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Application of Finite Volume Method Based on the C-N Format with Parallel Algorithms in Metal Thermal Phase Transition Coupled Field

Bing Su , Zeyu Gong * and Han Li 

School of Mechatronics Engineering, Henan University of Science and Technology, Luoyang 471003, China; subing@haust.edu.cn (B.S.); li_han0013@163.com (H.L.)

* Correspondence: zeyugong@outlook.com

Abstract: The numerical simulation of metal heat treatment is of great significance for the development of new materials or processes. The complex coupling relationship in metal heat treatment poses a huge challenge to the overall solution. A finite volume method based on the Crack–Nicolson (C-N) scheme with parallel algorithms is proposed for solving the coupling of metal thermal phase transition. Taking the quenching simulation problem of bearing rings as an example, the calculation accuracy and speed of this method were evaluated by comparing the results of the temperature field and phase transformation field obtained from the literature. In addition, grid independence analysis was conducted to demonstrate the rationality of the established model. The finite volume method using a hexahedral mesh C-N format improves the accuracy of calculations. Optimizing the equation form and using parallel algorithms have improved computational speed. In the trial example, in the 850 °C quenching simulation of GCr15, the outer ring of the bearing is composed of 89.1% martensite and 10.9% residual austenite. The results compare with test verification, which shows that the model performs well in the numerical simulation of metal heat treatment.

Keywords: metal heat treatment; finite volume method; numerical simulation; parallel algorithm; coupled field calculation



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1. Introduction

Metal heat treatment is a complex multidisciplinary research field in which three physical fields, heat, phase transition, and stress/strain, are coupled to each other. In the process of simulation, the complexity of this nonlinear coupling can bring great challenges to the solution. Numerical simulation study of metal heat treatment began in the middle of the last century. In 1978, Hildenwall [1] calculated the residual stress of carburizing steel quenching. Then, the Scheil superposition rule was used to disperse the continuous cooling into different small segments, and the amount of tissue transition during continuous cooling was successfully predicted using the TTT curve. After Fernandes performed a similar study on pearlitic transition in XC80 [2] Rammerstorfer et al. [3] conducted thermal elastic plasticity analysis of quenching and compared the influence of creep and phase change plasticity on the simulation results. The test results showed that phase change plasticity influenced stress change, while creep affected it less. Inoue [4,5] conducted a numerical simulation study on the quenching process, and their developed numerical simulation software, “HEARTS”, can conduct numerical simulation of water quenching, carburization quenching, and induction quenching of parts, and the simulation value showed high

reliability in comparison with the actual test results. Later, Denis et al. [6] considered the effect of phase change plasticity on the phase transition dynamics of martensite in the simulation process of martensite steel quenching and compared the calculated residual stress with the actual measured stress state. Gergely [7] and Sjostrom [8] also analyzed the influence of the part size and process conditions on the quenching process. In recent years, Dongying Ju and Ishikawa [9] have used the “metal-heat-machinery” theory and the multi-field coupling simulation method to study the elliptical deformation mechanism after the quenching of the bearing ring. Samuel [10] summarizes previous studies and summarizes the rules of stress and deformation in quenching. In addition, the calculation method of multi-field coupling during quenching is also described in the literature [11].

To sum up, current scholars have deeply studied the different metals and different phase transition forms, and have obtained many mathematical models. However, there is little research on the computational method of the model. Based on the Fourier equation (Fourier’s Law) and the phase transition model, we propose a C-N scheme finite volume method with a parallel algorithm. To further determine the accuracy of the calculation method and the high speed of the calculation speed, the simulated bearing outer ring quenching was compared with the literature. It is shown that the proposed calculation method can effectively solve the calculation problem of the metal heat-phase transition coupling field.

2. The Coupled-Field Model

The metal heat treatment process includes three physical fields: heat, phase transition, and strain. The relationship of three-field coupling is shown in Figure 1 [12]. The temperature distribution of the workpiece is affected by the external temperature, latent heat, and deformation heat generation. The thermal performance parameters of the material change with the change of temperature, the temperature change is caused by the phase transition, and the phase transition produces internal stress. The study shows that the effect of deformation heat generation on temperature change is more negligible than the effect of phase change latent heat on temperature change, so the effect of deformation heat generation is ignored in the simplified model.

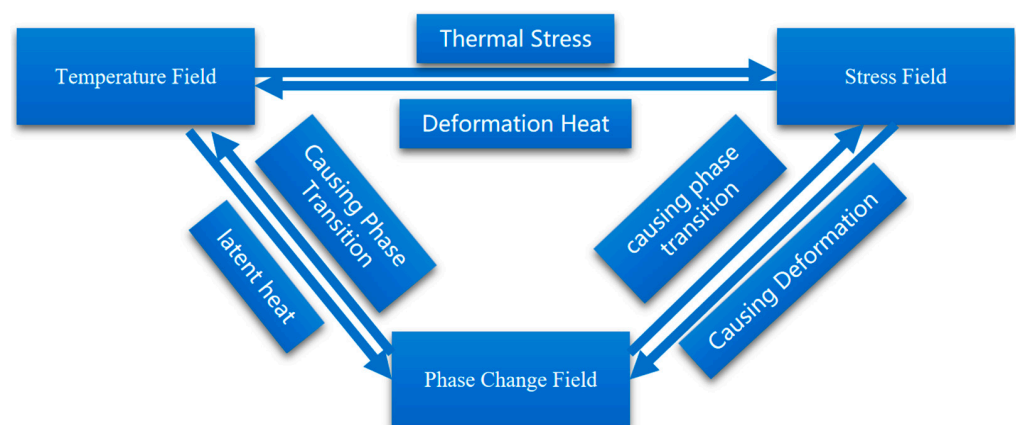


Figure 1. Diagram of the interaction relationship of quenching multi-physical fields [12].

