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# Channel Selection and Product Configuration Strategies of the Brander in Heterogeneous E-Commerce Platforms Under Consumption Subsidies

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**Abstract**

With the rise in social channels, branders have gained more sales opportunities, but at the same time, they have to face practical challenges such as channel introductions, product configuration, and product pricing. Although existing studies have discussed branders' product configuration, most of them confine product differentiation strategies to homogeneous e-commerce platforms, overlook the heterogeneity between traditional and social platforms, and rarely consider the impact of consumption subsidies. To this end, we consider a supply chain consisting of a brander and two heterogeneous e-commerce platforms. By incorporating both single and differentiated-product configurations, we developed a sequential Stackelberg game model to analyze the brander's optimal channel introduction, product configuration, and pricing strategies under government consumption subsidies. Under the model settings and assumptions of this paper, we derive the following findings: (1) when the quality of the general product is relatively high, the brander achieves higher profits after introducing social channels; (2) when the commission rate is high and the quality of the high-level product falls within a specific threshold, the brander achieves the maximum profit under the single-product strategy; (3) when the proportion of fans is high, the profit of the social platform is higher than that of the traditional platform; (4) when the quality of high-level products is relatively low, the platform supply chain can achieve Pareto optimality by adopting the single-product configuration. This paper elucidates the strategic roles of product configuration and consumption subsidies in the context of social channel introduction, thereby providing a theoretical basis for branders' operational decisions and for the government's formulation of consumption subsidy policies.

**Keywords:** product configuration; Heterogeneous platforms; consumption subsidy; channel selection; Stackelberg game

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## 1. Introduction

With the rapid development of e-commerce, online retail channels have evolved into a diversified landscape. Traditional e-commerce platforms (such as Tmall and JD.com) have long dominated the market, supported by mature supply chain systems, extensive user coverage, and comprehensive transaction guarantees. They serve as the primary destination for mainstream consumers and have become the preferred channel for most brands when entering the e-commerce sector. Meanwhile, driven by the rapid advancement of social media technologies, social e-commerce platforms centered on social relationship chains

(such as TikTok and Rednote) have risen rapidly, which leverage large fan bases to stimulate consumers' willingness to purchase through live streaming, short videos, and community interaction, creating distinctive consumption scenarios that differ from traditional search-based shopping. Such fan communities exhibit highly sticky consumption behavior rooted in their trust in the platform or content creators. Branders can leverage the spontaneous sharing and spread within these circles to reach many target consumers. The global social commerce market is projected to reach $688 billion in 2025, and driven by a 26% annual growth rate, the sector is forecast to surpass $1.2 trillion by 2028. China's market is expected to hit 3.5 trillion RMB, accounting for approximately 45% of the global total.

Faced with the traffic dividends of emerging social platforms, most branders choose to introduce social channels while retaining traditional ones to expand market coverage. However, dual-channel operations also introduce complex trade-offs regarding product configuration strategies for branders. On the one hand, branders may distribute identical general products across both platforms, simplifying operations, generating economies of scale, and reducing unit costs; however, homogeneous offerings intensify cross-platform price competition and erode overall profits. On the other hand, branders can adopt a differentiated-product configuration, offering products with distinct quality levels tailored to the characteristics of the two platforms. For example, targeting business-oriented users on JD.com who prioritize performance and service, Lenovo primarily promotes high-level configurations of the ThinkPad X1 Carbon series, which meet the demands of corporate clients and professionals for stable device performance. Meanwhile, on Douyin, which targets younger consumers focused on cost performance, Lenovo launches the lightweight version of the Xiaoxin Pro 14. Bundled with exclusive student discounts available in live streams, it precisely matches the budget and basic office needs of young users. However, although a differentiated strategy can better align with platform-specific user preferences and expand the potential market, it may also lead to higher research and production costs, as well as increased operational complexity. Therefore, optimizing product configuration in response to platform heterogeneity has become an urgent strategic issue for branders.

At the same time, in response to the prosperity of the e-commerce economy and the release of consumption potential, governments have introduced consumption subsidy policies to stimulate consumer demand and guide industrial upgrading. Local governments across China are frequently introducing policies such as e-commerce consumption vouchers and subsidies for the research and development of high-quality products. For example, during the Spring Festival of 2026, Jiangsu Province organized over 1100 digital RMB consumption promotion activities, issuing digital RMB consumption vouchers and red envelopes totaling approximately 36 million yuan. These activities covered more than 200,000 merchants, involved over 2 million participants, and drove nearly 500 million yuan in consumption. The digital consumption vouchers issued by Hangzhou achieved a consumption multiplier effect ranging from 3.5 to 5.8 within seven days, with robust results. Specifically, one yuan of government fiscal subsidy generated 3.5 yuan of new consumption, an increase solely induced by the vouchers. At the international level, the South Korean government officially launched applications for livelihood-oriented consumption vouchers on 21 July to stimulate grassroots consumption, boost domestic demand, and revitalize regional economic activity. These vouchers are available to all residents in South Korea, and each person can receive between 150,000 and 450,000 Korean won. While such subsidies stimulate consumption, they also exert a certain influence on branders' decisions regarding channel layout and product configuration.

Although extensive research has been conducted on channel selection, product configuration, and consumption subsidies, there remain three important research gaps:

(1) Most channel studies assume platform symmetry and consider only traditional e-commerce platforms, thereby overlooking the differentiated demand-driving mechanism formed by social platforms through fan interaction and trust transmission.
(2) The literature on product differentiation typically assumes homogeneous sales channels and overlooks how brands allocate their products across different channels.
(3) Few studies have examined how such subsidies affect branders' product configuration decisions, and even fewer have established a linkage between consumption subsidies and heterogeneous e-commerce platforms.

Therefore, this study attempts to answer the following questions: (1) What if the brander decides to introduce a social channel? (2) How should the brander select product configuration strategies and set product prices when faced with heterogeneous platforms? (3) How do consumption subsidies and product configuration influence the supply chain system and social welfare? To address the above issues and to comply with the objective requirement of sequential decision-making among the parties in reality (the government first determines the subsidy, and then the brander sets the prices), we construct a Stackelberg game model in this paper to conduct an analysis of the brander's channel selection and product configuration strategies. The theoretical and practical implications of this study are as follows: (1) This study extends vertical differentiation theory to the context of heterogeneous platforms (traditional and social platforms) and identifies the conditions under which the single-product and differentiated-product strategies are respectively optimal. The findings further offer a reference for designing product configuration schemes that enable cooperation between branders and heterogeneous platforms. (2) In practice, the brander typically distributes their products through multiple channels rather than being confined to a single platform. Therefore, this study provides theoretical and practical foundations for the brander's channel introduction strategies, as well as an analysis of channel competition balance and supply chain efficiency after channel introduction. (3) This paper focuses on the impact of consumption subsidy policies on product configuration choices and on competition between heterogeneous platforms. These findings refine the theoretical basis for designing subsidy policies and offer theoretical implications for consumption subsidy practices.

The remainder of the paper is organized as follows: the next section reviews the related literature; Section 3 describes the model and provides the assumptions; Sections 4 and 5 set up the model without and with consumption subsidies, respectively; Section 6 compares and discusses the influence of consumption subsidies; and Section 7 summarizes the conclusion of this research and discusses the limitations. All derivations are shown in Appendix A.

## 2. Literature Review

Our study primarily engages with three streams of the literature: (a) channel selection, (b) differentiation strategies, and (c) consumption subsidies. In this section, we will conduct a literature review of these three areas.

### *2.1. Channel Selection Strategies*

Research related to the channel selection primarily concentrates on the issues detailed below.

The dual-channel decision-making problem involving agency selling and direct selling has long attracted extensive attention in traditional supply chain research. Chu et al. [1] investigate the strategic interaction between the introduction of private labels by online retail platforms and manufacturers' choices between reselling and agency selling channels under different pricing strategies. Guan et al. [2] examine the impact of introducing channel

competition on suppliers' information disclosure strategies, showing that after introducing a direct sales channel, suppliers disclose information more frequently, but this also leads to a decline in their profits. Therefore, the introduction of a direct sales channel is not beneficial to suppliers. Luo et al. [3] investigate manufacturers' distribution channel decisions when selling products through agency or reselling channels on a hybrid platform, and explore the impact of product quality on these decisions. Zhang et al. [4] develop a three-tier game-theoretic analysis model involving the government, manufacturers, and online retail platforms to investigate the optimal dual-channel structure model for green supply chains. In addition, attention has been paid to channel strategy choices in various supply chain scenarios, such as online stores versus offline physical stores, and direct online sales versus distribution channels. Lu et al. [5] investigate a platform's sales mode selection strategy under four scenarios: agency selling alone, agency selling combined with reselling, agency selling combined with private labels, and reselling combined with agency selling under the introduction of private labels.

With the development of online retail platforms, issues related to channel competition and channel selection in different online retail platform scenarios have been extensively studied. For example, scenarios such as a manufacturer and two competing online retailers [6], a supplier with a traditional distribution channel and a competitive online platform [7], and channel expansion strategies for competing asymmetric platforms [8] have been considered. With the rapid development of the global platform economy, the online retail platform scenarios have become increasingly complex, making it imperative that research on channel decisions and contract selection in platform contexts accounts for the influence of supply chain competition factors. Zhou et al. [9] construct a supply chain consisting of a manufacturer, a traditional online retailer, and a social e-commerce retailer, and explore the manufacturer's optimal strategy choice. Qiu et al. [10] construct a dual-channel supply chain consisting of a traditional online retailer and an e-tailer, finding that as the price sensitivity of the e-tailer increases, customers' market sensitivity, both the e-tailer and the traditional online retailer, reduce their retail prices to offset the impact of declining sales.

Prior research mainly investigates channel choices between online and offline, or between direct selling and agency selling, while relying on the restrictive assumption of symmetric platforms with identical features, user bases, and market traits [4,6,7]. Such an assumption prevents these studies from capturing channel introduction and coordination strategies across heterogeneous platforms. Against this backdrop, we use the traditional e-commerce platform as the benchmark and examine whether a brander should add a social e-commerce platform as a second channel, allowing us to further analyze supply chain decision-making in a context with two heterogeneous platforms.

### 2.2. Differentiation Strategies

Research related to differentiation strategies primarily concentrates on the two aspects detailed below.

First, there is the issue of the degree of product differentiation. Geng et al. [11] discovered that the Market Expansion Effect (MEE) attenuates the quality disparities among products, whereas the Value Enhancement Effect (VEE) amplifies such differentiations. Wong et al. [12] conduct a study considering a scenario where some consumers exhibit a preference for high-quality products while others are solely price-sensitive. Their findings confirm that the common strategy adopted by manufacturers is conducive to rectifying the quality distortion issues within the low-valuation market segment and minimizing the efficiency losses in the distribution channel. Mandal and Jain [13] indicate that a supply chain practicing a product quality differentiation strategy can mitigate competition and promote consumption. Wang et al. [14] construct a two-period game model to analyze the

impact of product quality on the sales patterns of manufacturers, considering strategic customer behavior.

Second, some scholars have investigated distribution channel issues. Zhang et al. [15] contend that in the absence of the influence of potential alternative suppliers, the manufacturer's distribution strategy is contingent upon the product type. Chen et al. [16] state that in the presence of product quality variations, the introduction of new channels can enhance product quality and the performance of the supply chain system. Dai et al. [17] find that the reference quality effect has a negative influence on the manufacturer's profit, and the optimal channel selection remains indeterminate. Zheng et al. [18] examine the effect of product substitutability on the equilibrium profits in a distribution channel of two symmetric manufacturers selling substitutable products to consumers through a nonexclusive retailer.

Most existing studies only explore product configuration design under the assumption of homogeneous platforms, thus failing to combine product differentiation with platform heterogeneity and systematically examine how to match products to platform characteristics to achieve differentiation advantages [12,18]. Therefore, this paper introduces product configuration strategies into a heterogeneous platform environment. By matching product attributes with platform features, we offer theoretical support and decision guidelines for branders' product configuration decisions.

### 2.3. Consumption Subsidies

Research on consumption subsidy policies predominantly converges on two key aspects.

The first is research on the subsidy mechanism. For example, Cohen et al. [19] explore the issue of how the government utilizes fixed-amount subsidies for consumers purchasing green products. Chen et al. [20] analyze two types of subsidies employed by the government for supply chain participants: those for production efforts and for innovation endeavors. Chung et al. [21] address the question of how the government should combine two distinct subsidy types—technology-push subsidies, which motivate manufacturers' research and development efforts to cut costs, and demand-pull subsidies, which directly reward customers—when multiple firms are engaged in competitive scenarios with spillover effects. Bai et al. [22] conduct research on two subsidy policies—fixed trade-in subsidies and shared trade-in subsidies—and recommend that the government opt for shared subsidies. Guo et al. [23] analyze the issue of how to enhance the participation of stakeholders in adopting environmental practices in an appropriate platform supply chain environment by constructing a tripartite evolutionary game model involving the government, manufacturers, and consumers.

Secondly, there is research on subsidy recipients. Zeng et al. [24] analyze and develop four closed-loop supply chain models for recycling to explore how different government subsidy recipients influence recycling efficiency, profitability, and the overall supply chain architecture under diverse power structures. He et al. [25] examine the interplay between government subsidy arrangements and information asymmetry in relation to manufacturers' emission-reduction initiatives and retailers' promotion of low-carbon products. Zaman and Zaccour [26] determine the optimal scrapping subsidy level in the vehicle trade-in market by considering a two-stage game between strategic consumers and the government. Li et al. [27] explore whether the government should provide subsidies to producers or customers to stimulate corporate innovation investment under different power-dominant scenarios in the supply chain, finding that subsidizing customers is more conducive to promoting social welfare. Some scholars have also focused on government subsidies for the recycling and remanufacturing sectors. He et al. [28] consider the manufacturer's channel structure and pricing decisions for new and remanufactured products in a dual-channel closed-loop supply chain with government subsidies for manufacturers, along

with relevant government subsidy policies. Gao et al. [29] construct a dual-channel online sales system, which comprises a self-operated channel of an e-commerce firm and a green platform channel, and explore how government green subsidies affect the supply chain channel selection strategy of e-commerce enterprises.

While prior research has extensively examined subsidy formats and subsidy recipients, it overlooks the link between consumption subsidies and heterogeneous e-commerce platforms and their influence on branders' product configuration [26,28,29]. In contrast, we embed branders' product configuration decisions into a government consumption subsidy and dual platform setting and identify how consumption subsidies affect their channel and product configuration strategies. In doing so, we significantly extend the research scope of consumption subsidies. A detailed comparison of this paper with related studies is provided in Table 1.

**Table 1.** Comparison of this paper with related studies.

| Literature | Consumption Subsidy | Product Configuration | | Channel Selection | |
|---|---|---|---|---|---|
| | | Homogeneous | Heterogeneous | Single | Dual |
| Zhou et al. [9] | | √ | | | √ |
| Luo et al. [3] | | | √ | | √ |
| Zhang et al. [15] | | | √ | | √ |
| He et al. [28] | √ | √ | | | √ |
| Mandal and Jain [13] | | | √ | √ | |
| Li et al. [27] | √ | | | √ | |
| Dai et al. [17] | | | √ | | √ |
| Zhang et al. [4] | | √ | | | √ |
| Wang et al. [14] | | | √ | | √ |
| Present study | √ | √ | √ | √ | √ |

Note: √ indicates that the corresponding feature is considered in the study.

## 3. Problem Description and Assumptions

### *3.1. Problem Description*

In this paper, we construct a supply chain system consisting of a brander and a traditional e-commerce platform (referred to as the traditional platform). To further expand market demand, a social e-commerce platform (referred to as the social platform) is introduced based on the traditional platform. The two platforms allow the brander to sell products, and they charge a fraction $\lambda$, where $0 < \lambda < 1$. In practice, the brander usually produces similar products with different quality levels [30]; thus, the brander is considered to have two types of product configuration models in this study: the single-product strategy and the differentiated-product strategy. When the brander chooses the single-product strategy, they sell general products (product M) on both traditional platforms and social platforms. However, when they choose the differentiated-product strategy, there are two specific product configuration schemes: first, the brander sells high-level products (product H) on traditional platforms and low-level products (product L) on social platforms; second, they sell product H on traditional platforms and product L on social platforms. Therefore, four sales strategies are investigated in this paper, namely, strategy M, strategy AM, strategy AH, and strategy AL, thus summarizing the product configuration of a brander under dual platforms into four typical strategies, see Table 2. This categorization is a modeling simplification and theoretical abstraction based on platform heterogeneity and relative

product quality differences, and it does not imply that branders operate in only four modes in practice.

(1) Strategy M: The brander sells product M with quality level $e_M$ on the traditional platform, as shown in Figure 1a.
(2) Strategy AM: The brander sells product M with quality level $e_M$ on the traditional platform and the social platform, as shown in Figure 1b.
(3) Strategy AH: The brander sells product H with quality level $e_H$ on the traditional platform and product L with quality level $e_L$ on the social platform, as shown in Figure 1c.
(4) Strategy AL: The brander sells product L with quality level $e_L$ on the traditional platform and product H with quality level $e_H$ on the social platform, as shown in Figure 1d.

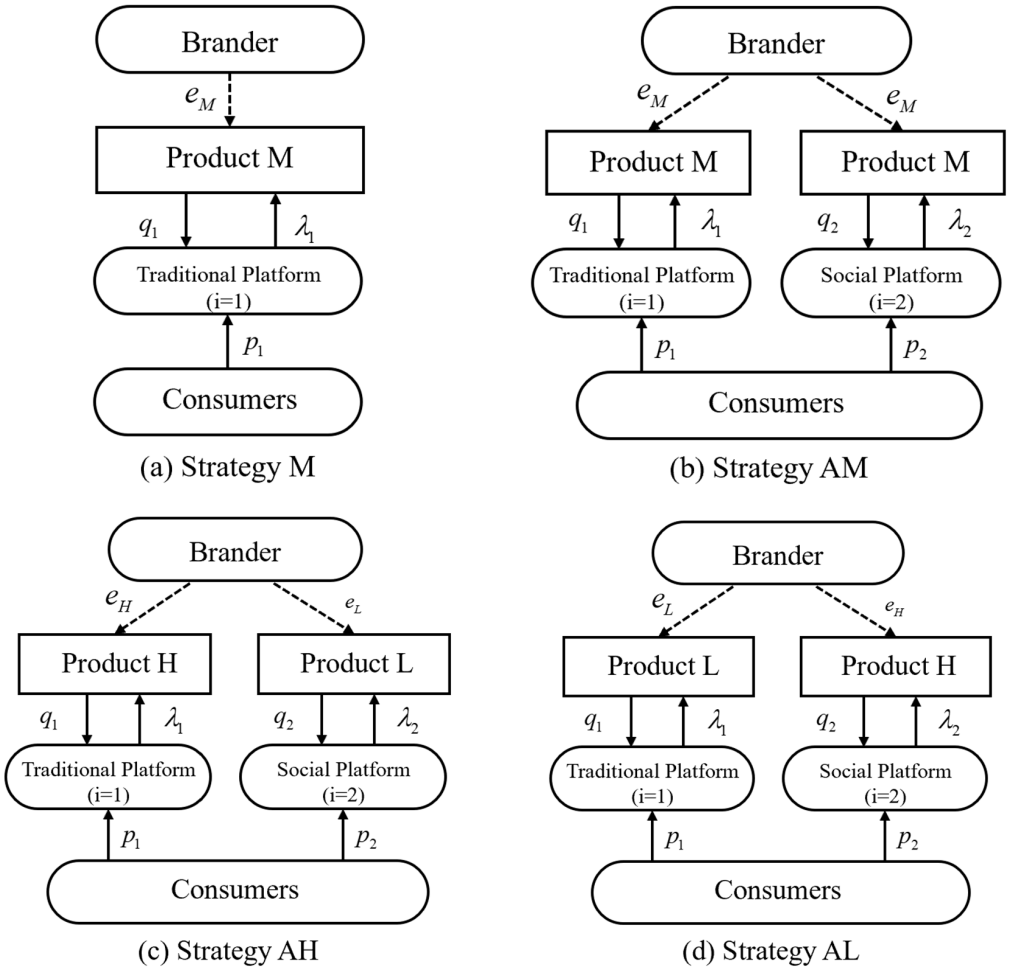



**Figure 1.** The structure of the supply chain.

**Table 2.** The brander's product configuration strategies and specific scenarios.

| | Strategy | Traditional Platform | Social Platform |
|---|---|---|---|
| Brander | M | Product M | — (Not introduced) |
| | AM | Product M | Product M |
| | AH | Product H | Product L |
| | AL | Product L | Product H |

### 3.2. Customer Demand

The brander's market demand functions on the traditional and social platforms are expressed as follows:

$$q_1 = a + (1 - r)mat - p_1 + \theta p_2 + \beta e_n (n = \mathrm{M}, \mathrm{H}, \mathrm{L}) \quad (1)$$

$$q_2 = rma + (1 - r)ma(1 - t) - p_2 + \theta p_1 + \beta e_n (n = \mathrm{M}, \mathrm{H}, \mathrm{L}) \quad (2)$$

In this study, the market demand of the traditional e-commerce channel is $a$; to simplify calculations, we adopt a dimensionless normalization procedure and reduce it to 1. $m$ denotes the ratio of the social channel's potential demand to that of the traditional channel, $ma$ represents the potential market demand of the social e-commerce channel, and $r$ represents the proportion of potential consumers among the fan users in the market demand of the social media channel ($0 < r < 1$). Therefore, $rma$ represents the potential market demand of fan consumers in the social media channel, and the non-fan consumers $(1 - r)ma$ are part of the potential market demand of the traditional e-commerce channel. $t$ denotes the spillover degree of the potential demand from ordinary users in the social channel to the potential demand in the traditional channel ($0 < t < 1$), and $t$ also reflects the degree of difference between fans and ordinary users. $p_i$ denotes the product prices of the brander's products on platform $i$ ($i = 1, 2$, where $i = 1$ represents the traditional platform and $i = 2$ represents the social platform). The price cross-coefficient $\theta$ represents the degree of price competition between competitive heterogeneous platforms ($0 < \theta < 1$). Given that the product quality of product M typically lies between that of product H and product L, it is assumed that $e_{\mathrm{L}} < e_{\mathrm{M}} < e_{\mathrm{H}}$. The product quality levels $e_{\mathrm{L}}, e_{\mathrm{M}}, e_{\mathrm{H}}$ defined in this paper do not represent absolute quality, but rather relative quality tiers. $\beta$ denotes consumers' preference for quality $e_n$; to fully concentrate consumer heterogeneity at the platform level, we normalize $\beta$ (set $\beta = 1$) [16,31]. Economically, a one-unit quality increase directly raises market demand by one unit. The production cost of the product is $e_n^2/2$ ($n = \mathrm{M}, \mathrm{H}, \mathrm{L}$) [32,33]. The government acts as a comprehensive coordinating body that balances environmental and social factors, and its utility function is given by $\mathrm{SW} = \pi_{\mathrm{b}} + \pi_1 + \pi_2 + CS - g(q_1 + q_2)$. In this study, the platforms are assumed to be domestic operators whose profits accrue to domestic residents. The game sequence is as follows: the brander determines the prices $p_i (i = 1, 2)$ of products sold by the platforms [34,35]. The basic notations are summarized in Table 3.

