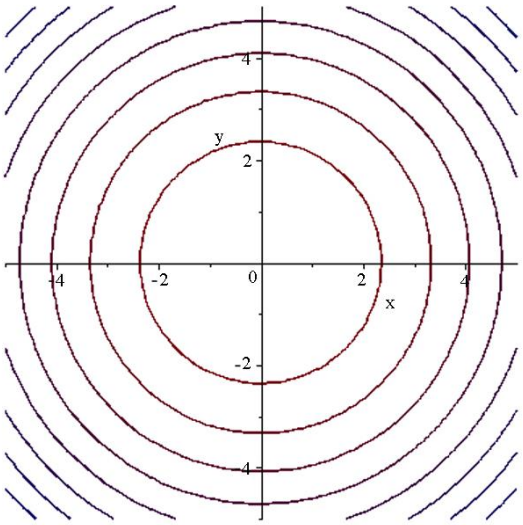
$$D_{\upsilon} = \begin{bmatrix} -y \\ x \end{bmatrix}^T \begin{bmatrix} \frac{\partial}{\partial x} \\ \frac{\partial}{\partial y} \end{bmatrix} \psi = (-y \frac{\partial}{\partial x} + x \frac{\partial}{\partial y}) \psi \tag{5}$$

For

$$(-y \frac{\partial}{\partial x} + x \frac{\partial}{\partial y}) \psi = 0 \tag{6}$$

We have that $\psi = x^2 + y^2 = R^2$ so circles with different radius are the new coordinates that design the intrinsic geometry.

Figure 2. Intrinsic geometry of circles which derivative is $D\psi = (-y \frac{\partial}{\partial x} + x \frac{\partial}{\partial y}) \psi$.



When we move on the intrinsic geometry circles (Figure 2), the derivative is equal to zero so we have no stress or virtual forces. When we consider the Cartesian coordinates and we move on the circle we have the relation between the partial derivatives for x and y

$$\frac{\partial}{\partial x} = \frac{x}{y}\frac{\partial}{\partial y} \tag{7}$$

and

$$\begin{aligned} &\left[\frac{\partial}{\partial x}, \frac{\partial}{\partial y}\right]\psi = (\frac{\partial}{\partial x}\frac{\partial}{\partial y} - \frac{\partial}{\partial y}\frac{\partial}{\partial x})\psi = (\frac{x}{y}\frac{\partial}{\partial y}\frac{\partial}{\partial y} - \frac{\partial}{\partial y}\frac{x}{y}\frac{\partial}{\partial y})\psi \\ &= (\frac{x}{y}\frac{\partial^2}{\partial y^2} - (-\frac{x}{y^2}\frac{\partial}{\partial y} + \frac{x}{y}\frac{\partial^2}{\partial y^2}))\psi = \frac{x}{y^2}\frac{\partial \psi}{\partial y} \neq 0 \end{aligned} \tag{8}$$

The Cartesian coordinates and geometry are not the intrinsic geometry because include the (0,0) point that is a singular point or defect. In this situation, the commutator is different from zero. Because the direction derivative on the tangent vector to the circle is a derivative for polar coordinates that is the intrinsic geometry, we have no particular problem to introduce the singular point. The derivative is denoted as the Lie derivative. We remark that the Lie derivative can be obtained also by the differential form

$$xdx + ydy = 0 \tag{9}$$

In fact, we have

$$x\frac{dx}{dt} + y\frac{dy}{dt} = 0\ ,\ \frac{dx}{dt} = -\frac{y}{x}\frac{dy}{dt} \tag{10}$$

but

$$\frac{d\psi}{dt} = \frac{\partial \psi}{\partial x}\frac{dx}{dt} + \frac{\partial \psi}{\partial y}\frac{dy}{dt} = -\frac{y}{x}\frac{dy}{dt}\frac{\partial \psi}{\partial x} + \frac{\partial \psi}{\partial y}\frac{dy}{dt} = (-y\frac{\partial \psi}{\partial x} + x\frac{\partial \psi}{\partial y})\frac{1}{x}\frac{dy}{dt} = 0$$

and

$$-y\frac{\partial \psi}{\partial x} + x\frac{\partial \psi}{\partial y} = \frac{d\psi}{dt} = 0$$

where the invariant form for the intrinsic geometry is the circle $\psi = x^2 + y^2 = R^2$.

2.1. Change of Intrinsic Geometry by Moving Reference

In the Cartesian reference and geometry, the equation for inertial movement is

$$\frac{d^2 x^i}{dt^2} = 0 \tag{11}$$

We can see that no force appears so the system is in the stress-free state. Given the transformation of the curvilinear coordinates q in the Cartesian coordinates x

$$x^i = x^i(q^\mu) = x^i(q) \tag{12}$$

We have the change of the velocity

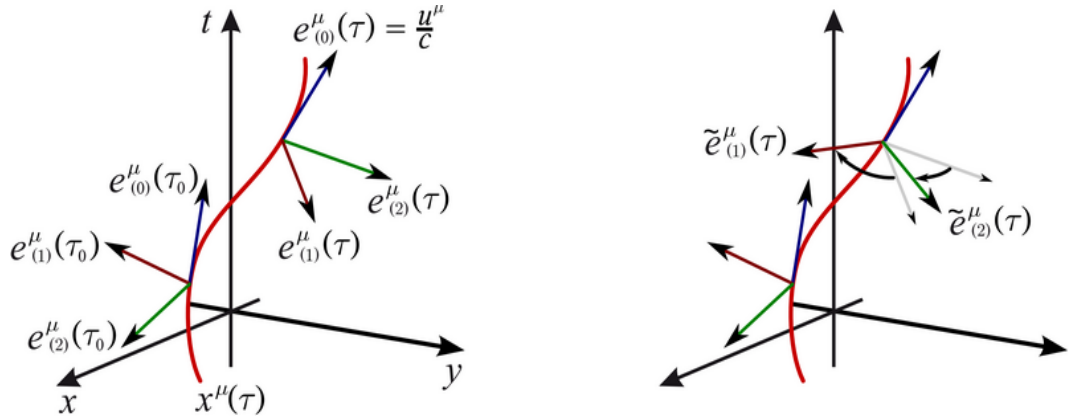
$$\begin{aligned} &\frac{dx^i}{dt} = \frac{dx^i(q)}{dt} = \frac{\partial x^i(q)}{\partial q^1}\frac{dq^1}{dt} + \frac{\partial x^i(q)}{\partial q^2}\frac{dq^2}{dt} + ... + \frac{\partial x^i(q)}{\partial q^p}\frac{dq^p}{dt} = \sum_\mu \frac{\partial x^i}{\partial q^\mu}\frac{dq^\mu(t)}{dt} = e^i_\mu \frac{dq^\mu(t)}{dt} \\ &e^i_\mu = \frac{\partial x^i}{\partial q^\mu} = \partial_\mu x^i(q) = J \end{aligned} \tag{13}$$

In many cases this is true of only the local transformation of the derivative but, in general, it is impossible to write a global expression. So, it is true of only the transformation

$$\frac{dx^i}{dt} = e^i_\mu \frac{dq^\mu(t)}{dt} \tag{14}$$

The reference $e^i_\mu(q(t))$ is the basis moving reference that is a function of the new coordinates q changing in time as we can see in Figure 3.

Figure 3. The basis of the new reference is a function of the position q and time t in the Euclidean space.



We remark that the new moving reference in a Cartesian space loses the commutative property

$$\frac{\partial e^i_\mu(q(t))}{\partial q_\upsilon} - \frac{\partial e^i_\upsilon(q(t))}{\partial q_\mu} \neq 0 \tag{15}$$

In the new reference, the acceleration takes this form

$$\begin{aligned} \frac{d^2x^i}{dt^2} &= \frac{d}{dt}\left(\frac{dq^\mu}{dt}\, e^i_\mu(q(t))\right) = \frac{d^2q^\mu}{dt^2}\, e^i_\mu(q(t)) + \frac{dq^\mu}{dt}\,\frac{de^i_\mu(q(t))}{dt} = 0 \\ &= \frac{d^2q^\mu}{dt^2}\, e^i_\mu(q(t)) + \frac{\partial e^i_\mu(q(t))}{\partial q_\upsilon}\frac{dq^\upsilon}{dt}\frac{dq^\mu}{dt} = 0 \end{aligned} \tag{16}$$

Because the basis is orthonormal and complete, as we can see in Figure 2; therefore, we have

$$e^\mu_i e^i_\upsilon = \delta^\mu_\upsilon \tag{17}$$

where

$$\delta^\mu_\upsilon = \begin{bmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & 1 \end{bmatrix}$$

