

NS-Cross Entropy-Based MAGDM under Single-Valued Neutrosophic Set Environment

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Abstract: A single-valued neutrosophic set has king power to express uncertainty characterized by indeterminacy, inconsistency and incompleteness. Most of the existing single-valued neutrosophic cross entropy bears an asymmetrical behavior and produces an undefined phenomenon in some situations. In order to deal with these disadvantages, we propose a new cross entropy measure under a single-valued neutrosophic set (SVNS) environment, namely NS-cross entropy, and prove its basic properties. Also we define weighted NS-cross entropy measure and investigate its basic properties. We develop a novel multi-attribute group decision-making (MAGDM) strategy that is free from the drawback of asymmetrical behavior and undefined phenomena. It is capable of dealing with an unknown weight of attributes and an unknown weight of decision-makers. Finally, a numerical example of multi-attribute group decision-making problem of investment potential is solved to show the feasibility, validity and efficiency of the proposed decision-making strategy.

Keywords: neutrosophic set; single-valued neutrosophic set; NS-cross entropy measure; multi-attribute group decision-making

1. Introduction

To tackle the uncertainty and modeling of real and scientific problems, Zadeh [1] first introduced the fuzzy set by defining membership measure in 1965. Bellman and Zadeh [2] contributed important research on fuzzy decision-making using max and min operators. Atanassov [3] established the intuitionistic fuzzy set (IFS) in 1986 by adding non-membership measure as an independent component to the fuzzy set. Theoretical and practical applications of IFSs in multi-criteria decision-making (MCDM) have been reported in the literature [4–12]. Zadeh [13] introduced entropy measure in the fuzzy environment. Burillo and Bustince [14] proposed distance measure between IFSs and offered an axiomatic definition of entropy measure. In the IFS environment, Szmidt and Kacprzyk [15] proposed a new entropy measure based on geometric interpretation of IFS. Wei et al. [16] developed an entropy measure for interval-valued intuitionistic fuzzy set (IVIFS) and presented its applications in pattern recognition and MCDM. Li [17] presented a new multi-attribute decision-making (MADM) strategy combining entropy and Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) in an IVIFS environment. Shang and Jiang [18] introduced the cross entropy in the fuzzy environment. Vlachos and Sergiadis [19] presented intuitionistic fuzzy cross entropy by extending fuzzy cross entropy [18]. Ye [20] defined a new cross

entropy under an IVIFS environment and presented an optimal decision-making strategy. Xia and Xu [21] put forward a new entropy and a cross entropy and employed them for multi-attribute criteria group decision-making (MAGDM) strategy under an IFS environment. Tong and Yu [22] defined cross entropy under an IVIFS environment and applied it to MADM problems.

The study of uncertainty took a new direction after the publication of the neutrosophic set (NS) [23] and single-valued neutrosophic set (SVNS) [24]. SVNS appeals more to researchers for its applicability in decision-making [25–54], conflict resolution [55], educational problems [56,57], image processing [58–60], cluster analysis [61,62], social problems [63,64], etc. The research on SVNS gained momentum after the inception of the international journal “Neutrosophic Sets and Systems”. Combining with the neutrosophic set, a number of hybrid neutrosophic sets such as the neutrosophic soft set [65–72], the neutrosophic soft expert set [73–75], the neutrosophic complex set [76], the rough neutrosophic set [77–86], the rough neutrosophic tri complex set [87], the neutrosophic rough hyper complex set [88], the neutrosophic hesitant fuzzy sets/multi-valued neutrosophic set [89–97], the bipolar neutrosophic set [98–103], the rough bipolar neutrosophic set [104], the neutrosophic cubic set [105–113], and the neutrosophic cubic soft set [114,115] has been reported in the literature. Wang et al. [116] defined the interval neutrosophic set (INS). Different interval neutrosophic hybrid sets and their theoretical development and applications have been reported in the literature, such as the interval-valued neutrosophic soft set [117], the interval neutrosophic complex set [118], the interval neutrosophic rough set [119–121], and the interval neutrosophic hesitant fuzzy set [122]. Other extensions of neutrosophic sets, such as trapezoidal neutrosophic sets [123,124], normal neutrosophic sets [125], single-valued neutrosophic linguistic sets [126], interval neutrosophic linguistic sets [127,128], simplified neutrosophic linguistic sets [129], single-valued neutrosophic trapezoid linguistic sets [130], interval neutrosophic uncertain linguistic sets [131–133], neutrosophic refined sets [134–139], linguistic refined neutrosophic sets [140] bipolar neutrosophic refined sets [141], and dynamic single-valued neutrosophic multi-sets [142] have been proposed to enrich the study of neutrosophics. So the field of neutrosophic study has been steadily developing.

Majumdar and Samanta [143] defined an entropy measure and presented an MCDM strategy under SVNS environment. Ye [144] proposed cross entropy measure under the single-valued neutrosophic set environment, which is not symmetric straight forward and bears undefined phenomena. To overcome the asymmetrical behavior of the cross entropy measure, Ye [144] used a symmetric discrimination information measure for single-valued neutrosophic sets. Ye [145] defined cross entropy measures for SVNSs to overcome the drawback of undefined phenomena of the cross entropy measure [144] and proposed a MCDM strategy.

The aforementioned applications of cross entropy [144,145] can be effective in dealing with neutrosophic MADM problems. However, they also bear some limitations, which are outlined below:

- i. The strategies [144,145] are capable of solving neutrosophic MADM problems that require the criterion weights to be completely known. However, it can be difficult and subjective to offer exact criterion weight information due to neutrosophic nature of decision-making situations.
- ii. The strategies [144,145] have a single decision-making structure, and not enough attention is paid to improving robustness when processing the assessment information.
- iii. The strategies [144,145] cannot deal with the unknown weight of the decision-makers.

Research gap:

MAGDM strategy based on cross entropy measure with unknown weight of attributes and unknown weight of decision-makers.

This study answers the following research questions:

- i. Is it possible to define a new cross entropy measure that is free from asymmetrical phenomena and undefined behavior?
- ii. Is it possible to define a new weighted cross entropy measure that is free from the asymmetrical phenomena and undefined behavior?

- iii. Is it possible to develop a new MAGDM strategy based on the proposed cross entropy measure in single-valued neutrosophic set environment, which is free from the asymmetrical phenomena and undefined behavior?
- iv. Is it possible to develop a new MAGDM strategy based on the proposed weighted cross entropy measure in the single-valued neutrosophic set environment that is free from the asymmetrical phenomena and undefined behavior?
- v. How do we assign unknown weight of attributes?
- vi. How do we assign unknown weight of decision-makers?

Motivation:

The above-mentioned analysis describes the motivation behind proposing a comprehensive NS-cross entropy-based strategy for tackling MAGDM under the neutrosophic environment. This study develops a novel NS-cross entropy-based MAGDM strategy that can deal with multiple decision-makers and unknown weight of attributes and unknown weight of decision-makers and free from the drawbacks that exist in [144,145].

The objectives of the paper are:

1. To define a new cross entropy measure and prove its basic properties, which are free from asymmetrical phenomena and undefined behavior.
2. To define a new weighted cross measure and prove its basic properties, which are free from asymmetrical phenomena and undefined behavior.
3. To develop a new MAGDM strategy based on weighted cross entropy measure under single-valued neutrosophic set environment.
4. To develop a technique to incorporate unknown weight of attributes and unknown weight of decision-makers in the proposed NS-cross entropy-based MAGDM under single-valued neutrosophic environment.

To fill the research gap, we propose NS-cross entropy-based MAGDM, which is capable of dealing with multiple decision-makers with unknown weight of the decision-makers and unknown weight of the attributes.

The main contributions of this paper are summarized below:

1. We define a new NS-cross entropy measure and prove its basic properties. It is straightforward symmetric and it has no undefined behavior.
2. We define a new weighted NS-cross entropy measure in the single-valued neutrosophic set environment and prove its basic properties. It is straightforward symmetric and it has no undefined behavior.
3. In this paper, we develop a new MAGDM strategy based on weighted NS cross entropy to solve MAGDM problems with unknown weight of the attributes and unknown weight of decision-makers.
4. Techniques to determine unknown weight of attributes and unknown weight of decisions makers are proposed in the study.

The rest of the paper is presented as follows: Section 2 describes some concepts of SVN. In Section 3 we propose a new cross entropy measure between two SVN and investigate its properties. In Section 4, we develop a novel MAGDM strategy based on the proposed NS-cross entropy with SVN information. In Section 5 an illustrative example is solved to demonstrate the applicability and efficiency of the developed MAGDM strategy under SVN environment. In Section 6 we present comparative study and discussion. Section 7 offers conclusions and the future scope of research.

