



# Article Evaluating Evidence Reliability on the Basis of Intuitionistic Fuzzy Sets

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**Abstract:** The evaluation of evidence reliability is still an open topic, when prior knowledge is unavailable. In this paper, we propose a new method for evaluating evidence reliability, in the framework of intuitionistic fuzzy sets. The reliability of evidence was evaluated, based on the supporting degree between basic probability assignments (BPAs). The BPAs were first transformed to intuitionistic fuzzy sets (IFSs). By the similarity degree between the IFSs, we can get the supporting degree between the BPAs. Thus, the reliability of evidence can be evaluated, based on its connection with supporting degree. Based on the new evidence reliability, we developed a new method for combining evidence sources with different reliability degrades. Comparison with other methods was carried out to illustrate the effectiveness of the new method.

Keywords: intuitionistic fuzzy set; evidence theory; reliability evaluation; sensor fusion

# 1. Introduction

Multi-sensor data fusion has been applied in many fields, such as pattern recognition, target identification [1–4], decision making [5,6], and so on. When fusing information from multiple sensors, the information provided by different sensors may be uncertain, imprecise, or even contradictory with each other. Many theories including the probability theory, fuzzy theory, and evidence theory, have been applied in data fusion [7–11]. The theory of intuitionistic fuzzy sets, as an important tool to model uncertainty, have attracted much attention from researchers [12–18]. Research on intuitionistic fuzzy set, mainly focus on the intuitionistic fuzzy measures [12–19], its mathematical properties [20–22], its application in decision-making [23,24], optimization [25,26], and so on. In all theories that deal with uncertainty, the evidence theory is usually regarded as an extension of the probability theory. In the evidence theory, we can assign belief degree to all subsets of the set of the discernment frame. Moreover, Dempster's combination rule for basic probability assignments (BPAs) can provide an easy way to fuse the uncertain information, from different sensors. So evidence theory can be used to handle more kinds of uncertainty and has been widely used in the application of data fusion [27–29].

However, the classical combination rule may lead to counter-intuitive results, when the BPAs are in high conflict. If the BPAs are in complete conflict, the classical combination rule may be inefficient. To solve the problems in the application of evidence theory, researchers have proposed many modified methods for combining BPAs. Some of them are developed by improving the combination rule, which may lose the good properties of the classical Dempster's combination rule. Others are proposed by modifying the original BPA, before combining them by Dempster's combination rule. These two kinds of methods are different from each other, since their focus on the source of counter-intuitive results are different. Researchers have proposed that the first kind of this method implies that the counter-intuitive results are caused by the process of normalization in Dempster's combination rule, and the second type

of methods hold the perspective that unreliable evidence sources lead to unreasonable results. We hold that unreliable evidence sources cause the problem in the application of evidence theory. So we must evaluate the reliability of all BPAs, before combining them. Then we can use the discounting operation to modify the original BPAs. Hence, the evaluation of evidence is important for the combination of BPAs.

Some methods have been proposed to evaluate the reliability of evidence sources, but most of these methods are developed to evaluate evidence reliability when prior knowledge is available. The method proposed by Elouedi et al. [30] assessed the reliability of an evidence source, in the model of transferable belief, which is developed from evidence theory. In this method, reliability is obtained by an optimization model. The goal function of the optimization model is the square error between the modified BPAs and the actual identification of the data. So, the information about the real identification of data should be known. The evaluation method proposed by Elouedi et al. was extended in two aspects, by Guo et al. [31]. The evaluation method introduced in Reference [30] was first developed into a new method called the static evaluation, which is also implemented on the basis of supervised training. Another evaluation method is the dynamic evaluation of evidence reliability. The dynamic reliability can be obtained by adaptive learning and regulation, in real-time situations, which depends on the contexts of sensor readings and dynamic performance.

When assessing evidence reliability, the crucial difficulty is the absence of prior knowledge. In such cases, the principle of majority [32] is usually used to facilitate evaluation. The conflict measure, similarity measure, and distance measure can be applied to depict the relation between BPAs. For example, Klein and Colot [32] used Jousselme's [33] distance measure to propose the dissent degree. The dissent degree between a BPA and the averaged BPA is used to estimate the evidence reliability. Liu et al. [34] combined the distance measure and divergence degree to define a new dissimilarity measure, based on which the reliability of evidence sources can be evaluated.

We can see that the definition of similarity or dissimilarity measures for BPAs can help the evaluation of evidence reliability. So, we proposed a new method for evaluating the evidence reliability based on the new similarity measure of BPAs. Motivated by the rich kinds of intuitionistic fuzzy similarity measures and the relation between BPA and intuitionistic fuzzy set (IFS), we have calculated the similarity degree between BPAs, in the framework of intuitionistic fuzzy sets. Based on the new reliability evaluation method and evidence discounting operation, we proposed a new evidence combination method. Numerical examples were used to validate the performance of the proposed method.

