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Homogeneous–Heterogeneous Chemical Reactions of Radiation Hybrid Nanofluid Flow on a Cylinder with Joule Heating: Nanoparticles Shape Impact

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Abstract: The current analysis aims to exhibit the nanoparticles of Al_2O_3 + Cu-water hybrid nanofluid flow for Darcy–Forchheimer with heterogeneous–homogeneous chemical reactions and magnetic field aspects past a stretching or shrinking cylinder with Joule heating. This paper performed not only with the hybrid nanofluid but also the shape of Al_2O_3 and Cu nanoparticles. The model of single-phase hybrid nanofluid due to thermophysical features is utilized for the mathematical formulation. In the present exploration equal diffusions factors for reactants and auto catalyst are instituted. The system of governing equations has been simplified by invoking the similarity transformation. The numerical computations are invoked due to the function `bvp4c` of Matlab, with high non-linearity. Numerical outcomes illustrated that; sphere shape nanoparticles presented dramatic performance on heat transfer of hybrid nanofluid movement; an opposite behavior is noticed with lamina shape. The local Nusselt number strengthens as the transverse curvature factor becomes larger. In addition, the homogeneous–heterogeneous reactions factors lead to weaken concentration fluctuation.

Keywords: hybrid nanofluid; Darcy–Forchheimer; homogeneous–heterogeneous reactions; cylinder; nanoparticles shape; radiation

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1. Introduction

In recent times, nanofluids have been an active field of research due to its greatly enhanced thermal properties and numerous significant applications in many fields as heat transfer fluids, ferromagnetic fluids, superwetting fluids and detergents, biomedical fluids, polymer nanocomposites, gain media in random lasers, and as building blocks for electronics and optoelectronics devices [1–3]. Choi [4] was the first, who proposed the term “nanofluid”. It is obtained by dispersing a small amount of certain nanometer-sized particles (called nanoparticles) of metals, metallic oxides, carbides or carbon nanotubes stably and uniformly in base fluids such as water, ethylene glycol, oil, etc. There are several refrigeration systems based on the absorption phenomenon that are used in many applications such as air conditioning, automobiles [5], solar water distillation [6], freezing of foods [7], etc. Additionally, convection in porous media saturated with nanofluids has extensive experimental and theoretical importance because of its natural occurrence and wide range of applications in many practical situations such as chemical engineering, geothermal energy utilization, oil reservoir modeling, solar energy, building of thermal insulation, nuclear waste disposal, lubrication, biological processes, etc. [8,9]. The experimental study on convective heat transport in nanofluid has been analyzed by Baïri and Laraqi [10] and Torki and Etesami [11]. It is studied theoretically by many researchers for various situations in porous media using Buongiorno’s transport model [12]. Many

of them are Khan and Alzahrani [13], Li et al. [14] and Rawat and Kumar [15]. Ibrahim and Khan [16] examined the influence of viscous dissipation on the mixed convective flow of nanofluids in a porous medium past a stretchable surface. Kumar et al. [17] explored the impacts of thermal radiation on stagnation point polar nanofluid flow over stretchable surface in a porous medium. Loghmani et al. [18] worked on heat transport of nanofluids flow through a porous medium. Khan et al. [19] studied the problem of magneto–nanofluid flow in non-Darcy porous medium.

On other side, hybrid nanoliquids are a new kind of nanofluid. Generally, these kinds of fluid can be made by two separate approaches: (a) dispersion of two or even more nano-sized particles to a base fluid, and (b) by suspending the so-called hybrid nanoparticles to a host fluid. The efficacy of hybrid nano-suspensions through their chemo-physical characteristics was examined and proved by several numerical and experimental investigations. The applicability and properties of hybrid nano-suspensions were extensively studied in the literature [19–24]. The magneto-flow and heat transport of hybrid nanoparticles suspended in a non-Newtonian fluid were analyzed by Ghadikolaei et al. [25]. The magneto-natural convection flow of hybrid nanofluid inside a double porous medium was analyzed by Mehryan et al. [26]. Ghadikolaei and Gholinia [27] worked on heat transport and magneto-natural convection flow of hybrid nanofluid over a vertical porous stretchable surface. Suganya et al. [28] evaluated the hybrid nanofluid flow the effect of Darcy–Forchheimer porous medium.