2. Preliminaries

This section presents a short list of mostly known definitions pertaining to this paper.

Definition 1 [23] NS. Let U be a space of points (objects) with a generic element in U denoted by u , i.e., $u \in U$. A neutrosophic set A in U is characterized by truth-membership measure $T_A(u)$, indeterminacy-

membership measure $I_A(u)$ and falsity-membership measure $F_A(u)$, where $T_A(u)$, $I_A(u)$, $F_A(u)$ are the measures from U to $]^{-}0, 1^{+}[$ i.e., $T_A(u), I_A(u), F_A(u):U \rightarrow]^{-}0, 1^{+}[$ NS can be expressed as $A = \{<u; (T_A(u), I_A(u), F_A(u))>: \forall u \in U\}$. Since $T_A(u), I_A(u), F_A(u)$ are the subsets of $]^{-}0, 1^{+}[$ there the sum $(T_A(u) + I_A(u) + F_A(u))$ lies between $]^{-}0$ and 3^{+} .

Example 1. Suppose that $U = \{u_1, u_2, u_3, \dots\}$ be the universal set. Let R_1 be any neutrosophic set in U . Then R_1 expressed as $R_1 = \{<u_1; (0.6, 0.3, 0.4)>: u_1 \in U\}$.

Definition 2 [24] SVNNS. Assume that U be a space of points (objects) with generic elements $u \in U$. A SVNNS H in U is characterized by a truth-membership measure $T_H(u)$, an indeterminacy-membership measure $I_H(u)$, and a falsity-membership measure $F_H(u)$, where $T_H(u), I_H(u), F_H(u) \in [0, 1]$ for each point u in U . Therefore, a SVNNS A can be expressed as $H = \{u, (T_H(u), I_H(u), F_H(u)) \mid \forall u \in U\}$, whereas, the sum of $T_H(u), I_H(u)$ and $F_H(u)$ satisfy the condition $0 \leq T_H(u) + I_H(u) + F_H(u) \leq 3$ and $H(u) = \langle T_H(u), I_H(u), F_H(u) \rangle$ call a single-valued neutrosophic number (SVNN).

Example 2. Suppose that $U = \{u_1, u_2, u_3, \dots\}$ be the universal set. A SVNNS H in U can be expressed as: $H = \{u_1, (0.7, 0.3, 0.5) \mid u_1 \in U\}$ and SVNN presented $H = \langle 0.7, 0.3, 0.5 \rangle$.

Definition 3 [24] Inclusion of SVNNSs. The inclusion of any two SVNNS sets H_1 and H_2 in U is denoted by $H_1 \subseteq H_2$ and defined as follows:

$$H_1 \subseteq H_2, T_{H_1}(u) \leq T_{H_2}(u), I_{H_1}(u) \geq I_{H_2}(u), F_{H_1}(u) \geq F_{H_2}(u) \text{ iff for all } u \in U.$$

Example 3. Let H_1 and H_2 be any two SVNNs in U presented as follows: $H_1 = \langle 0.7, 0.3, 0.5 \rangle$ and $H_2 = \langle 0.8, 0.2, 0.4 \rangle$ for all $u \in U$. Using the property of inclusion of two SVNNs, we conclude that $H_1 \subseteq H_2$.

Definition 4 [24] Equality of two SVNNSs. The equality of any two SVNNS H_1 and H_2 in U denoted by $H_1 = H_2$ and defined as follows:

$$T_{H_1}(u) = T_{H_2}(u), I_{H_1}(u) = I_{H_2}(u) \text{ and } F_{H_1}(u) = F_{H_2}(u) \text{ for all } u \in U.$$

Definition 5 Complement of any SVNNSs. The complement of any SVNNS H in U denoted by H^c and defined as follows:

$$H^c = \{u, 1 - T_H, 1 - I_H, 1 - F_H \mid u \in U\}.$$

Example 4. Let H be any SVNN in U presented as follows: $H = \langle 0.7, 0.3, 0.5 \rangle$. Then compliment of H is obtained as $H^c = \langle 0.3, 0.7, 0.5 \rangle$.

Definition 6 [24] Union. The union of two single-valued neutrosophic sets H_1 and H_2 is a neutrosophic set H_3 (say) written as

$$H_3 = H_1 \cup H_2.$$

$$T_{H_3}(u) = \max \{T_{H_1}(u), T_{H_2}(u)\}, I_{H_3}(u) = \min \{I_{H_1}(u), I_{H_2}(u)\}, F_{H_3}(u) = \min \{F_{H_1}(u), F_{H_2}(u)\}, \forall u \in U.$$

Example 5. Let H_1 and H_2 be two SVNNSs in U presented as follows:

$H_1 = \langle 0.6, 0.3, 0.4 \rangle$ and $H_2 = \langle 0.7, 0.3, 0.6 \rangle$. Then union of them is presented as:

$$H_1 \cup H_2 = \langle 0.7, 0.3, 0.4 \rangle.$$

Definition 7 [24] Intersection. The intersection of two single-valued neutrosophic sets H_1 and H_2 denoted by H_4 and defined as

$$H_4 = H_1 \cap H_2$$

$$T_{H_4}(u) = \min \{ T_{H_1}(u), T_{H_2}(u) \}, I_{H_4}(u) = \max \{ I_{H_1}(u), I_{H_2}(u) \}$$

$$F_{H_4}(u) = \max \{ F_{H_1}(u), F_{H_2}(u) \}, \forall u \in U.$$

Example 6. Let H_1 and H_2 be two SVNNS in U presented as follows:

$$H_1 = \langle (0.6, 0.3, 0.4) \rangle \text{ and } H_2 = \langle (0.7, 0.3, 0.6) \rangle.$$

Then intersection of H_1 and H_2 is presented as follows:

$$H_1 \cap H_2 = \langle (0.6, 0.3, 0.6) \rangle$$

3. NS-Cross Entropy Measure

In this section, we define a new single-valued neutrosophic cross-entropy measure for measuring the deviation of single-valued neutrosophic variables from an a priori one.

Definition 8 NS-cross entropy measure. Let H_1 and H_2 be any two SVNNSs in $U = \{ u_1, u_2, u_3, \dots, u_n \}$. Then, the single-valued cross-entropy of H_1 and H_2 is denoted by $CE_{NS}(H_1, H_2)$ and defined as follows:

$$CE_{NS}(H_1, H_2) = \frac{1}{2} \left\{ \sum_{i=1}^n \left[\frac{2|T_{H_1}(u_i) - T_{H_2}(u_i)|}{\sqrt{1+|T_{H_1}(u_i)|^2} + \sqrt{1+|T_{H_2}(u_i)|^2}} + \frac{2|(1-T_{H_1}(u_i)) - (1-T_{H_2}(u_i))|}{\sqrt{1+|(1-T_{H_1}(u_i))|^2} + \sqrt{1+|(1-T_{H_2}(u_i))|^2}} \right] + \right. \\ \left[\frac{2|I_{H_1}(u_i) - I_{H_2}(u_i)|}{\sqrt{1+|I_{H_1}(u_i)|^2} + \sqrt{1+|I_{H_2}(u_i)|^2}} + \frac{2|(1-I_{H_1}(u_i)) - (1-I_{H_2}(u_i))|}{\sqrt{1+|(1-I_{H_1}(u_i))|^2} + \sqrt{1+|(1-I_{H_2}(u_i))|^2}} \right] + \\ \left. \left[\frac{2|F_{H_1}(u_i) - F_{H_2}(u_i)|}{\sqrt{1+|F_{H_1}(u_i)|^2} + \sqrt{1+|F_{H_2}(u_i)|^2}} + \frac{2|(1-F_{H_1}(u_i)) - (1-F_{H_2}(u_i))|}{\sqrt{1+|(1-F_{H_1}(u_i))|^2} + \sqrt{1+|(1-F_{H_2}(u_i))|^2}} \right] \right\} \quad (1)$$

Example 7. Let H_1 and H_2 be two SVNNSs in U , which are given by $H_1 = \{ u, (0.7, 0.3, 0.4) \mid u \in U \}$ and $H_2 = \{ u, (0.6, 0.4, 0.2) \mid u \in U \}$. Using Equation (1), the cross entropy value of H_1 and H_2 is obtained as $CE_{NS}(H_1, H_2) = 0.707$.

