

Article

# Modeling Emergency Logistics Location-Allocation Problem with Uncertain Parameters

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**Abstract:** In order to model the emergency facility location-allocation problem with uncertain parameters, an uncertain multi-objective model is developed within the framework of uncertainty theory. The proposed model minimizes time penalty cost, distribution cost and carbon dioxide emissions. The equivalents of the model are discussed via operational laws of uncertainty distribution. By employing the goal attainment technique, a series of Pareto-optimal solutions are generated that can be used for decision-making. Finally, several numerical experiments are presented to verify the validity of the proposed model and to illustrate decision-making strategy.

**Keywords:** uncertainty modeling; emergency logistics; location-allocation; sustainability; uncertainty theory



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## 1. Introduction

Emergency logistics is one of the most active fields in Operations Research and Management Science, which involve planning, managing, and controlling the flow of resources to provide relief to the people affected (Sheu [1]). Providing an adequate service level is a struggle for many emergency response systems, but it is of special concern in the field of emergency management. Despite the recent progress in hardware equipment, information access, human resources, some pressing issues are worth to be further concerned: the reduction of the urgency of the emergency response, the funding shortfalls, the carbon emission issues, to mention a few. As a critical part of emergency logistics, the location-allocation problem concerns to open a set of candidate emergency distribution centers such that the transportation cost from centers to demand points is minimized.

As its general practical application backgrounds, location-allocation problem has received considerable attention since Cooper [2] studied it for the first time. For reviews on optimization models of emergency location-allocation problem, we may consult Beaumont [3] and de Camargo and Miranda [4], where various cases were discussed. In Murtagh and Niwattisyawong [5], a capacitated location-allocation problem, in which the capacities of facilities are limited, was proposed and a new model was introduced. Then Badri [6] and Fang and Li [7] studied multi-objective location-allocation problems using goal programming. Recently, Fan et al. [8] formulated a reliable location-allocation model for hazardous materials by considering the depot disruption. Chu and Chen [9] proposed a multi-objective location-allocation model for a three-level logistics network. There is considerable research that has been carried out for emergency facility location-allocation problems; the interested readers may refer to Wang et al. [10].

A limitation of most existing studies on the location-allocation problem is that most work is done for the deterministic case. In practice, many parameters, such as the timing, location and magnitude of a natural disaster are unpredictable rather than deterministic.

The highly unpredictable of natural disasters causes the real decisions are usually surrounded in the state of indeterminacy. Some scholars thought that parameters in this case can be described as random variables. Accordingly, probability theory (Kolmogorov [11]) is introduced into location-allocation problems and a wide range of stochastic programming models and a series of heuristic algorithms have been developed. For instance, Zhou and Liu [12] formulated three kinds of stochastic programming models according to different criteria and presented a hybrid intelligent algorithm on the basis of stochastic simulation, network simplex algorithm and genetic algorithm. Alizadeh et al. [13] studied a location-allocation problem in the heterogeneous environment and employed the normal approximation method to assess the probability distribution of the demand. Cheng and Wang [14] developed a stochastic chance-constrained programming location model for capacitated alternative-fuel stations by considering road condition. Hu et al. [15] proposed a stochastic programming model for a capacitated single allocation hub location problem.

Although the stochastic programming models have been widely used, and tally with the facts in widespread cases, it is also not suitable in a great many situations. As a matter of fact, the probability distributions for indeterminate quantities are not easy to obtain due to the lack of sufficient data. Instead, experts' belief degrees are usually employed to describe the quantities with indeterministic information. Considering this, how can we deal with belief degrees? Fuzziologists believe belief degrees can be interpreted as fuzziness. This calls for the incorporation of fuzzy set theory (Zadeh [16]) into location-allocation problem. A fuzzy set  $A$  in  $X$  is characterized by its membership function which assigns to each element  $x$  a real number  $\mu_A(x) \in [0, 1]$ . The value of  $\mu_A(x)$  is interpreted as the degree of membership of  $x$  in fuzzy set  $A$  for each  $x \in X$  (Zadeh [16]). Within the framework of fuzzy set theory, location-allocation problem has been widely studied and a variety of fuzzy programming models have been carried out. For example, Zhou and Liu [17] investigated a capacitated location-allocation problem with fuzzy demands and proposed three types of fuzzy programming models. Wen and Iwamura [18] formulated an  $\alpha$ -cost model for fuzzy location-allocation problem under the Hurwicz criterion. Liu et al. [19] proposed a two-stage fuzzy 0–1 mixed integer programming model for a three-level location-allocation transfer center.

However, is it reasonable to use fuzzy set theory to deal with belief degrees? Through a lot of investigation and analysis, Kahneman and Tversky [20] concluded that human beings usually overweight some unlikely events. With the further research, Liu [21] reveals that paradoxes may appear when we use fuzzy set theory to handle belief degrees. That is to say, it is not suitable to employ fuzzy set theory to handle belief degrees. As a breakthrough to cope with indeterminate information, uncertainty theory, which was created by Liu [22] based on normality, duality, subadditivity and product axioms, provides a powerful alternative to address belief degrees. In turn, uncertain programming was proposed by Liu [23] as a spectrum of mathematical programming to handle optimization problems involving uncertain variables. Subsequently, uncertain programming has been gradually developed and successfully applied to a series of optimization problems, such as network optimization (Gao [24]), supply chain design (Ma and Li [25]), and optimal control problems (Sheng et al. [26]), and so forth.

In terms of emergency logistics network design, Gao [27] first proposed two types of uncertain models for a single facility location problem. Then Wen et al. [28] formulated an uncertain location-allocation model via chance-constraints, and designed a hybrid intelligent algorithm to solve the model. Wang and Yang [29] studied a hierarchical facility location problem from different decision criteria and proposed two types of uncertain models. Zhang et al. [30] developed three types of covering location models for emergency facilities and discussed the analytical solutions of the models based on operational laws of uncertainty distribution. Soltanpour et al. [31] investigated an inverse 1-median location problem with uncertain parameters, and discussed the necessary and sufficient condition for the  $\alpha$ -1-median on uncertain trees. Wang and Qin [32] studied a hub maximal covering location problem in the presence of partial coverage with uncertain travel times, proposed

two types of uncertain models and presented a greedy variable neighborhood search heuristic. Gradually, more researchers investigated emergency facility location problems, see Wen et al. [33], Gao and Qin [34], Zhang et al. [35] and Zhang et al. [36], for example.

The above discussion makes it clear that a whole range of research has been carried out to address location-allocation problem within the framework of uncertainty theory. However, most of studies did not take into account the urgency of emergency response. Additionally, sustainability is currently a societal concern for development, especially in the aspect of the environment. However, large-scale logistics activities will inevitably aggravate the environment. This fact provides our motivation to design a sustainable emergency logistics network, which can not only timely rescue the affected people but also effectively reduce the pollution to the environment.

Inspired by the above discussion, this article mainly concerns the following two issues: First, how can we quantitatively characterize urgency and environmental pollution? Second, how can we provide a good trade-off the relationship among urgency, cost and environmental pollution? To answer the questions, the time penalty cost function and carbon dioxide (CO<sub>2</sub>) emissions are used to describe urgency and environmental pollution, respectively. Then, we formulate a multi-objective uncertain programming model that includes (i) minimization of the time penalty cost, (ii) minimization of the total distribution cost, and (iii) minimization of the CO<sub>2</sub> emissions. To solve the model, we apply goal attainment technique to obtain Pareto-optimal solutions. As a consequence, a set of Pareto-optimal solutions is provided for decision-makers to seek the best strategy in accordance with their preferences.

In comparison to the existing works, this paper mainly focuses on optimizing facility location-allocation strategies with uncertain information. The main contributions of this paper can be summarized as follows. First, compared to the use of other approaches, such as probability theory and fuzzy set theory, for location-allocation with uncertain information, this paper provides an opportunity to advance uncertainty theory to address location-allocation problem. Second, compared to the existing works related to the uncertain facility location-allocation problem, this paper develops a novel multi-objective location-allocation model. This paper thus complements existing literature, and it will certainly further improve the efficiency of emergency response.

