

Review Overview of Friction and Wear Performance of Sliding Bearings

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Abstract: Sliding bearings are critical components of the internal combustion engine. Friction and wear occur in the contact area between the shaft and the bearing. Significant wear can occur in poor working conditions or after a long service time, leading to the failure of the sliding bearing and affecting the reliability of the machinery. It is essential to investigate the wear performance of sliding bearings, understand their wear mechanism, predict their service life, and select wear-resistant materials and surface treatments. This paper reviews the current status and prospects of sliding bearing wear research, focusing on the classification of sliding bearing wear tests, wear testing machines, wear test research, wear prediction models, and future research prospects.

Keywords: sliding bearings; friction; wear; lubricating oil



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1. Introduction

Sliding bearings are core components of machinery; they are characterized by high reliability, stable operation, and low noise. However, a large or eccentric load, abrasive particles in the lubricating oil, or an insufficient amount of lubricating oil can cause rapid wear and failure of the sliding bearing and affect its reliability. The wear degree of sliding bearings affects their normal operation and service life. Therefore, a wear analysis is required to ensure the bearing's reliability [1].

Before conducting a wear analysis of sliding bearings, it is necessary to determine the test type and select the wear testing machine and the appropriate parameters to characterize the degree of wear on the bearings. Subsequently, the influence of the working condition parameters, lubricating oil, surface coating, surface texture, and other factors on the wear of sliding bearings are analyzed to predict their wear performance and service life [2–5].

In addition to metal materials, composite materials can also be used to make sliding bearings. Some experimental methods of non-destructive testing and determination of micro-stresses in materials, such as X-ray and neutron diffraction are proposed, which could assess the surface tension, friction, and wear in bearing materials [6–8].

2. Loading Type of Bearings

In an engine, during normal conditions, the bearings are under hydrodynamic lubrication, which is shown in Figure 1. As the shaft rotates, it will bring lubricating oil into the bearing friction surface due to the viscosity of the lubricating oil. Lubricating oil is taken into the wedge clearance between the shaft and the bearing, forming a fluid dynamic pressure effect. When the pressure is balanced with the external load, a stable oil film forms between the shaft and the bearing bush, enabling hydrodynamic lubrication. The oil film supports the shaft, avoiding direct contact between the bearing and the shaft. The oil film thickness varies with load, speed, and temperature, so the bearing is acted on by oblique-type loads with periodically varying normal load during sliding motion.





Figure 1. Schematic diagram of hydrodynamic lubrication.

3. Classification of Sliding Bearing Wear Tests

The wear tests for sliding bearings can be categorized as whole machine tests, bench tests, component tests, and sample tests [9]. Whole machine tests and bench tests are the most reliable and provide the best simulation results. They are typically used for diesel engines. However, the test cycle is long, and the test costs are high. Component tests have the advantages of high control of the test parameters, easy simulation of contact conditions, short cycle, and good simulation results. Thus, they are suitable for research and development applications. However, the simulation results are not as good as those of bench and whole machine tests. In a sample test, the bearing is cut into small samples to study the friction and wear process mechanism. This test enables the control of various factors affecting friction and wear and is highly suitable for analyzing the influence of individual factors on friction and wear. The data obtained from the sample test have good repeatability and comparability, the test costs are low, and the cycle is short. However, the working conditions differ substantially from those in practical conditions, and the simulation accuracy is low.

4. Wear Testers for Sliding Bearings

Commonly used wear testers for sliding bearings include the four-ball friction and wear tester, pin-disc friction and wear tester, reciprocating friction and wear tester, and ring-block friction and wear tester. The four-ball friction and wear tester (Figure 2) is typically used to test the performance of the lubricating oil. It can be used to evaluate the bearing capacity, extrusion performance, extreme pressure, friction reduction, and wear resistance of lubricants under point contact pressure [10]. The pin-disc friction and wear tester (Figure 3) is used to study the friction and wear properties of various metal and non-metal materials. It is used to measure the friction coefficient of various materials in a sliding friction and wear test of the end contact and the wear resistance of various materials for different loads and speeds [11]. The ring-block friction and wear tester (Figure 4) is used to evaluate line contact friction and wear. The friction pair is composed of a standard rotating ring and a compressed rectangular block. The bearing capacity of the lubricant and the friction and wear performance of the friction pair materials are evaluated by measuring the width and depth of the strip wear marks on the rectangular block under different working conditions, as well as the friction and friction coefficient between the friction pair materials [12]. The reciprocating friction and wear tester (Figure 5) is used to determine the friction, wear, and scratch resistance of various coating surfaces. Tests are conducted



at different temperatures and for different loads and reciprocating speeds under different lubrication conditions [13].