2.1. Temperature Field

The Fourier thermal conductivity equation is the governing equation for solid thermal conductivity. When the temperature of an object varies with time and there is an internal

heat source, the heat transfer equation in a rectangular coordinate system can be derived from Fourier's law [13]:

$$\rho c \frac{\partial T}{\partial \tau} = \left[\frac{\partial}{\partial x} \left(\lambda \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left(\lambda \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left(\lambda \frac{\partial T}{\partial z} \right) \right] + \dot{\Phi} \quad (1)$$

where λ represents the thermal conductivity of the material; T represents the temperature of the object; c represents the specific heat capacity of the material; ρ represents the material density; and $\dot{\Phi}$ represents the internal heat source, which refers to the phase transition latent heat.

The heat transfer between bearing part surface and environment can be described by the third type of boundary conditions, namely:

$$-\lambda \frac{\partial T}{\partial n} = h(T - T_c) \quad (2)$$

where $\frac{\partial T}{\partial n}$ represents the temperature gradient of the surface layer of the part; h is the total heat transfer coefficient, which comprises the convective heat transfer coefficient h_k and the radiation heat transfer coefficient h_s ; T stands for the surface temperature of the work piece; and T_c is the ambient temperature, typically assumed to be 25 °C. The radiation heat transfer coefficient is as follows:

$$h_s = \sigma_0 \varepsilon (T^2 + T_c^2)(T + T_c) \quad (3)$$

where σ_0 is the Stefan–Boltzmann constant and its value is $5.768 \times 10^{-8} \text{ W}/(\text{m}^2 \text{gK}^4)$; ε is the surface radiation of the part.

2.2. Phase Field

In heat treatment, the phase transition field can be divided into diffusive and non-diffusion-type phase transition, and their phase transition model is different.

2.2.1. Diffusion Phase Transition

For diffusive phase transition, the common research model of Johnson and Avrami [14–16] and the expression of diffusive phase transition over time is as follows:

$$V = 1 - \exp(-bt^n) \quad (4)$$

$$\begin{cases} n = \frac{\ln \left[\frac{\ln(1 - V_1)}{\ln(1 - V_2)} \right]}{\ln \left[\frac{t_1}{t_2} \right]} \\ b = -\frac{\ln(1 - V_1)}{t_1^n} \end{cases} \quad (5)$$

In Equations (4) and (5), t is a time and V is the isothermal transformation amount. However, the phase transition model proposed by Johnson and Avrami describes the phase transition process in the isothermal case. For the continuous cooling quenching, the time needs to be discretized under the continuous cooling condition. The differential time period is considered as the isothermal transition process, and the Scheil superposition rule is used to calculate the transition amount.

2.2.2. Non-Diffusive Phase Transition

For the non-diffusion phase transition, the amount of transition depends on the change of temperature rather than the length of the transition time. Koistinen and Marburger [17] show that the relationship of the transition amount on temperature can be expressed as:

$$V = 1 - \exp[-\alpha(M_s - T)] \quad (6)$$

In Equation (6), M_s is the temperature point where martensite transition begins, and α is a constant, which reflects the transition rate of martensite and varies with different steel types.

2.3. Coupled Field

To ensure accuracy, the hexahedral meshing method was used. The finite-volume method was used to solve the Fourier equation. The boundary grid needs to be subdivided when dividing the irregular artifacts. In this paper, the grid is preprocessed by combining the grid method and the sweep method [18]. The domain building method for the bearing outer ring is shown in Figure 2.

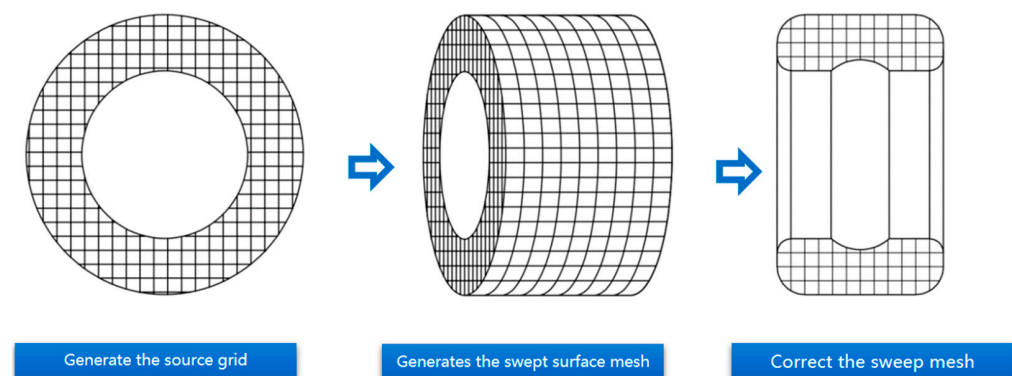


Figure 2. Finite volume method mesh generation and boundary mesh processing.

First, with a disjointed grid (cell) covering the end surface of the bearing outer ring, keep the grid falling inside the target area and delete the grid outside the area. This step is called source surface grid generation. Then, based on the source mesh, a circular cylinder composed of hexahedrons is obtained by sweeping in the radial direction of the outer ring of the bearing. The grid principle formed by this grid method and sweep method is simple, and the grid-forming speed is fast. It is a simple and effective method to solve the rectangular area (rectangular coordinate system) and cylindrical region (polar coordinate system). However, the grid generated by this method has poor cell quality of the boundary grid, so the boundary grid needs correction. In this paper, we propose a new boundary grid reconstruction method for the characteristics of the thermal gradient in the heat transfer equation, as shown in Figure 3.

When the grid is generated by the above method, the length, width and height of a single differential unit are set, and each differential unit has the same unit attribute. However, in this case, the boundary of the object is the structure of the surface. If the properties of this cell are not adjusted at this time, then the impact on the accuracy of the calculation results is very large. In Figure 3a is the actual grid and Figure 3b is the corrected grid. Considering the definition of the temperature gradient in the heat transfer equation, we need to find the temperature of the unit represented by the discretization of the partial differential equation in the calculation. In the dimensional direction, we need to know the distance between the current cell and the two adjacent cells, which is twice the length of the sum of the model. In the boundary grid of the two-dimensional model, the boundary arc to

the line average can be taken as the distance between the cell and the boundary. Similarly, in the 3D model, the average of the vertical surface of the boundary arc surface relative to the direction of the point can be taken as the distance between the cell and the boundary. Adjusting the properties of the boundary grid according to this method can improve the accuracy of the calculation results.

