



Article Developing a Stochastic Two-Tier Architecture for Modelling Last-Mile Delivery and Implementing in Discrete-Event Simulation

Zichong Lyu *^(D), Dirk Pons *^(D), Jiasen Chen and Yilei Zhang ^(D)

Department of Mechanical Engineering, University of Canterbury, Kirkwood Ave, Christchurch 8140, New Zealand

* Correspondence: zichong.lyu@pg.canterbury.ac.nz (Z.L.); dirk.pons@canterbury.ac.nz (D.P.)

Abstract: Modelling freight logistics is challenging due to the variable consignments and diverse customers. Discrete-event Simulation (DES) is an approach that can model freight logistics and incorporate stochastic events. However, the flexible delivery routes of Pickup and Delivery (PUD) are still problematic to simulate. This research aims to develop last-mile delivery architecture in DES and evaluate the credibility of the model. A two-tier architecture was proposed and integrated with a DES model to simulate freight operations. The geographic foundation of the model was determined using Geographic Information Systems (GIS), including identifying customer locations, finding cluster centres, and implementing Travelling Salesman Problem (TSP) simulation. This complex model was simplified to the two-tier architecture with stochastic distances, which is more amenable to DES models. The model was validated with truck GPS data. The originality of the work is the development of a novel and simple methodology for developing a logistics model for highly variable last-mile delivery.



1. Introduction

Freight transportation plays an important role in supporting economic growth [1]. It is required to transport goods from the origin to the destination, including trade, logistics and transportation, including linehaul and urban pickup and delivery (PUD). The operations are highly complex, especially in transport activities, because variable customer addresses and freight consolidation affect route planning [2]. These factors cause the delivery route to be difficult to model. In New Zealand (NZ), the country under examination, the New Zealand Business Council estimates freight will increase by 75% in the next 30 years [3], and this is broadly consistent with elsewhere in the world. Hence there is a need for optimization and continuous improvement regarding logistics efficiency [4]. In turn, this requires modelling and simulation.

Computer simulation techniques are primarily discrete-event simulation (DES), and agent-based modelling (ABM). DES readily accommodates stochastic variability, and it also has the means to model conditional events [5]. It has been widely applied to logistics as well as plant layout and service queuing problems [6,7]. ABM has the advantage of more easily accommodating the choices of individual agents and including their behaviours and interactions. Examples of applications to logistics are the agents being trucks to optimise freight distribution [8] and being locations to solve routing problems [9]. Another avenue of simulation is provided by geographic information systems (GIS). While DES and ABM primarily operate in the time domain, GIS is in the spatial domain. Typical logistics applications are to determine routes (and hence distance) between physical addresses [10], with the travel time a secondary determination by introducing speeds. GIS requires precise



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Copyright: © 2022 by the authors. Licensee MDPI, Basel, Switzerland. This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution (CC BY) license (https:// creativecommons.org/licenses/by/ 4.0/). identification of the addresses, and hence offers a specific rather than generalised simulation method. The present paper focuses on DES as applied to freight last-mile delivery.

However, the delivery route is challenging to incorporate in DES models due to the dynamic nature of operations, especially the daily variability of consignments and the variability in route [11,12]. An intersection-based DES model has been developed [13], but it is difficult to apply to changeable conditions. This is where models with simpler architectures can be advantageous, especially from an operational perspective.

The motivation of this work is to develop an architecture to represent last-mile delivery suitable for integration with stochastic models. The present paper shows how clustering may be applied to develop a two-tier architecture capable of representing the complexity of last-mile delivery routes for a variable number and address-set of consignments. A case study is presented using representative industry data. The method is demonstrated by a case study.

The paper is structured as follows: Section 2 is a literature review of last-mile delivery modelling methods. Section 3 introduces the proposed two-tier delivery model. Section 4 describes the implementation of the proposed method through a case study for real freight operations.

2. Literature Review

Last-mile delivery models may be an approach in three main ways: mathematical models, computer simulation models, and GIS models. These are briefly reviewed below.