The rest of this paper unfolds as follows. Basic knowledge on the evidence theory is presented in Section 2. A new method of reliability evaluation is developed, based on intuitionistic fuzzy sets, in Section 3, where a new combination method is also proposed, based on evidence discounting and Dempster's combination rule. Numerical examples and comparative analysis are presented in Section 4 to show the performance of the new combination method. This paper is concluded in Section 5.

#### 2. Brief Review on Evidence Theory

## 2.1. Basic Concepts

The evidence theory, initiated by Dempster and developed by Shafer, was modeled based on the frame of discernment denoted by  $\Theta$ , which is a finite set with mutually exclusive elements. The power set of  $\Theta$ , denoted by  $2^{\Theta}$ , contains all the possible unions of the sets in  $\Theta$  including  $\Theta$  itself. Singleton sets in a frame of discernment  $\Theta$  are called atomic sets because they do not contain nonempty subsets. The following terminologies are central in the Dempster-Shafer theory [35,36].

Let  $\Theta = \{\theta_1, \theta_2, \dots, \theta_n\}$  be the frame of discernment. A basic probability assignment (BPA) is a function  $m: 2^{\Theta} \to [0, 1]$ , satisfying the two following conditions:

$$\sum_{A \subseteq \Theta} m(A) = 1 \tag{2}$$

where  $\emptyset$  is the empty set, and A denotes the subset of  $\Theta$ . A BPA is also called as a belief structure. For  $A \subseteq \Theta$ , the value assigned by the BPA on A is the basic probability mass of A, expressed by m(A).

For  $A \subseteq \Theta$ , if m(A) > 0, A is the focal element of m. The set of all focal elements is expressed by  $\{A | A \subseteq \Theta, m(A) > 0\}$ . If the focal elements of a BPA m are all atomic sets with only one element, the BPA is called Bayesian belief structure (BBS). The BPA with the following form: m(A) = 1,  $\forall A \subseteq \Theta$  and m(B) = 0,  $\forall B \subseteq \Theta$ ,  $B \neq A$ , is called as a categorical belief structure. The BPA with  $m(\Theta) = 1$  and m(A) = 0,  $\forall A \neq \Theta$ , is called as a vacuous BPA.

Given a BPA *m* defined on  $\Theta$ , its belief function and plausibility function can be, respectively, defined as:

$$Bel(A) = \sum_{B \subseteq A} m(B) \tag{3}$$

$$Pl(A) = \sum_{B \cap A \neq \emptyset} m(B) = 1 - \sum_{B \cap A = \emptyset} m(B)$$
(4)

Bel(A) quantifies all basic probability masses exactly assigned to A and its subsets. Pl(A) measures all possible basic probability masses that could be assigned to A and its subsets. In such sense, Bel(A) and Pl(A) can be regarded as the lower bound and upper bound of the probability to which A is supported. So, the belief degree of A can be considered as an interval number BI(A) = [Bel(A), Pl(A)].

The pignistic transformation [37] is defined to transform a belief structure *m* to the so-called pignistic probability function, which is helpful for decision making. For a BPA *m* defined on  $\Theta = \{\theta_1, \theta_2, \dots, \theta_n\}$ , the pignistic transformation is expressed by

$$BetP(A) = \sum_{B \subseteq \Theta} \frac{|A \cap B|}{|B|} \frac{m(B)}{1 - m(\emptyset)}, \forall A \subseteq \Theta$$
(5)

where |A| is the number of elements in set *A*, which is also called as the cardinality of set *A*.

Particularly, given  $m(\emptyset) = 0$  and  $\theta \in \Theta$ , we have

$$BetP(\{\theta\}) = \sum_{\theta \in B} \frac{m(B)}{|B|}, \theta = \theta_1, \dots, \theta_n, B \subseteq \Theta$$
(6)

## 2.2. Dempster's Combination Rule

Given two BPAs  $m_1$  and  $m_2$  defined on  $\Theta$ , the BPA that results from their combination, denoted as  $m_1 \oplus m_2$ , or  $m_{12}$  for short, can be obtained by Dempster's combination rule [35], shown as:

$$m_{12}(A) = \begin{cases} \sum_{\substack{B \cap C = A \\ 1 - \sum_{\substack{B \cap C = \emptyset \\ B \cap C = \emptyset \\ 0 \ , A = \emptyset}}} m_1(B)m_2(C), \forall A \subseteq \Theta, A \neq \emptyset \end{cases}$$
(7)

For more than two BPAs to be combined, the combination results of all BPAs can be obtained as:

$$m(A) = \begin{cases} \sum_{\substack{\cap A_i = A \\ n \in \mathbb{Z}}} \prod_{i=1}^n m_i(A_i) \\ 1 - \sum_{\substack{\cap A_i = \emptyset \\ n \in \mathbb{Z}}} \prod_{i=1}^n m_i(A_i)}, \forall A \subseteq \Theta, A \neq \emptyset \\ 0, A = \emptyset \end{cases}$$
(8)