**Table 3.** Notation information.

| Notations | Descriptions |
| --- | --- |
| $i$ | $i = 1$ represents the traditional platform; $i = 2$ represents the social platform; $i = b$ represents the brander. |
| $a$ | The market demand of the traditional e-commerce channel. |
| $\beta$ | The consumers' preference for product quality. |
| $p_i$ | The selling prices of the products of platform $i$ $(i = 1, 2)$. |
| $q_i$ | The market demand of the brander's products on platform $i$ $(i = 1, 2)$. |
| $Q$ | The total market demand of the brander. |
| $\lambda$ | The commission rate charged by platform $i$ $(i = 1, 2)$. |
| $g$ | The consumption subsidy amount. |

**Table 3.** *Cont.*

| Notations | Descriptions |
|---|---|
| $e_n$ | The product quality of the products, where $n = M, H, \mathrm{L}$. |
| $\theta$ | The price cross-coefficient among competitive platforms. |
| $m$ | The latent market demand in social media channels. |
| $t$ | The spillover degree of the potential demand from ordinary users in the social channel to the potential demand in the traditional channel. |
| $r$ | The proportion of potential consumers who are fan users in the market demand of social media channels ($0 < r < 1$). |
| $\pi_b$ | The total profits achieved by the brander. |
| $\pi_i$ | The total profits obtained by platform $i$ $(i = 1, 2)$. |
| SW | The utility of the government. |

## 4. Model

### *4.1. Model Settings*

This section considers the model construction and derivation of equilibrium outcomes under two scenarios: the case where the brander introduces only the traditional channel, and the case where the brander introduces the social channel, leading to three product configuration strategies. We first consider the brander's optimal decisions under the strategy where they only use the traditional channel (strategy M); second, we analyze the optimal decisions after the social channel launch when they adopt the single-product strategy (strategy AM); then, we explore the brander's optimal decisions when they choose the differentiated-product strategies (strategy AH and strategy AL). The profit functions of the brander, the traditional platform, and the social platform are shown below.

#### 4.1.1. Strategy M

Under strategy M, the profit functions of the brander and the traditional platform are as follows:

$$\begin{cases} \pi_b^M = (1-\lambda)p_1q_1 - \frac{e_M^2}{2} \\ \pi_1^M = \lambda p_1 q_1 \end{cases}$$

Using the backward induction method, the optimal decisions of the brander can be obtained, as detailed in Proposition 1.

**Proposition 1.** *Under strategy M, the demand and product prices of the brander on the traditional platform are* $p_1^M = \frac{1+e_M}{2}$, *the profit of the brander is* $\pi_b^M = \frac{\left[-2{e_M}^2-(1+e_M)^2(\lambda-1)\right]}{4}$, *and the profit of the traditional platform is* $\pi_1^M = \frac{(1+e_M)^2\lambda}{4}$.

#### 4.1.2. Strategy AM

Under strategy AM, the profit functions of the brander, the social platform, and the traditional platform are as follows:

$$\begin{cases} \pi_b^{AM} = (1-\lambda)p_1q_1 + (1-\lambda)p_2q_2 - \frac{e_M^2}{2} \\ \pi_1^{AM} = \lambda p_1 q_1 \\ \pi_2^{AM} = \lambda p_2 q_2 \end{cases}$$

Using the backward induction method, the optimal decisions of the brander can be obtained, as detailed in Proposition 2.

**Proposition 2.** *Under strategy AM, the product prices of the brander on the traditional platform and the social platform are*

$$\begin{cases} p_1^{AM} = \frac{1+e_M(1+\theta)+m[(r-1)t(\theta-1)+\theta]}{2(1-\theta^2)} \\ p_2^{AM} = \frac{m+\theta+e_M(1+\theta)+mt(r-1+\theta-r\theta)}{2(1-\theta^2)} \end{cases}$$

*and the profit of the brander, the traditional platform, and the social platform is*

$$\begin{cases} \pi_b^{AM} = \frac{\left\{ \begin{array}{l} \left(m^2[2(r-1)t(1+rt-t)(\theta-1)-1]-1-2m[(r-1)t(\theta-1)+\theta]\right)(\lambda-1)+ \\ 2{e_M}^2(1+\theta)(\theta-\lambda)-2e_M(1+m)(1+\theta)(\lambda-1) \end{array} \right\}}{4(1-\theta^2)} \\ \pi_1^{AM} = \frac{[1+e_M+m(t-rt)][1+e_M(1+\theta)+(r-1)tm(\theta-1)+m\theta]\lambda}{4(1-\theta^2)} \\ \pi_2^{AM} = \frac{[e_M+m+m(r-1)t][m+\theta+e_M(1+\theta)+mt(r-1+\theta-r\theta)]\lambda}{4(1-\theta^2)} \end{cases}.$$

4.1.3. Strategy AH

Under strategy AH, the profit functions of the brander and the traditional platform are as follows:

$$\begin{cases} \pi_b^{AH} = (1-\lambda)p_1q_1 + (1-\lambda)p_2q_2 - (\frac{e_H^2}{2} + \frac{e_L^2}{2}) \\ \pi_1^{AH} = \lambda p_1q_1 \\ \pi_2^{AH} = \lambda p_2q_2 \end{cases}$$

Using the backward induction method, the optimal decisions of the brander can be obtained, as detailed in Proposition 3.

**Proposition 3.** *Under strategy AH, the product prices of the brander on the traditional platform and the social platform are*

$$\begin{cases} p_1^{AH} = \frac{1+e_H+e_L\theta+m[(r-1)t(\theta-1)+\theta]}{2(1-\theta^2)} \\ p_2^{AH} = \frac{e_L+m+(1+e_H)\theta+mt(-1+r+\theta-r\theta)}{2(1-\theta^2)} \end{cases},$$

*and the profits of the brander, the traditional platform, and the social platform are*

$$\begin{cases} \pi_b^{AH} = \frac{\left( \begin{array}{l} -2{e_H}^2-2{e_L}^2+\frac{1}{\theta^2-1}(1+e_H+mt-mrt)[1+e_H+e_L\theta+(r-1)tm(\theta-1)+m\theta](\lambda-1)+ \\ \frac{1}{\theta^2-1}(e_L+m+mrt-mt)[e_L+m+(1+e_H)\theta+mt(r-1+\theta-r\theta)](\lambda-1) \end{array} \right)}{4} \\ \pi_1^{AH} = \frac{(1+e_H+mt-mrt)(1+e_H+e_L\theta+(r-1)tm(\theta-1)+m\theta)\lambda}{4(1-\theta^2)} \\ \pi_2^{AH} = \frac{(e_L+m+mrt-mt)[e_L+m+(1+e_H)\theta+mt(-1+r+\theta-r\theta)]\lambda}{4(1-\theta^2)} \end{cases}.$$

4.1.4. Strategy AL

Under strategy AL, the profit functions of the brander and the traditional platform are as follows:

$$\begin{cases} \pi_b^{AL} = (1-\lambda)p_1q_1 + (1-\lambda)p_2q_2 - (\frac{e_H^2}{2} + \frac{e_L^2}{2}) \\ \pi_1^{AL} = \lambda p_1q_1 \\ \pi_2^{AL} = \lambda p_2q_2 \end{cases}.$$

Using the backward induction method, the optimal decisions of the brander can be obtained, as detailed in Proposition 4.

**Proposition 4.** *Under strategy AH, the product prices of the brander on the traditional platform and the social platform are*

$$\begin{cases} p_1^{AL} = \frac{1+e_L+e_H\theta+m[(r-1)t(\theta-1)+\theta]}{2(1-\theta^2)} \\ p_2^{AL} = \frac{e_H+m+(1+e_L)\theta+mt(r-1+\theta-r\theta)}{2(1-\theta^2)} \end{cases},$$

*and the profit of the brander, the traditional platform, and the social platform is*

$$\begin{cases} \pi_b^{AL} = \frac{\left( \begin{array}{l} -2{e_H}^2 - 2{e_L}^2 + \frac{1}{\theta^2-1}[1+e_L+m(t-rt)][1+e_L+e_H\theta+(r-1)tm(\theta-1)+m\theta](\lambda-1)+ \\ \frac{1}{\theta^2-1}[e_H+m+m(r-1)t][e_H+m+(1+e_L)\theta+mt(r-1+\theta-r\theta)](\lambda-1) \end{array} \right)}{4} \\ \pi_1^{AL} = \frac{[1+e_L+m(t-rt)][1+e_L+e_H\theta+(r-1)tm(\theta-1)+m\theta]\lambda}{4(1-\theta^2)} \\ \pi_2^{AL} = \frac{[e_H+m+m(r-1)t][e_H+m+(1+e_L)\theta+mt(-1+r+\theta-r\theta)]\lambda}{4(1-\theta^2)} \end{cases}.$$

*The complete proof can be found in Appendix A.*

**Lemma 1.** *The impacts of fan proportion $(r)$, the intensity of price competition between channels $(\theta)$, the potential demand of the social channel $(m)$, and the difference between fans and ordinary users $(t)$ on the retail prices, demand, and profit are shown in Tables 4 and 5.*

**Table 4.** Comparative statics analysis.

| | $q_1^{AM/AH/AL}$ | $q_2^{AM/AH/AL}$ | $p_1^{AM/AH/AL}$ | $p_2^{AM/AH/AL}$ | $\pi_1^{AM/AH/AL}$ | $\pi_2^{AM/AH/AL}$ | $\pi_b^{AM/AH/AL}$ |
|---|---|---|---|---|---|---|---|
| $r$ | ↓ | ↑ | ↓ | ↑ | ↓ | ↑ | ↓↑ |
| $\theta$ | — | — | ↑ | ↑ | ↑ | ↑ | ↑ |
| $m$ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ | ↑ |
| $t$ | ↑ | ↓ | ↑ | ↓ | ↑ | ↓ | ↓↑ |

Note: The symbol "—" implies that the optimal decision and demand are not influenced by the parameters. "↑" and "↓" denote that the optimal decision and demand increase and decrease, respectively, as the parameter increases.

As the proportion of fans increases, the traditional platform's demand and pricing, together with profit, decrease, while those of the social platform increase. This is because a higher fan proportion expands the loyal user base of the social platform, enhances its demand and pricing power, and thereby raises its profit. In contrast, the traditional platform suffers from user loss and intensified price competition, which leads to a corresponding decline in its profit. The brander's profit first decreases and then increases as the fan proportion rises for the following reasons: When r is low, an increase in the fan proportion gradually diverts some potential consumers away from the traditional platform, leading to a decline in its demand. At the same time, the fan base on the social platform remains limited, and the brander sets relatively low prices there. Consequently, the loss from the traditional channel outweighs the gain from the social channel, and the brander's total profit decreases. When r is high, fans have a stronger willingness to purchase at given prices, allowing the brander to raise prices on the social platform and achieve higher profits. Moreover, as the fan proportion continues to rise, the demand on the social platform expands; thus, the profit increase from the social channel begins to compensate for, and even surpass, the profit loss from the traditional channel, driving the brander's total profit upward.

The brander's pricing on both channels, as well as the profits of the platforms and the brander, increases with the intensity of price competition between channels. Platforms do not make pricing decisions; instead, product prices are set by the brander. Therefore, when platform competition intensifies, it is more profitable for the brander to raise prices on both platforms, thus preventing a price war between them. When prices rise simultaneously

on both platforms, consumers who comparison-shop across platforms cannot find lower prices elsewhere, which weakens their resistance to the price hike, allowing the brander to maintain a higher price level. Platform profits mainly come from commission income; an increase in prices directly expands transaction volume, which, in turn, leads to a simultaneous rise in commission income. When channel competition becomes intense, the brander should avoid being trapped in passive price wars; instead, they should actively coordinate prices across the two platforms and convert competitive pressure into overall profit growth through synchronized price increases.

**Table 5.** The computational results of strategies AM, AH, and AL.

| The computational results of strategy AM | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Baseline results | $\pi_1^{AM} = 0.143$ | | $p_1^{AM} = 1.429$ | | $q_1^{AM} = 1.000$ | | $\pi_2^{AM} = 0.143$ | | $p_2^{AM} = 1.429$ | | $q_2^{AM} = 1.000$ | | $\pi_b^{AM} = 2.446$ | |
| $r$ (0.4; 0.6) | 0.154 | 0.132 | 1.467 | 1.390 | 1.050 | 0.950 | 0.132 | 0.154 | 1.390 | 1.467 | 0.950 | 1.050 | 2.450 | 2.450 |
| $\theta$ (0.2; 0.4) | 0.125 | 0.167 | 1.250 | 1.667 | — | — | 0.125 | 0.167 | 1.250 | 1.667 | — | — | 2.125 | 2.875 |
| $m$ (1.5; 2.5) | 0.122 | 0.166 | 1.298 | 1.559 | 0.938 | 1.063 | 0.098 | 0.197 | 1.202 | 1.655 | 0.813 | 1.188 | 1.849 | 3.135 |
| $t$ (0.4; 0.6) | 0.132 | 0.154 | 1.390 | 1.467 | 0.950 | 1.050 | 0.154 | 0.132 | 1.467 | 1.390 | 1.050 | 0.950 | 2.450 | 2.450 |
| The computational results of strategy AH | | | | | | | | | | | | | | |
| Baseline results | $\pi_1^{AH} = 0.166$ | | $p_1^{AH} = 1.505$ | | $q_1^{AH} = 1.100$ | | $\pi_2^{AH} = 0.122$ | | $p_2^{AH} = 1.352$ | | $q_2^{AH} = 0.900$ | | $\pi_b^{AH} = 2.295$ | |
| $r$ (0.4; 0.6) | 0.178 | 0.154 | 1.544 | 1.467 | 1.150 | 0.950 | 0.112 | 0.132 | 1.313 | 1.390 | 0.850 | 0.950 | 2.313 | 2.285 |
| $\theta$ (0.2; 0.4) | 0.147 | 0.191 | 1.333 | 1.738 | — | — | 1.105 | 0.144 | 1.167 | 1.595 | — | — | 1.975 | 2.723 |
| $m$ (1.5; 2.5) | 0.143 | 0.190 | 1.375 | 1.636 | 1.038 | 1.163 | 0.080 | 0.172 | 1.125 | 1.578 | 0.713 | 1.088 | 1.715 | 2.966 |
| $t$ (0.4; 0.6) | 0.154 | 0.176 | 1.467 | 1.544 | 1.050 | 1.150 | 0.132 | 0.112 | 1.390 | 1.313 | 0.950 | 0.850 | 2.285 | 2.313 |
| The computational results of strategy AL | | | | | | | | | | | | | | |
| Baseline results | $\pi_1^{AL} = 0.122$ | | $p_1^{AL} = 1.352$ | | $q_1^{AL} = 0.900$ | | $\pi_2^{AL} = 0.166$ | | $p_2^{AL} = 1.505$ | | $q_2^{AL} = 1.100$ | | $\pi_b^{AL} = 2.295$ | |
| $r$ (0.4; 0.6) | 0.132 | 0.112 | 1.390 | 1.313 | 0.950 | 0.850 | 0.154 | 0.178 | 1.467 | 1.544 | 1.050 | 1.150 | 2.285 | 2.313 |
| $\theta$ (0.2; 0.4) | 0.105 | 0.144 | 1.167 | 1.595 | — | — | 0.147 | 0.191 | 1.333 | 1.738 | — | — | 1.975 | 2.723 |
| $m$ (1.5; 2.5) | 0.102 | 0.143 | 1.221 | 1.482 | 0.838 | 0.963 | 0.117 | 0.223 | 1.279 | 1.732 | 0.913 | 1.288 | 1.681 | 3.001 |
| $t$ (0.4; 0.6) | 0.112 | 0.132 | 1.313 | 1.390 | 0.850 | 0.950 | 0.178 | 0.154 | 1.544 | 1.467 | 1.150 | 1.050 | 2.313 | 2.285 |

The growth of potential demand in social channels has the following effects: the demand in both traditional and social channels increases because more consumers enter the purchase decision process through social recommendations; the retail prices in both channels increase because the expansion of demand scale means that the brander can fully capitalize on increased consumer interest to raise prices; the profits of both platforms and the brander increase. Therefore, when the potential demand in the social channel rises, the brander can moderately increase the retail prices in both channels and convert enhanced consumer interest and willingness to pay into tangible profits.

The higher the spillover degree of ordinary users from the social channel to the traditional channel, the higher the demand and product prices on the traditional platform, and the lower the demand and product prices on the social platform. A high spillover level indicates that ordinary users tend to choose the traditional channel, driving up its demand and prices; conversely, fewer ordinary users remain on the social channel, resulting in a decline in its demand and prices. With the increase in the spillover effect, the profits of the traditional platforms rise; moreover, the brander's profit first decreases and then increases for the following reasons: When the spillover effect is low, it mainly diverts non-fan users

from the social platform, reducing its demand, while the traditional platform gains only a limited benefit, leading to a profit loss. When the spillover effect is high, the demand on the traditional platform expands significantly, and its pricing power strengthens; meanwhile, the remaining users on the social platform are mostly loyal fans with high stickiness and stable profit contribution. As a result, the profit increment from the traditional channel outweighs the loss from the social channel, and the total profit recovers.

### *4.2. Equilibrium Analysis*

#### 4.2.1. Social Channel Selection Strategy

**Corollary 1.** *Comparison of the total market demand under strategy M and strategy AM.*

$$Q^{AM} > Q^{M};\ p_1^{AM} > p_1^{M}.$$

After the brander introduces the social channel, the corresponding market demand is higher than that under the traditional channel alone, as the social platform taps potential users and consumption scenarios that the traditional platform fails to cover. By leveraging social fission and word-of-mouth communication, it stimulates latent demand and expands the overall market size.

After introducing a social channel, the product prices in the traditional channel are higher than those when only the traditional channel exists, because the social channel primarily targets fans, while the traditional channel serves the general consumer base. When the proportion of fans is low, ordinary users tend to favor the positioning and image of the traditional platform when making cross-platform comparisons. As a result, the traditional channel does not need to lower prices to compete for customers and can instead maintain a higher pricing level.

**Corollary 2.** *Comparison of the brander's profits under strategy M and strategy AM.*

$$\pi_b^{AM} > \pi_b^{M}.$$

The brander achieves a higher profit after introducing the social platform than they do under the traditional platform alone because the dual channels jointly expand the sales scale. In addition, fans on the social platform exhibit a higher willingness to pay, so the brander can implement differentiated pricing; customer operations on the social platform reduce customer acquisition and marketing costs; and spontaneous word-of-mouth dissemination among fans replaces part of the advertising investment of the brander. Therefore, the brander should actively deploy the social e-commerce channel to maximize market coverage and profit.