Figure 2. Schematic diagram of four-ball friction and wear tester.



Figure 3. Schematic diagram of pin-disc friction and wear tester. Reprinted with permission from Ref. [11]. 2019, IOP Publishing Ltd.



Figure 4. Schematic diagram of ring-block friction and wear tester. Reprinted with permission from Ref. [12]. 2017, The Japan Society of Mechanical Engineers.



Figure 5. Schematic diagram of reciprocating friction and wear tester. Reprinted with permission from Ref. [13]. 2013, Elsevier.

5. Wear Performance of Sliding Bearings

Sliding bearings have different wear properties for different hardness, friction coefficients, test conditions, lubricants, bearing materials, and bearing surface textures. Appropriate evaluation methods and wear characterization methods are crucial.

5.1. Effect of Hardness on Sliding Bearing Wear

Hardness is an important index of material properties, which can be understood as the ability of the material to resist elastic deformation, plastic deformation, or damage. Jia et al. [14] investigated the wear behavior of thermoplastic polyurethane (TPU) materials with different Shore hardness under sediment conditions and found that the wear resistance of the materials increased significantly with increasing hardness and concluded that the hardness of the materials was an important factor affecting the tribological performance of water-lubricated bearings under sediment conditions. Hasan et al. [15] prepared titanium carbide (TiC) and graphite particle-reinforced copper alloy B-RG10 composites and performed Rockwell hardness, wear, and microstructure tests on the fabricated composites and found that the wear resistance was significantly better at greater hardness levels of the composites. Krishnakumar et al. [16] investigated the effect of nickel on the microstructure, hardness, and wear behavior of aluminum-12% Si alloy surface alloying, and calculated the hardness and wear rate of the modified layer using a microhardness tester and a pin-disc wear tester, and found that the addition of nickel increased the hardness and decreased the wear rate. The hardness of a sliding bearing material has a great influence on the wear resistance of the bearing. In many cases excessive bearing wear can lead to premature bearing failure. Improving the surface hardness of the bearing material can improve the wear resistance of the bearing [17,18]. The wear resistance of the sliding bearing material has a decisive influence on the service life of the bearing. Improving the wear resistance of sliding bearings can effectively improve their service life.

5.2. Effect of Coefficient of Friction of Materials on Sliding Bearing Wear

The coefficient of friction in the operation of a sliding bearing can be affected by many factors. The size of the coefficient of friction depends mainly on the material of the friction sub-surface, etc., and is independent of the size of the contact area. Under operating conditions, the friction coefficient can be used to determine the friction reduction performance of the sliding bearing [19]. In the case of dry friction, a sliding bearing with excellent performance will show excellent self-lubricating characteristics [20]. In the case of oil lubrication, generally, the smaller the coefficient of friction the better the lubrication between the friction pairs [21]. The frictional wear performance varies with the surface material of the frictional substrate. Nuruzzaman et al. [22] conducted an experimental study

on the variation of friction coefficient and wear rate with load for stainless steel (SS 304) pins sliding on different types of materials and observed that the friction coefficient values for glass fiber, nylon, and PTFE decreased with increasing normal load. For gear fibers, a different trend was observed, i.e., the coefficient of friction increased with increasing normal load. It was also found that the coefficient of friction differs for different materials. TUfekci et al. [23] studied the variation of coefficient of friction of 90% Cu + 10% Sn bronze and 1% C + 99 % balance Fe iron-based self-lubricating P/M bearings at different sliding speeds, loads, and temperatures. The test results showed that Cu-based bearings had better friction and wear properties than Fe-based bearings. Zhao et al. [24] conducted friction and wear tests on 45# steel (carbon steel) with aluminum bronze, aluminum bronze-based inlaid solid self-lubricating bearing (ISSLB) material, tin bronze, and tin bronze-based ISSLB material under different loads. The test results showed that under different loads, the average friction coefficients of aluminum bronze-based ISSLB material were the lowest and the average friction coefficients of copper alloy were the highest. The friction coefficients of friction pairs with different materials were different in magnitude, and improving the self-lubricating characteristics of bearings was also an important research direction.