So, we have

$$\frac{d^2q^{\mu}}{dt^2}\, e^{i}_{\mu}(q(t)) + \frac{\partial e^{i}_{\mu}(q(t))}{\partial q_{\upsilon}} \frac{dq^{\upsilon}}{dt} \frac{dq^{\mu}}{dt} = 0$$

that can be written in this way

$$\frac{d^2q^{\mu}}{dt^2}\, e^{i}_{\mu}(q(t)) + \frac{\partial e^{i}_{s}(q(t))}{\partial q_{p}} \frac{dq^{p}}{dt} \frac{dq^{s}}{dt} = 0$$

and

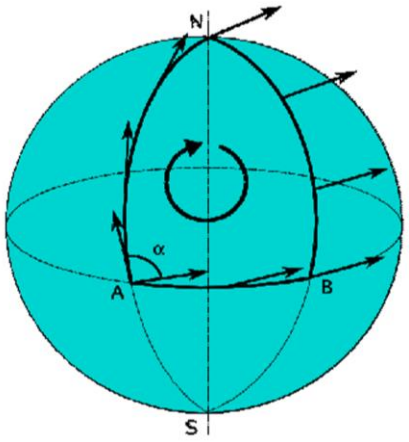
$$\frac{d^2q^{\mu}}{dt^2}\, e^{i}_{\mu}(q(t))\, e^{\mu}_{i}(q(t)) + e^{\mu}_{i}(q(t)) \frac{\partial e^{i}_{s}(q(t))}{\partial q_{p}} \frac{dq^{p}}{dt} \frac{dq^{s}}{dt} = \frac{d^2q^{\mu}}{dt^2} + e^{\mu}_{i}(q(t)) \frac{\partial e^{i}_{s}(q(t))}{\partial q_{p}} \frac{dq^{p}}{dt} \frac{dq^{s}}{dt} = \frac{d^2q^{\mu}}{dt^2} + \Gamma^{\mu}_{p,q} \frac{dq^{p}}{dt} \frac{dq^{s}}{dt} = 0 \quad (18)$$

By the inertial movement of the basis, we compute the movement in a stress-free intrinsic geometry whose geodesic equation computes the connection on the manifold where the basis moves, which value is given by the variables $\Gamma^{\mu}_{p,q}$. Because the previous equation can be written in this way

$$\frac{d^2q^{\mu}}{dt^2} = -\Gamma^{\mu}_{p,q} \frac{dq^{p}}{dt} \frac{dq^{s}}{dt} = F^{\mu} \quad (19)$$

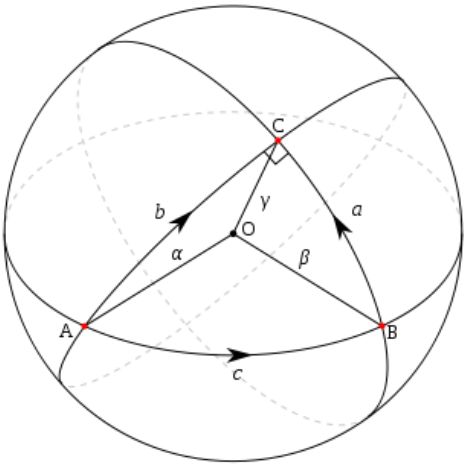
The force F^{μ} is the stress force that we must use to compute the kinematic movement in the Cartesian coordinates where the basis moves. In the Cartesian space, we have to treat stress as an external element, *i.e.*, "it breaks the system", but when we use the intrinsic geometry with curve and torsion, the kinematic movement is stress-free. This occurs, for example, when the basis moves on the spherical surface as intrinsic geometry, as we can see in Figure 4.

Figure 4. Spherical intrinsic geometry. Locally, the space is flat but globally we have a curvature for which the basis moving on the sphere is not commutative.



The dynamical equation of a Geodesic movement on a sphere is given by the previous equation, and this can be represented by Figure 5.

Figure 5. Geodesic triangle and geodesic trajectories. On the surface of the intrinsic geometry (spherical geometry), the geodesics are straight lines without curvature and so are stress-free.



Now, we have the problem to compute the derivative from the Cartesian coordinates to the general moving basis e_ν. We solve the previous problem by projecting the vector A^ν on the moving basis so we have the vector field

$$A = A^i e_i \tag{20}$$

The derivative is

$$\frac{\partial A}{\partial x_j} = \frac{\partial A^i e_i}{\partial x_j} = \frac{\partial A^i}{\partial x_j} e_i + A^i \frac{\partial e_i}{\partial x_j} \tag{21}$$

we can write the term $\frac{\partial e_i}{\partial x_j}$ as the linear combination of the basis

$$\frac{\partial e_i}{\partial x_j} = \Gamma^k_{i,j} e_k \tag{22}$$

We remark that if

$$\frac{\partial e_i}{\partial x_j} - \frac{\partial e_j}{\partial x_i} \neq 0 \tag{23}$$

The connection terms $\Gamma^k_{i,j}$ are not commutative in the index $\Gamma^k_{i,j} \neq \Gamma^k_{j,i}$ we have that $\Gamma^k_{i,j}$ are not Christoffel symbols but are simple connections with torsion $T^k_{i,j} = \Gamma^k_{i,j} - \Gamma^k_{j,i}$. Now, we have

$$\frac{\partial A}{\partial x_j} = \frac{\partial A^i}{\partial x_j} e_i + A^i \Gamma^k_{i,j} e_k \tag{24}$$

but

$$\Gamma^k_{i,j} e_k = \Gamma^i_{k,j} e_i$$

$$\frac{\partial A^i}{\partial x_j} e_i + A^k \Gamma^i_{k,j} e_i = \left(\frac{\partial A^i}{\partial x_j} + A^k \Gamma^i_{k,j}\right) e_i$$

Now, in index notation, the covariant derivative of A^i is given by

$$D_j A^i = (\frac{\partial A^i}{\partial x_j} + A^k \Gamma^i_{k,j}) \tag{25}$$

and

$$\left[D_\alpha, D_\beta \right] A^\mu = (D_\alpha D_\beta - D_\beta D_\alpha) A^\mu = R^\mu_{\nu\alpha\beta} A^\nu \tag{26}$$

where A^μ is a vector and $R^\mu_{\nu\alpha\beta}$ is the Riemann tensor curvature.

If we have a point moving on a curve in time, we have

$$x^j = x^j(t) \tag{27}$$

and the directional derivative to the tangent vector is

$$\begin{aligned} &\frac{dA}{dt} = \frac{\partial A}{\partial x_j}\frac{dx_j}{dt} = \frac{\partial A^i e_i}{\partial x_j}\frac{dx_j}{dt} = (\frac{\partial A^i}{\partial x_j} + A^k \Gamma^i_{k,j})\, e_i \frac{dx_j}{dt} \\ &= (\frac{\partial A^i}{\partial x_j}\frac{dx_j}{dt} + A^k \Gamma^i_{k,j}\frac{dx_j}{dt})\, e_i = (\frac{dA^i}{dt} + A^k \Gamma^i_{k,j}\frac{dx_j}{dt})\, e_i \end{aligned} \tag{28}$$

In the geodesic line we have

$$\frac{dA^i}{dt} + A^k \Gamma^i_{k,j}\frac{dx_j}{dt} = 0 \tag{29}$$

The derivative in the direction of the tangent vector is equal to zero. So, the geodesic is a line without stress.

2.2. Electrical Circuit and Moving Reference

Given the electrical circuit inertial equation (free from stress) for the voltages

$$\frac{d^2 v^i}{dt^2} = 0 \tag{30}$$

When we change the reference from fixed and inertial movement for the voltage to the current moving reference we know that we have the relation

$$dv^i = R^i_\mu di^\mu = e^i_\mu di^\mu \tag{31}$$

or

$$dv^i - R^i_\mu di^\mu = 0 \text{ and } dv^i - e^i_\mu di^\mu = 0$$

The first equation is the phenomenological relation between currents and voltage by the resistor tensor R^i_μ. The second is the geometry representation of the movement by the movie reference tensor without stress e^i_μ. The previous relations can be written by the tangent vectors in this way

$$\frac{dv^i}{dt}dt = e^i_\mu \frac{di^\mu}{dt}dt = R^i_\mu \frac{di^\mu}{dt}dt \tag{32}$$

or

$$\frac{dv^i}{dt} = e^i_\mu (i(t))\frac{di^\mu}{dt} = R^i_\mu (i(t))\frac{di^\mu}{dt}$$

For the compatibility between the phenomenological equation and intrinsic geometry, we have the identity

$$\frac{\partial R_i}{\partial x_j} = \Gamma^k_{i,j} R_k \tag{33}$$

and

$$\frac{\partial R_i}{\partial x_j} R^k = \Gamma^k_{i,j} R_k R^k = \Gamma^k_{i,j}$$

Given the connection term, we can design the resistor tensor in a way to have geodesic transformation and covariant derivative in the wanted space of the currents. For example, given the spherical geometry by the transformation

$$\begin{cases} x_1 = \rho \sin(\alpha)\cos(\beta) \\ x_2 = \rho \sin(\alpha)\sin(\beta) \\ x_3 = \rho \cos(\alpha) \end{cases} \tag{34}$$

The tangent vector is

$$\begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \\ \frac{dx_3}{dt} \end{bmatrix} = \begin{bmatrix} \frac{\partial x_1}{\partial R} & \frac{\partial x_1}{\partial \alpha} & \frac{\partial x_1}{\partial \beta} \\ \frac{\partial x_2}{\partial R} & \frac{\partial x_2}{\partial \alpha} & \frac{\partial x_2}{\partial \beta} \\ \frac{\partial x_3}{\partial R} & \frac{\partial x_3}{\partial \alpha} & \frac{\partial x_3}{\partial \beta} \end{bmatrix} \begin{bmatrix} \frac{dR}{dt} \\ \frac{d\alpha}{dt} \\ \frac{d\beta}{dt} \end{bmatrix} \tag{35}$$

