systems

MDPI

Article

Operational Hedging Against Supply Disruptions: The Strategic Value of Partial Vertical Ownership

Baichuan Gong [1], Xiaobing Liu [2,*] and Yanlei Guo [3,*]

[1] School of Economics and Management, Beijing Jiaotong University, Beijing 100044, China; 20120588@bjtu.edu.cn
[2] School of Systems Science, Beijing Jiaotong University, Beijing 100044, China
[3] School of Emergency Management and Safety Engineering, North China University of Science and Technology, Tangshan 063210, China
* Correspondence: lxiaobing@bjtu.edu.cn (X.L.); guoyanleimax@163.com (Y.G.)

Highlights

Please indicate how your work links to systems science via your contributions to systems practice, theory, and or methodology.

- This study conceptualizes supply disruption mitigation as a governance–information–capacity system, linking partial vertical ownership, supplier-side risk information, and backup capacity reservation.
- The model provides a systems-based analytical framework for comparing equity-based information acquisition with contractual flexibility under supply disruption risk and information asymmetry.

What are the main findings and/or the implications of the main findings?

- Partial vertical ownership can generate an information dividend when disruption risk heterogeneity is high, enabling more adaptive backup capacity decisions than a non-ownership benchmark.
- Compared with option contracts, PVO is more attractive under high disruption risk dispersion and severe stockout consequences, whereas contractual hedging remains preferable when disruption risk and stockout exposure are relatively limited.

Abstract

In an era characterized by frequent global supply chain disruptions, manufacturers increasingly explore strategies to safeguard operational continuity, including equity participation in upstream suppliers. This paper investigates partial vertical ownership (PVO), defined as a manufacturer's partial equity stake in an upstream supplier that supports strategic cooperation and risk information access without full vertical integration. We model a single manufacturer, an unreliable primary supplier with private disruption risk information, and a reliable backup supplier. Relative to a non-ownership (NO) baseline, PVO is modeled as a benchmark governance mechanism that can improve the manufacturer's information set before backup capacity is reserved. The analytical results show that expected profit is convex in the unit stockout penalty under the specified convex capacity cost structure and deterministic demand assumptions. They also show that the information dividend from PVO is governed by the variance of the disruption probability, while its economic magnitude depends on the scale of stockout losses, backup procurement costs, and the equity return condition. By benchmarking PVO against a clarified option contract strategy, we identify parameter regions in which equity-based information access can outperform contractual hedging. The study reframes PVO as a conditional operational hedging instrument and clarifies the assumptions under which it can strengthen supply chain resilience.

Check for updates

Academic Editor: Jianhua Zhu

Received: 1 June 2026
Revised: 10 July 2026
Accepted: 17 July 2026
Published: 20 July 2026

Copyright: © 2026 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license.

Keywords: partial vertical ownership; option contracts; supply disruption risk; information asymmetry; operational hedging

1. Introduction

In recent years, frequent natural disasters and geopolitical conflicts, coupled with increasingly complex supply chain structures, have led to frequent supply chain disruptions, causing significant losses to enterprises [1]. For instance, on 11 March 2011, a magnitude 9.0 earthquake struck northeastern Japan, triggering a tsunami and the Fukushima nuclear disaster [2]. This catastrophe damaged the production facilities of numerous key automotive component suppliers, halting production lines and preventing on-time deliveries. Consequently, Toyota Motor Corporation was forced to suspend partial production lines, resulting in massive order delays, a damaged market reputation, and ultimately billions of dollars in lost revenue.

The factors causing supply disruptions are multifaceted, encompassing external triggers such as natural disasters and political conflicts, as well as internal operational failures such as equipment breakdowns and production accidents [1,2]. Because manufacturers and suppliers are often geographically and organizationally separated, the probability of a supply disruption generally remains the supplier's private information. For example, information about the 2000 fire at a Philips plant in the United States was not transmitted to Ericsson in time, and Ericsson suffered severe losses after failing to take appropriate countermeasures [1]. To acquire such risk information and reduce suboptimal decisions caused by information asymmetry, some manufacturers purchase equity in upstream suppliers, and industry evidence also suggests that equity investment and vertical integration can improve supply chain transparency [3,4]. We define this disruption mitigation strategy as partial vertical ownership (PVO), whereby a manufacturer holds a partial equity stake in a supplier without fully integrating or controlling the supplier. This is an operational definition developed for the present study on the basis of the broader literature on partial ownership and vertical governance [5–9]. In this study, PVO is interpreted as an equity-based governance arrangement that can be accompanied by board-level communication, audit access, data-sharing agreements, or long-term strategic partnership clauses. These governance channels can improve supply chain transparency and coordination while preserving the supplier as an independent firm. This definition distinguishes PVO from full vertical integration and from arm's length business cooperation.

On the other hand, investing in a supplier requires substantial capital. Instead of purchasing supplier equity, some enterprises allocate these funds to buy options for the corresponding products in the options market to mitigate the impact of potential disruptions. Consequently, the manufacturer faces a critical decision: should this capital be invested in supplier equity or used to purchase options? In other words, compared to strategies like option contracts, under what conditions should a manufacturer adopt PVO to mitigate supply disruption risks?

Despite extensive research on dual sourcing, emergency procurement, and option contracts, less is known about whether partial equity participation can operate as an information-acquisition mechanism that improves disruption risk decisions before disruption occurs. This gap motivates our comparison of PVO with non-ownership and option contract benchmarks.

Building on this motivation, we develop a governance-based information-acquisition framework for disruption risk management. The framework links three mechanisms. First, partial equity ownership can create governance channels through which the manufacturer

may obtain more accurate information about the supplier's disruption risk, especially when the ownership relationship is supported by board-level communication, audit access, data-sharing agreements, formal reporting rights, or long-term strategic partnership clauses. Second, improved information changes the manufacturer's backup capacity decision before uncertainty is resolved. Third, better capacity calibration reduces both stockout losses in high-risk states and excessive reservation costs in low-risk states. This framework clarifies why PVO may affect disruption risk not only through financial returns, but also through changes in the manufacturer's information set and operational response. Other mechanisms, such as information-sharing clauses, audits, screening contracts, and performance-based incentives, can also reduce information asymmetry. The present study focuses on PVO because it represents an equity-based governance channel whose information effect can be compared with contractual hedging.

This paper investigates the strategic use of PVO by manufacturers to mitigate upstream supply disruption risks under information asymmetry. We consider a supply chain comprising one manufacturer, a low-cost but unreliable primary supplier, and a reliable but high-cost backup supplier. The disruption risk of the primary supplier is private information. If the manufacturer does not invest in the primary supplier, its knowledge is limited to a public prior belief. If the manufacturer adopts PVO, we model the governance channel as revealing the supplier's disruption risk type to the manufacturer. The game unfolds as follows. First, the manufacturer decides whether to acquire partial ownership of the primary supplier. Second, based on its information state, the manufacturer determines the reserved capacity at the backup supplier. Third, the disruption state of the primary supplier is realized. If no disruption occurs, the manufacturer fulfills market demand through the primary supplier. If a disruption occurs, the manufacturer triggers emergency procurement from the backup supplier up to the reserved capacity. When PVO is adopted, the manufacturer also receives a financial return or loss proportional to its equity stake.

This study makes three contributions. First, it contributes to the supply disruption literature by introducing PVO as a governance-based information-acquisition mechanism rather than treating equity ownership only as a financial arrangement. Second, it develops an analytical model that links information access, backup capacity reservation, and stockout penalties under asymmetric disruption risk information. The model shows that the value of PVO depends on the variance of the disruption probability, the cost of supply interruption, and the financial-return condition. Third, it compares PVO with an option contract benchmark and identifies parameter regions in which equity-based information access can be more valuable than contractual hedging. These results provide theoretical guidance on when manufacturers should consider PVO as a conditional operational hedging instrument for supply chain resilience.

2. Literature Review

This section builds the theoretical foundation for analyzing PVO as an operational hedging mechanism. The related literature can be organized into three connected streams: supply disruption risk management, option contract hedging, and partial ownership among firms. Reviewing these streams together allows us to identify how existing studies manage disruption risk, how they address information asymmetry, and why partial equity ownership has not been fully examined as a disruption risk information mechanism.

The supply disruption literature has established several important mechanisms for coping with unreliable supply, including inventory, emergency sourcing, dual sourcing, supplier reliability improvement, incentive contracts, and flexible procurement arrangements [10–31]. These studies show that firms can reduce disruption losses by diversifying supply sources, improving supplier reliability, or designing contracts that allocate disrup-

tion risks more efficiently. However, most of this literature treats the manufacturer and supplier as arm's length parties and focuses on how the manufacturer reacts to uncertain reliability after the information barrier remains in place. Even studies that explicitly consider asymmetric supplier reliability information mainly examine screening contracts, backup production options, or sourcing choices [22–24,31]. They provide limited insight into whether an ownership-based governance arrangement can change the manufacturer's information set before backup capacity decisions are made. This limitation motivates our focus on PVO as a mechanism that links supplier governance, disruption risk information, and operational hedging.

Within this stream, dual-sourcing and multi-sourcing studies examine order allocation between reliable and unreliable suppliers, random capacity risk, supplier competition, process improvement, and incentives for capacity restoration [17,19,25–31]. These studies clarify how operational flexibility can reduce exposure to disruption. However, their main focus remains on procurement allocation or supplier improvement under a given relationship structure. They do not examine whether a manufacturer's equity stake in an unreliable supplier can create an information channel that changes the backup capacity decision itself. Our model complements this stream by studying how ownership-based information access interacts with operational hedging.

Option contracts represent another important stream of supply chain flexibility and disruption risk hedging research [32–36]. Existing studies show that option contracts can help firms reserve access to backup supply, coordinate procurement decisions, and mitigate the negative consequences of demand uncertainty, supplier disruption, or spot-market price volatility. For example, recent research on multi-channel procurement considers a primary supplier, a backup supplier with an option contract, and a spot market under disruption risk, uncertain demand, and price uncertainty. Other studies examine whether option contracts can improve profitability, product availability, and service performance when supply-side disruptions occur. These studies demonstrate that option contracts are useful contractual flexibility tools for managing disruption consequences. However, this literature generally treats option contracts as mechanisms that provide procurement flexibility under a given information structure. It pays less attention to whether alternative governance mechanisms can change the manufacturer's disruption risk information before capacity or procurement decisions are made. Therefore, option contracts provide a natural benchmark for evaluating PVO. The key distinction is that option contracts hedge disruption consequences through contractual flexibility, whereas PVO may create value by improving disruption risk information before the backup capacity decision is made.

The partial ownership and vertical governance literature provides the governance foundation for this study. Recent studies have examined how partial vertical ownership or partial vertical integration affects capacity investment, information exchange, coordination, downstream competition, and strategic interaction in supply chains [5–9,37–41]. In particular, Fang et al. [37] examine PVO in the presence of downstream competition, further showing that partial ownership can reshape strategic incentives in vertically related markets. Pishchulov et al. [40] model partial vertical integration under asymmetric information by allowing the buyer to hold an equity stake in the supplier while Avinadav and Shamir [41] show that partial vertical ownership can affect both capacity investment and information exchange. These studies demonstrate that partial ownership is not merely a financial investment; it can also reshape interfirm governance and operational decisions. Nevertheless, the existing literature has paid limited attention to the role of partial ownership in supply disruption risk management. In particular, it has not fully examined how a manufacturer's partial equity stake in an unreliable supplier may create an information channel for disruption risk assessment before backup capacity decisions are made.

We address this gap by modeling PVO as a governance-based mechanism that enables the manufacturer to obtain supplier-side disruption risk information and adjust backup capacity accordingly.

