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Improving Ride-Hailing Platform Operations in Dynamic Markets: A Drivers' Switching Perspective

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Abstract: Improving the performance of the operations of ride-hailing platforms (RHPs) by adequately considering drivers' switching behaviors is becoming crucial for their profitability and sustainability. This study explores how to optimize the operations of RHPs by investigating the impact of commission rates on drivers' switching behaviors in a dynamic mobility market. Two queue-theory-based mathematical models have been developed to explore the relationship between commission rates, drivers' switching behaviors, and critical platform parameters in optimizing the operations of RHPs. Numerical examples are presented to demonstrate the applicability of such models in determining the best commission rate to optimize the operations of RHPs in duopoly and fully competitive market conditions. The findings suggest that understanding the intricate relationship between commission rates, drivers' switching behaviors, and critical platform parameters is significant for RHPs in formulating appropriate strategies and policies to ensure their sustainable operations.

Keywords: shared mobility; ride-hailing platforms; platform operations; commission rates; drivers' switching behaviors; market competition

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1. Introduction

Ride-hailing is an emerging shared mobility solution focusing on the delivery of prearranged mobility services that connect drivers of private vehicles with customers through ride-hailing platforms (RHPs) [1,2]. It is becoming increasingly popular due to the rapid development of digital technologies [3] and the increasing availability of mobile applications for facilitating booking, paying, and rating mobility services in a holistic manner [4–6]. This results in growing competition between these platforms [7–9].

There are increasing initiatives in the development of RHPs in China for improving the mobility of individuals [10], exemplified by the average daily order volume of about 29 million trips on more than 349 RHPs [11]. This leads to fierce competition between and among RHPs for their profitability and sustainability in a dynamic mobility market. More and more platforms have joined the ride-hailing market due to the increasing demand for shared mobility services after the pandemic [12], further exacerbating the competition. As a result, determining how to improve the performance of ride-hailing operations of these digital platforms is becoming critical [13].

The operations of RHPs are running in a two-sided market by connecting drivers with customers in a dynamic environment [14,15]. Improving the operations of RHPs, therefore, is becoming complex and challenging [16,17]. There are various variables that must be

adequately considered in formulating appropriate strategies and policies for optimizing the operations of RHPs in a profitable and sustainable manner [13,18–20].

Drivers are the vital resource in the mobility market for providing ride-hailing services through their employment in RHPs [10,21]. They often change their employment from one platform to another, commonly referred to as drivers' switching behaviors [22], in pursuing better working conditions including higher income [23,24]. Such switching behaviors adversely affect the sustainable operations of RHPs due to the potential shortage of platform drivers, resulting in increased customer waiting time and poor customer service quality, ultimately leading to the deterioration of the profitability and sustainability of individual RHPs [8,25,26].

Understanding how drivers' switching behaviors affect the profitability and sustainability of RHPs is critical for improving their operations [19,27]. Such behaviors are usually related to the determination of the commission rate between RHPs and drivers. Setting a high commission rate, i.e., the proportion of the fare to the platform, can increase the revenue of the platform in the short term. This, however, may lead to more frequent drivers' switching behaviors. It, therefore, weakens the overall profitability and sustainability of individual RHPs in the long term [17,26]. This suggests that exploring the interplay between commission rates, drivers' switching behaviors, and critical platform parameters is crucial for ensuring the profitable and sustainable operations of RHPs.

Many studies have been conducted to explore how to develop effective operations strategies through leveraging various pricing mechanisms and non-monetary incentives for optimizing the operations of RHPs [13]. Cohen and Zhang [28], for example, develop an endogenous model to simulate the operations of two-sided RHPs, leading to the determination of the optimal service price and the most appropriate commission rate in an analytical manner. Sun and Ertz [26] propose a system dynamic modeling framework to explore the ride-hailing operations, discovering that commission rates, order prices, and investment levels are the critical variables for optimizing the operations of RHPs. Zhong et al. [22] examine how RHPs compete with traditional taxi companies, resulting in designing specific pricing strategies under unregulated and regulated environments in optimizing the operations of RHPs. The studies above have provided insights into how to improve the operations of RHPs from different perspectives.

Existing studies concentrate on understanding how to improve the operations of RHPs by maximizing platform revenue [29], improving customer service utility [30], and increasing customers' social welfare [24,28]. While commission rates are widely considered in most studies, the findings on the impact of commission rates on the profitability and sustainability of platform operations are inconclusive [26]. There is a lack of understanding of the relationship between commission rates, drivers' switching behaviors, and critical platform parameters in optimizing the operations of RHPs. To address these gaps, a research question that this study aims to address is formulated as follows: How do commission rates affect drivers' switching behaviors in optimizing the operations of RHPs in the dynamic mobility market?

This study examines the impact of commission rates on drivers' switching behaviors for optimizing the operations of RHPs. The related literature has been comprehensively reviewed, leading to the development of two queue-theory-based mathematical models for exploring the relationship between commission rates, drivers' switching behaviors, and critical platform parameters in optimizing the operations of RHPs. Numerical examples are presented to demonstrate the applicability of the two models in optimizing the operations of RHPs. The study reveals that there is an intricate relationship between commission rates, drivers' switching behaviors, and critical platform parameters in optimizing platform operations. These findings highlight the importance of understanding such a complex

relationship for formulating appropriate strategies and policies to ensure the profitability and sustainability of RHPs in a dynamic mobility market.

In what follows, Section 2 presents a literature review, followed by the formulation of the research problem in Section 3. Section 4 develops two queue-theory-based mathematical models. Section 5 describes three examples for demonstrating the applicability of the two models in optimizing the operations of RHPs in the dynamic ride-hailing market. Section 6 presents the research findings followed by the concluding remarks.

2. Literature Review

Ride-hailing is operated in a dynamic mobility market [13,19]. Optimizing ride-hailing operations involves making timely decisions with respect to specific demand and supply issues while considering the requirements and expectations of various stakeholders [10]. This brings substantial complexity and challenges to individual RHPs [19,26,31–34].

Much research has been conducted to understand the operations of RHPs from different perspectives. An examination of such research shows that existing studies can be classified into three streams including (a) operations, (b) critical factors, and (c) market evolution and platform competition. Such studies have provided insights into how to optimize the operations of RHPs in the dynamic ride-hailing market. Table 1 presents an overview of these studies from these three perspectives.

Table 1. An overview of the related studies on optimizing ride-hailing operations.