And the moving basis is

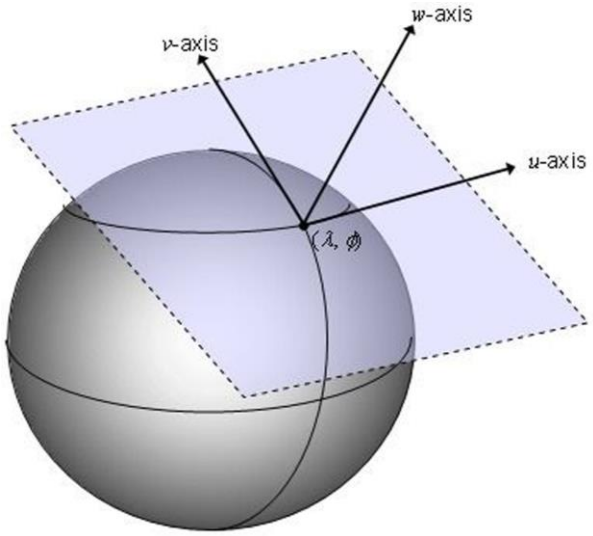
$$e^i_\mu = \begin{bmatrix} \frac{\partial x_1}{\partial R} & \frac{\partial x_1}{\partial \alpha} & \frac{\partial x_1}{\partial \beta} \\ \frac{\partial x_2}{\partial R} & \frac{\partial x_2}{\partial \alpha} & \frac{\partial x_2}{\partial \beta} \\ \frac{\partial x_3}{\partial R} & \frac{\partial x_3}{\partial \alpha} & \frac{\partial x_3}{\partial \beta} \end{bmatrix} = \begin{bmatrix} \sin(\alpha)\cos(\beta) & \rho\cos(\alpha)\cos(\beta) & -\rho\sin(\alpha)\sin(\beta) \\ \sin(\alpha)\sin(\beta) & \rho\cos(\alpha)\sin(\beta) & \rho\sin(\alpha)\cos(\beta \\ \rho\cos(\alpha) & -\rho\sin(\alpha) & 0 \end{bmatrix} \tag{36}$$

For the phenomenological identity, we have the resistor matrix

$$R^i_\mu = \begin{bmatrix} \sin(\alpha)\cos(\beta) & \rho\cos(\alpha)\cos(\beta) & -\rho\sin(\alpha)\sin(\beta) \\ \sin(\alpha)\sin(\beta) & \rho\cos(\alpha)\sin(\beta) & \rho\sin(\alpha)\cos(\beta \\ \rho\cos(\alpha) & -\rho\sin(\alpha) & 0 \end{bmatrix} \tag{37}$$

And the electrical circuits with current-controlled voltage sources (CCVS) and resistors (Figure 6).

Figure 6. Moving reference on the sphere.



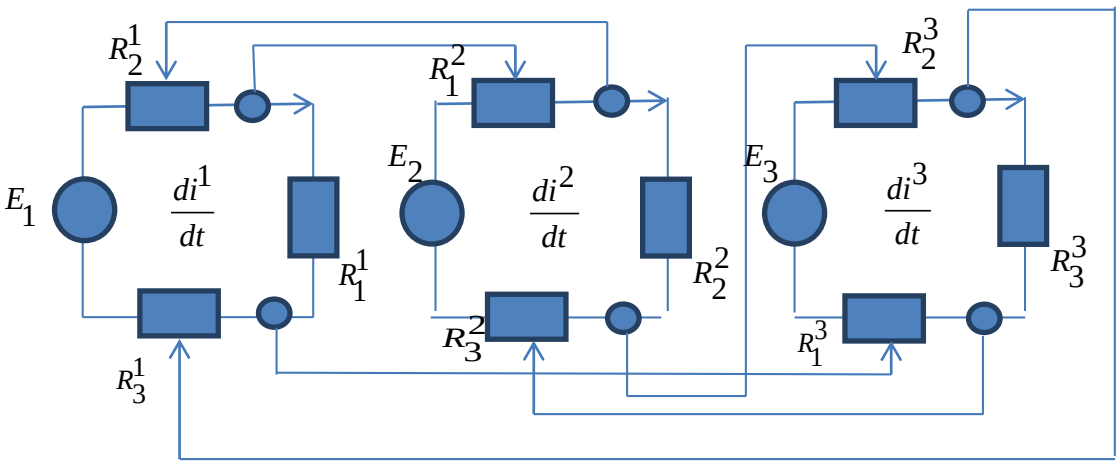
We know that the connection terms of the intrinsic geometry are

$$\frac{\partial R^i_\mu}{\partial i_j} = \Gamma^k_{\mu,j} R^i_\mu \tag{38}$$

Now, we represent the circuit with current-controlled voltage source (CCVS) R^i_μ where $i \neq \mu$ and variable resistor for $i = \mu$.

In Figure 7, we have three circuits providing the derivative in time of the current. The big circle represents the sources of the voltage that are constant or change proportionally to the time. The term R^α_α is ordinary resistors, while the other is sources of voltages ${}_v{}^\beta$ controlled by current i^α in other circuits by the proportional value R^β_α

Figure 7. Moving reference in the electrical circuit.



To complete possible electrical circuits, we have another three derivative transformations

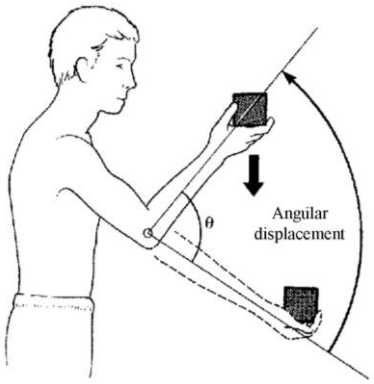
$$\begin{aligned}\frac{dq^j}{dt} &= e^j_\eta \frac{dv^\eta}{dt} = C^j_\eta \frac{dv^\eta}{dt} \\ \frac{d\phi^i}{dt} &= e^i_\xi \frac{di^\xi}{dt} = L^i_\xi \frac{di^\xi}{dt} \\ \frac{dq^k}{dt} &= e^k_\beta \frac{d\phi^\beta}{dt} = M^k_\beta \frac{d\phi^\beta}{dt}\end{aligned} \tag{39}$$

where C^j_η , L^i_ξ, M^k_β are the capacitor tensor, the induct tensor and the memristor tensor [2,3]. The variables i, v, q, ϕ are the currents, the voltages, the charges and the magnetic fluxes.

2.3. Deformation and Displacement in Media with Defects for Rotation (Disclination) and Translation (Dislocation)

Given a space where the general coordinates are $q = \left\{q^1, q^2, ..., q^n\right\}$, the bases are the vectors $e_\mu = \frac{\partial s}{\partial q^\mu}$ where s is the displacement vector (Figure 8).

Figure 8. Angular displacement q = θ and mind control of initial and final positions.



With the basis vectors $e_\mu = \frac{\partial s}{\partial q^\mu}$ we can compute the affine connection $\Gamma^\mu_{\alpha,\beta}$ in this way $\frac{\partial e_\alpha}{\partial x^\beta} = \Gamma^\mu_{\alpha,\beta}\, e_\mu$. Now, when $\Gamma^\mu_{\alpha,\beta} = \Gamma^\mu_{\beta,\alpha}$, we have curvature and metric $g_{\alpha,\beta} = (e_\alpha)^T (e_\beta)$ but no torsion. When $\Gamma^\mu_{\alpha,\beta} \neq \Gamma^\mu_{\beta,\alpha}$, we have the torsion

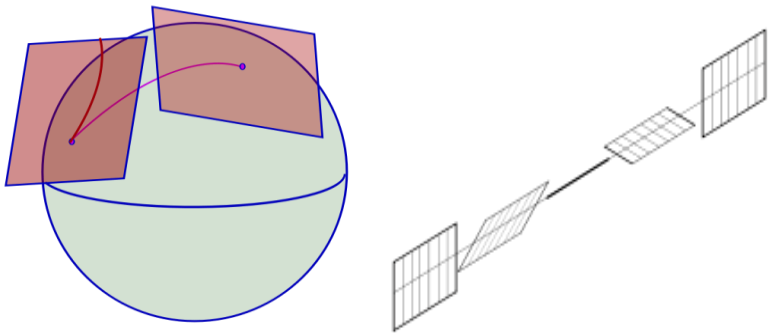
$$S^\mu_{\alpha,\beta} = \Gamma^\mu_{\alpha,\beta} - \Gamma^\mu_{\beta,\alpha} \tag{40}$$

Intrinsic geometry can have curvature and torsion that can be seen as an external element as we can see in the Euler Lagrange equation with torsion

$$\begin{aligned}&\frac{d}{dt}\frac{\partial L}{\partial v^\mu} - \frac{\partial L}{\partial q^\mu} = 2S^\nu_{\mu,\sigma}\frac{\partial L}{\partial v^\nu} v^\alpha \\ &L = \int_{t_1}^{t_2} dt \sqrt{g_{\mu,\nu}\frac{dq^\mu}{dt}\frac{dq^\nu}{dt}} \quad , \quad v^\mu = \frac{dq^\mu}{dt}\end{aligned} \tag{41}$$

Examples of curvature and torsion (Figure 9):

Figure 9. Rotation and torsion geometry.



