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Review

Lot-Streaming Workshop Scheduling with Operation Flexibility: Review and Extension

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Abstract: Lot-streaming scheduling methods with operation flexibility have been widely used in aerospace, semiconductor, automotive, pharmaceutical and other manufacturing enterprises. Lot-splitting scheduling methods have attracted much more attention from academia and industry due to an urgent requirement for an effective way to improve the productivity of the flexible workshop scheduling. During the past decade, many works have been made on the different lot-streaming scheduling methods of the flexible workshop scheduling. The scope of this review focuses on the journal publications collected in the Web of Science database, among which 80% are from high-ranked journals. This paper aims to provide a comprehensive survey on the lot-streaming workshop scheduling with operation flexibility. First, the lot-streaming methods of jobs are discussed and the objectives as well as constraints in applications are summarized. Then, the problem models and their solution approaches are reviewed. Next, the research trends of problem applications, modeling and solution approaches are recalled. Finally, the potential future research directions are concluded.

Keywords: lot-streaming; operation flexibility; workshop scheduling; manufacturing enterprise

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1. Introduction

Recently, to satisfy the increasingly diverse and individualized needs of customers in the manufacturing market, the traditional standardized and mass production mode has gradually changed into a non-standardized and rapid customization production mode with multi-variety, small and medium-sized batches and high flexibility. It is essential to deliver orders in a high-quality, efficient and low-cost manner to cope with the fierce market competition. Workshop scheduling optimizes the allocation of resources to meet specific production target requirements. Some common optimization objectives are makespan, energy consumption, production costs, machine utilization and average machine load. Empirical evidence shows that an efficient shop scheduling solution not only reduces energy consumption but also achieves synergistic optimization of makespan, production cost, machine utilization and other indicators.

The lot-streaming workshop scheduling with operation flexibility (LWSOF) can be considered as an extension of the traditional shop scheduling, which consists of three sub-problems: lot splitting, machine assignment and operation sequencing. The workpieces are often produced in batches in the actual production of enterprises. Lot splitting is aimed at increasing the flexibility of scheduling, dividing the whole batch of workpieces into several smaller sub-lots during actual processing, which allows batch production

and batch transmission. Thus, the post-procedure operations can be executed in advance, such that the purposes of shortening the makespan, promoting the rational allocation of production resources and improving production efficiency are achieved. Machine assignment is to select a machine from a candidate set for each operation while operation sequencing is to schedule all operations on all machines to obtain satisfactory schedules [1]. The performance of scheduling schemes is affected by the method and lot-size of lot-splitting, and differs significantly from one solution to another. A smaller lot-size will result in frequent workpiece clamping, tool changes and inter-process transportation, and hence increased completion time, higher energy consumption and production costs. While larger batches may suffer from increased bottlenecks, underutilization of machine tools, it may lead to longer product delivery times, increased energy consumption, and failure of key equipment. Thus, a reasonable lot-splitting scheme is critical to ensure the performance of the LSWOF problem.

A reasonable and accurate model considering various production factors on a shop floor is the basis and prerequisite for optimization. At present, the workshop modeling methods include mathematical program-based modeling, graph-based modeling, simulation-based modeling and others (e.g., neural networks, digital-twins and continuous-time Markov chain). Among them, mathematical program-based modeling is widely applied to workshop scheduling problems owing to its simplicity, efficiency, and ease of decision making as well as verification. By using a variety of mathematical numbers, letters and symbols, the whole process of shop floor production scheduling is described, and the objective function to be optimized is formulated based on reasonable assumptions and constraints. However, it often requires the setting of numerous assumptions and lacks the consideration of multiple practical constraints, which makes it difficult to fit the production reality and leads to a certain limitation of the practical application performance of the obtained optimization scheme. Since graph theory was introduced by Euler in 1735, it has been gradually applied to a wide range of fields such as production scheduling [2], assembly or disassembly [3,4], computer science [5], and transportation [6].

The graph-based modeling method defines nodes and edges to model the relationships between workshop production resources. According to the existence of directed and undirected edges in different graph classes, graph theory methods can be classified into three categories: undirected graphs, directed graphs, and directed and undirected co-existing graphs; the representative graph theory methods include social networks, Markov random field models, directed acyclic graphs, Petri nets, disjunctive graphs, and AND/OR graphs [7]. Due to its ability to simplify the analysis of complex systems and to provide a more comprehensive response to each production element on a shop floor, the graph-based modeling has received a lot of attention from scholars and practitioners. For example, Knopp et al. [2] applied the disjunctive graph model to the LWSOF in 2017. However, the difficulty in rationalizing the characterization of complex practical constraints leads to some limitations in its application. The simulation-based modeling approach represents the real production environment on a shop floor by virtually modeling the production elements to discover and understand how a complex production scheduling system works by applying different conditions. Therefore, it is often used by practitioners to verify the actual performance of optimized scheduling solutions. Well-known software tools for workshop scheduling simulation are Flexsim [8], Plant Simulation [9], Anylogic [10], Delmia [11], and Wisdom [12].

Flexible workshop scheduling has been proven, in a general case, as an NP-hard problem; accordingly, LWSOF is no exception [2]. Currently, the methods for solving LWSOF can be divided into exact approaches and approximate algorithms. Among them, exact algorithms are applied to address polynomial-time problems and small-scale workshop

scheduling problems, which can obtain the optimal solution of the problem. Some well-known exact methods are linear programming [13], branch-and-bound [14], and cutting plane [15]. However, approximation algorithms are usually utilized to solve large-scale complex scheduling problems, which can give high-performance solutions to the optimization problems in polynomial time, which include swarm intelligence and evolutionary algorithms, heuristic algorithms, and deep learning methods. Due to the complexity of LW-SOF, obtaining better optimization solutions in a limited time by approximation algorithms has received many concerns [14].

The motivation of this paper is the wide application of flexible workshop scheduling and lot streaming technology in the manufacturing industry. We conducted statistics and analysis of about 80 publications could be found under "Web of Science Core Collection" database in Web of Science with keywords "flexible job-shop scheduling" or "hybrid flow-shop scheduling" and "lot splitting" or "lot streaming" in topics between 2015 and March 2025 for the convenience of researchers related to academia and industry. Furthermore, the Preferred Reporting Items for Systematic Reviews and Meta-Analyses (PRISMA) method is introduced to ensure transparency and reproducibility of the systematic review. The number of records identified, screened, excluded, and included, along with reasons for exclusions, are summarized in the PRISMA flow diagram, as shown in Figure 1. The aim of this research is to conduct a systematic review of literature focused on lot-streaming scheduling optimization, with an emphasis on operation flexibility. This work serves as an extensive resource for professionals and researchers in both academic and industrial fields. By synthesizing prior advancements, evaluating current and emerging studies, and identifying potential research trajectories, this paper highlights the application of swarm intelligence and evolutionary computation in addressing such complex NP-hard problems.

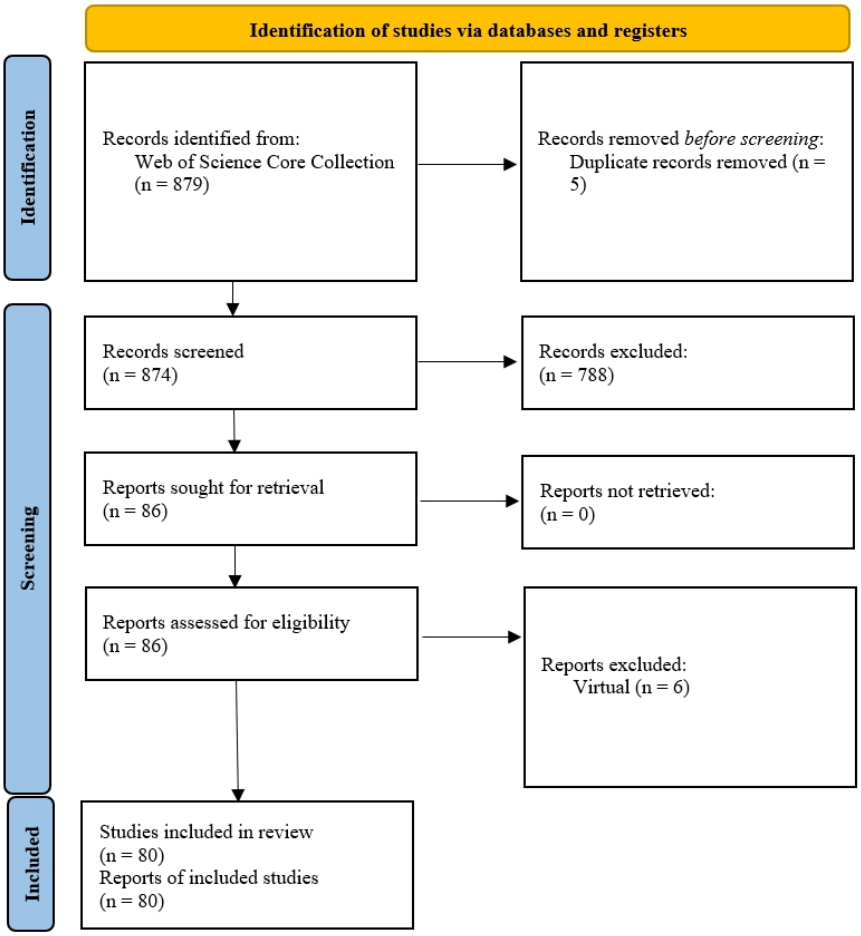


Figure 1. The PRISMA steps of the database search.