Themes	References	Approaches/Theories	Findings/Critical Factors
Operations	Bahrami et al. (2022) [8]	Nonlinear optimization and queueing theory	Returns-to-scale, marginal fleet operating cost are the crucial variables for underpinning the ride-hailing operations
	Lin et al. (2024) [9]	Game theory	An asset-heavy model on ride-hailing operations for a win–win outcome
	Guo et al. (2024) [13]	Game theory	Derive the conditions that bring the optimal platform revenue
	Chen et al. (2020) [30]	Dynamic programming	A dynamic vacant car–passenger meeting model
	Ke and Qian (2023) [34]	Bi-level optimization	A novel optimal ride-hailing pricing scheme
	Alonso-Mora et al. (2017) [35]	Integer linear optimization	A general mathematical model for real-time high-capacity ridesharing
	Di and Ban (2019) [36]	Quasi-variational inequality	A theoretical framework to unify driving solo, ridesharing, and e-hailing services
	Ke et al. (2020) [37]	Macroscopic fundamental diagram	Passenger demand and matching window range affect traffic congestion and riders' travel time
	Sun et al. (2020) [38]	Queueing theory	Explore the optimal operations on two matching systems: Inform and Assign
	Xu et al. (2021) [39]	A fluid modeling framework	A macroscopic fluid modeling framework to operate a large-scale on-demand app-based ride-hailing system
	Bandiera et al. (2024) [40]	Mathematical model	A mathematical approach to capture the interaction between multiple providers
	Xi et al. (2024) [41]	Mathematical model	A multi-leader multi-follower game model to maximize profits and balance profitability with environmental issues
	Yao and Zhang (2024) [42]	Mathematical model	Study the assignment and pricing problem for a MaaS platform
Critical factors	Zhou et al. (2024) [10]	Survey	Perceived high workload, salary satisfaction, job satisfaction
	Mourad et al. (2019) [14]	Literature review	Demand attributes, drivers' behavior, available driver number, and platform service capacity
	Wang and Yang (2019) [19]	Literature review	Service quality, driver income, passenger demand, driver supply
	Zhong et al. (2022) [22]	Nonlinear optimization	Competition intensity, commission rate, drivers' opportunity cost
	Acheampong et al. (2020) [43]	Survey	Socio-demographic factors, perceived benefits, ease of use, perceived safety risks, and car-dependent lifestyles
	Dean and Kockelman (2021) [44]	Spatial linear panel model	Socioeconomic, built environment, temporal, and trip-specific variables
	Li et al. (2022) [45]	Multinomial logistic models	Personal and family attributes, travel information, perceived performance, perceived environment
	Chen et al. (2023) [46]	Markov model framework	Income satisfaction

Table 1. *Cont.*

Themes	References	Approaches/Theories	Findings/Critical Factors
Market evolution and platform competition	Wang and Li (2024) [2]	Game theory	The transfer of AVs between two platforms can lead to a win–win situation
	Zhang et al. (2022) [15]	Game theory	Driver selection behavior has an impact on platform price competition and wage
	Wang and Yang (2019) [19]	Literature review	Different types of digital platforms can use strategic or operational tools to improve their operations
	Sun and Etrz (2021) [26]	System dynamics modeling	Commission rate, order price, and investment level affect sustainable growth
	Cachon (2019) [47]	Theory of the firm boundary	Pricing, timing, location, and control affect competition
	Benjaafar and Hu (2020) [48]	Inventory theory, and queueing theory	The number of platforms has a substantial impact on the competition
	Rietveld and Schilling (2020) [49]	Literature review	Pricing, quality, vertical integration, and diversification influence platform competition
	Sun et al. (2023) [50]	System dynamics	Policy restriction, social acceptability, and market competition intensity affect the operations of platforms

Operations-based studies focus on the formulation of specific strategies and policies for improving the performance of RHPs [8,9,13,30,35–42]. Bahrami et al. [8], for example, establish an analytical model to understand the impact of operational variables in matching supply with demand to maximize the profit of RHPs. Lin et al. [9] examine the impact of an asset-heavy model on the profitability of RHPs, revealing that such a model can provide a "win–win–win" outcome for platforms, customers, and drivers in the competitive mobility market. Guo et al. [13] analyze the impact of three service mode selection strategies on customers and drivers, leading to the identification of the conditions that bring the optimal platform revenue. Chen et al. [30] develop a dynamic model for characterizing the matching process between waiting customers and vacant cars in ride-sourcing for optimizing the operations of RHPs. Alonso-Mora et al. [35] present an algorithm for optimizing the operations of large-scale RHPs focusing on more passenger assignments and less waiting time via an experiment using real-world data. Di and Ban [36] propose a conceptual framework for exploring the interaction between private vehicles and service fleets for developing sustainable transport. Ke et al. [37] investigate the benefits of ride-pooling with respect to different levels of customer demands using specific strategies, finding that the level of customer demand and the range of the matching window are the two critical platform parameters for optimizing their operations. Sun et al. [38] discover that two types of order assignment strategies have distinct impacts on the operations of RHPs. Xu et al. [39] propose a generalized fluid framework incorporating appropriate commission rate setting for modeling ride-hailing to improve the operations of RHPs. Ke and Qian [34] develop an optimal pricing scheme for improving the operations of RHPs. Bandiera et al. [40] present a mathematical model to simulate the interaction between service providers and customers. Xi et al. [41] develop a multi-leader–multi-follower model in which platforms compete to optimize their operations. Yao and Zhang [42] adopt a many-to-many matching framework for solving the pricing problem that RHPs face in a multi-modal transport network.

Critical-factor-aligned studies concentrate on understanding the critical factors affecting the operations of RHPs in the dynamic ride-hailing market [10,14,19,22,43–46]. Zhou et al. [10], for example, offer an empirical explanation for the turnover intention of ride-hailing drivers in which drivers' income satisfaction is critical for affecting the operations of RHPs. Mourad et al. [14] discover that demand attributes, drivers' behaviors, driver supply, and platform service capacity are critical for optimizing the operations of RHPs. Wang and Yang [19] reveal that service quality, driver income, customer demand, and driver supply are crucial for influencing the efficiency of the operations of RHPs. Zhong et al. [22] find that the intensity of the competition between platforms, commission rates, and drivers' opportunity costs are critical for affecting the operations of RHPs.

Acheampong et al. [43] show that socio-demographic factors, perceived benefits, ease of use, perceived risks, and car-dependent lifestyle influence the utilization of ride-hailing services. Dean and Kockelman [44] find that trip length, day of the week, pedestrian density, multi-modal infrastructure, and socioeconomic characteristics have a substantial impact on ride-hailing operations. Li et al. [45] examine the critical factors for using different types of ride-hailing services, showing that there are different critical factors that influence the utilization of respective ride-hailing services. Chen et al. [46] argue that drivers' income satisfaction is critical for the operations of RHPs.

Market-evolution-and-platform-competition-oriented studies try to understand the interaction between platform operations, service competition, and market evolution in delivering ride-hailing services [2,15,19,26,47–50]. Wang and Li [2], for example, study the competition between an emerging autonomous RHP and a traditional RHP by characterizing the equilibrium of their competition, finding that the transfer of autonomous vehicles between the two types of platforms can lead to a win–win outcome in an increasingly competitive mobility market. Zhang et al. [15] examine the competition between RHPs in a shared economy, revealing that the competition is more intense in a shared economy than in a traditional one. Wang and Yang [19] examine the strategic and operational issues with respect to different types of digital platforms for understanding how to improve the operations of RHPs. Sun and Ertz [26] explore the growth mechanism of a simulated digital platform from a systematic perspective for investigating the sustainable growth mechanism in optimizing the operations of RHPs. Cachon [47] states that pricing mechanism, location, and control of the supply side are the substantial factors influencing the competition between RHPs and market evolution. Benjaafar and Hu [48] argue that understanding how incentives are affected by multisided platforms is critical for optimizing platform operations. Rietveld and Schilling [49] present a comprehensive review of the platform competition literature for developing sustainable operations of RHPs. Sun et al. [50] investigate the influence of various regulatory policies on the operations of RHPs with respect to the growth of individual platforms.

