

Technical Note

Estimations of the Optical Equivalence Theorem for Opto-Mechanical Systems for Investigation in General Relativity and High-Energy Physics

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Abstract: The optical equivalence principle is analyzed according to the possibility of describing unbounded states, and the suitable approximations are calculated for highly energetic phenomena. Among these possibilities, the relevance for laser fields, interferometers, and optomehcanical systems are implemented. Their suitableness for research in General Relativity, Cosmology, and High-Energy Physics are outlined.

Keywords: optical equivalence principle; optical systems; quantum systems; optomechanics; quantum optics; general relativity; quantum optical systems

1. Introduction

Quantum and optical systems are described at different spacetime distances, for which different measurements describe different features of the spacetime as well as of those of highly energetic phenomena. The implementation of laser systems and of opto-mechanical systems are developed after its foundation on the quantum theory associated with the pertinent systems. Their relevance for General Relativity and High-Energy Physics is of pertinence in the analysis of several measurement operations and estimations, in the modellization of the research results. The feature of laser fields, optical systems and optomechanical devices suited for such research guidelines are to be outlined on the basis of the quantum properties they rely on. The optical equivalence theorem [1] allows on to single out the features of a quantum system, which can be studied also by means of the examination of the wavepackets. The optical theorem can also be implemented for unbounded observables [2].

The Optical Equivalence Principle allows one to construct a sequence for the density operator for laser beams. Such a definition results in unbounded operators for an arbitrary number of quantum states. Several demonstration of the Optical Equivalence Theorem were proposed, which rely on different features of quantum-optic systems and of quantum systems [3–7]. It is, therefore, possible to compare the density operator, the expressions for the sequences of the density operator, the expectation values of the density operators and the sequences for the expectation values of the density operators at the quantum description, at the semiclassical description, for quantum systems, for quantum optical systems, for semi-classical optical systems and for optical systems.

As a result, it is possible to establish a relation between the sequences of the density matrix operators expansions and their expectation values. More in particular, it is possible to establish the pertinent coordinate transformations and the majorizations both for the sequences of the density operators as well as for the weighting functions for the states of the considered systems to obtain a consistent descriptions of the observables of the systems. As a result, the description of the observables of the quantum systems can be compared to those of the optical systems, the latter being considered at the quantum level, at the semi-classical level and at the classical one. The possibility to extend the



techniques for other operators is also considered. In this way, it is possible to describe optical systems, whose dimensions are larger than the Plank length, but which are constituted by elements of quantum nature, at the the quantum level, at the semiclassical approximation and at the classical scheme. Furthermore, the calculations hold exactly also for optical systems containing particles of very large (but not infinite) momentum, whose probability distribution is non-trivial. Fourier decomposition can be also obtained, by extending the proper quantization techniques and then by evaluating all the orders of the quantum operators as a sequence of operators. The proper quantization techniques are those requested by the analysis of the partition function of systems of quantum states of infinite momentum on a Minkowski spacetime. The expectation values are therefore expressed not as a sequence, but at their quantum values plus the summands of the correction terms for the semiclassical states. The Fourier decomposition of the spectral modes as well consists of the Quantum expectation values plus the correction terms. The advantage of these examinations of the Optical Equivalence Theorem are therefore appreciated by the availability of exact terms plus the exact expansion of the correction terms for different

quantum states also at the semiclassical description. The necessity for well-defined-temporal-modes

photon states in quantum metrology was outlined in [8,9] for quantum networks. The paper is organized as follows. The Introduction is aimed at exposing the motivations of the analysis of the paper. In Section 2, the particular features of the optical equivalence theorem are revised. In Section 3 some peculiarities of the semiclassical states of laser optical systems are recalled. In Section 4, the features of laser systems useful in this analysis are reviewed. In Section 5, the optical equivalence theorem for laser fields is recalled. The approximations necessary for the application to the investigated systems are therefore calculated. In particular, the evaluation of the approximations of the weighting function necessary for the expansion of the density matrix are performed. The expansion of the density matrix is calculated at the requested approximation order. Section 6 is devoted to the description of the quantum systems and the corresponding optical ones, for which the quantization techniques necessitate the approximations calculated. In Section 7, semiclassical optical systems to which the approximations calculated are of pertinence, are outlined. In Section 8, applications to optical systems are envisaged, for which the properties of the density matrix analyzed are explicative. The main results of the majorizations for the density matrix operator calculated in the present analysis for the Optical Equivalence Theorem are therefore applied for an intense laser-beam fields within the framework of the particular representation of the Optical Equivalence theorem, which does not rely on the Fock occupation space, to the analyses of the power spectrum noise for a quantum-mechanical (optical) system of an intense laser-beam field. In the Concluding Remarks Section, the main subjects of General Relativity and of High-Energy Physics are recalled, for which the quantum systems investigated and the optical systems considered, as well as the optomechanical devices taken into account, are connected within the Heisenberg limit of the considered devices.

2. Some Special Cases for the Optical Equivalence Theorem

The optical equivalence principle in based on the comparison if the expectation values of the measure of operator measurements in the Hilbert space and in the phase space formulation. More precisely, the optical equivalence theorem [2] demonstrates the connection between the classical probability distribution formalism and the density matrix for a quantum-mechanical system. Among the several possible demonstrations, in particular, in [1], one of the proofs of the theorem are not based on the number operator: this allows one to construct a diagonal density operator ρ for the coherent states $|z\rangle$, i.e.,

$$\rho \equiv \pi^{-1} \int \phi(z) \mid z > < z \mid d^2 z \tag{1}$$

For the majorization techniques applied in the present paper, the problems evidentiated by the unapplicability [1,10] of the definitions for the Fock occupation space are avoided; in particular it is possible to proceed also for the other degrees of freedom, where the integration is extended on the subregions not available Fock occupation space, where annihilation operators and creation ones are not allowed to be defined [11].

The interest in this demonstration relies on the particular hypotheses assumed for the states | z > of the system. For unbounded observables, the function $\phi(z)$ is replaced by a suitable function, i.e.,

$$\phi(z) \to \phi_{\beta}(z) \equiv S_{\beta}\phi(z), \tag{2}$$

where S_{β} is a suitable support-controlling function for $\phi(z)$ in the definition of $\phi_{\beta}(z)$.

To proceed further, a weighting function of compact support S_M can be further defined, which acts on $\phi_\beta(z)$ as $\phi_{\beta,M}(z)$ and defines the corresponding density matrix.

The change of variable

$$z'e^{-\beta/2} \to z \tag{3}$$

leads to the definition of the function ϕ_M of compact support, such that the corresponding density operator ρ_M , which is finally diagonal.

This definition of the density operator ρ_M is a well-defined approximating sequence for unbounded operators for the definition of a density operators (which is, on the contrary, directly defined for bounded operators. Differently from other demonstrations, this procedure is not based on any hypotheses for the Fock representation.).