Here, *n* is the number of evidence pieces in the process of combination, *i* denotes the *i*th piece of evidence, and  $m_i(A_i)$  is the BPA of hypothesis  $A_i$  supported by BPA *i*. The amount of conflict among *n* 

mutually independent pieces of evidence is equal to the mass of the empty set after the conjunctive combination and before the normalization step. It represents contradictory evidence. It is calculated as:

$$k = \sum_{\bigcap A_i = \emptyset} \prod_{i=1}^n m_i(A_i) \tag{9}$$

The case of k = 0 indicates that there is no conflict among BPAs, while k = 1 indicates that all BPAs are in complete conflict.

Dempster's rule has many good properties, such as commutativity and associativity. So, it has been widely applied in many areas. However, when the BPAs to be combined are completely contradictory, i.e., k = 1, the combination rule cannot be performed. When they are in high conflict, i.e.,  $k \rightarrow 1$ , we may get counter-intuitive combination results, which do not coincide with the actual situation. This can be demonstrated by the following example [38].

**Example 1.** Two BPAs  $m_1$  and  $m_2$ , defined on the frame of discernment  $\Theta = \{\theta_1, \theta_2, \theta_3\}$ , are to be considered. *These two BPAs are given as following:* 

$$m_1 : m_1(\{\theta_1\}) = 0.9, m_1(\{\theta_2\}) = 0.1$$
$$m_2 : m_2(\{\theta_2\}) = 0.1, m_2(\{\theta_3\}) = 0.9$$

Using Dempster's rule to combine these BPAs, we can get  $m(\{\theta_1\}) = m(\{\theta_3\}) = 0$ ,  $m(\{\theta_2\}) = 1$ . It is shown that  $m_1$  and  $m_2$  assigned a low support degree to  $\theta_2$ , but the final result completely support  $\theta_2$ . On the other hand, we can see that  $m_1$  support  $\theta_1$  in a high degree, and  $m_2$  supports  $\theta_3$  in a high degree, but neither  $\theta_1$  nor  $\theta_3$  is supported in the combination result. This is counter-intuitive and unreasonable. This indicates that Dempster's rule cannot be used to combine evidence bodies in high conflict.

When the evidence source is not reliable, and its reliability degree is assigned as  $\lambda$  with  $\lambda \in [0, 1]$ , we can use the discounting operation introduced by Shafer [36] to modify the original BPA. Based on Shafer's discounting operation, the BPA  $m^{\lambda}$  obtained by discounting is expressed as:

$$\begin{cases} m^{\lambda}(A) = \lambda m(A), A \subset \Theta\\ m^{\lambda}(\Theta) = 1 - \lambda + \lambda m(\Theta) \end{cases}$$
(10)

We note that if the evidence source is totally reliable, i.e.,  $\lambda = 1$ , then the BPA  $m^{\lambda}$  is identical to the original BPA m. If the evidence source is completely unreliable, i.e.,  $\lambda = 0$ , we can get  $m^{\lambda}(\Theta) = 1$ , which means that the discounted BPA is a vacuous one providing no information.

### 3. Evaluating the Evidence Reliability

#### 3.1. The Relation between BPA and IFS

The intuitionistic fuzzy set was developed from Zadeh's fuzzy set. Zadeh's fuzzy set can be descripted as follows.

**Definition 1 [39].** Let  $X = \{x_1, x_2, \dots, x_n\}$  be the universe of discourse. A fuzzy set A, defined in X, is expressed as:

$$A = \{ \langle x, \mu_A(x) \rangle | x \in X \}$$
(11)

where  $\mu_A(x) : X \to [0,1]$  is the membership degree.

**Definition 2** [40]. An intuitionistic fuzzy set A in X can be written as:

$$A = \{ \langle x, \mu_A(x), v_A(x) \rangle | x \in X \}$$
(12)

where  $\mu_A(x) : X \to [0,1]$  is the membership degree, and  $v_A(x) : X \to [0,1]$  is the non-membership degree. They are constrained by the condition:

$$0 \le \mu_A(x) + v_A(x) \le 1$$
 (13)

The hesitancy degree is expressed as:

$$\pi_A(x) = 1 - \mu_A(x) - v_A(x) \tag{14}$$

Obviously, we have  $\pi_A(x) \in [0, 1]$ ,  $\forall x \in X$ . Greater  $\pi_A(x)$  implies more vagueness on x with respect to A. Particularly, when  $\pi_A(x) = 0$ ,  $\forall x \in X$ , the IFS A degenerates into Zadeh's fuzzy set.

In the sequel, IFSs(X) denotes the set of all IFSs in X. If |X| = 1, i.e., there is only one element x in X, the IFS A in X can be expressed by  $A = \langle \mu_A, v_A \rangle$  for short, which is also named as an intuitionistic fuzzy value (IFV).