We provide that any deformation of the reference as a crystal totally ordered is given by the transformation

$$y^i = x^i + s^i(x) \tag{42}$$

The difference of the distance L between points (atoms) before and after the deformation is given by the expression

$$dL_y^2 - dL_x^2 = 2\varepsilon_{ij} x^i x^j \tag{43}$$

where ε_{ij} is the strain tensor

$$\varepsilon_{ij} = \frac{1}{2}\left(\frac{\partial s_j}{\partial x_i} + \frac{\partial s_i}{\partial x_j} + \frac{\partial s_k}{\partial x_i}\frac{\partial s^k}{\partial x_j}\right) \tag{44}$$

In the work by Ruggero and Tartaglia [4], we find some important remarks. After the deformation, we may have two different situations:

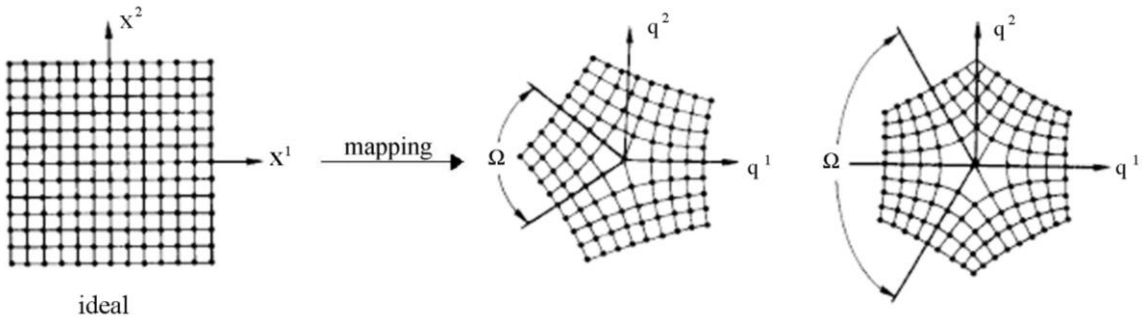
(1) The deformed elements fit perfectly or they do not. In the latter case, we must apply a further deformation to re-compact the body. In the first case, we speak of a compatible deformation.

(2) In the second case, we have an incompatible deformation. Let us imagine that during the deformation the coordinates are dragged with the medium. In the compatible deformation, the internal or intrinsic observer cannot see any difference as the Galileo internal observer for inertial system. In the incompatible deformation, the internal observer notices a change in the number of particles along a cycle in the medium as excess of holes or particles. The internal point of view is useful to find an incompatible deformation, due to the presence of defects. Mathematically, an incompatible deformation corresponds to the non-integrability of the differential form ds_j where s_j is the displacement. The non-integrability means that the displacement field $s_j(x)$ is multivalued, and thus discontinuities or defects arise when passing from one point to another. This fact is expressed by,

$$\left[\frac{\partial}{\partial x_k}, \frac{\partial}{\partial x_h}\right] s^j \neq 0 \tag{45}$$

In this situation, intrinsic geometry will no longer be Euclidean. The intrinsic view suggests that relations can be found between the geometric properties and the densities of defects that influence them. From ideal crystal or ideal reference as Cartesian reference, after the deformation we have Crystal incompatibility or disclination where there is deformation in the rotation and without torsion (Figure 10).

Figure 10. Change of reference or crystal medium by curved system where the center is a singularity or defect in the disclination.

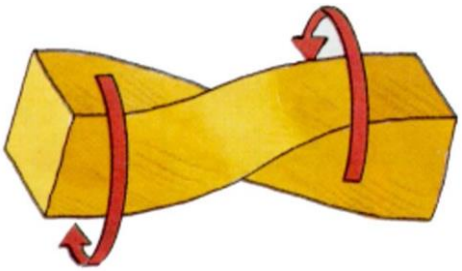


With the scalar *f* there is the torsion connection [5]

$$\left[D_\alpha, D_\beta\right] f = (D_\alpha D_\beta - D_\beta D_\alpha) f = T^{\mu}_{\alpha,\beta} D_\mu f \tag{46}$$

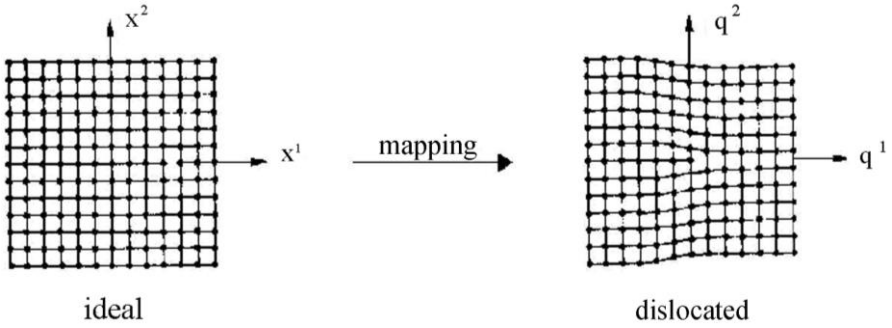
Tensor $T^{\mu}_{\alpha,\beta}$ is the torsion tensor. The torsion is given by Figure 11.

Figure 11. Torsion as defects in translation.



The torsion is a defect in translation (dislocation) as we can see in Figure 12.

Figure 12. Defects in translation or crystal dislocation.



The defect or singularity is given by defect in the reference due to translation transformation.

3. Incompatible Condition for Commutators and Wave Field Control by Active Secondary Sources

For the wave equation, we have

$$\sum_j \frac{\partial^2 \eta}{\partial x_j^2} - c^2 \frac{\partial^2 \eta}{\partial t^2} = 0 \qquad (47)$$
$$\eta = \psi e^{-ie\varphi}$$

where η is the field with noise and ψ is the field without noise or incompatibility.

$$\sum_j \frac{\partial^2 \psi e^{-ie\varphi}}{\partial x_j^2} - c^2 \frac{\partial^2 \psi e^{-ie\varphi}}{\partial t^2} = 0$$
$$\sum_j (\frac{\partial^2 \psi}{\partial x_j^2} e^{-ie\varphi} - \frac{\partial \psi}{\partial x_j} e^{-ie\varphi} ie \frac{\partial \varphi}{\partial x_j}) + (-\frac{\partial \psi}{\partial x_j} e^{-ie\varphi} ie \frac{\partial \varphi}{\partial x_j} + \psi e^{-ie\varphi} ie \frac{\partial \varphi}{\partial x_j} ie \frac{\partial \varphi}{\partial x_j} - \psi e^{-ie\varphi} ie \frac{\partial^2 \varphi}{\partial x_j^2}) -$$
$$c^2 (\frac{\partial^2 \psi}{\partial t^2} e^{-ie\varphi} - \frac{\partial \psi}{\partial t} e^{-ie\varphi} ie \frac{\partial \varphi}{\partial t}) + (-\frac{\partial \psi}{\partial t} e^{-ie\varphi} ie \frac{\partial \varphi}{\partial t} + \psi e^{-ie\varphi} ie \frac{\partial \varphi}{\partial t} ie \frac{\partial \varphi}{\partial t} - \psi e^{-ie\varphi} ie \frac{\partial^2 \varphi}{\partial t^2} = 0$$
$$\sum_j (\frac{\partial^2 \psi}{\partial x_j^2} - \frac{\partial \psi}{\partial x_j} ie \frac{\partial \varphi}{\partial x_j}) + (-\frac{\partial \psi}{\partial x_j} ie \frac{\partial \varphi}{\partial x_j} + \psi ie \frac{\partial \varphi}{\partial x_j} ie \frac{\partial \varphi}{\partial x_j} - \psi ie \frac{\partial^2 \varphi}{\partial x_j^2}) - \qquad (48)$$
$$c^2 (\frac{\partial^2 \psi}{\partial t^2} - \frac{\partial \psi}{\partial t} ie \frac{\partial \varphi}{\partial t}) + (-\frac{\partial \psi}{\partial t} ie \frac{\partial \varphi}{\partial t} + \psi ie \frac{\partial \varphi}{\partial t} ie \frac{\partial \varphi}{\partial t} - \psi ie \frac{\partial^2 \varphi}{\partial t^2} = 0$$
$$\sum_j \frac{\partial^2 \psi}{\partial x_j^2} - 2 \frac{\partial \psi}{\partial x_j} ie \frac{\partial \varphi}{\partial x_j} - \psi e^2 \frac{\partial \varphi}{\partial x_j} \frac{\partial \varphi}{\partial x_j} - \psi ie \frac{\partial^2 \varphi}{\partial x_j^2}) -$$
$$c^2 (\frac{\partial^2 \psi}{\partial t^2} - 2 \frac{\partial \psi}{\partial t} ie \frac{\partial \varphi}{\partial t} - \psi e^2 \frac{\partial \varphi}{\partial t} \frac{\partial \varphi}{\partial t} - \psi ie \frac{\partial^2 \varphi}{\partial t^2}) = 0$$
$$\sum_j D_j D_j \psi - c^2 D_t D_t \psi = 0$$

where

$$D_j = \frac{\partial}{\partial x_j} - ieA_j, D_t = \frac{\partial}{\partial t} - ie\Theta \qquad (49)$$

where

$$A_j = \frac{\partial \varphi}{\partial x_j}, \Theta = \frac{\partial \varphi}{\partial t}$$

When we use the space time reference $y_j = (x_1, x_2, x_3, ict)$ we have

$$\sum_j \frac{\partial^2 \eta}{\partial x_j^2} - c^2 \frac{\partial^2 \eta}{\partial t^2} = \sum_{k=1}^{4} \frac{\partial^2 \eta}{\partial y_k^2} = 0 \qquad (50)$$