The discussion above has provided a better understanding of how to improve the operations of RHPs from three perspectives. There is, however, a lack of studies on how to achieve a subtle trade-off between platform profitability and customer service quality in optimizing the operations of RHPs. The interplay between commission rates, drivers' switching behaviors, and critical platform parameters in optimizing the operations of RHPs is unclear. This demonstrates an urgent need to address the current research problem as detailed above in this study.

3. Problem Formulation

There are various platform variables that affect the operations of RHPs in the dynamic ride-hailing market [15,26]. Such variables can be classified into three perspectives including the supply side, the demand side, and the operations side. Supply-side factors include drivers' commission rates, drivers' quantity, drivers' switching behaviors, and drivers' income. Demand-side variables consist of customers' socio-demographics, trip attributes, and psychological or behavioral factors. Operations-side factors include price and income-related factors in which commission rates are employed as a main income lever for the profitability of individual RHPs.

Commission rates are critical for optimizing the operations of RHPs. Increasing the commission rate can lead to higher platform profitability while introducing drivers' switching behaviors. This negatively affects the platform's customer service quality, thus reducing the number of orders and leading to a decline in platform profitability [15,26].

This section formulates the research problem to facilitate the development of the mathematical model for optimizing the operations of RHPs. To pave the way for the problem formulation process, a comprehensive overview of the operations of RHPs is discussed. How the commission rate affects the platform performance is elaborated. Some basic assumptions are proposed together with notations and specifications employed in the study. All these underpin the development of the mathematical model for addressing the research problem formulated in this section.

3.1. The Mechanism and Evolution of the Operations of RHPs

There are three stages in the development of the ride-hailing market including (a) a monopolistic market, (b) an oligarchic market, and (c) a competitive market [4,31–33]. Stage one is characterized by the use of conventional taxis in the market, referred to as hailing systems R_T. Stage two is exemplified by the introduction of RHPs, indicated as hailing systems R_1. Stage three features the addition of more RHPs in the market, referred to as RHP follow-up systems R_2.

Determining the commission rate directly affects the relationship between RHPs and their drivers, leading to drivers' switching behaviors [17,33]. Excessive commission rates encourage drivers to transfer from one platform to another, therefore reducing the availability of drivers in specific RHPs. As a result, the service capacity of such RHPs suffers. This leads to customers waiting for a longer time for expected services [19]. The profitability of RHPs and their sustainability in the long term are, therefore, adversely affected.

As critical platform performance indicators, customer waiting time and platform productivity have a crucial impact on the customer service quality of RHPs [19,21]. Excessive customer waiting time or poor productivity have a negative impact on platform customer service quality. Customers can cancel orders due to long waiting time, leading to a reduction in platform revenue. The decrease in platform revenue affects the determination of the commission rate in the platform. RHPs, therefore, have an evident motivation to increase the commission rate to pursue higher operational revenue, while contemplating their commission rate decisions for achieving the trade-off between operations revenue in the short term and profitable operations in the long term, especially when facing increasingly fierce market competition [22,26,34]. Figure 1 presents an overview of the mechanism of the operations of RHPs in a dynamic ride-hailing market.

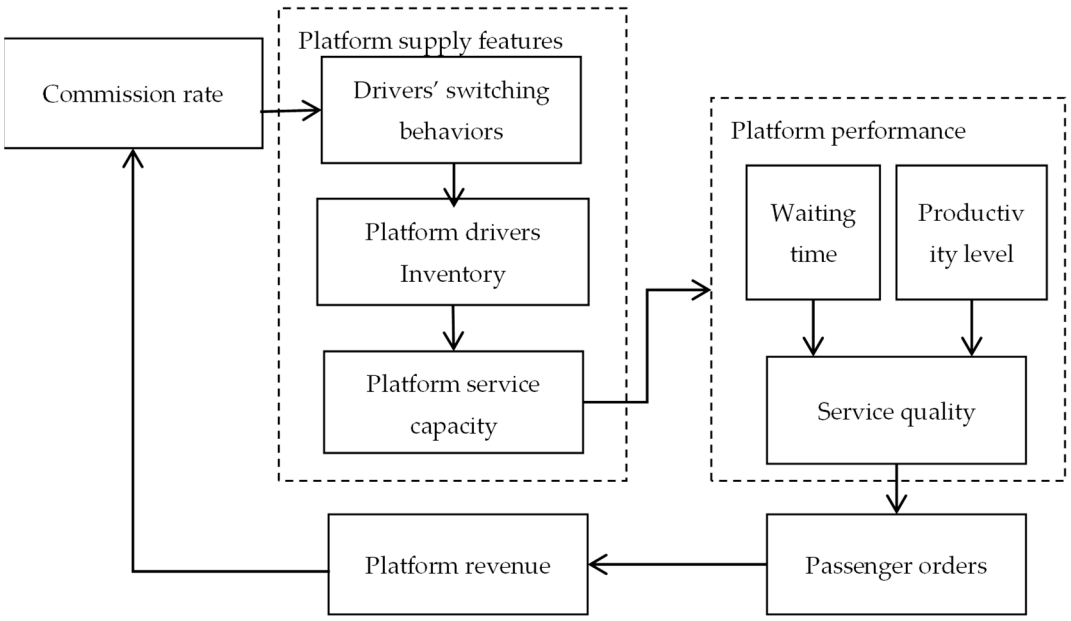


Figure 1. An overview of the mechanism of the operations of RHPs.

3.2. Drivers' Switching Behaviors

This study explores the impact of commission rates on drivers' switching behaviors for optimizing the operations of RHPs in a dynamic mobility market. Without the loss of generality, the study assumes that the ride-hailing market is open and dynamic in which drivers are free to move from one platform to another to pursue individual benefits.

The availability of drivers influences customer service quality and platform profitability [10,50]. As a result, the leading platform needs to set up a reasonable commission rate to mitigate the loss of drivers in ensuring its customer service quality. Platform customer service quality is measured by customer waiting time, which is a function of the number of drivers in the platform. The number of platform drivers is correlated with the commission rate. Excessive commission rates reduce drivers' income, thus resulting in drivers' switching behaviors [21,46].

The total number of RHPs in the market is assumed as N. The starting time that drivers begin their switching behaviors is T_1. The ending time that drivers stop their switching behaviors from the leading RHP $\mathbf{R}_1$ to R_i is T_i. The number of drivers switching from R_1 to the other RHP R_i is x_{1i}. Here, subscript i denotes 2, 3, . . ., N. It is assumed that the leading RHP R_1 has sufficient operations information; i.e., the drivers' switching quantities x_{12}, x_{13}, . . ., x_{1N} are well known. The switching process of individual drivers between the leading RHP and other RHPs is illustrated in Figure 2.