Taken together, the literature suggests that disruption risk management requires both operational flexibility and credible information. Prior studies show how firms hedge supply risk through inventory, sourcing flexibility, incentives, and contractual backup mechanisms. However, these approaches generally preserve the information barrier between manufacturer and supplier. Existing contractual approaches improve flexibility, while partial ownership studies explain how equity links can reshape interfirm governance. Our study connects these two insights by analyzing when an equity-based information channel can improve disruption risk decisions relative to non-ownership and option contract benchmarks.

3. Model

Before formalizing the specific mathematical models, we first outline the sequence and the underlying information topology of our study, as illustrated in Figure 1. This sequence maps out the strategic divergence between the PVO strategy and the non-ownership (NO) strategy.

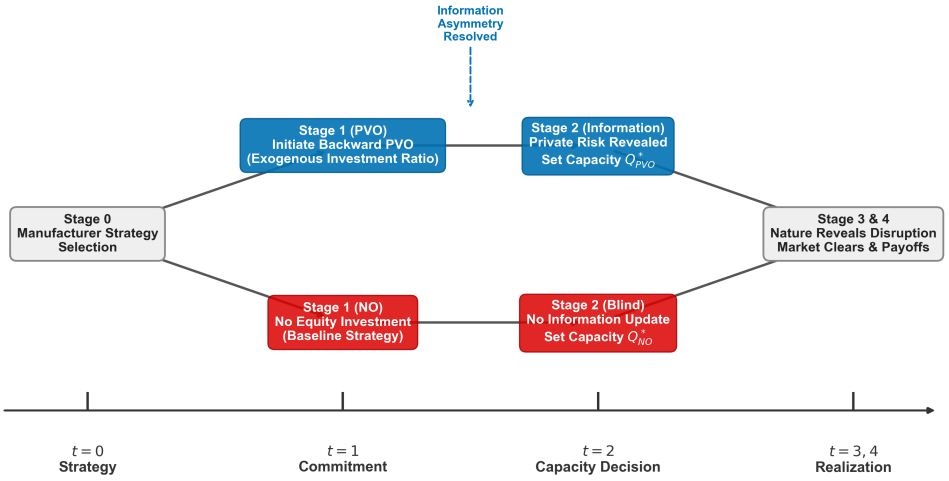


Figure 1. The sequence of events: A comparison between the PVO and the non-ownership (NO).

The sequence of events proceeds in four main stages. At the initial decision stage, the manufacturer determines the overarching hedging strategy against potential supply disruptions. If the manufacturer adopts the PVO strategy, it commits to an exogenous investment ratio that can provide access to the primary supplier's private disruption risk type through the governance channels discussed above. Alternatively, the manufacturer may proceed with the NO strategy, which serves as our baseline. The NO strategy requires no initial equity investment but leaves the information asymmetry unresolved.

The structural disparity in information directly impacts the backup capacity decisions at $t = 2$. Under the PVO strategy, the manufacturer leverages the revealed private risk to set an informed backup capacity Q^*_{PVO}. Conversely, under the NO strategy, the manufacturer commits to a backup capacity Q^*_{NO} relying merely on prior beliefs. Finally, at $t = 3$ and $t = 4$, the actual disruption state is realized by nature, the market clears, and the corresponding payoffs are realized.

The information structure should be interpreted as a benchmark rather than as an automatic consequence of equity ownership. In practice, partial equity may improve information access only when it is accompanied by governance, monitoring, or contractual channels that make supplier-side risk information observable to the manufacturer.

This paper investigates a supply chain consisting of a single manufacturer, an unreliable primary supplier (Supplier 1), and a reliable backup supplier (Supplier 2). We assume that market demand is known and deterministic. This assumption is used to isolate the informational value of PVO from demand-side uncertainty and is also consistent with settings in which demand is precommitted before the selling season.

To minimize procurement costs and mitigate supply disruption risk, the manufacturer must make a sequence of decisions prior to the selling season. First, the manufacturer decides whether to acquire PVO by investing in the primary supplier. At the time of this decision, the manufacturer does not know the primary supplier's exact disruption probability. Instead, the manufacturer holds a prior belief that the disruption probability, θ, is high ($\theta = \theta_h$) with probability β, and low ($\theta = \theta_l$) with probability $1 - \beta$. If the manufacturer acquires partial equity, the benchmark model assumes that governance channels reveal the supplier's disruption risk type to the manufacturer. Conversely, if the manufacturer decides against this investment, its information regarding the disruption risk remains unchanged. Subsequently, the manufacturer places an order with the primary supplier at a unit wholesale price of r_1.

This benchmark abstracts from partial information revelation and supplier strategic misreporting. It is most appropriate for settings in which the equity relationship provides credible access to risk-relevant information through formal reporting, audit rights, or strategic governance arrangements. To guard against potential stockouts resulting from a primary supplier disruption, the manufacturer reserves production capacity with the backup supplier before the selling season. The unit wholesale price for sourcing from the backup supplier r_2 is higher than that of the primary supplier, where $r_2 > r_1$. We assume that the capital required to reserve capacity Q_i follows the convex cost function $K(Q_i) = aQ_i^2 + bQ_i$. Following the common practice in the operations and supply chain management literature, we model the backup capacity investment cost as a convex cost function, $K(Q_i) = bQ_i + aQ_i^2$, where bQ_i captures the linear per-unit capacity reservation cost and aQ_i^2 captures the increasing marginal cost of reserving additional backup capacity. This quadratic specification reflects capacity-related diseconomies of scale: as the reserved backup capacity increases, additional capacity becomes increasingly costly due to limited production resources, tighter scheduling constraints, and higher coordination requirements. Similar linear-quadratic cost structures have been widely used to model production diseconomies and capacity-related costs in supply chain and operations management models [42,43].

The quadratic capacity-reservation cost is used to capture increasing marginal preparation costs at the backup supplier. The linear term represents the per-unit reservation cost, whereas the quadratic term captures the fact that reserving additional capacity becomes progressively more expensive when backup capacity is limited or requires advance resource preparation. The convexity results below should therefore be interpreted as implications of this specified cost structure rather than as claims for all possible capacity cost functions.

If no disruption occurs at the primary supplier, the manufacturer receives the ordered quantity D, and sells these products to consumers at the retail price P. However, if a supply disruption does occur, the manufacturer places an emergency order with the backup supplier, up to the maximum reserved capacity Q_i. Any unmet market demand $D - Q_i$, incurs a unit stockout penalty of l.

Furthermore, if the manufacturer acquires partial equity in the primary supplier, we assume the equity investment amount is C and the corresponding rate of return is $\lambda (\lambda \in \mathbb{R})$. The manufacturer's total profit comprises sales revenue, the capacity investment cost for the backup supplier, and the financial return on the equity investment. The manufacturer's

ultimate objective is to maximize its total expected profit. All the notations are summarized on Table 1.

Table 1. Notations and definitions.

Notations	Definition
i	Subscript, index of scenarios, $i \in \{OH, OL, NO, Option\}$
D	The market demand
P	The unit retail price of the product
r_1	The unit wholesale price for sourcing from the primary supplier
r_2	The unit wholesale price for sourcing from the backup supplier
r_3	The pre-agreed strike price to procure components under the option contract
F	The upfront premium cost for the option contract
a	Parameter governing the increasing marginal cost of reserving backup capacity
b	Linear per-unit cost of reserving backup capacity
l	The unit stockout penalty for unmet market demand
θ_h	The high disruption probability of the primary supplier
θ_l	The low disruption probability of the primary supplier
β	The prior belief probability that the disruption probability is high
C	The equity investment amount under the PVO strategy
λ	The rate of return on the equity investment
Q_i	The reserved capacity at the backup supplier or option quantity in scenario i
Q	The regular order quantity under the option contract strategy
π_i	The manufacturer's expected profit in scenario i

The return rate on the equity investment is treated as an exogenous net financial-return parameter. This assumption separates the operational information value of PVO from a full financial valuation of supplier equity. If the return rate were endogenously linked to the supplier's disruption probability, the profitability threshold for PVO would change. We therefore interpret the return-rate condition as a benchmark result.

3.1. Manufacturer's Decisions and Profit with PVO

In this section, we analyze the optimal decisions and equilibrium profits when the manufacturer chooses to acquire PVO of the primary supplier. Under this scenario, the manufacturer is fully informed of the primary supplier's specific disruption probability—whether high or low—prior to determining the order quantity and the backup supplier's reserved capacity. Specifically, when the primary supplier's disruption probability is high (θ_h), for a given reserved capacity Q_{OH}, the manufacturer's total profit is expressed as

$$\Pi_O^H(Q_{OH}) = (1-\theta_h)[(P-r_1)D - K(Q_{OH})] + \theta_h[(P-r_2)Q_{OH} - K(Q_{OH}) - l(D-Q_{OH})] + \lambda C \quad (1)$$

The first and second terms represent the manufacturer's operational profits under the scenarios of supply disruption and normal supply, respectively, while the third term denotes the financial return obtained from holding equity in the supplier. To maximize the manufacturer's total profit, the optimal reserved capacity at the backup supplier, Q_{OH}^*, is the solution to the following optimization problem:

$$max_{Q_{OH} \geq 0} \Pi_O^H(Q_{OH}) \quad (2)$$

When Supplier 1's disruption probability is low ($\theta = \theta_l$), given the reserved capacity Q_{OL}, the manufacturer's total profit is given by

$$\Pi_O^L(Q_{OL}) = (1-\theta_l)[(P-r_1)D - K(Q_{OL})] + \theta_l[(P-r_2)Q_{OL} - K(Q_{OL}) - l(D-Q_{OL})] + \lambda C \quad (3)$$

The manufacturer's optimal reserve capacity Q_{OL}^* for Supplier 2 is the solution to the following optimization problem:

$$max_{Q_{OL}\geq 0}\Pi_O^L(Q_{OL}) \quad (4)$$

The manufacturer's optimal profit under PVO scenario is

$$\Pi_O^* = \beta\Pi_O^H(Q_{OH}^*) + (1-\beta)\Pi_O^L(Q_{OL}^*) \quad (5)$$

To simplify the exposition and highlight the underlying economic intuition of the analytical results, we first define the following notation:

$\Pi_0 = (P-r_1)D + \lambda C$: The baseline expected profit when no disruption occurs, which consists of the normal operating profit and the financial return from the equity.

$\Delta_1 = l + P - r_2$: The marginal benefit of utilizing the backup supplier (i.e., the avoided stockout penalty plus the retail price, minus the higher backup procurement cost).

$\Delta_2 = l + P - r_1$: The marginal loss incurred from unmet demand when the primary supplier experiences a disruption.

$\overline{\theta} = \beta\theta_h + (1-\beta)\theta_l$: The expected disruption probability of the primary supplier based on the manufacturer's prior belief.

$\overline{\theta}^2 = \beta\theta_h^2 + (1-\beta)\theta_l^2$: The expected squared disruption probability.

All proofs are presented in Appendix A.