Theorem 1. Single-valued neutrosophic cross entropy $CE_{NS}(H_1, H_2)$ for any two SVNNSs H_1, H_2 , satisfies the following properties:

- i. $CE_{NS}(H_1, H_2) \geq 0$.
- ii. $CE_{NS}(H_1, H_2) = 0$ if and only if $T_{H_1}(u_i) = T_{H_2}(u_i)$, $I_{H_1}(u_i) = I_{H_2}(u_i)$, $F_{H_1}(u_i) = F_{H_2}(u_i)$, $\forall u_i \in U$.
- iii. $CE_{NS}(H_1, H_2) = CE_{NS}(H_1^c, H_2^c)$
- iv. $CE_{NS}(H_1, H_2) = CE_{NS}(H_2, H_1)$

Proof. (i) For all values of $u_i \in U$, $|T_{H_1}(u_i)| \geq 0$, $|T_{H_2}(u_i)| \geq 0$, $|T_{H_1}(u_i) - T_{H_2}(u_i)| \geq 0$, $\sqrt{1+|T_{H_1}(u_i)|^2} \geq 0$, $\sqrt{1+|T_{H_2}(u_i)|^2} \geq 0$, $|(1-T_{H_1}(u_i))| \geq 0$, $|(1-T_{H_2}(u_i))| \geq 0$, $|(1-T_{H_1}(u_i)) - (1-T_{H_2}(u_i))| \geq 0$, $\sqrt{1+|(1-T_{H_1}(u_i))|^2} \geq 0$, $\sqrt{1+|(1-T_{H_2}(u_i))|^2} \geq 0$.

$$\text{Then, } \left[\frac{2|T_{H_1}(u_i) - T_{H_2}(u_i)|}{\sqrt{1+|T_{H_1}(u_i)|^2} + \sqrt{1+|T_{H_2}(u_i)|^2}} + \frac{2|(1-T_{H_1}(u_i)) - (1-T_{H_2}(u_i))|}{\sqrt{1+|(1-T_{H_1}(u_i))|^2} + \sqrt{1+|(1-T_{H_2}(u_i))|^2}} \right] \geq 0.$$

$$\text{Similarly, } \left[\frac{2|I_{H_1}(u_i) - I_{H_2}(u_i)|}{\sqrt{1+|I_{H_1}(u_i)|^2} + \sqrt{1+|I_{H_2}(u_i)|^2}} + \frac{2|(1-I_{H_1}(u_i)) - (1-I_{H_2}(u_i))|}{\sqrt{1+|(1-I_{H_1}(u_i))|^2} + \sqrt{1+|(1-I_{H_2}(u_i))|^2}} \right] \geq 0, \quad \text{and}$$

$$\left[\frac{2|F_{H_1}(u_i) - F_{H_2}(u_i)|}{\sqrt{1+|F_{H_1}(u_i)|^2} + \sqrt{1+|F_{H_2}(u_i)|^2}} + \frac{2|(1-F_{H_1}(u_i)) - (1-F_{H_2}(u_i))|}{\sqrt{1+|(1-F_{H_1}(u_i))|^2} + \sqrt{1+|(1-F_{H_2}(u_i))|^2}} \right] \geq 0.$$

Therefore, $CE_{NS}(H_1, H_2) \geq 0$.

Hence complete the proof.

$$(ii) \quad \left[\frac{2|T_{H_1}(u_i) - T_{H_2}(u_i)|}{\sqrt{1+|T_{H_1}(u_i)|^2} + \sqrt{1+|T_{H_2}(u_i)|^2}} + \frac{2|(1-T_{H_1}(u_i)) - (1-T_{H_2}(u_i))|}{\sqrt{1+|(1-T_{H_1}(u_i))|^2} + \sqrt{1+|(1-T_{H_2}(u_i))|^2}} \right] = 0, \quad \Leftrightarrow T_{H_1}(u_i) = T_{H_2}(u_i),$$

$$\left[\frac{2|I_{H_1}(u_i) - I_{H_2}(u_i)|}{\sqrt{1+|I_{H_1}(u_i)|^2} + \sqrt{1+|I_{H_2}(u_i)|^2}} + \frac{2|(1-I_{H_1}(u_i)) - (1-I_{H_2}(u_i))|}{\sqrt{1+|(1-I_{H_1}(u_i))|^2} + \sqrt{1+|(1-I_{H_2}(u_i))|^2}} \right] = 0 \quad \Leftrightarrow I_{H_1}(u_i) = I_{H_2}(u_i), \quad \text{and,}$$

$$\left[\frac{2|F_{H_1}(u_i) - F_{H_2}(u_i)|}{\sqrt{1+|F_{H_1}(u_i)|^2} + \sqrt{1+|F_{H_2}(u_i)|^2}} + \frac{2|(1-F_{H_1}(u_i)) - (1-F_{H_2}(u_i))|}{\sqrt{1+|(1-F_{H_1}(u_i))|^2} + \sqrt{1+|(1-F_{H_2}(u_i))|^2}} \right] = 0, \quad \Leftrightarrow F_{H_1}(u_i) = F_{H_2}(u_i)$$

Therefore, $CE_{NS}(H_1, H_2) = 0$, iff $T_{H_1}(u_i) = T_{H_2}(u_i)$, $I_{H_1}(u_i) = I_{H_2}(u_i)$, $F_{H_1}(u_i) = F_{H_2}(u_i)$, $\forall u_i \in U$.

Hence complete the proof.

(iii) Using Definition 5, we obtain the following expression

$$\begin{aligned} CE_{NS}(H_1^c, H_2^c) &= \frac{1}{2} \left\{ \sum_{i=1}^n \left[\frac{2|(1-T_{H_1}(u_i)) - (1-T_{H_2}(u_i))|}{\sqrt{1+|(1-T_{H_1}(u_i))|^2} + \sqrt{1+|(1-T_{H_2}(u_i))|^2}} + \frac{2|T_{H_1}(u_i) - T_{H_2}(u_i)|}{\sqrt{1+|T_{H_1}(u_i)|^2} + \sqrt{1+|T_{H_2}(u_i)|^2}} \right] + \right. \\ &\quad \left[\frac{2|(1-I_{H_1}(u_i)) - (1-I_{H_2}(u_i))|}{\sqrt{1+|(1-I_{H_1}(u_i))|^2} + \sqrt{1+|(1-I_{H_2}(u_i))|^2}} + \frac{2|I_{H_1}(u_i) - I_{H_2}(u_i)|}{\sqrt{1+|I_{H_1}(u_i)|^2} + \sqrt{1+|I_{H_2}(u_i)|^2}} \right] + \\ &\quad \left. \left[\frac{2|(1-F_{H_1}(u_i)) - (1-F_{H_2}(u_i))|}{\sqrt{1+|(1-F_{H_1}(u_i))|^2} + \sqrt{1+|(1-F_{H_2}(u_i))|^2}} + \frac{2|F_{H_1}(u_i) - F_{H_2}(u_i)|}{\sqrt{1+|F_{H_1}(u_i)|^2} + \sqrt{1+|F_{H_2}(u_i)|^2}} \right] \right\} \\ &= \frac{1}{2} \left\{ \sum_{i=1}^n \left[\frac{2|T_{H_1}(u_i) - T_{H_2}(u_i)|}{\sqrt{1+|T_{H_1}(u_i)|^2} + \sqrt{1+|T_{H_2}(u_i)|^2}} + \frac{2|(1-T_{H_1}(u_i)) - (1-T_{H_2}(u_i))|}{\sqrt{1+|(1-T_{H_1}(u_i))|^2} + \sqrt{1+|(1-T_{H_2}(u_i))|^2}} \right] + \right. \\ &\quad \left[\frac{2|I_{H_1}(u_i) - I_{H_2}(u_i)|}{\sqrt{1+|I_{H_1}(u_i)|^2} + \sqrt{1+|I_{H_2}(u_i)|^2}} + \frac{2|(1-I_{H_1}(u_i)) - (1-I_{H_2}(u_i))|}{\sqrt{1+|(1-I_{H_1}(u_i))|^2} + \sqrt{1+|(1-I_{H_2}(u_i))|^2}} \right] + \\ &\quad \left. \left[\frac{2|F_{H_1}(u_i) - F_{H_2}(u_i)|}{\sqrt{1+|F_{H_1}(u_i)|^2} + \sqrt{1+|F_{H_2}(u_i)|^2}} + \frac{2|(1-F_{H_1}(u_i)) - (1-F_{H_2}(u_i))|}{\sqrt{1+|(1-F_{H_1}(u_i))|^2} + \sqrt{1+|(1-F_{H_2}(u_i))|^2}} \right] \right\} = CE_{SN}(H_1, H_2) \end{aligned}$$

Therefore, $CE_{NS}(H_1, H_2) = CE_{NS}(H_1^c, H_2^c)$.