3. Quantum States, Semiclassical States, Laser Fields and Optical Systems

The optical equivalence principle is straightforward extended to the semiclassical description by making use of semiclassical wavepackets in (1); the case for Gaussian wavepackets is illustrated in [2]. In the semiclassical description, the support controlling function S_{β} is supposed therefore to act on the semiclassical wavepackets. The statistical states of a quantum-mechanical system are described as equivalent by the one-to one correspondence of the degrees of freedom; the definition of the partition function also holds. The definition of a support controlling weighting function for the function $\phi(z)$ has therefore its effects on the definition of the partition function. In particular, the partition function must be therefore well-defined also for unbounded-observable states. The definition of the weighting-support-controlling function must therefore be suitably act on the density-matrix definition leading to the partition function on those states, which have an absolute infinite value for the expectation value of those operators, which lead to an infinite value for the expectation value intended (inserted) as a physical state in the definition of the classical density matrix (1).

Quantum optical systems for spatially non-Gaussian states of light [12], the output modes are characterized as superpositions of Laguerre-Gauss (LG) modes for numerically generated orbital angular momentum (OAM) degree-of-freedom under the hypothesis of external noise also for models of radial mode index both for a deep neural network and for a convolutional neural network. At varying the integer *l*—the argument of the LG polynomials-' corresponds to one 2π phase oscillation with different radial-mode index *p* by analyzing the twisted superpositions, as

$$|\Psi_{p}^{l,-l}(r,\phi)|_{LG}^{2} \simeq r^{2|l|} L_{p}^{|l|} \left(\frac{2r^{2}}{w^{2}}\right) exp\left(\frac{2r^{2}}{w^{2}}\right) \left(1 + \cos\left(2 \mid l \mid \phi - \theta\right)\right)$$
(4)

$$|\Psi_{p}^{n,-n}(r,\phi)|_{BG}^{2} \simeq J_{|n|}(\beta r)^{2} exp\left(\frac{2r^{2}}{w^{2}}\right) \left(1 + (-1)^{n}\cos\left(2 \mid n \mid \phi - \theta\right)\right),$$
(5)

with *BG* the Bessel-Gauss polynomials [13]. The numerically generated external noise is not specified whether to be ascribed with gravitational effects and/or quantum-gravitational effects or interactions.

Applications in metrology are ensured by the validity of the analysis for many kinds of interferometers, including hybrid interferometers.

4. Intense Fields and Highly Energetic Particles

In an intense laser filed, several many couples of electron-positron pairs occupy the vacuum available for the experimental setting [14]. Experimental availability is ensured by the Compton scattering producing a high rate at harmonic range.

The relativistic analysis of quantum electrodynamics in intense laser fields allows one for the Relativistic investigation of Compton scattering in the collision with proton scattering [15]. As in [15], in this case, the cross-section is evaluated after the series expansion of the pertinent $J_{N-1}(z)$ Bessel functions, which became relevant only at the first order, as

$$d\sigma \sim r_0^2 N^4 \left(\frac{I}{I_C}\right)^{N-1} 2^{-2(N-1)} d\Omega,\tag{6}$$

with r_0 the electron radius, v the velocity of the motion of a (non-)relativistic electron of mass m and charge e, I the unperturbed intensity, I_C the critical intensity of the laser field, at which the ratio v/c becomes $|v/c| \simeq 1$, $d\Omega$ being the solid angle integration region corresponding tho the experimental detector apparatus; here $v \equiv -\mu c c cos(\omega t)$, with $\mu \equiv \sqrt{I/I_C}$, ω the frequency of the field, and e the linear-polarization versor: $\mu c/\omega$ is thus the amplitude of the classical electron- oscillations in the radiation field and μ therefore the corresponding velocity amplitude (in units of c- the speed of light). The electron-positron pair production under intense laser field with highly charged ions is studied in the distribution, correlation and propagation direction of the production of electron-positron pair, which can be analyzed also as anti-correlated. Multiphoton scattering has also been investigated in [16].

5. Applications for the States with an Almost-Infinite-Expectation-Valued Operators

The optical equivalence principle can be stated, as in [2], at the semiclassical level, within the framework of the Fock occupation space.

Any quantum-mechanical system can be described over the complex plane by a classical probability distribution, for which the density operator can be recast as Hermitian, endowed with a probability distribution function ϕ non necessarily positive-definite. Such a quantum-mechanical system can be considered to be consisting of an arbitrary number of states *n* in the Fock representation.

While for external thermal fields the probability distribution is described as Gaussian, for laser beams the sequence of Fock states n can be non-trivial. In particular, not all phase-angle sequences might not have the same weight; this peculiarity leads to the possibility of a non-diagonal density operator. The calculation of the partition function has to be performed for the sequence of Fock states n, at the semiclassical description, for the quantum-optics description and for the optical-systems descriptions, for n the occupation number sequence. In this case, the partition function is calculated as [2]

$$\rho(n;n') \equiv \Pi_{\lambda} \int d^2 z_{\lambda} S_{\beta} \phi(z) exp^{|z_{\lambda}|^2} \frac{z_{\lambda}^{n_{\lambda}}(z_{\lambda}^*) n_{\lambda}'}{\sqrt{n_{\lambda}! n_{\lambda}'}}$$
(7)

This density operator describes therefore a quantum-mechanical system for which the sequence of the n Fock states can be discontinuous and consisting of states with non-trivial probabilities, and therefore is not regularly leading to a diagonal density operator, as for a quantum-system. The density operator in Equation (7) consists therefore of a sequence for the non-trivial Fock states considered.

Further Approximations for the Density Operators

It is, therefore, possible to discuss the sequence of expectation values of the density operators obtained in [1,2] with the sequence obtained for the expressions of the density operators.

In all the approximation here described, it is the purpose of the present Section to show that the weighting support function *S* in (7) from [2] therefore satisfies, by definition, the properties

$$S_{\lambda}\rho(n;n') \equiv \rho(z;z)[\phi_{\beta}(z)] + \mathcal{O}(n;n';\beta;\lambda)$$
(8)

for unbounded observables. It is, therefore, important to compare this result with the implications of Equation (2). The implications of such an approximation are to be developed within the framework of almost-infinite-valued-operator eigenstates for extremely high energetic processes, such as those taking place in the case, such as, but not only, of interactions extremely intense laser-field background, in highly energetic quantum processes and semiclassical ones, as well as for infinite-momentum classical states, for which the high-energetic process is the correct suitable phenomenological experimental approximation , and therefore exhibits the same high-energy limits.

Furthermore, the weighting-support function has the properties to straightforwardly be extended to the convergence properties in the trace class norm as well through the definition for the orthonormal basis for the unbounded observables by transitivity, where suitable higher-order corrections are evaluated as

$$S_{\lambda} \{ \psi_k \} \equiv \{ \psi_k \} + \mathcal{O}(n; n'; \beta; \lambda).$$
⁽⁹⁾

6. Null-Hypersurface Quantization and Laser Systems

High-energy interactions of matter fields with laser beams was analyzed in [17]. Within this frame, applications for the evaluation of neutrino oscillations have been adduced in [18] The approximation of high-energy particles as particles with an infinite momentum was proposed and studied in [18,19] and related literature, such as [19,20], where, in the latter, the renormalization conditions and the rules for Feymann-graphs-procedure in the limit of an infinite momentum have been exactly stated; furthermore, massive quantum electrodynamics was formulated in [21]. The renormalizability for Electrodynamics on null-plane two-dimensional hypersufaces was controlled in [22] and the pertinent version of the Standard Model was built in [23]. In [24], the production of electron/positron pairs due to heavy-ions collisions is revised in several energy ranges and approximations. In [25], the main features of the QED processes in the presence of strong background laser fields are outlined: in the complete evaluation, the polarisation tensor has to be determined at all orders of the external momentum, while in the low-energy approximation as electron fields with both a real part and an imaginary one are found, as crossed fields [25–27].