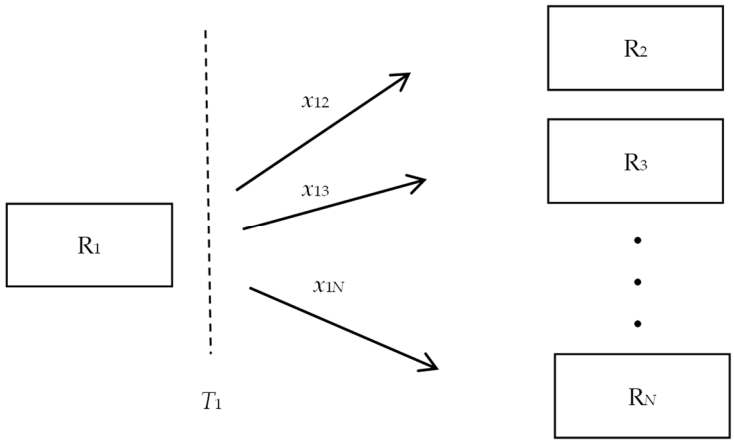


Figure 2. Overview of drivers' switching behaviors in a dynamic ride-hailing market.

3.3. Theoretical Foundation and Assumptions

This study treats the ride-hailing market as a queueing system [19,27]. It is assumed that there are no restrictions on the number of customers and platform capacity. Ride-hailing services are provided according to the principle of first-in and first-served [8,19]. Customers waiting for rides on the roadside or sending orders on their smart devices, such as mobile phones or tablets, are regarded as the "generation" of customers' demand [27]. Drivers pick up passengers on the roadside or accept customer orders on their smart devices through ride-hailing apps, which are regarded as starting to provide the services. Customers are regarded as "leaving" when they arrive at their destinations.

With the assumptions above, the ride-hailing market is treated as an M/M/S queueing system [27,51]. Based on the premise of knowing the arrival rate of customer orders and the vehicle service rate, crucial platform parameters such as customer queue length, customer waiting time, and average queue length can be derived. See Appendix A for the details.

Drivers' switching behaviors have a substantial impact on the operations of RHPs [14,19]. To better understand the relationship between commission rates, drivers' switching behaviors, and critical platform parameters in optimizing the operations of RHPs, several further assumptions are proposed in this study as follows:

Assumption 1: *The average trip distance and the unit price for the average trip distance are exogenous, determined by the travel demand and the current ride-hailing market properties.*

Assumption 2: *The generation of customer demand orders is a Poisson process with parameter $\lambda > 0$, and the time interval for drivers to provide services follows the exponential distribution with parameter $\mu > 0$.*

Assumption 3: *Drivers' switching rate between RHPs is proportional to the commission rate r and inversely proportional to platform productivity p.*

There are some notations that this study uses in addressing the problem above. Such notations are detailed in Table 2.

Table 2. Notations and specifications.

Notation	Description	Range	Notes
λ	Average arrival rate of order generation in a unit time	≥ 0	
μ	Average service rate of drivers completing orders in a unit time	≥ 0	
N	Total number of platforms in the market	$\geq$0, integer	
M	Total number of drivers in the market	$\geq$0, integer	$M = \sum_{i=1}^{N} S_i$
S_i	Number of drivers of the ith platform	$\geq$0, integer	$i = 1, \cdots, N$
V_i	Total order volume in a specific time of the ith platform	$\geq$0, integer	
r_i	Drivers' market share proportion of the ith platform	[0, 1]	$S_i = M \cdot r_i, r_1 \geq r_j$
c_i	Commission rate in the ith platform	[0, 1]	$i = 1, \cdots, N$
η_{ij}	Unit price of customer's trip distance of the jth order in the ith platform	≥ 0	$j = 2, \cdots, N$
l_{ij}	Customer's trip distance of the jth order in the ith platform	≥ 0	
p_i	Productivity level coefficient of the ith platform	[0, 1]	$i = 1, \cdots, N$
W_i	Average waiting time of the ith platform	≥ 0	
x_{1j}	Driver transfer quantity from the leading platform to the jth platform	$\geq$0, integer	
T_1	Starting time that a driver begins switching in the market	≥ 0	
T_i	Ending time that a driver stops switching to the *i*th platform	$\geq T_1$	$i = 2, \cdots, N$

4. The Model

Two queue-theory-based mathematical models are developed for exploring the relationship between commission rates, drivers' switching behaviors, and crucial platform parameters including platform drivers' market share proportion r_i, platform productivity p_i, and driver switching lasting time period $T_i - T_1$ from the leading RHP R_1 to the *ith* RHP R_i in optimizing the operations of RHPs. The operational revenue of the *ith* RHP is formulated as the cumulative order income as follows:

$$B_i = \sum_{j=1}^{V_i} c_i \cdot \eta_{i,j} \cdot l_{i,j} \tag{1}$$

This operational revenue function characterized as in Equation (1) has general applicability for different shared mobility modes such as ride-sourcing and ride-splitting [7]. The presence of ride-splitting means a decrease in the amount of customer orders in the market. All these situations can be addressed by changing the value of variable V_i in the operational revenue function as in Equation (1).

A duopoly competitive market in which there are two platforms, R_1 and R_2, is considered first. Here, R_1 is the leader, and R_2 is the follower. Drivers switch from R_1 to R_2 in the period $[T_1, T_2]$. To ensure the customer service quality of the platform, platform R_1 must set an appropriate commission rate to ensure that the average waiting time is not higher than that of R_2. As a result, the number of drivers switched x must satisfy the following condition:

$$W_1(Mr_1 - x) \leq W_2(Mr_2 + x) \tag{2}$$

To optimize the operations of RHPs, the best commission rate must be determined as follows:

Model 1: $\max_{c_i} B_i$

$$\text{s. t. } W_1(Mr_1 - x) \leq W_2(Mr_2 + x)$$

A fully competitive mobility market is considered here in which there are N platforms R_1, …, R_N existing in the market. R_1 is the leading platform, and R_2, …, R_N are the followers. Drivers switch from R_1 to R_2 in the period $[T_1, T_2]$. Drivers switch from R_1 to R_3 in the period $[T_1, T_3]$, …, and from R_1 to R_N in the period $[T_1, T_N]$. To ensure the level of customer service quality, R_1 must set an appropriate commission rate to guarantee that the average waiting time is not higher than all other platforms R_2~R_N. As a result, the number of drivers switched x_i must satisfy the condition as follows:

$$\begin{cases} W_1(Mr_1 - \sum_{i=2}^{N} x_{1i}) \leq W_2(Mr_2 + x_{12}) \\ \vdots \\ W_1(Mr_1 - \sum_{i=2}^{N} x_{1i}) \leq W_N(Mr_N + x_{1N}) \end{cases} \tag{3}$$

To optimize the ride-hailing operations of RHPs, the optimal commission rate must be determined as follows:

Model 2: $\max_{c_i} B_i$

$$\text{s. t. } \begin{cases} W_1(Mr_1 - \sum_{i=2}^{N} x_{1i}) \leq W_2(Mr_2 + x_{12}) \\ \vdots \\ W_1(Mr_1 - \sum_{i=2}^{N} x_{1i}) \leq W_N(Mr_N + x_{1N}) \end{cases}$$

To obtain the optimal solution of Model 1 and Model 2, the objective function $B_i = \sum_{j=1}^{V_i} c_i \cdot \eta_{i,j} \cdot l_{i,j}$ must be examined first. Indeed, $B_i = \sum_{j=1}^{V_i} c_i \cdot \eta_{i,j} \cdot l_{i,j} = \left(\sum_{j=1}^{V_i} \eta_{i,j} l_{i,j} \right) c_i$. This shows that the objective function is a non-negative linear function with a single independent variable c_i with Assumption 1.