5.3. Effect of Test Conditions on Sliding Bearing Wear

Sadatomic et al. [25] studied the influence of various working conditions on sliding bearing wear and found that the wear depth decreased with an increase in the oil pressure. As the oil temperature, load, and rotation speed increased, the minimum oil film thickness decreased, and the wear increased. Ravikiran et al. [26] designed a bench wear test to investigate the influence of the load, rotation speed, and running time on the wear rate of a sliding bearing. Under normal conditions, the performance was stable, and the sliding bearing exhibited almost no wear. Li et al. [27] studied the wear behavior of tin-based sliding bearings under different working conditions and found that the critical speed of the transition from mixed lubrication to dynamic lubrication increased with an increase in the load. The effect of the load on the friction coefficient was negligible in hydrodynamic lubrication because the journal and bearing were separated by the lubricating oil film. In contrast, the wear rate of the bearing was much higher in boundary lubrication or mixed lubrication than in stable lubrication. The wear-induced weight loss of the bearing was much higher during the start-up and stop conditions than in the steady-state conditions, significantly affecting the engine's service life. The main wear types of the bearing were abrasive wear under stable working conditions and adhesive wear, abrasive wear, and fatigue wear under start-up and stop conditions. Zhang et al. [28] found that axial misalignment led to increased wear. An optimal bearing clearance that resulted in minimal wear existed between the journal and the plain bearing. The wear degree was low when the smoothness and roughness of the shaft and bearing surfaces were low. Guo et al. [29] found that the bearing wear intensified or the bearing seized when the lubricating oil amount decreased. Liu et al. [30] selected sliding bearings with large surface roughness for a wear test. Substantial abrasion or abrasive wear occurred, resulting in increased wear. Meng et al. [31] compared the wear performance of different sliding bearing coating materials under oil lubrication and dry friction; the wear rate was higher under dry friction (Figure 6). Surface pre-treatment was performed on the surface of AISI 1045 steel to prepare linear micro-groove structures with different spacing. Subsequently, AlTiN coatings were deposited on the polished and textured AISI 1045 steel surface to prepare the specimens. The polishing steel coating (PSC) and the ultrasonic rolling textured coating with a groove spacing of 150 µm (URTC-150) were tested under dry friction and oil lubrication. Under oil lubrication conditions, the lubricant reduced the area of direct contact between the surfaces of the friction partners compared to dry friction, so the wear rate was lower. At the same time, the micro-groove spacing on the surface of URTC-150 was larger, and the local hydrodynamic pressure lubrication effect was more easily formed inside the micro-grooves, which promoted the generation of hydrodynamic pressure lubrication and improved the load-bearing capacity, so the wear rate was lower than that of PSC.



Figure 6. The wear rate of different materials under dry friction and oil lubrication. Reprinted with permission from Ref. [31]. 2021, Elsevier.

5.4. Effect of Lubricating Oil on Sliding Bearing Wear

In modern industry, the bearing lubricating oil has to meet higher wear- and frictionreduction requirements. The oil's performance can be significantly improved by adding appropriate additives.