#### 4.2.2. Comparison Under Strategies AM, AH, and AL

**Corollary 3.** *Comparison of demand and prices between the traditional and social platforms under the strategies AM, AH, and AL.*

*(1) When* $r > \widetilde{r}_1$, $q_2^{AM} > q_1^{AM}$; *when* $r > \widetilde{r}_2$, $q_2^{AH} > q_1^{AH}$; *when* $r > \widetilde{r}_3$, $q_2^{AL} > q_1^{AL}$.
*(2) When* $r > \widetilde{r}_1$, $p_2^{AM} > p_1^{AM}$; *when* $r > \widetilde{r}_2$, $p_2^{AH} > p_1^{AH}$; *when* $r > \widetilde{r}_3$, $p_2^{AL} > p_1^{AL}$.

The expressions of the thresholds $\widetilde{r}_1$, $\widetilde{r}_2$, and $\widetilde{r}_3$ are provided in detail in Appendix A.

When the proportion of fans is high, the demand on the social platform exceeds that on the traditional platform; this phenomenon is intuitive, as fans constitute the primary source of product demand on the social platform. Therefore, a higher proportion of fans leads to a larger market share for the social platform. Meanwhile, due to the inherent sharing and interactive features of the social platform, fan groups are encouraged to distribute content

and attract attention from ordinary users. Positive feedback from fans further amplifies the demand effect.

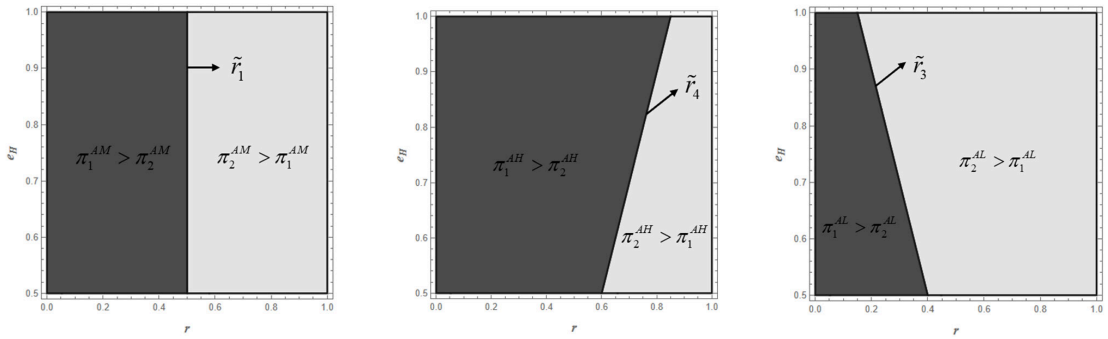
The high proportion of fans also results in the product pricing on the social platform being higher than that on the traditional one, since the core users of the social platform are mainly fans, who maintain high user stickiness and emotional attachment to the platform and whose consumption decisions are not primarily driven by cross-platform price comparisons. Therefore, the brander can leverage the characteristics of the fan group to raise product prices and achieve higher returns.

**Corollary 4.** *Comparison of profits between the traditional and social platforms under the strategies AM, AH, and AL.*

*When* $r > \widetilde{r}_1$, $\pi_2^{AM} > \pi_1^{AM}$; *when* $r > \widetilde{r}_4$, $\pi_2^{AH} > \pi_1^{AH}$; *and when* $r > \widetilde{r}_3$, $\pi_2^{AL} > \pi_1^{AL}$.

The expressions of the thresholds $\widetilde{r}_1$, $\widetilde{r}_3$, and $\widetilde{r}_4$ are provided in detail in Appendix A.

When the proportion of fans is high, the profit of the social platform is higher than that of the traditional one, as shown in Figure 2a–c. Combined with the above corollaries, when the proportion of fans is high, the demand on the social platform and the prices set by the brander on the social platform are both higher, resulting in the revenue of the social platform exceeding that of the traditional platform. In addition, the social platform enables targeted marketing by leveraging abundant social interaction data. For example, by analyzing users' browsing behavior, TikTok Shop can identify users' preferences, browsing paths, dwell time, and other key metrics, which helps the brander understand users' shopping preferences and needs. Accordingly, intelligent algorithms are adopted to recommend products aligned with user interests and demands, improving the user shopping experience and conversion rates. Therefore, social platforms should attach importance to the pivotal role of fans, leverage their inherent advantages to reduce costs and improve efficiency, and thereby capture excess returns. The relevant parameter settings for Figure 2 are specified as follows: $\lambda = 0.1, \theta = 0.3, e_L = 0.3; e_M = 0.5, t = 0.5, m = 2$.




(**a**) Strategy AM. (**b**) Strategy AH. (**c**) Strategy AL.

**Figure 2.** The impact of platform competition on equilibrium profits.

#### 4.2.3. Product Configuration Strategy

**Corollary 5.** *Comparison of prices and demand between strategy AM and strategies AH/AL.*

*(1) Strategy AM versus Strategy AH:*

$q_1^{AH} > q_1^{AM}$, $q_2^{AH} < q_2^{AM}$; *when* $e_L < e_M < \frac{e_H + e_L\theta}{1+\theta}$, $p_1^{AH} > p_1^{AM}$; *when* $e_L < e_M < \frac{e_L + e_H\theta}{1+\theta}$, $p_2^{AH} > p_2^{AM}$.

*(2) Strategy AM versus Strategy AL:*

$q_1^{AL} < q_1^{AM}$, $q_2^{AL} > q_2^{AM}$; *when* $e_L < e_M < \frac{e_H + e_L\theta}{1+\theta}$, $p_2^{AL} > p_2^{AM}$; *when* $e_L < e_M < \frac{e_L + e_H\theta}{1+\theta}$, $p_1^{AL} > p_1^{AM}$.

When a platform sells product H (L), its demand is higher (lower) than that when it sells product M. Compared with the general product M with moderate quality, product H features distinct, differentiated selling points and easily attracts consumers' attention through content display and performance presentation on the platform, thereby further expanding market demand. Moreover, offering product H tends to enhance the platform's overall quality image, attracting more quality-sensitive users to browse and make purchases. Even if these users eventually buy other products, the overall traffic and demand of the platform will increase accordingly.

When the quality of the general product M is relatively low, the prices on both traditional and social platforms under the differentiated-product strategy are higher than those under the single-product strategy, because under the latter, both platforms sell the identical general product M. When it is low quality, the product lacks differentiated selling points, and consumers make their choices mainly based on prices; thus, to compete for limited demand, platforms will attract users by issuing coupons and other means. Therefore, when the quality of the general product is low, the brander should adopt a differentiated-product strategy to create room for prices to increase.

**Corollary 6.** *Comparisons of optimal profits for the brander and platforms.*

*(1) The profit of the traditional platform is as follows: when* $e_H > \Delta e_H^2$, $\pi_1^{AH} > \max\{\pi_1^{AM}, \pi_1^{AL}\}$.

*(2) The profit of the social platform is as follows: when* $e_H > \Delta e_H^4$, $\pi_2^{AL} > \max\{\pi_2^{AM}, \pi_2^{AH}\}$.

*(3) The profit of the brander is as follows: when* $\lambda > 2\theta^2 - 1$, *if* $\max\{\Delta e_H^5, \Delta e_H^7, 0\} < e_H < \min\{\Delta e_H^6, \Delta e_H^8, 1\}$, *then* $\pi_b^{AM} > \max\{\pi_2^{AH}, \pi_2^{AL}\}$.

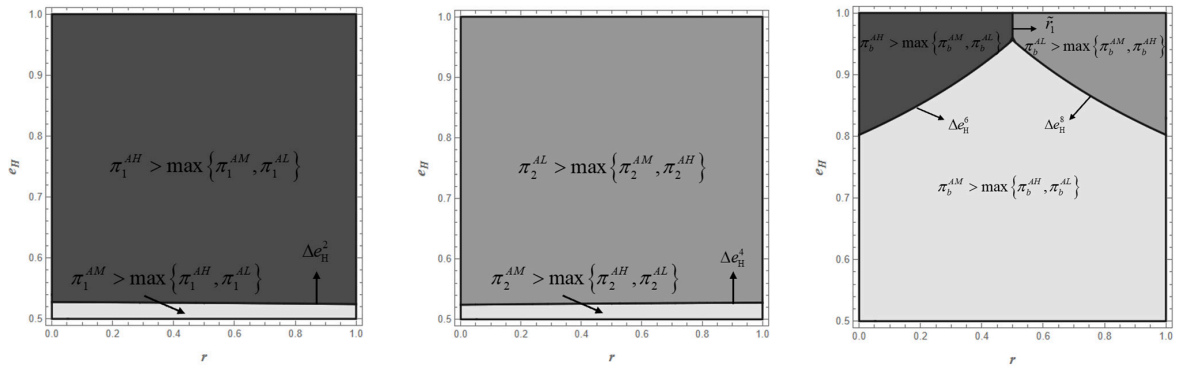
The expressions of the thresholds $\Delta e_H^2$, $\Delta e_H^4$, $\Delta e_H^5$, $\Delta e_H^6$, $\Delta e_H^7$, and $\Delta e_H^8$ are provided in detail in Appendix A.

It can be derived from Corollary 6 (1) and (2) that when the quality level of product H is high, the platform achieves the highest revenue by selling product H, as illustrated in Figure 3a,b. Essentially, product H, whose quality exceeds the threshold, can meet the demand for consumption upgrading. Particularly on social platforms, word-of-mouth dissemination among users amplifies the multiplier effect of the fan economy. The presence of product H sends a high-level signal to consumers, reducing their decision-making costs and boosting repurchase rates and customer loyalty, thereby consolidating the foundation for the platform's long-term profits. In addition, platforms generate revenue from commissions and bear no product production costs.

When the commission rate is high, and the quality level of product H is within a specific threshold ($\max\{\Delta e_H^5, \Delta e_H^7\} < e_H < \min\{\Delta e_H^6, \Delta e_H^8\}$), the brander maximizes their profits under strategy AM. Currently, the quality of product H is neither markedly high nor significantly below its lower bound, and the quality difference between product H and product M is insufficient to support a noticeable differentiated price premium; meanwhile, a high commission rate also poses challenges to the brander. Therefore, adopting strategy AM enables them to control costs and cover the largest user base, thereby achieving profit maximization. As demonstrated in Figure 3, for the platform supply chain, adopting the single-product strategy can achieve Pareto optimality when the quality of product H is low. The relevant parameter settings in Figure 3 remain the same as those above.

**Observation 1.** *We fix the fan proportion to investigate the impact of changes in the commission rate and set it to* $r = 0.4$ *and* $r = 0.6$, *respectively, to verify the conclusions of Corollary 6. The research observation reveals that under a given combination of commission rate and fan proportion,*

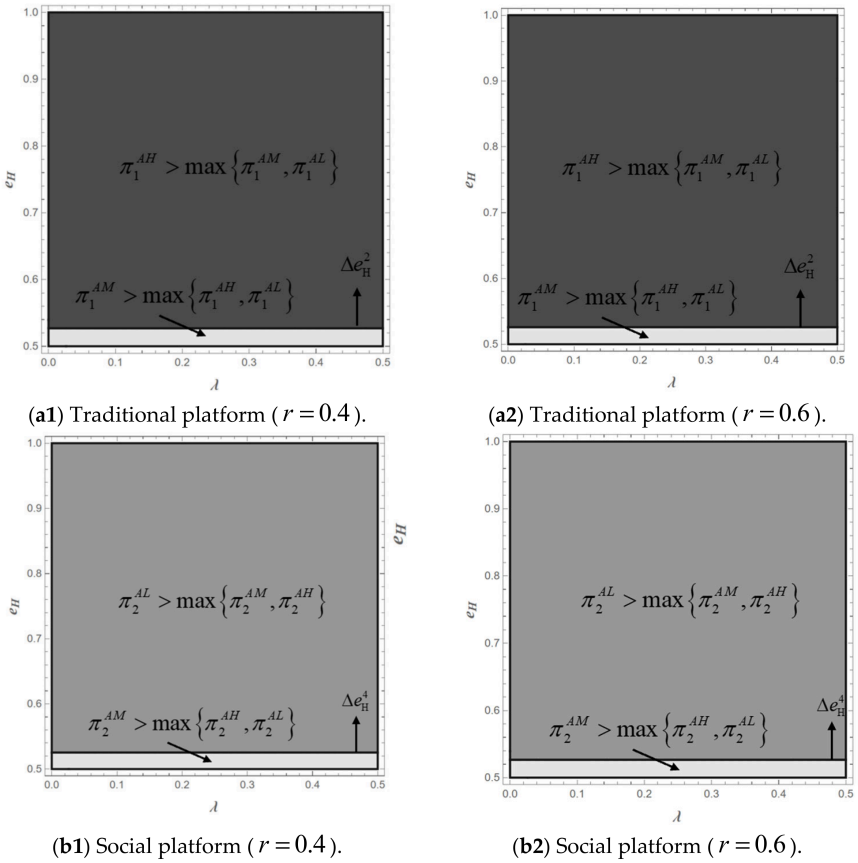
*and when the prerequisites of Corollary 6 are satisfied, the optimal strategy is AH for the traditional platform, AL for the social platform, and AM for the brander.*




(**a**) Traditional platform. (**b**) Social platform. (**c**) Brander.

**Figure 3.** The optimal strategy selection for platform retail systems.

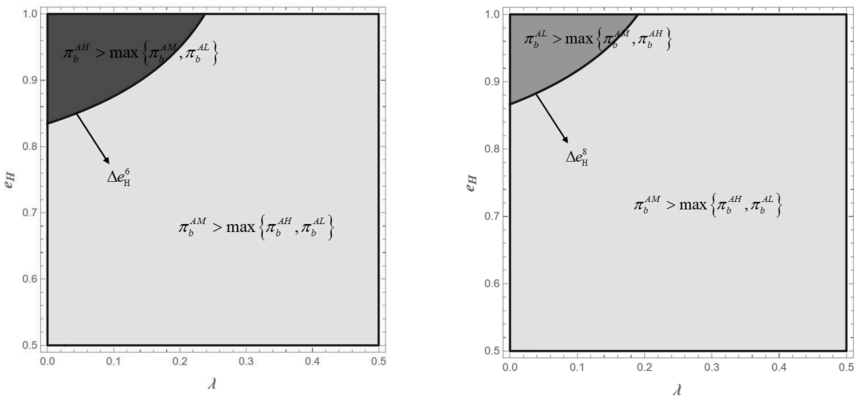
This result is consistent with Corollary 6: when the quality level of product H is high, the platform achieves the highest revenue by selling product H; under the condition of Corollary 6, the brander's optimal strategy is the single-product model, as shown in Figure 4. This verifies the good robustness of the research conclusions.




(**a1**) Traditional platform ( $r = 0.4$ ). (**a2**) Traditional platform ( $r = 0.6$ ).

(**b1**) Social platform ( $r = 0.4$ ). (**b2**) Social platform ( $r = 0.6$ ).

**Figure 4.** *Cont.*




(**c1**) Brander ( $r = 0.4$ ). (**c2**) Brander ( $r = 0.6$ ).

**Figure 4.** The optimal strategy selection for platform retail systems with varying fan proportion.

## 5. The Case of Consumption Subsidies

This section considers the model construction and derivation of equilibrium outcomes under government consumption subsidies. First, we consider the optimal decisions of the government, platforms, and the brander when only the traditional channel is used (strategy MS); second, we analyze the optimal decisions of both parties after the social channel is introduced and the brander adopts the single-product strategy (strategy AMS); next, we explore the optimal decisions of all relevant parties when the brander chooses the differentiated-product strategies (strategy AHS and strategy ALS).

Furthermore, the fact that the government may provide promotional subsidies for consumers is considered. Under different product configuration strategies, the decision-making order of the government, brander, and platforms is as follows: Firstly, as the leader of the game, the government determines the subsidy amount $g$ to maximize social welfare, based on its anticipation of the strategic responses of the brander. Secondly, branders decide on the selling prices $p_i (i = 1, 2)$ of products on traditional platforms and social platforms. The demand functions of traditional platforms and social platforms are as follows [19,26]:

$$q_1 = 1 + (1 - r)mt - p_1 + \theta p_2 + e_n + g(n = \mathrm{M}, \mathrm{H}, \mathrm{L}) \tag{3}$$

$$q_2 = rm + (1 - r)m(1 - t) - p_2 + \theta p_1 + e_n + g(n = \mathrm{M}, \mathrm{H}, \mathrm{L}) \tag{4}$$

The profit functions of the brander, the traditional platform, and the social platform under consumption subsidies are similar to those presented in Section 3.1.

Employing the backward induction method, the optimal decisions of the government and brander can be derived. The complete equilibrium solutions and corresponding proof derivations are presented in Appendix A.

The consumer surplus and the government's optimization problem can be formulated as follows:

$$CS = \frac{{q_1}^2 + {q_2}^2}{2} \tag{5}$$

$$\mathrm{SW} = \pi_\mathrm{b} + \pi_1 + \pi_2 + CS - g(q_1 + q_2) \tag{6}$$

Solving by means of the backward induction method, the equilibrium solutions under different product configuration strategies can be obtained. Based on this, the following proposition is derived.

**Assumption 1.** To ensure the strict concavity of the profit functions of the brander and the platforms, and thereby the existence and uniqueness of the Stackelberg equilibrium, we impose the following constraint on $\theta$:

$$0 < \theta < \frac{1}{3}$$

This bound is derived from the negative definiteness of the Hessian matrix of the brander's profit function (see Appendix A for details).

### *5.1. Equilibrium Analysis Under Consumption Subsidies*

**Corollary 7.** *Comparison of total market demand under strategies MS and AMS under consumption subsidies.*

$$Q^{AMS} > Q^{MS}. \text{ When } r < \widetilde{r}_5,\ p_1^{AMS} > p_1^{MS}.$$

The expression of the threshold $\widetilde{r}_5$ is provided in detail in Appendix A.

Under consumption subsidies, the market demand after adding a social channel still generates higher profits than the single-channel because consumption subsidies only generate a horizontal boost in market demand and corporate profits, without altering the demand spillover between the dual channels. Therefore, under consumption subsidies, the total demand available to the brander after introducing a social channel remains higher than that under the single-channel strategy.

Compared with Corollary 1, Corollary 7 shows a different trend. Under consumption subsidies, when the fan base is small, the prices in the traditional channel after introducing a social channel are higher than those when only the traditional channel exists, because in the presence of consumption subsidies, a smaller proportion of fans implies limited market share for the social channel and weaker price competition across channels. Meanwhile, subsidies are difficult to effectively pass through to the social channel and instead mostly support the profits of the traditional channel. Therefore, after introducing a social channel, the traditional channel can still maintain a higher price than that in a single traditional channel.

**Corollary 8.** *Comparison of profits under strategies MS and AMS under consumption subsidies.*

*When* $e_M \geq \frac{\sqrt{6}}{4}$, $\pi_b^{AMS} > \pi_b^{MS}$.

Under consumption subsidies, when the quality of product M is high, the brander obtains higher profits by introducing a social channel than by adopting the traditional single-channel strategy. On one hand, superior product quality can effectively enhance the market demand and consumers' willingness to pay, forming a positive incentive synergy with consumption subsidies, thereby further expanding the brander's profit margin. On the other hand, the dual-channel structure not only caters to consumers in the traditional channel who focus on product attributes but also attracts loyal fan groups with established brand trust through content-driven operations in the social channel. It fully taps potential consumer segments uncovered by the traditional channel, achieving an expansion of the overall market scale. Based on the above reasons, when the quality of the general product M is high, the total profit of the brander is significantly higher than that in the single-channel scenario, suggesting that the brander should introduce social channels in this case.

**Corollary 9.** *Comparison of demand and pricing between traditional and social platforms.*

*(1)* *When* $r > \widetilde{r}_1$, $q_2^{AMS} > q_1^{AMS}$; *when* $r > \widetilde{r}_2$, $q_2^{AHS} > q_1^{AHS}$; *when* $r > \widetilde{r}_3$, $q_2^{ALS} > q_1^{ALS}$.

*(2)* *When* $r > \widetilde{r}_1$, $p_2^{AMS} > p_1^{AMS}$; *when* $r > \widetilde{r}_2$, $p_2^{AHS} > p_1^{AHS}$; *when* $r > \widetilde{r}_3$, $p_2^{ALS} > p_1^{ALS}$.