To determine the optimal commission rate based on Model 1 and Model 2, it is required to articulate the explicit expression of the commission rate according to the constraint condition of Model 1 and Model 2. The constraint conditions of the two models comprise a set of inequalities which are described as a customer waiting time function $W_i(x)$. If the monotonic features of the function $W_i(x)$ could be garnered, the explicit expression of the commission rate can be derived analytically. Proposition 1 shows that the function $W_i(x)$ is monotonic with the independent variable x.

Proposition 1. *It is claimed that $W_q(x)$ is a monotonic descent function. Here, independent variable $x \in$ N, parameters $\lambda > 0, \mu > 0$, and satisfies $x\mu - \lambda > 0$. See* Appendix B *for the proof.*

Both Model 1 and Model 2 are examined with respect to their characteristics and functionalities. With respect to Model 1, $W_1(Mr_1 - x_{12}) \leq W_2(Mr_2 + x_{12})$, drivers' switching quantity x satisfies $Mr_1 - x_{12} \geq Mr_2 + x_{12}$ based on Proposition 1. It is easy to know $x_{12} \leq \frac{1}{2}M(r_1 - r_2)$. From Assumption 3, the driver switching rate $f_1 = \frac{1}{p_1}c_1$. In the time period $[T_1, T_2]$, the cumulative drivers' switching quantity

$x_{12} = \int_{T_1}^{T_2} f_1 dt = \int_{T_1}^{T_2} \frac{1}{p_1} c_1 dt = \frac{c_1}{p_1}(T_2 - T_1)$. This means that $x_{12} = \frac{c_1}{p_1}(T_2 - T_1) \leq \frac{1}{2}M(r_1 - r_2)$. The commission rate can then be derived immediately as follows:

$$c_1 \leq \frac{M(r_1 - r_2)}{2(T_2 - T_1)} \cdot p_1 \tag{4}$$

The optimal commission rate in Model 1 is $c* = \frac{M(r_1 - r_2)}{2(T_2 - T_1)} \cdot p_1$ from Equation (4). This means that the optimal commission rate $c*$ is related to four operation parameters including (a) M: the total number of drivers in the market, (b) $r_1 - r_2$: the difference between drivers' market share proportion of the leading platform and that of the following platform, (c) p_1: the productivity level of the leading platform, and (d) $T_2 - T_1$: the time period between the start time and the end time of drivers' switching behaviors.

For a given ride-hailing market with the total number of drivers fixed, the optimal commission rate $c*$ is positively proportional to the difference between the drivers' market share proportion $r_1 - r_2$ between two RHPs and the productivity p_1 of the leading platform R_1. It is inversely proportional to the period between the start time and the end time of drivers' switching behavior $T_2 - T_1$.

An examination of Model 2, $W_1(Mr_1 - \sum_{i=2}^{N} x_{1i}) \leq W_i(Mr_i + x_{1i})$, finds that drivers' switching quantity x satisfies $Mr_1 - \sum_{i=2}^{N} x_{1i} \geq Mr_i + x_{1i}$ based on Proposition 1. Assumption 3 indicates that the drivers' transferring rate $f_1 = \frac{1}{p_1}c_1$. In the period $[T_1, T_i]$, the cumulative drivers' switching quantity $x_{1i} = \int_{T_1}^{T_i} f_1 dt = \int_{T_1}^{T_i} \frac{1}{p_1} c_1 dt = \frac{c_1}{p_1}(T_i - T_1)$.

Substituting $x_{1i} = \frac{c_1}{p_1}(T_i - T_1)$ into $Mr_1 - \sum_{i=2}^{N} x_{1i} \geq Mr_i + x_{1i}$, this can lead to $Mr_1 - \frac{c_1}{p_1}(T_2 - T_1 + T_3 - T_1 + \cdots + T_N - T_1) \geq Mr_i + \frac{c_1}{p_1}(T_i - T_1), i = 2, \cdots, N$. That means $\frac{c_1}{p_1}\left(\sum_{j=2}^{N} T_j + T_i - NT_1\right) \leq M(r_1 - r_i), i = 2, \cdots, N$.

Based on this relationship, the upper boundary of the commission rate can be obtained as follows:

$$\begin{cases} c_1 \leq \dfrac{M(r_1 - r_2)}{(T_2 + \sum_{i=2}^{N} T_i - NT_1)} p_1 \\ \vdots \\ c_1 \leq \dfrac{M(r_1 - r_N)}{(T_N + \sum_{i=2}^{N} T_i - NT_1)} p_1 \end{cases} \tag{5}$$

It is easy to know that if the number of RHPs equals 2 ($N = 2$), Model 2 is equivalent to Model 1. There are N platforms in a general ride-hailing market in which $T = \sum_{i=2}^{N} T_i - NT_1$. As a result, the optimal commission rate in Model 2 can be presented as follows:

$$c* = \min\left\{\frac{(r_1 - r_j)}{T_j + T} M p_1\right\}, j = 2, \cdots, N$$

This means that the leading platform R_1 should adopt the operations strategy by setting the commission rate as $c* = \min\left\{\frac{(r_1 - r_j)}{T_j + T} M p_1\right\}$, $j = 2, \cdots, N$. This can ensure that the leading platform R_1 is more competitive than other following RHPs.

The solution algorithm for calculating the optimal commission rate upper boundary in Model 1 is straightforward and simple. The solution algorithm for Model 2 is presented as follows:

Step 1: Initialization. Given the total available number of drivers in the market M, the productivity level p_1 of platform R_1, the market share information of RHP $\{r_i\}$, $i = 1, \cdots, N$. The starting time that a driver begins switching in the market is T_1, and the ending times that a driver stops switching to the ith RHP are $\{T_i\}$, $i = 2, \cdots, N$;

Step 2: Calculate the real number sequence $\mathbf{c} = \left\{ \frac{(r_1 - r_j)}{T_j + T} M p_1 \right\}$, $j = 2, \cdots, N$, where $T = \sum_{i=2}^{N} T_i - N T_1$;

Step 3: Search the minimum value of sequence $\mathbf{c} = \left\{ \frac{(r_1 - r_j)}{T_j + T} M p_1 \right\}$, $j = 2, \cdots, N$;

Step 4: Set the optimal commission rate as $c* = \min\left\{ \frac{(r_1 - r_j)}{T_j + T} M p_1 \right\}$, $j = 2, \cdots, N$.

5. Examples

Three numerical examples are presented to demonstrate the applicability of the models above for optimizing the operations of RHPs in the dynamic ride-hailing market. The first one is based on Model 1 with a focus on obtaining the commission rate upper boundary and exploring the relationship between commission rates, drivers' switching behaviors, and the critical platform parameters under the duopoly competitive condition. The second one is based on Model 2 for demonstrating how the optimal commission rate is determined for optimizing the operations of RHPs in the fully competitive ride-hailing market. The third one is used to demonstrate the validity of the two models for different shared mobility modes.

