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Numerical Study of Thermal Radiation Phenomenon and Its Influence on Amelioration of the Heat Transfer Mechanism through MHD Non-Newtonian Casson Model

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Abstract: The present study's main focus is regarding the physical properties of a two-dimensional (2D) magneto-hydrodynamic boundary layer non-Newtonian Casson fluid flow that moves due to an exponentially expanding surface with a mixed convection heat transfer mechanism. In the hydrodynamic flow and heat transmission process, the combined impact of thermal radiation and magnetic field influence is explored. The internal heat generation owing to the fluid motion or a very fluid viscosity is not taken into account. The Chebyshev spectral method (CSM) is employed in this work due to its ability, accuracy, and ease of obtaining the solution for non-linear system of ordinary differential equations (ODEs). This method is an approximate method that can usually obtain the solution in a series form. The mixed convection impact is incorporated in our problem. The results are graphed to help comprehend the many physical parameters that arise in the problem. Graphical results uncover that the speed liquid stream is lessened when reinforcing both the Casson boundary and the Hartmann number, while converse attributes are applied for the Grashof number and the radiation boundary. Finally, a comparison of our current results with previously published work on several particular situations of the problem reveals that they are in excellent agreement.

Keywords: non-Newtonian Casson fluid; MHD; thermal radiation; Chebyshev spectral method

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1. Introduction

Research issues in regards to the non-Newtonian fluid movement and heat transport in the Casson model as a result of an extending surface with a given temperature have an assortment of uses in the design and a few modern assembly processes, particularly in the most recent couple of years. A non-Newtonian fluid with yield stress is known as a Casson fluid [1–4]. These manufacturing processes may include hot rolling, glass fiber, wire drawing, the expulsion of plastic sheets, extraction of raw petroleum from oil-based commodities and paper creation. Hence, studies of the non-Newtonian Casson fluid have their importance in engineering, as well as in scientific areas [5–10]. Most of the related studies look at heat transfer mechanisms as well as mass transfer aspects and their importance in engineering fields using various types of fluids, such as the viscoelastic nanofluid, ferrofluid, non-Newtonian blood fluid, the flow of copper oxide nanoparticles, Bingham fluid, Jeffrey fluid, micropolar fluid, Newtonian and non-Newtonian nanofluid [11–21].

Because Fourier transforms for Chebyshev multinomials are available to properly compute matrix–vector products, they have been more widely employed than additional groups of orthogonal polynomials. The linear and non-linear differential equations, the integral equations, and the integro-differential equations have all been solved using these

polynomials [22]. This method is also widely used to explain the fractional diffusion equation [23], fractional-order integro-differential equations and others ([24–27]). The key benefit of this strategy is that it can obtain correct results by using only a few terms from the series solution. The proposed method is significantly faster than the alternatives. Chebyshev's functions are commonly utilized in function approximation because of their good features. Spurred by the above significant examinations, the current exploration is centered around the investigation of the mathematical arrangement by the CSM for the MHD fluid flow and heat transfer for the non-Newtonian Casson liquid over an extending surface with viscous dissipation and thermal radiation.

2. Governing Equations

In this section, we investigated the influence of heat radiation on the non-Newtonian Casson model, which could be described as follows, based on the model provided previously by Pal [28]:

$$\left(1 + \frac{1}{\beta}\right) f''' + f f'' - 2f'^2 - 2 \frac{Ha^2}{Re} e^{-X} f' + 2Gr e^{\frac{aX}{2}} e^{-2X} \theta = 0, \quad (1)$$

$$\left(\frac{1+R}{Pr}\right) \theta'' + f\theta' - af'\theta + Ec \left(1 + \frac{1}{\beta}\right) \left(2 \frac{Ha^2}{Re} f'^2 + f''^2 e^X\right) e^{\frac{X(2-a)}{2}} + 2\lambda e^{-X} \theta = 0, \quad (2)$$

with the following set of constraints:

$$f(0) = 0, \quad f'(0) = 1, \quad \theta(0) = 1, \quad (3)$$

$$f' \rightarrow 0, \quad \theta \rightarrow 0, \quad \text{at} \quad \eta \rightarrow \infty, \quad (4)$$

where f represents the dimensionless stream function, f' represents the dimensionless velocity, θ represents the dimensionless temperature, β represents the Casson parameter and X represents the dimensionless location along the plate. Additionally, the governing parameters Gr , Ha , R , Re , Pr , a , λ and Ec can be defined as the thermal buoyancy parameter, the Hartmann number, the radiation parameter, the Reynolds number, the Prandtl number, the parameter of the temperature distribution in the stretching surface, the dimensionless heat generation/absorption parameter and the Eckert number, respectively.