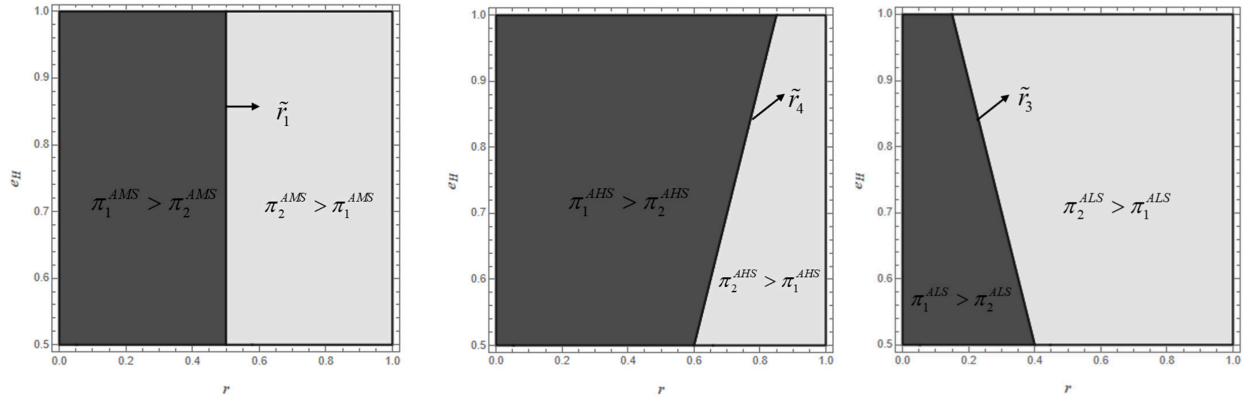
The expressions of the thresholds $\widetilde{r}_1$, $\widetilde{r}_2$, and $\widetilde{r}_3$ are provided in detail in Appendix A.

In the same strategy as consumption subsidies, the comparative results of demand and pricing between the two platforms are consistent with Corollary 3, meaning that consumption subsidies do not alter the trends of demand and pricing across the two platforms. When the proportion of fans is high, the following effects emerge: the demand and prices on the social platform exceed those on the traditional platform, as consumption subsidies only exert a positive incentive effect on overall market demand and profitability, without altering the channel preferences between fan and ordinary consumers or the cross-channel demand spillover effects; supported by fans' strong loyalty and high willingness to pay, the social platform attracts more consumers, thereby sustaining a higher equilibrium price. Therefore, even with consumption subsidies, the proportion of fans remains the core driving factor underlying the differences in demand and prices across channels.

**Corollary 10.** *Comparison of profits between traditional and social platforms.*
*When* $r > \tilde{r}_1$, $\pi_2^{AMS} > \pi_1^{AMS}$; *when* $r > \tilde{r}_4$, $\pi_2^{AHS} > \pi_1^{AHS}$; *when* $r > \tilde{r}_3$, $\pi_2^{ALS} > \pi_1^{ALS}$.

The expressions of the thresholds $\tilde{r}_1$, $\tilde{r}_3$, and $\tilde{r}_4$ are provided in detail in Appendix A.

When the proportion of fans is high, the profits of the social platform exceed those of the traditional platform, as shown in Figure 5. As indicated in Corollary 9, a higher proportion of fans leads to greater demand and higher product prices on the social platform relative to those on the traditional platform. Furthermore, platform profits are derived from commissions; therefore, driven by the dual advantages of higher demand and prices, the profitability of the social platform surpasses that of the traditional one.




(**a**) Strategy AMS (**b**) Strategy AHS (**c**) Strategy ALS

**Figure 5.** The impact of platform competition on equilibrium profits under consumption subsidies.

**Corollary 11.** *Comparison of demand and prices between strategy AMS and strategies AHS/ALS.*

*(1) Strategy AMS versus Strategy AHS:*

*When* $e_H > \frac{4e_M - e_L - e_L\theta - 4e_M\theta}{3-5\theta}$, $q_1^{AHS} > q_1^{AMS}$; *when* $e_H > \frac{4e_M - 3e_L - 4e_M\theta + 5e_L\theta}{\theta+1}$, $q_2^{AHS} > q_2^{AMS}$ *and* $q_1^{ALS} > q_1^{AMS}$.

*When* $e_H > \frac{e_L(1+5\theta) - 4e_M(1+\theta)}{\theta-3}$, $p_1^{AHS} > p_1^{AMS}$; *when* $e_H > \frac{e_L(\theta-3) + 4e_M(1+\theta)}{1+5\theta}$, $p_2^{AHS} > p_2^{AMS}$.

*(2) Strategy AMS versus Strategy AHS:*

*When* $e_H > \frac{4e_M - e_L - e_L\theta - 4e_M\theta}{3-5\theta}$, $q_2^{ALS} > q_2^{AMS}$; *when* $e_H > \frac{4e_M - 3e_L - 4e_M\theta + 5e_L\theta}{\theta+1}$, $q_1^{ALS} > q_1^{AMS}$.

*When* $e_H > \frac{e_L(1+5\theta) - 4e_M(1+\theta)}{\theta-3}$, $p_2^{ALS} > p_2^{AMS}$; *when* $e_H > \frac{e_L(\theta-3) + 4e_M(1+\theta)}{1+5\theta}$, $p_1^{ALS} > p_1^{AMS}$.

Under consumption subsidies, when the quality of product H is high, the demand and prices are higher when platforms sell differentiated products than those when they sell product M. This is because product H caters to the demands of consumers with high quality requirements, and its performance or brand image can stimulate consumers' stronger

willingness to purchase, meaning that even at a higher price point, the target consumer segment remains willing to pay a premium. Under the differentiation strategy, the products sold by one platform differ from those of the other; thus, consumers cannot directly compare the prices of identical products across platforms, which avoids price wars triggered by homogenization. Consequently, platforms can maintain higher pricing without losing customers. However, under the single-product strategy, both platforms market the same product; this homogeneity intensifies cross-platform price comparisons among consumers, and platforms reduce their prices accordingly, which ultimately lowers both demand and equilibrium pricing.

**Corollary 12.** *Comparisons of optimal profits for the brander and platforms.*

*(1) The profit of the traditional platform is as follows: when* $e_H > \Delta e_H^{10}$, $\pi_1^{AHS} > \max\{\pi_1^{AMS}, \pi_1^{ALS}\}$.

*(2) The profit of the social platform is as follows: when* $e_H > \Delta e_H^{12}$, $\pi_2^{ALS} > \max\{\pi_2^{AMS}, \pi_2^{AHS}\}$.

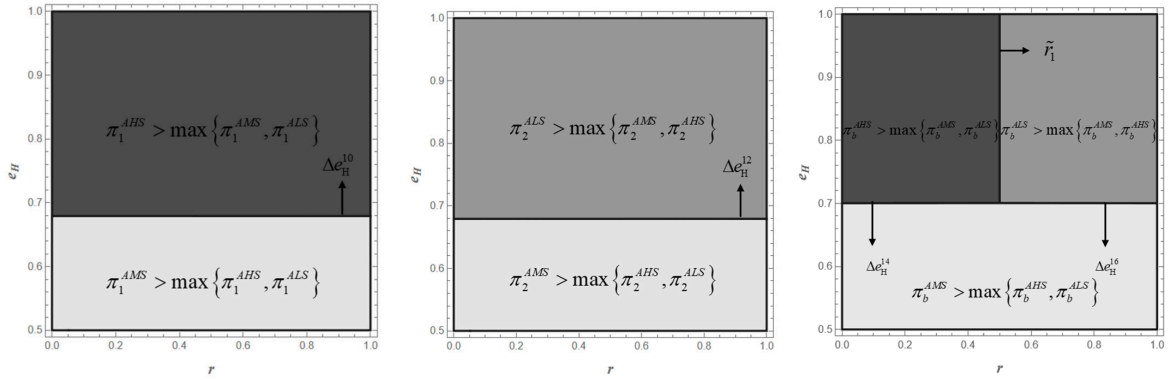
*(3) The profit of the brander is as follows: when* $\lambda < \frac{7\theta^2+36\theta^3-1-14\theta}{6\theta-5-5\theta^2}$, *if* $\max\{\Delta e_H^{13}, \Delta e_H^{15}, 0\} < e_H < \min\{\Delta e_H^{14}, \Delta e_H^{16}, 1\}$, *then* $\pi_b^{AMS} > \max\{\pi_b^{AHS}, \pi_b^{ALS}\}$.

The expressions of the thresholds $\Delta e_H^{10}$, $\Delta e_H^{12}$, $\Delta e_H^{13}$, $\Delta e_H^{14}$, $\Delta e_H^{15}$, and $\Delta e_H^{16}$ are provided in detail in Appendix A.

Under consumption subsidies, the optimal strategic choices for all participants align with Corollary 6. However, the threshold values for selecting optimal strategies differ: the threshold $\Delta e_H$ for platforms to adopt their optimal strategy increases, whereas the threshold $\Delta e_H$ for the brander to select their optimal strategy decreases, as shown in Figure 6. Only when the quality of product H is sufficiently elevated can the brand premium of differentiated products outweigh that of subsidized product M. Meanwhile, latent demand for product H is stimulated, enabling platforms to obtain higher commission revenues. For the brander, lower commission rates expand their profit margins, which partly offset the relatively high fixed costs associated with differentiated products. Meanwhile, the demand expansion induced by subsidies helps the brander spread their fixed costs more rapidly, so that even though product H does not achieve an extremely high quality level, scale effects can still sustain profit growth. Subsidies drive higher sales volumes, which partially offset cost pressures. When product quality is low, costs rise modestly while demand expands substantially; accordingly, the net returns under the differentiation strategy surpass those under the single-product strategy at an earlier threshold. Therefore, the brander faces a relaxed threshold requirement for the quality of product H and adopts the differentiated strategy at an earlier stage. It can be clearly observed that, under certain conditions, the single-product strategy achieves Pareto optimality. The parameter settings in Figure 6 remain consistent with those specified above.

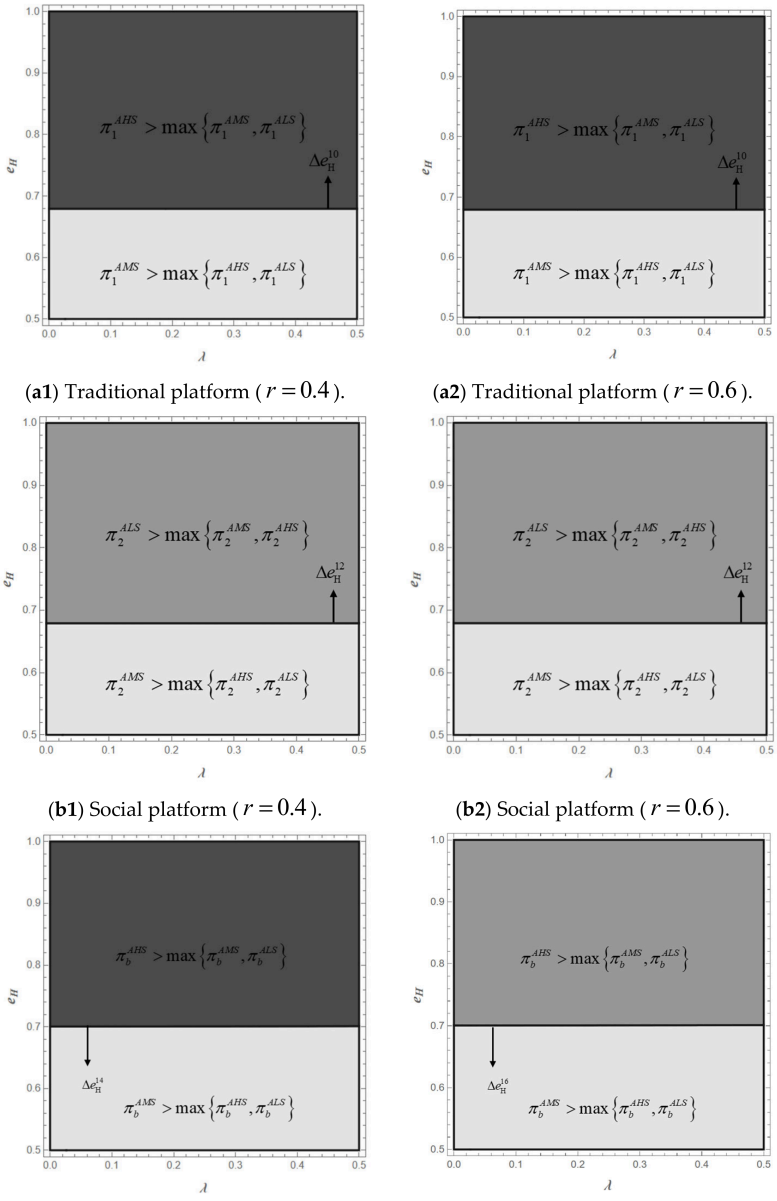
**Observation 2.** *We fix the fan proportion to investigate the impact of changes in the commission rate and set it to* $r = 0.4$ *and* $r = 0.6$, *respectively, to verify the conclusions of Corollary 12. It is observed that under the combination of commission rate and fan proportion, and subject to the prerequisites of Corollary 12, the optimal strategy is AHS for traditional platforms, ALS for social platforms, and AMS for branders. These findings are consistent with the results of Corollary 12.*

It can be observed that even with a different set of parameters, the optimal strategies of all participants remain unchanged under the same conditions, as shown in Figure 7, which verifies the good robustness of the research conclusions.




(**a**) Traditional platform. (**b**) Social platform. (**c**) Brander.

**Figure 6.** The optimal strategy selection for platform retail systems under consumption subsidies.




(**a1**) Traditional platform ( $r = 0.4$ ). (**a2**) Traditional platform ( $r = 0.6$ ).

(**b1**) Social platform ( $r = 0.4$ ). (**b2**) Social platform ( $r = 0.6$ ).

(**c1**) Brander ( $r = 0.4$ ). (**c2**) Brander ( $r = 0.6$ ).

**Figure 7.** The optimal strategy selection for platform retail systems under consumption subsidies with varying fan proportion.

**Corollary 13.** *Comparison of consumption subsidy amounts under different product configuration strategies.*

*When* $e_M < \frac{e_H+e_L}{2}$, $g^{BH} = g^{BL} > g^{BM}$.

When the average quality of differentiated products launched by the brander exceeds that of product M, the required consumption subsidy level becomes higher. In essence, under the differentiated-product strategy, consumers cannot accurately evaluate true product quality based on short-term market signals. Persistent information asymmetry prevails, which may trigger adverse selection and a lemon-market effect. Accordingly, consumer acceptance of product H remains constrained. To mitigate risks in the market introduction phase, governments provide higher subsidy levels, which lower consumers' entry barriers for trialing the product and signal to branders, through subsidy amounts, that the market risk of launching product H is reduced, thereby boosting their confidence to enter the market.

### *5.2. The Impacts of Consumption Subsidies*

**Corollary 14.** *Comparison of demand and prices with and without consumption subsidies.*

*(1)* $q_1^{AnS} > q_1^{An}$, $q_2^{AnS} > q_2^{An}$ $(n = M, H, L)$;
*(2)* $p_1^{AnS} > p_1^{An}$, $p_2^{AnS} > p_2^{An}$ $(n = M, H, L)$.

After a consumption subsidy, both the demand and product prices on each platform are consistently higher than those without a consumption subsidy. The subsidy provided by the government is directly granted to consumers, which is equivalent to increasing consumers' willingness to pay at every price level, as subsidies reduce the net prices actually paid by consumers, thereby stimulating a higher demand volume. Even if the brander moderately raises nominal prices, as long as the demand-expanding effect of the subsidy consistently dominates the demand-suppressing effect of the price increase, the equilibrium will feature a simultaneous rise in prices and demand, allowing the brander to further expand their profits by leveraging the subsidy policy. When designing subsidy policies, the government should consider the strategic responses of branders and platforms. This prevents subsidy effects from being partially offset by price increases and ensures that the policy genuinely benefits consumers.

**Corollary 15.** *Comparison of profits: with and without consumption subsidies.*

$$\pi_1^{AnS} > \pi_1^{An},\ \pi_2^{AnS} > \pi_2^{An}, \text{ and } \pi_b^{AnS} > \pi_b^{An}\ (n = M, H, L).$$

After the consumption subsidy, the profits of both platforms and the brander are consistently higher than those without the consumption subsidy. According to Corollary 15, both the market demand and product prices on each platform are consistently higher with consumption subsidies than in the baseline without subsidies. Since platform profits stem from commissions, higher prices and expanded demand correspondingly raise platform earnings. To maximize their profit, the brander will moderately raise the prices while keeping the demand higher than the level without the subsidy, thereby converting part of the subsidy into their own profit. Subsidies amplify demand, but long-term profitability depends on product competitiveness. The brander should continuously invest in research and development to ensure that it can maintain its market share after the subsidies are withdrawn.

**Corollary 16.** *Comparison of consumer surplus and social welfare with and without consumption subsidies.*

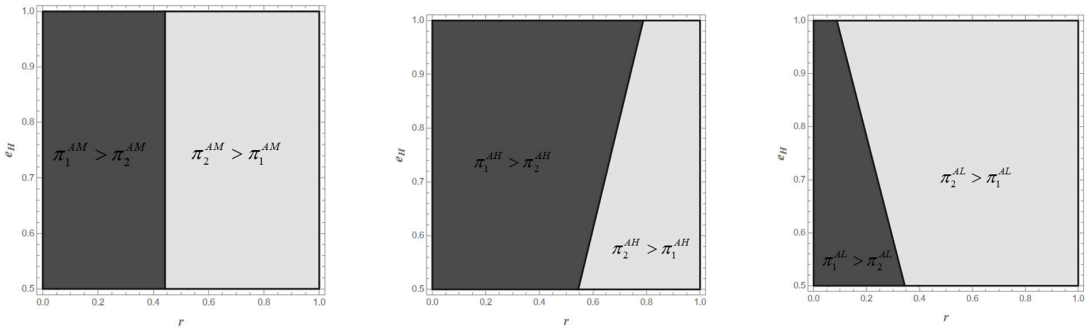
$$CS^{AnS} > CS^{An} \ (n = M, H, L);$$
$$SW^{AnS} > SW^{An} \ (n = M, H, L).$$

Consumer surplus after the consumption subsidy is higher than that without the consumption subsidy. Although the brander and platforms may capture part of the subsidy gains through strategic pricing, consumers remain one of the primary beneficiaries of the subsidy policy, as subsidies reduce the net prices they pay. This price reduction brings consumers who previously refrained from purchasing due to high prices into the market, while existing consumers may increase their purchase quantities, leading to demand expansion. However, policymakers still need to pay attention to subsidy efficiency and long-term impacts to ensure the coordinated enhancement of consumer welfare and market vitality. Social welfare under consumption subsidies is higher than that without subsidies. This is because these subsidies can directly stimulate consumer demand and increase market transaction volume; meanwhile, consumer surplus rises, the profits of the brander and platforms improve, and resource allocation is optimized. Therefore, the overall level of social welfare is higher than in the no-subsidy scenario.

## 6. Extension

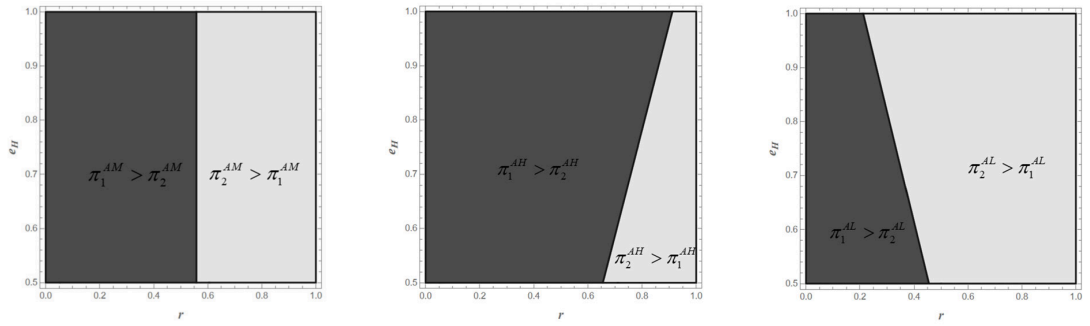
Commission rate is a relevant parameter that affects platform heterogeneity. This section further relaxes the assumption of equal commission rates and analyzes the model results under different commission rate scenarios to test the reliability of our core findings under variations in key exogenous parameters.

The results in Figures 8 and 9 indicate that changes in the commission rate do not alter the profit trends between the traditional and social platforms, consistent with the findings of Corollaries 4 and 10. We assign multiple sets of commission rates: in Figure 8a–c, the commission rate of the traditional platform $(\lambda_1 = 0.09)$ is set lower than that of the social platform $(\lambda_2 = 0.1)$; in Figure 8d–f, the commission rate of the traditional platform $(\lambda_1 = 0.1)$ is set higher than that of the social platform $(\lambda_2 = 0.09)$. A comparison of the results shows that the relative profit trends of the two platforms do not change fundamentally under either strategy, indicating that the level of the commission rate has no substantive impact on our core conclusions and further validating the robustness of the model. This is because the fan proportion is the key internal factor determining the relative profit levels of the two platforms. When the fan proportion is low, the traditional platform achieves higher profits by leveraging its basic market demand advantage. When the fan proportion exceeds a critical threshold, the traffic and user preference advantages of the social platform begin to dominate, allowing it to surpass the traditional platform in profit. The commission rate, as an exogenous revenue-sharing parameter, raises or lowers the absolute profit levels of both parties but does not alter the relative profit ranking determined by consumer groups. Figure 9 further tests the robustness of the model under varying commission rates in the context of government consumption subsidies, and the conclusions are consistent with the above analysis.