Often, we encounter several processes in nature and industries that also witness mass transport due to variations in concentration. The impact of a chemical reaction is determined by whether it is homogeneous or heterogeneous. The inclusion of pure water and air is impossible in nature. It is possible that any outer matter is naturally there, or that it is combined with air or water. As an outer mass is present in air or liquids, it induces a chemical reaction. Many chemical technologies, such as the manufacture of food processing, glassware or ceramics, and the production of polymers, benefit from the study of related chemical reactions. Lately, many investigations have been published regarding the importance of chemical reaction on hybrid nanofluids flow in porous medium showing their importance in several fields of science and technology; see [29–34].

The main objective of this investigation is to construct a mathematical model Cu-Al₂O₃/water hybrid nanofluid flow for Darcy–Forchheimer with homogeneous–heterogeneous reactions, non-linear thermal radiation, Joule heating, and heat transfer towards a permeable radially stretching/shrinking cylinder. Five different types of nanoparticle shapes, i.e., sphere, hexahedron, tetrahedron, column and lamina, are taken into account through this contribution. A classical transformation is employed to convert the PDEs into a non-linear system of ODEs.

2. Materials and Methods

Consider a steady laminar boundary layer flow of viscous incompressible hybrid nanofluid Cu and Al₂O₃ towards on a stretching/shrinking cylinder with radius h as illustrated in Figure 1. Here, (x, r) is the cylindrical polar coordinates which are assigned in the axial and radial directions, respectively. The velocity of the cylinder is given as $u_w(x) = u_0 x/d$, where constant u_0 is a characteristic velocity and d is the characteristic length of the cylinder. The nanofluid saturates the given porous medium through Darcy–Forchheimer relation. The magnetic field of strength B_0 is imposed normal to the x -axis. The interaction of homogeneous and heterogeneous reactions is given as [35]:

in which the concentration of chemical species A_1 and B_1 are symbolized by A and B , respectively, while k_c and k_s denote the constant rates. Using these assumptions together

with usual boundary layer approximations, the equations of fluid motion can be expressed as [36,37]:

$$\frac{\partial(ru)}{\partial x} + \frac{\partial(rv)}{\partial r} = 0 \quad (2)$$

$$\rho_{\text{hnf}} \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial r} \right) = \mu_{\text{hnf}} \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right) - \left(\sigma_{\text{hnf}} B_0^2 + \frac{\mu_{\text{hnf}}}{K_p} \right) u - \frac{\tilde{F}}{K_p^{1/2}} u^2 \quad (3)$$

$$\begin{aligned} (\rho c_p)_{\text{hnf}} \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial r} \right) &= k_{\text{hnf}} \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \sigma_{\text{hnf}} B_0^2 u^2 - \frac{1}{r} \frac{\partial}{\partial r} (r q^r) + \mu_{\text{hnf}} \left(\frac{\partial u}{\partial r} \right)^2 \\ &+ Q_0 (T - T_\infty) \end{aligned} \quad (4)$$

$$u \frac{\partial A}{\partial x} + v \frac{\partial A}{\partial r} = D_A \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial A}{\partial r} \right) - k_c A B^2 \quad (5)$$

where u and v denote the fluid velocity in x and r directions, T is the hybrid nanofluid temperature, D_A and D_B are the diffusion coefficient. The term of radiative heat flux is extended by assisting the Rosseland approximation as [38]:

$$q^r = -\frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial r} = -\frac{16\sigma^*}{3k^*} T^3 \frac{\partial T}{\partial r} \quad (6)$$

here σ^* and k^* denote the constant of Stefan–Boltzman and mean absorption factors, respectively.

Figure 1. Schematic diagram of the flow problem.