In the presence of a (superposition of) non-monochromatic laser fields (sources) background, the main QED processes can be described as after the interaction with infinitely massive (atom) nuclei [28]. The strong intensity of laser fields also allows for the analysis of external fields [29], once the other relativistic properties of the matter involved has been sampled. The need for quantum optical systems can be understood as an improvement for quantum metrology, in those cases, for which the resolution of the detection apparati is not fully consistent with the quantities and properties of quantum-matter(-spacetime) systems aims of the experiment, and therefore the experimental techniques requires different qualities for the measurements devices. The propagation of photons in intense magnetic fields can allow one to gain insight about the refraction index, for which, at different energy scales, different photon phenomena can be observed [30,31].

Quantum optical systems might offer, in these cases, [32] the descriptions of the quantum phases and that that of the possible phase shifts.

7. More about Semiclassical Optical Systems

As a result, in Relativistic Quantum Field Theory, quantum fields on light-like hyperplanes have an irreducible the free-field algebra [10]. The algebras for quantized fields with different masses become unitarily equivalent. The Fock representation space allows for a vacuum state stable, in the Heisenberg picture, under the interaction Hamiltonian. The 3-dimensional Poincaré transformations are not defined, but only the Poincaré transformations which leave the hyperplane invariant.

For quantization on null-planes, no vacuum polarization is possible [33]. On a null-plane, the stability group of a null plane has non-trivial unitary one-dimensional representations: the Lorentz transformations delineate Wightman functions which are not well defined: the Lorentz transformations are comprehended in the Poincaré transformations, which contain subsurface terms; the surface terms are not eliminated, but the one-dimensional quantization operators contain dumping factors, which outline the definition of the operators. As an application, on null hyper-planes, the average transverse momentum of the quarks composing mesons [34] is described as with strict constraints in opposition with the free-quarks model.

For two-dimensional electrodynamics on two-dimensional null-planes, the quantized spin-zero field on an unquantized background field is investigated. In the presence of an unquantized background laser field, exact closed-form Volkov solutions are found [35]. Commutation relations and the vacuum definition are consistent; the wavefunctions are prepared and constructed as L_2 functions, instead of plane waves not defined in a Hilbert space, and can be expressed in the Heisenberg picture. The infinite-momentum-limit for Lorentz transformations is solved by considering wavepackets which are valid both outside and outside the two-dimensional laser-beam region.

In [36], condensates in the light-cone Hamiltonian are included, after considering a Gaussian approximation for the wavefunctions.

For a relativistic three-dimensional two-body equations and for three-body ones [37] on a null-plane on a null plane, the features of the relativistic Regge formalisms are extrapolated, and the pertinent phenomenological information are stressed out. Differently from the Schroedinger approach, the three-body Hamiltonian satisfies cluster separability the for two-body forces; the corresponding Regge formalism allows one to extract information about three-body mesons, baryons, and quarks (and the corresponding fields) by the definition of Kernels for the Regge trajectories. In the case of a two-body system, the kernels for the Regge trajectories are finite, rotationally invariant and satisfy the proper angular conditions. The formalisms is equivalent to the null-plane constructions, under suitable assumptions, only if the interaction between the two body are not negligible. In the case of a three-body system, the kernel satisfies the cluster separability conditions. Both for the two-body case, the Bethe-Salpeter equations are defined and for the three-body system, the Zero-Range Approximation holds and allows one to eliminate the unwanted time dependences for the wave equations. The choice of a proper covariant Hamiltonian ensures that the angular momentum operators, chosen the proper representation, commute with the suitable kernel.

8. Applications to Optical Systems

Spectral singularities can be studied for the analysis of the behavior of paired photons whose interaction is ruled by a potential implying such a behavior [38]. The states which exhibit a behaviour of the wavepackets are described, for which a suitable approximation of the density matrix, such as those calculated in Section 5, are necessitated.

A simplification of dispersion characterization was proposed in [31] for neural networks in dispersive media for confrontation with the experimental data by the spectral analysis with respect to distorted output pulses; in quantum optical system, the discrepancies accepted are to be ascribed to the interaction of matter with the possible non-flat background metrics, as in [39]. The differences of a one-channel-input to a two-channel input for the chosen dispersive atomic medium (nonlinear four-wave mixing in rubidium vapor) are analyzed in the output by the construction of convolutional neural networks (CNN) for a Ti:Sapphire laser on a beam-splitter, for which the single-peak-center

output requires no spectral analysis for the centers of the output frequencies through a large range of test frequencies, while nonlinearities are observed for the non-peak-center frequencies.

Artificial neural networks are analyzed in [40] for the analysis of the intensity profile of distorted modes, for which the output-center-peaks are with near-zero mean square error indices with respect to the non-perturbed cases for turbulence corrections at different superpositions of intensities of OAM modes chosen at given ranges of refractive indices, for example as due to the atmosphere. In particular, the input Gaussian signal has to be converted in the Gauss-Laguerre output signal. Some features of the wave-front corrector and of the input-data processing are described in [41].

In [42], the dynamics of cold atomic ensembles is investigated, where the variation of the known adiabatic solutions of the standard absorption formula

$$\frac{I_t}{I_0} = exp\left[OD\frac{\gamma^2/4}{\Delta^2 + \gamma^2/4}\right]$$
(10)

in magnetic fields of cooling and trapping of neutral atoms devices for better understanding the effects of perturbations, where *OD* is the optical depth, γ is the exited-state decay rate, and Δ is the probe detuning.

Optical mutations and resonant transitions for many-level atoms were compared in [43] for an unquantized field theory, and comparisons can be accomplished with the Dirac variation-of-constants method perturbation theory [44].

8.1. Radiative Effects in Semiclassical Theory

The coincidence rates for the photoelectric effects in photomultipliers devices for the classical theory and for the semiclassical approach are compared in [45]. The introduction of a Berry topological phase fermions and for solitons in a magnetic field in chiral gauge field theories is approached in [46].

In [47], the properties of time-symmetric theory of radiation is reviewed. Quantum technologies are developing as far as the application of control protocols to quantum metrology is extending [48]. Recombination terms for photon-based interferometers are studied in [49] for the use of non-linear interactions in quantum metrology, for which the signal-to-noise ratio *STN* reads

$$STN \equiv \frac{B_{S0}}{N_0 [1 + \frac{L}{N_0 (1 - L)}]},$$
(11)

with *N* the photon number experiencing the phase change, *B* the signal enhancement, *L* the loss of the attenuator, and the pedix 0 indicates the values before the attenuator.