Proposition 1. *Under the PVO strategy, the optimal decisions and expected profits are as follows: (1) When the primary supplier's disruption probability is high (θ_h), the optimal reserved capacity at the backup supplier is $Q_{OH}^* = \frac{\theta_h\Delta_1 - b}{2a}$. The manufacturer's corresponding optimal profit is given by*

$$\Pi_O^H(Q_{OH}^*) = \Pi_0 - \theta_h\Delta_2 D + \frac{(\theta_h\Delta_1 - b)^2}{4a} \quad (6)$$

(2) When the primary supplier's disruption probability is low (θ_l), the optimal reserved capacity at the backup supplier is $Q_{OL}^* = \frac{\theta_l\Delta_1 - b}{2a}$. The manufacturer's corresponding optimal profit is given by

$$\Pi_O^L(Q_{OL}^*) = \Pi_0 - \theta_l\Delta_2 D + \frac{(\theta_l\Delta_1 - b)^2}{4a} \quad (7)$$

(3) Overall, the manufacturer's optimal expected profit under PVO is

$$\Pi_O^* = \Pi_0 - \overline{\theta}\Delta_2 D + \frac{\Delta_1^2\overline{\theta}^2 - 2b\Delta_1\overline{\theta} + b^2}{4a} \quad (8)$$

The closed-form capacities in Proposition 1 are interior solutions. They apply when $0 < \theta_i\Delta_1 - b < 2aD$. Outside this range, the economically feasible reserved capacity is boundary-adjusted as $Q_i^* = min\left\{D, max\left\{0, \frac{\theta_i\Delta_1 - b}{2a}\right\}\right\}$. Therefore, if $\theta_i\Delta_1 \leq b$, the manufacturer reserves no backup capacity; if $\theta_i\Delta_1 - b \geq 2aD$, the reserved capacity is capped at market demand D.

Proposition 2 characterizes the impact of the unit stockout penalty on the manufacturer's expected profit.

Proposition 2. *(1) When the supply disruption risk is higher, the optimal reserved capacity at the backup supplier increases more rapidly with respect to the unit stockout penalty.*

(2) The manufacturer's expected profit Π_O^* is a convex function of the stockout penalty l. To ensure a non-trivial interior solution, we assume the baseline parameters satisfy $\frac{\overline{\theta}(2aD+b)}{\widetilde{\theta}^2} > P + r_2$. Under this condition, there exists a strictly positive critical threshold $\widetilde{l} = \frac{\overline{\theta}(2aD+b)}{\widetilde{\theta}^2} - P + r_2$, such that the expected profit first decreases and then increases as the stockout penalty l increases.

Figure 2 illustrates this convexity result is conditional on the specified convex capacity-reservation cost and deterministic demand assumptions. It should be interpreted as a model-specific implication of the analytical setting rather than as a general property of all disruption risk models. Proposition 2(1) indicates that as the unit stockout penalty increases, the manufacturer increases the reserved capacity at the backup supplier, and the increase is faster when the primary supplier's disruption probability is higher. Proposition 2(2) should not be interpreted as implying that a higher stockout penalty is beneficial. Rather, the stockout penalty represents the economic severity of unmet demand, such as lost sales, contractual penalties, service failures, or reputational losses. When this penalty becomes sufficiently high, the manufacturer responds by reserving more backup capacity. The upward part of the convex profit curve therefore reflects an endogenous operational adjustment that reduces expected disruption losses under the specified cost and demand assumptions. From a managerial perspective, the critical threshold should be interpreted as a warning point at which stockout exposure becomes severe enough to justify more aggressive backup capacity reservation, not as evidence that firms benefit from higher stockout penalties.

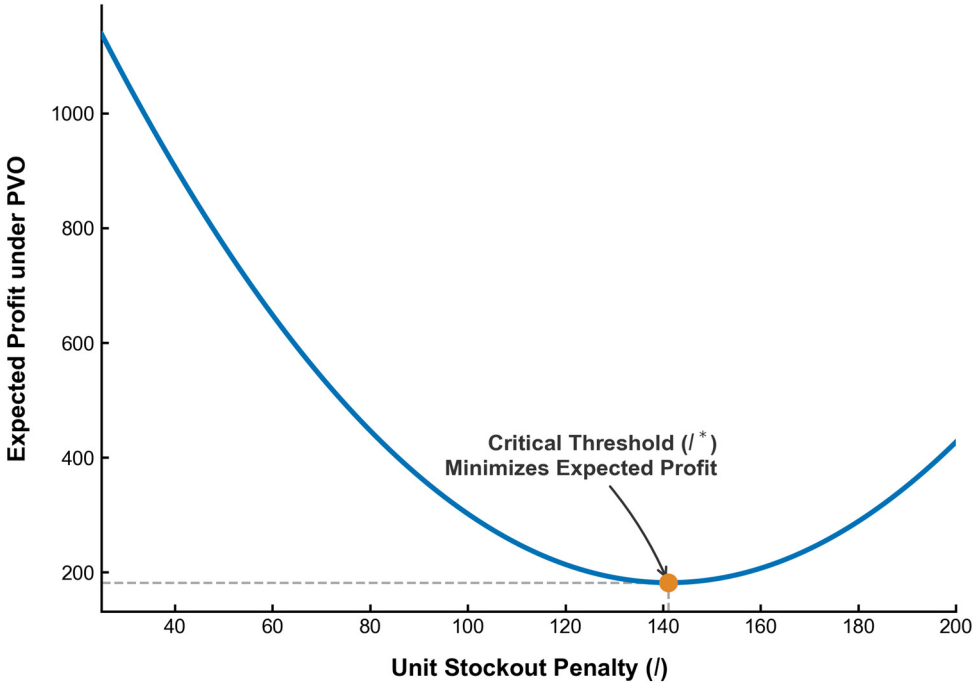


Figure 2. Convexity of the expected profit under the PVO strategy with respect to the unit stockout penalty.

3.2. Manufacturer's Decisions and Profit with NO

As a benchmark, this section discusses the manufacturer's decisions and expected profit under the scenario where no equity investment is made in the primary supplier. Because the manufacturer does not hold PVO, it remains uninformed about whether the primary supplier's actual disruption probability is θ_h or θ_l at the time of decision-making. Instead, the manufacturer must rely on prior knowledge derived from publicly available

market information regarding supply disruptions. Specifically, it holds a prior belief that the primary supplier's disruption probability is θ_h with probability β, and θ_l with probability $1-\beta$. Under these conditions of information asymmetry, the manufacturer determines the reserved capacity at the backup supplier, which we denote as Q_{NO}. Therefore, given the reserved capacity Q_{NO}, the manufacturer's total expected profit is expressed as

$$\begin{aligned}\Pi_{NO}(Q_{NO}) = &[\beta(1-\theta_h)+(1-\beta)(1-\theta_l)][(P-r_1)D-K(Q_{NO})] \\ &+[\beta\theta_h+(1-\beta)\theta_l][(P-r_2)Q_{NO}-K(Q_{NO})-l(D-Q_{NO})]\end{aligned} \tag{9}$$

The first term represents the manufacturer's expected profit when no supply disruption occurs at the primary supplier, while the second term denotes the operational profit when a disruption does occur. Because the manufacturer does not hold PVO as the primary supplier, there are no equity investment costs or corresponding financial returns from holding shares.

The optimal reserved capacity at the backup supplier, Q_{NO}^*, is the solution to the following optimization problem:

$$max_{Q_{NO}\geq 0}\Pi_{NO}(Q_{NO}) \tag{10}$$

Consequently, the manufacturer's optimal expected profit without PVO is given by

$$\begin{aligned}\Pi_{NO}(Q_{NO}^*) = &[\beta(1-\theta_h)+(1-\beta)(1-\theta_l)]\left[(P-r_1)D-K(Q_{NO}^*)\right] \\ &+[\beta\theta_h+(1-\beta)\theta_l]\left[(P-r_2)Q_{NO}^*-K(Q_{NO}^*)-l(D-Q_{NO}^*)\right]\end{aligned} \tag{11}$$

Proposition 3 presents the optimal strategy and the corresponding expected profit for the manufacturer when it does not hold PVO of the primary supplier.

Proposition 3. *Under the scenario with NO, (1) The optimal reserved capacity at the backup supplier is given by*

$$Q_{NO}^* = \frac{\overline{\theta}(l+P-r_2)-b}{2a} = \frac{\overline{\theta}\Delta_1-b}{2a} \tag{12}$$

(2) The manufacturer's optimal expected profit is given by

$$\Pi_{NO}^*(Q_{NO}^*) = (P-r_1)D-\overline{\theta}\Delta_2 D+\frac{\left(\overline{\theta}\Delta_1-b\right)^2}{4a} \tag{13}$$

The optimal reserved capacity under NO is subject to the same feasibility logic. The interior expression applies only when $0<\overline{\theta}\Delta_1-b<2aD$. Otherwise, the feasible solution is $Q_N^*=min\left\{D,max\left\{0,\frac{\overline{\theta}\Delta_1-b}{2a}\right\}\right\}$.

From Propositions 1, 2 and 3, the following corollary is derived.

Corollary 1. *The optimal reserved capacities at the backup supplier under the different information scenarios satisfy the following relationship:*

$$Q_{OH}^* > Q_{NO}^* > Q_{OL}^* \tag{14}$$

Based on Corollary 1, we observe that once the manufacturer acquires partial equity in the primary supplier and discovers that the actual disruption probability is high, it will reserve a larger capacity at the backup supplier ($Q_{OH}^* > Q_{NO}^*$). Conversely, if it discovers that the disruption probability is low, it will reserve a smaller capacity ($Q_{OL}^* < Q_{NO}^*$). This conclusion demonstrates that holding partial equity allows the manufacturer to obtain precise disruption probability information and subsequently adjust its capacity reservation

at the backup supplier accordingly. When the primary supplier's disruption probability is high, the manufacturer increases the reserved capacity to hedge against potential supply disruption risks. Conversely, when the disruption probability is low, the manufacturer decreases the reserved capacity to reduce upfront investment costs. In contrast, a manufacturer without partial ownership lacks specific disruption information and can only adopt a more conservative procurement strategy, resulting in an intermediate reserved capacity.

From Propositions 2 and 3, we can derive the following proposition:

Proposition 4. *(1) The marginal sensitivity of the optimal reserved capacity to the stockout penalty under the different scenarios satisfies the following relationship:*

$$\frac{\partial Q_{OL}}{\partial l} < \frac{\partial Q_{NO}}{\partial l} < \frac{\partial Q_{OH}}{\partial l} \tag{15}$$

(2) The manufacturer's optimal expected profit with NO, Π^*_{NO}, is a strictly convex function of the stockout penalty l. Similar to the PVO scenario, under the regular interior solution condition, there exists a strictly positive critical threshold $\widetilde{l}_{NO} = \frac{2aD+b}{\bar{\theta}} - P + r_2$, such that the expected profit first decreases and then increases with the stockout penalty.

Proposition 4(1) indicates that the manufacturer's backup capacity response to the stockout penalty depends on its information state. When PVO reveals a high disruption risk type, the manufacturer reacts most strongly to an increase in the stockout penalty by reserving more backup capacity. When PVO reveals a low disruption risk type, the response is weaker because excessive reservation would mainly increase preparation costs. Under NO, the manufacturer relies on the prior expected disruption probability and therefore adopts an intermediate response. Proposition 4(2), like Proposition 2(2), as shown in Figure 3, does not suggest that higher stockout penalties are desirable. It shows that, within the model's feasible domain, adaptive capacity reservation can partially offset expected disruption losses when the stockout penalty becomes sufficiently high. The managerial implication is that information access changes not only the level of backup capacity, but also the responsiveness of capacity planning to the economic severity of stockouts. Without supplier-specific risk information, the manufacturer may respond too moderately to high-risk states and too aggressively to low-risk states. While this strategy inflates investment costs, the expanded reserved capacity substantially minimizes stockout losses, ultimately driving an increase in expected profit. Combining Propositions 2 and 4, we obtain Proposition 5.