The corresponding boundary conditions for a given problem is [38]:

$$\text{at } r = h, \begin{cases} u = \lambda u_w, v_w(r) = -\frac{h}{r} \left(\frac{u_0 v_t}{d} \right)^{1/2} S, T_w(x) = T_\infty + T_0 (x/d)^2 \\ D_A \frac{\partial A}{\partial r} = k_s A, D_B \frac{\partial B}{\partial r} = -k_s A, \\ \text{as } r \rightarrow \infty, u \rightarrow 0, T \rightarrow T_\infty, A \rightarrow A_0, B \rightarrow 0 \end{cases} \quad (7)$$

Here $v_w(r)$ is the constant mass flux velocity through the permeable surface such that $v_w > 0$ for mass injection and $v_w < 0$ for mass suction. For the case of an impermeable surface $v_w = 0$. In addition, the constant stretching/shrinking parameter λ signifies the stretching cylinder when $\lambda > 0$ and shrinking cylinder when $\lambda < 0$. The static cylinder is symbolized by $\lambda = 0$.

2.1. Thermophysical Features

The modified thermo-physical features regard to the single-phase pattern by Tiwari and Das [39] was imposed early by Devi and Devi [40,41] to exhibit the boundary layer and

energy equations of hybrid Cu–Al₂O₃–H₂O nanofluid around a stretching sheet with variant various factors. Besides, because the boundary layer is analytically formulated, a small number of assumptions were recognized; namely, the main liquid and hybrid nanoparticles were kept in a thermal equilibrium state. The hybrid nanofluid was considered to be stable; thus the impact of hybrid nanoparticles aggregation and sedimentation was ignored. The hybrid nanoparticles are assumed to be spherical, lamina, column, shape.

This portion is devoted to displaying the mathematical expressions of thermophysical features of the hybrid nanofluids and primary fluid which are portrayed in Table 1 due to Gorla et al. [42] and Devi and Devi [40,41]. The physical properties of the base fluid, water, copper Cu (as first nanoparticle) and alumina Al₂O₃ (second nanoparticle) are invoked in Table 2. In addition, the values of the shape factor m are illustrated in Table 3.

Table 1. Thermophysical properties of hybrid nanofluids as Gorla et al. [42].

Properties	Hybrid Nanofluid	Nanofluid
Dynamic viscosity	$\mu_{hnf} = \mu_{bf}(1 - \varphi_2)^{-2.5}$,	$\mu_{bf} = \mu_f(1 - \varphi_1)^{-2.5}$
Density	$\rho_{hnf} = (1 - \varphi_2)\rho_{bf} + \varphi_2\rho_2$,	$\rho_{bf} = (1 - \varphi_1)\rho_f + \varphi_1\rho_1$
Heat capacity	$(\rho c_p)_{hnf} = (1 - \varphi_2)(\rho c_p)_{bf} + \varphi_2(\rho c_p)_2$,	$(\rho c_p)_{bf} = (1 - \varphi_1)(\rho c_p)_f + \varphi_1(\rho c_p)_1$
Thermal conduc.	$\frac{k_{hnf}}{k_{bf}} = \frac{(k_2 + (m-1)k_{bf}) - (m-1)\varphi_2(k_{bf} - k_2)}{(k_2 + (m-1)k_{bf}) + \varphi_2(k_{bf} - k_2)}$,	$\frac{k_{bf}}{k_f} = \frac{(k_1 + (m-1)k_f) - (m-1)\varphi_1(k_f - k_1)}{(k_1 + (m-1)k_f) + \varphi_1(k_f - k_1)}$
Electrical conduc.	$\frac{\sigma_{hnf}}{\sigma_{bf}} = \left(1 + \frac{3\left(\frac{\sigma_2}{\sigma_{bf}} - 1\right)\varphi_2}{\left(\frac{\sigma_2}{\sigma_{bf}} + 2\right) - \left(\frac{\sigma_2}{\sigma_{bf}} - 1\right)\varphi_2}\right)$,	$\frac{\sigma_{bf}}{\sigma_f} = \left(1 + \frac{3\left(\frac{\sigma_1}{\sigma_f} - 1\right)\varphi_1}{\left(\frac{\sigma_1}{\sigma_f} + 2\right) - \left(\frac{\sigma_1}{\sigma_f} - 1\right)\varphi_1}\right)$

Table 2. Thermophysical features of base fluid, alumina, and copper [40,42].