2.1. Mathematical Modelling of Vehicle Routing Problems (VRP)

Freight last-mile delivery is highly variable and requires flexible route planning, because of daily variability in customers being served and freight volume. Conventional approaches to modelling last-mile delivery have primarily taken the consolidation perspective whereby consignments are aggregated to delivery runs within geographic territories. This results in the need to solve vehicle routing problems (VPR). There are various forms of VPR, including node routing problem (NRP) [14,15], travelling salesman problem (TSP) [16–18], arc routing problem (ARP) [19–21], rural postman problem (RPP) [22,23] and Chinese postman problem (CPP) [24–26]. All these methods apply a mathematical optimisation to find the most efficient route given the constraints applied.

There are innovative methods to simulate last-mile delivery, including agent-based models [27], system dynamics models [28], the fuzzy analytic hierarchy process model [29], Petri net models [30], crowd shipping models [31], Hamiltonian cycle model [32], continuous approximation models [33] and the dynamic traffic simulation [34]. Applications of mathematical route models to freight last-mile delivery are numerous. However, there are limitations in these models. The simplifications in the models make it difficult to include all operational constraints [13]. These methods may have some but limited ability to represent stochastic variability because of the complexity of metaheuristic calculations [35,36]. However, they fare poorly with the epistemic uncertainty that arises from variability in simulating real-life problems. Many of these methods do not generalise well—they are specific solutions for a given set of customer addresses, and for a different set, the model has to be re-run in its entirety. This limitation makes it difficult to apply in real operational situations where real-time modelling is appreciated.

2.2. Computer Simulation

Computer simulation techniques include discrete-event simulation (DES), agent-based simulation (ABS) and system dynamics (SD). Compared with conventional mathematical models, simulation models tend to describe systems and measure system performance [37]. DES is a versatile simulation technique to solve practical problems [38]. It imitates systems with predefined architecture and resources. The process is divided into sequential discrete events. This attribute reduces the complexity of computations on constraints and allows probability distributions to be incorporated with the Monte Carlo sampling method.

Examples of DES applications in logistics include rail freight models [39], multimodal transport [40], underground logistics transportation [41], container port operations [42], logistics planning [43] and fleet management [44,45].

Perhaps unexpectedly, there is a scarcity of literature on the application of computer modelling for freight last-mile delivery. For an exception, see an agent-based model which was applied to evaluate delivery cutoff times (the responsiveness question) but not distances [46].

As applied to modelling freight last-mile delivery, all the computer simulation methods offer the positive features of coping with stochastic parameters, and can represent complex systems, including decisions, specifically model resource utilisation & report on productivity metrics. The ABS methods have the further advantage of being to represent individual behaviour, such as specific route choices of vehicles. Agency is also possible with DES, although it requires more effort to model the necessary decision blocks.

Disadvantages of computer simulation as applied to freight last-mile delivery are the greater effort to create the model, and the difficulty of representing specific routes. Also, the models are descriptive rather than the basis for optimisation. Another challenge is the difficulty in the validation of the model, and this becomes increasingly difficult as the model becomes more complex. The model requires prior assumptions about the architecture of the problem (the underlying structure of the decision problem), which tend to be obscured within the computer models. Furthermore, supply chain models require many assumptions, and it is difficult to obtain accurate estimates for real-world situations [47].

The underlying problem is that optimisation of final mile routes is a hard combinatorial problem that these types of simulation cannot solve or even represent.

2.3. Geographic Information System

GIS can manage, display and analyse large spatial data sets. It has also been used to model last-mile delivery, and has some attributes that make it very different to the mathematical models. In particular, GIS is good at representing addresses and road routes, including road information and traffic conditions (including congestion, and time-of-day variability therein), and road suitability for freight trucks. This can make for powerful models, and indeed GIS may be used to solve TSP problems. Applications of GIS to logistics and transportation generally include formulation of future transportation plans [48], multimodal freight [49], routes for forest firefighters [50], oil and gas transportation [51] and e-commerce logistics distribution [52].

GIS has benefits in implementing simple TSP simulations. It can accurately identify locations and incorporate geographical data. Therefore, the route results can be an authentic representation of real truck routes selected by experienced drivers.

An application to last-mile delivery and PUD is evident in [13], where GIS was used to find the routes for two models (a simple suburb model and an intersection-based model), which were subsequently represented in DES with predefined routing decisions and stochastic simulation.

Some comparable work is available on the use of GIS to determine routes for waste collection [53–56]. However, waste collection is a different and simpler problem: in waste collection, the need is to determine a route that takes every street, and once determined the truck always follows the same route. In contrast, last-mile delivery does not require every street to be covered every day, and new optimisation is needed every day.