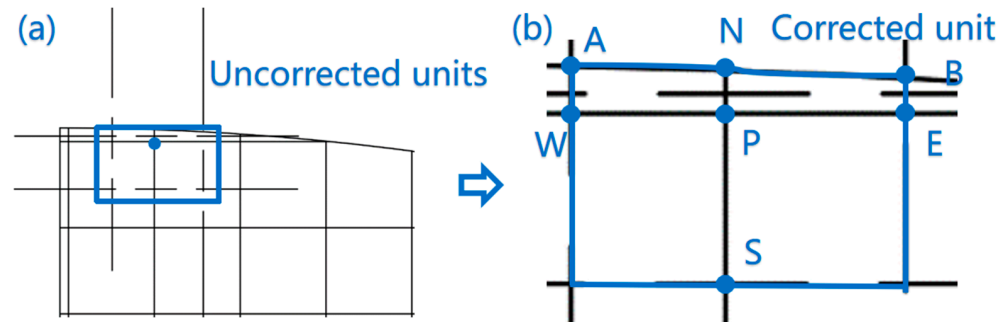


Figure 3. Boundary mesh correction scheme. (a) Boundary mesh shape and (b) Enlarged version of boundary mesh shape.

As shown in Figure 4, different modeling areas are divided according to the shape of the boundary. Regions 1 and 7 correspond to the inner and outer chamfers. Regions 2 and 6 correspond to the outer chamfer and outer inner diameter. Regions 3 and 5 correspond to the outer diameter and inner diameter. Region 4 is the area corresponding to the outer diameter and channel.

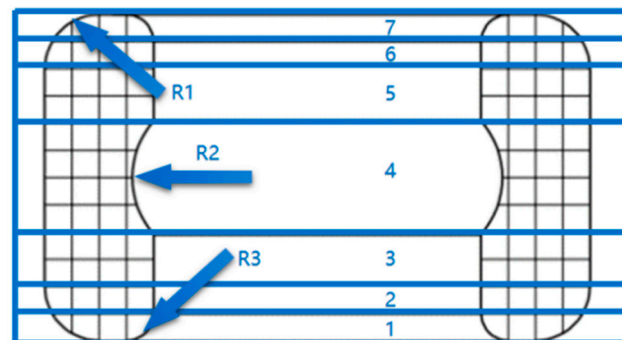


Figure 4. Schematic diagram of modeling area division.

The bearing ring shown above should be divided into 7 regions to model, and the boundary equations satisfied by the nodes in different regions are shown in Table 1.

Table 1. Boundary conditions of each region.

Area	Boundary Condition
1	$R^3 = [(x, y, z) R3 \leq x^2 + y^2 \leq R1, z \in \text{area1}]$
2	$R^3 = [(x, y, z) d \leq x^2 + y^2 \leq R1, z \in \text{area2}]$
3	$R^3 = [(x, y, z) d \leq x^2 + y^2 \leq D, z \in \text{area3}]$
4	$R^3 = [(x, y, z) R2 \leq x^2 + y^2 \leq D, z \in \text{area4}]$
5	$R^3 = [(x, y, z) d \leq x^2 + y^2 \leq D, z \in \text{area5}]$
6	$R^3 = [(x, y, z) d \leq x^2 + y^2 \leq R1, z \in \text{area6}]$
7	$R^3 = [(x, y, z) R3 \leq x^2 + y^2 \leq R1, z \in \text{area7}]$

The geometry of the bearing outer ring is a highly symmetric structure, with the X- and Y-axes representing the circumferential plane and the Z-axis representing the direction of the axis. In Table 1, R1, R2, and R3 are the equations represented by the arc segments, d

is the inner diameter, and D is the outer diameter. All the points that fall within these seven regions are considered as grid nodes. The mesh size near the boundary is represented by the following equation:

$$X^+ = \begin{cases} mesh_x, & \left| \sqrt{D^2 - y^2} - x \right| > mesh_x \\ \left| \sqrt{D^2 - y^2} - x \right|, & \left| \sqrt{D^2 - y^2} - x \right| \leq mesh_x \end{cases} \quad (7)$$

Equation (7) expresses the grid boundary condition in area 3 of Figure 3/Table 1. Here is the distance of the current node from the next node in the positive direction of the X-axis, and the initial grid spacing set for the X-axis direction. When the distance is close to the boundary, and the distance from the next grid is less than $mesh_x$, the actual distance is taken as the node spacing. Similarly, the node distances in the y, z, and negative directions can be obtained.

3. Computational Method

This chapter explains the calculation method of the coupling field in detail, discusses the stability of the model, and also introduces the parallel calculation and the accelerated calculation method.

3.1. Calculation of the Temperature Field

In the calculation of the transient heat-phase transition coupling field, the temperature convergence is an important basis to judge whether the calculation process is completed, so the accuracy of the temperature field is particularly important. Before calculating the temperature field, the divided grid is first identified, as shown in Figure 5, and can be any cell in the current calculation model. P is the center of the cell, S and N are the centers of the adjacent cells for the cell's X-axis, W and E are the centers of the adjacent cells, and T and B are the centers of the adjacent cells in the cell's Z-axis.

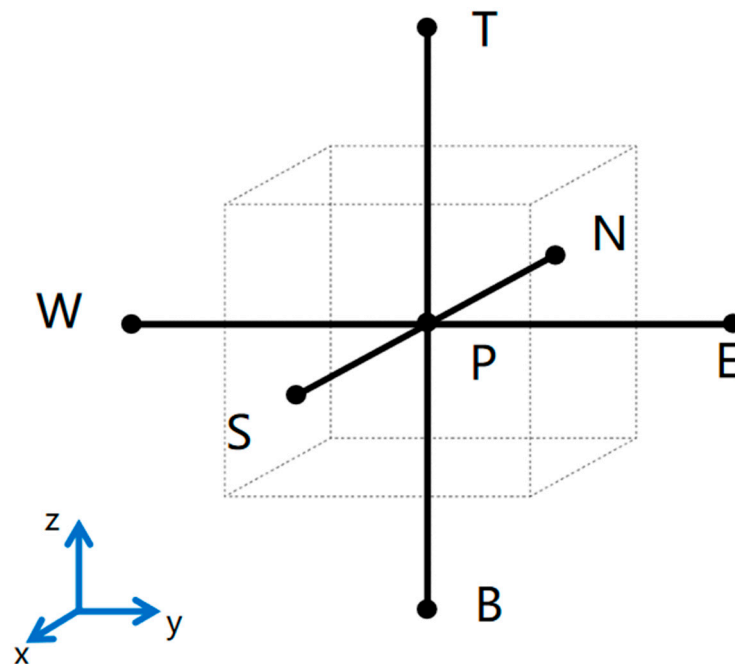


Figure 5. Schematic diagram of differential cell coordinate orientation.