$\lambda_1 = 0.09, \lambda_2 = 0.1$

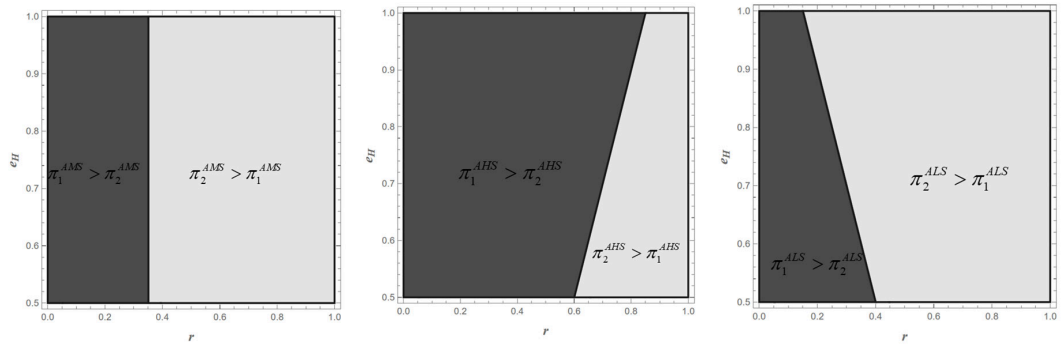
(**a**) Strategy AM. (**b**) Strategy AH. (**c**) Strategy AL.




$\lambda_1 = 0.1, \lambda_2 = 0.09$

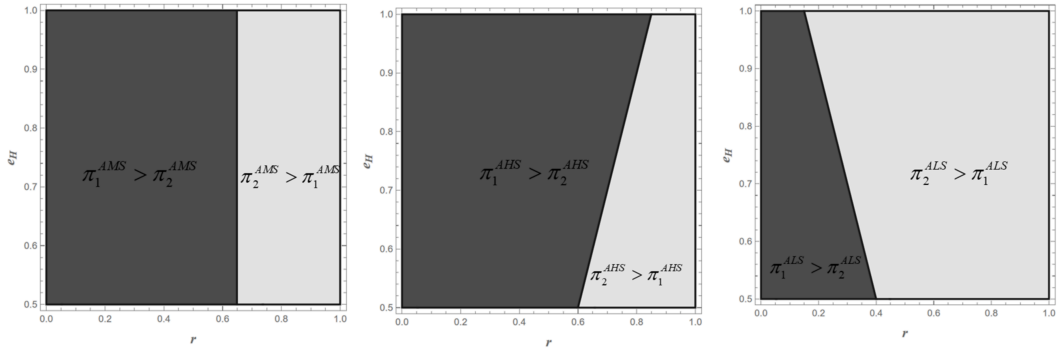
(**d**) Strategy AM. (**e**) Strategy AH. (**f**) Strategy AL.

**Figure 8.** Profit comparison between traditional and social platforms with different commission rates.




$\lambda_1 = 0.09, \lambda_2 = 0.1$

(**a**) Strategy AMS. (**b**) Strategy AHS. (**c**) Strategy ALS.

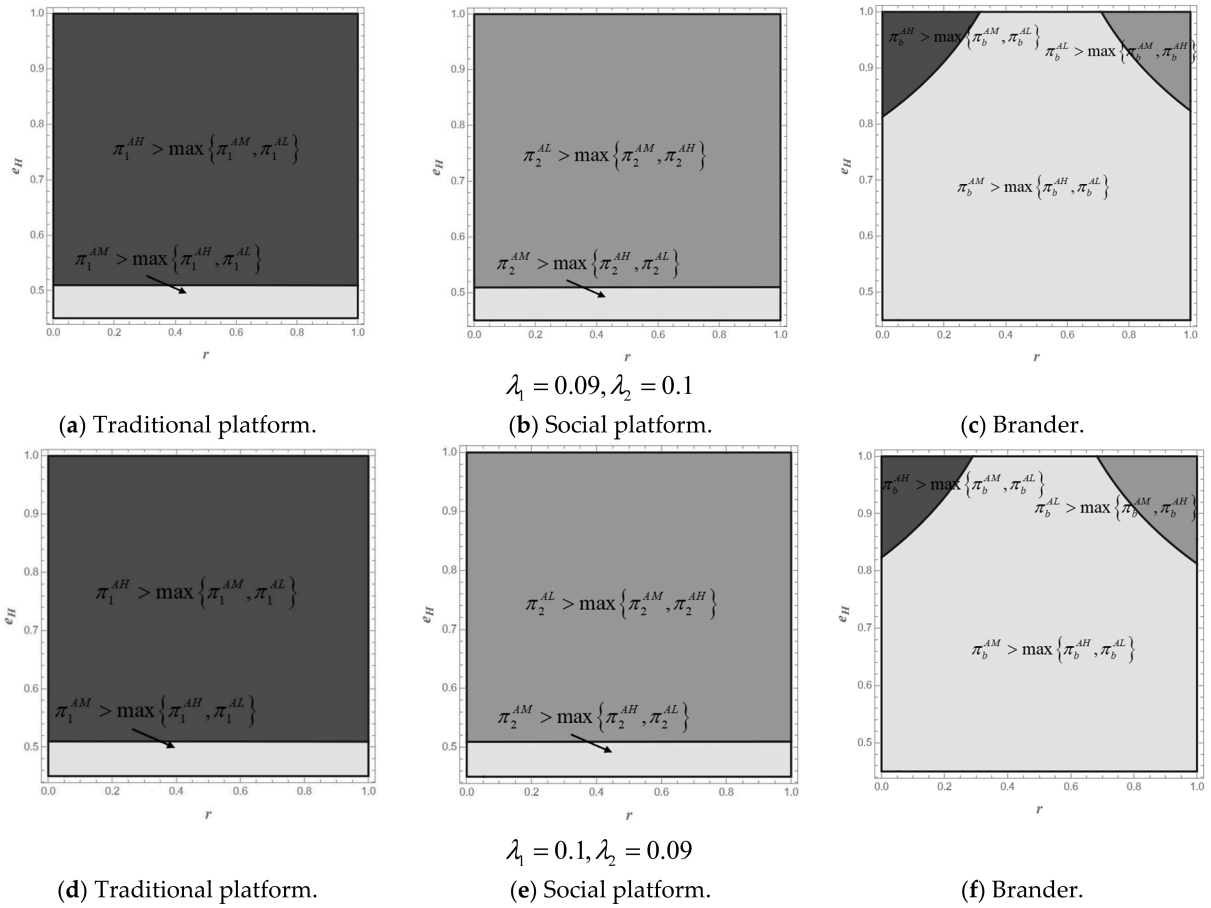



$\lambda_1 = 0.1, \lambda_2 = 0.09$

(**d**) Strategy AMS. (**e**) Strategy AHS. (**f**) Strategy ALS.

**Figure 9.** Profit comparison between traditional and social platforms with different commission rates under consumption subsidies.

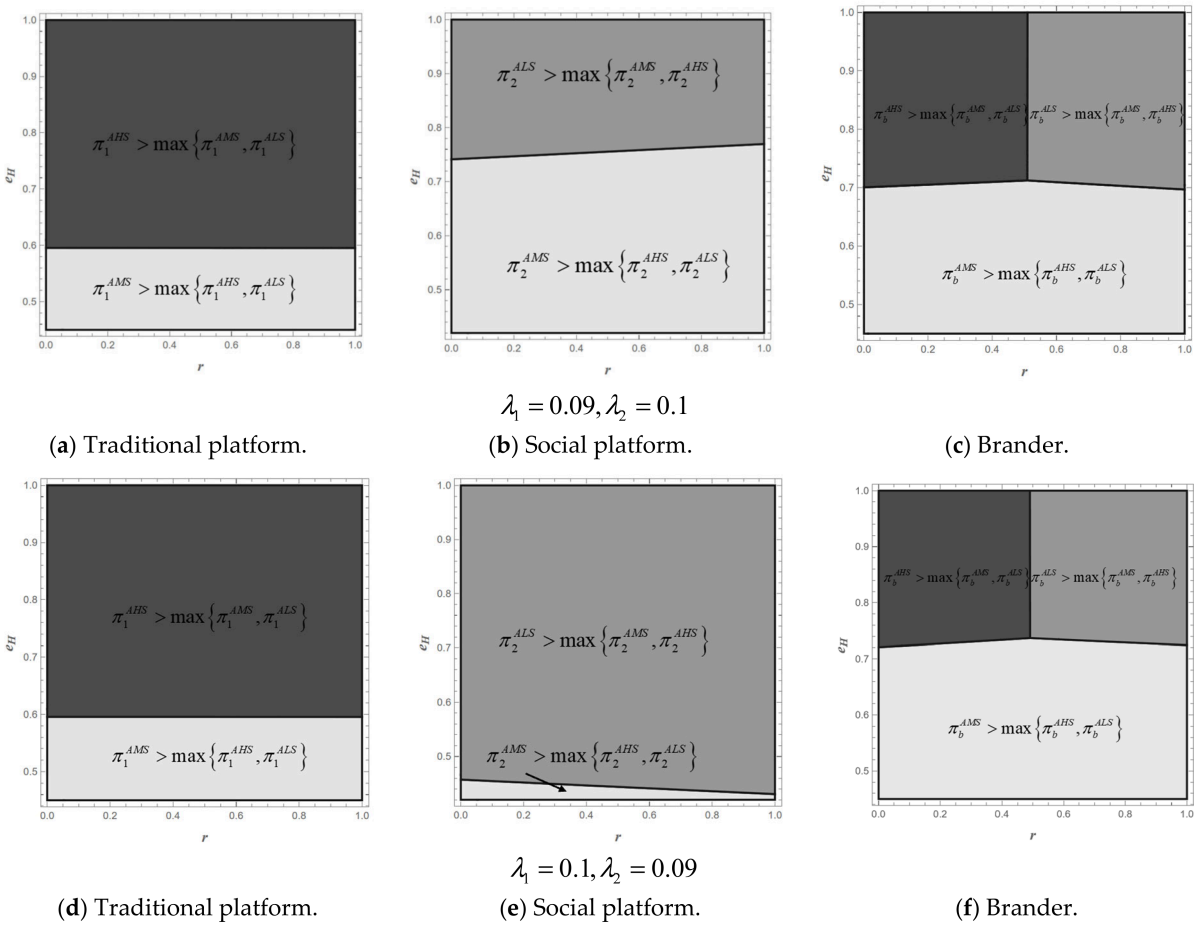
Furthermore, we relax the assumption of equal commission rates between the two platforms and re-examine the optimal strategy selection patterns of each party. As shown in Figures 10 and 11, the differentiated adjustment of commission rates does not alter the trends of the optimal strategies of the traditional platform, social platform, or brander, and the observed patterns are highly consistent with the core conclusions of Corollaries 6 and 11. We conduct simulation analyses with multiple sets of differentiated commission values: in Figure 8a–c, the commission rate of the traditional platform is set lower than that of the social platform; in Figure 8d–f, it is set higher. The simulation results show that under both asymmetric commission scenarios, the relative trends of the optimal strategies of all parties remain essentially unchanged, indicating that differences in commission rate levels do not undermine the core findings of this study and effectively validate the robustness of the model results. The fan proportion and the high-level product quality jointly determine which platform enjoys stronger market demand and whether it is necessary for the brander to adopt a differentiation strategy. In contrast, the commission rate only plays a profit-allocation role: it can adjust the absolute level of profit that each party obtains from transactions, but it cannot reverse the relative ranking of competitive advantages determined by fans and product quality. Furthermore, Figure 11 examines the impact of commission rate variations on the model's conclusions in the context of government consumption subsidies, and the results are consistent with the previous analysis.




(**a**) Traditional platform. (**b**) Social platform. (**c**) Brander.

(**d**) Traditional platform. (**e**) Social platform. (**f**) Brander.

**Figure 10.** The optimal strategy selection with different commission rates.




(**a**) Traditional platform. (**b**) Social platform. (**c**) Brander.

(**d**) Traditional platform. (**e**) Social platform. (**f**) Brander.

**Figure 11.** The optimal strategy selection with different commission rates under consumption subsidies.

## 7. Conclusions

### *7.1. Main Findings*

For the brander, the demand after accessing the social platform is higher than that under the traditional platform alone. When the quality of product M is relatively high, they achieve a higher profit by introducing the social channel. Furthermore, their channel pricing and profits rise with intensified cross-channel price competition. When the quality of general product M is low, the prices on both traditional and social platforms under the differentiated-product strategies exceed those under the single-product strategy. When the commission rate is high, and the quality level of product H falls within a specific threshold, the brander maximizes their profits under strategy AM. Moreover, after a consumption subsidy, both the product prices and profits of the brander are consistently higher than those without a consumption subsidy.

For platforms, profits increase with the intensity of inter-channel price competition. The demand for both traditional and social channels increases with the potential demand of the social channel. The higher the fan proportion, the lower the demand and pricing of the traditional platform, and the higher the demand and pricing of the social platform; nevertheless, the profits of both platforms increase. The greater the spillover effect of ordinary users in the social channel on the traditional channel, the higher the demand and product prices of the traditional platform, while those of the social platform decrease. When the fan proportion is high, the demand, prices, and profit of the social platform are higher than those of the traditional platform. When a platform sells product H (L), its demand is higher (lower) than when selling product M, and when the quality level of product H is high, the platform achieves the highest profit by selling product H. Under

consumption subsidies, the demand and prices of the platform when selling differentiated products are higher than when selling general products when the quality of product H is high; moreover, the demand and profit of each platform are consistently higher than those without consumption subsidies.

For the government, when the quality level of product H is low, choosing the single-product strategy in the platform supply chain can achieve Pareto optimality. Consumption subsidies do not alter the comparative trends in demand and prices between the two platforms. When the average quality level of the differentiated products introduced by the brander exceeds that of the general product, the amount of the consumption subsidy is higher. Consumer surplus and social welfare after a consumption subsidy are higher than those without a consumption subsidy.

### *7.2. Managerial Insights*

For the brander, when the quality of the general product is high, introducing a social channel is more conducive to increasing profits. Under the profit maximization objective, they can moderately raise prices while keeping demand higher than the level without subsidies, thereby converting part of the subsidy into their own profit.

For platforms, when the quality of product H is high, striving to secure the allocation of high-level products from the brander is more conducive to increasing profits. The traditional platform should strengthen it's ability to capture spillover users, while social platforms should strive to reduce spillovers to sustain their own demand and pricing.

For the government, when the fan base is small, direct subsidies to encourage social platform entry are ineffective. Policies should instead focus on building digital marketing, fan management, and channel capabilities; otherwise, subsidies merely translate into short-term price discounts. When the fan base is large, the marginal effect of subsidies diminishes, and they risk being captured by branders via price hikes. Policies should therefore shift from subsidizing entry to preventing subsidy dilution (e.g., requiring price transparency). Policymakers may link the subsidy intensity to the average quality level of the brander's product portfolio: when the average quality of its differentiated products exceeds the market benchmark, the subsidy rate can be moderately increased; otherwise, the subsidy rate remains at the baseline level. Regional heterogeneity also matters: in areas with strong traditional e-commerce but weak social channels, direct consumption subsidies are less effective; infrastructure and capacity building should take priority. In mature social commerce regions, policies can support brander upgrading and conversion onto social channels.

All recommendations are proposed under the assumptions of the model in this paper, and firms should assess whether to adopt them based on their own actual circumstances.

### *7.3. Limitations and Future Research*

This study explored the synergistic effects of a brander's channel introduction and product configuration strategies on supply chain profits. However, several limitations remain. In this paper, the proportion of fans is taken as the key characteristic to distinguish social platforms from traditional platforms. In future work, we will attempt to introduce variables of brand loyalty and parameters, such as platform promotion efforts and consumer preferences, to more accurately capture the role of these key factors in achieving supply chain equilibrium. Meanwhile, the cost structure in this paper is simplified, and future research will incorporate additional cost components. Secondly, in this paper, the focus is on the decision-making of the brander in the supply chain without considering the autonomous decisions of platforms as followers. Future research could extend the model to incorporate platform decisions into the analysis of the supply chain. In addition, we

will also consider extending the model to other scenarios, such as multi-SKU product configuration. Finally, mathematical modeling is conducted in this study based on a linear demand framework. In the future, empirical tests using real industry data or large-sample econometric analyses could be conducted to validate the theoretical propositions, thereby enhancing the practical relevance of the research.

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## Appendix A

**Proof of Proposition 1.** M Strategy: In this case, the objective function of the brander and the traditional platform is, respectively,

$$\pi_b^M = (1-\lambda)p_1q_1 - \frac{e_M^2}{2}$$

$$\pi_1^M = \lambda p_1 q_1$$

Since $\frac{\partial^2 \pi_b}{\partial p_1^2} = -2 + 2\lambda < 0$, $\pi_b^M$ is a concave function in $p_1^M$. Solving the first-order condition of the traditional platform's profit yields the following equilibrium decision

$$p_1^M = \frac{1+e_M}{2}$$

Substituting the optimal prices back into the relevant functions yields the optimal demand and profits for the brander and the platform. □

**Proof of Proposition 2.** AM Strategy: In this case, the objective functions of the brander, traditional platforms, and social platforms are

$$\pi_b^{AM} = (1-\lambda)p_1q_1 + (1-\lambda)p_2q_2 - \frac{e_M^2}{2}$$

$$\pi_1^{AM} = \lambda p_1 q_1$$

$$\pi_2^{AM} = \lambda p_2 q_2$$

The brander simultaneously determines the product prices $p_i^{AM}$ to maximize its own profit. The Hessian matrix of $\pi_b^{AM}$ with respect to $p_i^{AM}$ is given by

$$H = \begin{pmatrix} \frac{\partial^2 \pi_b}{\partial {p_1}^2} & \frac{\partial^2 \pi_b}{\partial p_1 p_2} \\ \frac{\partial^2 \pi_b}{\partial p_2 p_1} & \frac{\partial^2 \pi_b}{\partial {p_2}^2} \end{pmatrix} = \begin{pmatrix} 2(\lambda-1) & -2\theta(\lambda-1) \\ -2\theta(\lambda-1) & 2(\lambda-1) \end{pmatrix}$$

It is easy to verify that this matrix is negative definite. Solving the first-order optimality conditions yields the equilibrium prices $p_i^{AM}(i = 1, 2)$. Substituting the optimal prices back into the relevant functions yields the optimal demand and profits for the brander and the platform. □

**Proof of Proposition 3.** AH Strategy: The proof process is the same as above. □

**Proof of Proposition 4.** AL Strategy: The proof process is the same as above.

The consumer surplus formula $CS = \frac{{q_1}^2 + {q_2}^2}{2}$ is derived using the integration method under a linear demand system, with the detailed process as follows:

In the original model, the demand function without government subsidy is $\begin{cases} q_1 = 1 + (1-r)mt - p_1 + \theta p_2 + e_n \\ q_2 = rm + (1-r)m(1-t) - p_2 + \theta p_1 + e_n \end{cases}$. The cross partial derivatives satisfy $\frac{\partial q_1}{\partial p_2} = \frac{\partial q_2}{\partial p_1} = \theta$, and are strictly symmetric.

Thus, the Slutsky matrix is symmetric, and the line integral of consumer surplus is path independent. We will explicitly state this property in Appendix A: the integrability condition is satisfied.

The intercept term is $\alpha_1 = 1 + (1-r)mt + e_n$, $\alpha_2 = rm + (1-r)m(1-t) + e_n$

Therefore, $\begin{cases} q_1 = \alpha_1 - p_1 + \theta p_2 \\ q_2 = \alpha_2 - p_2 + \theta p_1 \end{cases}$

Then the demand system can be written as $q_1 = \alpha_1 - p_1 + \theta p_2$, $q_2 = \alpha_2 - p_2 + \theta p_1$

We adopt a stepwise price adjustment path to computing CS:

1. First, fix the social platform price at the baseline value $p_2^0$, and then calculate the change in CS when the traditional platform price is adjusted from $p_1^0$ to the equilibrium level $p_1^*$: $CS_1 = \int_0^{p_1^*} (\alpha_1 - p_1 + \theta * 0) dp_1 = \alpha_1 p_1^* - \frac{(p_1^*)^2}{2}$
2. Then, fix the traditional platform price at its equilibrium value $p_1^*$, and calculate the change in CS when the social platform price is adjusted from its baseline $p_2^0$ to its equilibrium level $p_2^*$: $CS_2 = \int_0^{p_2^*} (\alpha_2 - p_2 + \theta p_1^*) dp_2 = (\alpha_2 + \theta p_1^*) p_2^* - \frac{(p_2^*)^2}{2}$
3. The total consumer surplus is $CS = CS_1 + CS_2 = \alpha_1 p_1^* - \frac{(p_1^*)^2}{2} + (\alpha_2 + \theta p_1^*) p_2^* - \frac{(p_2^*)^2}{2}$, and this result is independent of the price adjustment path, which ensures the uniqueness and rigor of the calculation.