**Definition 3 [40].** *For*  $A \in IFSs(X)$  *and*  $B \in IFSs(X)$ *, some relations between them are defined as:* 

(R1)  $A \subseteq B$  iff  $\forall x \in X \ \mu_A(x) \le \mu_B(x), v_A(x) \ge v_B(x);$ (R2) A = B iff  $\forall x \in X \ \mu_A(x) = \mu_B(x), v_A(x) = v_B(x);$ (R3)  $A^C = \{\langle x, v_A(x), \mu_A(x) \rangle | x \in X \}, where A^C \text{ is the complement of } A.$ 

**Definition 4 [40].** Let  $A = \{ \langle x, \mu_A(x), v_A(x) \rangle | x \in X \}$ ,  $B = \{ \langle x, \mu_B(x), v_B(x) \rangle | x \in X \}$  be two IFSs in X, then the following operations can be defined:

$$A \cap B = \{ \langle x, \min(\mu_A(x), \mu_B(x)), \max(v_A(x), v_B(x)) \rangle | x \in X \},\$$

$$A \cup B = \{ \langle x, \max(\mu_A(x), \mu_B(x)), \min(v_A(x), v_B(x)) \rangle | x \in X \},\$$

$$A \oplus B = \{ \langle x, \mu_A(x) + \mu_B(x) - \mu_A(x) \cdot \mu_B(x), v_A(x) \cdot v_B(x) \rangle | x \in X \},\$$

$$A \otimes B = \{ \langle x, \mu_A(x) \cdot \mu_B(x), v_A(x) + v_B(x) - v_A(x) \cdot v_B(x) \rangle | x \in X \},\$$

$$\gamma \cdot A = \{ \langle x, 1 - (1 - \mu_A(x))^{\gamma}, (\gamma_A(x))^{\gamma} \rangle | x \in X \},\$$

$$A^{\gamma} = \{ \langle x, (\mu_A(x))^{\gamma}, 1 - (1 - v_A(x))^{\gamma} \rangle | x \in X \}.$$

As discussed earlier, for a BPA m,  $[Bel(\theta), Pl(\theta)]$  can be regarded as the confidence interval of  $\theta$ . We can use  $[Bel(\theta), Pl(\theta)]$  to represent the lower bound and upper bound of the belief on  $\theta$ . Here,  $Bel(\theta)$  and  $Pl(\theta)$  are the lower probability and the upper probability, respectively. Hence, the probability  $P(\theta)$  lies in an interval  $[Bel(\theta), Pl(\theta)]$ . If m is regarded as an IFS A defined in  $\Theta = \{\theta_1, \theta_2, \dots, \theta_n\}$ ,  $Bel(\theta)$  is the degree of membership, while  $1 - Pl(\theta)$  is the degree of non-membership. Based on these analysis, the BPA m, defined on the discernment frame  $\Theta = \{\theta_1, \theta_2, \dots, \theta_n\}$ , can be expressed as an IFS A defined on  $\Theta = \{\theta_1, \theta_2, \dots, \theta_n\}$ . The IFS A is written as:

$$A = \{ \langle \theta, \mu_A(\theta), v_A(\theta) \rangle | \theta \in \Theta \} = \{ \langle \theta_1, Bel(\theta_1), 1 - Pl(\theta_1) \rangle, \langle \theta_2, Bel(\theta_2), 1 - Pl(\theta_2) \rangle, \cdots, \langle \theta_n, Bel(\theta_n), 1 - Pl(\theta_n) \rangle \}$$
(15)

The relation between BPA and IFS has its physical interpretation from the viewpoint of target identification. Let the discernment frame be  $\Theta = \{\theta_1, \theta_2, \theta_3\}$ , i.e., all possible classes of the target are contained in the set  $\Theta = \{\theta_1, \theta_2, \theta_3\}$ . The output of the sensor

expressed by a BPA *m* indicates that the target is identified as an IFS *A* with *A* =  $\{\langle \theta_1, Bel(\theta_1), 1 - Pl(\theta_1) \rangle, \langle \theta_2, Bel(\theta_2), 1 - Pl(\theta_2) \rangle, \langle \theta_3, Bel(\theta_3), 1 - Pl(\theta_3) \rangle\}.$ 

Specially, if a sensor identifies the target as a singleton subset of  $\Theta$ , taking  $\{\theta_1\}$  as an example, the BPA can be written as:  $m(\{\theta_1\}) = 1, m(\{\theta_2\}) = 0, m(\{\theta_3\}) = 0$ .

Then the corresponding IFS is  $A = \{ \langle \theta_1, 1, 0 \rangle, \langle \theta_2, 0, 1 \rangle, \langle \theta_3, 0, 1 \rangle \}$ , which is same as the set  $\{ \theta_1 \}$ .