For the three-dimensional transient thermal conductivity problem [19], integrate Equation (1):

$$\begin{aligned} \int_w^e \int_n^s \int_b^t \left(\int_t^{t+\Delta t} \rho c_p \frac{\partial T}{\partial t} dt \right) dx dy dz = \\ \int_1^{t+\Delta t} \left[\int_w^e \int_n^s \int_b^t \frac{\partial}{\partial x} \left(\lambda \frac{\partial T}{\partial x} \right) dx dy dz \right] dt \\ + \int_t^{t+\Delta t} \left(\int_w^e \int_n^s \int_b^t \Phi dx dy dz \right) dt \end{aligned} \quad (8)$$

In Equation (8), s and n are, respectively, two interfaces for the X-axis direction of the cell; w and e are two interfaces in the Y-axis direction of the cell; and t and b are, respectively, two interfaces for the Z-axis direction of the cell. The left side of the equation is the transient term, and the two terms on the right side of the equation are the diffusion and source terms, respectively. Discretization of these three terms is available:

$$\begin{aligned} \rho c_p \frac{\Delta x \Delta y}{\Delta t} (T_P - T_P^0) = f [-(a_W + a_E + a_S + a_N \\ + a_B + a_T - S_P \Delta x \Delta y \Delta z)] T_P + f (a_W T_W + a_E T_E \\ + a_S T_S + a_N T_N + a_B T_B + a_T T_T + S_C \Delta x \Delta y \Delta z) \\ + (1 - f) [-(a_W + a_E + a_S + a_N + a_B + a_T \\ - S_P \Delta x \Delta y \Delta z)] T_P^0 + (1 - f) (a_W T_W^0 + a_E T_E^0 \\ + a_S T_S^0 + a_N T_N^0 + a_B T_B^0 + a_T T_T^0 + S_C \Delta x \Delta y \Delta z) \end{aligned} \quad (9)$$

After finishing, you can get:

$$\begin{aligned} a_P T_P = f (a_W T_W + a_E T_E + a_S T_S + a_N T_N \\ + a_B T_B + a_T T_T) + (1 - f) (a_W T_W + a_E T_E \\ + a_S T_S + a_N T_N + a_B T_B + a_T T_T) + [a_P^0 \\ - (1 - f) (a_W + a_E + a_S + a_N + a_B \\ + a_T - S_P \Delta x \Delta y \Delta z)] T_P^0 + b \end{aligned} \quad (10)$$

In equation:

$$\left\{ \begin{aligned} a_P &= a_P^0 + f (a_W + a_E + a_S + a_N + a_B \\ &\quad + a_T - S_P \Delta x \Delta y \Delta z) \\ b &= S_C \Delta x \Delta y \Delta z \\ a_P^0 &= \rho c_p \frac{\Delta x \Delta y \Delta z}{\Delta t} \\ a_W &= \frac{k_w \Delta y \Delta z}{\delta x_{WP}} \\ a_E &= \frac{k_e \Delta y \Delta z}{\delta x_{PE}} \\ a_S &= \frac{k_s \Delta x \Delta z}{\delta y_{SP}} \\ a_N &= \frac{k_n \Delta x \Delta z}{\delta y_{PN}} \\ a_B &= \frac{k_b \Delta x \Delta y}{\delta z_{BP}} \\ a_T &= \frac{k_t \Delta x \Delta y}{\delta z_{PT}} \\ \dot{\Phi} &= S_C + S_P T_P \end{aligned} \right. \quad (11)$$

where T_*^0 (* represents any letter contained in the equation) is the temperature of each discrete cell in the current time step, and T_* is the temperature of each discrete cell in the next time step. f is the parameter of the iterative method, $f = 0$ is in implicit format, $f = 1$ is the display format, $f = 0.5$ is in C-N format, Δ^* is the size of the current discrete cell, k_* is the thermal conductivity of the current cell in the * direction, and δ^* is the distance between the center-to-center of two adjacent cells. The source term $\dot{\Phi}$ is a function of temperature.

Here, the source term will be linearized to the first order, S_C is the constant part, and S_P is the coefficient of T_P .

3.2. Calculation of the Coupled Field

The phase transformation field is calculated by the tissue field based on the temperature field. Special attention should be paid to the fact that the temperature change will affect the phase change, and the latent heat of the phase change will, in turn, affect the temperature change. When treating the phase transition latent heat, the latent heat generated inside each unit is treated as the internal heat source of this unit. In the quenching process, the latent heat of the phase transition is the heat generated by the decomposition of austenite, and the calculation equation is [20]:

$$q_v = \Delta H \frac{f_{n+1} - f_n}{\Delta t} \quad (12)$$

In Equation (12), it is the phase change volume fraction at time, the volume fraction at time, the corresponding heat enthalpy of ferrite (F), pearlite (P), bainite (B), and martensite (M), which is the degree of phase change per unit time. The thermal enthalpies of austenite decomposition are shown in Table 2.

Table 2. The enthalpy value of austenite decomposition [20].

Phase	Ferrite	Pearlite	Bainite	Martensite
ΔH (J/m ³)	5.9×10^8	6.0×10^8	6.2×10^8	6.5×10^8

The calculated temperature field is considered below. If you need to pursue a fast solution, then directly use the display format; the calculation speed is fast and simple. If the complex problem of heat-phase transition coupled field is considered, then the C-N format or implicit format is needed to ensure a good calculation result. This is necessary to solve a system of linear equations related to Equation (10). The common methods of solving linear equations are Cramer's rule method, the tridiagonal matrix algorithm (TDMA) method, the direct iteration method, the Jacobi iteration method, and the alternating direction implicit method.