Quantum vacuum fluctuations in interferometers and the possible reduction of the phenomenon, The non-relativistic scattering theory admits the same limit of that of are studied in [50]. the cross-section, as the quantum constraint Hamiltonian dynamics and quantum field theory perturbation-expansion approach admits comparable limits for a relativistic quantum scattering theory [51]. A feedback amplification method for gravitational-wave detectors is used in [52] in combination with quantum information methods such as entanglement generation and analogue information processing also for the further sake of creating new quantum machines by means of optical quantum communication channels and non-reciprocal amplifiers. In an anti-symmetric medium, for a PT-symmetric coupler consisting of two wavegiudes, there can exist two mechanisms of the transition from a purely-real to complex one, with splitting of a degenerate semi-simple eigenvalue [38]. Odd-PT couplers and even-PT-symmetric couplers can therefore be compared. Bargman-Fock particle states of finite norm can be demonstrated to admit local solutions, and asymptotical solutions can be calculated in particular cases [53] The density of photonic states can be used to probe and analyze the properties of Minkowski-flat spacetime [54] as a limit for inflationary scenarios by analyzing the microscopic degrees of freedom bu making use of standard optical tools after the study means of the diffraction limit of optical imaging, as several Cosmological-Singularity models and the consequent

thermal evolution history of the Universe can be reproduced.Indeed, in particular ferrofluids, there can potentially be yet unknown microscopic degrees of freedom, which are nevertheless still limited by the low-energy scales available at terrestrial experiments and for astronomical observations.

For a a quantum spindensity-wave transition for dynamically generated Landau damping of spin fluctuations [55] can describe fermionic self-energy. The self-energy of cold fermions and its scaling as the fermion coupling is weak can be compared with an increasing one by means of the calculation of the numerical coefficients arising from the data analysis of the optical conductivity of a two-dimensional metal.

For a field theory of the spin-density wave quantum phase transition in two dimensional metals [56], where scattering electrons and the spin-density wave. The wave-order parameter allows one to describe the fermion damping by a full set of composite operators in the corresponding quantum-optical limit of the related field theory. Optical potentials for the Fadeev equations are studied in [57]. The terms of the rearrangement scattering are explicitly solved in particular cases.

New designs for high-accuracy photon-number resolving detectors have been proposed in [58] and related Literature. For gravimeters, the improvements of the measurement results descending from wave-front aberration has been afforded in [59].

The contributions arising from the presence of an external test mass in atom interferometers can be pointed out by examining the related terms of the density matrix in the Wigner representation [60]. The numerical calculations for the approximated expressions for the related quantum field theory are based on the hypothesis of almost-homogeneity for the consequent phase shift.

A moving refractive index medium in presence of a gravitational field [61] exhibits a non-trivial sequence of emission peaks. The several spectra of spontaneous emissions and the photon-number correlations are evaluated both for the lab frame and for the co-moving frame. The dispersion coefficients and the medium dispersions are identified in the spectral analysis.

Quantum technologies exploit entanglement to revolutionize computing, measurements, and communications. This has stimulated research in different areas of physics to engineer and manipulate fragile many-particle entangled states. Progress has been particularly rapid for atoms. Thanks to the large and parameterizable nonlinearities and the well-developed techniques for trapping, controlling, and counting, many groundbreaking experiments have demonstrated the generation of entangled states of trapped ions, cold, and ultracold gases of neutral atoms. Moreover, atoms can strongly couple to external forces and fields, which makes them ideal for ultraprecise sensing and time keeping. All these factors call for generating nonclassical atomic states designed for phase estimation in atomic clocks and atom interferometers, exploiting many-body entanglement to increase the sensitivity of precision measurements.

The parameter estimation in optomechanical-systems experiments can be tested using the generalized likelihood-ratio test; the assumption of static parameters and that of time-varying parameters can be compared for the Gauss-Markov model for quantum systems [62].

In [63], the preparation of macroscopic objects as pure quantum-mechanical states is described, according to the possibility to keep the mechanical degrees of freedom from decoherence caused by the environment by linearizing the dynamics of in-states and out-states.

In [64], optomechanics experiments concerning optical cavities and mechanical resonators are revised, where the underlying basics concepts are reviewed in [65].

Photon-pairs sources [9] can provide one with spectrally correlated two-photons states. The symmetries SU(2) and SU(1,1) in [66] are examined with respect to the amplification techniques. The exact superposition of optical fields can be decomposed as a superposition of eigenmodes [67] whose temporal spectrum is not changes by amplification techniques; in the case of parametric amplifiers, the analysis of the spectra allows distinguishing the features of the amplification. In particular, the spectra of two entangled photons can be reconstructed [68].

Non-classical photon statistics alternative to entanglement are studied in [70] for the implementation of precision measurements in quantum metrology. High-resolution and remote-measurement for entangled systems are analyzed in [71]. The observation of temporally entangled photon pairs in their temporal odes can be achieved by a single-valued decomposition of the spectral modes [68] to study the correlation, or by analyzing the vector field corresponding to the decomposition [67].

Among the possible control paradigms for fundamental tests of quantum mechanics, the long-time limit for the error estimation [62] can be formulated theoretically and by numerical methods as with a power spectrum $S(\omega)$

$$S(\omega) = \frac{C^2}{(\omega - \Omega)^2 + \gamma^2}$$
(12)

with *C* a real parameter, Ω is the mechanical resonance frequency, γ the dumping rate.

The quantum correlation of multispatial modes can be examined by means of the experimental errors due to experimental noise and those due to the attenuation of the experimental apparati, for which the optimization methods for the analysis are described in [72].

8.2. The Optical Equivalents for Quantum-Mechanical Operators

It is, therefore, now possible to apply the results found for the expressions of sequences of observables of the unbounded operators such as the density matrix found in the previous investigation to the calculations of other operators defined in quantum-mechanical systems as far as their optical equivalent can be needed. For operators *A*, with weighted density matrix and the spectral component $\phi_R(z)$ on compact support,

$$A_R \equiv \frac{1}{\pi} \int d^2 z \phi_R(z) \mid z > < z \mid + \mathcal{O}(n; n'; \beta; \lambda),$$
(13)

respectively, in the projector operator |z > < z|.

The definition of the first-approximation correction orders therefore very importantly depend on the definition of the parameters β and λ in the definitions of the weighting-support-control function S_{β} and S_{λ} in Equations (2) and (8), respectively.

The related results are obtained by considering the properties of quantization on null-hypersurface quantization techniques. The investigation is consistent for systems constituted of intense, non-monochromatic laser fields. The power spectrum of the operators is therefore decomposed as a sequence obtained after the majorization of the operators after those of the weighting function. The power spectrum is therefore not needed to be expressed as a sequence (of majorizations), where such majorization do not apply to pure states.

In the comparison with the quasi-probability distributions for the density operator for an infinite momentum which involves the Fock representation, infinite-momentum states can be studied as suitable approximation for states in extremely-high intesity laser fields, whose energy can be compared as its limit going to infinity. The observables for interaction processes ca be schemtazied within the approximation of the expectation values of the density matrix corresponding to the infinite-energy (momentum) laser fileds, and the corresponding eigenstates. The following remarks are in order, after the inconveniences evidentiated by [11,73,74].