Hence complete the proof.

$$\begin{aligned}
 \text{(iv)} \quad \text{Since,} \quad & |T_{H_1}(u_i) - T_{H_2}(u_i)| = |T_{H_2}(u_i) - T_{H_1}(u_i)|, \quad |I_{H_1}(u_i) - I_{H_2}(u_i)| = |I_{H_2}(u_i) - I_{H_1}(u_i)|, \\
 & |F_{H_1}(u_i) - F_{H_2}(u_i)| = |F_{H_2}(u_i) - F_{H_1}(u_i)|, \quad |(1 - T_{H_1}(u_i)) - (1 - T_{H_2}(u_i))| = |(1 - T_{H_2}(u_i)) - (1 - T_{H_1}(u_i))|, \\
 & |(1 - I_{H_1}(u_i)) - (1 - I_{H_2}(u_i))| = |(1 - I_{H_2}(u_i)) - (1 - I_{H_1}(u_i))|, \quad |(1 - F_{H_1}(u_i)) - (1 - F_{H_2}(u_i))| = |(1 - F_{H_2}(u_i)) - (1 - F_{H_1}(u_i))|, \text{ then,} \\
 & \sqrt{1 + |T_{H_1}(u_i)|^2} + \sqrt{1 + |T_{H_2}(u_i)|^2} = \sqrt{1 + |T_{H_2}(u_i)|^2} + \sqrt{1 + |T_{H_1}(u_i)|^2}, \\
 & \sqrt{1 + |I_{H_1}(u_i)|^2} + \sqrt{1 + |I_{H_2}(u_i)|^2} = \sqrt{1 + |I_{H_2}(u_i)|^2} + \sqrt{1 + |I_{H_1}(u_i)|^2}, \\
 & \sqrt{1 + |F_{H_1}(u_i)|^2} + \sqrt{1 + |F_{H_2}(u_i)|^2} = \sqrt{1 + |F_{H_2}(u_i)|^2} + \sqrt{1 + |F_{H_1}(u_i)|^2}, \\
 & \sqrt{1 + |(1 - T_{H_1}(u_i))|^2} + \sqrt{1 + |(1 - T_{H_2}(u_i))|^2} = \sqrt{1 + |(1 - T_{H_2}(u_i))|^2} + \sqrt{1 + |(1 - T_{H_1}(u_i))|^2}, \\
 & \sqrt{1 + |(1 - I_{H_1}(u_i))|^2} + \sqrt{1 + |(1 - I_{H_2}(u_i))|^2} = \sqrt{1 + |(1 - I_{H_2}(u_i))|^2} + \sqrt{1 + |(1 - I_{H_1}(u_i))|^2}, \\
 & \sqrt{1 + |(1 - F_{H_1}(u_i))|^2} + \sqrt{1 + |(1 - F_{H_2}(u_i))|^2} = \sqrt{1 + |(1 - F_{H_2}(u_i))|^2} + \sqrt{1 + |(1 - F_{H_1}(u_i))|^2}, \quad \forall u_i \in U.
 \end{aligned}$$

Therefore, $CE_{NS}(H_1, H_2) = CE_{NS}(H_2, H_1)$.

Hence complete the proof. $\square$

Definition 9 Weighted NS-cross entropy measure. We consider the weight w_i ($i = 1, 2, \dots, n$) for the element u_i ($i = 1, 2, \dots, n$) with the conditions $w_i \in [0, 1]$ and $\sum_{i=1}^n w_i = 1$.

Then the weighted cross entropy between SVNss H_1 and H_2 can be defined as follows:

$$\begin{aligned}
 CE_{NS}^w(H_1, H_2) = & \frac{1}{2} \left\langle \sum_{i=1}^n w_i \left\{ \left[\frac{2|T_{H_1}(u_i) - T_{H_2}(u_i)|}{\sqrt{1 + |T_{H_1}(u_i)|^2} + \sqrt{1 + |T_{H_2}(u_i)|^2}} + \frac{2|(1 - T_{H_1}(u_i)) - (1 - T_{H_2}(u_i))|}{\sqrt{1 + |(1 - T_{H_1}(u_i))|^2} + \sqrt{1 + |(1 - T_{H_2}(u_i))|^2}} \right] + \right. \right. \\
 & \left. \left[\frac{2|I_{H_1}(u_i) - I_{H_2}(u_i)|}{\sqrt{1 + |I_{H_1}(u_i)|^2} + \sqrt{1 + |I_{H_2}(u_i)|^2}} + \frac{2|(1 - I_{H_1}(u_i)) - (1 - I_{H_2}(u_i))|}{\sqrt{1 + |(1 - I_{H_1}(u_i))|^2} + \sqrt{1 + |(1 - I_{H_2}(u_i))|^2}} \right] + \right. \\
 & \left. \left[\frac{2|F_{H_1}(u_i) - F_{H_2}(u_i)|}{\sqrt{1 + |F_{H_1}(u_i)|^2} + \sqrt{1 + |F_{H_2}(u_i)|^2}} + \frac{2|(1 - F_{H_1}(u_i)) - (1 - F_{H_2}(u_i))|}{\sqrt{1 + |(1 - F_{H_1}(u_i))|^2} + \sqrt{1 + |(1 - F_{H_2}(u_i))|^2}} \right] \right\} \right\rangle \quad (2)
 \end{aligned}$$

Theorem 2. Single-valued neutrosophic weighted NS-cross-entropy (defined in Equation (2)) satisfies the following properties:

- i. $CE_{NS}^w(H_1, H_2) \geq 0$.
- ii. $CE_{NS}^w(H_1, H_2) = 0$, if and only if $T_{H_1}(u_i) = T_{H_2}(u_i)$, $I_{H_1}(u_i) = I_{H_2}(u_i)$, $F_{H_1}(u_i) = F_{H_2}(u_i)$, $\forall u_i \in U$.
- iii. $CE_{NS}^w(H_1, H_2) = CE_{NS}^w(H_1^c, H_2^c)$
- iv. $CE_{NS}^w(H_1, H_2) = CE_{NS}^w(H_2, H_1)$

Proof. (i). For all values of $u_i \in U$, $|T_{H_1}(u_i)| \geq 0$, $|T_{H_2}(u_i)| \geq 0$, $|T_{H_1}(u_i) - T_{H_2}(u_i)| \geq 0$, $\sqrt{1 + |T_{H_1}(u_i)|^2} \geq 0$, $\sqrt{1 + |T_{H_2}(u_i)|^2} \geq 0$, $|(1 - T_{H_1}(u_i))| \geq 0$, $|(1 - T_{H_2}(u_i))| \geq 0$, $|(1 - T_{H_1}(u_i)) - (1 - T_{H_2}(u_i))| \geq 0$, $\sqrt{1 + |(1 - T_{H_1}(u_i))|^2} \geq 0$, $\sqrt{1 + |(1 - T_{H_2}(u_i))|^2} \geq 0$, then,

$$\left[\frac{2|T_{H_1}(u_i) - T_{H_2}(u_i)|}{\sqrt{1 + |T_{H_1}(u_i)|^2} + \sqrt{1 + |T_{H_2}(u_i)|^2}} + \frac{2|(1 - T_{H_1}(u_i)) - (1 - T_{H_2}(u_i))|}{\sqrt{1 + |(1 - T_{H_1}(u_i))|^2} + \sqrt{1 + |(1 - T_{H_2}(u_i))|^2}} \right] \geq 0.$$

Similarly,
$$\left[\frac{2|I_{H_1}(u_i) - I_{H_2}(u_i)|}{\sqrt{1+|I_{H_1}(u_i)|^2} + \sqrt{1+|I_{H_2}(u_i)|^2}} + \frac{2|(1-I_{H_1}(u_i)) - (1-I_{H_2}(u_i))|}{\sqrt{1+|(1-I_{H_1}(u_i))|^2} + \sqrt{1+|(1-I_{H_2}(u_i))|^2}} \right] \geq 0, \text{ and}$$

$$\left[\frac{2|F_{H_1}(u_i) - F_{H_2}(u_i)|}{\sqrt{1+|F_{H_1}(u_i)|^2} + \sqrt{1+|F_{H_2}(u_i)|^2}} + \frac{2|(1-F_{H_1}(u_i)) - (1-F_{H_2}(u_i))|}{\sqrt{1+|(1-F_{H_1}(u_i))|^2} + \sqrt{1+|(1-F_{H_2}(u_i))|^2}} \right] \geq 0.$$

Since $w_i \in [0, 1]$ and $\sum_{i=1}^n w_i = 1$, therefore, $CE_{NS}^w(H_1, H_2) \geq 0$.

Hence complete the proof.