In this paper, we used the idea of the C-N scheme to achieve the purpose of phase transition and temperature equilibrium. We defined $T_{1,1}$ as the temperature value of the current cell in the first time step and first iteration. We defined $f_{1,1}$ as the phase transition volume fraction of the current cell in the first time step and first iteration. We defined $q_{1,1}$ as the phase change thermogeneration of the current cell in the first time step and first iteration. The iterative process is:

$$\begin{array}{llll}
 1 & T_{1,1} = F(T_{0,0}, T_{0,0}) & f_{1,1} = G(T_{1,1}) & q_{1,1} = H(f_{1,1}) \\
 2 & T_{1,2} = F(T_{0,0}, T_{1,1}, q_{1,1}) & f_{1,2} = G(T_{1,2}) & q_{1,2} = H(f_{1,2}) \\
 \dots & \dots & \dots & \dots \\
 N & T_{1,N} = F(T_{0,0}, T_{1,N-1}, q_{1,N-1}) & f_{1,N} = G(T_{1,N}) & q_{1,N} = H(f_{1,N})
 \end{array} \quad (13)$$

In the above process, F , G and H are the solution functions of temperature, phase transition, and latent heat, respectively. In the first iteration step, the temperature $T_{1,1}$ is calculated using the display format, the phase transition volume fraction $f_{1,1}$ of the next time step is calculated by $T_{1,1}$, and then the latent heat $q_{1,1}$ of the next time step is calculated according to $f_{1,1}$. Then in the second iteration step, $T_{0,0}$, $T_{1,1}$ and $q_{1,1}$ are included as parameters to calculate the temperature $T_{1,2}$ by using C-N format, and then $f_{1,2}$ and $q_{1,2}$

are calculated according to the same principle. When the difference between F , G , H in step $N - 1$ and F , G , H in step N is less than the error ξ , it is considered to be complete iteration.

The SDL flow chart of the overall quenching simulation calculation process is shown in Figure 6. Since the focus of this article is on the model of thermal phase transition, the stress–strain model and hardness model are not the main focus of this article. Therefore, the stress–strain model and hardness model are omitted in this article.

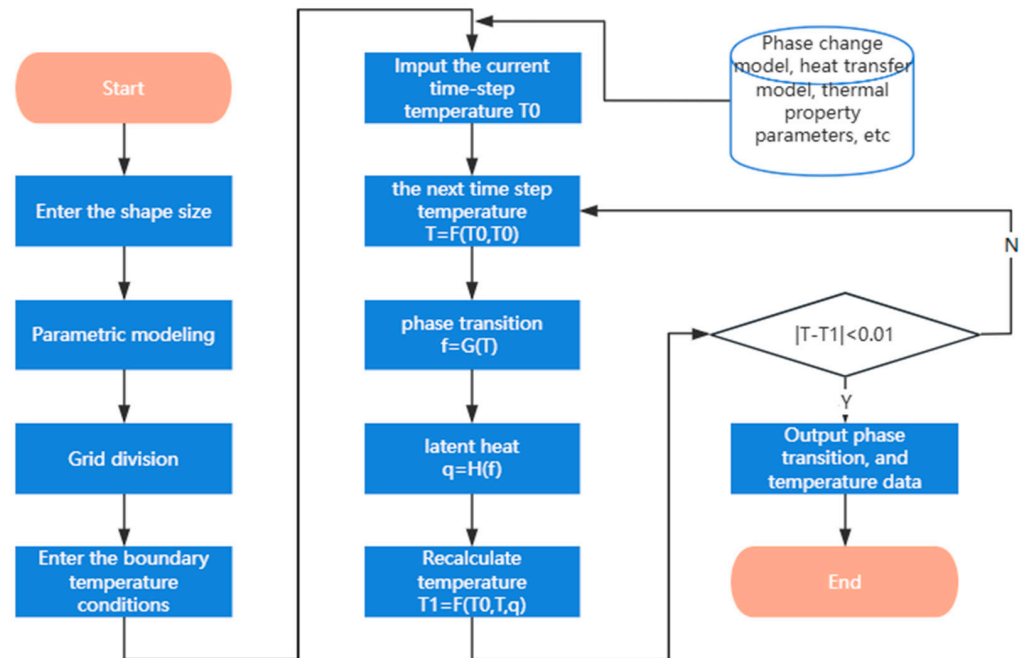


Figure 6. SDL flowchart of quenching simulation calculation process.

The above flowchart corresponds to one iteration of Formula (13), which illustrates how to use the temperature of the current unit to calculate the temperature for the next time step. The overall calculation process can be iterated in this way.

3.3. Stability and Convergence Domain of Model Computation

In the simulation calculation, the calculation accuracy directly determines the reliability of the calculation results. The central difference scheme used in this paper is a discrete method with second-order accuracy, with the step size reduced by half and the error reduced to 1/4 of the original. Reducing the time step with reducing the grid size is a common method to improve the calculation accuracy, but considering the actual computing resources consumed, we should not indefinitely reduce the time step and grid size, it is very important to select the appropriate time step and grid size. Considering the discrete form of the Fourier equation and the physical significance of the equation, the solution domain of Equation (10) is not a real range. When selecting the time step and grid size, unphysical shock will occur if it is not suitable. From the expression of Equation (10), the unknown T_p is a weighted function about T_*^0 . From the basic physics, the temperature change of cells is also positively correlated with the temperature change of adjacent cells. In terms of expression, the coefficient of T_*^0 must be positive, and here only the coefficient of T_p^0 may not be positive, so to avoid the non-physical shock phenomenon, we need to ensure that:

$$a_p^0 - (1 - f)(a_W + a_E + a_S + a_N + a_B + a_T - S_P \Delta x \Delta y \Delta z) > 0 \quad (14)$$

When Equation (14) always holds true ($f = 1$), Equation (10) is absolutely stable, regardless of the step size and grid size. When $f \neq 1$, we should consider adjusting the step size and the grid size to meet the Equation (14).