The remainder of this paper is organized as follows. Section 2 describes the framework of LWSOF and the existing achievements. Then, research trends on the problem extension, modeling and solution approaches are summarized in Section 3. Section 4 presents some future directions. Finally, concluding remarks are given in Section 5. Table 1 summarizes the relevant literature in the last decade in terms of workshop types, lot-splitting strategies, modeling and solution methods, objectives as well as constraints, respectively.

Table 1. Literature classification [4,16–93].

Article	Shop Floor Category	Lot Splitting Method	Model	Objective	Approach (Algorithm)	Constraint
Jalilvand-Nejad et al., 2015 [16]	Job-shop	Unknown	MILP	Cost	GA and SA	Setup times and costs
Defersha et al., 2015 [17]	Flow-shop	UCS	MILP	Makespan	Parallel multiple-search path SA	Setup times
Liu et al., 2015 [18]	Job-shop	Unknown	Others	Processing energy consumption and makespan	Modified GA	Routing problem
Mohsen et al., 2016 [19]	Flow-shop	ECS	MINLP	Weighted mean makespan	GA and SA	Setup times
Chen et al., 2016 [20]	Job-shop	ECS	Unknown	Makespan	Modified GA	None
Cheng et al., 2016 [21]	Flow-shop	UCS	MIP	Makespan	Heuristic	None
Li et al., 2016 [22]	Flow-shop	Unknown	IP	Makespan	Heuristic	Periodical job
Shahvari et al., 2016 [23]	Flow-shop	UVS	MILP	Total makespan and weighted tardiness	TS/path-relinking algorithms	Setup times
Vivek et al., 2016 [24]	Flow-shop	UCS	Simulation-based	Makespan	Extend V6	None
Lalitha et al., 2017 [25]	Flow-shop	UCS	MILP	Makespan	Heuristic and mathematical programming	None
Yu et al., 2017 [26]	Flow-shop	ECS	MIP	Total job tardiness	Iterative algorithms	Due-date and setup times
Zhang et al., 2017 [27]	Flow-shop	ECS	MIP	Total flow time	Effective modified MBO	Overlapping in operations
Zhong et al., 2017 [28]	Job-shop	UCS	Unknown	Makespan, labour distribution, equipment compliance and production cost	Improved NSGA-II	Worker allocation
Bozek et al., 2018 [29]	Job-shop	ECS	MILP and Graph-based	Makespan and sizes of the sub-lots	TS and Greedy constructive algorithm	None
Defersha et al., 2018 [13]	Job-shop	Unknown	LP	Makespan	LP assisted GA	None
Liu et al., 2018 [30]	Flow-shop	UCS	Unknown	Makespan and TEC	NSGA-II	Composite recycling and energy consumption
Meng et al., 2018 [31]	Job-shop	ECS	Others	Total flowtime	Enhanced fruit fly optimization	None
Meng et al., 2018 [32]	Job-shop	ECS	Others	Total flowtime	Enhanced monarch butterfly optimization	None
Romero et al., 2018 [33]	Job-shop	UCS	IP	Makespan	TS and heuristic	None
Shahvari et al., 2018 [34]	Flow-shop	UVS	MILP	Total weighted makespan and tardiness	PSO and TS	Machine availability times, job release times, machine capability and eligibility, stage skipping, learning effect and setup times

Table 1. *Cont.*

Article	Shop Floor Category	Lot Splitting Method	Model	Objective	Approach (Algorithm)	Constraint
Zhang et al., 2018 [35]	Job-shop	ECS	Others	Makespan and cost	Binary PSO	Alternative process plans
Gong et al., 2018 [36]	Flow-shop	ECS	Unknown	Makespan and earliness time	Hybrid multi-objective discrete ABC	Due-date and operation blocking
Gu et al., 2019 [37]	Flow-shop	Unknown	IP	Total weighted tardiness	Mathematical programming	Capacity constraints
Novas et al., 2019 [38]	Job-shop	UCS	CP	Makespan	Mathematical programming	Setup times
Wang et al., 2019 [39]	Flow-shop	UVS	MILP	Total weighted makespan	Mathematical programming and heuristic	Setup times
Yang et al., 2019 [40]	Job-shop	ECS	Unknown	Makespan	GA	Setup times
Zacharias et al., 2019 [41]	Flow-shop	Unknown	MILP	Makespan	NN, SVM and heuristic	Transportation resource
Chen et al., 2020 [42]	Flow-shop	UCS	MIP	Makespan and TEC	GA	Energy consumption and setup times
Li et al., 2020 [43]	Flow-shop	EVS	Unknown	Penalty caused by the average sojourn time, energy consumption in the last stage and earliness and tardiness values	Right-shift heuristic and multi-objective evolutionary algorithm based on decomposition	Energy consumption
Wang et al., 2020 [44]	Flow-shop	ECS	Others	Makespan	MBO and heuristic	None
Zhang et al., 2020 [45]	Job-shop	UCS	Unknown	Makespan	Competitive and cooperative MBO algorithm	Setup times
Zhang et al., 2020 [46]	Job-shop	Unknown	Unknown	Makespan, total tardiness and total workload	ACO	Setup times and transportation resource
Hadi et al., 2021 [47]	Job-shop	UVS	MILP	Total production, setup, and tardiness penalty costs	Self-adaptive cuckoo optimisation algorithm	Setup times, initial inventory and safety stock levels
Wu et al., 2021 [48]	Job-shop	UVS	MILP	Makespan and transportation time	Improved multi-objective optimization algorithm	None
Zhang et al., 2021 [49]	Flow-shop	ECS	MILP	Makespan	Collaborative VND	Setup times
Zhang et al., 2021 [50]	Job-shop	EVS	Unknown	Makespan	Discrete grey wolf optimizer	Setup and transportation times
Chiu et al., 2022 [51]	Job-shop	ECS	Simulation-based	Expected flow time	Enhanced GA and heuristic	Processing time variability and setup times
Daneshamooz et al., 2022 [4]	Job-shop	ECS	MILP	Makespan	Mathematical programming and VNS	Parallel assembly and setup times
Han et al., 2022 [52]	Job-shop	ECS	Unknown	Makespan, TEC and total costs	Improved NSGA-II	Intracellular transportation, energy consumption and setup times
Jiang et al., 2022 [53]	Job-shop	ECS	MILP and Simulation-based	TEC, makespan, and processing cost	Improved crossover ABC	Setup times and energy consumption
Li et al., 2022 [54]	Job-shop	UVS	MILP	Makespan	Hyper-heuristic improved GA	Setup times
Zhang et al., 2022 [55]	Flow-shop	ECS	Unknown	Makespan, starting time deviations of operations, and average adjustment of sublot sizes	Multi-objective MBO algorithm based on decomposition	Machine breakdown
Li et al., 2022 [56]	Job-shop	UCS	MILP	Average flow time	Improved ABC	Setup times