The rest of the paper is organized as follows. Section 2 briefly reviews some relevant basic knowledge about uncertainty theory for readers to better understand this paper. Section 3 introduces the location-allocation problem and presents a novel multi-objective model for the deterministic case. In Section 4, an uncertain multi-objective location-allocation model is developed. After that, some crisp equivalents of the model are discussed and a solution methodology is introduced. Section 5 discusses the main contributions of the paper by comparing it with the existing works. Section 6 presents the computational experiments and illustrates the results of the analysis in detail. Finally, conclusion remarks and future research directions are provided in Section 7.

## 2. Preliminaries

Since its introduction in 2007 by Liu [22], uncertainty theory has been well studied and applied in a wide variety of fields. In the following, we briefly review the basic concepts, such as uncertainty space, uncertain variable, and uncertainty distribution, of uncertainty theory.

Let  $\Gamma$  be a nonempty set,  $\mathcal{L}$  a  $\sigma$ -algebra over  $\Gamma$ , and  $\mathcal{M}\{\Lambda\} \in [0, 1]$  an uncertain measure. Then the triplet  $(\Gamma, \mathcal{L}, \mathcal{M})$  is said to be an uncertainty space. The uncertain measure  $\mathcal{M}$  satisfies the following three axioms (Liu [22]): (i)  $\mathcal{M}\{\Gamma\} = 1$  for the universal set  $\Gamma$ ; (ii)  $\mathcal{M}\{\Lambda\} + \mathcal{M}\{\Lambda^c\} = 1$  for any event  $\Lambda$ ; (iii) For every countable sequence of events  $\Lambda_1, \Lambda_2, \dots$ , we have  $\mathcal{M}\{\bigcup_{i=1}^{\infty} \Lambda_i\} \leq \sum_{i=1}^{\infty} \mathcal{M}\{\Lambda_i\}$ .

To obtain an uncertain measure of compound event, Liu [37] defined a product uncertain measure on multiple uncertainty spaces: Let  $(\Gamma_k, \mathcal{L}_k, \mathcal{M}_k)$  be uncertainty spaces for  $k = 1, 2, \dots$ . The product uncertain measure  $\mathcal{M}$  is an uncertain measure satisfying

$$\mathcal{M}\left\{\prod_{k=1}^{\infty} \Lambda_k\right\} = \bigwedge_{k=1}^{\infty} \mathcal{M}_k\{\Lambda_k\}$$

where  $\Lambda_k$  are arbitrarily chosen events from  $\mathcal{L}_k$  for  $k = 1, 2, \dots$ , respectively.

An uncertain variable (Liu [22]) is defined as a measurable function  $\xi$  from an uncertainty space to the set of real numbers such that  $\{\xi \in B\}$  is an event for any Borel set  $B$  of real numbers. The uncertainty distribution  $\Phi$  (Liu [22]) of an uncertain variable  $\xi$  is defined as

$$\Phi(x) = \mathcal{M}\{\xi \leq x\}, \quad \text{for any } x \in \mathfrak{R}.$$

If an uncertain variable  $\xi$  has a linear uncertainty distribution

$$\Phi(x) = \begin{cases} 0, & \text{if } x \leq a \\ \frac{x-a}{b-a}, & \text{if } a < x \leq b \\ 1, & \text{if } b < x \end{cases}$$

where  $a$  and  $b$  are both real numbers with  $a < b$ , we name it linear and denote this by  $\xi \sim L(a, b)$  (Liu [22]). It is easy to verify that the inverse uncertainty distribution of  $\xi \sim L(a, b)$  is

$$\Phi^{-1}(\alpha) = (1 - \alpha)a + \alpha b, \quad \alpha \in (0, 1).$$

**Theorem 1** (Liu [38]). *Let  $\xi_1, \xi_2, \dots, \xi_n$  be independent uncertain variables with regular uncertainty distributions  $\Phi_1, \Phi_2, \dots, \Phi_n$ , respectively. If  $f(\xi_1, \xi_2, \dots, \xi_n)$  is a continuous and strictly increasing function, then  $\xi = f(\xi_1, \xi_2, \dots, \xi_n)$  is an uncertain variable with inverse uncertainty distribution*

$$\Psi^{-1}(\alpha) = f(\Phi_1^{-1}(\alpha), \Phi_2^{-1}(\alpha), \dots, \Phi_n^{-1}(\alpha)).$$

**Definition 1** (Liu [22]). *Let  $\xi$  be an uncertain variable. Then, the expected value of  $\xi$  is defined by*

$$E[\xi] = \int_0^{+\infty} \mathcal{M}\{\xi \geq x\}dx - \int_{-\infty}^0 \mathcal{M}\{\xi \leq x\}dx$$

*provided that at least one of the two integrals is finite.*

It can be verified that the expected value of the linear uncertain variable  $\xi \sim L(a, b)$  is  $(a + b)/2$ , i.e.,  $E[\xi] = \frac{a+b}{2}$ . For detailed expositions, the interested reader may refer to Liu [21,22,38].

### 3. Problem Description and Formulation

In this paper, the location-allocation problem contains two levels (emergency distribution centers and demand points) and aims to open some centers to supply resources to demand points. In consideration of urgency, relief cost and CO<sub>2</sub> emissions, the goals of the problem are (i) to determine the subset of candidate emergency distribution centers to open and (ii) to draw up the resource allocation plan from the centers to the demand points.

#### 3.1. Notations

To clearly describe the problem by a mathematical model, Table 1 lists some parameters and variables which are used to formulate the problem.

**Table 1.** List of notations.

Parameters	
$I$	The set of candidate emergency distribution centers with index $i = 1, 2, \dots, n$
$J$	The set of demand points with index $j = 1, 2, \dots, m$
$C_i$	The $i$ th candidate emergency distribution center, $i \in I$
$D_j$	The $j$ th demand point, $j \in J$
$p_{ij}$	The unit material time penalty cost from $C_i$ to $D_j$ , $i \in I, j \in J$
$c_{ij}$	The unit material distribution cost from $C_i$ to $D_j$ , $i \in I, j \in J$
$d_{ij}$	The distance from $C_i$ to $D_j$ , $i \in I, j \in J$
$e_{ij}$	The unit material CO <sub>2</sub> emissions per kilometer from $C_i$ to $D_j$ , $i \in I, j \in J$
$s_j$	The minimal demand of materials in $D_j$ , $j \in J$
$q_i$	The capacity of $C_i$ , $i \in I$
$p$	The maximum number of emergency distribution centers selected
$f_i$	The operation cost of opening $C_i$ , $i \in I$
$W$	The budget for opening the emergency distribution centers
Variables	
$y_i$	1 if $C_i$ is selected to open, 0 otherwise, $i \in I$
$x_{ij}$	The material distribution volume from $C_i$ to $D_j$ , $i \in I, j \in J$
$Y$	The set of $y_i$ , $i \in I$
$X$	The set of $x_{ij}$ , $i \in I, j \in J$

### 3.2. Multi-Objective Location-Allocation Model

In our model, three objectives are considered: (i) urgency, i.e., the urgency of emergency response; (ii) cost, i.e., the distribution costs for supplying the demand points with relief goods; and (iii) sustainability, i.e., the carbon dioxide emissions from transportation.

#### 3.2.1. Objective 1: Minimization of the Time Penalty Cost

The urgency of demand points often has a significant effect on emergency response decision-making. Now, the critical issue is how to measure urgency. It follows from Wan et al. [39] that the time penalty cost approach can be used for the quantization of urgency. Denote  $p_{ij}$  as the unit material time penalty cost from the candidate emergency distribution center  $C_i$  to the demand point  $D_j$ , and use  $x_{ij}$  to denote the material distribution volume from  $C_i$  to  $D_j$ . Therefore, the time penalty cost from  $C_i$  to  $D_j$  is  $p_{ij}x_{ij}$ . Then, the objective 1 can be formulated as:

$$\min F_1(X, Y) = \max_{j \in J} \sum_{i \in I} p_{ij}x_{ij}, \quad (1)$$

which tries to minimize the maximal time penalty cost.