For $\eta = \psi e^{-ie\varphi}$ we have

$$\sum_j \frac{\partial^2 \eta}{\partial x_j^2} - c^2 \frac{\partial^2 \eta}{\partial t^2} = \sum_{k=1}^{4} \frac{\partial^2 \eta}{\partial y_k^2} = \sum_{k=1}^{4} D_k D_k \psi = 0 \qquad (51)$$

where

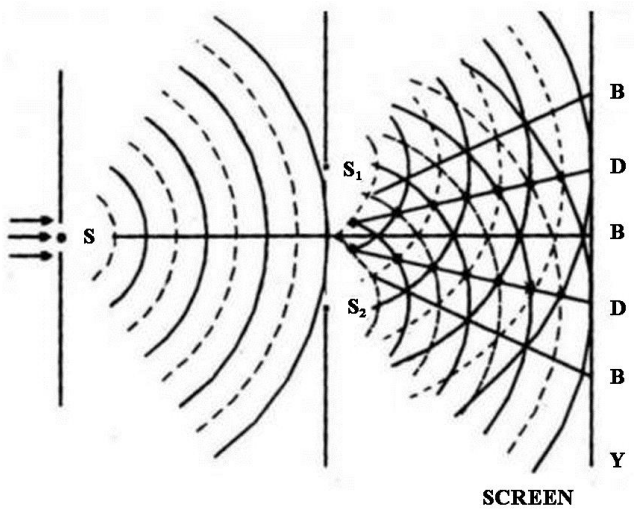
$$D_k = \frac{\partial}{\partial y_k} - ieC_k, \text{ where } C_k = \frac{\partial \varphi}{\partial y_k} \tag{52}$$

We remark that

$$\begin{aligned}
&[D_\mu, D_\nu]\psi = (\partial_\mu - ieC_\mu)(\partial_\nu - ieC_\nu)\psi - (\partial_\nu - ieC_\nu)(\partial_\mu - ieC_\mu)\psi \\
&= \partial^2{}_{\mu\nu}\psi - ie(\partial_\mu C_\nu)\psi - ieC_\nu(\partial_\mu\psi) - ieC_\mu(\partial_\nu\psi) - e^2 C_\mu C_\nu\psi \\
&-\partial^2{}_{\nu\mu}\psi + ie(\partial_\nu C_\mu)\psi + ieC_\mu(\partial_\nu\psi) + ieC_\nu(\partial_\mu\psi) + e^2 C_\mu C_\nu\psi \\
&= ie(\frac{\partial C_\nu}{\partial x_\mu} - \frac{\partial C_\mu}{\partial x_\nu})\psi = -ieW_{\mu\nu}\psi
\end{aligned} \tag{53}$$

For the wave equation, the change of the wave function generates incompatible medium where there are defects as we can see in Figure 13.

Figure 13. From compatible medium on the left, there is incompatible medium on the screen.



The new derivative does not commute but the wave equation does not change its form and the wave sources are always the same. In fact, for

$$\sum_j \frac{\partial^2 \eta}{\partial x_j^2} - c^2 \frac{\partial^2 \eta}{\partial t^2} = \sum_{k=1}^{4} \frac{\partial^2 \eta}{\partial y_k^2} = S \tag{54}$$

We have

$$\sum_{k=1}^{4} D_k D_k \psi = S \tag{55}$$

In the Jessel book [6] we found the connection between sources and transformation of field variable. Now, we use this method to explain better the meaning of the non-commutativity of the derivates and the incompatibility.

In fact, the transformation $\eta = \psi e^{-ie\varphi}$ of the equation $\sum_j \frac{\partial^2 \eta}{\partial x_j^2} - c^2 \frac{\partial^2 \eta}{\partial t^2} = S$ we have

$$\sum_j \frac{\partial^2 \psi e^{-ie\varphi}}{\partial x_j^2} - c^2 \frac{\partial^2 \psi e^{-ie\varphi}}{\partial t^2} = \sum_j \frac{\partial^2 \psi}{\partial x_j^2} - 2\frac{\partial \psi}{\partial x_j} ie\frac{\partial \varphi}{\partial x_j} - \psi e^2 \frac{\partial \varphi}{\partial x_j}\frac{\partial \varphi}{\partial x_j} - \psi ie\frac{\partial^2 \varphi}{\partial x_j^2}) -$$

$$c^2(\frac{\partial^2 \psi}{\partial t^2} - 2\frac{\partial \psi}{\partial t} ie\frac{\partial \varphi}{\partial t} - \psi e^2 \frac{\partial \varphi}{\partial t}\frac{\partial \varphi}{\partial t} - \psi ie\frac{\partial^2 \varphi}{\partial t^2}) = S$$

or

$$\sum_j \frac{\partial^2 \psi}{\partial x_j^2} - c^2 \frac{\partial^2 \psi}{\partial t^2}$$

$$= S + \sum_j (2\frac{\partial \psi}{\partial x_j} ie\frac{\partial \varphi}{\partial x_j} + \psi e^2 \frac{\partial \varphi}{\partial x_j}\frac{\partial \varphi}{\partial x_j} + \psi ie\frac{\partial^2 \varphi}{\partial x_j^2}) - c^2(2\frac{\partial \psi}{\partial t} ie\frac{\partial \varphi}{\partial t} + \psi e^2 \frac{\partial \varphi}{\partial t}\frac{\partial \varphi}{\partial t} + \psi ie\frac{\partial^2 \varphi}{\partial t^2}) = S + S^* \quad (56)$$

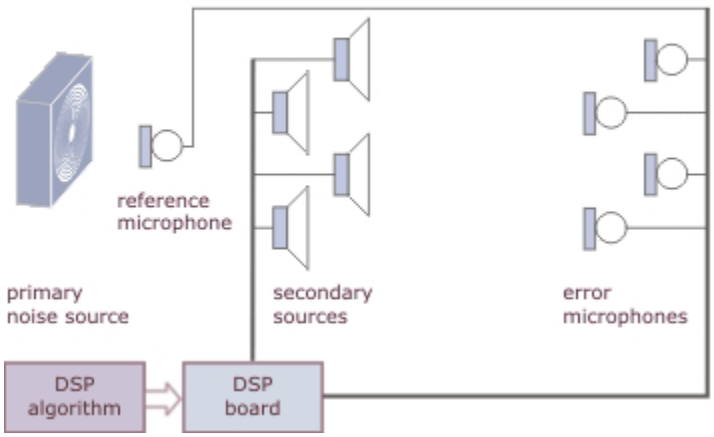
In the transformation, the derivatives are those in the compatible medium but we must change the source from S as the original sources of the wave to new artificial sources or secondary sources

$$S^* = \sum_j (2\frac{\partial \psi}{\partial x_j} ie\frac{\partial \varphi}{\partial x_j} + \psi e^2 \frac{\partial \varphi}{\partial x_j}\frac{\partial \varphi}{\partial x_j} + \psi ie\frac{\partial^2 \varphi}{\partial x_j^2}) - c^2(2\frac{\partial \psi}{\partial t} ie\frac{\partial \varphi}{\partial t} + \psi e^2 \frac{\partial \varphi}{\partial t}\frac{\partial \varphi}{\partial t} + \psi ie\frac{\partial^2 \varphi}{\partial t^2})$$

That gives us the physical image of the incompatibility in the medium when we transform one field to another. By the new sources we can generate the new field with the same derivatives. M. Jessel uses the new sources to design a wanted field from the initial one. This is the beginning of a new computation where we design a new intrinsic geometry in the field by artificial sources. This is an example of field control by the active or secondary sources S* [7].

InVuksanovic and Nikolic [8] we have the multichannel active noise control (see Figure 14).

Figure 14. Active noise control (ANC) by the algorithm or DSP for S*.



The active noise control (ANC) is the process of reducing an unwanted or incompatible sound by combining it with a sound of the same amplitude but of opposite phase. The proposed ANC system uses an active sound barrier of secondary sources S* to cancel the unwanted sound or incompatibility from the primary source at an array of error microphones. By cancelling the sound at the error microphones distributed across the controlled region, the secondary sources create a zone of reduced noise over this area as we can see in Figure 14 where the DSP algorithm uses the expression for S* to generate a wanted field without noise.

4. Schrödinger and Maxwell Equations Commutators and Incompatible Equations

In the work by Russer [9], we can see that the Maxwell equation can be represented by exterior differential forms. Now, in this chapter, because of a suggestion by Pessa [10], we show the invariance of the Schrodinger equation for a given transformation of the wave function; therefore, we obtain the Maxwell equations by commutators that are connected by the incompatibility of the medium. So, given the celebrate Schrodinger equation,

$$i\frac{\partial \eta}{\partial t} = -\frac{1}{2}m\sum_j \frac{\partial^2 \eta}{\partial x_j^2} + U\eta$$
$$\eta = \psi e^{-ie\varphi} \tag{57}$$

When we substitute the new variable, we have

$$-\frac{1}{2}m\sum_j\left(\frac{\partial^2\psi}{\partial x_j^{\,2}} - \frac{\partial\psi}{\partial x_j}ie\frac{\partial\varphi}{\partial x_j}\right) + \left(-\frac{\partial\psi}{\partial x_j}ie\frac{\partial\varphi}{\partial x_j} + \psi ie\frac{\partial\varphi}{\partial x_j}ie\frac{\partial\varphi}{\partial x_j} - \psi ie\frac{\partial^2\varphi}{\partial x_j^{\,2}}\right) + U\psi$$
$$i\left(\frac{\partial\psi}{\partial t}\right) = -\frac{1}{2}m\left[\sum_j\left(\frac{\partial^2}{\partial x_j^{\,2}} - 2ie\frac{\partial\varphi}{\partial x_j}\frac{\partial}{\partial x_j}\right) - \left(e^2\left(\frac{\partial\varphi}{\partial x_j}\right)^2 + ie\frac{\partial^2\varphi}{\partial x_j^{\,2}}\right)\right]\psi + \left(U - e\frac{\partial\varphi}{\partial t}\right)\psi \tag{58}$$