Table 1. *Cont.*

Article	Shop Floor Category	Lot Splitting Method	Model	Objective	Approach (Algorithm)	Constraint
Yilmaz et al., 2022 [57]	Flow-shop	ECS	Unknown	Makespan	Mathematical	None
Meng et al., 2021 [58]	Flow-shop	ECS	MIP	Makespan	Enhanced ABC	Setup times
Meng et al., 2019 [59]	Flow-shop	ECS	MIP	Makespan	Discrete ABC	Order constraint
Omid et al., 2022 [14]	Flow-shop	UVS	MILP	Total weighted job makespan and tardiness	Random forest and branch-and-price	Setup times, dynamic machine availability and job release times
Wang et al., 2022 [60]	Flow-shop	Unknown	MIP	Cost	Fuzzy-GA	Multilevel capacitated
Li et al., 2023 [61]	Flow-shop	UCS	MILP	TEC	Improved cooperative coevolutionary algorithm and VND	Energy consumption and setup times
Yang et al., 2023 [62]	Job-shop	UVS	MILP	Makespan	Guided shuffled frog-leaping algorithm	Overlapping in operations and setup times
Lu et al., 2023 [63]	Flow-shop	UCS	MILP	Makespan	Heuristic-based adaptive iterated greedy algorithm	Setup times
Zhu et al., 2023 [64]	Flow-shop	UCS	MILP	Makespan and due time deviations	Improved multi-objective ABC	Due-date
Tian et al., 2024 [65]	Job-shop	ECS and UCS	MILP	Makespan and TEC	Knowledge-based lot-splitting method	Energy consumption and setup times
Tian et al., 2023 [66]	Job-shop	ECS	MILP	Makespan, TEC and Cost	Bi-population differential ABC	Energy consumption and setup times
Shao et al., 2024 [67]	Flow-shop	ECS	MILP	Total tardiness	Learning-driven iterated local search algorithm	Setup times
Tutumlu et al., 2023 [68]	Job-shop	UVS	MIP	Makespan	Hybrid GA	Setup times
Chen et al., 2023 [69]	Flow-shop	ECS	MIP	Makespan, idle time of machines, total production cost and total flow time	Modified adaptive switching-based many-objective evolutionary algorithm	Transportation resource and setup times
Rohaninejad et al., 2023 [70]	Job-shop	UCS	MILP	The sum of setup, production, and inventory holding costs	Decomposition heuristic	Setup times and capacitated machines
Wang et al., 2023 [71]	Flow-shop	UCS	MILP	Makespan and TEC	Multi-objective discrete ABC	Energy consumption and setup times
Tian et al., 2023 [72]	Flow-shop	UCS	MILP	Makespan and TEC	Hybrid multi-objective fruit fly optimization algorithm	Energy consumption
Li et al., 2024 [73]	Flow-shop	UVS	MILP	Total penalty values	Novel collaborative iterative greedy	Setup and transportation times
Li et al., 2023 [74]	Job-shop	UCS and ECS	Unknown	Makespan	Reinforcement learning and ABC	None
Liu et al., 2023 [75]	Job-shop	ECS	Unknown	Makespan	GA and SA	Transportation resource
Yunusoglu et al., 2023 [76]	Job-shop	ECS	CP	Makespan	Mathematical programming and large neighbourhood search	Setup times and transport resource
Hadi et al., 2023 [77]	Job-shop	UCS	MINLP	Total production costs and total maintenance costs	Self-adaptive cuckoo optimization algorithm	Dynamic opportunistic maintenance and setup times
Shao et al., 2023 [78]	Flow-shop	ECS	MILP	Makespan	Iterated local search algorithm and heuristic	Setup times

Table 1. *Cont.*

Article	Shop Floor Category	Lot Splitting Method	Model	Objective	Approach (Algorithm)	Constraint
Chen et al., 2025 [79]	Flow-shop	ECS	MILP	Makespan, TEC, and total tardiness time	TS-ISMO and heuristic	Energy consumption
Duan et al., 2024 [80]	Flow-shop	UCS	MILP	Maximum tardiness, total idle energy consumption, and makespan	CHS-GPHH	Arrival of new workpieces, machine breakdown, and setup times
Fan et al., 2024 [81]	Job-shop	UVS	MILP	Makespan	ER-GA and heuristic	Setup times
Fan et al., 2024 [82]	Job-shop	UCS	MILP	Total weighted tardiness	GA-based matheuristic, VNS, and heuristic	Setup times
Lu et al., 2024 [83]	Flow-shop	UCS	MILP	Makespan	GA, Q-learning, and heuristic	None
Singh et al., 2024 [84]	Flow-shop	UCS	MIP	Makespan and small penalty proportional to the total number of machines	Mathematical programming	None
Zhu et al., 2024 [85]	Flow-shop	UCS	MILP	Makespan and due time deviation	KMA	Setup times
Chen et al., 2024 [86]	Flow-shop	ECS	MILP	Makespan, total earliness and total energy consumption	knowledge-driven many-objective optimization evolutionary algorithm	Transportation resource, due-date, energy consumption and setup times
Chen et al., 2024 [87]	Job-shop	ECS	MINLP	Tardiness,makespanand total setup time	Hybrid multi-objective GA	Alternative process plans, due-date and setup times
Yilmaz et al., 2024 [88]	Flow-shop	ECS	Unknown	Makespan, average flow time and total workload imbalance	Improved NSGA-II	Limited waiting time
Shao et al., 2025 [89]	Job-shop	EVS	Unknown	Makespan and utilization rate of machines	Adaptive job scheduling NSGA-II and heuristic	Setup times and operator limitations
Tang et al., 2025 [90]	Flow-shop	ECS	Markov chain	Makespan, total machine idle time and total travel distance	Multi-objective double deep Q-network	Transportation resource
Wang et al., 2025 [91]	Job-shop	EVS	MILP and simulatio-based	Makespan	Self-repair GA	Transportation resource and setup times
Yilmaz et al., 2025 [92]	Job-shop	ECS and EVS	MILP	Makespan, average flow time and total workload imbalance	Improved NSGA-II	Worker allocation
Zhu et al., 2025 [93]	Flow-shop	UCS	MILP	Makespan and due time deviation	Cooperative coevolutionary algorithm with global and local-oriented cooperative mechanisms, heuristic	Two types of time-overlaps

2. The Framework of LWSOF

Suppose that in the LWSOF, there are $n \in \mathbb{N}$ jobs J_1, J_2, …, J_n to be processed in $m \in \mathbb{N}$ machines M_1, M_2, …, M_m, where $\mathbb{N}$ is the set of natural numbers. Each job J_i is characterized by Q_{i1}, Q_{i2}, …, Q_{i,q_i} operations and a batch n_i, where $q_i, n_i \in \mathbb{N}$. All jobs may be divided into an arbitrary batch size larger than the minimum lot size to be set. Any operation can be processed on one or more machines with different kinds of production resource consumption such as electrical energy, processing time and cost. Then, we select the most suitable machine for each operation of all products and determine the optimal sequence of all processes on each machine to optimize certain specified production targets, including makespan, machine utilization, processing energy consumption, total weighted tardiness, etc. According to the specific production characteristics of workshops, the LWSOF can be divided into a lot-splitting flexible flow-shop scheduling problem and a lot-splitting

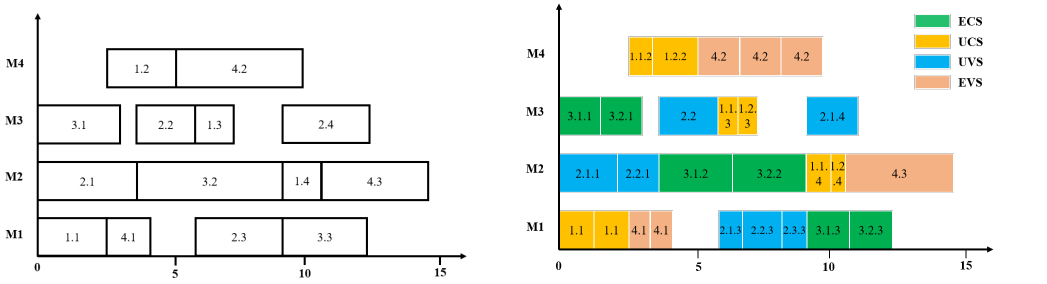
flexible job-shop scheduling problem. In addition, based on the different optimization objectives and actual constraints, the LWSOF can be expanded into multi-objective LWSOF, LWSOF with multiple resource constraints, dynamic LWSOF, etc. Then, to solve these problems, researchers developed a variety of methods to model the problem and designed various efficient algorithms based on the model characteristics.

2.1. Lot-Splitting Methods

At present, a series of lot-splitting methods have been proposed for different scheduling problems, including the ECS [1,26,65], UCS [24,25,30], EVS [43,50], and UVS [23,73,81]. Among them, the sizes of different sub-lots of a job are the same in the case of ECS, while the opposite is true for UCS. It is worth noting that the size of each sub-lot obtained from both remains the same in all periods of processing. In contrast, the number of each sub-lot may be different under UVS and EVS and may also change in different stages under UVS. The detailed comparison results between these lot-splitting methods are provided in Table 2. An illustration of LWSOF based on different lot-splitting methods is provided in Figure 2. Figure 2b demonstrates the statistical results of the lot-splitting strategies based on different classification methods.

Table 2. Comparisons between different lot-splitting methods.

Lot-Splitting Strategy	Sizes of Different Sub-Lots of a Job	Sub-Lot Sizes of a Job in Different Stages
ECS	Same	Consist
UCS	Different	Consist
EVS	Same	Variable
UVS	Different	Variable



(**a**) (**b**)

Figure 2. An example of LWSOF: (**a**) before lot-splitting, (**b**) after lot-splitting.