Feature	ρ (kg/m ³)	c_p (J/kg·K)	k (W/m·K)	σ (S/m)
H ₂ O	997.1	4179	0.613	0.05
Cu	8933	385	401	5.96×10^7
Alumina	3970	765	40	3.69×10^7

Table 3. The empirical shape nanoparticles factor m [43].

Nanoparticles	Sphere	Hexahedron	Tetrahedron	Column	Lamina
Shape
m	3.0000	3.7221	4.0613	6.3698	16.1576

2.2. Similarity Solution

Here, we apply the following similarity transformation to convert the considered physical model into non-dimensional mathematical expression:

$$\eta = \frac{r^2 - h^2}{2h} \left(\frac{u_0}{v_f d} \right)^{1/2}, \quad u = \frac{u_0 x}{d} F'(\eta), \quad v = -\frac{h}{r} \left(\frac{u_0 v_f}{d} \right)^{1/2} F(\eta) \quad (8)$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad A = A_0 N(\eta), \quad B = A_0 \tilde{N}(\eta)$$

Equation (2) is satisfied automatically and Equations (3)–(6) yield:

$$(1 + 2\alpha\eta)F''' + 2\alpha F'' + \frac{\rho_{hnf}}{\rho_f} \frac{\mu_f}{\mu_{hnf}} (FF'' - F'^2) - \frac{\sigma_{hnf}}{\sigma_f} \frac{\mu_f}{\mu_{hnf}} M_a F' - \lambda F' - \frac{\mu_f}{\mu_{hnf}} F^* F'^2 = 0 \quad (9)$$

$$\frac{1}{Pr} \left(\frac{k_{hnf}}{k_f} + R_d((Nr-1)\theta + 1)^3 \right) ((1+2\alpha\eta)\theta'' + 2\alpha\theta') + \frac{(\rho c_p)_{hnf}}{(\rho c_p)_f} (F\theta' - 2F'\theta) + \frac{3R_d}{Pr} (Nr-1)(1+2\alpha\eta)((Nr-1)\theta + 1)^2 \theta'^2 \quad (10)$$

$$+ \frac{\mu_{hnf}}{\mu_f} (1+2\alpha\eta) E_c F''^2 + \frac{\sigma_{hnf}}{\sigma_f} M_a E_c F'^2 + Q\theta$$

$$(1+2\alpha\eta)N'' + 2\alpha N' + ScFN' - ScK_c N\tilde{N}^2 = 0 \quad (11)$$

$$\delta \left((1+2\alpha\eta)\tilde{N}'' + 2\alpha\tilde{N}' \right) + ScF\tilde{N}' + ScK_c N\tilde{N}^2 = 0 \quad (12)$$

With the boundary conditions:

$$F(0) = S, F'(0) = \lambda, \theta(0) = 1, N'(0) = K_s N(0), \delta\tilde{N}'(0) = -K_s N. \quad (13)$$

$$F'(\infty) \rightarrow 0, \theta(\infty) \rightarrow 0, N(\infty) \rightarrow 1, \tilde{N}(\infty) \rightarrow 0.$$

where the obtained parameters are defined as:

$\alpha = (\nu_f d / (u_0 h^2))^{1/2}$	curvature factor	$\lambda = \nu_f d / (u_0 K_p)$	stretching or shrinking parameter
$E_c = u_0^2 x^2 / (d^2 (c_p)_f (T_w - T_\infty))$	Eckert number	$F^* = \tilde{F}x / (\rho_f K_p^{1/2})$	local inertia factor
$M_a = \sigma_f B_0^2 d / (\rho_f u_0)$	magnetic field parameter	$Q = Q_0 d / ((\rho c_p)_f u_0)$	heat generation factor
$Nr = T_w / T_\infty$	surface temperature excess	$Sc = \nu_f / D_A$	Schmidt number
$R_d = 16\sigma^* T_\infty^3 / (3k^* k_f)$	radiation factor	$Pr = \mu_f (c_p)_f / k_f$	Prandtl number
S	suction/injection factor	$\delta = D_B / D_A$	mass diffusion coefficients ratio
$K_s = (k_s / D_A) \sqrt{\nu_f d / u_0}$	heterogeneous reaction factors	$K_c = k_c A_0^2 d / u_0$	homogeneous reaction factors

Now, if we put $\delta = 1$, i.e., $D_A = D_B$, thus $N(\eta) + \tilde{N}(\eta) = 1$.

Again, Equations (11) and (12) yield:

$$(1+2\alpha\eta)N'' + 2\alpha N' + ScFN' - ScK_c N(1-N)^2 = 0 \quad (14)$$

with, the corresponding boundary condition

$$N'(0) = K_s N(0), N(\infty) \rightarrow 1 \quad (15)$$

The skin friction coefficient C_f and the Nusselt number Nu , are defined as:

$$C_f = \frac{\mu_{hnf}}{\rho_f u_w^2} \left(\frac{\partial u}{\partial r} \right)_{r=h}, Nu = \frac{xq_w}{k_f (T_w - T_\infty)} \quad (16)$$

where:

$$q_w = \left(-k_{hnf} \frac{\partial T}{\partial r} + q^r \right)_{r=h}$$

Substituting Equation (8) into Equation (16), we get the following formula for skin-friction factor and Nusselt number,

$$Re^{\frac{1}{2}} C_f = \frac{\mu_{hnf}}{\mu_f} F''(0), Re^{-1/2} Nu = - \left(\frac{k_{hnf}}{k_f} + R_d((Nr-1)\theta(0) + 1)^3 \right) \theta'(0) \quad (17)$$

where $Re = \frac{xu_w}{\nu_f}$ shows the local Reynolds number, $f''(0)$, $\theta'(0)$ indicate the velocity and temperature gradients.

3. Results

The salient parameters of the flow and heat transfer behavior are exhibited for the hydromagnetic hybrid Al_2O_3 -Cu nanofluid flow and heat transfer for Darcy–Forchheimer with homogeneous–heterogeneous reactions by a permeable stretching or shrinking cylinder. The exhibited non-linear flow differential Equations (9), (10) and (14) with the four

proper boundary conditions expressed in Equations (13) and (15) are a strenuous way to obtain a solution. Hence, these flow equations are solved numerically, due to the function `bvp4c` from MATLAB R2008b for representative values of the dimensionless factors. The imposed Matlab code, `bvp4c`, is developed implementing a FDM that use the three-stage Lobatto IIIa expression. This is a collocation method with fourth-order accuracy. Through this technique, the ODEs. (9), (10) and (14) are first converted to a system of first order by presenting new transformations, i.e., initial value problem (IVP). The mesh select and error control are related to the residual of the continuous solution. Comprehensive outcomes were performed for miscellaneous values of the factors describing fluid motion. Validation of numerical computations is achieved by comparing the outcomes of our investigation with Khashi'ie et al. [36] and Waini et al. [44], which are displayed in Table 4 for various values of φ_{Cu} , when $\varphi_{Al_2O_3} = 0.1$, $M_a = S = \alpha = E_c = R_d = 0$, $\lambda = 1$ and $Pr = 6.135$. From the presented data it is noticed that the calculated outcomes are completely in agreement with previous published computations. The physical parameters were fixed as $m = 6.3698$, i.e., column nanoparticle shape, $E_c = 0.1$, $M_a = 0.5$, $R_d = 0.2$, $Nr = 0.1$, $\alpha = 1$, $S = 1$, $\lambda = 1$, $Sc = 0.62$, $Q = 0.2$, $F^* = 0.4$, $K_c = K_s = 0.5$, $\varphi_{Al_2O_3} = \varphi_{Cu} = 0.03$. Figure 2 illustrates the impact of miscellaneous nanoparticles shapes of the hybrid nanofluid through the boundary layer flow of $Cu+Al_2O_3-H_2O$, at a fixed volume fraction $\varphi_{Al_2O_3} = \varphi_{Cu} = 0.03$.