Consequently, although GIS has the advantage of simulating freight delivery routes, a general limitation of GIS as regards modelling logistics operations is that it mainly focuses on standalone cases. In an operational situation with high daily variability in customers, the model has to be re-run for each situation. Also, GIS is challenging to include freight operations such as queuing models and the freight consolidation process. It is possible to integrate ad-hoc plug-ins with GIS to extend the functionality, but the existing applications are limited.

2.4. Gaps in the Modelling of Freight Last-Mile Delivery

The integration of mathematical, simulation and GIS models has been shown in recent literature. GIS was utilised to obtain natural hazard data and support mathematical model development for optimising post-disaster relief distribution and network restoration [57]. GIS can also support discrete-event model development, which has been presented in [13]. In addition, a Monte Carlo simulation has been combined with a multi-objective model to analyse an electronic reverse logistics network with stochasticity [58]. It is also possible to incorporate simulation models into mathematical models. A DES model was integrated with a mixed-integer programming model to allocate vessels [59]. The mathematical model assisted in optimizing the process.

There are extensive works of literature on clustering design approaches and route approximation models. The works of literature are summarised in Table 1. However, these models rarely incorporate a logistics model to represent realistic operations and address a large number of stochastic variables. DES has the strength to measure operational performance, and compare different scenarios [60], but struggles with the optimisation of final mile routes that change on a daily basis. Therefore, there is a gap in developing an architecture to represent last-mile delivery suitable for integration with stochastic models.

 Table 1. Summary of last-mile delivery modelling methods.

Methods	Description	Examples of Approaches That Have Been Attempted	
Mathematical models (VRP)	Typical VPR models include NRP, TSP and ARP models. Mathematical models can be developed easily and rapidly. However, these models need to simplify the real system with little stochasticity.	[14-21]	
Computer simulation models	These models can simulate complex systems with a large number of uncertainties. The disadvantages are the long modelling period and the difficulty of representing specific routes.	[39,40]	
GIS models	GIS is able to find routes by implementing VRP algorithms, but it is limited to standalone cases.	[53–56]	
Integration of mathematical, simulation and GIS models	High daily variability in consignment/customer addresses, and number of consignments requires that the model be recomputed each time. The integration tends to be applied manually, rather than completely automated, so this takes time. For this reason, existing integration systems may not yet be ready for routine operational use in the industry.	[57,58]	

3. Methodology

3.1. Research Objective

The objectives of this research were to find an architecture that can reflect the freight delivery route with high flexibility, suitable for incorporation into a DES model. This is worth attempting, for the benefit of being able to include stochastic elements in the model. DES models are relatively quick to implement in an operational sense, and have the potential to be aggregated into larger simulation models.

3.2. Approach

For logistics modelling, the delivery route is challenging to simulate in terms of travel distance and travel time. A two-tier architecture was developed to represent the delivery route and incorporate stochasticity for a concentrated cluster to estimate the travel distance. The accuracy of the architecture was validated by a case study. A freight logistics model was developed by DES and conducted in Arena[®] (version 16.00, Rockwell Automation, Milwaukee, WI, USA). ArcGIS Pro[®] (version 2.6.3, Esri, Redlands, CA, USA) was applied to find cluster centres, calculate distances and run TSP simulation. TSP simulation is generally used to find the optimal route by organising the sequence of customer locations. ArcGIS was used for TSP simulation because it can provide more accurate routes and distances in terms of customer locations and road information with the existing database.

3.2.1. Developing a Two-Tier Architecture

To simplify the delivery route and apply it to a DES model, the route was divided into two tiers, as Figure 1 shows, with the first tier from the depot to the cluster centre and the second tier from the cluster centre to each customer. Hence in this architecture, the first-tier distance is fixed, and the second-tier distance is stochastic since the customer location varies. All parameters for the architecture are calculated based on TSP results. Then the architecture is developed in a DES model accordingly. For N consignments, the estimated route distance is the sum of the first-tier distance and all second-tier distances. In practice, the truck does a tour and travels back to the depot at the end, rather than shuttling between the cluster centre and the customer addresses.