It can be seen from Figure 3a that over 71% of publications adopt ECS and UCS strategies, while only 13.1% and 6% publications use UVS and EVS strategy, respectively. It can be explained as the frequent changes in the number of sub-lots under the variable splitting strategy. In this way, machine allocation and job sequencing are hard, which makes it difficult to encode and decode the problem reasonably. As a result, some treatment methods are designed to cope with this problem. For example, Andrzej introduced the maximum or minimum batch method based on two stages [29], and Hadi et al. [47] proposed a variable lot-splitting method based on the maximum number of allowed batches. However, although the above methods have achieved some success in solving small-scale cases, they are still difficult to apply for handling large-scale issues in actual production due to the complex batch processing strategy. In addition, in actual production, complete equal splitting is an ideal situation. After all the jobs are split based on the minimum lot-size constraint, it often occurs that the size of the last sub-lots is not equal to that of other batches of the job. In this way, the ECS strategy can also be regarded as a special case of the UCS strategy.

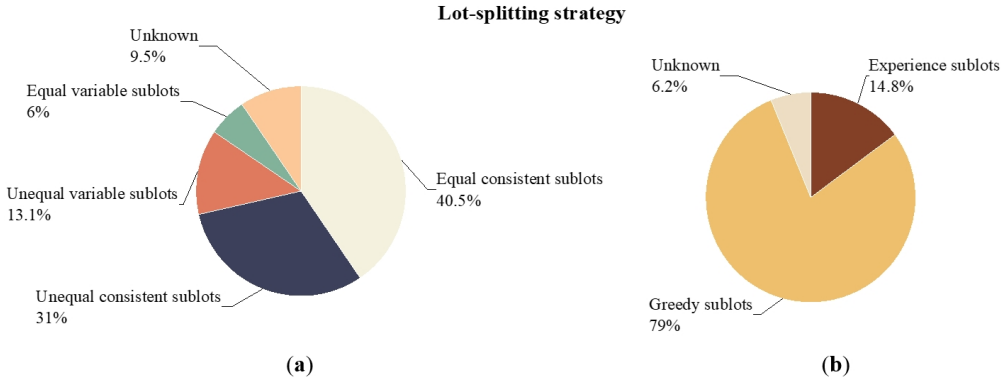


Figure 3. Statistical results of the lot-splitting strategy based on different classification methods: (**a**) lot-splitting based on ECS/UCS/EVS/UVS, (**b**) lot-splitting based on empirical methods and greedy methods.

In addition, the lot-splitting methods can be divided into empirical methods and greedy methods according to the ability to optimize the lot-splitting process continuously; the statistical results are shown in Figure 3b. Among them, the empirical methods are easy to implement. Through setting an optimal minimum lot-size according to the experience of decision makers, the lot-streaming scheduling problem can be transformed into an ordinary scheduling problem, which greatly reduces the complexity of the problem [58,59]. Therefore, empirical methods are usually adopted to solve the LWSOF on a large scale. However, due to the issues of strong randomness, large search space for the optimal scheme, unstable actual performance, and high dependence on the prior knowledge of decision makers, the optimization effect of the empirical methods is difficult to achieve expectations.

Compared with empirical methods, the greedy methods are more objective, and they obtain satisfactory lot-splitting schemes through a continuous optimization process, which mainly contains separate optimization [21,22,65] or integrated optimization [15,42,43] of the lot-splitting and job scheduling. Separate optimization refers to the two-stage feedback optimization; that is, the sub-lots are optimized first, and then the optimization result is taken as the known condition to transform the lot-streaming scheduling problem into a traditional scheduling problem. The performance evaluation of the lot-splitting scheme is based on the final optimization scheduling results obtained under this scheme. In this way, two-stage optimization can effectively reduce the search space of the optimal scheme so that it can achieve good results when solving large-scale problems. However, for some smaller problems, it is easy to fall into a local optimum due to the ignorance of optimization conflict between lot-splitting and job shop scheduling. As a result, it is challenging to obtain the global optimal solution. On the contrary, collaborative optimization of the lot-splitting and job scheduling can obtain the optimal solution of two sub-problems at the same time from a global perspective. Owing to the full search of the solution space, it performs better in solving small and medium-sized scheduling problems. Nevertheless, limited by the large search space of the optimal solution of large-scale problems, an excessive number of evaluation is often required to fully assess the solution space, which makes the algorithm time-consuming and hard to obtain satisfactory schemes in a finite time.

2.2. Objectives and Constraints

Based on the different numbers of optimized objectives, LWSOF can be divided into single-objective LWSOF and multi-objective LWSOF. By analyzing the documented studies in the literature as shown in Table 1, from 2015 to 2021, the research on LWSOF mostly aimed at the economic indicators, such as the shortest makespan and the lowest production cost. In the past four years, as a result of the global energy shortage and rising

production costs, the research focus has gradually shifted from single-objective optimization to multi-objective optimization of both economic indicators and green indicators. The major objectives are as follows:

Makespan (Maximum completion time): Makespan is the most classic criterion in scheduling, and LWSOF is not an exception. The makespan of LWSOF refers to the completion time of the last operation of all sub-lots, which can be calculated by Equation (1) [53].

$$makespan = \max\{ET_{i,q_i,k}\}, i \in [1, n'], k \in [1, m] \quad (1)$$

where n' refers to the number of jobs after splitting.

Processing cost: As for the cost criterion, existing works mainly focus on the machining process. For the LWSOF, the processing cost can be considered as the cutting cost of operations of all sub-lots, as shown in Equation (2):

$$Processing\ cost = \sum_{i=1}^{n'} \sum_{j=1}^{q_i} \sum_{k=1}^{m} n_i C_{ijk} X_{ijk} \quad (2)$$

where C_{ijk} refers to the unit cutting cost of Q_{ij} on M_m, and X_{ijk} is a decision variable which indicates that whether Q_{ij} working on M_m ($X_{ijk} = 1$), or not (X_{ijk}=0).

Total flow time/total tardiness: The total flow time of workshop scheduling problem can be evaluated by the sum of completion time of all jobs ($Total\ flowtime = \sum_{i=1}^{n} \sum_{k=1}^{m} ET_{i,q_i,k}$). In this case, with a specific due date d_i, the total tardiness can be obtained ($Total\ tardiness = \sum_{i=1}^{n} \sum_{k=1}^{m} |ET_{i,q_i,k} - d_i|$). In the LWSOF, the total flow time and total tardiness are shown in Equations (3) and (4), respectively.

$$Total\ flowtime = \sum_{i=1}^{n'} \sum_{k=1}^{m} ET_{i,q_i,k} \quad (3)$$

$$Total\ tardiness = \sum_{i=1}^{n'} \sum_{k=1}^{m} |ET_{i,q_i,k} - d_i| \quad (4)$$

Energy consumption/carbon emission: The workshop energy consumption is produced by the direct production-related parts including machining, set-up and transportation, and indirect parts such as air-conditioning and lighting. The various states of the machine tool are subject to fluctuations in power due to load variations. Thus, to access the energy consumption, the power in different states is considered to be stable. Then, the workshop energy consumption can be calculated based on the state energy consumption characteristics [94]. On this basis, by introducing a carbon emission factor for the local power grid, the energy consumption can be converted to the carbon emission. For example, the carbon emission factors (φ) for each region in China are shown in Table 3 [95].

Table 3. The carbon emission factors (φ) for each region in China.

Region	Carbon Emission Factor ($kgCO_2/kw{\cdot}h$)
North China	0.4578
Northeast China	0.3310
East China	0.4923
Central China	0.3112
Northwest China	0.3232

For the LWSOF, the energy consumption and carbon emission can be obtained by Equations (5) and (6), respectively.

$$Energy\ consumption = \sum_{k=1}^{m} P_k^m \times t_k^m + P_k^s \times t_k^s + P_k^t \times t_k^t + E_{fix} \tag{5}$$

where P_k^m, P_k^s, and P_k^t represent the machining power, set-up power and transportation power, respectively. Notations t_k^m, t_k^s, and t_k^t correspond to the time of these states, respectively. The E_{fix} refers to the total indirect energy consumption.

$$Carbon\ emission = \varphi \sum_{k=1}^{m} P_k^m \times t_k^m + P_k^s \times t_k^s + P_k^t \times t_k^t + E_{fix} \tag{6}$$

To facilitate the analysis, the multi-objective optimization strategies involved in this paper are classified into three categories: weighted summation, non-domination, and others (normalization or as constraints) according to the relationship among different objectives [96]. The visualization results of relevant statistics are shown in Figure 4. It can be summarized that the ratio of non-dominated and weighted summation method for dealing with multi-objective optimization problems ranked one or two, which are 52.9% and 17.6%, respectively. And 11.8% of the publications handle the objectives as constraints, while less than 2% of the publications adopt the normalization method. Furthermore, some works that do not mention multi-objective processing strategies are summarized in the "Unknown" category. This phenomenon can be explained that, in comparison with the weighted summation and other multi-objective processing techniques, the non-dominant method can provide decision makers multiple alternatives simultaneously. Then, the decision makers can choose satisfactory scheduling schemes according to the actual production and objective preferences.