Proposition 5. *The critical stockout penalty threshold that minimizes the expected profit under PVO ($\widetilde{l}$) is strictly lower than the threshold under the no-ownership scenario ($\widetilde{l}_{NO}$). That is, $\widetilde{l} < \widetilde{l}_{NO}$.*

As visually illustrated in Figure 4, comparing the profit trajectories of the two models across varying stockout penalties reveals important differences in their operational dynamics. Consistent with Proposition 5, the critical threshold for the NO model is located to the right of that for the PVO model under the maintained parameter values. The PVO curve can exceed the NO curve when the information value and equity return condition are sufficiently strong relative to the capacity-reservation and investment costs. This dominance is therefore conditional on the equity return, the equity investment cost, and the parameter region, rather than being a universal property.

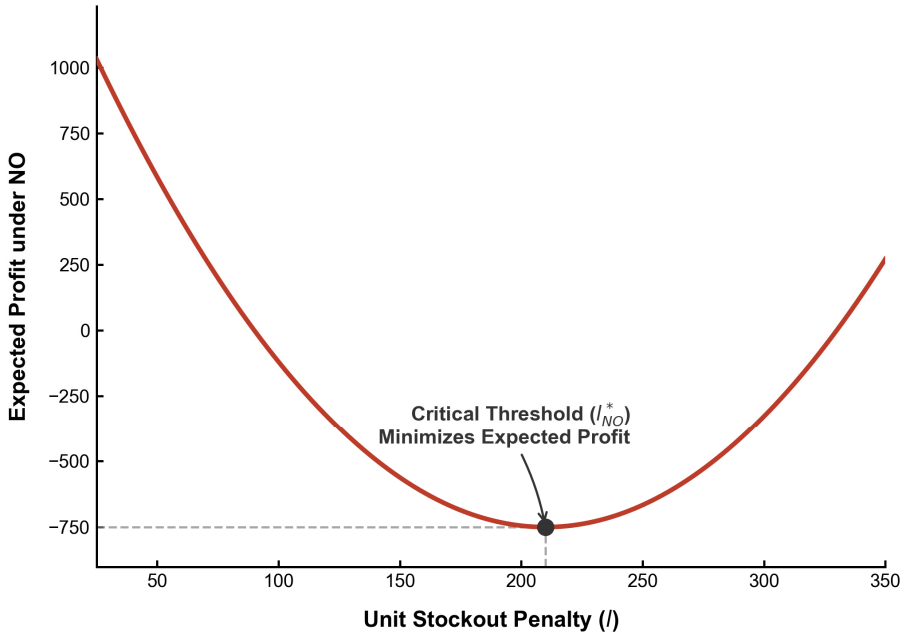


Figure 3. Convexity of the expected profit under the NO strategy with respect to the unit stockout penalty.

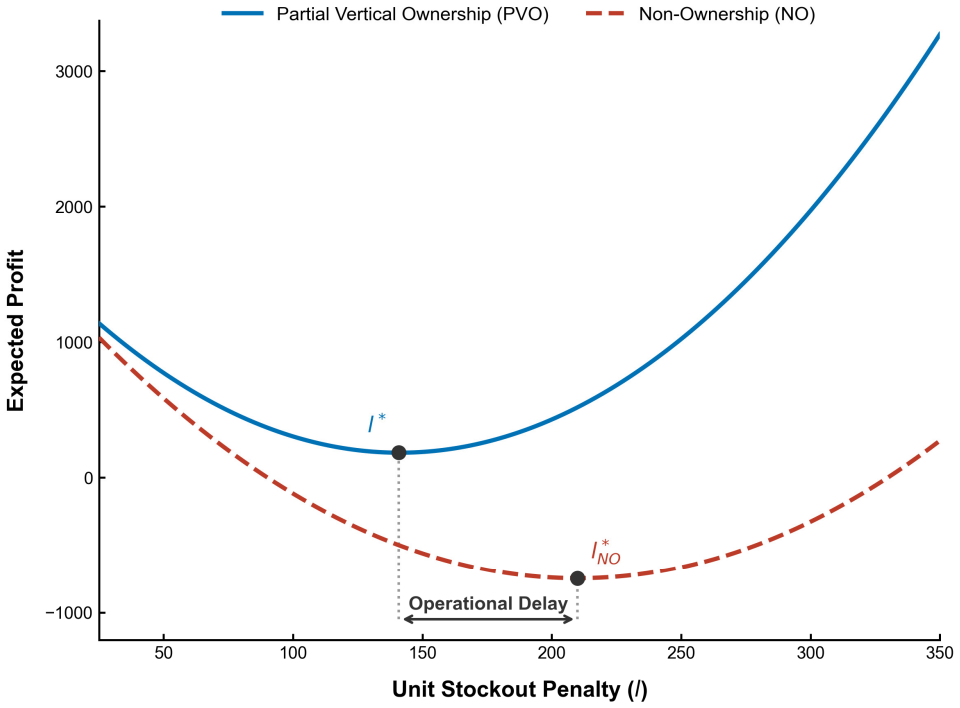


Figure 4. Comparison of expected profit convexity between the PVO and NO strategies.

3.3. The Value of PVO

The previous two sections analyzed the manufacturer's optimal decisions and corresponding expected profits under both the PVO and no-ownership scenarios. By comparing the profits between these two scenarios, we can derive the manufacturer's optimal equity investment strategy, as formalized in Proposition 6.

Proposition 6. *(1) As the difference between the high disruption probability and the low disruption probability ($\theta_h - \theta_l$) increases, the additional profit gained by the manufacturer from acquiring PVO in the primary supplier increases.*

(2) When the manufacturer's prior belief regarding the primary supplier's disruption probability is uncertain ($\beta = 0.5$), the additional profit gained from acquiring partial ownership is maximized.
(3) There exists a critical threshold for the equity investment return rate, denoted as $\widetilde{\lambda}$. The manufacturer should invest in the primary supplier if and only if $\lambda > \widetilde{\lambda}$.

Proposition 6(1) indicates that when the difference between the high and low supply disruption probabilities is substantial, acquiring PVO can generate a larger benefit for the manufacturer. The mechanism is informational: if the actual disruption risk is high but the manufacturer remains uninformed, it may reserve insufficient backup capacity; if the actual disruption risk is low, an uninformed manufacturer may reserve excessive capacity. Proposition 6(2) shows that the informational benefit is largest when the prior belief creates the highest ambiguity about the supplier's disruption type. Proposition 6(3) further gives the equity return threshold under which the information benefit can offset the investment cost. These results should be interpreted as theoretical implications of the benchmark model rather than as empirical validation of PVO in all settings. The variance term identifies the source of the information dividend, but the economic magnitude of the effect also depends on the scale of avoided stockout losses and backup procurement costs. Consequently, Proposition 6 explains the structure and direction of the benefit, whereas numerical or empirical calibration is needed to evaluate its magnitude in a specific industry setting. An industry example can help illustrate this governance logic, although it should not be interpreted as empirical validation of the model. In 2022, Knorr-Bremse acquired a 13% stake in Shanghai Bobang Automotive Technology, a Chinese e-motor specialist, and became its second-largest shareholder [4]. This investment was reported as a way to strengthen the partnership, improve responsiveness in the Chinese market, and support access to critical e-motor capabilities. In the context of our model, such a case illustrates why a manufacturer may value partial equity participation not only for financial returns, but also for closer supplier relationships and improved visibility into supply-side conditions. The example is therefore used only to motivate the practical relevance of PVO as a governance-based information channel, while the analytical propositions remain theoretical results derived from the benchmark model.

4. PVO and Option Contract

Both option contracts and the PVO strategy require ex ante commitments to mitigate supply disruption risk, but they operate through different mechanisms. PVO is modeled as an equity-based governance arrangement that can improve risk information access before backup capacity is reserved. An option contract is modeled as a contractual flexibility mechanism in which the manufacturer pays an upfront premium to reserve access to backup supply and may exercise the option after the disruption state is realized. This section compares these two mechanisms to identify their respective conditions of use.

Building upon the baseline NO model, this section formally introduces the option contract strategy. As illustrated in the sequence of events in Figure 5, the option contract provides an alternative pathway for the manufacturer to mitigate supply disruption risks without seeking an information advantage.

Unlike the PVO strategy, which is modeled as a benchmark governance mechanism for improving risk information access, the option contract operates within the manufacturer's prior information state. The manufacturer signs the option contract by paying an upfront premium to reserve option capacity before uncertainty is realized. This financial commitment relies on the public prior belief rather than the supplier's private risk type. If disruption occurs, the manufacturer can use the pre-reserved option capacity at the pre-agreed strike price according to the fulfillment structure specified in the benchmark model.

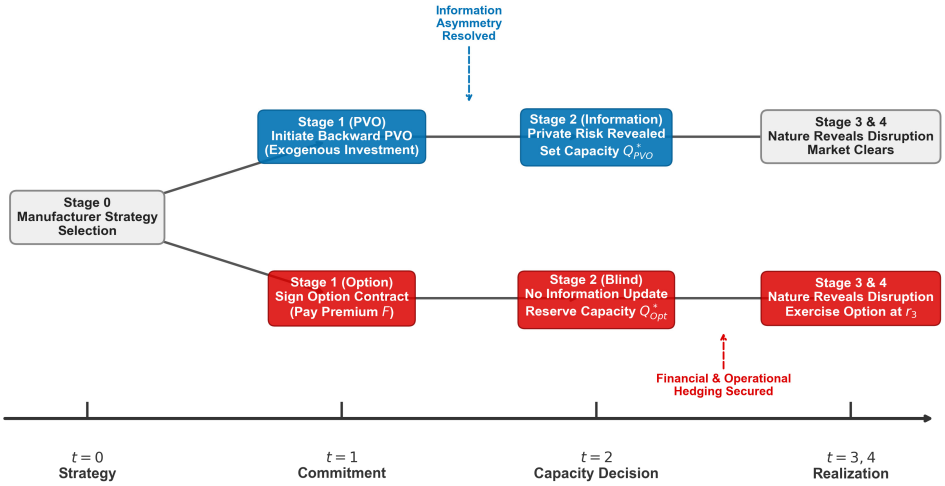


Figure 5. The sequence of events: A comparison between the PVO and the option contract.

In this comparison, PVO functions as an information hedge under the benchmark governance assumption, whereas the option contract functions as a contractual flexibility hedge. The following formulation is therefore intended as a tractable benchmark comparison between information acquisition and contractual flexibility, not as a complete model of all real-world option contract designs.

We first investigate the optimal decisions and expected profit under the option strategy. Note that under this strategy, the manufacturer cannot access the primary supplier's private disruption probability information; hence, its information state is identical to the no-ownership scenario. The manufacturer's decision-making process consists of two stages: first, it pays a deterministic option premium, FQ_{Option}, to lock in the backup supplier's capacity; subsequently, it decides whether to exercise the option based on the actual occurrence of a supply disruption. When the primary supplier operates normally, the manufacturer prioritizes using the regular order quantity, Q, to satisfy market demand, and any excess demand is fulfilled by exercising the option, Q_{Option}. Conversely, if a supply disruption occurs, the demand is entirely satisfied by exercising the option, and any unmet demand incurs a stockout penalty, $l\left(D - Q_{Option}\right)$. Throughout this process, the manufacturer must comprehensively balance the option premium F, the exercise price r_3, and the stockout cost l to determine the optimal option quantity Q_{Option} and the optimal regular order quantity Q.

The option quantity in our benchmark represents the option capacity reserved before the selling season. For tractability, we do not introduce a separate continuous exercise-quantity decision after the disruption state is realized. Instead, the realized use of the option follows the state-contingent fulfillment structure specified in the profit function. Therefore, the option contract strategy should be interpreted as a simplified contractual-flexibility benchmark rather than a complete model of all possible option-exercise arrangements.