#### 3.2.2. Objective 2: Minimization of the Total Distribution Cost

Our model tries to simultaneously determine the locations of emergency distribution centers and the material distribution volume such that the total material distribution cost is minimized. Denote  $c_{ij}$  as the unit material distribution cost from  $C_i$  to  $D_j$ . Objective 2 is as follows:

$$\min F_2(X, Y) = \max_{j \in J} \sum_{i \in I} c_{ij}x_{ij}. \quad (2)$$

#### 3.2.3. Objective 3: Minimization of the CO<sub>2</sub> Emissions

In the past few years, a growing number of authors have paid attention to address the issue of carbon emissions in emergency response. Road transportation has become a major factor causing the rapid growing of carbon emissions. Denote  $e_{ij}$  and  $d_{ij}$  as the unit material CO<sub>2</sub> emissions per kilometer and distance from  $C_i$  to  $D_j$ , respectively. Thus, the formulation for objective 3 is given by

$$\min F_3(X, Y) = \max_{j \in J} \sum_{i \in I} d_{ij}e_{ij}x_{ij}. \quad (3)$$

### 3.3. Constraints

By considering some realistic constraints, the constraints shown in (4)–(8) are established. Constraint (4) requires that the total goods transported from  $n$  emergency distribution centers should satisfy the demand of  $D_j$ . Constraint (5) guarantees that the amount of the relief distributed from  $C_i$  does not exceed its capacity; it also prevents materials from being distributed from an emergency distribution center that is not open. Constraint (6) ensures that all expenditures on the operations of the distribution centers cannot exceed the budget. Constraint (7) limits the maximum number of emergency distribution centers opened at  $p$ . Constraint (8) provides the range of the decision variables  $x_{ij}$  and  $y_i$ .

$$\sum_{i \in I} x_{ij} \geq s_j, \quad \forall j \in J, \tag{4}$$

$$\sum_{j \in J} x_{ij} \leq q_i y_i, \quad \forall i \in I, \tag{5}$$

$$\sum_{i \in I} f_i y_i \leq W, \tag{6}$$

$$\sum_{i \in I} y_i \leq p, \tag{7}$$

$$x_{ij} \geq 0, y_i \in \{0, 1\}, \quad \forall i \in I, j \in J. \tag{8}$$

### 4. Uncertain Location-Allocation Model

Notice that the parameters in the above model are all assumed to be constants. However, in the real world, especially in the emergency response system, indeterminate factors may exist, and it is difficult for us to describe them as random variables due to the lack of history data. In this case, we usually invite some domain experts to evaluate the values of the parameters. As a result, we may get states such as “about 10 km”, “about 5 tons” and “between 3 and 5 h”. Uncertainty theory is a mathematical subject specially used to address this kind of expert’s experimental data. To formulate the location-allocation problem with uncertain information, we assume that  $p_{ij}$ ,  $c_{ij}$ ,  $d_{ij}$ ,  $s_j$ ,  $q_i$ , and  $f_i$  are all uncertain variables, and rewrite them as  $\zeta_{ij}$ ,  $\eta_{ij}$ ,  $\zeta_{ij}$ ,  $\psi_j$ ,  $\varphi_i$ , and  $\omega_i$ , respectively. That is,

$\zeta_{ij}$ : Uncertain unit material time penalty cost from  $C_i$  to  $D_j$ ,  $i \in I, j \in J$

$\eta_{ij}$ : Uncertain unit material distribution cost from  $C_i$  to  $D_j$ ,  $i \in I, j \in J$

$\zeta_{ij}$ : Uncertain distance from  $C_i$  to  $D_j$ ,  $i \in I, j \in J$

$\psi_j$ : Uncertain minimal demand of materials in  $D_j$ ,  $j \in J$

$\varphi_i$ : Uncertain capacity of  $C_i$ ,  $i \in I$

$\omega_i$ : Uncertain operation cost of opening  $C_i$ ,  $i \in I$ .

Accordingly, the model formulated above becomes an uncertain model. Employing expected value and chance-constraint, the uncertain location-allocation model can be developed as follows:

$$\min \tilde{F}_1(X, Y) = \max_{j \in J} E \left[ \sum_{i \in I} \zeta_{ij} x_{ij} \right] \tag{9}$$

$$\min \tilde{F}_2(X, Y) = \max_{j \in J} E \left[ \sum_{i \in I} \eta_{ij} x_{ij} \right] \tag{10}$$

$$\min \tilde{F}_3(X, Y) = \max_{j \in J} E \left[ \sum_{i \in I} \zeta_{ij} \varphi_i x_{ij} \right] \tag{11}$$

$$\text{s.t. } \mathcal{M} \left\{ \sum_{i \in I} x_{ij} \geq \psi_j \right\} \geq \alpha_j, \quad \forall j \in J, \tag{12}$$

$$\mathcal{M} \left\{ \sum_{j \in J} x_{ij} \leq \varphi_i y_i \right\} \geq \beta_i, \quad \forall i \in I, \tag{13}$$

$$\mathcal{M} \left\{ \sum_{i \in I} \omega_i y_i \leq W \right\} \geq \gamma, \tag{14}$$

$$\sum_{i \in I} y_i \leq p, \tag{15}$$

$$x_{ij} \geq 0, y_i \in \{0, 1\}, \quad \forall i \in I, j \in J, \tag{16}$$

where  $\alpha_j, \beta_i$  and  $\gamma$  are predetermined confidence levels.

#### 4.1. Deterministic Transformations

To solve the proposed uncertain model, it is necessary for us to discuss the equivalence of the model. Using  $\theta, \delta$  and  $\mu$  as auxiliary variables, which are defined as follows:

$$\theta = \max_{j \in J} E \left[ \sum_{i \in I} \xi_{ij} x_{ij} \right],$$

$$\delta = \max_{j \in J} E \left[ \sum_{i \in I} \eta_{ij} x_{ij} \right],$$

$$\mu = \max_{j \in J} E \left[ \sum_{i \in I} \zeta_{ij} e_{ij} x_{ij} \right].$$

Then, Models (9)–(16) is reformulated as

$$\min \theta \tag{17}$$

$$\min \delta \tag{18}$$

$$\min \mu \tag{19}$$

$$\text{s.t. } E \left[ \sum_{i \in I} \xi_{ij} x_{ij} \right] \leq \theta, \quad \forall j \in J, \tag{20}$$

$$E \left[ \sum_{i \in I} \eta_{ij} x_{ij} \right] \leq \delta, \quad \forall j \in J, \tag{21}$$

$$E \left[ \sum_{i \in I} \zeta_{ij} e_{ij} x_{ij} \right] \leq \mu, \quad \forall j \in J, \tag{22}$$

$$\text{s.t. } \mathcal{M} \left\{ \sum_{i \in I} x_{ij} \geq \psi_j \right\} \geq \alpha_j, \quad \forall j \in J, \tag{23}$$

$$\mathcal{M} \left\{ \sum_{j \in J} x_{ij} \leq \varphi_i y_i \right\} \geq \beta_i, \quad \forall i \in I,$$

$$\mathcal{M} \left\{ \sum_{i \in I} \omega_i y_i \leq W \right\} \geq \gamma,$$

$$\sum_{i \in I} y_i \leq p,$$

$$x_{ij} \geq 0, y_i \in \{0, 1\}, \quad \forall i \in I, j \in J,$$

**Theorem 2.** Let  $\xi_{ij}, \eta_{ij}$ , and  $\zeta_{ij}$  be independent uncertain variables with finite expected values. If  $\psi_j, \varphi_i$ , and  $\omega_i$  are independent uncertain variables with regular uncertainty distributions  $\Phi_j, \Psi_i$ , and  $\Omega_i$ , respectively. Then, Models (17)–(23) can be converted into the following form:

$$\min \theta \tag{24}$$

$$\min \delta \tag{25}$$

$$\min \mu \tag{26}$$

$$\text{s.t. } \sum_{i \in I} E[\xi_{ij}]x_{ij} \leq \theta, \quad \forall j \in J, \tag{27}$$