If the target is totally unknown by the sensor, i.e., the sensor provides no helpful knowledge about the target, the BPA *m* is a vacuous one with  $m(\Theta) = 1$ . Thus, we have:  $Bel(\theta_1) = Bel(\theta_2) = Bel(\theta_3) = 0$ ,  $Pl(\theta_1) = Pl(\theta_2) = Pl(\theta_3) = 1$ .

So the IFS can be written as  $A = \{ \langle \theta_1, 0, 0 \rangle, \langle \theta_2, 0, 0 \rangle, \langle \theta_3, 0, 0 \rangle \}$ , which indicates total ignorance.

#### 3.2. Supporting Degree of BPAs

Supporting degree of BPAs has been introduced to develop the modified combination rules [34,41]. Generally, the supporting degree is calculated on the basis of the similarity or distance measures between BPAs. If we use *Sup* to express the supporting degree, we have  $Sup(m_1, m_2) = Sup(m_2, m_1)$ . Taking *Sim* and *Dis* as the similarity and distance measures between BPAs, respectively, we can get the following relations:

$$Sup(m_1, m_2) \propto Sim(m_1, m_2), Sup(m_1, m_2) \propto 1 - Dis(m_1, m_2)$$

In other words, the higher similarity degree between the two BPAs indicates the higher supporting degree between them. The lower distance between the two BPAs also indicates higher supporting degree between the BPAs. For clarity, the supporting degree between BPAs can be usually considered as consistent to the similarity degree between BPAs.

The relation between BPA and IFS allow us to calculate the supporting degree of BPAs in the framework of IFS. Thus, the supporting degree  $Sup(m_1, m_2)$  can be obtained by calculating the supporting degree between IFSs  $A_1$  and  $A_2$ , where  $A_1$  and  $A_2$  are IFSs derived from  $m_1$  and  $m_2$ , respectively. So we have:

$$Sup(m_1, m_2) = Sup(A_1, A_2) = Sim(A_1, A_2)$$
 (16)

In recent years, a lot of similarity measures of IFSs have been proposed [12–18]. This provide us much convenience in calculating the supporting degree of BPAs. In the following, we use the Euclidian-distance-based similarity measure of IFSs [16]. The Euclidian-distance-based similarity measure is defined as following:

Let  $A = \{\langle x, \mu_A(x), v_A(x) \rangle | x \in X\}$  and  $B = \{\langle x, \mu_B(x), v_B(x) \rangle | x \in X\}$  be two IFSs defined in  $X = \{x_1, x_2, \dots, x_n\}$ . The similarity degree between *A* and *B* are calculated by:

$$S_E(A,B) = 1 - \frac{1}{n} \sum_{i=1}^n \sqrt{\frac{(\mu_A(x_i) - \mu_B(x_i))^2 + (v_A(x_i) - v_B(x_i))^2}{2}}$$
(17)

It has been proved that the similarity measure  $S_E(A, B)$  satisfies all axiomatic properties of intuitionistic fuzzy similarity measure [16].

Based on the above analysis, we can obtain the supporting degree between two BPAs  $m_1$  and  $m_2$ , by the following steps:

**Step 1.** From Equations (3) and (4), we can get the values of the belief function and plausibility function of all singleton subsets, corresponding to the BPAs  $m_1$  and  $m_2$ .

**Step 2.** From Equation (15), we can get the two IFSs  $A_1$  and  $A_2$ , according to  $m_1$  and  $m_2$ .

**Step 3.** Following Equation (17), we can calculate the similarity degrees  $S_E(A_1, A_2)$ .

Finally, we can get the degree to which  $m_1$  supports  $m_2$  is  $Sup(m_1, m_2) = S_E(A_1, A_2)$ , the degree of  $m_2$  supporting  $m_1 Sup(m_2, m_1) = Sup(m_1, m_2) = S_E(A_1, A_2)$ .

Based on the axiomatic properties of  $S_E(A, B)$ , we have  $m_1 = m_2 \Rightarrow Sup(m_1, m_2) = Sup(m_2, m_1) = 1$ .

#### 3.3. Evidence Reliability

Suppose that there are N BPAs expressed as  $m_1, m_2, \dots, m_N$ . Based on the supporting degree between any two BPAs, we can construct the supporting degree matrix (SDM) as:

$$SDM = \begin{pmatrix} Sup(m_1, m_1) & Sup(m_1, m_2) & \cdots & Sup(m_1, m_N) \\ Sup(m_2, m_1) & Sup(m_2, m_2) & \cdots & Sup(m_2, m_N) \\ \vdots & \vdots & \vdots \\ Sup(m_N, m_1) & Sup(m_N, m_2) & \cdots & Sup(m_N, m_N) \end{pmatrix}$$
(18)

We can see that the elements in the *i*th row represent the degree to which  $m_i$  is supported by other BPAs. So the total supporting degree of  $m_i$  can be calculates as:

$$Total\_Sup(m_i) = \sum_{\substack{j=1\\ j \neq i}}^{N} Sup(m_i, m_j)$$
(19)