To obtain the first-tier distance (D_f) , the cluster centre (*C*) should be determined first. There are several methods to find the cluster centre, including cluster mean centres and cluster median centres with data point attributes. The second-tier distance (D_s) for each customer location is stochastic, which can be calculated by multiplying the Euclidean distance (D_e) from the cluster centre to the customer locations and a factor *R*. The total travel distance (D_{total}) for the journey is:

$$D_{total} = 2 * D_f + \sum D_s(i) \quad \forall \ i \in N \tag{1}$$

$$D_s(i) = R * D_e(i) \quad \forall i \in N$$
(2)

The factor *R* adjusts D_e and hence D_s , so D_{total} is close to the real route distance.

3.2.2. Determining *R*

The first-tier distance (D_f) can be calculated once the cluster centre (*C*) is defined. The *R* value is comparatively difficult to obtain, which determines the second-tier distance (D_s) .

The intention of introducing *R* is to adjust the total second-tier distance and approach to the real delivery distance. To acquire the *R* value, TSP simulation is a technique to simulate delivery routes and obtain the total route distance (D_t). Then the imaginary second-tier distance (D_m) can be calculated by Equation (3) and one *R* value can be determined by Equation (4) regarding one TSP simulation.

$$D_m = D_t - 2 * D_f \tag{3}$$

$$R = \frac{D_m}{\sum D_s} \tag{4}$$

The *R* values can be obtained through implementing TSP simulation with different customer locations.

3.2.3. Implementation Stages

GIS was applied to develop the two-tier architecture, which is a part of the logistics model. The holistic logistics model was created by DES. The model development process is illustrated in Figure 2.



Figure 2. Simulation model development process.

Step 1: Consignment data and truck GPS data were obtained from the industry. Operational data were manually recorded.

Step 2: Consignment data were input in ArcGIS and customer locations were identified. Different types of centres are mean and median, for the number of consignments, weight, and volume. The algorithms were the standard implementations in ArcGIS. The centre selected was the mean by consignments.

Step 3: The first-tier distance (D_f) and the second-tier distances (D_e) were computed in ArcGIS and the second distances were formed into a Gamma distribution.

Step 4: Random sets of addresses were generated to simulate different delivery routes. The consignment range considered is from 10 to 15, which is common for daily delivery. Hence, cases for n = 10 and n = 15 consignments were analysed. There was a total of ten ensembles for each. The corresponding addresses were input in ArcGIS. TSP simulations were conducted in ArcGIS to find each delivery tour and obtain (D_t). These tours were not from the cluster centre, but from the depot, and were believed to closely represent real routes [13].

Step 5: The *R* values were calculated by D_m and D_s and formed into Normal distributions for each consignment number.

Step 6: The DES model for freight logistics was created in Arena by incorporating the two-tier architecture. Freight operations, including freight loading, unloading and consolidation were developed in the model, see Figure 3.



Figure 3. Freight delivery operations.

Step 7: Consignment data and operational data were formed into distributions and input into the DES model. The truck speed was calculated based on the GPS data.

Step 8: Stochastic simulation was conducted by the DES model. Results about truck travel distance, total time, truck utilisation and truckload were produced.

4. Results

4.1. Description of the Case Study

The specific case under investigation is freight delivery. The truck delivered freight consignments by a sequence in an intensive area. One-year consignment data and the truck GPS data were obtained from the industry partner. Data included customer address, freight attributes and truck speeds. Two cases were assumed, which are 10 delivery consignments (n = 10) and 15 delivery consignments (n = 15). Operations of freight delivery were observed by the first author. The freight loading time was recorded on-site and the unloading time was obtained from the GPS data. Operations were validated with the industry partner through face validation. Embedded communication was conducted to solicit tacit knowledge [12]. Key assumptions are shown in Table 2.

Table 2. Key assumptions.

Parameters	Assumptions
Number of consignments	10 and 15
Addresses	Randomly selected
Truck capacity	One truck is sufficient for assumed consignment numbers
Truck speed	Average speed including truck stop and start times
Backhaul	None

4.2. Determining the Parameters for Two-Tier Architecture

Suburbs with weight and volume less than 3% of the total were neglected to simplify the model. The driver's main service areas are concentrated in Wigram and Sockburn. Therefore, addresses in Wigram and Sockburn were considered as one cluster. The proportion of consignment quantities, weight and volume for suburbs in Christchurch is shown in Table 3.