$$\sum_{i \in I} E[\eta_{ij}]x_{ij} \leq \delta, \quad \forall j \in J, \tag{28}$$

$$\sum_{i \in I} E[\zeta_{ij}]e_{ij}x_{ij} \leq \mu, \quad \forall j \in J, \tag{29}$$

$$\sum_{i \in I} x_{ij} \geq \Phi_j^{-1}(\alpha_j), \quad \forall j \in J, \tag{30}$$

$$\sum_{j \in J} x_{ij} \leq y_i \Psi_i^{-1}(1 - \beta_i), \quad \forall i \in I, \tag{31}$$

$$\sum_{i \in I} y_i \Omega_i^{-1}(\gamma) \leq W, \tag{32}$$

$$\sum_{i \in I} y_i \leq p, \tag{33}$$

$$x_{ij} \geq 0, y_i \in \{0, 1\}, \quad \forall i \in I, j \in J,$$

**Proof.** It follows from the linearity of expected value operator of uncertain variable that

$$E \left[ \sum_{i \in I} \xi_{ij} x_{ij} \right] = \sum_{i \in I} E[\xi_{ij}]x_{ij},$$

which shows that constraint (27) holds for any  $j \in J$ . Similarly, we may verify that constraints (21) and (22) can be equivalently transformed into constraints (28) and (29), respectively.

Next, we prove that constraint (13) can be transformed into constraint (31). Denote  $Y_i$  as the uncertainty distribution of  $\varphi_i y_i$ . That is,

$$Y_i(x) = \mathcal{M}\{\varphi_i y_i \leq x\}$$

for any real number  $x$ . It follows from Theorem 1 that  $Y_i$  is regular since  $\varphi_i$  has a regular uncertainty distribution  $\Psi_i$ . In other words, for any  $\beta_i \in (0, 1)$ , we have

$$Y_i^{-1}(\beta_i) = y_i \Psi_i^{-1}(\beta_i).$$

In addition,

$$\mathcal{M}\{\varphi_i y_i \leq y_i \Psi_i^{-1}(1 - \beta_i)\} = \mathcal{M}\{\varphi_i y_i \leq Y_i^{-1}(1 - \beta_i)\} = 1 - \beta_i, \tag{34}$$

since  $Y_i$  is regular. According to constraint (13), for each  $i$ , we know

$$\mathcal{M}\left\{\varphi_i y_i \leq \sum_{j \in J} x_{ij}\right\} \leq 1 - \beta_i. \tag{35}$$

As a result, it follows from (34) and (35) that constraint (31) holds. Similarly, we may also verify that constraints (12) and (14) can be converted into constraints (30) and (32), respectively. Thus, the proof is completed.  $\square$

#### 4.2. Solution Methodology

As mentioned above, to solve the proposed uncertain Models (9)–(16), the most important but most difficult step is to solve the Models (24)–(33), which is essentially a multi-objective programming model. Typically, no single optimal solution can be found to optimize all of the objectives at the same time. Instead, the efficient solution/Pareto optimal is commonly used for multi-objective optimization. To date, an assortment of methods, such as heuristic algorithms (Zhang and Xiong [40], Majumder et al. [41]), goal programming methods (Liu and Chen [42]) and multi-objective evolutionary algorithms

(Alcaraz et al. [43]), have been developed to solve multi-objective problems. Goal attainment technique (Azaron et al. [44]) is one of the multi-objective techniques, and has been successfully applied to production systems (Azaron et al. [45]) and supply chain design (Azaron et al. [44]). In the following, we will use the goal attainment technique to solve Models (24)–(33) and to generate Pareto-optimal solutions. Different from interactive multi-objective technology, the goal attainment technique is a one-stage method, so it will be computationally faster.

By using the goal attainment technique, Models (24)–(33) can be reformulated as follows:

$$\min \quad \pi \quad (36)$$

$$\text{s.t.} \quad \theta - w_1\pi \leq g_1, \quad (37)$$

$$\delta - w_2\pi \leq g_2, \quad (38)$$

$$\mu - w_3\pi \leq g_3, \quad (39)$$

$$\text{s.t.} \quad \sum_{i \in I} E[\xi_{ij}]x_{ij} \leq \theta, \quad \forall j \in J, \quad (40)$$

$$\sum_{i \in I} E[\eta_{ij}]x_{ij} \leq \delta, \quad \forall j \in J,$$

$$\sum_{i \in I} E[\zeta_{ij}]e_{ij}x_{ij} \leq \mu, \quad \forall j \in J,$$

$$\sum_{i \in I} x_{ij} \geq \Phi_j^{-1}(\alpha_j), \quad \forall j \in J,$$

$$\sum_{j \in J} x_{ij} \leq y_i \Psi_i^{-1}(1 - \beta_i), \quad \forall i \in I,$$

$$\sum_{i \in I} y_i \Omega_i^{-1}(\gamma) \leq W,$$

$$\sum_{i \in I} y_i \leq p,$$

$$x_{ij} \geq 0, y_i \in \{0, 1\}, \quad \forall i \in I, j \in J,$$

In Models (36)–(40),  $g_i$  and  $w_i$ ,  $i = 1, 2, 3$ , are goals and weights for the three objective functions. The values of  $w_i$  are generally normalized such that  $\sum_{i=1}^3 w_i = 1$  and  $w_i \geq 0$  for  $i = 1, 2, 3$ . The weights  $w_i$  relate the relative under-attainment of the goals  $g_i$ . In other words, the more important the goal  $g_i$ , the smaller the weight  $w_i$ .

Notice that if  $(X^*, Y^*)$  is Pareto-optimal, then there exists a pair of  $(g = \{g_i\}, w = \{w_i\})$  such that  $(X^*, Y^*)$  is an optimal solution to the optimization Problems (36)–(40) (Azaron et al. [44]). Obviously, the optimal solution to model (36)–(40) is sensitive to  $(g, w)$ . In the numerical experiments, we will generate different Pareto-optimal solutions by changing the values of  $g$  and  $w$ .

## 5. Innovations and Comparisons

In order to highlight the contributions of the proposed work, we will compare the article with existing works in two main aspects. First, the modeling method of this paper is discussed by comparing it with stochastic optimization models. Second, the innovations of the paper are further illustrated by comparing it with the existing works within the framework of uncertainty theory.

As previously discussed, we are usually in the state of indeterminacy since the highly unpredictable nature of emergencies. To date, a wide range of innovative studies on facility location and resource allocation problem have been conducted within the framework of probability theory. Although stochastic models have been widely accepted and applied to practical problems, it is not suitable to regard every indeterminate factor as a random factor. It is universally acknowledged that a premise of applying probability theory is that we can obtain sufficient historical data to estimate probability distribution. However, we often have no access to get the required observational data due to economical or technical reasons. In this case, we have no choice but to rely on the subjective-intuitive opinions of

experts, which is named “belief degree”. Obviously, it is not suitable to deal with belief degree by probability theory. In order to address belief degree rationally, uncertainty theory was founded by Liu [22] based on normality, duality, subadditivity and product axioms. Liu [22] also pointed out that belief degree follows the laws of uncertainty theory. Subsequently, uncertainty theory has been successfully applied to the fields of information science, management science and engineering. In this paper, uncertainty theory is employed to address location-allocation problem with uncertain parameters, which are described as uncertain variables. An uncertain multi-objective model is proposed by using of uncertainty theory. In more detail, the main difference between the proposed model and stochastic location-allocation models are demonstrated in Table 2.

**Table 2.** Comparison between the proposed model and stochastic models.

	Stochastic Location-Allocation Models	The Proposed Model
Sample size	Large enough	Too small (even no-sample)
Type of indeterminacy	Stochastic factors	Subjective-intuitive opinions of experts
Uncertain parameter	Random variable	Uncertain variable
Theoretical basis	Probability theory	Uncertainty theory

As a powerful tool to address belief degrees, uncertainty theory has been applied to the field of location problem and resource allocation problem. Compared with existing articles within the framework of uncertainty theory, the main innovation of the proposed paper is to introduce an approach to quantitatively characterizing urgency, and further develop a novel multi-objective model. The main difference among them are illustrated in Table 3 for better readability.