Generally, the larger support degree of a BPA indicates that this BPA is more reliable. Otherwise, the BPA is less reliable. So the reliability of each BPA can be calculated by its total support degree. In application, the reliability should be normalized. If we consider the relative reliability of all BPAs, they can be normalized to the reliability of  $m_i$  as:

$$R'(m_i) = \frac{Total\_Sup(m_i)}{\sum\limits_{j=1}^{N} Total\_Sup(m_j)}$$
(20)

If the reliability of the most reliable BPA is set as 1, the absolute dynamic reliability of  $m_i$  can be can be obtained as:

$$R(m_i) = \frac{Total\_Sup(m_i)}{\max_{j=1,2,\cdots,N} \{Total\_Sup(m_j)\}}$$
(21)

# 3.4. A New Method for Evidence Combination

Once the reliability of all BPAs are obtained, we can use evidence reliability to modify the original BPAs by the discounting operation. Then we can combine the discounted BPAs using Dempster's combination rule. So we can propose a new method for evidence combination. Suppose that there are N BPAs  $m_1, m_2, \dots, m_N$  to be combined, they can be combined as the following steps:

Step 1. Calculate the supporting degree of each BPA.

From Equations (3) and (4), we can get the value of the belief function and the plausibility function, for all singleton subsets with respect to  $m_i$ ,  $i = 1, 2, \dots, N$ .

From Equation (15), we can get IFSs corresponding to all BPAs.

Following Equation (17), we can calculate the similarity degrees  $S_E(A_i, A_k)$ ,  $i = 1, 2, \dots, N$ . Finally, we get the supporting degree between  $m_i$  and  $m_j$ , shown as:

$$Sup(m_i, m_j) = Sup(m_j, m_i) = S_E(A_i, A_j) = S_E(A_j, A_i)$$

Step 2. Calculate the reliability of each BPA.

From the supporting degree between every two BPAs, the support degree matrix can be constructed as Equation (18). Then the reliability of each BPA can be obtained based on Equation (19).

Step 3. Modify the original BPAs.

Using the evidence reliability and evidence discounting operation shown in Equation (10), we can modify the original BPAs  $m_1, m_2, \dots, m_N$ . The discounted BPAs are denoted by  $m_1^R, m_2^R, \dots, m_N^R$ .

**Step 4.** Evidence combination by Dempster's combination rule.

By Dempster's combination rule, the discounted BPAs  $m_1^R, m_2^R, \dots, m_N^R$  can be combined to an integrated BPA.

# 4. Illustrative Examples and Discussion

In this section, the proposed combination method will be applied to deal with the application of data fusion to validate its effectiveness and rationality.

First, we will use a numerical example to show the implementation of the proposed evidence combination, based on evidence reliability evaluation.

**Example 2.** In a target identification system based on multiple sensors, three sensors  $S_1$ ,  $S_2$ ,  $S_3$  are employed to recognize the identification of a target. Three possible types of the target are denoted by  $\theta_1$ ,  $\theta_2$ , and  $\theta_3$ . So the frame of discernment can be expressed as  $\Theta = \{\theta_1, \theta_2, \theta_3\}$ . The outputs of three sensors are expressed by three BBAs. They are listed as the following:

$$m_1(\{\theta_1\}) = 0.6, m_1(\{\theta_2\}) = 0.1, m_1(\{\theta_3\}) = 0.2, m_1(\Theta) = 0.1$$
$$m_2(\{\theta_1\}) = 0.2, m_2(\{\theta_2\}) = 0.5, m_2(\{\theta_3\}) = 0.1, m_2(\Theta) = 0.2$$
$$m_3(\{\theta_1\}) = 0.4, m_3(\{\theta_2\}) = 0.1, m_3(\{\theta_3\}) = 0.2, m_3(\{\Theta\}) = 0.3$$

Three IFSs in  $\Theta = \{\theta_1, \theta_2, \theta_3\}$  can be generated from these BPAs. They are expressed as:

 $A_{1} = \{ \langle \theta_{1}, 0.6, 0.3 \rangle, \langle \theta_{2}, 0.1, 0.8 \rangle, \langle \theta_{3}, 0.2, 0.7 \rangle \}$  $A_{2} = \{ \langle \theta_{1}, 0.2, 0.6 \rangle, \langle \theta_{2}, 0.5, 0.3 \rangle, \langle \theta_{3}, 0.1, 0.7 \rangle \}$  $A_{3} = \{ \langle \theta_{1}, 0.4, 0.3 \rangle, \langle \theta_{2}, 0.1, 0.6 \rangle, \langle \theta_{3}, 0.2, 0.5 \rangle \}$ 

The supporting degree matrix (SDM) for the three BPAs is:

$$SDM = \begin{pmatrix} 1 & S_E(A_1, A_2) & S_E(A_1, A_3) \\ S_E(A_2, A_1) & 1 & S_E(A_2, A_3) \\ S_E(A_3, A_1) & S_E(A_3, A_2) & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0.7058 & 0.8492 \\ 0.7058 & 1 & 0.7445 \\ 0.8492 & 0.7445 & 1 \end{pmatrix}$$

Based on Equation (19), the total supporting degree of the BPA can be calculated:

 $Total\_Sup(m_1) = 1.5550$ ,  $Total\_Sup(m_2) = 1.4503$ ,  $Total\_Sup(m_1) = 1.5936$ .