This is due to $\begin{cases} q_1 = \alpha_1 - p_1 + \theta p_2 \\ q_2 = \alpha_2 - p_2 + \theta p_1 \end{cases}$, that is, $\begin{cases} \alpha_1 = q_1 + p_1 - \theta p_2 \\ \alpha_2 = q_2 + p_2 - \theta p_1 \end{cases}$.

Substituting this into the CS calculation yields $CS = (q_1^* + p_1^* - \theta p_2^*) p_1^* - \frac{(p_1^*)^2}{2} + (q_2^* + p_2^*) p_2^* - \frac{(p_2^*)^2}{2}$.

By combining the equations $\begin{cases} q_1 = \alpha_1 - p_1 + \theta p_2 \\ q_2 = \alpha_2 - p_2 + \theta p_1 \end{cases}$, we obtain the price $\begin{cases} p_1^* = \frac{\alpha_1 - q_1^* + \theta(\alpha_2 - q_2^*)}{1-\theta^2} \\ p_2^* = \frac{\alpha_2 - q_2^* + \theta(\alpha_1 - q_1^*)}{1-\theta^2} \end{cases}$.

Substituting this into the CS calculation and simplifying yields $CS = \frac{{q_1^*}^2 + {q_2^*}^2}{2}$.

Under the assumed parameter constraints, all demand expressions are proven to be non-negative $(q_i^{An} > 0)$, ensuring the rationality and internal consistency of the model. □

**Lemma A1.**

*(1) Parameter t:*

$$\frac{\partial q_1^{AM}}{\partial t} = \frac{\partial q_1^{AH}}{\partial t} = \frac{\partial q_1^{AL}}{\partial t} = -\frac{1}{2}m(r-1) > 0; \frac{\partial q_2^{AM}}{\partial t} = \frac{\partial q_2^{AH}}{\partial t} = \frac{\partial q_2^{AL}}{\partial t} = \frac{1}{2}m(r-1) < 0.$$

$$\frac{\partial p_1^{AM}}{\partial t} = \frac{m-mr}{2+2\theta} > 0; \frac{\partial p_2^{AM}}{\partial t} = \frac{m(r-1)}{2(1+\theta)} < 0; \frac{\partial p_1^{AH}}{\partial t} = \frac{m-mr}{2+2\theta} > 0; \frac{\partial p_2^{AH}}{\partial t} = \frac{m(r-1)}{2(1+\theta)} < 0; \frac{\partial p_1^{AL}}{\partial m} = \frac{m-mr}{2+2\theta} > 0; \frac{\partial p_2^{AL}}{\partial t} = \frac{m(r-1)}{2(1+\theta)} < 0.$$

$$\frac{\partial \pi_1^{AM}}{\partial t} = \frac{m(r-1)(2+2e_M-\theta+m(2(r-1)t(\theta-1)+\theta))\lambda}{4(\theta^2-1)} > 0; \frac{\partial \pi_2^{AM}}{\partial t} = -\frac{m(r-1)(2e_M+\theta-m(-2+2(r-1)t(\theta-1)+\theta))\lambda}{4(\theta^2-1)} > 0; \frac{\partial \pi_b^{AM}}{\partial t} = -\frac{m(r-1)(-1+m+2m(r-1)t)(\lambda-1)}{2(1+\theta)}, \text{ when } t > \frac{1-m}{2m(r-1)}, \frac{\partial \pi_b^{AM}}{\partial t} > 0, \text{ and is the opposite otherwise. The profit differentiation methods under strategies AH and AL are the same as above and will not be repeated here; } \frac{\partial \pi_1^{AH}}{\partial t} > 0 \text{ and } \frac{\partial \pi_2^{AH}}{\partial t} > 0. \text{ When } t > \frac{e_H-e_L+1-m}{2m(r-1)}, \frac{\partial \pi_b^{AH}}{\partial t} > 0, \text{ and is the opposite otherwise; } \frac{\partial \pi_1^{AL}}{\partial t} > 0; \frac{\partial \pi_2^{AL}}{\partial t} > 0. \text{ When } t > \frac{e_L-e_H+1-m}{2m(r-1)}, \frac{\partial \pi_b^{AL}}{\partial t} > 0, \text{ and is the opposite otherwise.}$$

(2) *Parameter* $\theta$*:*

$$\frac{\partial p_1^{AM}}{\partial \theta} = \frac{1}{4}\left(\frac{1+2e_M+m}{(\theta-1)^2} + \frac{-1+m+2m(r-1)t}{(1+\theta)^2}\right) > 0; \frac{\partial p_2^{AM}}{\partial \theta} = \frac{1-m(r-1)t(\theta-1)^2+2m\theta+\theta^2+e_M(1+\theta)^2}{2(\theta^2-1)^2} > 0; \frac{\partial p_1^{AH}}{\partial \theta} = \frac{2(1+e_H)\theta+e_L\left(1+\theta^2\right)+m\left[1+(r-1)t(\theta-1)^2+\theta^2\right]}{2(\theta^2-1)^2} > 0; \frac{\partial p_2^{AH}}{\partial \theta} = \frac{1-m(r-1)t(\theta-1)^2+2e_L\theta+2m\theta+\theta^2+e_H\left(1+\theta^2\right)}{2(\theta^2-1)^2} > 0; \frac{\partial p_1^{AL}}{\partial \theta} = \frac{2(1+e_L)\theta+e_H\left(1+\theta^2\right)+m\left[1+(r-1)t(\theta-1)^2+\theta^2\right]}{2(\theta^2-1)^2} > 0; \frac{\partial p_2^{AL}}{\partial \theta} = \frac{1-m(r-1)t(\theta-1)^2+2e_H\theta+2m\theta+\theta^2+e_L\left(1+\theta^2\right)}{2(\theta^2-1)^2} > 0.$$

$$\frac{\partial \pi_1^{AM}}{\partial \theta} = \frac{(1+e_M+m(t-rt))\left(2\theta+e_M(1+\theta)^2+m\left(1+(-1+r)t(-1+\theta)^2+\theta^2\right)\right)\lambda}{4(\theta^2-1)^2} > 0;$$

$$\frac{\partial \pi_2^{AM}}{\partial \theta} = \frac{(e_M+m+m(r-1)t)\left(1-m(r-1)t(\theta-1)^2+2m\theta+\theta^2+e_M(1+\theta)^2\right)\lambda}{4(\theta^2-1)^2} > 0;$$

$$\frac{\partial \pi_b^{AM}}{\partial \theta} = -\frac{(1+e_M(1+\theta)+m((r-1)t(\theta-1)+\theta))(m+\theta+e_M(1+\theta)+mt(-1+r+\theta-r\theta))(\lambda-1)}{2(-1+\theta^2)^2} > 0;$$

*The profit differentiation methods under strategies AH and AL are the same as above and will not be repeated here:* $\frac{\partial \pi_1^{AH}}{\partial \theta} > 0; \frac{\partial \pi_2^{AH}}{\partial \theta} > 0; \frac{\partial \pi_b^{AH}}{\partial \theta} > 0; \frac{\partial \pi_1^{AL}}{\partial \theta} > 0; \frac{\partial \pi_2^{AL}}{\partial \theta} > 0; \frac{\partial \pi_b^{AL}}{\partial \theta} > 0.$

(3) *Parameter* $m$

$$\frac{\partial q_1^{AM}}{\partial m} = \frac{\partial q_1^{AH}}{\partial m} = \frac{\partial q_1^{AL}}{\partial m} = -\frac{1}{2}m(r-1) > 0; \frac{\partial q_2^{AM}}{\partial m} = \frac{\partial q_2^{AH}}{\partial m} = \frac{\partial q_2^{AL}}{\partial m} = \frac{1}{2}[1+(r-1)t] > 0;$$

$$\frac{\partial p_1^{AM}}{\partial m} = \frac{\partial p_1^{AH}}{\partial m} = \frac{\partial p_1^{AL}}{\partial m} = -\frac{(r-1)t(\theta-1)+\theta}{2(\theta^2-1)} > 0; \frac{\partial p_2^{AM}}{\partial m} = \frac{\partial p_2^{AH}}{\partial m} = \frac{\partial p_2^{AL}}{\partial m} = \frac{(r-1)t(\theta-1)-1}{2(\theta^2-1)} > 0;$$

$$\frac{\partial \pi_1^{AM}}{\partial m} = \frac{\left(2m(r-1)^2t^2(\theta-1)-(1+e_M)\theta+(r-1)t(2+2e_M-\theta+2m\theta)\right)\lambda}{4(\theta^2-1)} > 0;$$

$$\frac{\partial \pi_2^{AM}}{\partial m} = \frac{((1+(r-1)t)(2m(-1+(r-1)t(\theta-1))-\theta)\lambda-e_M(2+2(r-1)t+\theta)\lambda)}{4(\theta^2-1)} > 0;$$

$$\frac{\partial \pi_b^{AM}}{\partial m} = \frac{(m+(r-1)t(\theta-1)-2m(r-1)t(1+(r-1)t)(\theta-1)+\theta+e_M(1+\theta))}{2(\theta^2-1)}(\lambda-1) > 0;$$

*The profit differentiation methods under strategies AH and AL are the same as above and will not be repeated here:* $\frac{\partial \pi_1^{AH}}{\partial m} > 0; \frac{\partial \pi_2^{AH}}{\partial m} > 0; \frac{\partial \pi_b^{AH}}{\partial m} > 0; \frac{\partial \pi_1^{AL}}{\partial m} > 0; \frac{\partial \pi_2^{AL}}{\partial m} > 0; \frac{\partial \pi_b^{AL}}{\partial m} > 0.$

(4) *Parameter* $r$*:*

$$\frac{\partial q_1^{AM}}{\partial r} = \frac{\partial q_1^{AH}}{\partial r} = \frac{\partial q_1^{AL}}{\partial r} = -\frac{mt}{2} < 0; \frac{\partial q_2^{AM}}{\partial r} = \frac{\partial q_2^{AH}}{\partial r} = \frac{\partial q_2^{AL}}{\partial r} = \frac{mt}{2} > 0;$$

$$\frac{\partial p_1^{AM}}{\partial r} = \frac{\partial p_1^{AH}}{\partial r} = \frac{\partial p_1^{AL}}{\partial r} = -\frac{mt}{2+2\theta} < 0; \frac{\partial p_2^{AM}}{\partial r} = \frac{\partial p_2^{AH}}{\partial r} = \frac{\partial p_2^{AL}}{\partial r} = \frac{mt}{2+2\theta} > 0;$$

$$\frac{\partial \pi_1^{AM}}{\partial r} = \frac{mt(2+2e_M-\theta+m(2(r-1)t(\theta-1)+\theta))\lambda}{4(\theta^2-1)} > 0; \frac{\partial \pi_2^{AM}}{\partial r} = \frac{mt(-2e_M-\theta+m(-2+2(r-1)t(\theta-1)+\theta))\lambda}{4(\theta^2-1)} > 0;$$

$$\frac{\partial \pi_b^{AM}}{\partial r} = -\frac{mt(-1+m+2m(r-1)t)(\lambda-1)}{2(1+\theta)},$$

*when* $r > \frac{1-m}{2mt} + 1$, $\frac{\partial \pi_b^{AM}}{\partial r} > 0$, *and is the opposite otherwise. The profit differentiation methods under strategies AH and AL are the same as above and will not be repeated here:* $\frac{\partial \pi_1^{AH}}{\partial r} > 0; \frac{\partial \pi_2^{AH}}{\partial r} > 0.$ *When* $r > \frac{1+e_H-e_L-m}{2mt} + 1$, $\frac{\partial \pi_b^{AH}}{\partial r} > 0$, *and is the opposite otherwise;* $\frac{\partial \pi_1^{AL}}{\partial r} > 0; \frac{\partial \pi_2^{AL}}{\partial r} > 0.$ *When* $r > \frac{1-e_H+e_L-m}{2mt} + 1$, $\frac{\partial \pi_b^{AL}}{\partial r} > 0$, *and is the opposite otherwise.*

**Corollary A1.** *To compare the brander's demands after introducing the social channel, we take the difference.* $\left(q_1^{AM}+q_2^{AM}\right)-q_1^{M}=\frac{e_M+m}{2}>0$*; therefore,* $Q^{AM}>Q^{M}$

$$p_1^{AM}-p_1^{M}=-\frac{m((r-1)t(\theta-1)+\theta)+\theta(e_M+\theta+e_M\theta)}{2(\theta^2-1)}>0.\ \text{Therefore,}\ p_1^{AM}>p_1^{M}.$$

**Corollary A2.** *To compare the brander's profits after introducing the social channel, we take the difference.* $\pi_b^{AM}-\pi_b^{M}=\frac{\left(\begin{array}{l} m^2[1-2(r-1)t(1+rt-t)(\theta-1)]+\theta^2+ \\ {e_M}^2(1+\theta)^2+2e_M(1+\theta)(m+\theta)+2m[(r-1)t(\theta-1)+\theta]\end{array}\right)^{(\lambda-1)}}{4(\theta^2-1)}>0$*; therefore,* $\pi_b^{AM}>\pi_b^{M}$.

**Corollary A3.** *AM: To compare the magnitude of the platform's demands under each strategy, we take their difference.* $q_2^{AM}-q_1^{AM}=\frac{m-1+2m(r-1)t}{2}$*, when* $r>\frac{1-m}{2mt}+1$, $q_2^{AM}>q_1^{AM}$*. To compare the magnitude of the platform's product pricing under each strategy, we take the difference.* $p_2^{AM}-p_1^{AM}=\frac{-1+m+2m(r-1)t}{2(1+\theta)}$*; when* $r>\frac{1-m}{2mt}+1$, $p_2^{AM}>p_1^{AM}$.

AH: $q_2^{AH}-q_1^{AH}=\frac{(m-1-e_H+e_L)}{2}+m(r-1)t$; when $r>\frac{1-m+e_H-e_L}{2mt}+1$, $q_2^{AH}>q_1^{AH}$. $p_2^{AH}-p_1^{AH}=\frac{-1-e_H+e_L+m+2m(r-1)t}{2(1+\theta)}$; when $r>\frac{1+e_H-e_L-m}{2mt}+1$, $p_2^{AH}>p_1^{AH}$.

AL: $q_2^{AL}-q_1^{AL}=\frac{(m-1+e_H-e_L)}{2}+m(r-1)t$, when $r>\frac{1-m-e_H+e_L}{2mt}+1$, $q_2^{AL}>q_1^{AL}$. $p_2^{AL}-p_1^{AL}=\frac{-1+e_H-e_L+m+2m(r-1)t}{2(1+\theta)}$; when $r>\frac{1-e_H+e_L-m}{2mt}+1$, $p_2^{AL}>p_1^{AL}$.

**Corollary A4.** *AM: To compare the magnitude of the platform's profits under each strategy, we take their difference.* $\pi_2^{AM}-\pi_1^{AM}=-\frac{(1+2e_M+m)[m-1+2m(r-1)t]\lambda}{4(\theta^2-1)}$*; when* $r>\frac{1-m}{2mt}+1$, $\pi_2^{AM}>\pi_1^{AM}$.

*AH:* $\pi_2^{AH}-\pi_1^{AH}=\frac{(1+e_H+e_L+m)(1+e_H-e_L-m-2m(r-1)t)\lambda}{4(\theta^2-1)}$*; when* $r>\frac{-1-e_H+e_L+m}{2mt}+1$, $\pi_2^{AH}>\pi_1^{AH}$.

*AL:* $\pi_2^{AL}-\pi_1^{AL}=-\frac{(1+e_H+e_L+m)(-1+e_H-e_L+m+2m(r-1)t)\lambda}{4(\theta^2-1)}$*; when* $r>\frac{1-e_H+e_L-m}{2mt}+1$, $\pi_2^{AL}>\pi_1^{AL}$.

**Corollary A5.** *AM/AH: To compare the magnitude of demands under AM and AH, we take their difference.* $q_1^{AH}-q_1^{AM}=\frac{e_H-e_M}{2}>0$, $q_1^{AH}>q_1^{AM}$; $q_2^{AH}-q_2^{AM}=\frac{e_L-e_M}{2}<0$, $q_2^{AH}<q_2^{AM}$*. To compare the magnitude of product pricing under AM and AH, we take their difference.* $p_1^{AH}-p_1^{AM}=\frac{e_H+e_L\theta-e_M(1+\theta)}{2-2\theta^2}$*, and when* $e_L<e_M<\frac{e_H+e_L\theta}{1+\theta}$, $p_1^{AH}>p_1^{AM}$; $p_2^{AH}-p_2^{AM}=\frac{e_L+e_H\theta-e_M(1+\theta)}{2-2\theta^2}$*, and when* $e_L<e_M<\frac{e_L+e_H\theta}{1+\theta}$, $p_2^{AH}>p_2^{AM}$;

*AM/AL: To compare the magnitude of demands under AM and AL, we take their difference.* $q_1^{AL}-q_1^{AM}=\frac{e_L-e_M}{2}<0$, $q_1^{AL}<q_1^{AM}$; $q_2^{AL}-q_2^{AM}=\frac{e_H-e_M}{2}>0$, $q_2^{AL}>q_2^{AM}$; $p_1^{AL}-p_1^{AM}=\frac{e_L+e_H\theta-e_M(1+\theta)}{2-2\theta^2}$*, and when* $e_L<e_M<\frac{e_L+e_H\theta}{1+\theta}$, $p_1^{AL}>p_1^{AM}$; $p_2^{AL}-p_2^{AM}=\frac{e_H+e_L\theta-e_M(1+\theta)}{2-2\theta^2}$*, and when* $e_L<e_M<\frac{e_H+e_L\theta}{1+\theta}$, $p_2^{AL}>p_2^{AM}$.

**Corollary A6.** *The profit of the traditional platform:*

*To compare the magnitudes of the traditional platform's profits under the three strategies, we take their differences.* $\pi_1^{AL}-\pi_1^{AH}=\frac{(e_H-e_L)[2+e_H+e_L-\theta+m(2(r-1)t(\theta-1)+\theta)]\lambda}{4(\theta^2-1)}<0$, $\pi_1^{AL}<\pi_1^{AH}$; $\pi_1^{AH}-\pi_1^{AM}=\frac{\left(\begin{array}{l}\left[-e_L-e_He_L+e_M+{e_M}^2-e_Hm+e_Mm+(e_L-e_H)m(r-1)t\right]\theta- \\ \left[(e_H-e_M)(2+e_H+e_M-2m(r-1)t)\right]\end{array}\right)^{\lambda}}{4(\theta^2-1)}=f(e_H)$, where $f(e_H)$ *is a quadratic function opening upward, with the two roots* $\Delta e_H^1$, *and* $\Delta e_H^2$,

*and* $\Delta e_H^1 < \Delta e_H^2$. *Therefore, when* $e_H > \Delta e_H^2$, $\pi_1^{AH} > \pi_1^{AM}$. *In summary, when* $e_H > \Delta e_H^2$, $\pi_1^{AH} > \max\{\pi_1^{AM}, \pi_1^{AL}\}$.

*The profit of the social platform is as follows:*

$\pi_2^{AL} - \pi_2^{AH} = -\frac{(e_H - e_L)[e_H + e_L + \theta - m(-2 + 2(r-1)t(\theta-1) + \theta)]\lambda}{4(\theta^2-1)} > 0$, $\pi_2^{AL} > \pi_2^{AH}$; $\pi_2^{AL} - \pi_2^{AM} = \frac{\left(\begin{array}{l}[-e_H - e_H e_L + e_M + {e_M}^2 - e_L m + e_M m + (e_H - e_L)m(r-1)t]\theta - \\ (e_H - e_M)(e_H + e_M + 2m[1 + (r-1)t])\end{array}\right)\lambda}{4(\theta^2-1)} = f(e_H)$, *where* $f(e_H)$ *is a quadratic function opening upward, with the two roots* $\Delta e_H^3$ *and* $\Delta e_H^4$, *and* $\Delta e_H^3 < \Delta e_H^4$. *Therefore, when* $e_H > \Delta e_H^4$, $\pi_2^{AL} > \pi_2^{AM}$. *In summary, when* $e_H > \Delta e_H^4$, $\pi_2^{AL} > \max\{\pi_2^{AM}, \pi_2^{AH}\}$.