**Table 3.** Comparison between the proposed model and some related uncertain models.

References	Location Type	Uncertain Parameters	Modeling Approach	Critical Factors
Gao [27]	Single facility	Demand	Single objective	Satisfaction degree
Wen et al. [28]	Capacitated facility	Demand	Single objective; $\alpha$ -optimistic criterion	Transportation cost
Wang and Yang [29]	Hierarchical facility	Cost; the amount of waste	Single objective	Logistics cost
Zhang et al. [30]	Covering location	Demand; time	Single objective	Number of utilized facilities; the covered demand
Soltanpour et al. [31]	Inverse 1-median	Vertex weight; modification cost	Single objective	Total cost; tail value at risk
Wang and Qin [32]	Hub maximal covering	Travel time	Multi-objective	The total flows covered; total cost
Wen et al. [33]	Capacitated facility	Demand	Single objective; expected value	Transportation cost
Gao and Qin [34]	$p$ -hub center	Travel time	Minimax single objective	The maximal travel time
Zhang et al. [35]	Emergency facility	Demand; travel time; opening cost	Multi-objective	Travel time; relief cost; carbon dioxide emission
Zhang et al. [36]	Capacitated $p$ -center	Demand; distance	Minimax single objective	The maximal travel distance; capacity
The proposed model	Emergency distribution center	Demand; distance; time penalty cost; distribution cost; capacity; opening cost	Minimax multi-objective	Time penalty cost; distribution cost; carbon dioxide emission

### 6. Numerical Experiments

To verify the validity of the proposed model and test the efficiency of the solution method, numerical experiments are conducted in this section. To further illustrate the sensitivity of the optimal solution with respect to the values of goals  $g_i$  and weights  $w_i$ , more experiments with different values of  $g_i$  and  $w_i$  are provided.

We consider an instance with 12 demand points and 6 candidate emergency distribution centers, i.e.,  $m = 12$  and  $n = 6$ . Without loss of generality, the unit material time penalty cost  $\xi_{ij}$ , unit material distribution cost  $\eta_{ij}$  and distance  $\zeta_{ij}$  from candidate emergency distribution center  $C_i$  to demand point  $D_j$  are assumed to be independent linear uncertain variables, which are shown in Table 4, Table 5 and Table 6, respectively. The minimal demands  $\psi_j$  are given in Table 7. The capacities  $\varphi_i$  and the operation costs  $\omega_i$  are also assumed to be independent linear uncertain variables, which are shown in Table 8. In addition, set  $p = 5$ ,  $W = 120$ ,  $e_{ij} = 6$  for any  $i \in I, j \in J$ .

**Table 4.** Unit material time penalty cost  $\xi_{ij}$ .

		$D_j$					
		$D_1$	$D_2$	$D_3$	$D_4$	$D_5$	$D_6$
$C_i$	$C_1$	$\mathcal{L}(1,5)$	$\mathcal{L}(2,6)$	$\mathcal{L}(1,4)$	$\mathcal{L}(3,7)$	$\mathcal{L}(1,7)$	$\mathcal{L}(3,8)$
	$C_2$	$\mathcal{L}(2,5)$	$\mathcal{L}(1,6)$	$\mathcal{L}(1,5)$	$\mathcal{L}(2,7)$	$\mathcal{L}(4,7)$	$\mathcal{L}(2,8)$
	$C_3$	$\mathcal{L}(3,6)$	$\mathcal{L}(1,4)$	$\mathcal{L}(2,4)$	$\mathcal{L}(3,5)$	$\mathcal{L}(2,5)$	$\mathcal{L}(1,4)$
	$C_4$	$\mathcal{L}(2,4)$	$\mathcal{L}(3,6)$	$\mathcal{L}(3,6)$	$\mathcal{L}(1,5)$	$\mathcal{L}(2,7)$	$\mathcal{L}(1,5)$
	$C_5$	$\mathcal{L}(4,8)$	$\mathcal{L}(3,6)$	$\mathcal{L}(2,5)$	$\mathcal{L}(2,6)$	$\mathcal{L}(3,8)$	$\mathcal{L}(1,3)$
	$C_6$	$\mathcal{L}(3,7)$	$\mathcal{L}(2,4)$	$\mathcal{L}(2,6)$	$\mathcal{L}(1,5)$	$\mathcal{L}(2,6)$	$\mathcal{L}(2,5)$
		$D_j$					
		$D_7$	$D_8$	$D_9$	$D_{10}$	$D_{11}$	$D_{12}$
$C_i$	$C_1$	$\mathcal{L}(2,7)$	$\mathcal{L}(3,9)$	$\mathcal{L}(1,4)$	$\mathcal{L}(2,5)$	$\mathcal{L}(3,6)$	$\mathcal{L}(2,6)$
	$C_2$	$\mathcal{L}(2,4)$	$\mathcal{L}(1,5)$	$\mathcal{L}(2,6)$	$\mathcal{L}(1,5)$	$\mathcal{L}(1,4)$	$\mathcal{L}(1,6)$
	$C_3$	$\mathcal{L}(2,6)$	$\mathcal{L}(2,6)$	$\mathcal{L}(2,4)$	$\mathcal{L}(4,8)$	$\mathcal{L}(3,6)$	$\mathcal{L}(3,8)$
	$C_4$	$\mathcal{L}(2,5)$	$\mathcal{L}(2,4)$	$\mathcal{L}(3,5)$	$\mathcal{L}(3,8)$	$\mathcal{L}(1,4)$	$\mathcal{L}(1,6)$
	$C_5$	$\mathcal{L}(1,5)$	$\mathcal{L}(2,5)$	$\mathcal{L}(3,7)$	$\mathcal{L}(2,5)$	$\mathcal{L}(2,8)$	$\mathcal{L}(4,6)$
	$C_6$	$\mathcal{L}(4,8)$	$\mathcal{L}(2,7)$	$\mathcal{L}(5,8)$	$\mathcal{L}(1,5)$	$\mathcal{L}(2,6)$	$\mathcal{L}(1,3)$

**Table 5.** Unit material distribution cost  $\eta_{ij}$ .

		$D_j$					
		$D_1$	$D_2$	$D_3$	$D_4$	$D_5$	$D_6$
$C_i$	$C_1$	$\mathcal{L}(13,16)$	$\mathcal{L}(10,16)$	$\mathcal{L}(12,14)$	$\mathcal{L}(13,15)$	$\mathcal{L}(11,13)$	$\mathcal{L}(14,17)$
	$C_2$	$\mathcal{L}(12,16)$	$\mathcal{L}(11,17)$	$\mathcal{L}(12,14)$	$\mathcal{L}(10,16)$	$\mathcal{L}(13,18)$	$\mathcal{L}(13,16)$
	$C_3$	$\mathcal{L}(10,15)$	$\mathcal{L}(12,16)$	$\mathcal{L}(15,18)$	$\mathcal{L}(10,14)$	$\mathcal{L}(12,16)$	$\mathcal{L}(13,15)$
	$C_4$	$\mathcal{L}(15,18)$	$\mathcal{L}(12,16)$	$\mathcal{L}(14,16)$	$\mathcal{L}(10,15)$	$\mathcal{L}(10,13)$	$\mathcal{L}(12,15)$
	$C_5$	$\mathcal{L}(13,16)$	$\mathcal{L}(11,14)$	$\mathcal{L}(11,16)$	$\mathcal{L}(12,17)$	$\mathcal{L}(10,15)$	$\mathcal{L}(11,13)$
	$C_6$	$\mathcal{L}(10,15)$	$\mathcal{L}(12,14)$	$\mathcal{L}(12,16)$	$\mathcal{L}(11,16)$	$\mathcal{L}(10,16)$	$\mathcal{L}(14,16)$
		$D_j$					
		$D_7$	$D_8$	$D_9$	$D_{10}$	$D_{11}$	$D_{12}$
$C_i$	$C_1$	$\mathcal{L}(10,15)$	$\mathcal{L}(12,16)$	$\mathcal{L}(11,13)$	$\mathcal{L}(13,15)$	$\mathcal{L}(12,14)$	$\mathcal{L}(13,16)$
	$C_2$	$\mathcal{L}(15,18)$	$\mathcal{L}(10,14)$	$\mathcal{L}(13,17)$	$\mathcal{L}(17,19)$	$\mathcal{L}(11,16)$	$\mathcal{L}(11,15)$
	$C_3$	$\mathcal{L}(11,16)$	$\mathcal{L}(14,19)$	$\mathcal{L}(12,14)$	$\mathcal{L}(12,16)$	$\mathcal{L}(10,13)$	$\mathcal{L}(11,17)$
	$C_4$	$\mathcal{L}(15,17)$	$\mathcal{L}(12,14)$	$\mathcal{L}(11,14)$	$\mathcal{L}(15,18)$	$\mathcal{L}(11,14)$	$\mathcal{L}(15,18)$
	$C_5$	$\mathcal{L}(13,16)$	$\mathcal{L}(13,15)$	$\mathcal{L}(10,13)$	$\mathcal{L}(13,17)$	$\mathcal{L}(12,15)$	$\mathcal{L}(12,14)$
	$C_6$	$\mathcal{L}(14,18)$	$\mathcal{L}(12,14)$	$\mathcal{L}(13,16)$	$\mathcal{L}(12,16)$	$\mathcal{L}(12,14)$	$\mathcal{L}(11,16)$