3.4. Computational Efficiency and the Parallel Method

3.4.1. Handle Boundary Grids and Invalid Grids

When dividing the grid in modeling, there will be some invalid grid. The red area part of the figure and the cell with a value of 0 is an invalid grid.

Figure 7 shows a portion of the grid division in the axis plane, which is a complete circular ring, as shown in Figure 2. From here, we can see the temperature or metallographic changes at the core position. The red part is not within the scope of the workpiece modeling; therefore, it is an invalid mesh.

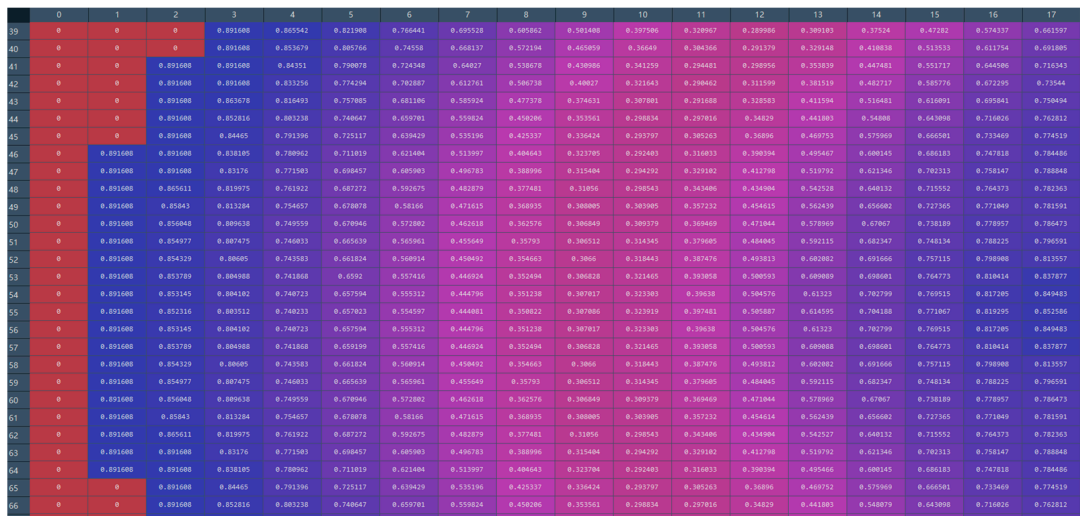


Figure 7. Axis plane slicing of matrices in the model (The red part is invalid mesh).

In the overall modeling, taking a 3-dimensional vector space, the expression is:

$$R^3 = [(x, y, z) | x \in X, y \in Y, z \in Z] \quad (15)$$

X , Y , and Z respectively represent the length, width, and height of this 3D space. Put the bearing outer ring that we need to model into this space, and the effective grid ratio in this space is expressed as follows:

$$\eta = \frac{V_{or}}{XYZ} \quad (16)$$

In equation V_{or} is the volume of the bearing outer ring. When using the computer calculation, all the cells in the whole space are traversed in each calculation, and the traversal of these invalid cells also increases the computation amount. This paper marks the invalid mesh during modeling, and automatically jumps the operation when the loop statement traverses to the position of the mark.

3.4.2. Separate the Variables

Since it is a 3D model, the amount of information stored in each valid cell is relatively large, and the calculation equation of the stored parameters can be expressed as:

$$Mesh = \begin{bmatrix} x, y, z, PE, PW, PN, PS, \\ PT, PB, \Delta x, \Delta y, \Delta z, \Delta H, \\ f, \Delta f, k_e, k_w, k_n, k_s, k_t, k_b \end{bmatrix} \quad (17)$$

All the information is stored for a single valid cell. Since the thermal conductivity of temperature changes with temperature during quenching, the storage parameters of one Equation (10) need to be recalculated at each iteration, which undoubtedly greatly increases the calculation amount. Therefore, the computation reduction is used on the algorithm. In the above Equation (17), the components that do not change with the number of iterations are stored in a list, and the components that change with the iteration step of the number of iterations are stored in another list. The separation can be expressed as:

$$\begin{aligned} Mesh1 &= \begin{bmatrix} x, y, z, PE, PW, PN, PS, \\ PT, PB, \Delta x, \Delta y, \Delta z, \Delta H, f \end{bmatrix} \\ Mesh2 &= [\Delta f, k_e, k_w, k_n, k_s, k_t, k_b] \end{aligned} \quad (18)$$

In iterative loop statements, only the relevant *Mesh2* part is recalculated, and the *Mesh1* part is introduced into the equation as a constant, which can greatly reduce the computational complexity of each loop.

3.4.3. Parallel Algorithm

With the development of computer technology, the advantages of computer multi-core and multi-thread can be used to improve the speed of computing. The three-dimensional vector space is cut and divided into regions of the same size (the remaining cells after the division are automatically incorporated into the last area), and the cells in this space are calculated simultaneously during the iterative calculation. The graph shows a logical schematic diagram of the parallel program.

As shown in Figure 8, A, B represents the cell number. If there is no parallelism, calculations can only be carried out in the order of numbering. Using parallel algorithms can divide the matrix into multiple calculation regions and calculate them simultaneously, greatly reducing the calculation time.

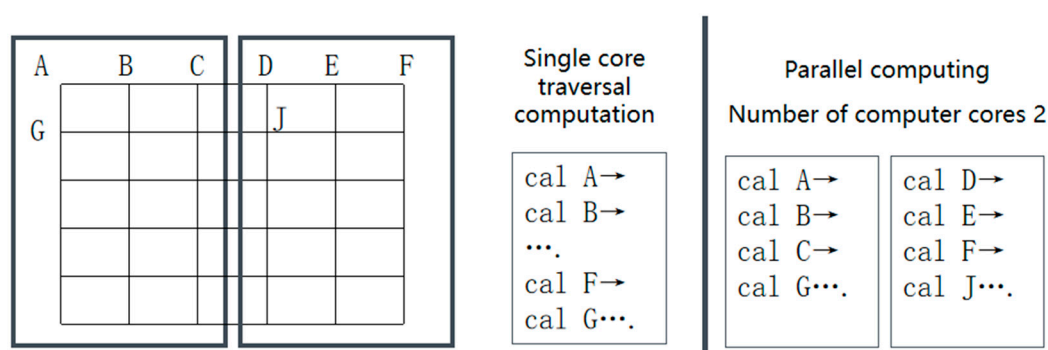


Figure 8. Logic diagram of parallel program.