*The profit of the brander is as follows:*

$\pi_b^{AH} - \pi_b^{AM} = \frac{\left(\begin{array}{l}2e_H[1 + e_L\theta + m(rt - t)(\theta - 1) + \mathrm{m}\theta](\lambda - 1) - \\ 2e_L[(r-1)t\mathrm{m}(\theta - 1) - \mathrm{m} - \theta](\lambda - 1) + \\ {e_H}^2(1 - 2\theta^2 + \lambda) + {e_L}^2(1 - 2\theta^2 + \lambda) + \\ 2e_M(1 + \theta)(1 + m + e_M\theta - (1 + e_M + m)\lambda)\end{array}\right)}{4(\theta^2-1)} = f(e_H)$, *and when* $\lambda > 2\theta^2 - 1$, $f(e_H)$ *is a quadratic function opening upward, with the two roots* $\Delta e_H^5$ *and* $\Delta e_H^6$, *and* $\Delta e_H^5 < \Delta e_H^6$. *Therefore, when* $\Delta e_H^5 < e_H < \Delta e_H^6$, $\pi_b^{AM} > \pi_b^{AH}$; $\pi_b^{AL} - \pi_b^{AM} = \frac{\left(\begin{array}{l}2e_L[1 + (r-1)tm(\theta - 1) + m\theta](\lambda - 1) - \\ 2e_H[(r-1)tm(\theta - 1) - m - (1 + e_L)\theta](\lambda - 1) + \\ {e_H}^2(1 - 2\theta^2 + \lambda) + {e_L}^2(1 - 2\theta^2 + \lambda) + \\ 2e_M(1 + \theta)[1 + m + e_M\theta - (1 + e_M + m)\lambda]\end{array}\right)}{4(\theta^2-1)} = f(e_H)$, *and when* $\lambda > 2\theta^2 - 1$, $f(e_H)$ *is a quadratic function opening upward, with the two roots* $\Delta e_H^7$ *and* $\Delta e_H^8$, *and* $\Delta e_H^7 < \Delta e_H^8$. *Therefore, when* $\Delta e_H^7 < e_H < \Delta e_H^8$, $\pi_b^{AM} > \pi_b^{AL}$.

*In summary, when* $\max\{\Delta e_H^5, \Delta e_H^7, 0\} < e_H < \min\{\Delta e_H^6, \Delta e_H^8, 1\}$, $\pi_b^{AM} > \max\{\pi_2^{AH}, \pi_2^{AL}\}$.

**Proof of Proposition 2.**

(1) MS Strategy: In this case, the objective functions of the government, the brander and the traditional platform are, respectively,

$$\mathrm{SW} = \pi_b + \pi_1 + CS - gq_1$$

$$\pi_b^{MS} = (1 - \lambda)p_1 q_1 - \frac{e_M^2}{2}$$

$$\pi_1^{MS} = \lambda p_1 q_1$$

Since $\frac{\partial^2 \pi_b}{\partial p_1^2} = -2 + 2\lambda < 0$, $\pi_b^{MS}$ is a concave function in $p_1^{MS}$. Solving the first-order condition of the traditional platform's profit yields the equilibrium decision:

$$p_1^{MS} = \frac{1 + e_M + g}{2}$$

Solving the first-order condition of the government's objective function gives the optimal consumption subsidy:

$$g^{MS} = 1 + e_M$$

Substituting the optimal consumption subsidy g back into the relevant functions yields the optimal pricing, demand, and profits for the government, the brand, and the platform:

The profit of the brander is $\pi_b^{MS} = \frac{-{e_M}^2 - 2(1 + e_M)^2(\lambda - 1)}{2}$, and the profit of the traditional platform is $\pi_1^{MS} = (1 + e_M)^2\lambda$.

(2) AMS Strategy: In this case, the objective functions of the government, the brander, the traditional and the social platform are, respectively,

$$SW = \pi_b + \pi_1 + \pi_2 + CS - g(q_1 + q_2)$$

$$\pi_b^{AMS} = (1-\lambda)p_1q_1 + (1-\lambda)p_2q_2 - \frac{e_M^2}{2}$$

$$\pi_1^{AMS} = \lambda p_1 q_1$$

$$\pi_2^{AMS} = \lambda p_2 q_2$$

The brander simultaneously determines the product prices $p_i^{AMS}$ to maximize its own profit. The Hessian matrix of $\pi_b^{AMS}$ with respect to $p_i^{AMS}$ is given by

$$H = \begin{pmatrix} \frac{\partial^2 \pi_b}{\partial {p_1}^2} & \frac{\partial^2 \pi_b}{\partial p_1 p_2} \\ \frac{\partial^2 \pi_b}{\partial p_2 p_1} & \frac{\partial^2 \pi_b}{\partial {p_2}^2} \end{pmatrix} = \begin{pmatrix} 2(\lambda-1) & -2\theta(\lambda-1) \\ -2\theta(\lambda-1) & 2(\lambda-1) \end{pmatrix}$$

It is easy to verify that this matrix is negative definite. Solving the first-order optimality conditions yields the equilibrium prices $p_i^{AMS}(i = 1, 2)$. Since $\frac{\partial^2 SW}{\partial g^2} = -\frac{3}{2} + \frac{1}{1-\theta}$, it is straightforward to show that this matrix is negative definite when $0 < \theta < \frac{1}{3}$. Solving the first-order condition of the government's objective function yields the optimal consumption subsidy:

$$g = \frac{(1 + 2e_M + m)(1+\theta)}{2 - 6\theta}$$

Substituting the optimal consumption subsidy $g$ back into the relevant functions yields the optimal pricing, demand, and profits for the government, the brander, and the platform.

Under strategy AM, the consumption subsidy is $g^{AMS} = \frac{(1+2e_M+m)(1+\theta)}{2-6\theta}$; the demand and product prices of the brander on the traditional platform are $q_1^{AMS} = \frac{-3+4e_M(\theta-1)+5\theta-m[1+\theta+2(r-1)t(3\theta-1)]}{12\theta-4}$, $p_1^{AMS} = \frac{\left(\frac{2+4e_M+2m}{1-3\theta} + \frac{1+m(-1-2(r-1)t)}{1+\theta}\right)}{2}$, respectively; the demand and product prices of the brander on the social platform are $q_2^{AMS} = \frac{-1+4e_M(\theta-1)-\theta+m[5\theta-3+2(r-1)t(3\theta-1)]}{12\theta-4}$, $p_2^{AMS} = \frac{\left(\frac{2+4e_M+2m}{1-3\theta} + \frac{-1+m+2m(-1+r)t}{1+\theta}\right)}{2}$, respectively; the profit of the brander is $\pi_b^{AMS} = \frac{\left(\begin{array}{l} 16e_M(1+m)(\theta-1)(\lambda-1) + 4{e_M}^2[3-4\lambda+\theta(2-9\theta+4\lambda)] - \\ \frac{1}{1+\theta}\left(\begin{array}{l} 5+\theta(5\theta-6)+m\left[6-4(r-1)t(1-3\theta)^2+2(6-13\theta)\theta\right] + \\ m^2\left[5+4(r-1)t(1-3\theta)^2+4(r-1)^2t^2(1-3\theta)^2+\theta(5\theta-6)\right] \end{array}\right)(\lambda-1) \end{array}\right)}{8(1-3\theta)^2}$; the profit of the traditional platform is $\pi_1^{AMS} = \frac{\left(\begin{array}{l} [3+m-\theta+5m\theta+4e_M(1+\theta)+2m(r-1)t(3\theta-1)] \\ [3-4e_M(\theta-1)-5\theta+m+m\theta+2m(r-1)t(3\theta-1)]\lambda \end{array}\right)}{16(1-3\theta)^2(1+\theta)}$; and the profit of the social platform is $\pi_2^{AMS} = -\frac{\left(\begin{array}{l} [(2+4e_M+2m)(1+\theta)+(m-1+2mtr-2mt)(1-3\theta)] \\ [4e_M(\theta-1)-1-\theta-3m+5\theta m+2m(r-1)t(3\theta-1)]\lambda \end{array}\right)}{16(1-3\theta)^2(1+\theta)}$.

(3) AHS Strategy: The proof process is the same as above.

Under strategy AHS, the consumption subsidy is $g^{AHS} = \frac{(1+e_H+e_L+m)(1+\theta)}{2-6\theta}$; the demand and product prices of the brander on the traditional platform are $q_1^{AHS} = \frac{(-3+5\theta-e_L(1+\theta)+e_H(5\theta-3)-m[1+\theta+2(r-1)t(3\theta-1)])}{12\theta-4}$, $p_1^{AHS} = \frac{1+e_H+e_L\theta+\frac{(1+e_H+e_L+m)(1+\theta)^2}{2-6\theta}+m((r-1)t(\theta-1)+\theta)}{2-2\theta^2}$, respectively; the demand and product prices of the brander on the social platform are $q_2^{AHS} = \frac{(-1-\theta-e_H(1+\theta)+e_L(5\theta-3)+m[-3+5\theta+2(r-1)t(3\theta-1)])}{12\theta-4}$,

$p_2^{AHS} = \frac{e_L+m+(1+e_H)\theta+\frac{(1+e_H+e_L+m)(1+\theta)^2}{2-6\theta}+mt(r-1+\theta-r\theta)}{2-2\theta^2}$, respectively; the profit of the brander is $\pi_b^{AHS} = \frac{\left(\begin{array}{c} -\frac{\left[\begin{array}{c} 1+e_H+3e_L+m(3+2rt-2t)+ \\ \theta+(e_H-5(e_L+m)-6m(r-1)t)\theta \end{array}\right]\left[\begin{array}{c} 1+e_H-e_L(\theta-3)+5\theta+5e_H\theta- \\ m(\theta-3+2(r-1)t(3\theta-1)) \end{array}\right]^{(\lambda-1)}}{(1-3\theta)^2(1+\theta)}- \\ [3+e_L+m-e_H(\theta-3)-\theta+5e_L\theta+5m\theta+2m(r-1)t(3\theta-1)] \\ \frac{[3+e_H(3-5\theta)-5\theta+e_L(1+\theta)+m(1+\theta+2(r-1)t(3\theta-1))](\lambda-1)}{(1-3\theta)^2(1+\theta)} \end{array}\right)}{16} - 8e_H{}^2 - 8e_L{}^2$, the profit of the traditional platform is $\pi_1^{AHS} = \frac{\left(\begin{array}{c} [3+e_L+m-e_H(-3+\theta)-\theta+5e_L\theta+5m\theta+2m(r-1)t(3\theta-1)] \\ [3+e_H(3-5\theta)-5\theta+e_L(1+\theta)+m+m\theta+2(r-1)mt(3\theta-1)]\lambda \end{array}\right)}{16(1-3\theta)^2(1+\theta)}$, and the profit of the social platform is $\pi_2^{AHS} = \frac{\left(\begin{array}{c} [1+e_H+3e_L+m(3+2rt-2t)+\theta+(e_H-5(e_L+m)-6m(r-1)t)\theta] \\ [1+e_H-e_L(-3+\theta)+5\theta+5e_H\theta-m(-3+\theta+2(r-1)t(3\theta-1))]\lambda \end{array}\right)}{16(1-3\theta)^2(1+\theta)}$.

(4) ALS Strategy: The proof process is the same as above.

Under strategy ALS, the consumption subsidy is $g^{ALS} = \frac{(1+e_H+e_L+m)(1+\theta)}{2-6\theta}$; the demand and product prices of the brander on the traditional platform are $q_1^{ALS} = \frac{-3+5\theta-e_H(1+\theta)+e_L(5\theta-3)-m[1+\theta+2(r-1)t(3\theta-1)]}{12\theta-4}$, $p_1^{ALS} = \frac{1+e_L+e_H\theta+\frac{(1+e_H+e_L+m)(1+\theta)^2}{2-6\theta}+m[(r-1)t(\theta-1)+\theta]}{2-2\theta^2}$, respectively; the demand and product prices of the brander on the social platform are $q_2^{ALS} = \frac{-1-\theta-e_L(1+\theta)+e_H(5\theta-3)+m[-3+5\theta+2(r-1)t(3\theta-1)]}{12\theta-4}$, $p_2^{ALS} = \frac{e_H+m+(1+e_L)\theta+\frac{(1+e_H+e_L+m)(1+\theta)^2}{2-6\theta}+mt(-1+r+\theta-r\theta)}{2-2\theta^2}$, respectively; the profit of the brander is $\pi_b^{ALS} = \frac{\left(\begin{array}{c} [3+e_H+3e_L+m+2mt-2mrt+(-5+e_H-5e_L+m+6m(r-1)t)\theta] \\ -8e_L{}^2-8e_L{}^2-\frac{[3+e_H+m-e_L(\theta-3)-\theta+5e_H\theta+5m\theta+2m(r-1)t(3\theta-1)](\lambda-1)}{(1-3\theta)^2(1+\theta)}- \\ [-1+e_H(-3+\theta)-5\theta-e_L(1+5\theta)+m(-3+\theta+2(r-1)t(3\theta-1))] \\ \frac{[-1-\theta-e_L(1+\theta)+e_H(5\theta-3)+m(-3+5\theta+2(r-1)t(3\theta-1))](\lambda-1)}{(1-3\theta)^2(1+\theta)} \end{array}\right)}{16}$, the profit of the traditional platform is $\pi_1^{ALS} = \frac{[3+e_H+3e_L+m+2mt-2mrt+(-5+e_H-5e_L+m+6m(r-1)t)\theta]}{16(1-3\theta)^2(1+\theta)}$ $\frac{[3+e_H+m-e_L(\theta-3)-\theta+5e_H\theta+5m\theta+2m(r-1)t(3\theta-1)]\lambda}{16(1-3\theta)^2(1+\theta)}$, and the profit of the social platform is $\pi_2^{ALS} = \frac{[1+e_H+3e_L+m(3+2rt-2t)+\theta+(e_H-5(e_L+m)-6m(r-1)t)\theta]}{16(1-3\theta)^2(1+\theta)}$ $\frac{[1+e_H-e_L(-3+\theta)+5\theta+5e_H\theta-m(-3+\theta+2(r-1)t(3\theta-1))]\lambda}{16(1-3\theta)^2(1+\theta)}$.

Under the assumed parameter constraints, all demand expressions are proven to be non-negative $\left(q_i^{AnS} > 0\right)$, ensuring the rationality and internal consistency of the model. □

**Corollary A7.** *The proof process is similar to that of Corollary 1 and is omitted here.*

**Corollary A8.** *The proof process is similar to that of Corollary 2 and is omitted here.*

**Corollary A9.** *The proof process is similar to that of Corollary 3 and is omitted here.*

**Corollary A10.** *The proof process is similar to that of Corollary 4 and is omitted here.*

**Corollary A11.** *The proof process is similar to that of Corollary 5 and is omitted here.*

**Corollary A12.** *The proof process is similar to that of Corollary 6 and is omitted here.*

**Corollary A13.** *To compare the magnitudes of the consumption subsidy amounts under the three strategies, we take their differences.* $g^{AHS} = g^{ALS} = \frac{(1+e_H+e_L)(1+\theta)}{2-6\theta}$, $g^{AHS} - g^{AMS} = \frac{(e_H+e_L-2e_M)(1+\theta)}{2-6\theta}$, *and when* $e_M < \frac{e_H+e_L}{2}$, $g^{AHS} > g^{AMS}$; $g^{ALS} - g^{AMS} = \frac{(e_H+e_L-2e_M)(1+\theta)}{2-6\theta}$, *and when* $e_M < \frac{e_H+e_L}{2}$, $g^{ALS} > g^{AMS}$. *In summary, when* $e_M < \frac{e_H+e_L}{2}$, $g^{BH} = g^{\mathrm{BL}} > g^{BM}$.

**Corollary A14.** *AM/AMS: To compare the magnitude of traditional platform's demands under AM and AMS,* $q_1^{AMS} - q_1^{AM} = \frac{(1+2e_M+m)(1+\theta)}{4-12\theta} > 0$, $q_1^{AMS} > q_1^{AM}$. *To compare the magnitude of traditional platform's product pricing under AM and AMS,* $p_1^{AMS} - p_1^{AM} = \frac{(1+2e_M+m)(1+\theta)}{4(\theta-1)(3\theta-1)} > 0$, $p_1^{AMS} > p_1^{AM}$; $p_2^{AMS} - p_2^{AM} = \frac{(1+2e_M+m)(1+\theta)}{4(\theta-1)(3\theta-1)} > 0$, $p_2^{AMS} > p_2^{AM}$; $q_2^{AMS} - q_2^{AM} = \frac{(1+2e_M+m)(1+\theta)}{4-12\theta} > 0$, $q_2^{AMS} > q_2^{AM}$;

*AH/AHS:* $q_1^{AHS} - q_1^{AH} = \frac{(1+e_H+e_L+m)(1+\theta)}{4-12\theta} > 0$, $q_1^{AHS} > q_1^{AH}$; $q_2^{AHS} - q_2^{AH} = \frac{(1+e_H+e_L+m)(1+\theta)}{4-12\theta} > 0$, $q_2^{AHS} > q_2^{AH}$; $p_1^{AHS} - p_1^{AH} = \frac{(1+e_H+e_L+m)(1+\theta)}{4(\theta-1)(3\theta-1)} > 0$, $p_1^{AHS} > p_1^{AH}$; $p_2^{AHS} - p_2^{AH} = \frac{(1+e_H+e_L+m)(1+\theta)}{4(\theta-1)(3\theta-1)} > 0$, $p_2^{AHS} > p_2^{AH}$;

*AL/ALS:* $q_1^{ALS} - q_1^{AL} = \frac{(1+e_H+e_L+m)(1+\theta)}{4-12\theta} > 0$, $q_1^{ALS} > q_1^{AL}$; $q_2^{ALS} - q_2^{AL} = \frac{(1+e_H+e_L+m)(1+\theta)}{4-12\theta} > 0$, $q_2^{ALS} > q_2^{AL}$; $p_1^{ALS} - p_1^{AL} = \frac{(1+e_H+e_L+m)(1+\theta)}{4(\theta-1)(3\theta-1)} > 0$, $p_1^{ALS} > p_1^{AL}$; $p_2^{ALS} - p_2^{AL} = \frac{(1+e_H+e_L+m)(1+\theta)}{4(\theta-1)(3\theta-1)} > 0$, $p_2^{ALS} > p_2^{AL}$;

**Corollary A15.** *AM/AMS: To compare the magnitude of the traditional platform's profits under AM and AMS, we take their difference.*

$$\pi_1^{AMS} - \pi_1^{AM} = \frac{\left((1+2e_M+m)\left(\begin{array}{l}-5-6e_M-m-4mt+4mrt+\\ 4(2+e_M+m(3t-1-3rt))\theta+5(1+2e_M+m)\theta^2\end{array}\right)\lambda\right)}{16(1-3\theta)^2(\theta-1)} > 0,\ \pi_1^{AMS} > \pi_1^{AM};\ \pi_2^{AMS} - \pi_2^{AM} = \frac{\left((1+2e_M+m)\left(\begin{array}{l}2e_M(1+\theta)(5\theta-3)+(\theta-1)(1+5\theta)+\\ m(-5+4(r-1)t(3\theta-1)+\theta(8+5\theta))\end{array}\right)\lambda\right)}{16(1-3\theta)^2(\theta-1)} > 0,$$
$$\pi_2^{AMS} > \pi_2^{AM};\ \pi_b^{AMS} - \pi_b^{AM} = -\frac{(1+2e_M+m)^2(1+\theta)(5\theta-3)(\lambda-1)}{8(1-3\theta)^2(\theta-1)} > 0,\ \pi_b^{AMS} > \pi_b^{AM};$$

$$\text{AH/AHS: } \pi_1^{AHS} - \pi_1^{AH} = \frac{\left((1+e_H+e_L+m)\left[\begin{array}{l}-5-5e_H-e_L+m(4(r-1)t-1)+8\theta-\\ 4(-2e_H+e_L+m+3m(r-1)t)\theta+5(1+e_H+e_L+m)\theta^2\end{array}\right]\lambda\right)}{16(1-3\theta)^2(\theta-1)} > 0,$$
$$\pi_1^{AHS} > \pi_1^{AH};\ \pi_2^{AHS} - \pi_2^{AH} = \frac{\left((1+e_H+e_L+m)\left[\begin{array}{l}-1-e_H-5e_L-5m+4mt-4mrt-\\ 4(1+e_H-2e_L-2m-3m(r-1)t)\theta+5(1+e_H+e_L+m)\theta^2\end{array}\right]\lambda\right)}{16(1-3\theta)^2(\theta-1)} > 0,$$
$$\pi_2^{AHS} > \pi_2^{AH};\ \pi_b^{AHS} - \pi_b^{AH} = -\frac{(1+e_H+e_L+m)^2(1+\theta)(5\theta-3)(\lambda-1)}{8(1-3\theta)^2(\theta-1)} > 0,\ \pi_b^{AHS} > \pi_b^{AH};$$

$$\text{AL/ALS: } \pi_1^{ALS} - \pi_1^{AL} = \frac{\left((1+e_H+e_L+m)\left[\begin{array}{l}-5-e_H-5e_L+m(-1+4(r-1)t)+8\theta-\\ 4(e_H-2e_L+m+3m(r-1)t)\theta+5(1+e_H+e_L+m)\theta^2\end{array}\right]\lambda\right)}{16(1-3\theta)^2(\theta-1)} > 0,$$
$$\pi_1^{ALS} > \pi_1^{AL};\ \pi_2^{ALS} - \pi_2^{AL} = \frac{\left((1+e_H+e_L+m)\left[\begin{array}{l}-1-5e_H-e_L-5m+4mt-4mrt+\\ 4(-1+2e_H-e_L+2m+3m(r-1)t)\theta+5(1+e_H+e_L+m)\theta^2\end{array}\right]\lambda\right)}{16(1-3\theta)^2(\theta-1)} > 0,$$
$$\pi_2^{ALS} > \pi_2^{AL};\ \pi_b^{ALS} - \pi_b^{AL} = -\frac{(1+e_H+e_L+m)^2(1+\theta)(5\theta-3)(\lambda-1)}{8(1-3\theta)^2(\theta-1)} > 0,\ \pi_b^{ALS} > \pi_b^{AL};$$