**Table 6.** Distance  $\zeta_{ij}$ .

		$D_j$					
		$D_1$	$D_2$	$D_3$	$D_4$	$D_5$	$D_6$
$C_i$	$C_1$	$\mathcal{L}(5, 10)$	$\mathcal{L}(4, 8)$	$\mathcal{L}(10, 15)$	$\mathcal{L}(3, 9)$	$\mathcal{L}(10, 15)$	$\mathcal{L}(8, 12)$
	$C_2$	$\mathcal{L}(10, 14)$	$\mathcal{L}(8, 10)$	$\mathcal{L}(6, 9)$	$\mathcal{L}(8, 12)$	$\mathcal{L}(12, 15)$	$\mathcal{L}(10, 14)$
	$C_3$	$\mathcal{L}(13, 16)$	$\mathcal{L}(8, 13)$	$\mathcal{L}(6, 10)$	$\mathcal{L}(10, 14)$	$\mathcal{L}(12, 14)$	$\mathcal{L}(5, 8)$
	$C_4$	$\mathcal{L}(11, 15)$	$\mathcal{L}(7, 10)$	$\mathcal{L}(6, 10)$	$\mathcal{L}(11, 16)$	$\mathcal{L}(7, 12)$	$\mathcal{L}(13, 16)$
	$C_5$	$\mathcal{L}(12, 15)$	$\mathcal{L}(9, 14)$	$\mathcal{L}(10, 13)$	$\mathcal{L}(10, 15)$	$\mathcal{L}(8, 10)$	$\mathcal{L}(12, 14)$
	$C_6$	$\mathcal{L}(6, 11)$	$\mathcal{L}(14, 19)$	$\mathcal{L}(13, 17)$	$\mathcal{L}(9, 13)$	$\mathcal{L}(13, 18)$	$\mathcal{L}(13, 17)$

		$D_j$					
		$D_7$	$D_8$	$D_9$	$D_{10}$	$D_{11}$	$D_{12}$
$C_i$	$C_1$	$\mathcal{L}(12, 16)$	$\mathcal{L}(6, 10)$	$\mathcal{L}(4, 10)$	$\mathcal{L}(7, 13)$	$\mathcal{L}(14, 17)$	$\mathcal{L}(8, 14)$
	$C_2$	$\mathcal{L}(8, 13)$	$\mathcal{L}(9, 12)$	$\mathcal{L}(8, 13)$	$\mathcal{L}(7, 9)$	$\mathcal{L}(13, 16)$	$\mathcal{L}(14, 17)$
	$C_3$	$\mathcal{L}(5, 7)$	$\mathcal{L}(9, 13)$	$\mathcal{L}(8, 13)$	$\mathcal{L}(8, 16)$	$\mathcal{L}(11, 14)$	$\mathcal{L}(7, 11)$
	$C_4$	$\mathcal{L}(12, 18)$	$\mathcal{L}(14, 17)$	$\mathcal{L}(9, 15)$	$\mathcal{L}(15, 18)$	$\mathcal{L}(7, 16)$	$\mathcal{L}(13, 17)$
	$C_5$	$\mathcal{L}(10, 17)$	$\mathcal{L}(7, 9)$	$\mathcal{L}(13, 17)$	$\mathcal{L}(10, 19)$	$\mathcal{L}(14, 17)$	$\mathcal{L}(10, 14)$
	$C_6$	$\mathcal{L}(6, 12)$	$\mathcal{L}(7, 12)$	$\mathcal{L}(13, 16)$	$\mathcal{L}(7, 13)$	$\mathcal{L}(7, 12)$	$\mathcal{L}(10, 13)$

**Table 7.** The minimal demand  $\psi_j$ .

$D_j$	$D_1$	$D_2$	$D_3$	$D_4$	$D_5$	$D_6$
$\psi_j$	$\mathcal{L}(25, 50)$	$\mathcal{L}(30, 40)$	$\mathcal{L}(25, 60)$	$\mathcal{L}(35, 65)$	$\mathcal{L}(20, 50)$	$\mathcal{L}(30, 55)$

$D_j$	$D_7$	$D_8$	$D_9$	$D_{10}$	$D_{11}$	$D_{12}$
$\psi_j$	$\mathcal{L}(35, 70)$	$\mathcal{L}(25, 55)$	$\mathcal{L}(30, 40)$	$\mathcal{L}(35, 50)$	$\mathcal{L}(20, 55)$	$\mathcal{L}(20, 35)$

**Table 8.** The capacity  $\varphi_i$  and the operation cost  $\omega_i$ .

$C_i$	$C_1$	$C_2$	$C_3$	$C_4$	$C_5$	$C_6$
$\varphi_i$	$\mathcal{L}(184, 194)$	$\mathcal{L}(190, 210)$	$\mathcal{L}(165, 185)$	$\mathcal{L}(220, 240)$	$\mathcal{L}(200, 210)$	$\mathcal{L}(175, 190)$
$\omega_i$	$\mathcal{L}(15, 30)$	$\mathcal{L}(20, 30)$	$\mathcal{L}(10, 30)$	$\mathcal{L}(25, 35)$	$\mathcal{L}(20, 40)$	$\mathcal{L}(20, 25)$

The problem attempts to minimize the time penalty cost, the total material distribution cost and the CO<sub>2</sub> emissions in the sense of expected value while making the following determinations: (i) which of the candidate emergency distribution centers to open; and (ii) for each demand point, which distribution centers are assigned to it? We use the multi-objective uncertain programming approach to this multi-objective optimization problem and employ the goal attainment technique to solve the multi-objective model. Then, a mathematical model can be formulated as model (36)–(40), which contains 82 variables and 132 constraints, is essentially a deterministic linear programming model.

To generate the Pareto-optimal solutions, the values of  $g_i$  and  $w_i$  ( $i = 1, 2, 3$ ) are varied manually. In order to illustrate the sensitivity of parameters, when one of the parameters is varied, the others are fixed. According to the obtained absolute minimum values for the maximal time penalty cost (i.e., “urgency”), the maximum material distribution cost and the maximum CO<sub>2</sub> emissions, by solving the corresponding single objective models,  $g_1$  is varied from 200 (close to the absolute minimum value for the maximal time penalty cost) to 400,  $g_2$  is varied from 850 (close to the absolute minimum value for the maximal material distribution cost) to 1700,  $g_3$  is varied from 3000 (close to the absolute minimum value for the maximal CO<sub>2</sub> emissions) to 6000. Similarly,  $w_i$  are varied from 0.001 to 0.998 for  $i = 1, 2, 3$ . Let  $\alpha_j = \beta_i = \gamma = 0.9$ , by using of LINGO 10, 30 Pareto-optimal strategies and the corresponding objective values for time penalty cost, material distribution cost and the CO<sub>2</sub> emissions are generated and are revealed in Table 9. Obviously, each strategy generates a location-allocation strategy. For example, according to strategy 1, the candidate emergency distribution centers 1, 3, 5 and 6 should be selected. In more detail, the resource allocation strategy is illustrated in Figure 1.