Table 4. Comparison of our results for skin friction.

φ_{Cu}	Khashi'ie et al. [36]	Waini et al. [44]	Present Study
0.005	−1.327097962	−1.327098	−1.327630
0.02	−1.40949019	−1.409490	−1.406323
0.04	−1.520721211	−1.520721	−1.512846
0.06	−1.634118687	−1.634119	−1.621717

Figure 2. Temperature distribution and Nusselt number for variant values of shape of nanoparticles.

Due to the graph, it is clarified that the dimensionless temperature of the hybrid nanofluid boosts as the values of factor m enhances. Spherical shape nanoparticles give the minimum temperature fluctuations, followed by hexahedron, tetrahedron, column, and lamina. This can be explained as the sphere shape nanoparticles have lowest thermal conductivity and viscosity of those lamina shape nanoparticles. The spherical shaped nanoparticle, due to its improved surface area, leads to dragging more heat from the boundary layer, whereas this impact is less evident for the other shapes. This accounts for the maximum rate of heat transfer at the boundary for the spherical-shaped nanoparticles as can be perceived in Figure 2 and Table 5.

Figure 3 obviously presents the hybrid nanofluid velocity fluctuations $F'(\eta)$ which start from maximum value at the surface of the cylinder and then weaken until they attain the lowest value of the boundary layer for miscellaneous positive values of magnetic field factor $M_a (= 0, 0.1, 0.5, 1$ and $2)$. This figure belays that the velocity curves minimize with higher values of M_a . In addition, the skin friction coefficient strengthens as M_a enhances.

An opposite influence is clear for temperature curves Figure 4, i.e., the hybrid nanofluid temperature fluctuations boost with the increase in magnetic field factor.

The rate of heat transfer variations for miscellaneous values of M_a , M_a yields the Nusselt number to weaken as shown. This is reality because of the boosting intensity of the magnetic field that outputs resistive force, i.e., an opposite force to the trend of fluid movement, that is named the “Lorentz force”. The created force has the susceptibility to minimize the velocity boundary layer and strengthen the thermal boundary-layer thickness.

Table 5. Velocity and temperature gradient as $\varphi_{Cu} = \varphi_{Al_2O_3} = 0.03$.

M_a	m	E_c	α	$-F''(0)$	$-\theta'(0)$
0.0	3.0	0.1	1.0	2.567727	6.276612
0.1	-	-	-	2.603955	6.243702
0.5	-	-	-	2.741170	6.119466
1.0	-	-	-	2.898269	5.978082
0.5	3.0	-	-	2.741170	6.119466
-	3.7221	-	-	2.741170	5.938033
-	4.0613	-	-	2.741170	5.857272
-	6.3698	-	-	2.741173	5.373093
-	3.0	0.0	-	2.741168	6.917546
-	-	0.1	-	2.741170	6.119466
-	-	0.3	-	2.741172	4.000560
-	-	0.5	-	2.741176	2.931343
-	-	0.1	0.0	2.356231	6.081831
-	-	-	0.1	2.395392	6.085194
-	-	-	0.5	2.550767	6.097921
-	-	-	1.0	2.741170	6.119466

Figure 3. Velocity distribution and skin friction coefficient for variant values of M_a parameter.

Figure 4. Temperature distribution and Nusselt number for variant values of M_a parameter.