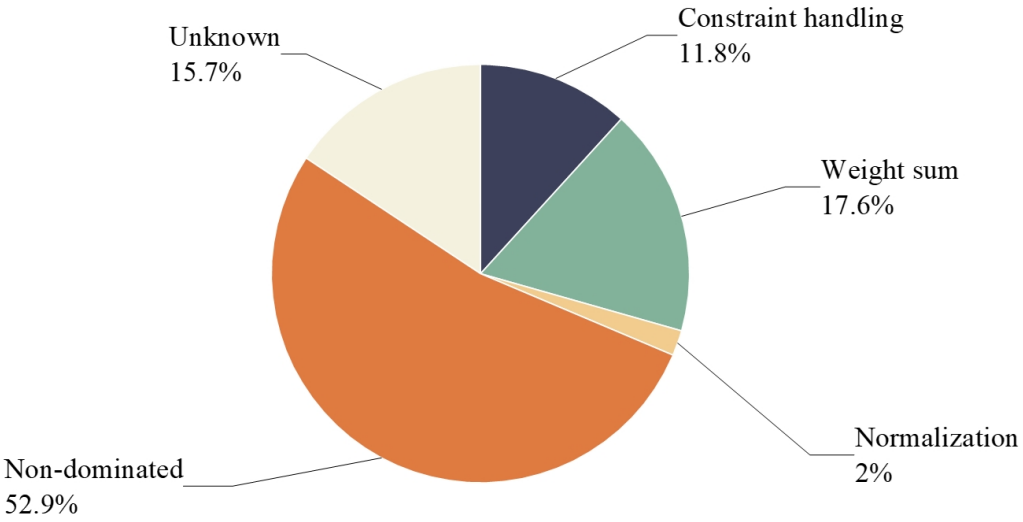


Figure 4. The visualization results of the multi-objective optimization strategies.

The lot-splitting process of the LWSOF is constrained by Equations (7) and (8), which ensure that the lot-size does not become zero for an existing sub-lot and the sum of the sub-lots of J_i equals the total batch of J_i, respectively.

$$M_{io} \leq n_i^o \leq n_i \times M_{io}, \forall i \in [1, n], o \in [1, l] \tag{7}$$

where n_i^o refers to the lot-size of the o-th sub-lot of J_i, and M_{io} is a decision variable which takes the value of 1 when $n_i^o > 0$, and 0 for another case, respectively.

$$\sum_{o=1}^{U_i} n_i^o = n_i, \forall i \in [1, n], o \in [1, l] \tag{8}$$

where U_i stands for the number of sub-lots after lot-splitting of J_i.

In addition, considering different additional constraints, these publications are classified by twelve categories: processing time variability, setup times, multi-constraint integration, dynamic events, energy consumption, overlapping in operations, resource recovery, transportation resource, due-date, alternative process plans, no additional constraints and others. The statistical results are summarized in Figure 5. With respect to the LWSOF, articles mainly focus on original problems (9.4%), multi-constraint integration (19.5%) and setup times (28.9%). Other constraints that account for less than 1% are grouped into the category "Others". In this way, to improve the practicability of the scheduling scheme, it is necessary to deeply analyze the conflict relationship of different scheduling indicators and explore the collaborative optimization mechanism among various sub-problems of LWSOF under complex and real conditions.

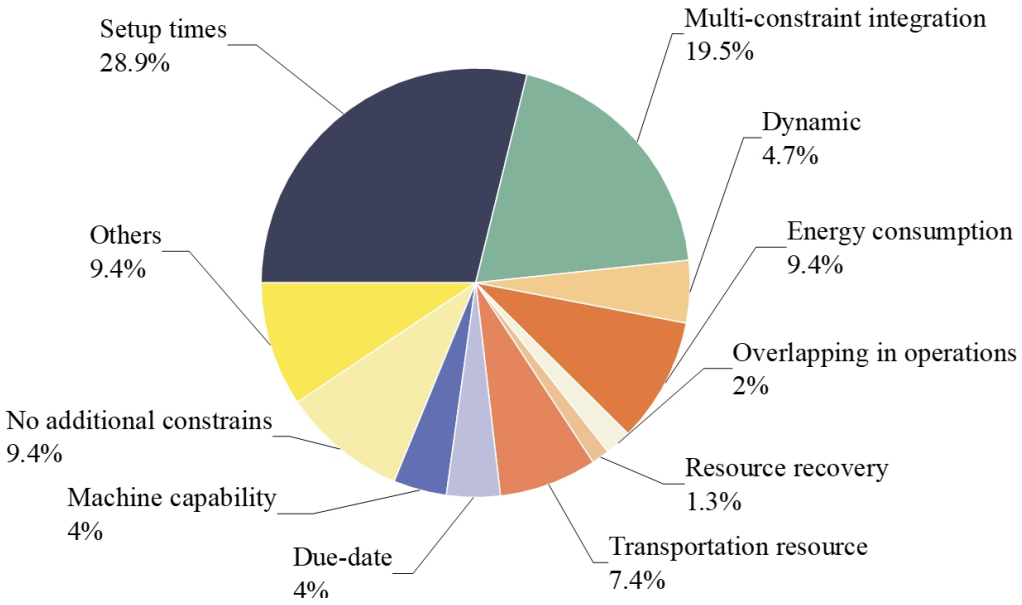


Figure 5. The visualization results of the constraints.

2.3. Problem Models

To analyze the advantages, disadvantages and applicable scenarios of different modeling methods, the works in Table 1 above are counted, and the statistical results are shown in Figure 6. It can be concluded that over 67% of the papers adopt mathematical programming methods, including MILP, MIP, MINLP, IP, LP, CP, etc., while less than 5% adopt simulation modeling methods or graph theory modeling methods, such as Markov chain, NN, and conjunctive graphs. Moreover, some models that the authors did not state the type clearly or cannot be converted to the above mainstream models, including the agent-based scheduling model and stochastic model, are categorized as the "Others" class. Among the mathematical programming methods, MILP and MIP methods are widely used (about 85%). On the one hand, this is determined by the characteristics of the problems. LWSOF is a combination optimization problem with equality and inequality constraints that includes both continuous and discrete variables in the decision variables, which has proved to be NP-hard [53]. On the other hand, MILP and MIP methods are widely followed in academia and industry for their wide range of application scenarios. Many sophisticated tools have emerged to enable us to solve MILP and MIP problems effectively. Therefore, if the target problem can be formulated into appropriate MILP and MIP models, it is close to the final optimal solution.

IP can be regarded as a special form of MIP, while MIP is considered a distinct form of MINLP. In addition, they can be deemed to be consistent if certain conditions are met. For example, if some integer decision variables are introduced into the IP model, then the IP model can be converted into an MIP model. And the MIP model is transformed

into the MINLP model if the constraint condition or objective function is nonlinear. For the complex large-scale LWSOF, the computational complexity of different mathematical programming models formulated from diverse perspectives often varies greatly. Therefore, it is necessary to establish appropriate mathematical programming models according to the problem characteristics and optimization objectives.

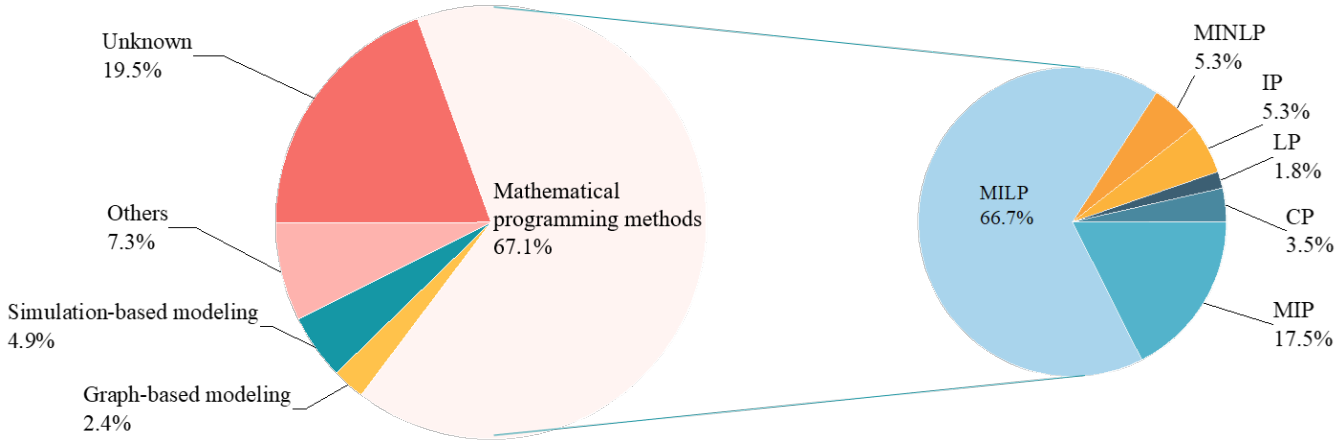


Figure 6. Statistical results of the different modeling methods.