For each pair of  $(g, w)$ , the Pareto-optimal solution is obtained. If we are not satisfied with any Pareto-optimal solution, then the value of  $w$  should be modified. As mentioned above, the values of  $w_i$  relate the relative under-attainment of the objective goals  $g_i$ . The more important the goal  $g_i$ , the smaller the value of  $w_i$ . For example, if the obtained value of objective 1 is much greater than 200,  $w_1$  should be decreased. Accordingly, both  $w_2$  and  $w_3$  should also be modified such that the summation of  $w_i$  equals to 1. Obviously, this process can be repeated with different pairs of  $(g, w)$  until the decision-maker obtains a satisfactory Pareto-optimal solution.

For example, strategy 1 in Table 9 implies that one unit deviation of the time penalty cost from 200 is about 1000 times as important as one unit deviation of the CO<sub>2</sub> emissions from 3000 and the same important as one unit deviation of the material distribution cost from 850. In this strategy, the weights for time penalty cost and material distribution cost are relatively low, which result in the strategy has low time penalty cost and material distribution cost. As shown in strategies 1–6, as the value of  $w_2$  increases, the value of material distribution cost also increases gradually. For better readability, the results are shown in Figure 2.

**Table 9.** Pareto-optimal solutions.

Strategy	$g_1$	$g_2$	$g_3$	$w_1$	$w_2$	$w_3$	Selected Centers	Obj. 1	Obj. 2	Obj. 3
1	200	850	3000	0.001	0.001	0.998	C <sub>1</sub> C <sub>3</sub> C <sub>5</sub> C <sub>6</sub>	248.68	898.68	5484.86
2	200	850	3000	0.001	0.005	0.994	C <sub>1</sub> C <sub>3</sub> C <sub>5</sub> C <sub>6</sub>	217.93	939.67	5423.37
3	200	850	3000	0.001	0.01	0.989	C <sub>1</sub> C <sub>3</sub> C <sub>5</sub> C <sub>6</sub>	210.02	950.22	5407.54
4	200	850	3000	0.001	0.05	0.949	C <sub>2</sub> C <sub>3</sub> C <sub>5</sub> C <sub>6</sub>	202.33	966.58	5212.65
5	200	850	3000	0.001	0.1	0.899	C <sub>2</sub> C <sub>3</sub> C <sub>5</sub> C <sub>6</sub>	201.83	1032.83	4643.60
6	200	850	3000	0.001	0.5	0.499	C <sub>1</sub> C <sub>2</sub> C <sub>3</sub> C <sub>6</sub>	202.24	1089.04	4115.64
7	200	850	3000	0.001	0.998	0.001	C <sub>1</sub> C <sub>3</sub> C <sub>5</sub> C <sub>6</sub>	255.13	995.30	3055.13
8	200	850	3000	0.01	0.989	0.001	C <sub>1</sub> C <sub>3</sub> C <sub>5</sub> C <sub>6</sub>	258.41	932.25	3005.84
9	200	850	3000	0.1	0.899	0.001	C <sub>1</sub> C <sub>3</sub> C <sub>5</sub> C <sub>6</sub>	258.76	927.74	3000.59
10	200	850	3000	0.998	0.001	0.001	C <sub>1</sub> C <sub>3</sub> C <sub>4</sub> C <sub>6</sub>	310.00	884.41	3034.41
11	200	850	3000	0.989	0.01	0.001	C <sub>1</sub> C <sub>3</sub> C <sub>4</sub> C <sub>6</sub>	297.96	885.05	3003.51
12	200	850	3000	0.899	0.1	0.001	C <sub>1</sub> C <sub>3</sub> C <sub>4</sub> C <sub>6</sub>	297.96	885.12	3000.35
13	400	850	3000	0.001	0.001	0.998	C <sub>1</sub> C <sub>2</sub> C <sub>3</sub> C <sub>6</sub>	289.07	851.61	4608.60
14	400	850	3000	0.001	0.005	0.994	C <sub>1</sub> C <sub>2</sub> C <sub>3</sub> C <sub>6</sub>	288.75	856.83	4358.09
15	400	850	3000	0.001	0.01	0.989	C <sub>1</sub> C <sub>2</sub> C <sub>3</sub> C <sub>6</sub>	288.75	861.48	4135.09
16	400	850	3000	0.001	0.05	0.949	C <sub>1</sub> C <sub>3</sub> C <sub>5</sub> C <sub>6</sub>	288.75	875.17	3477.76
17	400	850	3000	0.001	0.1	0.899	C <sub>1</sub> C <sub>3</sub> C <sub>5</sub> C <sub>6</sub>	295.83	879.58	3265.96
18	400	850	3000	0.001	0.5	0.499	C <sub>1</sub> C <sub>3</sub> C <sub>4</sub> C <sub>6</sub>	272.67	884.41	3034.34
19	200	1700	3000	0.001	0.998	0.001	C <sub>1</sub> C <sub>3</sub> C <sub>5</sub> C <sub>6</sub>	255.13	932.25	3055.13
20	200	1700	3000	0.01	0.989	0.001	C <sub>1</sub> C <sub>3</sub> C <sub>5</sub> C <sub>6</sub>	258.41	914.76	3005.84
21	200	1700	3000	0.1	0.899	0.001	C <sub>1</sub> C <sub>3</sub> C <sub>5</sub> C <sub>6</sub>	258.76	914.35	3000.59
22	200	1700	3000	0.998	0.001	0.001	C <sub>1</sub> C <sub>3</sub> C <sub>5</sub> C <sub>6</sub>	258.80	902.95	3000.06
23	200	1700	3000	0.989	0.01	0.001	C <sub>1</sub> C <sub>3</sub> C <sub>5</sub> C <sub>6</sub>	258.80	911.22	3000.06
24	200	1700	3000	0.899	0.1	0.001	C <sub>1</sub> C <sub>3</sub> C <sub>5</sub> C <sub>6</sub>	258.80	911.22	3000.07
25	200	850	6000	0.001	0.998	0.001	C <sub>1</sub> C <sub>2</sub> C <sub>5</sub> C <sub>6</sub>	200.11	963.43	5387.73
26	200	850	6000	0.01	0.989	0.001	C <sub>1</sub> C <sub>2</sub> C <sub>5</sub> C <sub>6</sub>	201.13	962.07	5389.77
27	200	850	6000	0.1	0.899	0.001	C <sub>1</sub> C <sub>3</sub> C <sub>5</sub> C <sub>6</sub>	211.00	948.91	5409.51
28	200	850	6000	0.998	0.001	0.001	C <sub>1</sub> C <sub>3</sub> C <sub>5</sub> C <sub>6</sub>	285.12	850.09	5557.75
29	200	850	6000	0.989	0.01	0.001	C <sub>1</sub> C <sub>3</sub> C <sub>5</sub> C <sub>6</sub>	284.55	850.85	5556.60
30	200	850	6000	0.899	0.1	0.001	C <sub>1</sub> C <sub>3</sub> C <sub>5</sub> C <sub>6</sub>	278.63	858.75	5544.76

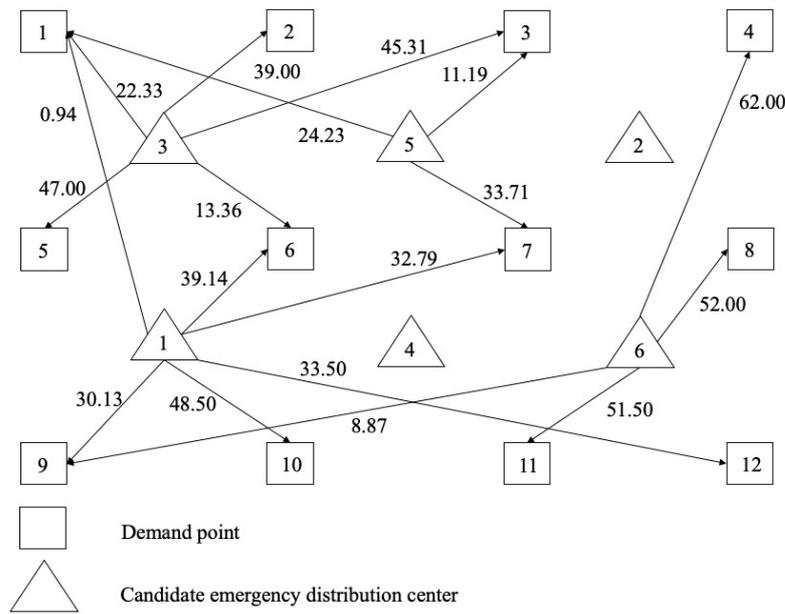


Figure 1. Location-allocation for strategy 1 in Table 9.