Given the option quantity Q_{Option} and the regular order quantity Q, utilizing our previously defined expected disruption probability $\overline{\theta} = \beta\theta_h + (1-\beta)\theta_l$, the manufacturer's expected profit can be expressed as

$$\begin{aligned} &\Pi_{option}\left(Q, Q_{Option}\right) \\ &= -FQ_{Option}\left(1-\overline{\theta}\right)\left[(P-r_1)Q + (P-r_3)(D-Q)\right] + \overline{\theta}\left[(P-r_3)Q_{Option} - l\left(D - Q_{Option}\right)\right] \end{aligned} \tag{16}$$

The option premium is incurred ex ante and is independent of the realized disruption state. The expected profit therefore separates the premium term from the no-disruption and disruption-state payoff components. The feasible set is $0 \le Q \le D$ and $0 \le Q_{\text{Option}} \le D$,

where Q denotes the regular order quantity and Q_{Option} denotes the option capacity reserved before the disruption state is realized.

The manufacturer's optimal decision is the solution to the following optimization problem:

$$max_{Q,Q_{Option}>0}\Pi_{option}(Q, Q_{Option}) \tag{17}$$

The following proposition provides the optimal solution to problem (17).

Proposition 7. *Under the option contract strategy, the manufacturer's optimal decisions and corresponding expected profit are as follows:*

(1) The optimal regular order quantity and option quantity are

$$Q^* = \begin{cases} D & if\ r_3 - r_1 > 0 \\ 0 & otherwise \end{cases} \tag{18}$$

$$Q^*_{Option} = \begin{cases} D & if\ \overline{\theta}(P - r_3 + l) > F \\ 0 & otherwise \end{cases} \tag{19}$$

(2) The manufacturer's optimal expected profit is

$$\Pi^*_{Option} = \begin{cases} \left[(1-\overline{\theta})(P-r_1) + \overline{\theta}(P-r_3) - F\right]D & \text{if } Q^* = D, Q^*_{Option} = D, \\ \left[(1-\overline{\theta})(P-r_1) - \overline{\theta}l\right]D & \text{if } Q^* = D, Q^*_{Option} = 0, \\ [P - r_3 - F]D & \text{if } Q^* = 0, Q^*_{Option} = D. \end{cases} \tag{20}$$

Here, we refer to the backup supplier under the option contract as the option supplier. Proposition 7 characterizes boundary solutions of the simplified linear benchmark. The feasible set also includes the zero-regular-order and zero-option boundary, $Q^* = 0$ and $Q^*_{Option} = 0$. Under the maintained positive-demand and positive-margin assumptions used for the benchmark comparison, this boundary is not active in the main parameter regions because it leaves demand unmet without creating option flexibility. It is nevertheless part of the feasible set and should be stated explicitly for completeness. Because the option contract benchmark is linear in the relevant decision variables, the resulting boundary solutions should be interpreted as a consequence of the simplified benchmark. More elaborate option models with convex exercise costs, risk aversion, minimum purchase commitments, or multi-period exercise decisions may generate interior trade-offs. These extensions are outside the scope of the present model. Given the complexity of the analytical expressions for Π^*_O and Π^*_{Option}, we conduct a numerical study to compare the two strategies and delineate their optimal conditions of use. We investigate how the retail price (P), the stockout penalty (l), the difference between the high and low disruption probabilities ($\theta_h - \theta_l$), and the prior probability (β) impact the strategic choice. The numerical analysis is illustrative rather than empirically calibrated. The baseline values are selected to satisfy the feasibility conditions of the model and to show how the analytical mechanisms operate across parameter regions. The figures should therefore be interpreted as theoretical sensitivity patterns, not as industry-specific predictions. The numerical results are illustrated in Figures 6–10, leading to the following key insights:

(1) When the retail price exceeds a certain critical threshold, the manufacturer should adopt the PVO strategy; otherwise, the option strategy is preferred under the examined parameter settings (Figure 6). As the retail price increases, the profit margin expands, increasing the opportunity cost of unmet demand. In this high-margin

regime, more accurate disruption risk information becomes more valuable because it supports better backup capacity calibration. In contrast, an option contract requires the upfront premium regardless of the realized disruption state.

(2) As the unit stockout penalty increases, the critical price threshold decreases, making the manufacturer more inclined to adopt PVO under the examined parameter settings (Figure 7). A severe stockout penalty magnifies the financial impact of a disruption. The informational advantage of PVO is valuable because it helps the manufacturer adjust backup capacity to the revealed risk state. Figure 7 illustrates this mechanism: the option contract benchmark is relatively stable, whereas the PVO surface becomes more favorable when market margin and stockout consequences are jointly high. Figure 8 further confirms that the manufacturer may pivot to PVO when the information dividend becomes large enough to offset the cost of equity-based information access.

(3) As the volatility of the disruption risk intensifies, the expected profit of the PVO strategy can increase relative to the option strategy under the examined parameter settings (Figure 8). To isolate the value of information, we keep the mean disruption probability fixed and symmetrically widen the risk spread. As illustrated in Figure 8, the option contract benchmark depends on the public average belief, whereas PVO uses the revealed risk type. As market ambiguity expands, the information dividend grows because the manufacturer can avoid under-reserving in high-risk states and over-reserving in low-risk states.

(4) To comprehensively delineate the optimal conditions for utilizing the PVO strategy versus the option contract strategy, we construct a two-dimensional strategy region map. As illustrated in Figure 9, we fix the mean disruption probability ($\bar{\theta} = \beta\theta_h + (1-\beta)\theta_l$) to control for the average risk, and simultaneously vary the disruption risk spread ($\Delta\theta = \theta_h - \theta_l$) on the horizontal axis and the unit stockout penalty (l) on the vertical axis. The indifference boundary (the solid black curve) represents the exact parameter combinations where the expected profits of both strategies are equal ($\Pi^*_{PVO} = \Pi^*_{Option}$), partitioning the decision space into two distinct operational regimes: Option Contract Dominance Zone (Red Shaded Region): This region emerges when the disruption risk spread ($\Delta\theta$) is relatively narrow and the stockout penalty (l) is low. In this regime, the market uncertainty is subdued, meaning the public average belief serves as a highly reliable proxy for the actual disruption risk. Consequently, the "information dividend" yielded by PVO is minimal. Furthermore, because the stockout penalty is not severe enough to threaten the firm's survival, purchasing an option contract serves as an economical and sufficient baseline hedge, outperforming the PVO strategy. PVO Dominance Zone (Blue Shaded Region): As the market environment moves toward the upper-right quadrant—characterized by high-risk volatility (large $\Delta\theta$) and severe stockout consequences (high l)—the PVO strategy achieves dominance. When the stockout penalty is high, the cost of being wrong (either facing a stockout without backup or over-reserving expensive capacity) becomes severe. Here, the profound value of private information is realized. PVO allows the manufacturer to synchronize its backup capacity with the true state of nature. The indifference curve explicitly reveals a substitution effect: as the market becomes more volatile (higher $\Delta\theta$), the manufacturer is willing to adopt the PVO strategy even at lower stockout penalties, as the operational savings from eliminating information asymmetry justify the strategic shift. The indifference boundary in Figure 9 is conditional on the baseline parameter values. Changes in the option premium, strike price, equity return, backup procurement cost, or capacity cost pa-

rameters may shift the boundary. Thus, the figure should be read as an illustration of the model mechanism rather than as a universal empirical decision map.

(5) The interaction between the prior probability (β) and the high disruption risk (θ_h) dictates the strategic trade-off between expensive emergency hedging and proactive capacity management (Figure 10). Reasoning: As demonstrated in Figure 10, the PVO strategy dominates the upper-right quadrant (high β, high θ_h), while the option contract strategy is preferred in the lower-left quadrant. This shift reflects the fundamental cost dynamics of severe supply chain risks. When the market anticipates frequent and severe disruptions (the upper-right region), relying on an option contract becomes a massive financial drain. The manufacturer is repeatedly forced to execute the option at a punitive emergency strike price ($r_3 = 90$). By adopting the PVO strategy, the manufacturer acquires precise risk information, allowing it to proactively and precisely reserve capacity with the standard backup supplier at a significantly lower operational cost ($r_2 = 50$). In this high-risk environment, the information value of PVO successfully shields the firm from emergency procurement costs. Conversely, in the lower-left region where severe disruptions are relatively rare, the manufacturer can efficiently utilize the option contract as a cheap "insurance policy" (paying only the upfront premium F). Since the punitive strike price is rarely triggered, the costly proactive capacity adjustments and potential equity investments associated with the PVO strategy become economically inefficient.

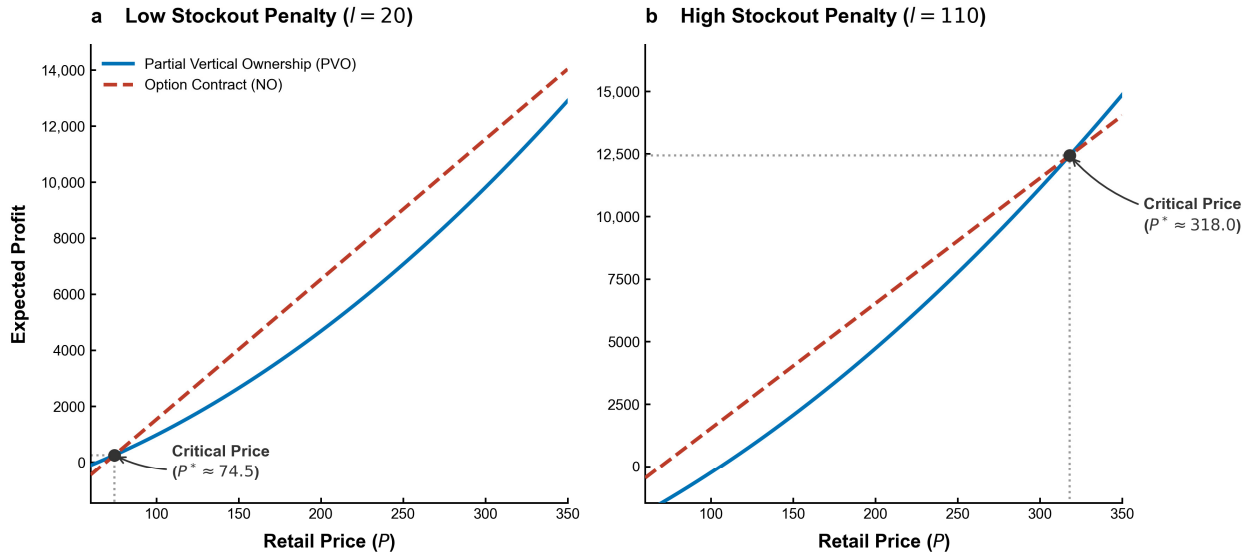


Figure 6. Comparison of expected profits between the PVO and option contract strategies under varying retail prices: (**a**) low stockout penalty scenario ($l = 20$); (**b**) high stockout penalty scenario ($l = 110$).

Overall, the numerical results support the qualitative mechanism of the analytical model: PVO is more attractive when information about disruption risk heterogeneity is valuable, whereas option contracts remain attractive when the cost of information acquisition or equity participation is high relative to the expected disruption loss. These results remain conditional on the baseline parameters and the simplified option contract benchmark. These findings also have implications for resilience governance. PVO should not be interpreted only as a dominance region in a numerical map. It can be understood as a governance mechanism that converts supplier-side uncertainty into actionable risk information before disruption losses materialize. This anticipatory role helps explain why equity-based governance may complement, rather than replace, contractual hedging mechanisms.