For

$$A_j = \frac{\partial\varphi}{\partial x_j}, \phi = -\frac{\partial\varphi}{\partial t} \tag{59}$$

$$i\left(\frac{\partial\psi}{\partial t}\right) = -\frac{1}{2}m\left[\sum_j\left(\frac{\partial^2}{\partial x_j^{\,2}} - 2ieA_j\frac{\partial}{\partial x_j}\right) - e^2(A_j)^2 - ie\frac{\partial A_j}{\partial x_j}\right]\psi + (U - e\phi)\psi \tag{60}$$

but

$$D_jD_j\psi = \left(\frac{\partial}{\partial x_j} - ieA_j\right)\left(\frac{\partial}{\partial x_j} - ieA_j\right)\psi =$$
$$= \frac{\partial^2\psi}{\partial x_j^{\,2}} - ie\left(\frac{\partial A_j}{\partial x_j}\right)\psi - ieA_j\left(\frac{\partial\psi}{\partial x_j}\right) - ieA_j\left(\frac{\partial\psi}{\partial x_j}\right) - e^2A_jA_j\psi \tag{61}$$
$$= \left[\frac{\partial^2}{\partial x_j^{\,2}} - 2ieA_j\left(\frac{\partial}{\partial x_j}\right) - \left(ie\left(\frac{\partial A_j}{\partial x_j}\right)\psi + e^2A_jA_j\right)\right]\psi$$

and

$$i\frac{\partial\psi}{\partial t} = -\frac{1}{2}m\sum_j D_jD_j\psi + \Phi\psi \tag{62}$$

where

$$\Phi = U - e\phi$$

When

$$\frac{\partial\varphi}{\partial x_j} = 0, \frac{\partial\varphi}{\partial t} = 0 \tag{63}$$

The phase is constant in space and time and we have the compatible condition

$$[D_\mu, D_\nu]\psi = [\partial_\mu \partial_\nu - \partial_\nu \partial_\mu]\ \psi = 0 \tag{64}$$

We have no curvature and torsion and the medium has no defects. However, when

$$\frac{\partial \varphi}{\partial x_j} = A_j, \frac{\partial \varphi}{\partial t} = \phi \tag{65}$$

We have the incompatible condition

$$\begin{aligned} &[D_\mu, D_\nu]\psi = (\partial_\mu - ieA_\mu)(\partial_\nu - ieA_\nu)\psi - (\partial_\nu - ieA_\nu)(\partial_\mu - ieA_\mu)\psi \\ &= \partial^2{}_{\mu\nu}\psi - ie(\partial_\mu A_\nu)\psi - ieA_\nu(\partial_\mu \psi) - ieA_\mu(\partial_\nu \psi) - e^2 A_\mu A_\nu \psi \\ &-\partial^2{}_{\nu\mu}\psi + ie(\partial_\nu A_\mu)\psi + ieA_\mu(\partial_\nu \psi) + ieA_\nu(\partial_\mu \psi) + e^2 A_\mu A_\nu \psi \\ &= ie(\frac{\partial A_\nu}{\partial x_\mu} - \frac{\partial A_\mu}{\partial x_\nu})\psi = -ieF_{\mu\nu}\psi \end{aligned} \tag{66}$$

For a reference with torsion, we obtain the incompatible equation

$$\left[D_\alpha, D_\beta\right] f = (D_\alpha D_\beta - D_\beta D_\alpha) f = -T^\mu_{\alpha,\beta} D_\mu f = -ieF_{\alpha,\beta}\ \psi \tag{67}$$

and

$$T^\mu_{\alpha,\beta} D_\mu f = ieF_{\alpha,\beta}\ \psi$$

When we solve this equation, we can provide a new geometric representation of the electromagnetic equation by torsion of the reference and defects in the medium. In crystal, there is a separation of the charges and the reference for the electromagnetic field is deformed by a torsion as in the dislocation of the crystal.

In the electromagnetic theory, we have that

$$\begin{aligned} &[D_\mu, [D_\nu, D_\rho]\psi = D_\mu([D_\nu, D_\rho]\psi) - [D_\nu, D_\rho]D_\mu\psi = ie[D_\mu(F_{\nu\rho}\psi) - F_{\nu\rho}D_\mu\psi] \\ &= ie[(D_\mu F_{\nu\rho})\psi + F_{\nu\rho}D_\mu\psi - F_{\nu\rho}D_\mu\psi] = ie(D_\mu F_{\nu\rho})\psi \end{aligned} \tag{68}$$

where

$$D_\mu F_{\nu\rho} = J_{\mu,\nu\rho} \tag{69}$$

$J_{\mu,\nu\rho}$ are the currents of the defects or electrical particles.

Because we have

$$[D_\mu, [D_\nu, D_\rho] + [D_\nu, [D_\rho, D_\mu] + [D_\rho, [D_\mu, D_\nu] = 0 \tag{70}$$

We have the invariant property for the currents

$$J_{\mu,\nu\rho} + J_{\nu,\rho\mu} + J_{\rho,\mu\nu} = 0 \tag{71}$$

Given the Maxwell equations in the tensor form

$$\begin{cases} \partial_\mu F^{\mu\upsilon} = \dfrac{4\pi}{c} J^\mu \\ \partial^\alpha F^{\mu\nu} + \partial^\mu F^{\nu\alpha} + \partial^\nu F^{\alpha\mu} = 0 \end{cases} \tag{72}$$

where the contravariant four-vector which combines electric current density and electric charge density $J^{\nu} = (cp, J_x, J_y, J_z)$ is the four-current, the electromagnetic tensor is $F^{????} = a?\,?^{??}\,A^{??} - a?\,?^{??}\,A^{??}$ that can be connected with the commutators' property and the incompatible condition that we have explained in the prrevious chapters.

The Maxwell-like scheme for an incompatible system is given by the set of equations

$$\begin{aligned} D_{\mu} &= \frac{\partial}{\partial x_{\mu}} + kA_{\mu}(x) \\ [D_{\mu}, D_{\nu}]\psi &= F_{\mu\nu}\psi \\ [D_{\mu},[D_{\nu}, D_{\rho}]\psi &= J_{\mu,\nu\rho} \end{aligned} \tag{73}$$

where the covariant derivative includes a connection term that is the potential, the commutator is the compensatory field for the incompatible system and the second commutator is the density current of the defects. The three equations can be used as models for all possible dynamical systems that include defects or sources. In the next chapter, we use this new scheme to improve the Einstein gravitational geometry.

5. Dynamic Equations with Torsion in Non-Conservative Gravity Maxwell-Like Equations

To introduce the new wave equation for gravity [11,12] and for the "constructive logic" of the gauges theories [13], we remember that

$$\left[\nabla_{\mu}, \nabla_{\nu}\right] V_{\alpha} = -R^{\lambda}{}_{\alpha\mu\nu} V_{\lambda} \tag{74}$$

where the Riemann tensor is

$$R^{\lambda}_{\alpha\mu\nu} = \partial_{\mu}\Gamma^{\lambda}_{\nu\alpha} - \partial_{\nu}\Gamma^{\lambda}_{\mu\alpha} + \Gamma^{\sigma}_{\mu\alpha}\Gamma^{\lambda}_{\upsilon\alpha} - \Gamma^{\sigma}_{\sigma\alpha}\Gamma^{\lambda}_{\mu\sigma} \tag{75}$$

With the double commutator we have the dynamic equation

$$\begin{aligned} &[\nabla_{\mu},[\nabla_{\alpha}, \nabla_{\beta}]]K_{\nu} = (\nabla_{\mu}[\nabla_{\alpha}, \nabla_{\beta}])K_{\nu} - [\nabla_{\alpha}, \nabla_{\beta}](\nabla_{\mu}K_{\nu}) \\ &= -(\nabla_{\mu}R^{\lambda}_{\nu\alpha\beta})K_{\lambda} + R^{\lambda}_{\mu\alpha\beta}(\nabla_{\lambda}K_{\nu}) \end{aligned} \tag{76}$$

where R is the Riemann tensor, $a?\,?_k$ is the covariant derivative and $K_{??}$ is the vacuum field. Now we connect the commutator with the gravity current in this way

$$\chi J_{\mu\alpha\beta}K_{\nu} = -(\nabla_{\mu}R^{\lambda}_{\nu\alpha\beta})K_{\lambda} + R^{\lambda}_{\mu\alpha\beta}(\nabla_{\lambda}K_{\nu}) \tag{77}$$

For the conservation of the current, after contractions, we have the equation

$$a?\,?_{??}\left[R^{????} + ?\,?\left(T^{????} + \tfrac{1}{2}g^{????}T\right)\right]K_{??} + R^{????}(a?\,?_{??}\,K_{??}) = 0 \tag{78}$$

when $a?\,?_k\,K_{??} = 0$ we have the Einstein equations.

Most applications of differential geometry, including General Relativity, assume that the connection is "torsion free": that is, vectors do not rotate during parallel transport. Because some extensions of GR do include torsion, it is useful to see how torsion appears in a modern geometrical language. The torsion corresponds intuitively to the condition that vectors must not be rotated by parallel transport.