Base oil is the main component of lubricating oil and determines its main characteristics. Additives account for a small proportion of the lubricating oil but are crucial for improving its performance. Lubricant base oils can be divided into mineral base oils, synthetic base oils, and vegetable base oils. Additives that improve the performance or properties of an oil include antioxidants, corrosion inhibitors, viscosity index improvers, friction modifiers, and anti-wear additives [32]. These additives perform different functions, such as reducing friction and wear, maintaining engine cleanliness, or improving fluid performance. Viscosity improvers, friction improvers, and anti-wear additives prevent excessive bearing wear, thereby improving fuel economy. Figure 7 shows a schematic diagram of the effect of lubricating oil on the sliding performance of bearings. The spherical shape of the nanoparticles in the lubricating oil improves the rolling mechanism between two sliding surfaces, changing the friction mode from sliding to rolling to reduce friction and wear [33]. Stable spherical nanoparticles can also improve the resistance to extreme conditions and the bearing capacity of lubricants [34–36]. Self-repair is another lubrication mechanism. The nanoparticles in the lubricating oil are deposited on the friction surface or fill the surface grooves to balance the mass loss. A single-layer or multi-layer film is formed on the contact surface to reduce the surface roughness and contact roughness [37–39]. The formation of a friction film is the most important mechanism to reduce friction and wear in sliding contact. The friction film is formed by the deposition and adsorption of nanoparticles on the sliding surface or by the tribochemical reaction between nanoparticles and the contact surface [40]. Abrasive, hard nanoparticles can be used as a polishing agent to reduce the surface roughness of the friction surface [41].

5.5. Effect of Surface Coating on Sliding Bearing Wear

The surface coating of sliding bearings can substantially improve the friction characteristics between friction pairs. New anti-friction and wear-resistant coatings are being developed to improve the wear performance of bearings. Ideal bearing coating materials have high fatigue strength, excellent wear resistance, embedment and corrosion resistance, high load capacity, and a high melting point.



Figure 7. Schematic illustration of the (**a**) rolling mechanism; (**b**) self-repair, or mending mechanism; (**c**) polishing mechanism; and (**d**) tribo-film formation. Reprinted with permission from Ref. [33]. 2019, Springer.

Alloy materials and polymer composites have been widely used as coating materials [42–44]. An alloy coating has finer crystallization, a smoother and brighter surface, higher wear, corrosion and high-temperature resistance, less friction, and higher hardness and strength than single-metal coatings [45–48]. The improved tribological properties of polymer composite coatings are attributed to the formation of a transfer film, resulting in a low interface shear strength, low friction, and a low wear rate. Adding functional fillers can overcome the limitations of a single-polymer coating and prolong the service life of the coating due to low friction, high wear resistance, high load capacity, high-temperature resistance, and high adhesion, as shown in Figure 8 [49,50].



Figure 8. Schematic of PDA/PTFE and PDA/PTFE + graphite. Reprinted with permission from Ref. [49]. 2016, Springer.

5.6. Effect of Surface Texture on Sliding Bearing Wear

The surface texture of sliding bearings affects the storage of the lubricating oil and debris. Therefore, an appropriate surface texture can improve the wear resistance of the bearing surface. Secondary dynamic pressure lubrication occurs due to the presence of abrasive particles and the lubricating oil, reducing abrasive cutting, adhesive wear, and the contact area between the friction pairs [51–53]. Thus, the surface texture is critical for improving the working conditions and performance of sliding bearings. A suitable texture can improve the bearing capacity and lubrication performance of journal bearings.

Common texture types include square [54], circular [55], triangular [56], and elliptical structures [57]. Many studies have investigated the effect of the surface texture type. A composite texture type can also improve the lubrication performance of sliding bearings [58]. Bionic texture simulates the texture of biological surfaces to achieve friction reduction and drag reduction on the friction pair surface. These texture types will be an important future

direction for the surface pattern design of sliding bearings [59]. Appropriate texture types, such as full texture, local texture, and combination texture types, have been investigated. The texture parameters can also determine the performance of sliding bearings. Figure 9 shows the surface textures of journal bearings. Common texture dimensions include the pit diameter, depth, depth-to-diameter ratio, groove width, polygon side length, convex body height, texture spacing, and substrate surface roughness [60–62].



Figure 9. Surface textures of plain bearings. Reprinted with permission from Ref. [60]. 2007, Springer.