The impact of suction/injection S factor on non-dimensional hybrid nanofluid velocity and temperature profiles is plotted in Figure 5. An increment in suction/injection factor has

the susceptibility to force the fluid moves into an unoccupied space that creates changes in the boundary layer. Therefore, the hybrid nanofluid velocity and temperature are dawdled for enhancing the S factor. Due to forcing the liquid through a permeable cylinder, it is clear that the hybrid nanofluid temperature weakens due to enlarging the S parameter. The influence of shrinking $\lambda < 0$ or stretching $\lambda > 0$ factor has been presented in Figure 6. For stretching cases with improving λ the hybrid nanofluid velocity curves boost but the temperature reduces. The opposite behavior is seen in the case of shrinking, i.e., with increasing λ the temperature enhances and the velocity reduces.

Figure 5. Velocity and temperature distributions for variant values of S parameter.

Figure 6. Velocity and temperature distributions for variant values of λ parameter.

The aspects of the curvature factor α on $F'(\eta)$ and $\theta(\eta)$ fluctuations are presented in Figure 7. From this figure, we perceive that the velocity of the hybrid nanofluid has a clear relation to the curvature factor α . This is due to α possessing a reverse relation with the curvature radius, so the contact horizontal region of the cylinder reduces with limited impedance to the hybrid nanofluid flow generated. Again, an enhancement in α lead to boost both in velocity and temperature distributions; physically, kinetic energy boosts with higher values of curvature factor because of the strengthening $\theta(\eta)$ profile of hybrid nanofluid. The influence of viscous dissipation factor (Eckert number) E_c on hybrid nanofluid temperature and rate of heat transfer is invoked in Figure 8. The thermal boundary-layer thickness strengthens for increasing values of E_c , whilst the gradient of hybrid nanofluid temperature at cylinder surface that formulated in terms of local Nusselt number weakens. This behavior can be interpreted as the viscous dissipation boosts thus the thermal conductivity of the flow uplifts, in which produces to strengthen the momentum and thermal boundary layers.

Figure 7. Velocity and temperature distributions for variant values of α parameter.

Figure 8. Temperature distribution and Nusselt number for variant values of E_c parameter.

Figure 9 reveals the impact of radiation R_d and Nr factors on hybrid nanofluid temperature curves. As illustrated, an increment in radiation factor results in boosts in thermal boundary layer thickness. The same behavior is gained due to the surface temperature factor Nr , that is the curves of hybrid nanofluid temperature enhance with increasing Nr . An increment in thermal radiation provides energy to the particles of the hybrid nanofluid which invokes an augment in both temperature and thermal boundary-layer thickness. Basically, this factor becomes obvious at the surface and is so helpful to strengthen the temperature distributions.

Figure 9. Temperature distribution for variant values of R_d and Nr parameters.

Aspects of the dimensionless intensity of homogeneous reaction factor K_c , heterogeneous reaction factor strength K_s on non-dimensional concentration fluctuations are highlighted in Figure 10. Due to an increment in values of homogeneous reaction strength K_c and heterogeneous reaction intensity K_s , a diminution in the concentration fluctuation $N(\eta)$ is noticed, since in homogeneous reaction the reactants are wasted, and after certain value of the similarity variable η this influence vanishes. In addition, the Schmidt number

Sc stands for the ratio of momentum to mass diffusivity. Thus, by enhancing Schmidt number, minimum mass diffusivity replaced yielding hybrid nanofluid concentration to weaken as depicted in Figure 11. The curvature factor α has the same aspect of the concentration curves, i.e., α leads to reducing the hybrid nanofluid concentration fluctuations, Figure 11.

Variations of skin friction coefficient, rate of heat transfer and gradient concentration at the surface of cylinder for miscellaneous parameters have been portrayed in Tables 5–7 for sphere shape nanoparticles. As given, the parameters M_a , m , E_c , Q , Nr and F^* lead to weakening the temperature gradient at the surface, whilst the α and R_d yield enhance it. The skin friction coefficient improves with escalating M_a , F^* and α . In addition, the gradient of concentration strengthens as Sc and K_s escalate and it reduces with increasing K_c .

Figure 10. Concentration distribution for variant values of K_c and K_s parameters.

Figure 11. Concentration distribution for variant values of Sc and α parameters.