Pure states *rho* in the momentum *P* representation define observables by measn of the density operator in the *P* representation $| \psi \rangle \langle \psi |$, whose weight function $P(\zeta)$ in the momentum *P* representation allows one to classify coherent states and incoherent states in a radiation background field, according to the properties of the radiation background field. Coherent states are represented by

a finite number of creation operators, with, as coefficients, arbitrary complex numbers. The weight functions $\tilde{P}(\zeta)$ are tempered distributions [11,73].

$$\tilde{\rho}_{\zeta} \equiv \int P(\zeta) \mid \zeta \rangle < \zeta \mid d^2 \zeta \tag{14}$$

with $d^2\zeta$ a real element of area (even in the complex plane), and coherent states are formedas a finite number ofcreation operators in the Fock representation. $P(\zeta)$ is a linear combination of 2-dimensional functions $\tilde{f}\beta$ and of a finite number of its derivatives. An infinitely-energetic *P* background laser filed for photons can be interpreted as a superposition of incoherent states, within the due hypotheses.

Density operators in the momentum representations [11] are weighted functionals $P(\zeta)$, which define Wigner distribution $W(\zeta)$, with $W(\zeta)$ continuous and uniformily bounded, as $\langle \zeta | \rho | \tilde{\zeta} \rangle$. Quasi-probability distributions are expectation values for the corresponding density operator(s). Integral representations for the density operators can therefore be found. $P(\zeta)$ are the expectation values of Hermitian operator(s), whose eigenvalues are infinite.

For the incoherent states [11], the density matrix $\tilde{\beta}'$ can be rewritten, in a limiting procedure, as a sum of neighbouring states

$$ilde{
ho}' \equiv (1 - ilde{\epsilon}) ilde{
ho} + ilde{\epsilon} ilde{
ho} \mid \zeta > < \zeta \mid;$$

in the limit $0 < \epsilon < 1$, $\tilde{\rho}'$ in the trace-class norm; the corresponding weight functions $P'(\zeta)$, which correspond to the density operators $\tilde{\rho}'$ are tempered distributions only for pure states. Differently, $P(\zeta)$ exhibits singularities not compatible with the form of the momentum *P* representation. It is therefore relevant to study a representation of the density operators, in the case of (almost-) infinite laser background field independent, of the Fock representation.

The discrepancies for quantum states in the power spectrum is expressed for optical systems by the terms $O(n; n'; \beta; \lambda)$, which depend both on the weighting function ϕ in (2) as well as the (non-equal) non-trivial weights characterizing the Fock states expressed by the parameters β and λ .

The examination of the power spectrum can be performed also by the Fourier decomposition, which takes into account the corrections at the proper order. This results ensures therefore to avoid the mixing and the superpositions of the corrections at different orders also in the Fourier decomposition of the spectral modes.

8.3. An Example: The Long-Time Limit for the Error Estimation

As an example, the long-time limit for the error estimation Equation (12) can be calculated exactly as

$$S(\omega) \simeq \frac{2C^2\Omega}{2\Omega^4 + 2\gamma^2\Omega^2 + \gamma^4}\omega + \frac{C^2}{\Omega^2 + \gamma^2} + \mathcal{O}(\omega^2, \Omega^2; C^2; \gamma^{-4})$$
(15)

where the correction term $\frac{C^2}{\Omega^2 + \gamma^2} + O(\omega^2, \Omega^2; C^2; \gamma^{-4})$ consists of a non-trivial summand plus the corrections due to the other parameters, i.e., *C* a real parameter, Ω the mechanical resonance frequency, γ the dumping rate. The Fourier decomposition of the modes corresponding to a systems of an intense laser beam can be approximated by one containing particles also with infinite momentum on Minkowski spacetime by means of standard quantization techniques. For the standard quantization techniques, the polarization tensor is evaluated at all orders (not expanded in Equation (12)). The peculiarities of intense laser fields, containing particle with very large values of the momentum but not an infinite momentum, can be ascribed to the properties of semiclassical optical systems, for which the dimensions of the system are larger than the Planck length, but whose constituents are of a quantum nature. For particles with very high value of the momentum, such as intense laser field, the expansion Equation (15), calculated after Equation's (13) for the intense laser beam modes.

9. Concluding Remarks

In [75], among the analysis of vacuum polarization for laser fields, the experimental vacuum space available for the experimental setting allows describing non-laser photon fields by two different complex refraction indices, differently by the index of refraction characterizing the vacuum polarization for inhomogeneous magnetic fields, both in the case of the strong-field approximation, and in that for the weak field approximation [30].

Quantum Electrodynamics in the presence of any external field, i.e., also a laser field, can be reformulated in terms of the corresponding free Green's function in presence of the external field [29].

The scattering of partially coherent radiation caused by non-Hermitian structures, such as those for which PT symmetry is not conserved, with coherent systems is studied in [76].

The spin-Hall effect in topological photonics were reviewed within the framework of topological insulator as far as the associated orbital angular momentum is concerned [77].

After the analysis of the properties of main laser devices, the implementation of hybrid interferometers offers further possibilities for the analysis and the spectral sampling, due to the particular features of the noise analysis, of interest also for the analysis gravitational-wave detection [49].

For detectors endowed with amplifiers rather than attenuators, it is possible to resolve the properties of a single photon in the quantum limit by the design of new detectors able to eliminate additional noise sources, within the specificities off the detector construction features [78].

By means of two optical amplifiers, it is possible to achieve an SU(1,1) interferometer, whose focusing properties allow one to separate spatial multimodes within a broad-angle resolution for quantum metrology, but also in remote sensing, and enable eliminating sub-shot-noises for the sake of quantum information processing [79,80]. Similar properties are exhibited by fiberoptic nonlinear interferometers [81].

Atomic ensembles can improve quantum-enhanced metrology for atom interferometers by collective spin systems and their phase estimations by providing upper bound and lower ones for the full probability distribution rather than some moments only [82].

The features of unsymmetrized optical potentials, whose states are described within a fully antisymmetrized Hilbert space, are useful for the study of pole singularities in the resonance structures for the elastic scattering amplitudes [83].

After the analysis of [84], it is possible to optically resolve in the spectral analysis [61] an optical analogue to waves under a suitable gravitational field by studying the properties of the refraction index of the medium by studying positive norm modes and negative-norms ones, useful for the analysis of exotic cosmological objects as well as classical ones. Quantum systems and their optical analogues can be investigated also for the description of quantum-gravitation properties of the spacetime close to the Planck semiclassicalization epoch after the Cosmological Singularity [85].

Via an SU(1,1) interferometer, the Heisenberg limit of the sensitivity [41,86] can be tested, as well as the parity properties of the states investigated [87].

It is possible to estimate quantum parameters in optical system via opto-mechanical devices [88], for which the quantum (Heisenberg) limit [41], given *N* the total particle number, is calculated by the precision of parameter estimation for the shot-noise limit is $1/\sqrt{N}$ while the Heisenberg limit is 1/N. The applications of the Heisenberg limit for the shot-noise valuation have been proposed in [70] for quantum-information entangled systems.

By numerical calculations, in optomechanical systems, it is possible to estimate the relations of between the number of macroscopic quantum states and the number of optical photons [89] by the analysis of the ground state among all the quantum states.

In the converse, [63], in opto-mechanical systems, it is possible to test the relation of macroscopic objects and pure quantum states after the analysis of the behaviour of the macroscopic objects with respect to quantum mechanics, for which the guidelines for the statistical analysis are outlined in [65].