(ii) Since,
$$\left[\frac{2|T_{H_1}(u_i) - T_{H_2}(u_i)|}{\sqrt{1+|T_{H_1}(u_i)|^2} + \sqrt{1+|T_{H_2}(u_i)|^2}} + \frac{2|(1-T_{H_1}(u_i)) - (1-T_{H_2}(u_i))|}{\sqrt{1+|(1-T_{H_1}(u_i))|^2} + \sqrt{1+|(1-T_{H_2}(u_i))|^2}} \right] = 0, \Leftrightarrow T_{H_1}(u_i) = T_{H_2}(u_i),$$

$$\left[\frac{2|I_{H_1}(u_i) - I_{H_2}(u_i)|}{\sqrt{1+|I_{H_1}(u_i)|^2} + \sqrt{1+|I_{H_2}(u_i)|^2}} + \frac{2|(1-I_{H_1}(u_i)) - (1-I_{H_2}(u_i))|}{\sqrt{1+|(1-I_{H_1}(u_i))|^2} + \sqrt{1+|(1-I_{H_2}(u_i))|^2}} \right] = 0, \Leftrightarrow I_{H_1}(u_i) = I_{H_2}(u_i),$$

$$\left[\frac{2|F_{H_1}(u_i) - F_{H_2}(u_i)|}{\sqrt{1+|F_{H_1}(u_i)|^2} + \sqrt{1+|F_{H_2}(u_i)|^2}} + \frac{2|(1-F_{H_1}(u_i)) - (1-F_{H_2}(u_i))|}{\sqrt{1+|(1-F_{H_1}(u_i))|^2} + \sqrt{1+|(1-F_{H_2}(u_i))|^2}} \right] = 0, \Leftrightarrow F_{H_1}(u_i) = F_{H_2}(u_i) \text{ and}$$

$w_i \in [0, 1], \sum_{i=1}^n w_i = 1, w_i \geq 0$. Therefore, $CE_{NS}^w(H_1, H_2) = 0$ iff $T_{H_1}(u_i) = T_{H_2}(u_i), I_{H_1}(u_i) = I_{H_2}(u_i),$

$F_{H_1}(u_i) = F_{H_2}(u_i), \forall u_i \in U$.

Hence complete the proof.

(iii) Using Definition 5, we obtain the following expression

$$\begin{aligned} CE_{NS}^w(H_1^c, H_2^c) &= \frac{1}{2} \left\{ \sum_{i=1}^n w_i \left\langle \left[\frac{2|(1-T_{H_1}(u_i)) - (1-T_{H_2}(u_i))|}{\sqrt{1+|(1-T_{H_1}(u_i))|^2} + \sqrt{1+|(1-T_{H_2}(u_i))|^2}} + \frac{2|T_{H_1}(u_i) - T_{H_2}(u_i)|}{\sqrt{1+|T_{H_1}(u_i)|^2} + \sqrt{1+|T_{H_2}(u_i)|^2}} \right] + \right. \right. \\ &\quad \left[\frac{2|(1-I_{H_1}(u_i)) - (1-I_{H_2}(u_i))|}{\sqrt{1+|(1-I_{H_1}(u_i))|^2} + \sqrt{1+|(1-I_{H_2}(u_i))|^2}} + \frac{2|I_{H_1}(u_i) - I_{H_2}(u_i)|}{\sqrt{1+|I_{H_1}(u_i)|^2} + \sqrt{1+|I_{H_2}(u_i)|^2}} \right] + \\ &\quad \left. \left[\frac{2|(1-F_{H_1}(u_i)) - (1-F_{H_2}(u_i))|}{\sqrt{1+|(1-F_{H_1}(u_i))|^2} + \sqrt{1+|(1-F_{H_2}(u_i))|^2}} + \frac{2|F_{H_1}(u_i) - F_{H_2}(u_i)|}{\sqrt{1+|F_{H_1}(u_i)|^2} + \sqrt{1+|F_{H_2}(u_i)|^2}} \right] \right\rangle \Bigg\} \\ &= \frac{1}{2} \left\{ \sum_{i=1}^n w_i \left\langle \left[\frac{2|T_{H_1}(u_i) - T_{H_2}(u_i)|}{\sqrt{1+|T_{H_1}(u_i)|^2} + \sqrt{1+|T_{H_2}(u_i)|^2}} + \frac{2|(1-T_{H_1}(u_i)) - (1-T_{H_2}(u_i))|}{\sqrt{1+|(1-T_{H_1}(u_i))|^2} + \sqrt{1+|(1-T_{H_2}(u_i))|^2}} \right] + \right. \right. \\ &\quad \left[\frac{2|I_{H_1}(u_i) - I_{H_2}(u_i)|}{\sqrt{1+|I_{H_1}(u_i)|^2} + \sqrt{1+|I_{H_2}(u_i)|^2}} + \frac{2|(1-I_{H_1}(u_i)) - (1-I_{H_2}(u_i))|}{\sqrt{1+|(1-I_{H_1}(u_i))|^2} + \sqrt{1+|(1-I_{H_2}(u_i))|^2}} \right] + \\ &\quad \left. \left[\frac{2|F_{H_1}(u_i) - F_{H_2}(u_i)|}{\sqrt{1+|F_{H_1}(u_i)|^2} + \sqrt{1+|F_{H_2}(u_i)|^2}} + \frac{2|(1-F_{H_1}(u_i)) - (1-F_{H_2}(u_i))|}{\sqrt{1+|(1-F_{H_1}(u_i))|^2} + \sqrt{1+|(1-F_{H_2}(u_i))|^2}} \right] \right\rangle \Bigg\} = CE_{NS}^w(H_1, H_2) \end{aligned}$$

Therefore, $CE_{NS}^w(H_1, H_2) = CE_{NS}^w(H_1^c, H_2^c)$.

Hence complete the proof.

$$(iv) \quad \text{Since} \quad |T_{H_1}(u_i) - T_{H_2}(u_i)| = |T_{H_2}(u_i) - T_{H_1}(u_i)|, \quad |I_{H_1}(u_i) - I_{H_2}(u_i)| = |I_{H_2}(u_i) - I_{H_1}(u_i)|, \\ |F_{H_1}(u_i) - F_{H_2}(u_i)| = |F_{H_2}(u_i) - F_{H_1}(u_i)|, \quad |(1 - T_{H_1}(u_i)) - (1 - T_{H_2}(u_i))| = |(1 - T_{H_2}(u_i)) - (1 - T_{H_1}(u_i))|, \\ |(1 - I_{H_1}(u_i)) - (1 - I_{H_2}(u_i))| = |(1 - I_{H_2}(u_i)) - (1 - I_{H_1}(u_i))|, \quad |(1 - F_{H_1}(u_i)) - (1 - F_{H_2}(u_i))| = |(1 - F_{H_2}(u_i)) - (1 - F_{H_1}(u_i))|,$$

$$\text{we obtain} \quad \sqrt{1 + |T_{H_1}(u_i)|^2} + \sqrt{1 + |T_{H_2}(u_i)|^2} = \sqrt{1 + |T_{H_2}(u_i)|^2} + \sqrt{1 + |T_{H_1}(u_i)|^2}, \\ \sqrt{1 + |I_{H_1}(u_i)|^2} + \sqrt{1 + |I_{H_2}(u_i)|^2} = \sqrt{1 + |I_{H_2}(u_i)|^2} + \sqrt{1 + |I_{H_1}(u_i)|^2}, \\ \sqrt{1 + |F_{H_1}(u_i)|^2} + \sqrt{1 + |F_{H_2}(u_i)|^2} = \sqrt{1 + |F_{H_2}(u_i)|^2} + \sqrt{1 + |F_{H_1}(u_i)|^2}, \\ \sqrt{1 + |(1 - T_{H_1}(u_i))|^2} + \sqrt{1 + |(1 - T_{H_2}(u_i))|^2} = \sqrt{1 + |(1 - T_{H_2}(u_i))|^2} + \sqrt{1 + |(1 - T_{H_1}(u_i))|^2}, \\ \sqrt{1 + |(1 - I_{H_1}(u_i))|^2} + \sqrt{1 + |(1 - I_{H_2}(u_i))|^2} = \sqrt{1 + |(1 - I_{H_2}(u_i))|^2} + \sqrt{1 + |(1 - I_{H_1}(u_i))|^2}, \\ \sqrt{1 + |(1 - F_{H_1}(u_i))|^2} + \sqrt{1 + |(1 - F_{H_2}(u_i))|^2} = \sqrt{1 + |(1 - F_{H_2}(u_i))|^2} + \sqrt{1 + |(1 - F_{H_1}(u_i))|^2}, \quad \forall u_i \in U \quad \text{and}$$

$$w_i \in [0, 1], \sum_{i=1}^n w_i = 1.$$

Therefore, $CE_{NS}^w(H_1, H_2) = CE_{NS}^w(H_2, H_1)$.