In recent years, some publications have focused on the application of simulation-based modeling and graph-based modeling methods in the field of LWSOF, such as [2,41,53]. Among them, Vivek et al. and Jiang et al. simulated and verified the optimized scheduling scheme based on Extend V6 and Plant Simulation, respectively [24,53]. Knopp et al. [2] addressed lot-splitting flexible job shop scheduling problems based on conjunctive graphs in 2017. In [41], a hybrid flow shop scheduling problem based on machine learning technology was modeled and optimized. The authors in [80] explored the application of the genetic programming method in LWSOF for the first time. However, such research is still at the starting stage, and the relevant theories for LWSOF are waiting to be further enriched.

2.4. Solution Approaches

Many feasible methods have been introduced to solve the LWSOF, including commercial solvers, e.g., CPLEX [97] and Lingo [98], evolutionary algorithms (EAs) or swarm intelligence algorithms (SIAs), e.g., SA and GA, heuristic algorithms, machine learning algorithms, and hybrid methods. Moreover, to overcome the shortcomings of these methods, diversified improvement strategies have been designed according to the characteristics of different types of problems, and achieved remarkable results. Figure 7 shows relevant statistics on the publications in Table 1 according to the different solution methods. More than 47% of them include variable EAs or SIAs, compared to 3.2% for single heuristics. Furthermore, the hybrid methods and mathematical optimization and control approaches are used in over 24.5% and 10.6% works, respectively. However, only six publications use machine learning algorithms. The number of works less than two is collected into the "Others" category. It is evident that EAs and SIAs are the most commonly used methods for solving LSWOF. Thus, we next highlight key areas of focus for solving the LSWOF by EAs and SIAs and discuss their limitations. Moreover, some optimization directions are provided in Section 3.3.

Since EAs or SIAs are robust and modular in their operational steps, only the problem-related operations of the algorithms need to be modified for quick application to address different problems with high solution quality. The general framework of EAs or SIAs is available in [1]. The following two aspects should be noted when applying it to solve the LWSOF: (1) Reasonable coding and decoding of each sub-problem. Coding and de-

coding is the basis and premise of algorithm optimization, which realize the bi-directional mapping between solution space and problem space. Unreasonable coding and decoding methods may produce infeasible solutions, which leads to low population diversity and thus affects the actual performance of the algorithm. Thus, many effective coding and decoding methods are designed to avoid this problem. For example, Jiang et al. [53] proposed a three-segment encoding and greedy insertion decoding method. Yang et al. [62] performed an operation-level batching encoding method, and Zhu et al. [64] presented a bi-layer fixed-length encoding approach. (2) Design of feature-oriented efficient search operators. The search operator is the core of EAs or SIAs, which directly determines the search performance of the algorithm. Some general search operators are crossover and mutation operators [3,5], neighborhood search operators [2,24], simulated annealing operators [6,16], differential evolution operators [53,66], tabu search operators [33,34]. However, the long search time has limited the practical application effect of EAs or SIAs for LWSOF.

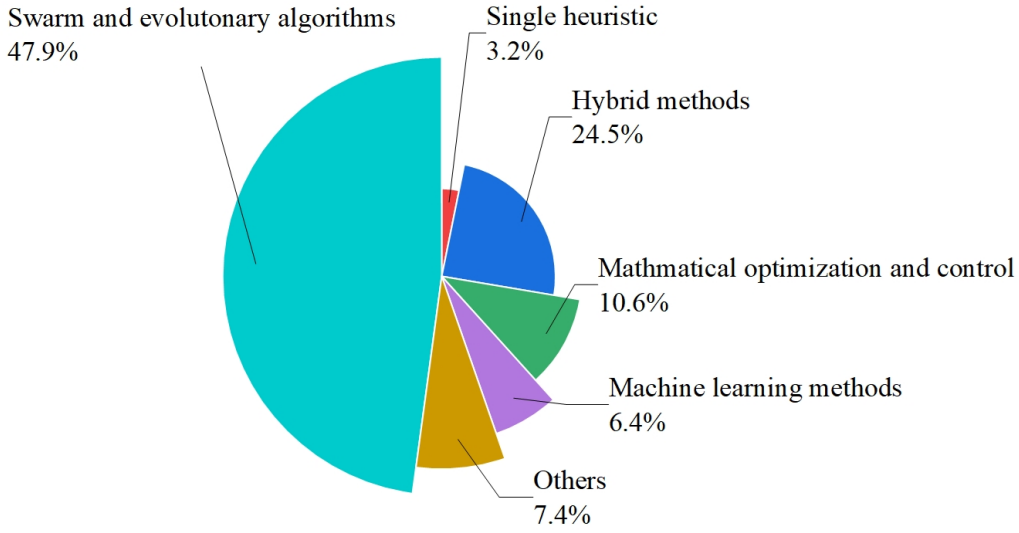


Figure 7. Statistical results of different solution approaches.

Compared with EAs or SIAs, deep learning methods such as reinforcement learning spend long times on the offline training, which greatly shortens the search time of practical problems. In general, for solving the LWSOF, a lot of effort is required to design and fine-tune since the manual reward depends on domain expertise. In addition, for the LWSOF with multi-variety and small batch rapid customization, the lack of data sample size greatly limits its solution accuracy. Thus, to handle this issue, some relevant explorations have been made. Lu et al. presented a multi-stage method based on GA, heuristic and Q-learning [83]. Omid et al. [14] addressed the uncorrelated parallel machine problem based on the random forest and branch-and-price algorithm. Considering the advantages of heuristic methods, such as high interpretability, easy implementation and fast solution speed, many related works have been carried out in LWSOF, and important progress has been made [21,22,70]. However, weaknesses of heuristics, such as low accuracy and poor continuous optimization capability in solving complex problems, also limit their applications. Thus, a series of hybrid approaches have been designed by combining heuristic methods with other evolutionary computation as well as machine learning methods and achieved remarkable results on different types of LWSOF [67,81,83]. The general framework of the reinforcement learning method for solving the LWSOF is illustrated in Figure 8 and described as follows.

First, the mathematical model of LSWOF (e.g., MILP) is constructed. Second, the state space and action space are defined, and the reward function is designed based on the optimization objectives and constraints of the proposed model, respectively. Subsequently, environmental parameters are set, and the presented model is transformed into a Markov decision

process. Furthermore, the network architecture is constructed, and the exploration strategy of the agent is determined. Additionally, a suitable deep reinforcement learning algorithm, such as Deep Q-Network (DQN), Proximal Policy Optimization (PPO), or Asynchronous Advantage Actor-Critic (A3C), is selected. Finally, the training process is executed.