In [90], the application of interferometers to the detection of gravitational waves are described as far as the improvement for the photon-counting errors and radiation-pressure errors, and the improvements for the measurement time and the laser power are outlined.

The necessity for these improvements for SU(2) and SU(1,1) interferometers has been pointed out in [66], also as far the number of quanta available for the device, as analyzed in [86].

The systems described in [76], i.e., which do not conserve PT symmetry, can also be affected by spectral singularities [38].

A geometrical phase for photons [46] can be investigated by cold atoms inside an optical cavity or in a microwave cavity [91].

Trapped systems are analyzed in [42,92].

The aim of the present paper has been to analyze the possible structures related to the equivalence optical principle without making use of the Fock representations space. Technical advantages of the implications studied can be outlined in the sistems described in [93] and [94].

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References

- Klauder, J.R.; Streit, L. Optical Equivalence Theorem for Unbounded Observables. J. Math. Phys. 1974, 15, 760–763. [CrossRef]
- 2. Sudarshan, E.C.G. Equivalence of Semiclassical and Quantum Mechanical Descriptions of Statistical Light Beams. *Phys. Rev.* **1963**, *10*, 277. [CrossRef]
- 3. Chaturvedi, S.; Agarwal, G.S.; Srinivasan, V. Positivization and regularization of quantum phase space distributions. *J. Phys. A Math. Gen.* **1994**, 27, L39. [CrossRef]
- 4. Lachs, G. Theoretical aspects of mixtures of thermal and coherent radiation. *Phys. Rev.* **1965**, *138*, B1012. [CrossRef]
- 5. Mehta, C.L.; Sudarshan, E.C.G. Relation between quantum and semiclassical description of optical coherence. *Phys. Rev.* **1965**, *138*, B274. [CrossRef]
- 6. Sudarshan, G. Quantum theory of partial coherence. J. Math. Phys. Sci. 1969, 3, 121.
- Perelomov, A.M. Generalized coherent states and some of their applications. Sov. Phys. Usp. 1977, 20, 703–720. [CrossRef]
- 8. Gisin, N.; Thew, R. Quantum communication. *Nat. Photonics* 2007, 1, 165–171. [CrossRef]
- 9. Su, J.; Cui, L.; Li, J.; Liu, Y.; Li, X.; Ou, Z.Y. Versatile and precise quantum state engineering by using nonlinear interferometers. *Opt. Express* **2019**, *27*, 20479–20492. [CrossRef]
- 10. Leutwyler, H.; Klauder, J.R.; Streit, L. Quantum field theory on lightlike slabs. *Nuovo Cim. A* **1970**, *66*, 536–554. [CrossRef]
- 11. Cahill, K.E.; Glauber, R.J. Density Operators and Quasiprobability Distributions. *Phys. Rev.* **1969**, 177, 1882–1902. [CrossRef]
- 12. Lohani, S.; Knutson, E.M.; O'Donnell, M.; Huver, S.D.; Glasser, R.T. On the use of deep neural networks in optical communications. *Appl. Opt.* **2018**, *57*, 4180–4190. [CrossRef] [PubMed]
- Abramowitz, M.; Stegun, I. Handbook of Mathematical Functions with Formulas Graphs and Mathematical Tables; Applied Mathematics Series; United States Department of Commerce, National Bureau of Standards: Washington, DC, USA; Dover Publications: New York, NY, USA, 1972; Volume 55.
- 14. Becker, W. Quantum electrodynamics in intense laser fields. Laser Part. Beams 1991, 9, 603–618. [CrossRef]
- 15. Ehlotzky, F.; Krajewska, K.; Kamiński, J.Z. Fundamental processes of quantum electrodynamics in laser fields of relativistic power. *Rep. Prog. Phys.* **2009**, *72*, 046401. [CrossRef]
- 16. Ehlotzky, F. Effect of laser light polarization on multiphoton scattering processes. *Acta Phys. Austriaca* **1972**, 36, 243–247.
- 17. Di Piazza, A.; Müller, C.; Hatsagortsyan, K.Z.; Keitel, C.H. Extremely high-intensity laser interactions with fundamental quantum systems. *Rev. Mod. Phys.* **2012**, *84*, 1177. [CrossRef]