Hence complete the proof. $\square$

4. MAGDM Strategy Using Proposed Ns-Cross Entropy Measure under SVNS Environment

In this section, we develop a new MAGDM strategy using the proposed NS-cross entropy measure.

Description of the MAGDM Problem

Assume that $A = \{A_1, A_2, A_3, \dots, A_m\}$ and $G = \{G_1, G_2, G_3, \dots, G_n\}$ be the discrete set of alternatives and attributes respectively and $W = \{w_1, w_2, w_3, \dots, w_n\}$ be the weight vector of attributes G_j ($j = 1, 2, 3, \dots, n$), where $w_j \geq 0$ and $\sum_{j=1}^n w_j = 1$. Assume that $E = \{E_1, E_2, E_3, \dots, E_\rho\}$ be the set of decision-makers who are employed to evaluate the alternatives. The weight vector of the decision-makers E_k ($k = 1, 2, 3, \dots, \rho$) is $\lambda = \{\lambda_1, \lambda_2, \lambda_3, \dots, \lambda_\rho\}$ (where, $\lambda_k \geq 0$ and $\sum_{k=1}^\rho \lambda_k = 1$), which can be determined according to the decision-makers' expertise, judgment quality and domain knowledge.

Now, we describe the steps of the proposed MAGDM strategy (see Figure 1) using NS-cross entropy measure.

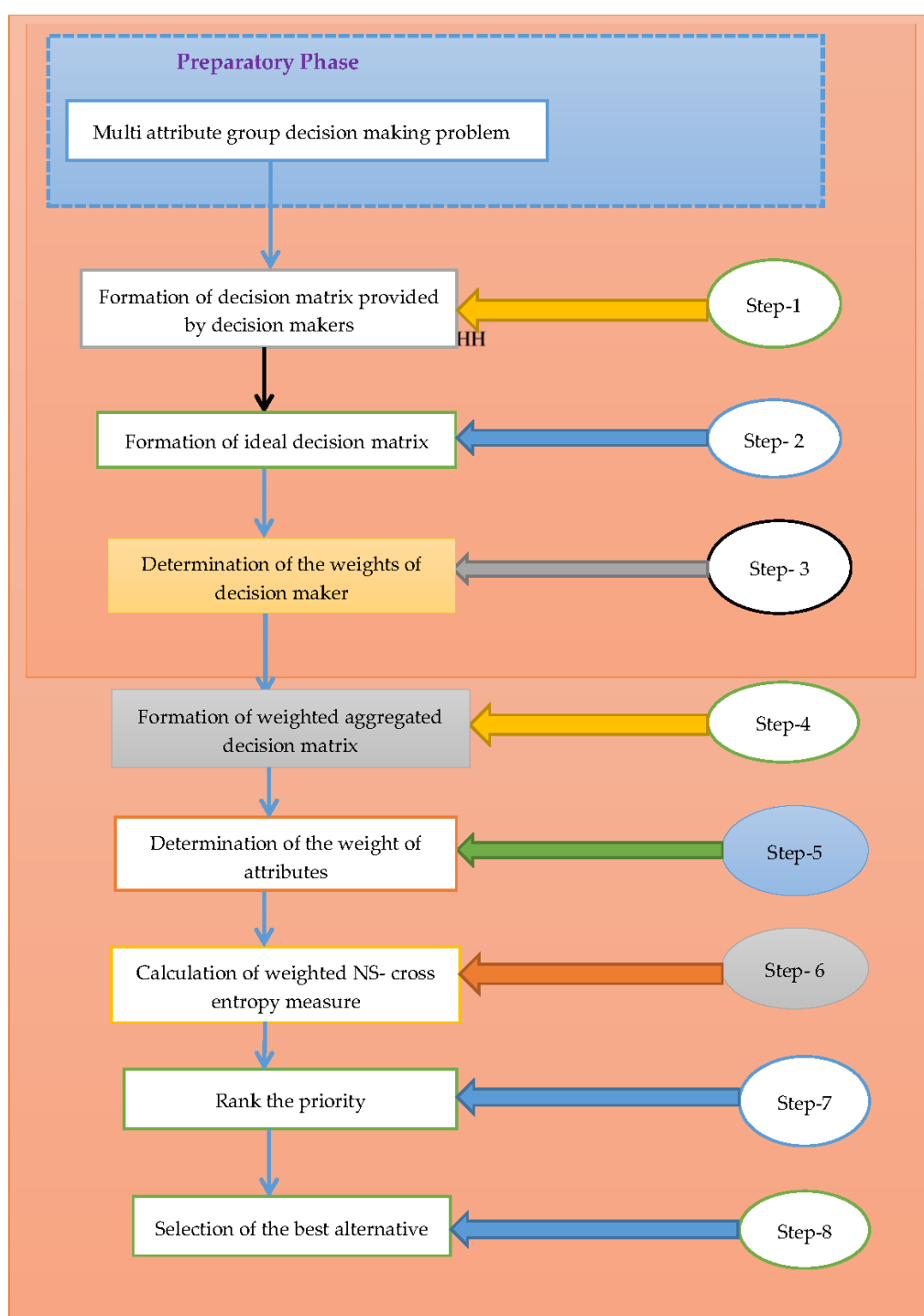


Figure 1. Decision-making procedure of the proposed MAGDM strategy.

MAGDM Strategy Using Ns-Cross Entropy Measure

Step 1. Formulate the decision matrices

For MAGDM with SVNss information, the rating values of the alternatives A_i ($i=1,2,3,\dots,m$) based on the attribute G_j ($j=1,2,3,\dots,n$) provided by the k -th decision-maker can be expressed in terms of SVN as $a_{ij}^k = \langle T_{ij}^k, I_{ij}^k, F_{ij}^k \rangle$ ($i = 1, 2, 3, \dots, m; j = 1, 2, 3, \dots, n; k = 1, 2, 3, \dots, \rho$). We present these rating values of alternatives provided by the decision-makers in matrix form as follows:

$$M^k = \begin{pmatrix} & G_1 & G_2 & \dots & G_n \\ A_1 & a_{11}^k & a_{12}^k & \dots & a_{1n}^k \\ A_2 & a_{21}^k & a_{22}^k & \dots & a_{2n}^k \\ \vdots & \vdots & \vdots & \dots & \vdots \\ A_m & a_{m1}^k & a_{m2}^k & \dots & a_{mn}^k \end{pmatrix} \quad (3)$$

Step 2. Formulate priori/ideal decision matrix

In the MAGDM, the a priori decision matrix has been used to select the best alternatives among the set of collected feasible alternatives. In the decision-making situation, we use the following decision matrix as a priori decision matrix.

$$P = \begin{pmatrix} & G_1 & G_2 & \dots & G_n \\ A_1 & a_{11}^* & a_{12}^* & \dots & a_{1n}^* \\ A_2 & a_{21}^* & a_{22}^* & \dots & a_{2n}^* \\ \vdots & \vdots & \vdots & \dots & \vdots \\ A_m & a_{m1}^* & a_{m2}^* & \dots & a_{mn}^* \end{pmatrix} \quad (4)$$

where, $a_{ij}^* = \langle \max_i(T_{ij}^k), \min_i(I_{ij}^k), \min_i(F_{ij}^k) \rangle$ corresponding to benefit attributes and $a_{ij}^* = \langle \min_i(T_{ij}^k), \max_i(I_{ij}^k), \max_i(F_{ij}^k) \rangle$ corresponding to cost attributes, and $(i = 1, 2, 3, \dots, m; j = 1, 2, 3, \dots, n; k = 1, 2, 3, \dots, \rho)$.

Step 3. Determinate the weights of decision-makers

To find the decision-makers' weights we introduce a model based on the NS-cross entropy measure. The collective NS-cross entropy measure between M^k and P (Ideal matrix) is defined as follows:

$$CE_{NS}^c(M^k, P) = \frac{1}{m} \sum_{i=1}^m CE_{NS}((M^k(A_i), P(A_i))) \quad (5)$$

where, $CE_{NS}((M^k(A_i), P(A_i))) = \sum_{j=1}^n CE_{NS}(M^k(A_i(G_j)), P(A_i(G_j)))$.

Thus, we can introduce the following weight model of the decision-makers:

$$\lambda_k = \frac{(1 \div CE_{NS}^c(M^k, P))}{\sum_{k=1}^{\rho} (1 \div CE_{NS}^c(M^k, P))} \quad (6)$$

where, $0 \leq \lambda_k \leq 1$ and $\sum_{k=1}^{\rho} \lambda_k = 1$ for $k = 1, 2, 3, \dots, \rho$.