Finally the absolute reliability of each BPA can be yielded according to Equation (21):

$$R(S_1) = 0.9758, R(S_2) = 0.9101, R(S_3) = 1.$$

Based on the reliability factor, we can modify three original BPAs by the discounting operation. We can get the discounted BPAs as:

$$m_1^{R}(\{\theta_1\}) = 0.5855, m_1^{R}(\{\theta_2\}) = 0.0976, m_1^{R}(\{\theta_3\}) = 0.1952, m_1^{R}(\Theta) = 0.1218$$
$$m_2^{R}(\{\theta_1\}) = 0.1820, m_2^{R}(\{\theta_2\}) = 0.4550, m_2^{R}(\{\theta_3\}) = 0.0910, m_2^{R}(\Theta) = 0.2719$$
$$m_3^{R}(\{\theta_1\}) = 0.4, m_3^{R}(\{\theta_2\}) = 0.1, m_3^{R}(\{\theta_3\}) = 0.2, m_3^{R}(\Theta) = 0.3$$

Combining these discounted BPAs by using Dempster's rule, we can get the final result:

$$m(\{\theta_1\}) = 0.6585, m(\{\theta_2\}) = 0.1601, m(\{\theta_3\}) = 0.1459, m(\Theta) = 0.0305$$

Based on the final fusion result, a comprehensive recognition on the target can be obtained. As shown in the result, the unknown target is identified as  $\theta_1$ , according to the outputs of three sensors.

This example demonstrates that the proposed method provides an alternative way to combine uncertain evidence sources with different reliability, when a priori knowledge is not available. In this example, it can be noted that the  $m_2$  is quite different from  $m_1$  and  $m_3$ . So  $m_2$  should be assigned to a low reliability. The proposed reliability evaluation method is sensitive to the BPA with low reliability. The reliability of  $m_2$ , obtained by the proposed evaluation method, is the lowest one, which is consistent with intuitive analysis. Using the reliability factor of each BPA, we can discount them by the discounting operation. Then the influence of unreliability BPA on the final fusion can be reduced, which is helpful for making a sound decision. It is indicated by this example that the proposed combination method can well deal with unreliable information and conflict in evidence sources.

Another example will be used to illustrate the performance of the new combination method, based on the comparison with other methods.

**Example 3.** In the information fusion system, based on multiple sensors, five sensors  $S_1$ ,  $S_2$ ,  $S_3$ ,  $S_4$ , and  $S_5$  are applied to identify a target. The outputs of the five sensors are expressed by five BPAs in  $\Theta = \{\theta_1, \theta_2, \theta_3\}$ , as shown in Table 1.

	$m_1$	$m_2$	$m_3$	$m_4$	$m_5$
$\{\theta_1\}$	0.8	0.4	0	0.3	0.45
$\{\theta_2\}$	0.1	0.2	0.95	0.2	0.1
$\{\theta_3\}$	0	0.1	0.05	0.25	0
$\{\theta_1, \theta_2\}$	0	0.3	0	0.2	0
$\{\theta_2, \theta_3\}$	0	0	0	0	0.15
Θ	0.1	0	0	0.05	0.3

Table 1. Five BPAs corresponding to five sensors.

From Table 1, we can see that the BPAs  $m_1$ ,  $m_2$ ,  $m_4$ , and  $m_5$  prefer  $\theta_1$ , by assigning the most basic probability masses to  $\theta_1$ , but  $m_3$  assigns most belief to  $\theta_2$ , which is quite different from other four BPAs. Based on the principle of majority,  $m_3$  should get the least supporting degree from other BPAs. The reliability of  $m_3$  should be the least one in all five BPAs.

Using the proposed evidence reliability evaluation method, we can get the reliability of five BPAs, as follows:

$$R(m_1) = 0.8684, R(m_2) = 0.9844, R(m_3) = 0.6645, R(m_4) = 0.9711, R(m_5) = 1.$$

It is shown that the reliability degree of  $m_3$  is 0.6645, which is the least one in all reliability factors. This coincides with the intuitive analysis. The information provided by  $S_3$  will be modified in a great degree in the combination.