- Głazek, S.D.; Trawiński, A.P. Neutrino oscillations in the front form of Hamiltonian dynamics. *Phys. Rev. D* 2013, *87*, 025002. [CrossRef]
- 19. Kogut, J.B.; Soper, D.E. Quantum electrodynamics in the infinite-momentum frame. *Phys. Rev. D* 1970, *1*, 2901. [CrossRef]
- 20. Brodsky, S.J.; Roskies, R.; Suaya, R. Quantum Electrodynamics and Renormalization Theory in the Infinite-Momentum Frame. *Phys. Rev. D* **1973**, *8*, 4574. [CrossRef]
- 21. Soper, D.E. Massive quantum electrodynamics in the infinite-momentum frame. *Phys. Rev. D* 1971, *4*, 1620. [CrossRef]
- 22. Rohrlich, F.; Ten Eyck, J.H. Null plane quantum electrodynamics. Phys. Lett. B 1973, 46, 102–104. [CrossRef]
- Bakker, B.L.G.; Bassetto, A.; Brodsky, S.J.; Broniowski, W.; Dalley, S.; Frederico, T.; Glazek, S.D.; Hiller, J.R.; Ji, C.-R.; Karmanov, V.; et al. Light-Front Quantum Chromodynamics: A framework for the analysis of hadron physics. *Nucl. Phys. B Proc. Suppl.* 2014, 251, 165–174. [CrossRef]
- 24. Baur, G.; Hencken, K.; Trautmann, D. Electron–positron pair production in ultrarelativistic heavy ion collisions. *Phys. Rep.* 2007, 453, 1–27. [CrossRef]
- 25. Heinzl, T. Strong-field QED and high-power lasers. Int. J. Mod. Phys. Conf. Ser. 2012, 14, 127–140. [CrossRef]
- 26. Seipt, D.; Sorbo, D.D.; Ridgers, C.P.; Thomas, A.G.R. Theory of radiative electron polarization in strong laser fields. *Phys. Rev. A* **2018**, *98*, 023417. [CrossRef]
- 27. Fedotov, A.M.; Narozhny, N.B.; Mourou, G.; Korn, G. Limitations on the attainable intensity of high power lasers. *Phys. Rev. Lett.* **2010**, *105*, 080402. [CrossRef]
- 28. Cajiao Vélez, F.; Kamiński, J.Z.; Krajewska, K. Electron Scattering Processes in Non-Monochromatic and Relativistically Intense Laser Fields. *Atoms* **2019**, *7*, 34. [CrossRef]
- 29. Mitter, H. Quantum Electrodynamics in Laser Fields. Acta Phys. Austriaca Suppl. 1975, 14, 397–498.
- 30. Tsai, W.; Erber, T. The Propagation of Photons in Homogeneous Magnetic Fields: Index of Refraction. *Phys. Rev. D* **1975**, *12*, 1132. [CrossRef]
- 31. Lohani, S.; Knutson, E.M.; Zhang, W.; Glasser, R.T. Dispersion Characterization and Pulse Prediction with Machine Learning. *arXiv* **2019**, arXiv:1909.02526.
- 32. Holland, M.J.; Burnett, K. Interferometric detection of optical phase shifts at the Heisenberg limit. *Phys. Rev. Lett.* **1993**, *71*, 1355–1358. [CrossRef] [PubMed]
- 33. Nakanishi, N.; Yabuki, H. Null-plane quantization and Haag's theorem. *Lett. Math. Phys.* **1977**, *1*, 371–374. [CrossRef]
- 34. Leutwyler, H.; Leutwyler, H. On the Average transverse momentum of the quarks within a meson. *Phys. Lett. B* **1974**, *48*, 45–47. [CrossRef]
- 35. Neville, R.A.; Rohrlich, F. Quantum electrodynamics on null planes and applications to lasers. *Phys. Rev. D* **1971**, *3*, 1692. [CrossRef]
- 36. Prokhvatilov, E.V.; Naus, H.W.L.; Pirner, H.J. Effective light-front quantization of scalar field theories and two-dimensional electrodynamics. *Phys. Rev. D* **1995**, *51*, 2933. [CrossRef] [PubMed]
- Di Salvo, E.; Kondratyuk, L.; Saracco, P. Relativistic three-dimensional two-and three-body equations on a null plane and applications to meson and baryon Regge trajectories. *Z. Phys. C Part. Fields* 1995, 69, 149–162. [CrossRef]
- Konotop, V.V.; Zezyulin, D.A. Spectral singularities of odd- PT -symmetric potentials. *Phys. Rev. A* 2019, 99, 013823. [CrossRef]
- 39. Faizal, M.; Momeni, D. Universality of short distance corrections to quantum optics. *arXiv* 2018, arXiv:1811.01934.
- 40. Lohani, S.; Glasser, R.T. Turbulence correction with artificial neural networks. arXiv 2018, arXiv:1806.07456.
- 41. Rukosuev, A.L.; Kudryashov, A.V.; Lylova, A.N.; Samarkin, V.V.; Sheldakova, Y.V. Adaptive optics system for real-time wavefront correction. *Atmos. Ocean. Opt.* **2015**, *28*, 381–386. [CrossRef]
- 42. Tranter, A.D.; Slatyer, H.J.; Hush, M.R.; Leung, A.C.; Everett, J.L.; Paul, K.V.; Vernaz-Gris, P.; Lam, P.K.; Buchler, B.C.; Campbell, G.T. Multiparameter optimisation of a magneto-optical trap using deep learning. *arXiv* **2018**, arXiv:1805.00654.
- 43. Crisp, M.D.; Jaynes, E.T. Radiative effects in semiclassical theory. *Phys. Rev.* **1969**, *179*, 1253–1261; Erratum in **1969**, *185*, 2046. [CrossRef]
- 44. Langhoff, P.W.; Epstein, S.T.; Karplus, M. Aspects of time-dependent perturbation theory. *Rev. Mod. Phys.* **1972**, 44, 602–644. [CrossRef]

- 45. Clauser, J.F. Experimental distinction between the quantum and classical field-theoretic predictions for the photoelectric effect. *Phys. Rev. D* **1974**, *9*, 853–860. [CrossRef]
- 46. Aitchison, I.J.R. Berry's topological phase in quantum mechanics and quantum field theory. *Phys. Scr.* **1988**, *T23*, 12–20. [CrossRef]
- 47. Krizan, J.E. Time-symmetric, approximately relativistic particle interactions and radiation. *Phys. Rev. D* **1980**, 22, 3017–3022. [CrossRef]
- 48. Dowling, J.P.; Milburn, G.J. Quantum technology: The second quantum revolution. Philosophical Transactions of the Royal Society of London. *Ser. A Math. Phys. Eng. Sci.* 2003, 361, 1655–1674. [CrossRef]
- 49. Hudelist, F.; Kong, J.; Liu, C.; Jing, J.; Ou, Z.Y.; Zhang, W. Quantum metrology with parametric amplifier-based photon correlation interferometers. *Nat. Commun.* **2014**, *5*, 3049. [CrossRef]
- 50. Bai, Y.; Venugopalan, G.; Kuns, K.; Wipf, C.; Markowitz, A.; Wade, A.R.; Chen, Y.; Adhikari, R.X. A phase-sensitive optomechanical amplifier for quantum noise reduction in laserinterferometers. *arXiv* **2019**, arXiv:1909.02264.
- 51. Arshansky, R.I.; Horwitz, L.P. Covariant phase shift analysis for relativistic potential scattering. *Phys. Lett. A* **1988**, 131, 222–226. [CrossRef]
- 52. Shimazu, R.; Yamamoto, N. Quantum functionalities via feedback amplification. *arXiv* 2019, arXiv:1909.12822.
- 53. Maciejewski, A.J.; Przybylska, M.; Stachowiak, T. An exactly solvable system from quantum optics. *Phys. Lett. A* **2015**, *379*, 1503. [CrossRef]
- 54. Smolyaninov, I.I.; Smolyaninova, V.N. Experimental observation of melting of the effective Minkowski spacetime in cobalt-based ferrofluids. In *Reviews in Plasmonics 2016*; Springer: Cham, Switzerland, 2017; pp. 137–158.
- 55. Chubukov, A.V.; Maslov, D.L.; Yudson, V.I. Optical conductivity of a two-dimensional metal at the onset of spin-density-wave order. *Phys. Rev. B* 2014, *89*, 155126. [CrossRef]
- 56. Hartnoll, S.A.; Hofman, D.M.; Metlitski, M.A.; Sachdev, S. Quantum critical response at the onset of spin-density-wave order in two-dimensional metals. *Phys. Rev. B* 2011, *84*, 125115. [CrossRef]
- 57. Grassberger, P.; Sandhas, W. Relation between the projection operator formalism and the Faddeev theory. *Z. Phys. Hadron. Nucl.* **1969**, 220, 29–44. [CrossRef]
- 58. Young, S.M.; Sarovar, M.; Léonard, F. Design of High-Performance Photon Number Resolving Photodetectors Based on Coherently Interacting Nanoscale Elements. *arXiv* **2020**, arXiv:1909.07911.
- 59. Schkolnik, V.; Leykauf, B.; Hauth, M.; Freier, C.; Peters, A. The effect of wavefront aberrations in atom interferometry. *Appl. Phys. B* 2015, *120*, 311–316. [CrossRef]
- 60. Dubetsky, B.; Libby, S.B.; Berman, P. Atom interferometry in the presence of an external test mass. *Atoms* **2016**, *4*, 14. [CrossRef]
- 61. Jacquet, M.J.; Koenig, F. Analytical description of quantum emission in optical analogues to gravity. *arXiv* **2019**, arXiv:1908.02060.
- 62. Ang, S.Z.; Harris, G.I.; Bowen, W.P.; Tsang, M. Optomechanical parameter estimation. *New J. Phys.* 2013, 15, 103028. [CrossRef]
- 63. Chen, Y. Macroscopic Quantum Mechanics: Theory and Experimental Concepts of Optomechanics. *J. Phys. B* **2013**, *46*, 104001. . [CrossRef]
- 64. Aspelmeyer, M.; Kippenberg, T.J.; Marquardt, F. Cavity optomechanics. *Rev. Mod. Phys.* **2014**, *86*, 1391. [CrossRef]
- 65. Tsang, M. Testing quantum mechanics: A statistical approach. *Quantum Meas. Quantum Metrol.* **2013**, *1*, 84–109. [CrossRef]
- 66. Yurke, B.; McCall, S.L.; Klauder, J.R. SU(2) and SU(1,1) interferometers. *Phys. Rev. A* **1986**, *33*, 4033–4054. [CrossRef]
- 67. Huo, N.; Liu, Y.; Li, J.; Cui, L.; Chen, X.; Palivela, R.; Xie, T.; Li, X.; Ou, Z.Y. Direct temporal mode measurement for the characterization of temporally multiplexed high dimensional quantum entanglement in continuous variables. *arXiv* **2020**, arXiv:1910.09102.
- 68. Chen, X.; Li, X.; Ou, Z.Y. Direct temporal mode measurement of photon pairs by stimulated emission. *arXiv* **2020**, arXiv:1910.09720.
- 69. Su, J.; Cui, L.; Li, J.; Liu, Y.; Li, X.; Ou, Z.Y. Quantum state engineering by nonlinear quantum interference. *arXiv* **2018**, arXiv:1811.07646.