Step 4. Formulate the weighted aggregated decision matrix

For obtaining one group decision, we aggregate all the individual decision matrices (M^k) to an aggregated decision matrix (M) using single valued neutrosophic weighted averaging (SVNWA) operator ([51]) as follows:

$$a_{ij} = SVNWA_{\lambda}(a_{ij}^1, a_{ij}^2, a_{ij}^3, \dots, a_{ij}^{\rho}) = (\lambda_1 a_{ij}^1 \oplus \lambda_2 a_{ij}^2 \oplus \lambda_3 a_{ij}^3 \oplus \dots \oplus \lambda_{\rho} a_{ij}^{\rho}) = < 1 - \prod_{k=1}^{\rho} (1 - T_{ij}^k)^{\lambda_k}, \prod_{k=1}^{\rho} (I_{ij}^k)^{\lambda_k}, \prod_{k=1}^{\rho} (F_{ij}^k)^{\lambda_k} > \quad (7)$$

Therefore, the aggregated decision matrix is defined as follows:

$$M = \begin{pmatrix} & G_1 & G_2 & \dots & G_n \\ A_1 & a_{11} & a_{12} & \dots & a_{1n} \\ A_2 & a_{21} & a_{22} & & a_{2n} \\ \vdots & \vdots & \vdots & \dots & \vdots \\ A_m & a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix} \quad (8)$$

where, $a_{ij} = \langle T_{ij}, I_{ij}, F_{ij} \rangle$, ($i = 1, 2, 3, \dots, m$; $j = 1, 2, 3, \dots, n$; $k = 1, 2, 3, \dots, \rho$).

Step 5. Determinate the weight of attributes

To find the attributes weight we introduce a model based on the NS-cross entropy measure. The collective NS-cross entropy measure between M (Weighted aggregated decision matrix) and P (Ideal matrix) for each attribute is defined by

$$CE_{NS}^j(M, P) = \frac{1}{m} \sum_{i=1}^m CE_{NS}((M(A_i(G_j)), P(A_i(G_j))) \quad (9)$$

where, $i = 1, 2, 3, \dots, m$; $j = 1, 2, 3, \dots, n$.

Thus, we defined a weight model for attributes as follows:

$$w_j = \frac{(1 \div CE_{NS}^j(M, P))}{\sum_{j=1}^n (1 \div CE_{NS}^j(M, P))} \quad (10)$$

where, $0 \leq w_j \leq 1$ and $\sum_{j=1}^n w_j = 1$ for $j = 1, 2, 3, \dots, n$.

Step 6. Calculate the weighted NS-cross entropy measure

Using Equation (2), we calculate weighted cross entropy value between weighted aggregated matrix and priori matrix. The cross entropy values can be presented in matrix form as follows:

$${}^{NS}M_{CE}^w = \begin{pmatrix} CE_{NS}^w(A_1) \\ CE_{NS}^w(A_2) \\ \dots\dots\dots \\ CE_{NS}^w(A_m) \end{pmatrix} \quad (11)$$

Step 7. Rank the priority

Smaller value of the cross entropy reflects that an alternative is closer to the ideal alternative. Therefore, the preference priority order of all the alternatives can be determined according to the increasing order of the cross entropy values $CE_{NS}^w(A_i)$ ($i = 1, 2, 3, \dots, m$). Smallest cross entropy value indicates the best alternative and greatest cross entropy value indicates the worst alternative.

Step 8. Select the best alternative

From the preference rank order (from step 7), we select the best alternative.

5. Illustrative Example

In this section, we solve an illustrative example adapted from [12] of MAGDM problems to reflect the feasibility, applicability and efficiency of the proposed strategy under the SVNS environment.

Now, we use the example [12] for cultivation and analysis. A venture capital firm intends to make evaluation and selection of five enterprises with the investment potential:

- (1) Automobile company (A_1)
- (2) Military manufacturing enterprise (A_2)
- (3) TV media company (A_3)

- (4) Food enterprises (A_4)
- (5) Computer software company (A_5)

On the basis of four attributes namely:

- (1) Social and political factor (G_1)
- (2) The environmental factor (G_2)
- (3) Investment risk factor (G_3)
- (4) The enterprise growth factor (G_4).

The investment firm makes a panel of three decision-makers.

The steps of decision-making strategy (4.1.1.) to rank alternatives are presented as follows:

Step: 1. Formulate the decision matrices

We represent the rating values of alternatives A_i ($i = 1, 2, 3, 4, 5$) with respects to the attributes G_j ($j = 1, 2, 3, 4$) provided by the decision-makers E_k ($k = 1, 2, 3$) in matrix form as follows:

Decision matrix for E_1 decision-maker

$$M^1 = \begin{pmatrix} & G_1 & G_2 & G_3 & G_4 \\ A_1 & (0.9, 0.5, 0.4) & (0.7, 0.4, 0.4) & (0.7, 0.3, 0.4) & (0.5, 0.4, 0.9) \\ A_2 & (0.7, 0.2, 0.3) & (0.8, 0.4, 0.3) & (0.9, 0.6, 0.5) & (0.9, 0.1, 0.3) \\ A_3 & (0.8, 0.4, 0.4) & (0.7, 0.4, 0.2) & (0.9, 0.7, 0.6) & (0.7, 0.3, 0.3) \\ A_4 & (0.5, 0.8, 0.7) & (0.6, 0.3, 0.4) & (0.7, 0.2, 0.5) & (0.5, 0.4, 0.7) \\ A_5 & (0.8, 0.4, 0.3) & (0.5, 0.4, 0.5) & (0.6, 0.4, 0.4) & (0.9, 0.7, 0.5) \end{pmatrix} \quad (12)$$

Decision matrix for E_2 decision-maker

$$M^2 = \begin{pmatrix} & G_1 & G_2 & G_3 & G_4 \\ A_1 & (0.7, 0.2, 0.3) & (0.5, 0.4, 0.5) & (0.9, 0.4, 0.5) & (0.6, 0.5, 0.3) \\ A_2 & (0.7, 0.4, 0.4) & (0.7, 0.3, 0.4) & (0.7, 0.3, 0.4) & (0.6, 0.4, 0.3) \\ A_3 & (0.6, 0.4, 0.4) & (0.5, 0.3, 0.5) & (0.9, 0.5, 0.4) & (0.6, 0.5, 0.6) \\ A_4 & (0.7, 0.5, 0.3) & (0.6, 0.3, 0.6) & (0.7, 0.4, 0.4) & (0.8, 0.5, 0.4) \\ A_5 & (0.9, 0.4, 0.3) & (0.6, 0.4, 0.5) & (0.8, 0.5, 0.6) & (0.5, 0.4, 0.5) \end{pmatrix} \quad (83)$$

Decision matrix for E_3 decision-maker

$$M^3 = \begin{pmatrix} & G_1 & G_2 & G_3 & G_4 \\ A_1 & (0.7, 0.2, 0.5) & (0.6, 0.4, 0.4) & (0.7, 0.4, 0.5) & (0.9, 0.4, 0.3) \\ A_2 & (0.6, 0.5, 0.5) & (0.9, 0.3, 0.4) & (0.7, 0.4, 0.3) & (0.8, 0.4, 0.5) \\ A_3 & (0.8, 0.3, 0.5) & (0.9, 0.3, 0.4) & (0.8, 0.3, 0.4) & (0.7, 0.3, 0.4) \\ A_4 & (0.9, 0.3, 0.4) & (0.6, 0.3, 0.4) & (0.5, 0.2, 0.4) & (0.7, 0.3, 0.5) \\ A_5 & (0.8, 0.3, 0.3) & (0.6, 0.4, 0.3) & (0.6, 0.3, 0.4) & (0.7, 0.3, 0.5) \end{pmatrix} \quad (14)$$

Step: 2. Formulate priori/ideal decision matrix

A priori/ideal decision matrix Please provide a sharper picture

$$P = \begin{pmatrix} & G_1 & G_2 & G_3 & G_4 \\ A_1 & (0.9, 0.2, 0.3) & (0.7, 0.4, 0.4) & (0.9, 0.3, 0.4) & (0.9, 0.4, 0.3) \\ A_2 & (0.7, 0.2, 0.3) & (0.9, 0.3, 0.3) & (0.9, 0.3, 0.3) & (0.9, 0.1, 0.3) \\ A_3 & (0.8, 0.3, 0.4) & (0.9, 0.3, 0.2) & (0.9, 0.3, 0.4) & (0.7, 0.3, 0.3) \\ A_4 & (0.9, 0.3, 0.3) & (0.6, 0.3, 0.4) & (0.7, 0.2, 0.4) & (0.7, 0.3, 0.4) \\ A_5 & (0.9, 0.3, 0.3) & (0.6, 0.4, 0.3) & (0.8, 0.3, 0.4) & (0.9, 0.3, 0.5) \end{pmatrix} \quad (95)$$