One-year consignment data were input in ArcGIS. The number of data points was 3195. Figure 4 exhibits hotspots of consignment data with attributes of the consignment number, weight and volume in ArcGIS. These hotspots were calculated based on the target features within Euclidean Distance [61].

All three hotspot maps show a similar distribution by different attributes. Therefore, the centre of the cluster can incorporate the feature of the number of consignments, consignment weight and volume in the two-tier architecture.

Area	Count of Consignments	Sum of Weight	Sum of Volume
Wigram	69.26%	70.16%	74.74%
Sockburn	26.23%	25.55%	21.63%
Central city	1.03%	2.23%	1.05%
Addington	0.94%	0.32%	0.78%
Hornby	0.57%	0.42%	0.42%
Halswell	0.54%	0.30%	0.47%
Upper Riccarton	0.26%	0.18%	0.10%
Sydenham	0.11%	0.05%	0.06%
Harewood	0.11%	0.10%	0.08%
Hillmorton	0.09%	0.06%	0.09%
Middleton	0.09%	0.06%	0.05%
Waltham	0.06%	0.02%	0.01%
Riccarton	0.06%	0.21%	0.22%
Templeton	0.03%	0.00%	0.01%
Islington	0.03%	0.00%	0.01%
Wainoni	0.03%	0.01%	0.02%
Bishopdale	0.03%	0.01%	0.03%
Airport	0.03%	0.00%	0.00%
Hoon Hay	0.03%	0.01%	0.04%
Hornby South	0.03%	0.07%	0.02%
Unknown data	0.46%	0.23%	0.20%

Table 3. The proportion of consignment data for suburbs.



Figure 4. Hotspots of consignment data with attributes (**a**) Attribute of consignment number; (**b**) Attribute of consignment weight; (**c**) Attribute of consignment volume.

Individual customer locations are challenging to embody in logistics models. Therefore, clusters may be considered in models instead of specific locations. The principle of cluster analysis is to make the objects in the same cluster have the greatest possible similarity. Then a representative centre is applied to express the cluster [62]. Cluster centres and centroids have emerged in research, including wireless sensor networks [63], the internet of things [64], biological systems [65] and the global navigation satellite system [66].

Cluster centres are also defined in the K-means clustering algorithm. K-means clustering is based on the concept of dividing *n* points into *k* clusters, so that each point belongs to the cluster corresponding to the nearest centre [67]. Combined with the K-means clustering algorithm, GIS may be used to identify the peak travel of residents and hot spots for taxis [68]. K-means clustering was used to establish hot spots where road accidents occur [69]. Similarly, a modified clustering method (DP-Dip) has been used to estimate the centre of the cluster [70]. DP-Dip does not make any assumptions about the data distribution and only admits that each cluster has a unimodal distribution. This method adaptively splits some clusters according to their density.

The benefits of clustering are that it provides a simple representation of the problem, which may be used to infer structure in a wider data set of addresses. However, applications of clustering to last-mile delivery are scarce. Successful applications include food delivery [71]. Costs were estimated by K-means clustering for restaurants in a city. However, the difficulty with the clustering approach for logistics is the high degree to which the operation reality has been simplified. Therefore, a cluster model can represent an average operational day, but does not correspond to the operational reality for any one day. Also, vehicles do not actually travel backwards and forwards from a cluster centre, but rather have a delivery run. In situations where the delivery addresses have high day-to-day variability, clustering methods are inadequate because the need is to represent vehicle routing.

Mean centres, median centres with volume and weight attributes were calculated in ArcGIS, as shown in Figure 5. It is observed that the multiple measures of centredness are all very similar.



Figure 5. Customer locations and cluster centres.

The mean centre identifies the geographic centre (or the centre of concentration) for a set of features. It is the average of x and y coordinates of all locations. The median centre identifies the location that minimizes the overall Euclidean distance to the location in a dataset. The mean centre is calculated by averaging x and y coordinates of all points. The mean centre (\overline{X} , \overline{Y}) is given as Equation (5) shows:

$$\overline{X} = \frac{\sum_{i=1}^{n} x_i}{n}, \overline{Y} = \frac{\sum_{i=1}^{n} y_i}{n}$$
(5)

where x_i , y_i are the coordinates for feature *i*, *n* is the total number of features. In comparison, the median centre (x^t, y^t) is typically found by minimising the Euclidean distance d^t from each point to the centre, as Equation (6) shows. Kuhn and Kuenne [72] introduced the method of calculating the median centre, and Burt and Barber [73] provided an iterative process.

$$\min \sum_{i \in n} d_i^t = \sqrt{(x_i - x^t)^2 + (y_i - y^t)^2}$$
(6)

It is observed that the six centres are geographically similar because weight values make small differences in hotspots, as Figure 3 shows.