5.1. Example One

The first numerical example is the simulation of a duopoly competitive market in which there are two platforms, R_1 and R_2. The objective of R_1 is to maximize its revenue while ensuring its customer service quality. The objective function and constraint conditions of Model 1 proposed in Section 4 characterize this situation.

To explore how various platform parameters affect the determination of the optimal commission rate in this numerical example, a comparative analysis between eight different scenarios is conducted, as shown in Table 3. Three platform parameters are incorporated in the proposed queue-theory-based mathematical model. When one parameter is changed every time, there are three independent scenarios produced. When two of the three parameters are changed every time, there are three more independent scenarios. When all three parameters are changed simultaneously, there is another scenario. With the consideration of the basic scenario as the comparative baseline, a total of eight scenarios are formulated in the study to investigate the impact of changing the platform parameters on the best commission rate in optimizing the operations of RHPs. Here, the values of these parameters refer to empirical observations of real platform operations.

Scenario 1 offers a fundamental benchmark to demonstrate how the change in related platform parameters influences the determination of the optimal commission rate. For the given drivers' market share proportion of two platforms $r_1(0.7)$, $r_2(0.3)$, the time period between the starting time and the ending time of drivers' switching behaviors $T_2 - T_1$ (5000), the productivity level of the platform p_1 (0.45), and the total available number of drivers in the market M(10,000), the optimal commission rate c* is calculated at 36%.

Scenario 2 investigates the impact of drivers' market share proportion on the optimal commission rate. When the difference in drivers' market share proportion between two

RHPs decreases ($r_1 - r_2$ is changed from 0.4 to 0.15), the optimal commission rate of leading platform R_1 is decreased from 36% to 13.5%. This reveals that if the platform has a higher drivers' market share proportion, it will have the chance to set a higher commission rate in optimizing its operations.

Table 3. Numerical results of Example One for Model 1 (M = 10,000) (Here the c^* means the optimal commission rate. Same as in Tables 4 and 5).

Scenarios	r_1	r_2	$r_1 - r_2$	p_1	$T_2 - T_1$	c^*
1	0.7	0.3	0.4	0.45	5000	36%
2	0.575	0.425	0.15	0.45	5000	13.5%
3	0.7	0.3	0.4	0.45	10,000	18%
4	0.575	0.425	0.15	0.45	10,000	6.75%
5	0.7	0.3	0.4	0.65	5000	52%
6	0.575	0.425	0.15	0.65	5000	19.5%
7	0.7	0.3	0.4	0.65	10,000	26%
8	0.575	0.425	0.15	0.65	10,000	9.75%

Scenario 3 examines the impact of drivers' switching behavior lasting time on the optimal commission rate. When the drivers' switching behavior lasting time R_1 increases ($T_2 - T_1$ is changed from 5000 to 10,000), the optimal commission rate of the leading platform R_1 is decreased from 36% to 18%. This shows that the drivers' switching time period affects adversely the optimal commission rate. The longer the period of drivers' switching behaviors lasts, the smaller the commission rate that the platform should adopt.

Scenario 4 explores the joint impact of drivers' market share proportion and drivers' switching behavior lasting time on the optimal commission rate. When the difference in the drivers' market share proportion between two RHPs is becoming smaller ($r_1 - r_2$ is changed from 0.4 to 0.15), and the drivers' switching behavior lasting time is longer ($T_2 - T_1$ is changed from 5000 to 10,000), the optimal commission rate of the leading platform R_1 is decreased from 36% to 6.75%. The optimal commission rate is smaller than that in Scenario 3 (6.75% compared to 18%). This demonstrates that drivers' market share proportion and drivers' switching behavior lasting time amplify the impact on the optimal commission rate of the leading platform.

Scenario 5 studies the impact of the productivity of the leading platform on the optimal commission rate. When the productivity of the leading platform R_1 is higher (0.45 to 0.65), the optimal commission rate of the leading platform R_1 is increased from 36% to 52%. This reveals that higher productivity allows the leading platform to set up higher commission rates in optimizing its operations. It means that RHPs should endeavor to improve their productivity so that they have more chances to set higher commission rates for optimizing their operations in the competitive mobility market.

Scenario 6 probes the joint impact of the productivity and drivers' market share proportion on the commission rate in optimizing the operations of RHPs. When the productivity of the leading platform R_1 is higher (0.45 to 0.65), and the difference in the drivers' market share proportion between two RHPs is becoming smaller (0.4 to 0.15), the optimal commission rate of the leading platform R_1 is decreased from 36% to 19.5%. This reveals that the increase in the platform productivity and the decrease in the drivers' market share proportion can cause the leading platform to reduce its commission rate to pursue more operations revenue.

Scenario 7 concentrates on investigating the joint effect of the platform productivity and the drivers' switching behavior lasting time on the optimal commission rate. When the productivity of the leading platform R_1 is higher (0.45 to 0.65), and the drivers' switching behavior lasting time is longer ($T_2 - T_1$ is changed from 5000 to 10,000), the optimal commission rate of the leading platform R_1 is decreased from 36% to 26%. This demonstrates

that the leading platform must reduce its commission rate to pursue more operations revenue when its productivity increases and drivers' switching behavior lasts longer.

Scenario 8 studies the joint effect of platform productivity, drivers' market share proportion and drivers' switching behavior lasting time on the optimal commission rate. When the productivity of the leader platform R_1 is higher (0.45 to 0.65), the difference in the drivers' market share proportion between two RHPs becomes smaller (0.4 to 0.15), and the drivers' switching behavior lasting time is longer (from 5000 to 10,000), the optimal commission rate of the leading platform R_1 is decreased from 36% to 9.75%, which is lower than that in Scenario 6 (19.5%) and Scenario 7 (26%), while higher than that in Scenario 4 (6.75%). There are two parameters changed in Scenarios 4, 6, and 7. This means that there are intricate relationships between platform productivity, platform drivers' market share proportion, and drivers' switching behavior lasting time and the optimal commission rate.

The discussion above demonstrates that Model 1 can be used as an informative instrument to understand how the optimal commission rate is determined in a duopoly competitive market. Multiple platform parameters including drivers' market share proportion, platform productivity, and drivers' switching behavior lasting time can be employed to garner the optimal commission rate in optimizing the operations of RHPs. The numerical experiment results above imply that the change in one or more of these platform parameters has a significant impact on the optimal commission rate. This suggests that RHPs should carefully consider these variables in optimizing their operations.

5.2. Example Two

Example Two has three RHPs competing in the ride-hailing mobility market. They are denoted as R_1, R_2, and R_3 respectively. This example is used to simulate a fully competitive market where drivers can switch from the leading R_1 to R_2, or from R_1 to R_3. Table 4 presents an overview of the simulation result.