Such a condition is natural to impose, the theory of General Relativity itself includes this assumption. However, differential geometry is equally well.defined with torsion as well as without, and some extensions of general relativity include torsion terms. The first of these was "Einstein-Cartan theory", as introduced by Cartan in 1922. We define the torsion tensor by the connection symbols in this way

$$\Gamma^{\lambda}_{\mu\nu} - \Gamma^{\lambda}_{\nu\mu} = T^{\lambda}_{\mu\nu} \tag{79}$$

We now show in an explicit way that it is possible to present the previous dynamical equation by a wave equation with a particular source where the variables include symmetric and anti-symmetric connection symbols as well as torsion in one geometric picture. In Appendix A, we define the new type of wave with an explicit computation of the commutator and of the double commutator.

6. Conclusions

Symmetry and its physical implications on conservation principles have a long history in physics. The same importance, if not higher, is shown by the concepts of symmetry breaking and local gauge as the constructive principle to characterize interactions as a "compensation mechanism". In particular, this was made possible by a unified geometrical vision of fundamental interactions in Gauge Theories [14]. The structural logics of these theories are not an exclusive prerogative of particle physics or relativistic geometrodynamics. In this work, we delineated a parallel development of such ideas we called "Compensative Geometry" which has old systemic roots. It is within such a context—at the crossroad of Theoretical Physics, Cybernetics, Category and Group Theory and Logical System Theory—that the constructive approach here introduced has been developed [3,7,11] (for some fundamental steps, see [15–17]; for the consequences on the computation concept, see [18,19]). Such class of theories is based on few principles related to different orders of commutators between covariant derivatives. Their physical meaning is very simple, and lies in stating that the local transformations of a suitable substratum (the space-time or a particular phase space) and the imposed constraints define a "compensative mechanism" or the "interaction" we want to characterize.

We stress the mathematical aspects which make this approach a "theory to build geometric-based unified theories".

The conceptual core of the procedure can be expressed in a five-point nutshell:

(a) The description of a suitable substratum and its global and local properties on invariance;
(b) The field potentials are compensative fields defined by a gauge covariant derivative. They share the global invariance properties with the substratum;
(c) The calculation of the commutators of the covariant derivatives in (b) provides the relations between the field strength and the field potentials;
(d) The Jacobi identity applied to commutators provides the dynamic equations satisfied by the field strength and the field potentials;
(e) The commutator between the covariant derivatives (b) and the commutator (c) (triple Jacobian commutator) fixes the relations between field strength and field currents.

We chose an example connected to the recent developments of the extended gravity theories in order to show the generality of the approach. Actually, the GR syntax seems to regenerate itself from

inside and to produce many schemes of classical coupling *Raum–Zeit–Materie*. This autopoietic feature is a distinctive and propulsive of Theoretical Physics considered as a totality of structures that fixes the conditions of thinkability for its entities and "beables".

In conclusion, the geometrical approach here delineated has significant potential in relation to the classical themes of systemics (system/environment; contextuality; computation; logical openness) thanks to the strategy allowing, in a simple and general way, to recognise the gauging as cognitive compensation between known and unknown domains.

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Author Contributions

The authors contributed in equal measure to the work.

Conflicts of Interest

The authors declare no conflict of interest.

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Appendix A

Given the general covariant derivative

$$D_\mu V_\alpha = \frac{\partial V_\alpha}{\partial x_\mu} - \Gamma^\lambda_{\mu\nu} V_\lambda \tag{1A}$$

where the connection terms are unknown variables that we define by the new gravitational equation obtained by the first and second commutator. We remark that the connection terms are not Christoffel elements but are a general connection which values will be defined by the new gravitational equations.

To know the connection terms we begin with the computation of the first commutator

$$F_{\mu\nu,\alpha} = \left[D_\mu, D_\nu \right] V_\alpha = D_\mu D_\nu V_\alpha - D_\nu D_\mu V_\alpha = D_\mu \left(\frac{\partial V_\alpha}{\partial x_\nu} - \Gamma^\lambda_{\nu\alpha} V_\lambda\right) - D_\nu \left(\frac{\partial V_\alpha}{\partial x_\mu} - \Gamma^\lambda_{\mu\nu} V_\lambda\right) \tag{2A}$$

So we have

$$F_{\mu\nu,\alpha} = \left[D_\mu, D_\nu\right] V_\alpha = \left(\frac{\partial B_{\nu,\alpha}}{\partial x_\mu} - \Gamma^\lambda_{\nu,\mu} B_{\nu,\lambda} - \Gamma^\lambda_{\alpha,\mu} B_{\alpha,\lambda}\right) - \left(\frac{\partial B_{\mu,\alpha}}{\partial x_\nu} - \Gamma^\lambda_{\mu,\nu} B_{\mu,\lambda} - \Gamma^\lambda_{\alpha,\nu} B_{\alpha,\lambda}\right) \tag{3A}$$

with

$$B_{\nu,\alpha} = \left(\frac{\partial V_\alpha}{\partial x_\nu} - \Gamma^\lambda_{\nu,\alpha} V_\lambda\right)$$

$$B_{\mu,\alpha} = \left(\frac{\partial V_\alpha}{\partial x_\mu} - \Gamma^\lambda_{\mu,\alpha} V_\lambda\right)$$

and

$$F_{\mu\nu,\alpha} = [D_\mu, D_\nu]V_\alpha = -(R^\lambda_{\alpha\mu\nu}V_\lambda + \mathrm{T}^\lambda_{\mu\nu}D_\lambda V_\alpha)$$

where

$$R^\lambda_{\alpha\mu\nu} = (\frac{\partial\Gamma^\lambda_{\nu,\alpha}}{\partial x_\mu} - \frac{\partial\Gamma^\lambda_{\mu,\alpha}}{\partial x_\nu} + \Gamma^\lambda_{\mu,q}\Gamma^q_{\nu,\alpha} - \Gamma^q_{\mu,\alpha}\Gamma^\lambda_{\nu,\alpha})$$

In conclusion, we have

$$-F_{\mu\nu,\alpha} = -[D_\mu, D_\nu]V_\alpha = \left(\frac{\partial\Gamma^\lambda_{\nu,\alpha}}{\partial x_\mu} - \frac{\partial\Gamma^\lambda_{\mu,\alpha}}{\partial x_\nu}\right)V_\lambda + \left(\Gamma^\lambda_{\mu,q}\Gamma^q_{\nu,\alpha} - \Gamma^q_{\mu,\alpha}\Gamma^\lambda_{\nu,q}\right)V_\lambda + \left(\Gamma^\lambda_{\mu,\nu} - \Gamma^\lambda_{\nu,\mu}\right)D_\lambda V_\alpha = G_{\mu\nu,\alpha} + \Omega_{\mu\nu,\alpha} \quad (4A)$$

where

$$G_{\mu\nu,\alpha} = \left(\frac{\partial\Gamma^\lambda_{\nu,\alpha}}{\partial x_\mu} - \frac{\partial\Gamma^\lambda_{\mu,\alpha}}{\partial x_\nu}\right)V_\lambda$$

and

$$\Omega_{\mu\nu,\alpha} = \left(\Gamma^\lambda_{\mu,q}\Gamma^q_{\nu,\alpha} - \Gamma^q_{\mu,\alpha}\Gamma^\lambda_{\nu,q}\right)V_\lambda + \left(\Gamma^\lambda_{\mu,\nu} - \Gamma^\lambda_{\nu,\mu}\right)D_\lambda V_\alpha$$

Now, we have that

$$\Gamma^\lambda_{\nu\alpha} \to \Gamma^\lambda_{\nu\alpha} + \frac{\partial\chi}{\partial x_\nu}$$

obtaining

$$G_{\mu\nu,\alpha} = (\frac{\partial\Gamma^\lambda_{\nu\alpha}}{\partial x_\mu} - \frac{\partial\Gamma^\lambda_{\mu\alpha}}{\partial x_\nu})V_\lambda + \frac{\partial^2\chi}{\partial x_\mu\partial x_\nu} - \frac{\partial^2\chi}{\partial x_\nu\partial x_\mu} = G_{\mu\nu,\alpha} \quad (5A)$$

So, we have that the field G is invariant. Now, we impose the Lorenz-like gauge condition in this way

$$\frac{\partial\Gamma^\lambda_{\nu\alpha}}{\partial x_\mu} = 0 \quad (6A)$$

Now, we have

$$\begin{aligned}
&\frac{\partial}{\partial x_\beta}G_{\mu\nu,\alpha} + \frac{\partial}{\partial x_\nu}G_{\beta\mu,\alpha} + \frac{\partial}{\partial x_\mu}G_{\nu\beta,\alpha} \\
&= \frac{\partial}{\partial x_\beta}(\frac{\partial\Gamma^\lambda_{\nu,\alpha}}{\partial x_\mu} - \frac{\partial\Gamma^\lambda_{\mu,\alpha}}{\partial x_\nu})V_\lambda + \frac{\partial}{\partial x_\nu}(\frac{\partial\Gamma^\lambda_{\mu,\alpha}}{\partial x_\beta} - \frac{\partial\Gamma^\lambda_{\beta,\alpha}}{\partial x_\mu})V_\lambda + \frac{\partial}{\partial x_\mu}(\frac{\partial\Gamma^\lambda_{\beta,\alpha}}{\partial x_\nu} - \frac{\partial\Gamma^\lambda_{\nu,\alpha}}{\partial x_\beta})V_\lambda \quad (7A) \\
&= \frac{\partial}{\partial x_\beta}\frac{\partial\Gamma^\lambda_{\nu,\alpha}}{\partial x_\mu} + \frac{\partial}{\partial x_\nu}\frac{\partial\Gamma^\lambda_{\mu,\alpha}}{\partial x_\beta} + \frac{\partial}{\partial x_\mu}\frac{\partial\Gamma^\lambda_{\beta,\alpha}}{\partial x_\nu} - \frac{\partial}{\partial x_\beta}\frac{\partial\Gamma^\lambda_{\mu,\alpha}}{\partial x_\nu} - \frac{\partial}{\partial x_\nu}\frac{\partial\Gamma^\lambda_{\beta,\alpha}}{\partial x_\mu} - \frac{\partial}{\partial x_\mu}\frac{\partial\Gamma^\lambda_{\nu,\alpha}}{\partial x_\beta} = 0
\end{aligned}$$