3. Implementation of the Proposed Numerical Method

In this section, we treated the obtained non-linear system of ODEs (1) and (2) with the corresponding boundary conditions (3) and (4) with the help of implementing the CSM. We applied this procedure with help from the following items:

1. We used the Gauss–Lobatto points which belong to $[-1, 1]$; in addition, we used the formula $\eta = \frac{\eta_\infty}{2}(x+1)$ to reduce Equations (1) and (2) into the forms:

$$\left(1 + \frac{1}{\beta}\right) \left(\frac{2}{\eta_\infty}\right)^3 f''' + \left(\frac{2}{\eta_\infty}\right)^2 f f'' - 2 \left(\frac{2}{\eta_\infty}\right)^2 f'^2 - \frac{Ha^2}{Re} e^{-X} \left(\frac{1}{\eta_\infty}\right) f' + 2Gr e^{\frac{aX}{2}} e^{-2X} \theta = 0, \quad (5)$$

$$\begin{aligned} &\left(\frac{1+R}{Pr}\right) \left(\frac{2}{\eta_\infty}\right)^2 \theta'' + \left(\frac{2}{\eta_\infty}\right) (f\theta' - af'\theta) + Ec \left(1 + \frac{1}{\beta}\right) \cdot \\ &\left(2 \frac{Ha^2}{Re} \left(\frac{2}{\eta_\infty}\right)^2 f'^2 + \left(\frac{2}{\eta_\infty}\right)^4 f''^2 e^X\right) e^{\frac{X(2-a)}{2}} + 2\lambda e^{-X} \theta = 0. \end{aligned} \quad (6)$$

2. Furthermore, the boundary conditions transformed as follows:

$$f(-1) = f'(1) = \theta(1) = 0, \quad f'(-1) = 0.5\eta_\infty, \quad f'(1) = 0, \quad \theta(-1) = 1, \quad (7)$$

the unknown functions $f(x)$, $\theta(x)$ belong to $C^m[-1, 1]$. Here, the derivatives in the previous system (5) and (6) were regarding the variable x .

3. We implemented the proposed technique by using a Chebyshev function to approximate $f^{(2)}(x)$, $f^{(1)}(x)$, $f(x)$, $\theta^{(1)}(x)$ and $\theta(x)$ by taking $f^{(3)}(x) = \Omega(x)$, and $\theta^{(2)}(x) = Y(x)$ and the integration as follows:

$$\begin{aligned} f^{(2)}(x) &= \int_{-1}^x \Omega(x) dx + c_0, \\ f^{(1)}(x) &= \int_{-1}^x \int_{-1}^x \Omega(x) dx dx + (x+1)c_0 + c_1, \\ f(x) &= \int_{-1}^x \int_{-1}^x \int_{-1}^x \Omega(x) dx dx dx + \frac{(x+1)^2}{2!}c_0 + \frac{(x+1)}{1!}c_1 + c_2, \end{aligned} \quad (8)$$

$$\begin{aligned} \theta^{(1)}(x) &= \int_{-1}^x Y(x) dx + d_0, \\ \theta(x) &= \int_{-1}^x \int_{-1}^x Y(x) dx dx + (x+1)d_0 + d_1. \end{aligned} \quad (9)$$

4. We implemented the boundary conditions (7) to find the values of c_j , d_j , $j = 0, 1, 2$ (the constants of integration) as follows:

$$\begin{aligned} c_0 &= -\frac{1}{2} - \frac{1}{2} \int_{-1}^1 \int_{-1}^x \Omega(x) dx dx, & c_1 &= 1, & c_2 &= 0, \\ d_0 &= -\frac{1}{2} - \frac{1}{2} \int_{-1}^1 \int_{-1}^x Y(x) dx dx, & d_1 &= 1. \end{aligned}$$