The purpose of the median centre is to find a centre that approximates each data point. The minimum distance was not involved in this research, so median centres were excluded. In addition, as Figure 5 shows, distributions of data points with weight, volume and consignment number are similar, which results in the mean centres with these factors being approximately coincident. From the consideration of data processing convenience, the mean centre with consignment number was selected to represent the cluster centre in the two-tier architecture, consistent with [74].

The first-tier distance (D_f) is 762.5 m, and all Euclidean distances (D_e) were fitted in a Gamma distribution, as shown in Figure 6. The result was computed statistically with Statistica.



Figure 6. The Euclidean distance (D_e) with a Gamma distribution fit with scale 107.3192 and shape 4.9074.

Ensembles of 10 and 15 consignment addresses were randomly selected from the consignment data. For example results of the TSP route determination for 10 and 15 consignment delivery, see Figures 7 and 8, respectively. The entire results are listed in Figures A1 and A2.



Figure 7. TSP results for *n* = 10 delivery routes. (**a**) Example one; (**b**) Example two; (**c**) Example three; (**d**) Example four.

Figure 8. TSP results for n = 15 delivery routes. (a) Example one; (b) Example two; (c) Example three; (d) Example four.

Note that the random selection was done by consignments, not addresses, to reflect actual operations. The difference is small, and causes the occasional duplicate address due to the consolidation process whereby one address may receive multiple consignments from different senders. The models allow the truck to economise in these cases, which is realistic. The effect is to broaden the uncertainty in the models, which is conservative.

The total distances were obtained from the two-tier architecture and TSP models. Table 4 presents the results of 10 and 15 consignment cases.

Table 4. Result of 10 and 15 consignment cases.

Number of Consignments	Delivery Case	Total Second-Tier Distance $\sum D_S$ (m)	Total Distance for Two-Tier Architecture D _{total} (m)	Total Distance for TSP D_t (m)	R-Value
	1	5265	6790	7645	1.163
	2	6176	7701	6066	0.735
	3	6152	7677	4906	0.550
	4	4557	6082	5417	0.854
n = 10	5	5248	6773	7459	1.131
	6	4191	5716	5175	0.871
	7	5840	7365	5241	0.636
	8	3712	5237	4115	0.698
	9	5439	6964	6397	0.896
	10	5905	7430	6906	0.911
	1	8337	9861	7956	0.771
	2	8126	9651	7242	0.704
	3	7999	9524	7416	0.736
	4	8385	9910	7311	0.690
<i>n</i> = 15	5	6694	8219	7131	0.837
	6	6988	8513	6426	0.701
	7	6951	8476	7599	0.874
	8	7750	9275	6584	0.653
	9	8099	9624	8674	0.883
	10	7701	9226	7912	0.829

The R-value for each run was used to form a normal distribution with a mean 0.806 and a standard deviation 0.153. It was applied to n = 10, 11, 12, 13, 14, 15 cases (number of consignments). Therefore, the second-tier distance was formed through multiplying the gamma distribution and the normal distribution. Table 5 indicates the parameters of the two-tier architecture for the cluster delivery.

Table 5. Parameters for the two-tier architecture of the cluster.

	Two-Tier Parameters		
D_f (m)	762.5		
D_s (m)	$D_e * R = \text{GAMM}(107.319, 4.907) * \text{NORM}(0.806, 0.153)$		

4.3. Results and Validation of Two-Tier Architecture

To validate the TSP simulation results, a delivery case for n = 10 was selected and the corresponding customer locations were input to ArcGIS. The TSP simulation was conducted and compared with real truck GPS data, see Figure 9.

Figure 9. TSP route and GPS data for an n = 10 case.

The close correspondence between GPS and TPS results indicated that the truck driver did actually and intuitively find the optimal route to conduct the delivery. This means the TSP result can generally reflect the actually delivery route.