The Lagrangian gravitational density is

$$L = F_{\mu\nu,\alpha}F^{\mu\nu,\alpha} = \left(G_{\mu\nu,\alpha} + \Omega_{\mu\nu,\alpha}\right)(G^{\mu\nu,\alpha} + \Omega^{\mu\nu,\alpha}) = G_{\mu\nu,\alpha}G^{\mu\nu,\alpha} + \Omega_{\mu\nu,\alpha}\Omega^{\mu\nu,\alpha} + 2G_{\mu\nu,\alpha}\Omega^{\mu\nu,\alpha} \quad (8A)$$

where $G_{\mu\nu,\alpha}G^{\mu\nu,\alpha}$ and $\Omega_{\mu\nu,\alpha}\Omega^{\mu\nu,\alpha} + 2G_{\mu\nu,\alpha}\Omega^{\mu\nu,\alpha}$ are the Lagrangian density for the free gravitational field and the reaction field of the vacuum. The interaction term $G_{\mu\nu,\alpha}\Omega^{\mu\nu,\alpha}$ connects the gravitation field with the field of the vacuum.

The dynamic equation for Non Conservative Gravity can be obtained in this way

$$\left[D_\beta,\left[D_\mu,D_\nu\right]\right]V_\alpha = D_\beta\left[D_\mu,D_\nu\right]V_\alpha - \left[D_\mu,D_\nu\right]D_\beta V_\alpha = D_\beta F_{\mu\nu,\alpha} - \left[D_\mu,D_\nu\right]D_\beta V_\alpha = \\ = -D_\beta\left(G_{\mu\nu,\alpha}+\Omega_{\mu\nu,\alpha}\right) - \left[D_\mu,D_\nu\right]D_\beta V_\alpha = J_{\mu\nu,\alpha\beta} \quad (9A)$$

and

$$D_\beta G_{\mu\nu,\alpha} = -J_{\mu\nu,\alpha\beta} - D_\beta\Omega_{\mu\nu,\alpha} - \left[D_\mu,D_\nu\right]D_\beta V_\alpha \quad (10A)$$

where

$$D_\beta G_{\mu\nu,\alpha} = \frac{\partial G_{\mu\nu,\alpha}}{\partial x_\beta} - G_{j\nu,\alpha}\Gamma^j_{\mu\beta} - G_{\mu j,\alpha}\Gamma^j_{\nu\beta} - G_{\mu\nu,j}\Gamma^j_{\alpha\beta} \quad (11A)$$

so

$$\frac{\partial G_{\mu\nu,\alpha}}{\partial x_\beta} - G_{j\nu,\alpha}\Gamma^j_{\mu\beta} - G_{\mu j,\alpha}\Gamma^j_{\nu\beta} - G_{\mu\nu,j}\Gamma^j_{\alpha\beta} = -J_{\mu\nu,\alpha\beta} - D_\beta\Omega_{\mu\nu,\alpha} - \left[D_\mu,D_\nu\right]D_\beta V_\alpha \quad (12A)$$

and

$$\frac{\partial G_{\mu\nu,\alpha}}{\partial x_\beta} = G_{j\nu,\alpha}\Gamma^j_{\mu\beta} + G_{\mu j,\alpha}\Gamma^j_{\nu\beta} + G_{\mu\nu,j}\Gamma^j_{\alpha\beta} - J_{\mu\nu,\alpha\beta} - D_\beta\Omega_{\mu\nu,\alpha} - \left[D_\mu,D_\nu\right]D_\beta V_\alpha \quad (13A)$$

$$\frac{\partial G_{\mu\nu,\alpha}}{\partial x_\beta} = J_{\mu\nu,\alpha\beta}$$

Now, we have

$$\frac{\partial G_{\mu\nu,\alpha}}{\partial x_\beta} = \frac{\partial}{\partial x_\beta}\left(\frac{\partial\Gamma^\lambda_{\nu,\alpha}}{\partial x_\mu} - \frac{\partial\Gamma^\lambda_{\mu,\alpha}}{\partial x_\nu}\right)V_\lambda = \left(\frac{\partial^2\Gamma^\lambda_{\nu,\alpha}}{\partial x_\beta\partial x_\mu} - \frac{\partial^2\Gamma^\lambda_{\mu,\alpha}}{\partial x_\beta\partial x_\nu}\right)V_\lambda \quad (14A)$$

For $x_{?2} = x_{??}$ we have

$$\frac{\partial G_{\mu\nu,\alpha}}{\partial x_\mu} = \left(\frac{\partial^2\Gamma^\lambda_{\nu,\alpha}}{\partial^2 x_\mu} - \frac{\partial^2\Gamma^\lambda_{\mu,\alpha}}{\partial x_\mu\partial x_\nu}\right)V_\lambda = \left(\frac{\partial^2\Gamma^\lambda_{\nu,\alpha}}{\partial^2 x_\mu} - \frac{\partial^2\Gamma^\lambda_{\mu,\alpha}}{\partial x_\nu\partial x_\mu}\right)V_\lambda \quad (15A)$$

However, for the Lorentz-like gauge we have

$$\frac{\partial^2\Gamma^\lambda_{\mu,\alpha}}{\partial x_\nu\partial x_\mu} = \frac{\partial}{\partial x_\nu}\left(\frac{\partial\Gamma^\lambda_{\nu,\alpha}}{\partial x_\mu}\right) = 0 \quad (16A)$$

and

$$\frac{\partial G_{\mu\nu,\alpha}}{\partial x_\mu} = \frac{\partial^2\Gamma^\lambda_{\nu,\alpha}}{\partial^2 x_\mu}V_\lambda = \left(\frac{\partial^2\Gamma^\lambda_{\nu,\alpha}}{\partial^2 x} - c^2\frac{\partial^2\Gamma^\lambda_{\nu,\alpha}}{\partial^2 t}\right)V_\lambda = J_{\nu\alpha} \quad (17A)$$

When the currents are equal to zero we have that

$$\frac{\partial G_{\mu\nu,\alpha}}{\partial x_\mu} = \frac{\partial^2\Gamma^\lambda_{\nu,\alpha}}{\partial^2 x_\mu}V_\lambda = \left(\frac{\partial^2\Gamma^\lambda_{\nu,\alpha}}{\partial^2 x} - c^2\frac{\partial^2\Gamma^\lambda_{\nu,\alpha}}{\partial^2 t}\right)V_\lambda = 0 \quad (18A)$$

The variable $\Gamma^\lambda_{\nu,\alpha}$ has a wave-like behaviour.

Now we look at the currents

$$J_{\mu\nu,\alpha\beta} = G_{j\nu,\alpha}\Gamma^j_{\mu\beta} + G_{\mu j,\alpha}\Gamma^j_{\nu\beta} + G_{\mu\nu,j}\Gamma^j_{\alpha\beta} - J_{\mu\nu,\alpha\beta} - D_\beta\Omega_{\mu\nu,\alpha} + \left[D_\mu,D_\nu\right]D_\beta V_\alpha = R_{\mu\nu,\alpha\beta} - J_{\mu\nu,\alpha\beta} \quad (19A)$$

where *R* is a reaction of a virtual matter or medium (vacuum) and *J* is the ordinary currents for the ordinary matter represented by the energetic tensor. The non-linear reaction of the self-coherent system

produces a current that justifies the complexity of the gravitational field and non-linear properties of the gravitational waves.

$$\left(\frac{\partial^2 \Gamma^{\lambda}_{\nu,\alpha}}{\partial^2 x} - c^2 \frac{\partial^2 \Gamma^{\lambda}_{\nu,\alpha}}{\partial^2 t}\right) V_{\lambda} = R_{\nu\alpha} - J_{\nu\alpha} \tag{20A}$$

In conclusion, we show that the non-conservative gravitational field is similar to a wave for 64 variables $\Gamma^{j}_{\nu\beta}$ in a non-linear material where we have complex non-linear phenomena inside the virtual material that represents the vacuum. In the previous equations, in the free field of the medium the Proca terms $\Gamma_{\rho}\,\Gamma_{\lambda}$, the Chern-Simons terms $(\partial_{\nu}\,\Gamma_{\rho})\,\Gamma_{\lambda}$ and the Maxwell-like terms $(\partial_{\nu}\,\Gamma_{\rho})\,(\partial_{\mu}\,\Gamma_{\lambda})$ are present. So, we have the mass terms, the topologic terms and the electromagnetic-like field terms. We can model the gravitational wave with torsion as a particle in a non-linear medium which gives the mass of the particle, in a way that can be compared to usual SSB processes of the standard model, for an orientation in extensive literature [20–22].