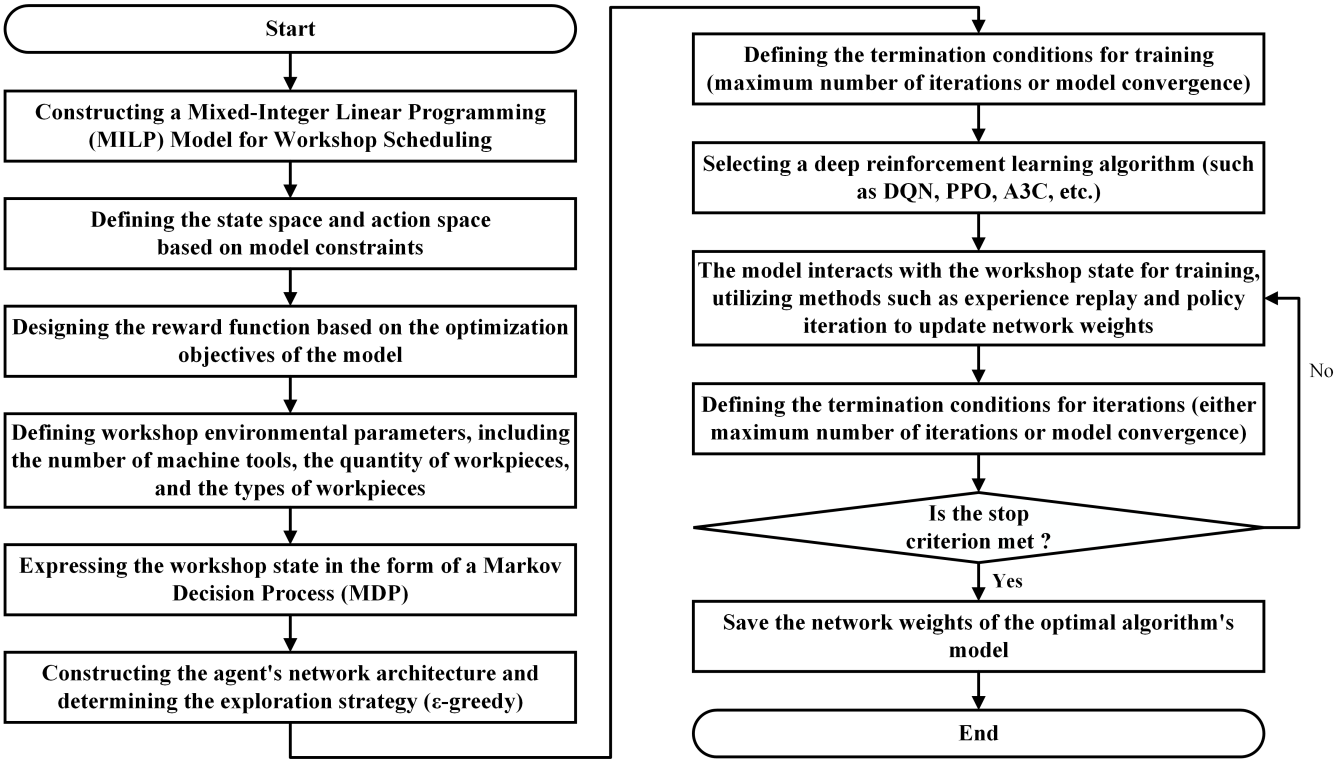


Figure 8. The general framework of reinforcement learning method for solving the LWSOF.

3. Research Trends

In the past 10 years, the optimization method of LWSOF has been widely studied. About 37 papers were published in the three years from 2022 to 2024, and the number of related articles has accounted for nearly half of the total number of papers. In particular, 17 and 10 related articles were published in 2023 and 2024, respectively. It can be inferred that the research on LWSOF in different real environments is gradually becoming a hot spot.

3.1. From Single Objective to Multi-Objective Considering Energy Consumption

Compared with the traditional empirical splitting methods, the application of greedy splitting methods has attracted more attention. Many publications explore the coupling relationship between the sub-problems of job splitting and machine assignment as well as operation sequencing, which provide the basis for implementing greedy optimization. Recently, with the rapid development of computer hardware, evolutionary computation, deep learning and other artificial intelligence approaches, the solution space of LWSOF is available to be fully explored. Many works have been gradually reported on the greedy optimization methods of complex LWSOF with large solution space, such as LWSOF with unequal variable lot-splitting and distributed LWSOF. The number of publications based on optimization objective from 2015 to 2025 is counted and shown in Figure 9. It can be deducted that rather than the single-objective LWSOF with traditional scheduling indicators such as makespan and processing cost, multi-objective LWSOF with energy consumption indicators has become a new research trend. Among the 27 related articles published in the five years from 2015 to 2019, there are 10 articles that refer to multi-objective LWSOF, while only two articles focus on the multi-objective LWSOF related to energy consumption.

However, among the 53 articles published in about five years from 2020 up to now, the ratio of multi-objective LWSOF considering energy consumption increased from 7% to 22.6%, reaching to 12 publications.

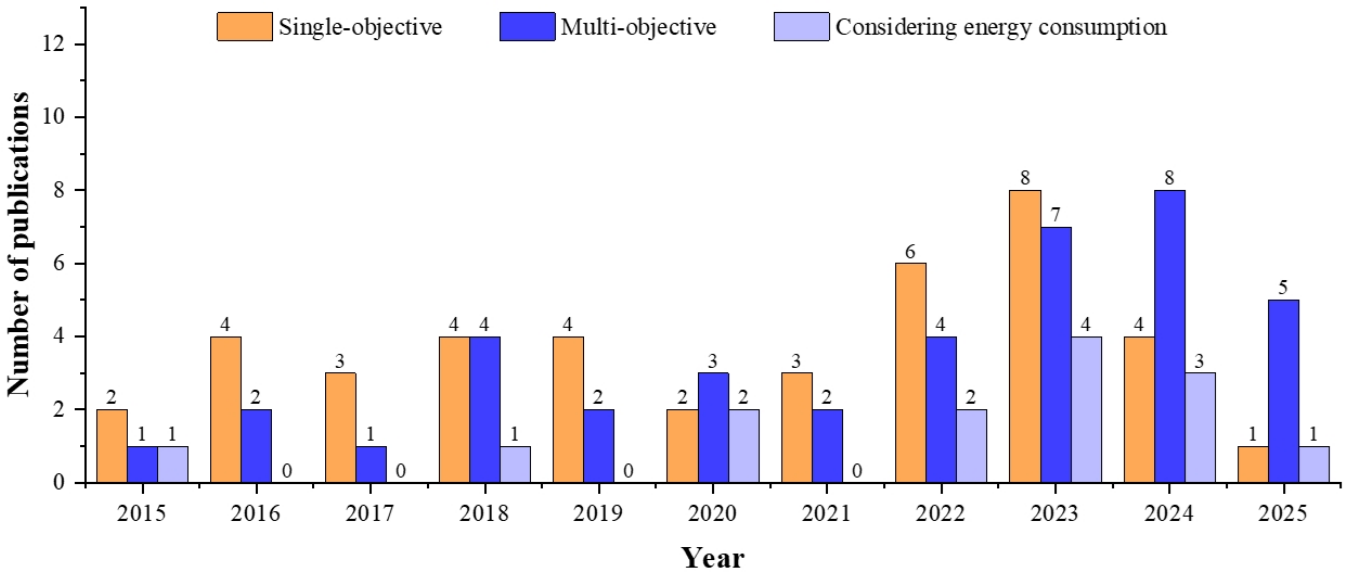


Figure 9. Publication counts of optimization objective from 2015 to 2025.

3.2. Perfecting of Mathematical Programming Models

From the statistical results in Figure 6, it can be found that the mathematical program is the main modeling method of LWSOF, and its related theories have been continuously developed and improved for many years. Moreover, it has been widely favored by researchers to formulate diversified and efficient mathematical programming models through analyzing problem characteristics. In addition, compared with mathematical program models, simulation-based and graph-based models provide a new way to solve the LWSOF. For graph-based modeling methods, most of the articles only focus on the LWSOF with a single optimization objective or relatively simple constraints, and its application is limited by the difficulty of rationalization representation of multi-objectives and complex constraints. As for the simulation-based modeling approaches, the models are less robust and difficult to be applied to complex and changeable production scheduling practices due to the large influence of problem structure and parameter changes.

3.3. Extending of Problem Feature-Oriented EAs or SIAs

According to the statistical results in recent years, EAs or SIAs remained the primary algorithms for solving LWSOF among all approaches. The present EAs or SIAs for LWSOF can be divided into seven categories, such as GA, SA, PSO, ABC, DE, TS and others, as shown in Figure 10. Among them, GA accounts for the highest proportion, reaching 45%, while ABC ranked second which reaches nearly 15%. The proportion of mainstream algorithms such as SA, MBO and TS is close to or slightly higher than 7%. Furthermore, the algorithms that account for less than 3% are classified into "Others", and the total proportion reaches 15%. For the different LWSOF, the variations in problem size and feature mechanism make it difficult for classical operators to maintain efficient search ability, and the improved operators for one problem are not generally effective when applied to other problems. Then, to handle this issue, a variety of efficient search operators are designed by analyzing the characteristic mechanism of the problem, including knowledge-driven operators [59,60], multi-stage [29,34], and multi-operator fusion [44,50,53]. At the same

time, the deep learning algorithms emerging in the past three years also provide a new way to solve LWSOF.