Table 4. Numerical results of Example Two for Model 2 (M = 10,000).

		p_1	r_1	r_2	r_3	T_1	T_2	T_3
		0.65	0.5	0.3	0.2	10,000	13,000	15,000
c	5.91%	7.5%						
c^*	5.91%							

The leading platform in the market R_1 has the largest drivers' market share at 50% of the ride-hailing market. The other two RHPs, R_2 and R_3, have 30% and 20% drivers' market share, respectively. The start time of drivers' switching behaviors in R_1 is at time point T_1 = 10,000. The stop time of drivers' switching behaviors from R_1 to R_2 is at time point T_2 = 13,000. The lasting time of drivers' switching behaviors from R_1 to R_2 equals $T_2 - T_1$ = 3000. The stop time of drivers' switching behaviors from R_1 to R_3 is at time point T_3 = 15,000. The lasting time of drivers' switching behaviors from R_1 to R_3 equals $T_3 - T_1$ = 5000. The total number of available drivers in the current market is M = 10,000. The productivity of the leading platform R_1 is p_1 = 0.65. The optimal commission rate is determined as 5.91%, which is calculated by using the solution algorithm presented in Section 4.

The results above show that the leading platform R_1 should set its commission rate as 5.91% to ensure that it can provide customers with better services than its competitors R_2 and R_3. Although the leading platform R_1 has more advantages than R_3 in drivers' market share proportion compared with R_2 ($r_1 - r_3 = 0.3 > r_1 - r_2 = 0.2$), the drivers' switching behavior lasting time is longer ($T_3 - T_1 = 5000 > T_2 - T_1 = 3000$), and the optimal commission rate between R_1 and R_3 (7.5%) is higher than that with R_1 and R_2

(5.91%). The optimal commission rate for the leading platform R_1 is 5.91%. This can ensure that platform R_1 is competitive in the fully competitive ride-hailing market.

5.3. Example Three

Example Three demonstrates the generality and validity of the proposed mathematical models between different ride-hailing modes. It considers two on-demand ride services including ride-sourcing and ride-splitting based on Model 1. The first scenario indicates ride-souring, and the second scenario indicates ride-splitting. The total order volume of Scenario 1 is V_i referring to the ride-sourcing mode, and the total order volume of Scenario 2 is V_j referring to the ride-splitting mode. Here, $V_j < V_i$ means ride-splitting mode will merge multiple orders, thus having similar passengers' routes into one order leading to a decrease in the total order volume. It reveals that the optimal commission rate is irrelevant to the type of ride-hailing modes. As a result, the optimal commission rates of two on-demand ride services are the same, as listed in Table 5.

Table 5. Numerical results of Example Three for two modes (M = 10,000).

Scenarios	r_1	r_2	$r_1 - r_2$	p_1	$T_2 - T_1$	c^*
Mode 1	0.7	0.3	0.4	0.45	5000	36%
Mode 2	0.7	0.3	0.4	0.45	5000	36%

The three numerical examples above provide some insights for optimizing the operations of RHPs. First, RHPs which have a dominant supply market share can set relatively higher commission rates for achieving higher revenue under the guarantee of customer service quality. Second, drivers' switching behaviors have a negative impact on the commission rate of RHPs. If drivers' switching behaviors last too long, RHPs must decrease their optimal commission rates to attract more drivers to join the platform to sustain their operations. Third, RHPs should endeavor to improve their productivity to obtain the opportunity to set higher commission rates. Fourth, RHPs should pay close attention to the relationship between commission rates, drivers' switching behavior, and critical platform parameters in optimizing their operations for profitability and sustainability. Fifth, the proposed two models could be used in different on-demand ride service modes such as ride-sourcing or ride-splitting.

6. Conclusions and Discussion

This study explores how to optimize the operations of RHPs with a focus on understanding the impact of the commission rate on drivers' switching behaviors in a dynamic ride-hailing market. This leads to the development of two specific mathematical models within the theoretical background of the queue theory for better explaining the interplay between commission rates, drivers' switching behaviors, and the critical platform parameters in optimizing platform operations. The study makes several theoretical contributions as follows.

First, this study formulates the problem of how to optimize the operations of RHPs with a focus on investigating the impact of the commission rate on drivers' switching behaviors, which has not been addressed in existing studies. Two queue-theory-based mathematical models are developed to better explain the impact of drivers' switching behaviors on the operations of RHPs. This addresses an existing research gap in optimizing the operations of RHPs in which drivers' switching behaviors have not been adequately considered. This is the first attempt to consider drivers' switching behaviors in optimizing the operations of RHPs. It enriches existing research in mobility operations management.

Second, the proposed models explore the trade-off between commission rates, drivers' switching behaviors, and critical platform parameters for optimizing the operations of

RHPs while sustaining customer service quality, which is not considered sufficiently in prior studies. RHPs strive to maximize their revenue by developing various pricing strategies in which commission rates are treated as a main component of such strategies. Two analytical models are formulated to maximize the operations revenue of RHPs while sustaining customer service quality in the platform.

Third, the analytical solutions of the models are derived in the examples given. This provides RHPs with feasible strategies for optimizing their operations in the dynamic ride-hailing market, which has not been addressed in existing studies. Numerical experiments provide a comprehensive analysis to examine how commission rate setting is associated with drivers' switching behaviors and crucial platform parameters in optimizing the ride-hailing operations of RHPs. This leads to a better understanding of the interplay between commission rates, drivers' switching behaviors, and critical platform parameters for optimizing the ride-hailing operations of RHPs.

The findings of this study have several practical implications. First, the findings of the study suggest that RHPs should strive to improve their drivers' market share so that they can set higher commission rates for optimizing their operations in the dynamic ride-hailing market, which is similar to the prior studies of [9,22]. Second, the findings state that RHPs should try to adopt proper incentives to shorten drivers' switching behavior lasting time as much as possible. If drivers' switching behavior occurs in a relatively short time interval, RHPs then could set higher commission rates to pursue more operations revenue. The drivers' switching behavior is associated with drivers' income satisfaction, which is verified in a recent study [10], while the relationship between drivers' switching behavior and commission rates shows that there is a complex interplay between them in the dynamic market conditions. Third, the findings advise that RHPs should commit to improving their productivity to optimize their operations. As a key platform parameter, the productivity of RHPs has a substantial influence on the optimal commission rate setting and the platform revenue, which are discussed in different service mode contexts [13]. Fourth, the study finds that all three operations instruments can be utilized for optimizing the operations of RHPs. RHPs should determine a rational configuration within drivers' market share, drivers' switching behavior lasting time, and platforms' productivity for developing competitive operations strategies.

There are some limitations in this study. First, there is specific simplification about drivers' behaviors in developing the two mathematical models for optimizing the operations of RHPs in the dynamic mobility market. The variation in the driver quantity for a long time, for example, is not considered. The total driver quantity in the ride-hailing market is assumed to be stable in this study. This can be further explored on the impact of the variation in the total driver quantity on the optimal commission rate. Second, the cooperation between RHPs is not considered. In practice, there usually exists not only competition but also cooperation between platforms, which is taken as a coopetition relationship in aggregated platform contexts [51,52]. The presence of the aggregation mode intensifies the complexity of drivers' supplies in the market. If the adoption of the aggregation mode is considered, how does it affect the determination of the optimal commission rate? This is an interesting problem that is worthy of further exploration. Third, no sensitivity analysis is conducted in the current study with the inclusion of more platform operations variables such as market density and price elasticity. Future studies can be carried out in further exploring the relationship between those critical platform operations variables in determining the best commission rate for optimizing platform operations. Fourth, emerging technological innovations in the transport domain such as the use of autonomous vehicles in the mobility market have a substantial impact on the operations of RHPs. Considering

these advances in optimizing the operations of RHPs in the dynamic mobility market is a direction worthy of attention in the future.