**Corollary A16.** *AM/AMS: To compare the magnitude of CS under AM and AMS, we take their difference.* $CS^{AMS} - CS^{AM} = -\frac{(1+2e_M+m)^2(1+\theta)(5\theta-3)}{16(1-3\theta)^2} > 0$, $CS^{AMS} > CS^{AM}$. *To compare the magnitude of SW under AM and AMS, we take their difference.* $SW^{AMS} - SW^{AM} = \frac{(1+2e_M+m)^2(1+\theta)^2}{16(\theta-1)(3\theta-1)} > 0$, $SW^{AMS} > SW^{AM}$.

$$AH/AHS: CS^{AHS} - CS^{AH} = -\frac{(1+e_H+e_L+m)^2(1+\theta)(5\theta-3)}{16(1-3\theta)^2} > 0, CS^{AHS} > CS^{AH}; SW^{AHS} - SW^{AH} = \frac{(1+e_H+e_L+m)^2(1+\theta)^2}{16(\theta-1)(3\theta-1)} > 0, SW^{AHS} > SW^{AH}.$$

$$AL/ALS: CS^{ALS} - CS^{AL} = -\frac{(1+e_H+e_L+m)^2(1+\theta)(5\theta-3)}{16(1-3\theta)^2} > 0, CS^{ALS} > CS^{AL}; SW^{ALS} - SW^{AL} = \frac{(1+e_H+e_L+m)^2(1+\theta)^2}{16(\theta-1)(3\theta-1)} > 0, SW^{ALS} > SW^{AL}.$$

**Table A1.** Threshold conditions and economic implications.

| Designation | Referent Content | Economic Implications of the Parameters |
|---|---|---|
| $\Delta e_H^1$ | $\frac{1}{2}\left(\begin{array}{l}-2-e_L\theta-m[(r-1)t(\theta-2)+\theta]- \\ \sqrt{\left(\begin{array}{l}4(1+e_M+mt-mrt)^2+(e_L+m+m(r-1)t)^2\theta^2+ \\ 4(e_M+m+m(r-1)t)(1+e_M+mt-mrt)\theta\end{array}\right)}\end{array}\right)$ | For the traditional platform, its profit is indifferent between the AH and AM strategies. |
| $\Delta e_H^2$ | $\frac{1}{2}\left(\begin{array}{l}-2-e_L\theta-m((r-1)t(\theta-2)+\theta)+ \\ \sqrt{\begin{array}{l}\left(4(1+e_M+mt-mrt)^2+(e_L+m+m(r-1)t)^2\theta^2\right)+ \\ 4(e_M+m+m(r-1)t)(1+e_M+mt-mrt)\theta\end{array}}\end{array}\right)$ | For the traditional platform, its profit is indifferent between the AH and AM strategies. |
| $\Delta e_H^3$ | $\frac{1}{2}\left(\begin{array}{l}m[-2+(r-1)t(\theta-2)]-(1+e_L)\theta- \\ \sqrt{\begin{array}{l}\left(4(e_M+m+m(r-1)t)^2+(1+e_L+mt-mrt)^2\theta^2\right)+ \\ 4(e_M+m+m(r-1)t)(1+e_M+mt-mrt)\theta\end{array}}\end{array}\right)$ | For the social platform, its profit is indifferent between the AL and AM strategies. |
| $\Delta e_H^4$ | $\frac{1}{2}\left(\begin{array}{l}m[-2+(r-1)t(\theta-2)]-(1+e_L)\theta+ \\ \sqrt{\begin{array}{l}\left(4(e_M+m+m(r-1)t)^2+(1+e_L+mt-mrt)^2\theta^2\right)+ \\ 4(e_M+m+m(r-1)t)(1+e_M+mt-mrt)\theta\end{array}}\end{array}\right)$ | For the social platform, its profit is indifferent between the AL and AM strategies. |
| A | $m((r-1)t(\theta-1)+\theta)(\lambda-1)+(1+e_L\theta)(\lambda-1)$ | New parameters after scaling $\Delta e_H^3$, $\Delta e_H^4$. |
| B | $\left(\begin{array}{l}4(1+e_L\theta+m((r-1)t(\theta-1)+\theta))^2(\lambda-1)^2- \\ 4(2\theta^2-1-\lambda)\left(\begin{array}{l}{e_L}^2(-1+2\theta^2-\lambda)+2e_L(m(-1+(r-1)t(\theta-1))-\theta)(\lambda-1)+ \\ 2e_M(1+\theta)(-1-e_M\theta+m(\lambda-1)+\lambda+e_M\lambda)\end{array}\right)\end{array}\right)$ | New parameters after scaling $\Delta e_H^3$, $\Delta e_H^4$. |
| $\Delta e_H^5$ | $\frac{1}{-1+2\theta^2-\lambda}\left(A+\frac{1}{2}\sqrt{B}\right)$ | For the brander, its profit is indifferent between the AH and AM strategies. |
| $\Delta e_H^6$ | $\frac{1}{-1+2\theta^2-\lambda}\left(A-\frac{1}{2}\sqrt{B}\right)$ | For the brander, its profit is indifferent between the AH and AM strategies. |
| C | $-m(-1+(r-1)t(\theta-1))(\lambda-1)+(1+e_L)\theta(\lambda-1)$ | New parameters after scaling $\Delta e_H^5$, $\Delta e_H^6$. |
| D | $\left(\begin{array}{l}4(m(-1+(r-1)t(\theta-1))-(1+e_L)\theta)^2(\lambda-1)^2- \\ 4(-1+2\theta^2-\lambda)\left(\begin{array}{l}{e_L}^2(-1+2\theta^2-\lambda)-2e_L(1+m((r-1)t(\theta-1)+\theta))(\lambda-1)+ \\ 2e_M(1+\theta)(-1-e_M\theta+m(\lambda-1)+\lambda+e_M\lambda)\end{array}\right)\end{array}\right)$ | New parameters after scaling $\Delta e_H^5$, $\Delta e_H^6$. |
| $\Delta e_H^7$ | $\frac{1}{-1+2\theta^2-\lambda}\left(C+\frac{1}{2}\sqrt{D}\right)$ | For the brander, its profit is indifferent between the AL and AM strategies. |
| $\Delta e_H^8$ | $\frac{1}{-1+2\theta^2-\lambda}\left(C-\frac{1}{2}\sqrt{D}\right)$ | For the brander, its profit is indifferent between the AL and AM strategies. |

**Table A1.** *Cont.*

| Designation | Referent Content | Economic Implications of the Parameters |
| --- | --- | --- |
| $\Delta e_H^9$ | $\frac{-3(3+e_L+m)+6m(r-1)t-6(-3+e_L+m+4m(r-1)t)\theta+(-5+13e_L+m(13+18(r-1)t))\theta^2-\frac{1}{2}\sqrt{\begin{array}{l}4\left(18\theta-9-5\theta^2+e_L(\theta(13\theta-6)-3)+m(-3+6(r-1)t(\theta-1)(3\theta-1)+\theta(13\theta-6))\right)^2-\\4(\theta-3)(5\theta-3)\left(\begin{array}{l}{e_L}^2(1+\theta)(1+5\theta)+-m(1+3\theta+2(r-1)t(3\theta-1))\\2e_L\left[3+(6-13\theta)\theta+m(1+\theta)(1+5\theta)+2m(r-1)t\left(9\theta^2-1\right)\right]+\\8e_M\left(-3-2e_M+2(e_M+m)\theta^2+\theta(3+2\theta)\right)\end{array}\right)\end{array}}}{(\theta-3)(5\theta-3)}$ | For the traditional platform, its profit under the consumption subsidy is indifferent between the AH and AM strategies. |
| $\Delta e_H^{10}$ | $\frac{-3(3+e_L+m)+6m(r-1)t-6(-3+e_L+m+4m(r-1)t)\theta+(-5+13e_L+m(13+18(r-1)t))\theta^2+\frac{1}{2}\sqrt{\begin{array}{l}4\left(18\theta-9-5\theta^2+e_L(\theta(13\theta-6)-3)+m(-3+6(r-1)t(\theta-1)(3\theta-1)+\theta(13\theta-6))\right)^2-\\4(\theta-3)(5\theta-3)\left(\begin{array}{l}{e_L}^2(1+\theta)(1+5\theta)+-m(1+3\theta+2(r-1)t(3\theta-1))\\2e_L\left[3+(6-13\theta)\theta+m(1+\theta)(1+5\theta)+2m(r-1)t\left(9\theta^2-1\right)\right]+\\8e_M\left(-3-2e_M+2(e_M+m)\theta^2+\theta(3+2\theta)\right)\end{array}\right)\end{array}}}{(\theta-3)(5\theta-3)}$ | For the traditional platform, its profit under the consumption subsidy is indifferent between the AH and AM strategies. |
| $\Delta e_H^{11}$ | $\frac{-3(1+e_L+m(3+2(r-1)t))-6(1+e_L+m(-3+4t-4rt))\theta+(13+13e_L+m(-5-18(r-1)t))\theta^2-\frac{1}{2}\sqrt{\begin{array}{l}4\left(\theta(13\theta-6)-3+m\left(-9+18\theta-5\theta^2-6(r-1)t(\theta-1)(3\theta-1)\right)+e_L(-3+\theta(13\theta-6))\right)^2-\\4(\theta-3)(5\theta-3)\left(\begin{array}{l}{e_L}^2(1+\theta)(1+5\theta)+8e_M\left(\begin{array}{l}-1-2e_M+3m(-1+\theta)+2(e_M+m)\theta^2+\\\theta(2\theta-3)+2m(r-1)t(3\theta-1)\end{array}\right)+\\2e_L\left((1+\theta)(1+5\theta)+m\left(3+(6-13\theta)\theta+2(r-1)t\left(1-9\theta^2\right)\right)\right)\end{array}\right)\end{array}}}{(\theta-3)(5\theta-3)}$ | For the social platform, its profit under the consumption subsidy is indifferent between the AL and AM strategies. |
| $\Delta e_H^{12}$ | $\frac{-3(1+e_L+m(3+2(r-1)t))-6(1+e_L+m(-3+4t-4rt))\theta+(13+13e_L+m(-5-18(r-1)t))\theta^2+\frac{1}{2}\sqrt{\begin{array}{l}4\left(\theta(13\theta-6)-3+m\left(-9+18\theta-5\theta^2-6(r-1)t(\theta-1)(3\theta-1)\right)+e_L(-3+\theta(13\theta-6))\right)^2-\\4(\theta-3)(5\theta-3)\left(\begin{array}{l}{e_L}^2(1+\theta)(1+5\theta)+8e_M\left(\begin{array}{l}-1-2e_M+3m(-1+\theta)+2(e_M+m)\theta^2+\\\theta(2\theta-3)+2m(r-1)t(3\theta-1)\end{array}\right)+\\2e_L\left((1+\theta)(1+5\theta)+m\left(3+(6-13\theta)\theta+2(r-1)t\left(1-9\theta^2\right)\right)\right)\end{array}\right)\end{array}}}{(\theta-3)(5\theta-3)}$ | For the social platform, its profit under the consumption subsidy is indifferent between the AL and AM strategies. |
| E | $e_L(\theta(13\theta-6)-3)(\lambda-1)+\left[(6-5\theta)\theta-5+m\left(2(r-1)t(1-3\theta)^2-3+\theta(13\theta-6)\right)\right](\lambda-1)$ | New parameters after scaling $\Delta e_H^{11}$, $\Delta e_H^{12}$. |
| F | $\begin{array}{l}4\left(\begin{array}{l}-5+(6-5\theta)\theta+e_L(-3+\theta(13\theta-6))+\\m\left(-3+2(r-1)t(1-3\theta)^2+\theta(13\theta-6)\right)\end{array}\right)^2(\lambda-1)^2-\\4(-1+5\lambda+\theta(-2(7+3\lambda)+\theta(7+36\theta+5\lambda)))\\\left(\begin{array}{l}2e_L\left(3+(6-13\theta)\theta+m\left(5+2(r-1)t(1-3\theta)^2+\theta(5\theta-6)\right)\right)(\lambda-1)-\\4e_M(1+\theta)(-4(1+m)(\theta-1)(\lambda-1)+e_M(\theta(-2+9\theta-4\lambda)-3+4\lambda))+\\{e_L}^2(-1+5\lambda+\theta(-2(7+3\lambda)+\theta(7+36\theta+5\lambda)))\end{array}\right)\end{array}$ | New parameters after scaling $\Delta e_H^{11}$, $\Delta e_H^{12}$. |
| $\Delta e_H^{13}$ | $\frac{1}{5\lambda-1+\theta[-2(7+3\lambda)+\theta(7+36\theta+5\lambda)]}\left(E+\frac{1}{2}\sqrt{F}\right)$ | For the brander, its profit under the consumption subsidy is indifferent between the AH and AM strategies. |
| $\Delta e_H^{14}$ | $\frac{1}{5\lambda-1+\theta[-2(7+3\lambda)+\theta(7+36\theta+5\lambda)]}\left(E-\frac{1}{2}\sqrt{F}\right)$ | For the brander, its profit under the consumption subsidy is indifferent between the AH and AM strategies. |
| G | $e_L(\theta(13\theta-6)-3)(\lambda-1)-\left(3+(6-13\theta)\theta+m\left(5+2(r-1)t(1-3\theta)^2+\theta(5\theta-6)\right)\right)(\lambda-1)+$ | New parameters after scaling $\Delta e_H^{13}$, $\Delta e_H^{14}$. |

**Table A1.** *Cont.*

| Designation | Referent Content | Economic Implications of the Parameters |
| --- | --- | --- |
| H | $4\left(-3-6\theta+13\theta^2+m\left(-5-2(r-1)t(1-3\theta)^2+(6-5\theta)\theta\right)+e_L(-3+\theta(13\theta-6))\right)^2(\lambda-1)^2-4(-1+5\lambda+\theta(-2(7+3\lambda)+\theta(7+36\theta+5\lambda)))$ $\left(\begin{array}{l}-2e_L\left(-5+(6-5\theta)\theta+m\left(-3+2(r-1)t(1-3\theta)^2+\theta(13\theta-6)\right)\right)(\lambda-1)-\\4e_M(1+\theta)(-4(1+m)(\theta-1)(\lambda-1)+e_M(-3+\theta(9\theta-2-4\lambda)+4\lambda))+\\{e_L}^2(-1+5\lambda+\theta(-2(7+3\lambda)+\theta(7+36\theta+5\lambda)))\end{array}\right)$ | New parameters after scaling $\Delta e_H^{13}$, $\Delta e_H^{14}$. |
| $\Delta e_H^{15}$ | $\frac{1}{5\lambda-1+\theta[-2(7+3\lambda)+\theta(7+36\theta+5\lambda)]}\left(G+\frac{1}{2}\sqrt{H}\right)$ | For the brander, its profit under the consumption subsidy is indifferent between the AL and AM strategies. |
| $\Delta e_H^{16}$ | $\frac{1}{5\lambda-1+\theta[-2(7+3\lambda)+\theta(7+36\theta+5\lambda)]}\left(G-\frac{1}{2}\sqrt{H}\right)$ | For the brander, its profit under the consumption subsidy is indifferent between the AL and AM strategies. |
| $\widetilde{r}_1$ | $\frac{1-m}{2mt}+1$ | The minimum fan proportion at which the demand, price, and profit of the social platform begin to surpass those of the traditional platform under the AM/AMS strategy. |
| $\widetilde{r}_2$ | $\frac{1-m+e_H-e_L}{2mt}+1$ | The minimum fan proportion at which the demand, price, and profit of the social platform begin to surpass those of the traditional platform under the AH/AHS strategy. |
| $\widetilde{r}_3$ | $\frac{1-e_H+e_L-m}{2mt}+1$ | The minimum fan proportion at which the demand, price, and profit of the social platform begin to surpass those of the traditional platform under the AL/ALS strategy. |
| $\widetilde{r}_4$ | $\frac{-1-e_H+e_L+m}{2mt}+1$ | The minimum fan proportion at which the demand, price, and profit of the social platform begin to surpass those of the traditional platform under the AH/AHS strategy. |

**Table A1.** *Cont.*

| Designation | Referent Content | Economic Implications of the Parameters |
| --- | --- | --- |
| $\widetilde{r}_5$ | $\frac{(1-3\theta)(-2-4e_M-2m)+(1+\theta)(-4-4e_M)+2m(1-3\theta)}{2mt(1-3\theta)(1+\theta)}$ | The minimum fan proportion at which the demand, price, and profit of the social platform begin to surpass those of the traditional platform under the AL/ALS strategy. |

**Table A2.** Numerical verification of the critical thresholds.

| | $m = 2, r = 0.4, t = 0.5, \lambda = 0.1, \theta = 0.3, e_L = 0.3, e_M = 0.5$ | $m = 2, r = 0.3, t = 0.5, \lambda = 0.05, \theta = 0.2, e_L = 0.3, e_M = 0.5$ |
| --- | --- | --- |
| $\Delta e_H^5$ | 0.9162667784475138 | 0.8971357351304233 |
| $\Delta e_H^6$ | 3.211994091117704 | 3.059565295797412 |
| $\Delta e_H^7$ | 1.0035236946718693 | 1.200998855633258 |
| $\Delta e_H^8$ | 2.8508241314150875 | 2.1288980515832368 |
| $\Delta e_H^{13}$ | −7.387771614133055 | −8.65728484375258 |
| $\Delta e_H^{14}$ | 0.7005461269694284 | 0.7063613609029766 |
| $\Delta e_H^{15}$ | −7.385654895475534 | −8.580583493615888 |
| $\Delta e_H^{16}$ | 0.7006564420267245 | 0.70987109256048 |

Note: After numerical verification, Corollary 6. $\max\{\Delta e_H^5, \Delta e_H^7, 0\} < e_H < \min\{\Delta e_H^6, \Delta e_H^8, 1\}$ and Corollary 12. $\max\{\Delta e_H^{13}, \Delta e_H^{15}, 0\} < e_H < \min\{\Delta e_H^{14}, \Delta e_H^{16}, 1\}$ are non-empty over the stated and non-degenerate parameter region.

**Table A3.** Calibration of key thresholds.

| Parameters | Benchmark Values | Calibration Rationales |
| --- | --- | --- |
| $\lambda$ | The baseline is set to 0.1. | According to industry reports, the commission rates charged by mainstream e-commerce platforms for most product categories typically range from 2% to 15%. |
| $m$ | The baseline is set to 2. | According to industry reports, sales of health supplements and tonics on Douyin have reached over 1.5 times those on Tmall, and in the snack food segment (such as Three Squirrels), the ratio has exceeded seven times. |
| $r$ | The baseline is set to 0.4. | According to a Deloitte industry report, users with private-domain relationships, such as followers and members, account for approximately 20% to 40% of e-commerce platform users. |
| $t$ | The baseline is set to 0.5. | According to industry research, in terms of purchase decision-making, 70% of users habitually use Xiaohongshu as a starting point to obtain product information and compare different brands by searching and browsing posts. However, 60% of these users eventually move to other platforms to complete their transactions after finishing product comparison. |

Note: The industry reports mentioned above are as follows: Deloitte. (2021). Upholding Long-termism, Creating Long-term Value: The FAST+ Methodology—Omnichannel Sustainable Consumer Operation Centered on Private Domain. Deloitte China; Deloitte. (2023). 2023 Douyin E-commerce Global Shopping Industry Trend White Paper. Deloitte China.

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