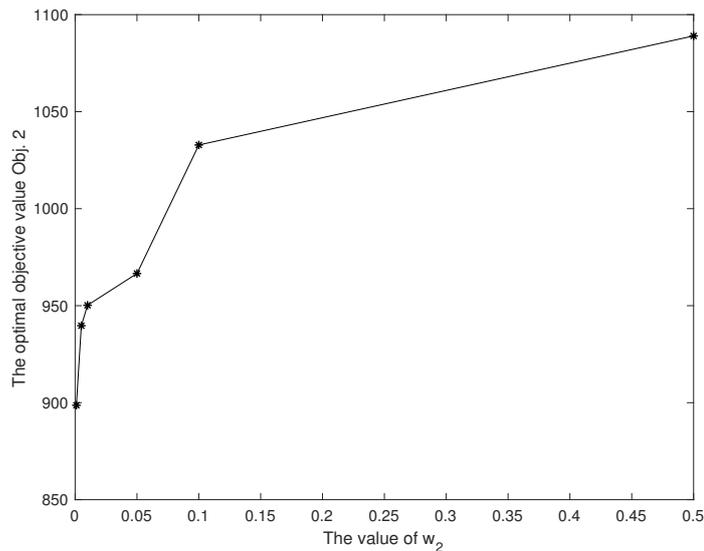
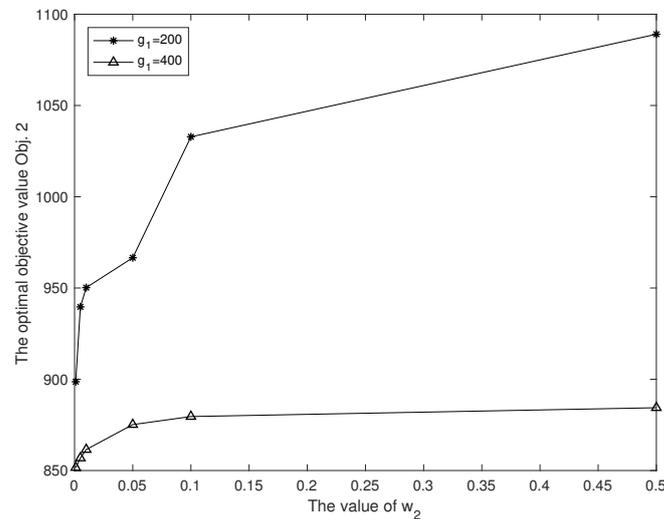


Figure 2. The value of material distribution cost with different  $w_2$ .

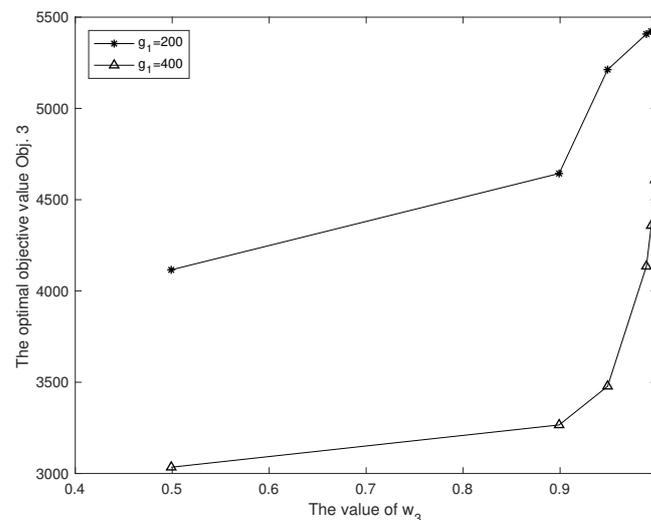
It follows from Table 9 that the lowest time penalty cost comes from strategy 25, with a value of 200.11. The strategy has relatively low material distribution cost, but high CO<sub>2</sub> emissions. The lowest CO<sub>2</sub> emissions is obtained in strategy 22 and strategy 23, with a time penalty cost of 258.80. The optimal material distribution cost is obtained in strategy 28, with a time penalty cost of 285.12. So, the time penalty cost ranges from 200.11 to 285.12. To find an appropriate solution, the decision-maker needs insight into this range of outcomes, to be able to trade-off different criteria in terms of the results. According to the numerical results, it is concluded that the relationship among time penalty cost, material distribution cost and CO<sub>2</sub> emissions is not clear and it is not easy to define a function to handle the relation among them. That is why we formulate a multi-objective model to solve this location-allocation problem.

It is also can be seen from Table 9 that increasing goal for the time penalty cost (i.e.,  $g_1$ ) and keeping other parameters unchanged cause the material distribution cost and CO<sub>2</sub> emissions to be decreased (see strategies 1–6 and strategies 13–18). In addition, the results are shown in Figures 3 and 4. Similarly, increasing goal for the material distribution cost

causes both the time penalty cost and the CO<sub>2</sub> emissions to be decreased (see strategies 7–12 and strategies 19–24). In addition, the increasing goal for the CO<sub>2</sub> emissions may cause time penalty cost or material distribution cost to be decreased (see strategies 7–12 and strategies 25–30). In other words, it seems that by increasing the goal of any one of the objectives, we can provide more space for other objectives to be improved.



**Figure 3.** The material distribution cost with different  $g_1$ .



**Figure 4.** The CO<sub>2</sub> emissions with different  $g_1$ .

## 7. Conclusions and Future Research

This paper studies the location-allocation problem within the framework of uncertainty theory. Its main contribution, the time penalty cost function, is introduced to characterize the urgency of emergency response and the formulation of a multi-objective model to consider urgency, relief costs and CO<sub>2</sub> emissions simultaneously. In the solving procedures, we transform the model from an uncertain programming problem to a deterministic programming problem.

In the numerical experiments, a series of Pareto-optimal solutions are generated by goal attainment technique. The results reveal that the goal attainment technique can provide an effective approach to trade-off urgency, relief costs and CO<sub>2</sub> emissions. It also shows that the goal attainment technique will help the decision-makers to find the best strategy based on their preferences by varying parameter  $(g, w)$ . However, for large-scale problems, this technique may no longer be effective, in terms of computational time. In this

case, some heuristic algorithms, such as genetic algorithms or simulated annealing, would be suitable.

Several directions for further studies are pointed out here. Future research could take the choices by beneficiaries into account, which will improve the allocation of relief goods and effectively enhance the use of scarce resources. The beneficiaries remain free in their choice of the distribution centers, so the preferences of beneficiaries rather than cost should be considered for decision-makers. Apart from that, for large-scale emergencies, the relief goods should be delivered to distant demand points. Therefore, it is necessary to develop a series of meta-heuristic approaches such as genetic algorithm or tabu search algorithm to solve large-scale problems. In addition, we could consider the problem under a more complex environment. For example, in an uncertain random environment, some indeterminate quantities can be interpreted as uncertain variables, while others may be interpreted as random variables.

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