5. Because of the previous assumption, we could introduce the approximations to Equations (5) and (6) in the following form:

$$\begin{aligned} f_i &= \sum_{j=0}^n \ell_{ij}^f \Omega_j + c_i^f, & f_i^{(1)} &= \sum_{j=0}^n \ell_{ij}^{f1} \Omega_j + c_i^{f1}, & f_i^{(2)} &= \sum_{j=0}^n \ell_{ij}^{f2} \Omega_j + c_i^{f2}, \\ \theta_i &= \sum_{j=0}^n \ell_{ij}^\theta Y_j + d_i^\theta, & \theta_i^{(1)} &= \sum_{j=0}^n \ell_{ij}^{\theta1} Y_j + d_i^{\theta1}, & \forall \quad i &= 0, 1, 2, \dots, n, \end{aligned} \quad (10)$$

where

$$\begin{aligned} \ell_{ij}^f &= s_{ij}^3 - \frac{1}{4}(x_i+1)^2 s_{nj}^2, & \ell_{ij}^{f1} &= s_{ij}^2 - \frac{1}{2}(x_i+1) s_{nj}^2, & \ell_{ij}^{f2} &= s_{ij} - \frac{1}{2} s_{nj}^2, \\ \ell_{ij}^\theta &= s_{ij}^2 - \frac{1}{2}(x_i+1) s_{nj}^2, & \ell_{ij}^{\theta1} &= s_{ij} - \frac{1}{2} s_{nj}^2, \\ c_i^f &= -\frac{1}{2}(x_i+1)x_i, & c_i^{f1} &= \frac{1}{2}(1-x_i), & c_i^{f2} &= -\frac{1}{2}, \\ d_i^\theta &= \frac{1}{2}(1-x_i), & d_i^{\theta1} &= -\frac{1}{2}. \end{aligned}$$

6. Here, we used the definition in [22] of the elements s_{ij} for the matrix S to define $s_{ij}^2 = (x_i - x_j)s_{ij}$, $s_{ij}^3 = \frac{(x_i - x_j)^2}{2!}s_{ij}$. This enabled us to substitute Equation (10), and in Equations (5) and (6) to construct the non-linear system of algebraic equations in the highest derivatives:

$$\begin{aligned} \left(1 + \frac{1}{\beta}\right) \Omega_i + \left(\frac{\eta_\infty}{2}\right) f_i f_i^{(2)} - 2 \left(\frac{\eta_\infty}{2}\right) \left(f_i^{(1)}\right)^2 - 2 \frac{Ha^2}{Re} e^{-X} \left(\frac{\eta_\infty}{2}\right)^2 f_i^{(1)} \\ + 2 Gr e^{\frac{aX}{2}} e^{-2X} \left(\frac{\eta_\infty}{2}\right)^3 \theta_i = 0, \end{aligned} \quad (11)$$

$$\left(\frac{1+R}{Pr}\right)Y_i + \left(\frac{\eta_\infty}{2}\right)\left(f_i\theta_i^{(1)} - a\theta_i f_i^{(1)}\right) + Ec\left(1 + \frac{1}{\beta}\right). \\ \left(2\frac{Ha^2}{Re}\left(f_i^{(1)}\right)^2 + \left(\frac{2}{\eta_\infty}\right)^2\left(f_i^{(2)}\right)^2 e^X\right)e^{\frac{X(2-a)}{2}} + 2\lambda e^{-X}\left(\frac{2}{\eta_\infty}\right)^2\theta_i = 0. \quad (12)$$

7. Finally, by solving the previous non-linear system of algebraic equations using the Newton iteration method, we obtained the required numerical scheme of the proposed model under study (1)–(4) by substituting Ω_i and Y_i in Equation (10).

4. Numerical Method Verification

To ensure the correctness and validity of the data acquired through the use of the Chebyshev spectral method, the numerical values of the Nusselt number estimated by the present approach for $Gr = Ec = Ha = R = \lambda = 0$ and $\beta \rightarrow \infty$ were compared to those of Pal [28]. As can be seen from Table 1, the current results agreed quite well with those produced by Pal [28], emphasizing the precision of the numerical method used.