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Appendix A

For the M/M/S queueing system with arrival rate λ and service rate μ, we have the following results [53]:

$$C_n = \begin{cases} \frac{(\frac{\lambda}{\mu})^n}{n!}, & n = 0, 1, 2, \cdots, s \\ \frac{(\frac{\lambda}{\mu})^n}{s!}\frac{1}{s^{n-s}}, & n = s+1, s+2, \cdots \end{cases},$$

$$P_0 = \frac{1}{\sum_{n=0}^{\infty} C_n} = \frac{1}{\sum_{n=0}^{s-1}\frac{(\frac{\lambda}{\mu})^n}{n!} + \sum_{n=s}^{\infty}\frac{(\frac{\lambda}{\mu})^n}{s!}\frac{1}{s^{n-s}}}$$
$$= \frac{1}{\sum_{n=0}^{s-1}\frac{(\frac{\lambda}{\mu})^n}{n!} + \frac{(\frac{\lambda}{\mu})^s}{s!}\sum_{n=s}^{\infty}\left(\frac{\lambda}{s\mu}\right)^{n-s}}$$
$$= \left(\sum_{n=0}^{s-1}\frac{(\frac{\lambda}{\mu})^n}{n!} + \frac{(\frac{\lambda}{\mu})^s}{s!}\frac{1}{1-\frac{\lambda}{s\mu}}\right)^{-1}$$

$$, P_n = \begin{cases} \frac{(\frac{\lambda}{\mu})^n}{n!}P_0, & n = 0, 1, 2, \cdots, s \\ \frac{(\frac{\lambda}{\mu})^n}{s!}\frac{1}{s^{n-s}}P_0, & n = s+1, s+2, \cdots \end{cases}.$$

According to the Little formula [54], the number of customers in the system queue can be calculated.

$$L_q = \sum_{n=s}^{\infty}(n-s)P_n = \sum_{n=0}^{\infty} jP_{s+j} = \frac{P_0(\frac{\lambda}{\mu})^s\rho}{s!(1-\rho)^2} \quad \text{(A1)}$$

where $\rho = \frac{\lambda}{s\mu} < 1$

Queue waiting time:

$$W_q = \frac{L_q}{\lambda} = \frac{P_0\lambda^{s-1}\rho}{s!(1-\rho)^2\mu^s} \quad \text{(A2)}$$

System waiting time:

$$W_s = W_q + \frac{1}{\mu} = \frac{P_0\lambda^{s-1}\rho}{s!(1-\rho)^2\mu^s} + \frac{1}{\mu} = \frac{P_0\lambda^{s-1}\rho + s!(1-\rho)^2\mu^{s-1}}{s!(1-\rho)^2\mu^s} \quad \text{(A3)}$$

Average number of customers in the system:

$$L_s = \lambda W_s = \frac{P_0 \lambda^s \rho + \lambda s!(1-\rho)^2 \mu^{s-1}}{s!(1-\rho)^2 \mu^s} \tag{A4}$$

Appendix B

Proof of Proposition 1. According to the Little formula [51,52], we can derive a variation form of waiting time function $W_q(x)$ from Equation (A2) as

$$W_q(x) = \left(\sum_{n=0}^{x-1} \frac{(\frac{\lambda}{\mu})^n}{n!} + \frac{(\frac{\lambda}{\mu})^x}{x!} \frac{1}{1-\frac{\lambda}{x\mu}} \right)^{-1} \frac{\frac{\lambda^x}{x\mu}}{x!(1-\frac{\lambda}{x\mu})^2 \mu^x}.$$

As a result,

$$\begin{aligned} W_q(x) &= \left(\sum_{n=0}^{x-1} \frac{(\frac{\lambda}{\mu})^n}{n!} + \frac{\lambda^x}{(x-1)!\mu^{x-1}(x\mu-\lambda)} \right)^{-1} \frac{\lambda^x}{(x-1)!\mu^{x-1}(x\mu-\lambda)^2} \\ &= \frac{\lambda^x}{(g(x)((x-1)!\mu^{x-1}(x\mu-\lambda)) + \lambda^x)(x\mu-\lambda)} \\ &= \frac{1}{\left(\frac{g(x)\left((x-1)!\mu^{x-1}(x\mu-\lambda)\right)}{\lambda^x} + 1 \right)(x\mu-\lambda)} \end{aligned}$$

where $g(x) = \sum_{n=0}^{x-1} \frac{\left(\frac{\lambda}{\mu}\right)^n}{n!}$. Assuming that $h(x) = \frac{g(x)\left((x-1)!\mu^{x-1}(x\mu-\lambda)\right)}{\lambda^x} > 0$, we have

$$\begin{aligned} h(x+1) - h(x) &= \tfrac{g(x+1)(x!\mu^x((x+1)\mu-\lambda))}{\lambda^{x+1}} - \tfrac{g(x)\left((x-1)!\mu^{x-1}(x\mu-\lambda)\right)}{\lambda^x} \\ &= \frac{\left(g(x)+\frac{\left(\frac{\lambda}{\mu}\right)^x}{x!}\right)(x!\mu^x((x+1)\mu-\lambda))}{\lambda^{x+1}} - \tfrac{g(x)\left((x-1)!\mu^{x-1}(x\mu-\lambda)\right)}{\lambda^x} \\ &= \tfrac{g(x)x!\mu^x((x+1)\mu-\lambda)+\lambda^x((x+1)\mu-\lambda)-g(x)\lambda\left((x-1)!\mu^{x-1}(x\mu-\lambda)\right)}{\lambda^{x+1}} \\ &> \tfrac{g(x)\lambda(x-1)!\mu^{x-1}((x+1)\mu-\lambda)+\lambda^x((x+1)\mu-\lambda)-g(x)\lambda\left((x-1)!\mu^{x-1}(x\mu-\lambda)\right)}{\lambda^{x+1}} \\ &> \tfrac{g(x)\lambda(x-1)!\mu^{x-1}((x+1)\mu-\lambda)-g(x)\lambda\left((x-1)!\mu^{x-1}(x\mu-\lambda)\right)}{\lambda^{x+1}} \\ &= \tfrac{g(x)\lambda(x-1)!\mu^x}{\lambda^{x+1}} > 0 \end{aligned}$$

That means $h(x+1) > h(x) > 0$ because $W_q(x) = \frac{1}{(h(x)+1)(x\mu-\lambda)}$ and $(x\mu-\lambda) > 0$. We have $\frac{1}{W_q(x+1)} = (h(x+1)+1)((x+1)\mu-\lambda) > \frac{1}{W_q(x)} = h(x)(x\mu-\lambda) > 0$; thus, $W_q(x+1) < W_q(x)$. The proof is completed. □

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