5.7. Evaluation Methods for Sliding Bearing Wear

5.7.1. Wear Amount

The most direct evaluation method for sliding bearing wear is the wear amount. The wear performance is typically measured by weight loss and wear depth. Tunay et al. [63] evaluated the wear performance of a copper-based bearing by measuring the wear weight in a test. The greater the weight loss, the worse the wear resistance of the material was. Jeon et al. [64] evaluated the wear depth of a sliding bearing and found that the greater the wear depth, the worse the wear resistance of the material was. When the wear depth reached the limit, the bearing failed. Chen et al. [65] analyzed the wear performance of polyimide/ultra-high molecular weight polyethylene (UHMWPE) composites and compared the wear resistance of composites for different polyimide mass fractions using the volume wear rate. The larger the volume wear rate, the worse the wear resistance of the materials was.

5.7.2. Wear Surface Structure

The change in the sliding bearing surface during wear is divided into three processes. The first is the surface interaction, including mechanical and molecular interactions. Mechanical interactions include elastic deformation, plastic deformation, and furrow formation. Surface molecular interactions include mutual attraction and adhesion effects. The second process is the change in the surface layer. Plastic deformation of the friction surface layer results in cold-work hardening and brittleness. If the surface undergoes repeated elastic deformation, fatigue failure will occur. The third process is the destruction of the surface layer, including abrasion caused by furrow marks and abrasive particles in the friction direction on the friction surface. Surface pitting corrosion is caused by the fatigue failure of metal under repeated contact stress. The metal surface becomes brittle due to deformation strengthening, and surface spalling of microcracks occurs under a load. Shear failure occurs when the surface bonding point formed by the adhesion effect has high strength [66,67]. Analyses of the worn surface generally include an evaluation of the worn surface morphology and structure and an analysis of the chemical composition of the worn surface. Commonly used detection methods include scanning electron microscopy (SEM) and energy-dispersive spectroscopy (EDS), as shown in Figure 10. Evaluating changes in the sliding bearing surface morphology, structure, and chemical composition is crucial to determining the wear mechanism. The cause of the wear morphology and the wear mechanism can be determined by comparing the surfaces of the worn samples before and after wear [68-71].



Figure 10. (a) SEM image of the wear scar and (b) EDS results of the sample. Adapted with permission from Ref. [71]. 2019, Elsevier.

5.7.3. Detection of Wear Debris in Oil

The performance of the lubricating oil is analyzed to detect its deterioration, pollution, and wear debris content and to understand the working conditions of sliding bearings. The type, size, and quantity of wear debris in the lubricating oil is investigated to infer the wear conditions of the sliding bearing. The presence of material elements in the oil only present in the sliding bearing demonstrates the wear loss of the sliding bearing. The lubrication state and wear condition can be evaluated by the quantity and size of the debris. The wear is more significant when the number and size of wear debris are large, as shown in Figure 11 [72–74]. Detection methods of debris in in-service lubricating oil can be divided into offline and online methods. Offline detection is performed by collecting in-service lubricating oil samples and carrying out appropriate post-processing in the laboratory. Ferrographic analysis and spectral analysis methods are widely used in offline detection. Ferrography provides data on the size, color, and outline of wear particles [75,76]. Spectral analysis has high sensitivity and a wide measurement range [77,78]. A common instrument for online detection is the lubricating oil wear analysis sensor, which can detect the type, size, and quantity of wear debris in the lubricating oil in real time [79].



Figure 11. Relationships between the friction coefficient, number of wear particles, and run-time. Reprinted with permission from Ref. [74]. 2016, Japanese Society of Tribologists.

6. Sliding Bearing Wear Prediction Methods

Predicting the wear of sliding bearings can prevent the occurrence of sliding bearing faults and predict sliding bearing life. Existing prediction methods can be divided into three categories: model-based, data-driven, and reliability-based prediction methods [80]. Model-based prediction uses physical models to represent the system's behavior. However, since most systems are complex, the modeling is computationally expensive, and various assumptions are required for establishing the model. Reliability-based prediction is also called experience-based prediction. It uses historical data from important periods to obtain the statistical distribution of the parameters. The Poisson, exponential, Weibull, and lognormal distributions have been proposed for the failure-time distribution. This

method is easy to implement when historical data of an important period are available. However, the prediction results are typically not as accurate as those of model-based and data-driven methods. The purpose of data-driven prediction is to obtain information from raw data, typically from experiments. This method uses artificial intelligence or statistical models to train and learn the wear responses and predict future conditions. The system runs automatically, based on the data-driven model without utilizing the parameters of the real system. Although the computational cost of data-driven methods is not high, they can usually provide good prediction results for bearing wear behavior or system failure.