In the actual calculation, run the same condition' single-core calculation time is 719.501 s, six-core calculation time is 123.940 s, and efficiency increases 5.8 times. The more grids there are and the smaller the time step, the higher the ratio of algorithm parallel time to overall computation time, and the efficiency improvement rate will be closer to the number of cores used.

4. Simulation Results with a Grid-Independent Analysis

Taking the outer ring of a certain type of horn contact ball bearing as an example, the initialization parameters of the outer ring are shown in the table.

Input the parameters from Table 3 into the Python program compiled using the above method to complete the initial model establishment.

Table 3. Simulation model initialization structural parameters.

Parameter	Value
Nominal outside diameter	220 mm
Outer diameter chamfer radius	3 mm
Inner diameter chamfer radius	2 mm
Inner diameter of outer ring	140 mm
Channel diameter	151.2 mm
Bearing width	19 mm
Curvature radius	6.13 mm
Material temperature	850 °C
Ambient temperature	25 °C

The change in the temperature of the ball bearing outer ring during quenching is depicted in Figure 9. The surface points and subsurface points of the bearing outer ring are taken as the characteristic points showing the simulated temperature change. It can be seen that within 0–2 s of just quenching, the cooling speed is small, and the heart cooling is slower than the surface. This is because, in the beginning of quenching, the temperature of the bearing of the outer ring is higher than the boiling point of the quenching oil, the surface of the bearing outer ring generates steam film, and the surface and the quenching oil contact is not sufficient; then the surface heat transfer coefficient is low. At the second stage of quenching, the overall cooling speed becomes significantly faster. At this stage, the stable steam film on the outer ring of the bearing begins to become unstable, forming small bubbles and gradually separating from the surface.. At this time, it is in the nuclear boiling stage, and the surface heat transfer coefficient increases sharply. Later, the cooling speed of the bearing is obviously reduced. At this time, the temperature of the outer ring of the bearing basically drops below the boiling point of the quenching oil, and the outer ring of the bearing enters the convection heat transfer stage.

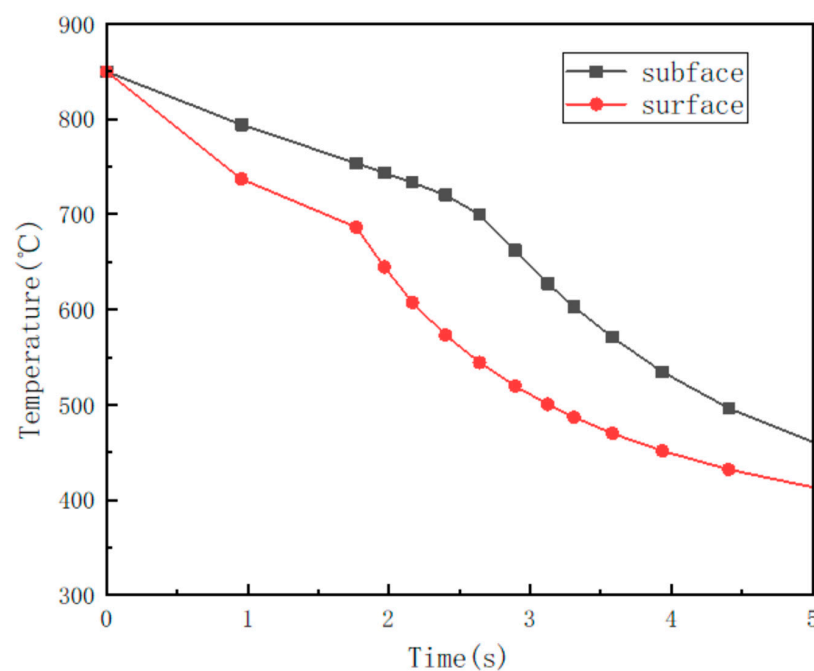
**Figure 9.** Temperature curves of subsurface and surface layers (short-term data).

Figure 10 clearly shows the temperature change of the bearing quenching in the long-term axis. At about 10 s, the overall temperature of the bearing outer ring is below the point at which martensite begins to form.

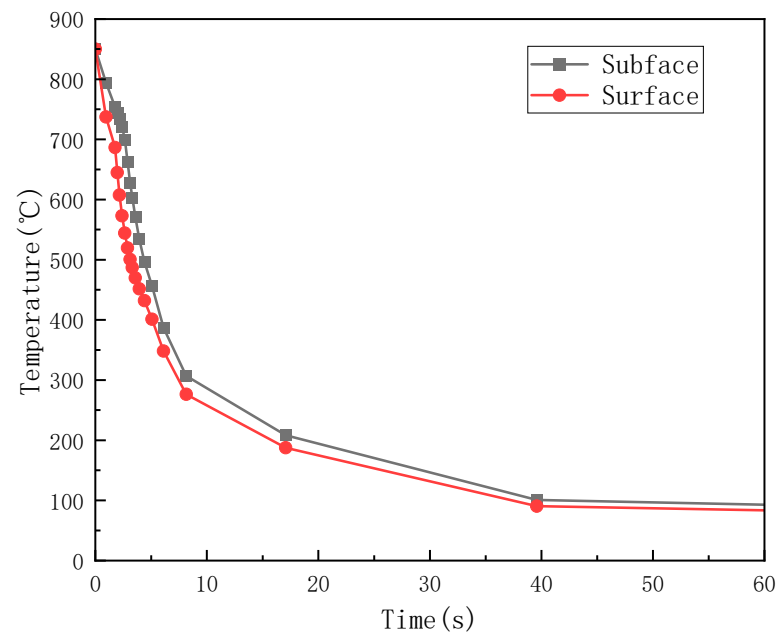


Figure 10. Temperature curves of subsurface and surface layers (long-term data).