Step: 3. Determine the weight of decision-makers

By using Equations (5) and (6), we determine the weights of the three decision-makers as follows:

$$\lambda_1 = \frac{(1 \div 0.9)}{3.37} \approx 0.33, \lambda_2 = \frac{(1 \div 1.2)}{3.37} \approx 0.25, \lambda_3 = \frac{(1 \div 0.7)}{3.37} \approx 0.42.$$

Step: 4. Formulate the weighted aggregated decision matrix

Using Equation (7) the weighted aggregated decision matrix is presented as follows:

Weighted Aggregated decision matrix

$$M = \begin{pmatrix} & G_1 & G_2 & G_3 & G_4 \\ A_1 & (0.8, 0.3, 0.4) & (0.6, 0.4, 0.4) & (0.8, 0.4, 0.4) & (0.7, 0.4, 0.5) \\ A_2 & (0.7, 0.3, 0.4) & (0.8, 0.3, 0.4) & (0.8, 0.4, 0.4) & (0.8, 0.2, 0.3) \\ A_3 & (0.8, 0.4, 0.4) & (0.8, 0.3, 0.3) & (0.9, 0.5, 0.5) & (0.7, 0.3, 0.4) \\ A_4 & (0.7, 0.5, 0.5) & (0.6, 0.3, 0.4) & (0.6, 0.2, 0.4) & (0.7, 0.4, 0.5) \\ A_5 & (0.8, 0.4, 0.4) & (0.6, 0.4, 0.4) & (0.7, 0.4, 0.4) & (0.8, 0.5, 0.5) \end{pmatrix} \quad (10)$$

Step: 5. Determinate the weight of the attributes

By using Equations (9) and (10), we determine the weights of the four attribute as follows:

$$w_1 = \frac{(1 \div 0.26)}{25} \approx 0.16, w_2 = \frac{(1 \div 0.11)}{25} \approx 0.37, w_3 = \frac{(1 \div 0.20)}{25} \approx 0.20, w_4 = \frac{(1 \div 0.15)}{25} \approx 0.27.$$

Step: 6. Calculate the weighted SVNS cross entropy matrix

Using Equation (2) and weights of attributes, we calculate the weighted NS-cross entropy values between ideal matrix and weighted aggregated decision matrix.

$${}^{NS}M_{CE}^w = \begin{pmatrix} 0.195 \\ 0.198 \\ 0.168 \\ 0.151 \\ 0.184 \end{pmatrix} \quad (11)$$

Step: 7. Rank the priority

The cross entropy values of alternatives are arranged in increasing order as follows:

$$0.151 < 0.168 < 0.184 < 0.195 < 0.198.$$

Alternatives are then preference ranked as follows:

$$A_4 > A_3 > A_5 > A_1 > A_2.$$

Step: 8. Select the best alternative

From step 7, we identify A_4 is the best alternative. Hence, Food enterprises (A_4) is the best alternative for investment.

In Figure 2, we draw a bar diagram to represent the cross entropy values of alternatives which shows that A_4 is the best alternative according our proposed strategy.

In Figure 3, we represent the relation between cross entropy values and acceptance values of alternatives. The range of acceptance level for five alternatives is taken by five points. The high acceptance level of alternatives indicates the best alternative for acceptance and low acceptance level of alternative indicates the poor acceptance alternative.

We see from Figure 3 that alternative A_4 has the smallest cross entropy value and the highest acceptance level. Therefore A_4 is the best alternative for acceptance. Figure 3 indicates that alternative A_2 has highest cross entropy value and lowest acceptance value that means A_2 is the worst alternative. Finally, we conclude that the relation between cross entropy values and acceptance value of alternatives is opposite in nature.

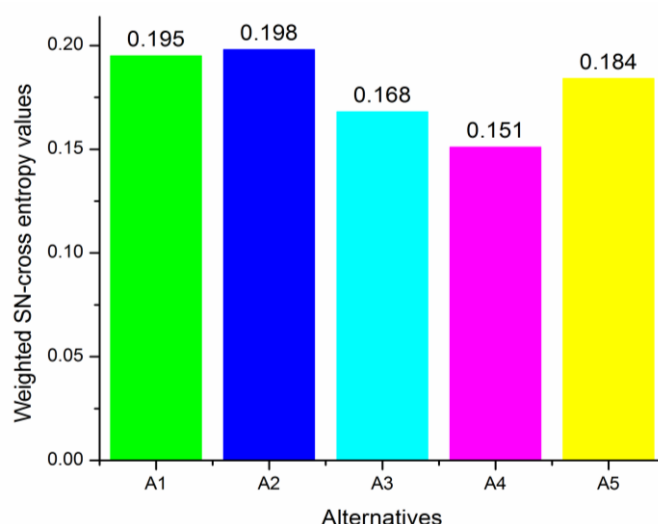


Figure 2. Bar diagram of alternatives versus weighted NS-cross entropy values of alternatives.

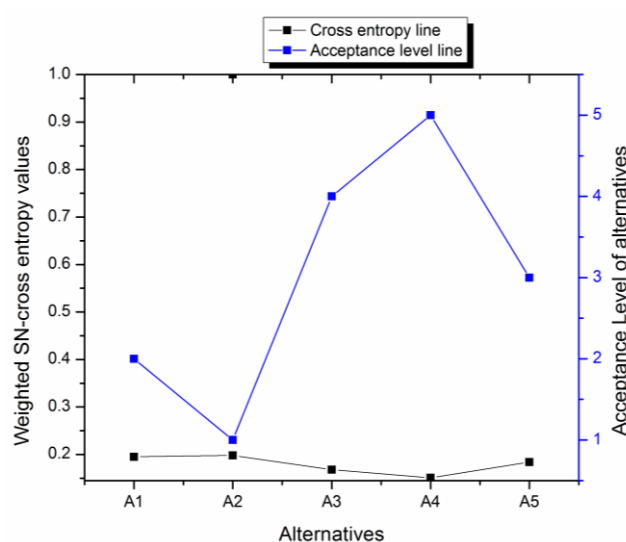


Figure 3. Relation between weighted NS-cross entropy values and acceptance level line of alternatives.

6. Comparative Study and Discussion

In literature only two MADM strategies [144,145] have been proposed. No MAGDM strategy is available. So the proposed MAGDM is novel and non-comparable with the existing cross entropy under SVNS for numerical example.

- The MADM strategies [144,145] are not applicable for MAGDM problems. The proposed MAGDM strategy is free from such drawbacks.
- Ye [144] proposed cross entropy that does not satisfy the symmetrical property straightforward and is undefined for some situations but the proposed strategy satisfies symmetric property and is free from undefined phenomenon.
- The strategies [144,145] cannot deal with the unknown weight of the attributes whereas the proposed MAGDM strategy can deal with the unknown weight of the attributes
- The strategies [144,145] are not suitable for dealing with the unknown weight of decision-makers, whereas the essence of the proposed NS-cross entropy-based MAGDM is that it is capable of dealing with the unknown weight of the decision-makers.

7. Conclusions

In this paper, we have defined a novel cross entropy measure in SVNS environment. The proposed cross entropy measure in SVNS environment is free from the drawbacks of asymmetrical behavior and undefined phenomena. It is capable of dealing with the unknown weight of attributes and the unknown weight of decision-makers. We have proved the basic properties of the NS-cross entropy measure. We also defined weighted NS-cross entropy measure and proved its basic properties. Based on the weighted NS-cross entropy measure, we have developed a novel MAGDM strategy to solve neutrosophic multi-attribute group decision-making problems. We have at first proposed a novel MAGDM strategy based on NS-cross entropy measure with technique to determine the unknown weight of attributes and the unknown weight of decision-makers. Other existing cross entropy measures [144,145] can deal only with the MADM problem with single decision-maker and known weight of the attributes. So in general, our proposed NS-cross entropy-based MAGDM strategy is not comparable with the existing cross-entropy-based MADM strategies [144,145] under the single-valued neutrosophic environment. Finally, we solve a MAGDM problem to show the feasibility, applicability and efficiency of the proposed MAGDM strategy. The proposed NS-cross entropy-based MAGDM can be applied in teacher selection, pattern recognition, weaver selection, medical treatment selection options, and other practical problems. In future study, the proposed NS-cross entropy-based MAGDM strategy can be also extended to the interval neutrosophic set environment.

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