6.1. Model-Based Prediction

Wear prediction models of sliding bearings include the finite element analysis model, the Archard wear prediction model, and the Fleischer wear prediction model. The finite element analysis model uses a friction pair model to simulate the operating conditions and conduct thermodynamic analysis [81–83]. The simulation results can be used as a reference for experiments. König et al. [84] used the Archard and Fleischer models to predict the amount of wear on sliding bearings according to the test load, sliding distance, hardness, roughness, and wear coefficients of the bearing materials. The model results were compared with the wear amount measured in the test. Pang et al. [85] proposed an improved Archard wear prediction model regarding the relationship between the torque, speed, and wear rate. A comparison between the predicted and actual wear showed that the model had higher accuracy than the standard model.

6.2. Data-Driven Prediction

Data-driven prediction models of sliding bearing wear include the grey prediction model and artificial neural network. The grey prediction model is a mathematical model that performs prediction using a small amount of incomplete information. It utilizes past and present information to predict future development trends and conditions using scientific methods and assumptions. An artificial neural network is a nonlinear and adaptive information processing system composed of many processing units. It is based on modern neuroscience research results. It processes information by simulating the brain's neural network processing and memorizing functions. It is suitable for analyzing nonlinear sliding bearing wear information. The grey prediction system and artificial neural network can quantitatively analyze the correlation between the anti-friction and wear resistance of sliding bearings for different working conditions, lubrication conditions, materials, and textures [86–92]. Common artificial neural networks include the back-propagation (BP) neural network. Figure 12 shows the double-input single-output BP neural network model. It has become a trend in wear prediction to use intelligent algorithms to optimize the neural network model to improve the prediction accuracy [93,94].

The hidden layer



Figure 12. Diagram of a BP neural network model. Adapted with permission from Ref. [91]. 2007, Elsevier.

6.3. Reliability-Based Prediction

Reliability-based prediction has been used for sliding bearing wear prediction. This method uses feedback data collected in a critical period. The models predict the time

of failure and service life by adjusting the parameters of the model. Several models with different distributions of the failure parameters have been proposed: the Poisson, exponential, Weibull, and lognormal distributions. Models using the Weibull distribution are most commonly used because they can predict the reliability performance of several important stages in the components' or system's life cycle and the failure rate, and reduce the costs [95].

7. Summary and Outlook

More in-depth research on sliding bearing wear is being conducted, resulting in higher measurement accuracy requirements. The measuring range of single-function testing machines is limited, and more multi-function friction and wear tests are being conducted. Therefore, the friction and wear tests must be improved. In addition, wireless measurement systems for friction and wear testing machines are being used more commonly than traditional wired connections. Thus, it is easier to collect data using sensors. Advanced measurement methods, such as ultrasonic and optical methods, provide higher measurement accuracy. Various modern detection methods should be considered to analyze the wear of sliding bearings to obtain comprehensive and objective results. The selection of lubricants and lubricating additives should consider various factors, such as the bearing's accuracy, hardness, and structural characteristics, as well as the test and environmental conditions. The selection of appropriate lubricants and additives is particularly critical. Excellent self-lubricating performance of sliding bearings is crucial to reducing wear. Therefore, coating the surface of bearings with a wear-resistant and friction-reducing coating or using a textured surface requires further studies. Important research directions include coating materials and the geometric parameters of the texture. The prediction of bearing wear has attracted significant attention. Various prediction methods have been used for solving nonlinear problems, such as tribological system design, material selection, wear performance analysis, friction fault diagnosis, and life cycle prediction. Therefore, the prediction accuracy has been improved by optimizing prediction models, and various prediction methods have been developed, providing a new direction for analyzing sliding bearing wear.

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