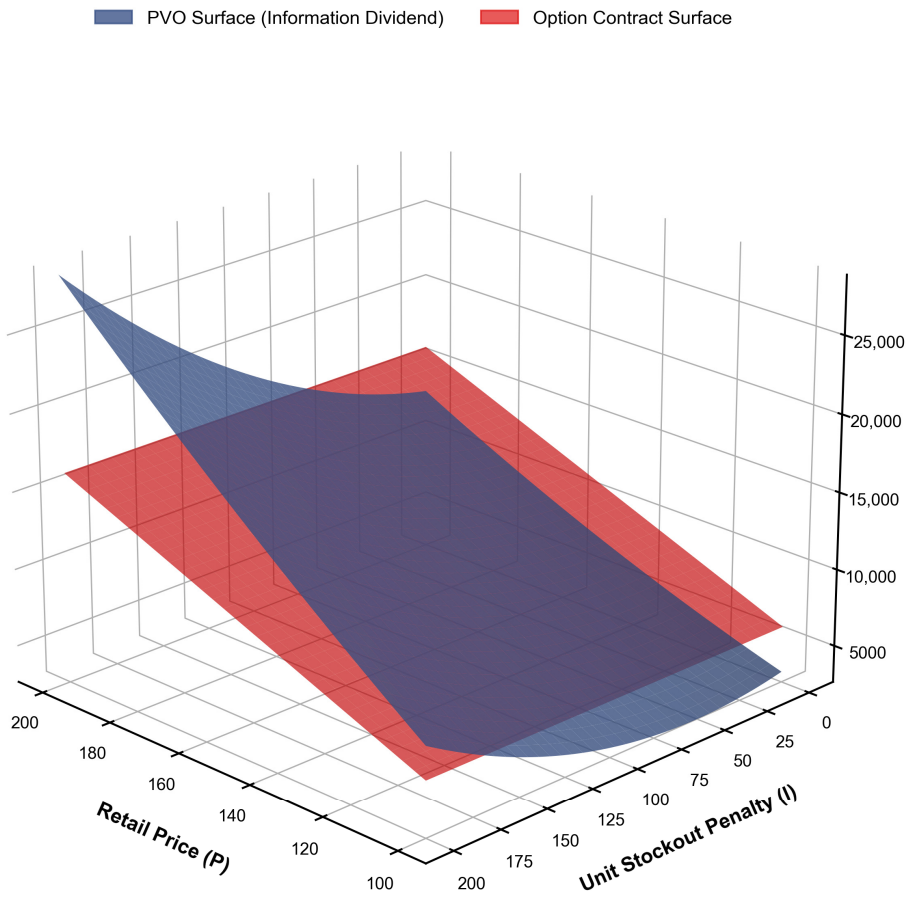


Figure 7. Comparison of expected profits between the PVO and option contract strategies under varying retail prices and unit stockout penalties.

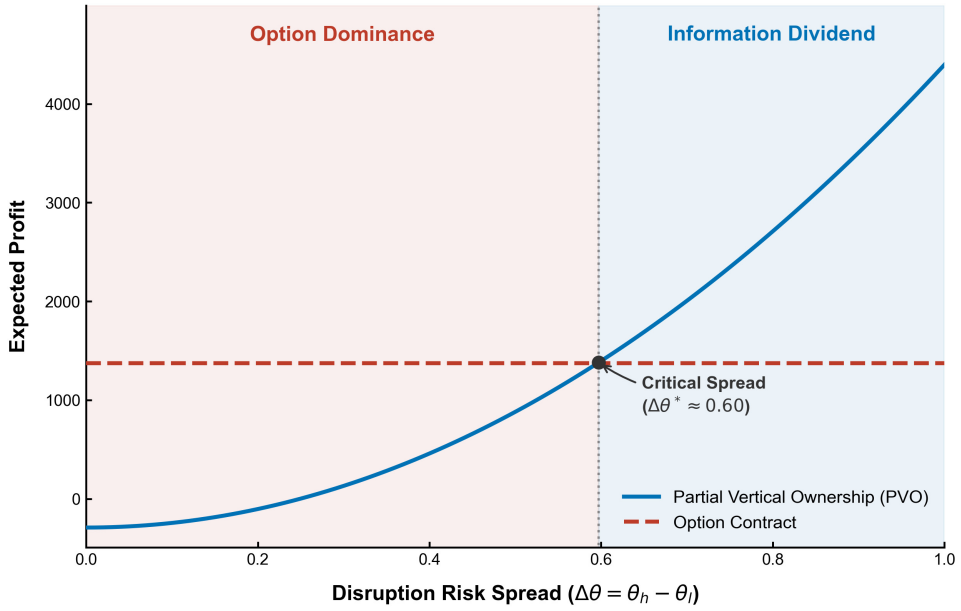


Figure 8. Impact of disruption risk Spread on expected profit and strategic choice.

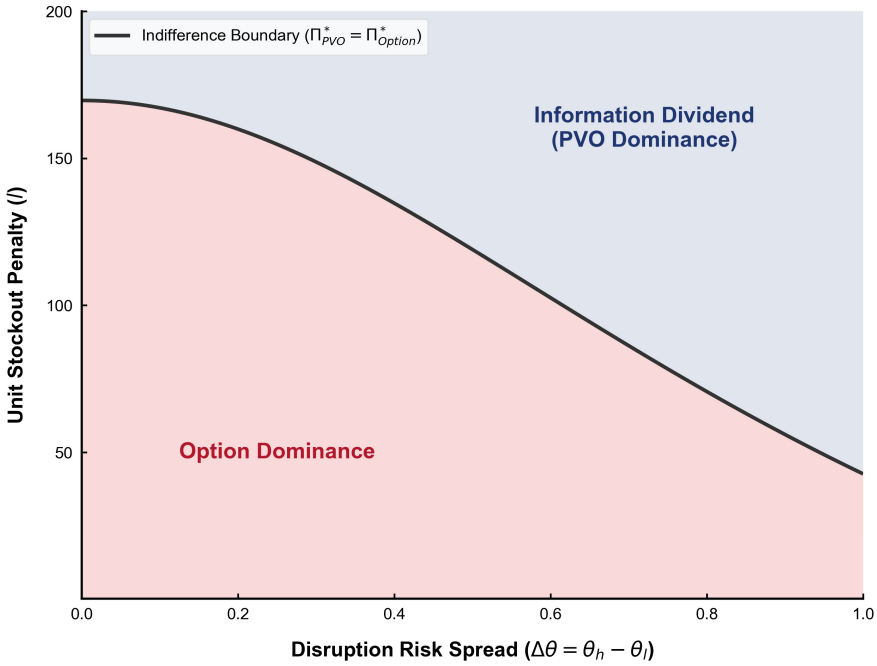


Figure 9. The interplay between disruption risk volatility and unit stockout penalty.

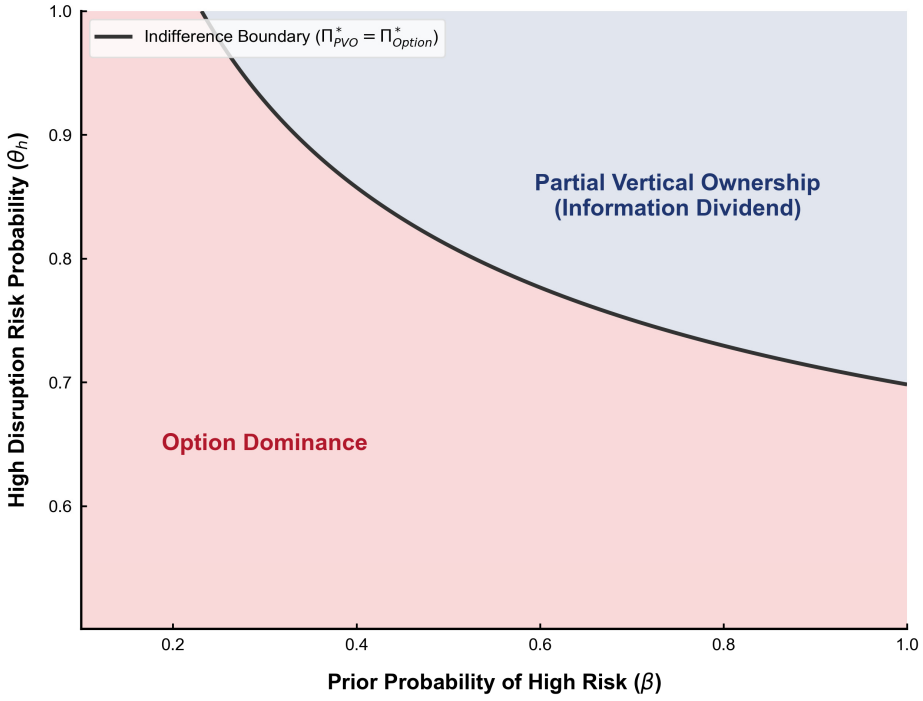


Figure 10. Strategic choice driven by prior probability and actual high disruption risk.

5. Conclusions

This paper investigates how a manufacturer can strategically mitigate upstream supply disruption risks under information asymmetry. While traditional supply chain literature extensively explores financial hedging tools such as option contracts, we propose and analyze an operational hedging mechanism: partial vertical ownership (PVO). By developing an analytical model, we characterize the manufacturer's optimal capacity reservation strategies at a backup supplier under three scenarios: NO, PVO, and option contracts. Our analytical results yield several theoretical insights. First, under the specified convex capacity cost structure, the manufacturer's expected profit exhibits a non-linear relationship with the

unit stockout penalty. Second, the extra expected profit generated by the PVO strategy is mathematically driven by the variance of the primary supplier's disruption probability, but its economic magnitude also depends on the scale of stockout losses, backup procurement costs, and the equity return condition.

Furthermore, by benchmarking the PVO strategy against an option contract strategy, we construct a two-dimensional strategy region map. The numerical analysis suggests that an option contract can serve as an efficient hedge in relatively stable environments with low disruption risk dispersion and minor stockout penalties. As the spread between high and low disruption probabilities widens and the stockout penalty increases, PVO can outperform the option contract benchmark under the examined parameter settings. This comparison should be interpreted as a conditional theoretical result rather than as evidence of universal PVO dominance.

The primary contribution of this study is to examine PVO not merely as a financial asset allocation, but as a possible operational hedging instrument for supply chain resilience. The findings provide theoretical guidance on when equity-based information access may be valuable and when contractual tools may remain preferable. They also highlight the importance of governance conditions, information credibility, and parameter calibration when translating the model into managerial decisions.

Several limitations should be noted. First, the model is a stylized analytical framework with one manufacturer, one unreliable primary supplier, and one reliable backup supplier. Second, demand is assumed to be deterministic so that the analysis can isolate the informational value of PVO from demand-side uncertainty. Third, the information effect of PVO is modeled as a benchmark case in which equity-based governance provides access to the supplier's disruption risk type. In practice, the validity of this mechanism may depend on country-specific legal rules, corporate governance arrangements, audit rights, data-sharing agreements, monitoring costs, and the strength of the strategic partnership. Fourth, the equity return rate is treated as exogenous, although in practice it may be correlated with supplier disruption risk. Fifth, the option contract benchmark is simplified and does not capture all possible forms of option exercise, convex option costs, risk aversion, or multi-period contracting. Finally, the numerical analysis is illustrative rather than empirically calibrated. Therefore, the findings should be interpreted as theoretical guidance under the stated assumptions rather than as universally valid prescriptions for all countries, industries, or contractual settings.

Future research could extend this framework by incorporating stochastic demand, partial information revelation, supplier agency behavior, endogenous equity returns, richer option contract structures, multi-period updating of disruption probabilities, and empirical calibration across different institutional contexts.