Stochastics simulations were conducted for each N case with 100 replications. All mean values of route distance are shown in Figure 10.

The results in Figure 10 show a reasonably close approximation for the two methods. The TSP method is the more accurate, but is not a practical method from an operational perspective because of the large amount of effort required to implement it. Hence the two-tier method has significant practical advantages. As the second stage of validation of the model, an additional set of ten TSP simulations were conducted for n = 12 using the method described above. These results were then compared to the corresponding two-tier result for n = 12. An ANOVA analysis shows the differences are not significant [F(1, 108) = 0.01745, p = 0.895], see box and whisker in Figure 11. The mean for two-tier architetcture is 6560m and for TPS 6595 m.

Figure 10. Mean values of route distance for different N cases for two-tier simulation (blue) compared to TSP results (orange). TSP results are only shown for n = 10 and 15.

Figure 11. Box plot with medians and percentiles for Two-tier and TSP for n = 12 cases.

The validation shows distances estimated by the two-tier architecture for two cases, and the mean value is approximate to the GPS data. Therefore, freight delivery with stochastic customer locations can be validly represented by the two-tier architecture.

Hence the entire delivery last-mile region can be simplified into a cluster model with a functional dependency on the number of consignments. This potentially moves the field forward, because it allows the complexity of a variable last-mile PUD situation to be reduced into a stochastic formulation. This can be used for operational planning purposes, in real-time, by use of a suitable stochastic engine. Such an engine might be @Risk, or DES software for a more comprehensive model. A DES model could, in principle, be expanded to include multiple such suburbs, as well as the additional complexity of consolidation and line haul.

While the resulting model is relatively simple, the key enabling method is GIS. This is because the TSP algorithm within GPS provides the virtual data representing the real route. The real route could alternatively be obtained from GPS, which would be superior. However, GPS data are limited and difficult to interpret, and not always available. Furthermore, the proposed scheme using GIS overcomes the problem when the delivery runs are not yet established, as occurs in a new PUD territory.

4.4. Discrete-Event Simulation (DES) in Arena®

DES gives the opportunity to incorporate not only the distance, but also other operational realities such as time taken, truck utilisation, etc.

The simulation model was developed in Arena in accordance with the two-tier architecture: (a) first-tier movement, (b) second-tier movement, (c) and the return to the depot. The architecture of the DES model is shown in Figure 12.

Figure 12. DES delivery model: (a) first-tier movement; (b) second-tier movement; (c) the return to the depot.

A description of the modelling approach follows. Firstly, in Figure 12a, consignments were generated with random addresses. Then, weight and volume were assigned to each consignment from probability distributions. Freight consignments were loaded by a forklift and consolidated on the truck. The forklift was assumed to carry one consignment for each movement. Secondly, in Figure 12b, when the truck finished the first-tier movement, the truckload sequentially completed the second-tier movement with stochastic distances. The freight was assumed to be unloaded by customers. Last, after completing the transportation of all consignments, a return module was applied to the truck. As Figure 12c shows, the truck was moved back to the depot.

Simulation inputs are shown in Table 6. The truck speed and freight unloading time were obtained from the GPS data. The speed includes the truck stop and start time on roads. Distributions of consignment weight and volume were fitted based on the consignment data. The freight loading time was recorded on-site. First-tier and second-tier distances were obtained from Section 4.2.

Parameter	Variable	Value
Consignment number	Ν	10 and 15
Truck speed (km/h)	S	30.893
Consignment weight (kg)	W	0.999 + EXPO(409)
Consignment volume (m ³)	V	-0.001 + EXPO(1.38)
Freight loading time (s)	T_l	26 + EXPO(21.6)
Freight unloading time (min)	T_u	0.5 + GAMM(7.81, 1.15)
First-tier distance (m)	D_f	762.5
Second-tier distance (m)	D_s	GAMM(107.319, 4.907) * NORM(0.806, 0.153)

Table 6. Simulation input variables.

The total time for the truck delivery T_{total} in the simulation is theoretically calculated by Equation (7).

$$T_{total} = \sum T_l(i) + \sum T_u(i) + \frac{D_f + \sum D_S(i)}{S} \quad \forall i \in N$$
(7)

4.5. Simulation Results

The simulation was run by 100 replications for 10 and 15 consignment cases, respectively. Figure 13 presents simulation results for time values.