Table 1. Comparison of Nusselt number $-\theta'(0)$ calculated by Pal [28] with $Gr = Ec = Ha = R = \lambda = 0$ and $\beta \rightarrow \infty$ for various values of a and Pr .

a	Pr	Pal ([28])	Present Work
−0.5	0.5	0.17582	0.17581998
0.0	0.5	0.33049	0.33048741
1.0	0.5	0.59434	0.59433910
3.0	0.5	1.00841	1.00840812
−0.5	1.0	0.29988	0.29987911
−0.5	3.0	0.63411	0.63410742
−0.5	5.0	0.87043	0.87042951
−0.5	8.0	1.15032	1.15031918
−0.5	10.0	1.30861	1.30860815

5. Results and Discussion

The system of ODEs (1) and (2) depends upon the conditions (3) and (4) was solved numerically after utilizing the Chebyshev spectral method. To obtain an actual understanding, a parametric report was generated for the current physical problem, and the mathematical results were displayed using graphical outlines. Diagrams in Figure 1 show the variety of velocity $f'(\eta)$ and the variety of temperature for distinct values of the Casson parameter β . It was noted that the impact of the Casson parameter played a prominent feature for both the velocity fluid flow and heat transfer. It very well may be seen from these figures that both the temperature and the velocity profiles decreased when expanding the value of the Casson parameter. Furthermore, the Casson parameter with the smallest value provided the most notable liquid velocity and the highest liquid temperature.

Figure 2a,b shows the influence of the velocity profile $f'(\eta)$ and the temperature profile $\theta(\eta)$ in the momentum and thermal boundary layers for various Hartmann numbers. It was noted that the impact of the Hartmann number played a prominent feature in the velocity fluid flow. The Hartmann number had the effect of decreasing the value of velocity profiles throughout the boundary layer, while the temperature diagram had the reverse effect.

To display the impacts of the Grashof number on both the velocity and the temperature fields, we plotted Figure 3a,b. It was found that both the velocity and the temperature of the fluid increased for the Grashof number. Furthermore, we can see that the impact of this parameter had a prime role near the stretching sheet. It is worth noting that the Hartmann number and the Grashof number exhibited opposite characteristics when it came to the velocity (see Figure 3a,b).

Figure 1. (a) Impact of β on $f'(\eta)$; (b) impact of β on $\theta(\eta)$.

Figure 2. (a) Impact of $\frac{Ha^2}{Re}$ on $f'(\eta)$; (b) impact of $\frac{Ha^2}{Re}$ on $\theta(\eta)$.

Figure 3. (a) Impact of Gr on $f'(\eta)$; (b) impact of Gr on $\theta(\eta)$.

Figure 4 is a plot for both the non-dimensional velocity $f'(\eta)$ and the non-dimensional temperature $\theta(\eta)$, with η for different upsides for X . From this plot, it was fascinating to see that as the value of X enhanced, the velocity of the non-Newtonian fluid increased, while the converse was noticed for $\theta(\eta)$. The impact of the dimensionless coordinate parameter X represented a critical role in both the hydrodynamic boundary layer and the velocity field.

Figure 4. (a) Impact of X on $f'(\eta)$; (b) impact of X on $\theta(\eta)$.

Figure 5a,b shows the effect of the thermal radiation parameter R on the velocity and temperature curves. It was observed that both the hydro-dynamical momentum layer and the thermal layer were increased when enhancing R . Physically, this was due to the radiation energy enlivening the related development of the boundary layer.

Figure 5. (a) Impact of R on $f'(\eta)$; (b) impact of R on $\theta(\eta)$.

The graph in Figure 6 illustrates the variation of temperature $\theta(\eta)$ for distinct values of the heat generation/absorption parameter λ . Figure 6 shows, as expected, that the escalation of the heat generation (absorption) parameter λ prompted an ascent of the temperature $\theta(\eta)$. We also saw that when λ grew, the thermal thickness increased.

Figure 6. Impact of λ on $\theta(\eta)$.

6. Conclusions

We demonstrated a numerical simulation of the non-Newtonian MHD Casson fluid flow and heat transfer due to an extensible sheet with thermal radiation. The current study looked into how heat transfer devices rely on temperature, which could signal that the Casson fluid's upgrading qualities are improving, making it more appealing for applications. The proposed ODEs were numerically solved after employing the Chebyshev spectral technique. The influence of various physical parameters on the velocity and temperature profiles was visually depicted and explained in detail. A portion of the significant findings of our examination was acquired by a graphical portrayal. Finally, the precious obtained results, which were confirmed through a validation of our data, reflected the reliability and the trust of the CSM, especially in this type of highly nonlinear ODEs that can be applied in the field of technological and industrial engineering.

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