Author Contributions: Conceptualization, B.G.; methodology, B.G.; validation, B.G., Y.G. and X.L.; formal analysis, B.G.; writing—original draft preparation, B.G.; writing—review and editing, Y.G. and X.L.; Resources, B.G. and Y.G.; Supervision, X.L.; Project administration, X.L.; Funding acquisition, X.L. All authors have read and agreed to the published version of the manuscript.

Funding: This research is funded by the National Natural Science Foundation of China, Grant No. 72304030.

Data Availability Statement: Data are contained within the article.

Conflicts of Interest: The authors declare no conflicts of interest.

Abbreviations

The following abbreviations are used in this manuscript:

PVO	Partial Vertical Ownership
NO	Non-Ownership

Appendix A

Proof of Proposition 1. Taking the second derivative of the manufacturer's profit function $\Pi_O^H(Q_{OH}^*)$ with respect to Q_{OH}, we obtain

$$\frac{\partial^2 \Pi_O^H}{\partial Q_{OH}^2} = -2a < 0$$

This indicates that Π_O^H is strictly concave in Q_{OH}, ensuring that a unique global maximum exists.

Next, taking the first derivative of Π_O^H with respect to Q_{OH} and setting it to zero yields

$$\frac{\partial \Pi_O^H}{\partial Q_{OH}} = -b - 2aQ_{OH} + (l + P - r_2)\theta_h = 0$$

Solving this equation for Q_{OH}, we obtain the optimal reserved capacity:

$$Q_{OH}^* = \frac{-b + (l + P - r_2)\theta_h}{2a} = \frac{\theta_h \Delta_1 - b}{2a}$$

Following the same logic, when the disruption probability is low, the optimal reserved capacity is

$$Q_{OL}^* = \frac{-b + (l + P - r_2)\theta_l}{2a} = \frac{\theta_l \Delta_1 - b}{2a}$$

Substituting the optimal capacities into the state-contingent profit functions and collecting terms in the operating margin, backup capacity cost, and stockout-loss components yield the following simplified expressions. By substituting Q_{OH}^* and Q_{OL}^* back into their respective profit functions, and applying our predefined parameters ($\Pi_0, \Delta_1, \Delta_2$), straightforward algebraic calculation simplifies the maximal profits to Equations (6) and (7) Consequently, the final optimal expected profit is the probability-weighted sum of the two scenarios:

$$\Pi_O^* = \beta \Pi_O^H(Q_{OH}^*) + (1 - \beta)\Pi_O^L(Q_{OL}^*)$$

Substituting the simplified expressions of $\Pi_O^H(Q_{OH}^*)$ and $\Pi_O^L(Q_{OL}^*)$ into the equation above, and factoring based on the expected probability $\bar{\theta}$ and expected squared probability $\bar{\theta}^2$, yield Equation (8).

This completes the proof. □

Proof of Proposition 2. (1) From the optimal reserved capacities derived in Proposition 1, taking the partial derivative with respect to l yields

$$\frac{\partial Q_{OH}}{\partial l} = \frac{\theta_h}{2a} \; and \; \frac{\partial Q_{OL}}{\partial l} = \frac{\theta_l}{2a}$$

Noting that $\theta_h > \theta_l$, it is straightforward to obtain

$$\frac{\partial Q_{OH}}{\partial l} > \frac{\partial Q_{OL}}{\partial l}$$

This indicates that a higher disruption probability leads to a higher marginal sensitivity of the reserved capacity to the stockout penalty.

(2) To analyze the concavity or convexity of the expected profit Π_O^* with respect to l, we take the first and second partial derivatives. Using the simplified notation introduced previously, the first derivative is

$$\frac{\partial \Pi_O^*}{\partial l} = -D\overline{\theta} + \frac{\Delta_1 \overline{\theta}^2 - b\overline{\theta}}{2a} = \frac{(l + P - r_2)\overline{\theta}^2 - \overline{\theta}(2aD + b)}{2a}$$

Taking the second derivative with respect to l yields

$$\frac{\partial^2 \Pi_O^*}{\partial l^2} = \frac{\overline{\theta}^2}{2a} > 0$$

Since the second derivative is strictly positive, we confirm that Π_O^* is a strictly convex function of l. By setting the first derivative $\frac{\partial \Pi_O^*}{\partial l} = 0$, we can find the global minimum point $\widetilde{l}$:

$$\widetilde{l} = \frac{\overline{\theta}(2aD + b)}{\overline{\theta}^2} - P + r_2$$

Based on our interior solution assumption, the parameters ensure that $\widetilde{l} > 0$. Therefore, within the practically meaningful domain $l \geq 0$, the partial derivative $\frac{\partial \Pi_O^*}{\partial l} < 0$ for $l \in \left[0, \widetilde{l}\right)$, and $\frac{\partial \Pi_O^*}{\partial l} > 0$ for $l \in \left(\widetilde{l}, \infty\right)$. Thus, the expected profit initially decreases and subsequently increases with l.

This completes the proof. □

Proof of Proposition 3. Taking the second derivative of the manufacturer's profit function Π_{NO} with respect to Q_{NO}, we obtain

$$\frac{\partial^2 \Pi_{NO}}{\partial Q_{NO}^2} = -2a < 0$$

This indicates that Π_{NO} is strictly concave in Q_{NO}. Therefore, the optimal solution to the maximization problem must satisfy the first-order condition. Setting the first derivative to zero yields

$$\frac{\partial \Pi_{NO}}{\partial Q_{NO}} = -b - 2aQ_{NO} + \beta(l + P - r_2)\theta_h + (1 - \beta)(l + P - r_2)\theta_l = 0$$

Using our predefined notation $\overline{\theta} = \beta\theta_h + (1 - \beta)\theta_l$ and $\Delta_1 = l + P - r_2$, this equation simplifies to

$$-b - 2aQ_{NO} + \overline{\theta}\Delta_1 = 0$$

From this, we obtain the optimal reserved capacity:

$$Q_{NO}^* = \frac{\overline{\theta}\Delta_1 - b}{2a}$$

Substituting Q_{NO}^* back into the original objective function and simplifying the algebraic expression, yield Equation (13)

This completes the proof. □

Proof of Corollary 1. It is straightforward to verify this relationship by calculating the differences between the optimal capacities. Using our established notation $\overline{\theta} = \beta\theta_h + (1-\beta)\theta_l$ and $\Delta_1 = l + P - r_2$, we have

$$Q_{OH}^* - Q_{OL}^* = \frac{\Delta_1(\theta_h - \theta_l)}{2a} > 0$$

$$Q_{OH}^* - Q_{NO}^* = \frac{\Delta_1(\theta_h - \overline{\theta})}{2a} = \frac{(1-\beta)\Delta_1(\theta_h - \theta_l)}{2a} > 0$$

$$Q_{NO}^* - Q_{OL}^* = \frac{\Delta_1(\overline{\theta} - \theta_l)}{2a} = \frac{\beta\Delta_1(\theta_h - \theta_l)}{2a} > 0$$

Therefore, the relationship holds. □

Proof of Proposition 4. (1) From the previous propositions, taking the partial derivative of Q_{NO}^* with respect to l yields

$$\frac{\partial Q_{NO}^*}{\partial l} = \frac{\overline{\theta}}{2a}$$

Using the expected disruption probability $\overline{\theta} = \beta\theta_h + (1-\beta)\theta_l$, we can calculate the differences between the derivatives

$$\frac{\partial Q_{NO}^*}{\partial l} - \frac{\partial Q_{OH}^*}{\partial l} = \frac{\overline{\theta} - \theta_h}{2a} = \frac{(1-\beta)(\theta_l - \theta_h)}{2a} < 0$$

$$\frac{\partial Q_{NO}^*}{\partial l} - \frac{\partial Q_{OL}^*}{\partial l} = \frac{\overline{\theta} - \theta_l}{2a} = \frac{\beta(\theta_h - \theta_l)}{2a} > 0$$

Therefore, the relationship $\frac{\partial Q_{OL}}{\partial l} < \frac{\partial Q_{NO}}{\partial l} < \frac{\partial Q_{OH}}{\partial l}$ holds.

(2) Taking the first and second partial derivatives of Π_{NO}^* with respect to l, we obtain

$$\frac{\partial \Pi_{NO}^*}{\partial l} = \frac{\overline{\theta}\left[\overline{\theta}(l + P - r_2) - (2aD + b)\right]}{2a}$$

$$\frac{\partial^2 \Pi_{NO}^*}{\partial l^2} = \frac{\overline{\theta}^2}{2a} > 0$$

Since the second derivative is strictly positive, Π_{NO}^* is a strictly convex function of l. Setting the first derivative $\frac{\partial \Pi_{NO}^*}{\partial l} = 0$, we find the global minimum point:

$$\tilde{l}_{NO} = \frac{2aD + b}{\overline{\theta}} - P + r_2$$

Under the regular parameter condition, $\tilde{l}_{NO} > 0$. Therefore, for $l \in \left[0, \tilde{l}_{NO}\right)$, the partial derivative is negative ($\frac{\partial \Pi_{NO}^*}{\partial l} < 0$), and for $l \in \left(\tilde{l}_{NO}, \infty\right)$, the partial derivative is positive. Thus, the expected profit first decreases and then increases.

This completes the proof. □

Proof of Proposition 5. To enhance the clarity of our mathematical notation and align with standard statistical conventions, we define $E[\theta] = \beta\theta_h + (1-\beta)\theta_l$ as the expected value of the primary supplier's disruption probability, and $E\left[\theta^2\right] = \beta\theta_h^2 + (1-\beta)\theta_l^2$ as the expectation of the squared disruption probability. Consequently, the variance of the disruption probability is given by $Var(\theta) = E\left[\theta^2\right] - \left(E[\theta]\right)^2$.

Let $\widetilde{l}$ and $\widetilde{l}_{NO}$ denote the critical thresholds derived in Propositions 2 and 4, rewritten with the expectation operators

$$\widetilde{l}_{NO} = \frac{2aD+b}{E[\theta]} - P + r_2 \text{ and } \widetilde{l} = \frac{E[\theta](2aD+b)}{E[\theta^2]} - P + r_2$$

Taking the difference between the two thresholds yields

$$\widetilde{l}_{NO} - \widetilde{l} = (2aD+b)\left(\frac{1}{E[\theta]} - \frac{E[\theta]}{E[\theta^2]}\right) = \frac{2aD+b}{E[\theta]E[\theta^2]}\left(E\left[\theta^2\right] - \left(E[\theta]\right)^2\right)$$

According to probability theory, $Var(\theta) = E\left[\theta^2\right] - \left(E[\theta]\right)^2$, which represents the variance of the primary supplier's disruption probability. Because the variance of a non-degenerate random variable is strictly positive ($Var(\theta) > 0$), and all other system parameters ($a, D, b, E[\theta], E\left[\theta^2\right]$) are strictly positive, it mathematically follows that $\widetilde{l}_{NO} - \widetilde{l} > 0$.

This completes the proof. □

Proof of Proposition 6. From Proposition 1 and Proposition 3, and applying our predefined variables $\Delta_1 = l + P - r_2$ and the variance of the disruption probability $Var(\theta) = \beta(1-\beta)(\theta_h - \theta_l)^2$, the profit difference between the two scenarios can be elegantly expressed as

$$\Pi_O^* - \Pi_{NO}^* = C\lambda + \frac{(1-\beta)\beta(l+P-r_2)^2(\theta_h-\theta_l)^2}{4a} = C\lambda + \frac{\Delta_1^2 Var(\theta)}{4a}$$

Based on this simplified expression, we can deduce the following:

(1) Since $\theta_h - \theta_l$ is on the numerator, the profit difference strictly increases as the gap between $\theta_h - \theta_l$ widens.
(2) The profit difference attains its global maximum at $\beta = 0.5$ since the term $(1-\beta)\beta$.
(3) By setting $\Pi_O^* - \Pi_{NO}^* = 0$, we solve for the critical return rate threshold:

$$\widetilde{\lambda} = -\frac{(1-\beta)\beta(l+P-r_2)^2(\theta_h-\theta_l)^2}{4aC} = -\frac{\Delta_1^2 Var(\theta)}{4aC}$$

Thus, when $\lambda > \widetilde{\lambda}$, $\Pi_O^* - \Pi_{NO}^* > 0$, making the investment profitable.