The different cooling speed of the various nodes lead to the different time of martensite formation, Figure 11 as a typical characteristic diagram depicting the volume fraction of the core temperature near the point. The trend of the martensite volume fraction along the wall thickness direction from the bottom of the channel to the outer diameter face is shown in Figure 11; 0 mm represents outer diameter, 35 mm represents groove bottom diameter, and 0–35 represents wall thickness. A grid independence analysis is done here, the trend line of the square node is the martensite volume fraction calculated at the maximum grid of 2.5 mm, and the trend line of the circular node is the martensite volume fraction calculated at the maximum grid of 2 mm. It can be seen that the trend is the same, but the overall error is about 10%, which is because the base surface is the end surface, the grid size is different, the position of the cell representing the bottom surface of the groove is also different, and the calculated martensitic volume fraction is also different. Each site of the final bearing outer ring consists of 89.1% martensite and 10.9% residual austenite.

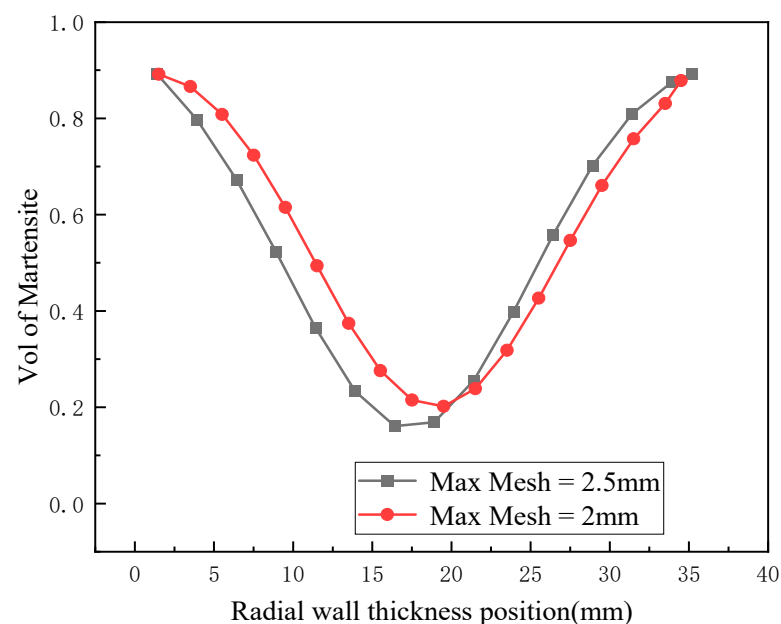


Figure 11. Distribution of martensite at the bottom section of the outer ring groove of the bearing.

5. Test Verification

The content of the outer ring of the deep-trench ball bearing was verified. Three points were evenly distributed on the outer ring roller, each point was 120° apart, on the center line of the outer ring roller. The average value of the measurement point was taken as the content value of the residual austenite of the workpiece. The position of the test point is shown in Figure 12. The measurement test equipment for residual austenite is an IXRD residual stress X-ray diffractometer.

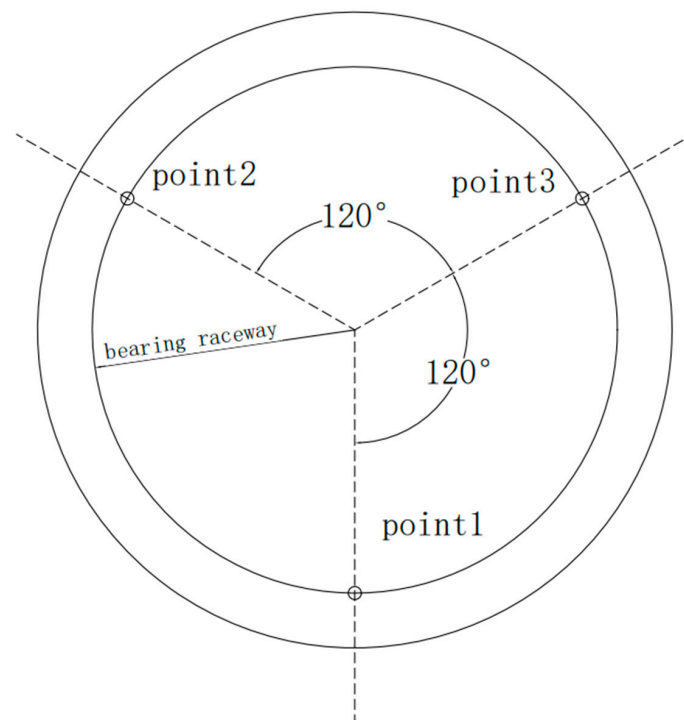


Figure 12. The position of the test point.

Table 4 shows the input parameters required for using an X-ray diffractometer. Table 5 is a comparison chart between the residual austenite content detected by the X-ray diffractometer and the residual austenite content calculated using the method provided in this article. It can be seen from Table 5 that the quenching test value of the outer ring of the deep-groove ball bearing is very close to the simulation value, and the error is about 10%, which shows that the numerical simulation of residual austenite and the test results are basically consistent, which can provide a theoretical basis for the actual engineering.

Table 4. Test parameters for residual austenite testing.

Phase	Ferrite	Austenite
2 θ scan starting angle	155.00°	135.00°
2 θ scan termination angle	142.00°	126.00°
2 θ scan step distance	0.10°	0.10°
Count time	2.00 s	4.00 s
Swing week	0	0

Table 5. Testing results of residual austenite after quenching.

Point	Residual Austenite/%	Simulation Value/%
1	12.1	10.9
2	11.4	10.9
3	11.8	10.9

6. Conclusions

1. The cooling is very fast, and there is a high temperature difference between the surface and the inside. When the surface forms martensite, the core is still in the state of austenite.
2. After quenching, the content of residual austenite is 11%, which is because the martensite transition is related to the final temperature and the final temperature (M_f) at 25 °C. The simulation results are close to the test results, indicating the accuracy of the model.
3. The C-N format and parallel calculation ensure the accuracy of the data and the speed of the calculation. Avoid the mistakes in using commercial software because of not seeing the calculation principles and algorithms.
4. The grid size has some influence on this model, but the grid size does not affect the trend of the results. The smaller the grid, the more accurate the model is.
5. The temperature–phase transition-coupled cloud map can also provide more accurate parameters and models for analyzing the quenched strain distribution.

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