- 70. Sahota, J.; Quesada, N. Quantum correlations in optical metrology: Heisenberg-limited phase estimation without mode entanglement. *Phys. Rev. A* **2015**, *91*, 013808. [CrossRef]
- 71. Lee, S.K.; Yoon, T.H.; Cho, M. Interferometric quantum spectroscopy with undetected photons via distinguishability modulation. *Opt. Express* **2019**, 27, 14853–14870. [CrossRef]
- 72. Du, J.; Cao, L.; Zhang, K.; Jing, J. Experimental observation of multi-spatial-mode quantum correlations in four-wave mixing with a conical pump and a conical probe. *Appl. Phys. Lett.* **2017**, *110*, 241103. [CrossRef]
- 73. Glauber, R.J. Coherent and incoherent states of the radiation field. Phys. Rev. 1963, 131, 2766–2788. [CrossRef]
- 74. Cahill, K.E. Pure states and the p representation. Phys. Rev. 1969, 180, 1239–1243. [CrossRef]
- 75. Becker, W.; Mitter, H. Vacuum polarization in laser fields. J. Phys. A Math. Gen. 1975, 8, 1638–1657. [CrossRef]
- 76. Brandão, P.A.; Cavalcanti, S.B. Scattering of partially coherent radiation by non-Hermitian localized structures having parity-time symmetry. *Phys. Rev. A* **2019**, *100*, 043822. [CrossRef]
- 77. Shi, P.; Du, L.; Li, C.; Zayats, A.V.; Yuan, X. Spin-momentum law for structured guided modes: The generalized quantum spin-Hall effect for light. *arXiv* **2019**, arXiv:1910.03904.
- 78. Zobrist, N.; Eom, B.H.; Day, P.; Mazin, B.A.; Meeker, S.R.; Bumble, B.; LeDuc, H.G.; Coiffard, G.; Szypryt, P.; Fruitwala, N.; et al. Wide-band parametric amplifier readout and resolution of optical microwave kinetic inductance detectors. *Appl. Phys. Lett.* **2019**, *115*, 042601. [CrossRef]
- 79. Frascella, G.; Mikhailov, E.E.; Takanashi, N.; Zakharov, R.V.; Tikhonova, O.V.; Chekhova, M.V. Wide-field SU(1,1) interferometer. *Optica* **2019**, *6*, 1233–1236. [CrossRef]
- Xin, J.; Lu, X.-M.; Li, X.; Li, G. Optimal phase point for SU(1,1) interferometer. J. Opt. Soc. Am. B 2019, 36, 2824–2833. [CrossRef]
- Lukens, J.M.; Pooser, R.C.; Peters, N.A. A broadband fiber-optic nonlinear interferometer. *Appl. Phys. Lett.* 2018, 113, 091103. [CrossRef]
- 82. Pezze, L.; Smerzi, A.; Oberthaler, M.K.; Schmied, R.; Treutlein, P. Quantum metrology with nonclassical states of atomic ensembles. *Rev. Mod. Phys.* **2018**, *90*, 035005. [CrossRef]
- 83. Kowalski, K.L.; Picklesimer, A. Modifications of the optical potential formalism arising from the pauli principle. *Nucl. Phys. A* **1981**, *369*, 336–364. [CrossRef]
- 84. Barker, B.M.; O'Connell, R.F. Gravitational Two-Body Problem with Arbitrary Masses, Spins, and Quadrupole Moments. *Phys. Rev. D* 1975, *12*, 329. [CrossRef]
- 85. Kuzmichev, V.E.; Kuzmichev, V.V. Behavior of the Gravitational System Close to the Planck Epoch. *Ukr. J. Phys.* **2017**, *62*, 545. [CrossRef]
- 86. Braunstein, S.L.; Caves, C.M.; Milburn, G.J. Generalized uncertainty relations: theory, examples, and Lorentz invariance. *Ann. Phys.* **1996**, 247, 135–173. [CrossRef]
- 87. Li, D.; Gard, B.T.; Gao, Y.; Yuan, C.H.; Zhang, W.; Lee, H.; Dowling, J.P. Phase sensitivity at the Heisenberg limit in an SU (1, 1) interferometer via parity detection. *Phys. Rev. A* **2016**, *94*, 063840. [CrossRef]
- 88. Zheng, Q.; Yao, Y.; Li, Y. Optimal quantum parameter estimation in a pulsed quantum optomechanical system. *Phys. Rev. A* **2016**, *93*, 013848. [CrossRef]
- Lin, Q.; He, B. Highly efficient cooling of mechanical resonator with square pulse drives. *Opt. Express* 2018, 26, 33830–33840. [CrossRef]
- 90. Caves, C.M. Quantum-mechanical noise in an interferometer. Phys. Rev. D 1981, 23, 1693–1981. [CrossRef]
- 91. Yu, Y.X.; Ye, J.; Zhang, C. Photon Berry phases, Instantons, Quantum chaos and quantum analog of Kolmogorov-Arnold-Moser (KAM) theorem in Dicke models. *arXiv* **2019**, arXiv:1903.02947.
- 92. Kartashov, Y.V.; Astrakharchik, G.E.; Malomed, B.A.; Torner, L. Frontiers in multidimensional self-trapping of nonlinear fields and matter. *Nat. Rev. Phys.* **2019**, *1*, 185–197. [CrossRef]
- 93. Gross, C.; Zibold, T.; Nicklas, E.; Esteve, J.; Oberthaler, M.K. Nonlinear atom interferometer surpasses classical precision limit. *Nature* **2010**, *464*, 1165–1169. [CrossRef] [PubMed]
- 94. Asano, M.; Ohta, R.; Aihara, T.; Tsuchizawa, T.; Okamoto, H.; Yamguchi, H. Optically probing Schwinger angular momenta in a micromechanical resonator. *arXiv* 2019, arXiv:1905.12178.



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