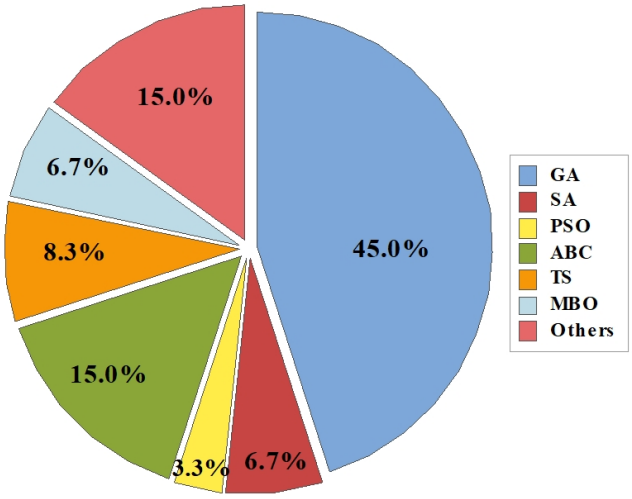


Figure 10. Publication counts of EAs or SIAs from 2015 to 2025.

4. Future Directions

Through summarizing the current studies and analyzing the research trends, the future research of LWSOF can be prospected from four aspects: lot-splitting strategy, problem extension, modeling methods and solution approaches.

4.1. Knowledge and Greedy-Based Lot-Splitting Strategies

Lot-splitting is an important sub-problem of LWSOF. At present, the empirical lot-splitting methods rely on the prior knowledge of decision makers, which leads to the problems of a large performance gap and difficult verification of lot-splitting schemes. The greedy lot-splitting methods encounter difficulties in balancing the search accuracy and time for the huge solution space of LWSOF, which makes it hard to obtain a well-performance lot-splitting scheme in a limited time. Therefore, a knowledge and greed-based lot-splitting method for LWSOFs is designed by exploring the mechanism of the impact of batching on problem size and scheduling objectives under the flexible production model and introducing appropriate algorithms as well as evaluation metrics to assess the effectiveness of the lot-splitting scheme. In this way, the convergence of the method and the diversity of the lot-splitting scheme are guaranteed, the search time and performance of the algorithm are balanced; hence, the appropriate lot-splitting is realized.

4.2. Practice in Actual Scheduling Environments with Multi-Constraint and Explore the Interaction Between Constraints

Reasonable assumptions are the foundation and prerequisite for scientific research. However, due to the diverse and complex constraints in actual production, scheduling schemes that consider a single constraint may not be directly applicable. In actual scheduling environments of discrete manufacturing enterprises, the production process generally involves the coordinated allocation of processing, set-up, and material handling resources, such as workpieces, machine tools, fixtures, and automated guided vehicles. The allocation results directly determine the production efficiency and smoothness. Meanwhile, uncertainties such as random job arrivals, machine degradation effects, and worker learning effects pose significant challenges to the construction of workshop scheduling models and their optimization. These uncertainties not only make it difficult to accurately predict the input parameters of the scheduling model but also cause fluctuations in objectives and conflicts in constraints during the actual execution of the optimal scheduling solution. As a result,

traditional optimization methods lack robustness and struggle to achieve high-quality scheduling solutions. However, there is currently limited research combining flexible production scheduling techniques under uncertain environments with lot-streaming technologies. Therefore, it is an important direction for future research to explore the possible interaction among constraints and investigate the mechanisms of multi-resource collaborative allocation for LSWOF problems under uncertain environments. Furthermore, it is also important to expand practical applications of the related approaches in aerospace, semiconductor, automobile, pharmaceutical and other fields. In addition, with the extensive attention of energy-saving and green scheduling indicators, it is of great theoretical significance and practical value to consider the green LWSOF problems under multi-objective and high-dimensional objective optimization based on practical application scenarios.

4.3. Developing of Highly Accurate Mathematical Planning Models and Graph-Based Models

First, for the LWSOF, the mathematical program modeling approach will still be the primary way. By extracting the problem features and reasonable simplification, the feasibility and efficiency of various mathematical planning models under different modeling ideas are analyzed; hence, an optimized mathematical programming model is formulated. Then, to enhance the usefulness of production scheduling solutions, it is essential to introduce flexibility into the model. In this way, the model will be able to adapt to different conditions and changes. In other words, the dynamics of the actual situation need to be reflected through adjustable parameters or constraints. Meanwhile, to facilitate the understanding of the model results by decision-makers, the optimization objectives and the explainability of the model should be clear and improved, respectively. Additionally, sensitivity analyses need to be performed to assess the response of the model to changes in input parameters to improve the robustness of the model. Moreover, with the enrichment of deep learning theories such as graph neural network and reinforcement learning, the related graph-based modeling methods including disjunctive graph model, petri net, AND/OR, and expression tree deserve further study. Finally, to cope with the problem that most graph-based modeling methods adopt simplified models, which are difficult to fit the actual production, the models should be reasonably expanded based on the analysis of problem characteristics and then establish various general graph models to provide a basis for the application of deep learning methods.

4.4. Improvement of EAs and SIAs as Well as Deep Learning Methods

Extended directions on solution approaches for solving LWSOF can be concluded as follows: First, by exploring the mechanism of different problem features, the effective rules and properties are refined, and the diversified efficient search operators are designed. Furthermore, with the combination of the advantages of heuristics and evolutionary algorithms, the hybrid optimization methods of problem knowledge-driven heuristics and evolutionary algorithms are proposed. In this way, the search efficiency and accuracy of the algorithm can be improved. Second, extending deep learning methods such as neural network, reinforcement learning and random forest in solving LWSOF. Moreover, digital-twins with deep learning methods are integrated to allow deep learning agents to interact with virtual workshops instead of physical ones. Otherwise, efficient data acquisition and preprocessing methods are used to meet the dependence of deep learning methods on high-quality data. In addition, through investigating the mapping relationship between the lot-splitting schemes and the possible scheduling results obtained by the optimization algorithm, the optimization results are used to guide the lot-splitting process. Based on the possible scheduling results of the lot-splitting scheme, the evolutionary algorithm

optimization or scheduling rule learning process is directional, resulting in a two-way feedback optimization of lot-splitting and workpiece scheduling.

4.5. Exploiting of Digital-Twin Based Optimization Frameworks

In addition, as a new method of simulation modeling, digital-twin technology has a very broad application prospect in complex LWSOF. With the information flow based on the simulation model, the production scheduling process of online monitoring and intelligent optimization decision-making can be carried out to realize the closed-loop of information flow and decision-making flow. Based on the five-dimensional model of digital-twin, a bidirectional mapping digital-twin architecture system is constructed. The LWSOF digital-twin model is established through using computer-aided software (NX [99], Technomatix platform [9]) and the acquired data information, which lays the foundation for the extended research. Digital-twin-based optimization methods allow for more accurate capture of the specific processes and constraints on the workshop, as well as more flexibility to react to unexpected events and changes, thereby improving the quality of real-time decision-making. Finally, no matter what modeling method is selected, it is necessary to ensure the high efficiency of the model and maximize the accurate description of the real production elements in the workshop.

5. Conclusions

Lot-streaming has been a powerful tool to optimize the operation of the manufacturing system. In the LWSOF, through the batch production and transmission of workpieces, the workshop operation efficiency is improved and the rational utilization of workshop production resources is realized. Although many works are reported in flexible workshop scheduling and lot-splitting scheduling, there is no comprehensive survey study in this area. Based on this, this paper reviews LWSOF from the perspective of lot-splitting, problem type, modeling and solution methods for the first time. First, through the classification statistics and analysis of the literature, the existing research progress and shortcomings are reviewed from four aspects: lot-splitting methods, objectives and constraints, modeling methods and solution approaches. Then, the current research trend of LFJSP is analyzed. Finally, based on the analysis and summary of the existing research results and trends, the possible future research directions of LWSOF are prospected.

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List of Abbreviations

ABC	Artificial bee colony	MILP	Mixed integer linear programming
ACO	Ant colony optimization	MINLP	Mixed integer non-Linear programming
CP	Constraint programming	MIP	Mixed integer programming
DE	Differential evolution algorithm	NILP	Non-integer linear programming
EA	Evolutionary algorithm	NN	Neural network
ECS	Equal consistent sub-lots	NSGA	Non-dominated sorting genetic algorithm
EDA	Estimation of distribution algorithm	PSO	Particle swarm optimization

GA	Genetic algorithm	MBO	Migrating birds optimization
IG	Iterated greedy	SA	Simulated annealing
KMA	Knowledge-based memetic algorithm	ER-GA	Early release GA
TS-ISMO	Two-stage improved spider monkey optimization	CHS-GPHH	Collaborative Harmony Searchbased Genetic Programming Hyper Heuristic
ILP	Integer linear programming	SVM	Support vector machine
IP	Integer programming	TS	Tabu search
LP	Linear programming	UCS	Unequal consistent sub-lots
UVS	Unequal variable sub-lots	VNS	Variable neighborhood search
VND	Variable neighborhood descent	EVS	Equal variable sub-lots

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