Table 6. Velocity and temperature gradient as $\varphi_{Cu} = \varphi_{Al_2O_3} = 0.03$.

Q	Nr	R_d	F^*	$F''(0)$	$-\theta'(0)$
0.0	0.1	0.2	0.4	2.741159	6.308612
0.2	-	-	-	2.741170	6.119466
0.5	-	-	-	2.741163	5.791617
1.0	-	-	-	2.741173	4.647444
0.2	0.1	-	-	2.741170	6.119466
-	0.2	-	-	2.741170	6.118418
-	0.5	-	-	2.741170	6.030476
-	1.0	-	-	2.741173	5.365267
-	0.1	0.0	-	2.741170	6.087991
-	-	0.4	-	2.741168	6.150117
-	-	0.8	-	2.741168	6.210376
-	-	1.2	-	2.741168	6.270762
-	-	0.2	0.0	2.682743	6.140906
-	-	-	0.5	2.755528	6.114194
-	-	-	0.9	2.812024	6.093441
-	-	-	1.0	2.853459	6.078212

Table 7. Concentration gradient as $\varphi_{Cu} = \varphi_{Al_2O_3} = 0.03$.

Sc	K_c	K_s	$N'(0)$
0.22	0.5	0.5	0.329924
0.62	-	-	0.358027
0.90	-	-	0.374613
1.2	-	-	0.389593
0.62	0.2	-	0.359505
-	0.8	-	0.356460
-	1.0	-	0.355361
-	1.5	-	0.352399
-	0.5	0.2	0.172884
-	-	0.8	0.488269
-	-	1.0	0.555509
-	-	1.5	0.680357

4. Conclusions

This contribution addresses the Cu + Al₂O₃ nanoparticles shape impact on MHD hybrid nanofluid flow and heat transfer past stretching and shrinking a horizontal permeable cylinder with isothermal heterogeneous and homogeneous reactions. In this regard, the striking attention of the present analysis is illustrated as follows

1. Factors K_s and K_c have similar impacts on concentration fluctuations. Higher values of K_c and K_s factors lead to weakened concentration curves.
2. Eckert number and magnetic field strengthen $\theta(\eta)$ distributions but weakens the local Nusselt number.
3. Sphere shape nanoparticles presented a dramatic performance on heat transfer of hybrid nanofluid movement whereas an opposite characteristic is noticed with lamina shape.
4. The maximum $\theta(\eta)$ is generated by the lamina-shaped nanoparticles and the minimum value is obtained by sphere shape nanoparticles, but column shape presented medium execution on heat transfer.

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Abbreviations

Nomenclature

A, B	Concentrations of chemical species
B_0	Strength of magnetic field
C_f	Skin friction coefficient
c_p	Specific heat
D_A, D_B	Diffusion coefficients of A, B species
Ec	Eckert number
F	Non-dimensional stream function
F^*	Local inertia factor
h	Radius of cylinder

k	Thermal conductivity
k^*	Mean absorption parameter
K_c	Homogeneous reaction strength
K_s	Heterogeneous reaction strength
m	Nanoparticle Empirical shape factor
M_a	Magnetic field parameter
N_r	Surface temperature excess
Nu	Nusselt number
Pr	Prandtl number
Q	Heat generation coefficient
q^r	Radiative heat flux
R_d	Radiation parameter
Re	Reynolds number
S	Mass transpiration parameter
Sc	Schmidt number
T	Dimensional temperature
(u, v)	Velocity components
v_w	Constant mass flux, ms^{-1}
(x, r)	Cylindrical polar coordinates
Greek Symbols	
μ	Dynamic viscosity
ρ	Fluid density
ψ	Stream function
φ	Nanoparticles
λ	Stretching/shrinking parameter
θ	Dimensionless temperature
ν	Kinematic viscosity
η	Similarity variable
α	Curvature parameter
σ	Electrical conductivity
σ^*	Stefan–Boltzman constant
Subscripts	
w	Conditions at the surface
∞	Conditions in the free stream
f, hnf	Regular fluid, hybrid nanofluid

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