This completes the proof. □

Proof of Proposition 7. Taking the first-order partial derivatives of the profit function Π_{Option}^* with respect to Q and Q_{Option}, we obtain

$$\frac{\partial \Pi_{option}}{\partial Q} = (1-\bar{\theta})(r_3 - r_1)$$

$$\frac{\partial \Pi_{option}}{\partial Q_{Option}} = \bar{\theta}(P - r_3 + l) - F$$

Since the objective function is linear with respect to the decision variables, the optimal solutions must lie at the boundaries. Thus, we have

If $r_3 - r_1 > 0$, then $Q^* = D$; otherwise, $Q^* = 0$. If $\bar{\theta}(P - r_3 + l) > F$, then $Q_{Option}^* = D$; otherwise, $Q_{Option}^* = 0$.

Therefore, there are three viable combinations of optimal strategies:

(1) $Q^* = D, Q_{Option}^* = D$,
(2) $Q^* = D, Q_{Option}^* = 0$,

(3) $Q^* = 0, Q^*_{Option} = D$.

Substituting these optimal strategy combinations back into the objective function, yield Equation (20).

This completes the proof. □

References

1. Jackley, M. 9 Ways to Reduce Supply Chain Disruptions. 2023. Available online: https://www.oracle.com/asean/scm/reduce-supply-chain-disruptions/ (accessed on 23 May 2026).
2. Establishment of Companywide Countermeasures Headquarters and Emergency Response Measures in Disaster Regions. Available online: https://www.toyota-global.com/company/history_of_toyota/75years/text/leaping_forward_as_a_global_corporation/chapter5/section5/item1.html (accessed on 23 May 2026).
3. iResearch Report on the Digital Transformation Trends of Chinese Automotive Enterprises in 2024: Supply Chain Digitization. 2024. Available online: https://www.iresearch.com.cn/Detail/report?id=4306&isfree=0 (accessed on 23 May 2026).
4. Knorr-Bremse Invests in the Chinese e-Motor Specialist Bobang and Strengthens the Partnership. 2022. Available online: https://newsroom.knorr-bremse.com/en/knorr-bremse-invests-in-the-chinese-e-motor-specialist-bobang-and-strengthens-the-partnership/ (accessed on 23 May 2026).
5. Hunold, M.; Stahl, K. Passive Vertical Integration and Strategic Delegation. *RAND J. Econ.* **2016**, *47*, 891–913. [CrossRef]
6. Fiocco, R. The Strategic Value of Partial Vertical Integration. *Eur. Econ. Rev.* **2016**, *89*, 284–302. [CrossRef]
7. Li, J.; Yang, S.; Shi, V.; Zhai, S. Partial Vertical Centralization in Competing Supply Chains. *Int. J. Prod. Econ.* **2020**, *224*, 107565. [CrossRef]
8. Levy, N.; Spiegel, Y.; Gilo, D. Partial Vertical Integration, Ownership Structure, and Foreclosure. *Am. Econ. J.-Microecon.* **2018**, *10*, 132–180. [CrossRef]
9. Chen, J.; Hu, Q.; Song, J.-S. Effect of Partial Cross Ownership on Supply Chain Performance. *Eur. J. Oper. Res.* **2017**, *258*, 525–536. [CrossRef]
10. Katsaliaki, K.; Galetsi, P.; Kumar, S. Supply Chain Disruptions and Resilience: A Major Review and Future Research Agenda. *Ann. Oper. Res.* **2022**, *319*, 965–1002. [CrossRef] [PubMed]
11. Pournader, M.; Kach, A.; Talluri, S. A Review of the Existing and Emerging Topics in the Supply Chain Risk Management Literature. *Decis. Sci.* **2020**, *51*, 867–919. [CrossRef] [PubMed]
12. Shekarian, M.; Mellat Parast, M. An Integrative Approach to Supply Chain Disruption Risk and Resilience Management: A Literature Review. *Int. J. Logist.-Res. Appl.* **2021**, *24*, 427–455. [CrossRef]
13. Sabbaghtorkan, M.; Batta, R.; He, Q. Prepositioning of Assets and Supplies in Disaster Operations Management: Review and Research Gap Identification. *Eur. J. Oper. Res.* **2020**, *284*, 1–19. [CrossRef]
14. Tomlin, B. On the Value of Mitigation and Contingency Strategies for Managing Supply Chain Disruption Risks. *Manag. Sci.* **2006**, *52*, 639–657. [CrossRef]
15. Tang, C.; Tomlin, B. The Power of Flexibility for Mitigating Supply Chain Risks. *Int. J. Prod. Econ.* **2008**, *116*, 12–27. [CrossRef]
16. Bakshi, N.; Kleindorfer, P. Co-Opetition and Investment for Supply-Chain Resilience. *Prod. Oper. Manag.* **2009**, *18*, 583–603. [CrossRef]
17. Wang, Y.; Gilland, W.; Tomlin, B. Mitigating Supply Risk: Dual Sourcing or Process Improvement? *MSOM-Manuf. Serv. Oper. Manag.* **2010**, *12*, 489–510. [CrossRef]
18. Dong, L.; Tomlin, B. Managing Disruption Risk: The Interplay Between Operations and Insurance. *Manag. Sci.* **2012**, *58*, 1898–1915. [CrossRef]
19. Tang, S.Y.; Gurnani, H.; Gupta, D. Managing Disruptions in Decentralized Supply Chains with Endogenous Supply Process Reliability. *Prod. Oper. Manag.* **2014**, *23*, 1198–1211. [CrossRef]
20. Ang, E.; Iancu, D.A.; Swinney, R. Disruption Risk and Optimal Sourcing in Multitier Supply Networks. *Manag. Sci.* **2017**, *63*, 2397–2419. [CrossRef]
21. Tao, Y.; Lai, X.; Zhou, S. Information Sharing in a Transparent Supply Chain with Transportation Disruptions and Supplier Competition. *Ann. Oper. Res.* **2023**, *329*, 307–329. [CrossRef]
22. Li, G.; Liu, M.; Bian, Y.; Sethi, S.P. Guarding against Disruption Risk by Contracting under Information Asymmetry. *Decis. Sci.* **2020**, *51*, 1521–1559. [CrossRef]
23. Yang, Z.; Aydin, G.; Babich, V.; Beil, D.R. Using a Dual-Sourcing Option in the Presence of Asymmetric Information About Supplier Reliability: Competition vs. Diversification. *MSOM-Manuf. Serv. Oper. Manag.* **2012**, *14*, 202–217. [CrossRef]
24. Yang, Z.; Aydin, G.; Babich, V.; Beil, D.R. Supply Disruptions, Asymmetric Information, and a Backup Production Option. *Manag. Sci.* **2009**, *55*, 192–209. [CrossRef]

25. Li, X.; Li, Y. On the Loss-Averse Dual-Sourcing Problem under Supply Disruption. *Comput. Oper. Res.* **2018**, *100*, 301–313. [CrossRef]
26. Qin, F.; Rao, U.S.; Gurnani, H.; Bollapragada, R. Role of Random Capacity Risk and the Retailer in Decentralized Supply Chains with Competing Suppliers. *Decis. Sci.* **2014**, *45*, 255–279. [CrossRef]
27. Kumar, M.; Basu, P.; Avittathur, B. Pricing and Sourcing Strategies for Competing Retailers in Supply Chains under Disruption Risk. *Eur. J. Oper. Res.* **2018**, *265*, 533–543. [CrossRef]
28. Gurnani, H.; Ramachandran, K.; Ray, S.; Xia, Y. Ordering Behavior Under Supply Risk: An Experimental Investigation. *MSOM-Manuf. Serv. Oper. Manag.* **2014**, *16*, 61–75. [CrossRef]
29. Hu, B.; Kostamis, D. Managing Supply Disruptions When Sourcing from Reliable and Unreliable Suppliers. *Prod. Oper. Manag.* **2015**, *24*, 808–820. [CrossRef]
30. Hu, X.; Gurnani, H.; Wang, L. Managing Risk of Supply Disruptions: Incentives for Capacity Restoration. *Prod. Oper. Manag.* **2013**, *22*, 137–150. [CrossRef]
31. Huang, H.; Shen, X.; Xu, H. Procurement Contracts in the Presence of Endogenous Disruption Risk. *Decis. Sci.* **2016**, *47*, 437–472. [CrossRef]
32. Saghafian, S.; Van Oyen, M.P. The Value of Flexible Backup Suppliers and Disruption Risk Information: Newsvendor Analysis with Recourse. *IIE Trans.* **2012**, *44*, 834–867. [CrossRef]
33. Kole, H.; Bakal, I.S. Value of Information through Options Contract under Disruption Risk. *Comput. Ind. Eng.* **2017**, *103*, 85–97. [CrossRef]
34. Yin, Z.; Wang, C. Strategic Cooperation with a Backup Supplier for the Mitigation of Supply Disruptions. *Int. J. Prod. Res.* **2018**, *56*, 4300–4312. [CrossRef]
35. Son, J. On the Effectiveness of Option Contracts under Supply Disruption. *Oper. Supply Chain Manag. Int. J.* **2023**, *16*, 378–388. [CrossRef]
36. Liu, A.; Wang, X.; Tang, J. Optimizing Multi-Channel Procurement Planning under Disruption Risks. *Int. J. Prod. Econ.* **2024**, *275*, 109346. [CrossRef]
37. Fang, F.; Jiang, B.; Sun, J. Partial Vertical Ownership in the Presence of Downstream Competition. *Prod. Oper. Manag.* **2023**, *32*, 1692–1704. [CrossRef]
38. Li, J.; Wang, Y. The Impact of Partial Vertical Integration on Supplier Encroachment. *RAIRO-Oper. Res.* **2025**, *59*, 3015–3040. [CrossRef]
39. Hua, C.; Zhao, L. Partial Vertical Ownership and Specific Investment of the Manufacturer under Demand Uncertainty: Utilisation, Risk, and Collaboration. *Int. J. Prod. Res.* **2026**, *64*, 1489–1513. [CrossRef]
40. Pishchulov, G.; Richter, K.; Golesorkhi, S. Supply Chain Coordination under Asymmetric Information and Partial Vertical Integration. *Ann. Oper. Res.* **2023**, *329*, 1315–1356. [CrossRef]
41. Avinadav, T.; Shamir, N. Partial Vertical Ownership, Capacity Investment, and Information Exchange in a Supply Chain. *Manag. Sci.* **2023**, *69*, 6038–6056. [CrossRef]
42. Ha, A.Y.; Tong, S.; Zhang, H. Sharing Demand Information in Competing Supply Chains with Production Diseconomies. *Manag. Sci.* **2011**, *57*, 566–581. [CrossRef]
43. Adida, E.; Perakis, G. The Effect of Supplier Capacity on the Supply Chain Profit. *Ann. Oper. Res.* **2014**, *223*, 1–52. [CrossRef]

Disclaimer/Publisher's Note: The statements, opinions and data contained in all publications are solely those of the individual author(s) and contributor(s) and not of MDPI and/or the editor(s). MDPI and/or the editor(s) disclaim responsibility for any injury to people or property resulting from any ideas, methods, instructions or products referred to in the content.