Figure 13. Mean values for times with logistics model using two-tier sub-model.

The travel time accounts for a small portion of the total time in two cases. This means most of the time was spent on freight loading and unloading. In addition, the average queueing time for each consignment is large in the delivery operations. The loading and unloading activities could be improved by adding more forklifts or optimising depot operations.

The freight volume is also of concern to freight companies. The truckload weight and volume were obtained from the simulation, see Table 7. The ideal weight limit and volume limit for the PUD truck are 11 tons and 40 m³. However, the actual volume limit is 70% of the ideal limit from the consideration of health and safety, which is 28 m³.

		<i>n</i> = 10	<i>n</i> = 15
Truckload weight (t)	mean	4.125	6.261
_	std dvn	1.207	1.511
Truckload volume (m ³)	mean	13.506	20.513
	std dvn	3.769	5.374
Capacity utilisation	mean	48%	73%

Table 7. Freight attribute results.

All truckload results, including maximum values are under the truck limits. When the consignment number is 10, the capacity utilisation is low.

5. Discussion

The two-tier architecture is a possible way to describe the last-mile delivery route and can be incorporated into operational models with stochasticity. The accuracy of the architecture is high, as shown by the validation. Possible practical implications of this are predicting travel distance for an unfamiliar territory where the truck route is unknown. Travel distance is a useful proxy for the delivery time, hence order fulfilment from a customer perspective.

The operational results can be used to evaluate the freight system under different consignment volumes. In practice, this could be used as follows:

Operators, specifically drivers and dispatchers, could estimate the travel distance and the travel time for the delivery tour. The truck dispatcher can assess the capability of the current system from the results of truckload weight, truckload volume and queuing time. It is also important from an operational perspective, because the dock at the depot typically needs to be cleared each day. Stochastic truck speeds and delay times could be included to reflect unexpected traffic conditions. However, these were not incorporated into this work since the intention was to investigate distances.

Another scenario where a simple calculator like this could be useful is when the dispatchers need to determine how to split a load among multiple trucks. In this work, only the consignment number limit was considered. However, a freight consolidation module with weight and volume limits could be added to further constrain the truck capacity. Then the model could include multiple trucks. Hence the trade-off between distance and relative parameters such as fuel consumption and emissions versus speediness of customer fulfilment can be explored. There are also implications for predicting truck repair and maintenance.

A city delivery network could be constructed by several two-tier architectures of clusters. All parameters need to change because the cluster centre and TSP results are varied for clusters. As a result, this is possible to analyse the above assumptions for the whole city's last-mile delivery.

GIS can support the construction of operational models. For specific cases, TSP can predict the truck route and distance with merely customer locations. The accuracy of this simulation is high from the comparison with real truck GPS data.

6. Conclusions

The originality of this work lies in the development of a complete workflow for highly variable PUD, incorporating a two-tier architecture informed by GIS & TSP, which is included in a DES operational model with stochasticity. The method could be used to either evaluate current freight operations or the operations for a new area. GIS was used to support the model development. TSP simulations were implemented in ArcGIS to obtain the factor R, and the accuracy was validated with real truck GPS data. A freight operations model was created and incorporated the two-tier architecture. Stochastic simulations were implemented for operational results were presented.

There are some limitations in this research. First, this research ignores pickups in the delivery run, which do occasionally occur in practice. In principle, it would not be

impossible to include pickups in the DES model, see also [13]. Second, it is recommended that any practitioners who want to apply this model could expand the size of the analysis. Last, the two-tier architecture is suitable for concentrated customers rather than sparse or scattered customers. However, sparse or scattered customers can be simulated by an intersection-based model, which is also presented by [13].

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Appendix A

Figure A1. TSP results for 10 consignments. (**a**) Route one; (**b**) Route two; (**c**) Route three; (**d**) Route four; (**e**) Route five; (**f**) Route six; (**g**) Route seven; (**h**) Route eight; (**i**) Route nine; (**j**) Route ten.

Figure A2. TSP results for 15 consignments. (**a**) Route one; (**b**) Route two; (**c**) Route three; (**d**) Route four; (**e**) Route five; (**f**) Route six; (**g**) Route seven; (**h**) Route eight; (**i**) Route nine; (**j**) Route ten.

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