Using the discounting operation to modify all BPAs, and combining all modified BPAs by Dempster's combination rule, we can achieve the fusion results as:

$$m(\{\theta_1\}) = 0.6923, m(\{\theta_2\}) = 0.2823, m(\{\theta_3\}) = 0.0100, m(\{\theta_1, \theta_2\}) = 0.0142, m(\Theta) = 0.0002$$

The fusion result assigns most belief to  $\{\theta_1\}$ . So the target is recognized as  $\theta_1$  by the fusion result. We can see that  $m_3$  was assigned to the least reliability and its influence on the final result is reduced greatly.

For comparison, we combine these BPAs by adding the BPA, one by one, based on other different methods. The results are presented in Table 2, where  $m_1^n (2 \le n \le 5)$  means the combination of  $m_1$  to  $m_n$ . We can see that if we use the classical Dempster's combination rule, the addition of  $m_3$  causes the basic probability assigned to  $\{\theta_1\}$  is 0. The addition of other BPAs supporting  $\{\theta_1\}$  cannot change such a situation. These results are unreasonable, since most of the BPAs support  $\{\theta_1\}$ , but the final result opposes  $\{\theta_1\}$ .

When the reliability of evidence sources is taken into consideration,  $m_3$  will be discounted, based on its evaluated reliability. If we use evidential distance measure  $d_J$  [41] and dissimilarity measure DismP [34] to evaluate evidence reliability,  $m_3$  will be assigned to a low-reliability degree. Thus, the information got from  $m_3$  slightly affects the final fusion result. This rapid discounting process may cause great loss of information, which will also brings much risk to decision-making.

In our proposed method, the addition of  $m_3$  will decrease the basic probability assigned to  $\{\theta_1\}$ , which is reasonable, as they were used in References [34,41]. With the addition of  $m_4$  and  $m_5$ , the basic probability on  $\{\theta_1\}$  increases and the basic probability on  $\{\theta_2\}$  decreases. In the proposed method, this process is slower. This is helpful for making a cautious decision.

	$m_1^2$	$m_1^3$	$m_1^4$	$m_{1}^{5}$
Classical Dempster's rule	$m(\{\theta_1\}) = 0.8451$ $m(\{\theta_2\}) = 0.0986$ $m(\{\theta_3\}) = 0.0140$ $m(\{\theta_1, \theta_2\}) = 0.0423$	$m(\{\theta_1\}) = 0$ $m(\{\theta_2\}) = 0.9948$ $m(\{\theta_3\}) = 0.0052$	$m(\{\theta_1\}) = 0$ $m(\{\theta_2\}) = 0.9965$ $m(\{\theta_3\}) = 0.0035$	$m(\{\theta_1\}) = 0$ $m(\{\theta_2\}) = 0.9971$ $m(\{\theta_3\}) = 0.0029$
d <sub>J</sub> & Dempster's rule [41]	$m(\{\theta_1\}) = 0.7659$ $m(\{\theta_2\}) = 0.1166$ $m(\{\theta_3\}) = 0.0294$ $m(\{\theta_1, \theta_2\}) = 0.0881$	$m(\{\theta_1\}) = 0.6239$ $m(\{\theta_2\}) = 0.2791$ $m(\{\theta_3\}) = 0.0252$ $m(\{\theta_1, \theta_2\}) = 0.0718$	$m(\{\theta_1\}) = 0.6858$ $m(\{\theta_2\}) = 0.2645$ $m(\{\theta_3\}) = 0.0146$ $m(\{\theta_1, \theta_2\}) = 0.0351$	$m(\{\theta_1\}) = 0.7528$ $m(\{\theta_2\}) = 0.2217$ $m(\{\theta_3\}) = 0.0096$ $m(\{\theta_1, \theta_2\}) = 0.0159$
Liu's method in [34]	$m(\{\theta_1\}) = 0.7503$ $m(\{\theta_2\}) = 0.1196$ $m(\{\theta_3\}) = 0.0319$ $m(\{\theta_1, \theta_2\}) = 0.0957$ $m(\Theta) = 0.0025$	$m(\{\theta_1\}) = 0.7157$ $m(\{\theta_2\}) = 0.1598$ $m(\{\theta_3\}) = 0.0308$ $m(\{\theta_1, \theta_2\}) = 0.0913$ $m(\Theta) = 0.0024$	$m(\{\theta_1\}) = 0.7670$ $m(\{\theta_2\}) = 0.11655$ $m(\{\theta_3\}) = 0.0194$ $m(\{\theta_1, \theta_2\}) = 0.0477$ $m(\Theta) = 0.0004$	$\begin{array}{l} m(\{\theta_1\}) = \textbf{0.8254} \\ m(\{\theta_2\}) = 0.1424 \\ m(\{\theta_3\}) = 0.0120 \\ m(\{\theta_1, \theta_2\}) = 0.0198 \\ m(\{\theta_2, \theta_3\}) = 0.0002 \\ m(\Theta) = 0.0002 \end{array}$
Proposed method	$m(\{\theta_1\}) = 0.8451$ $m(\{\theta_2\}) = 0.0986$ $m(\{\theta_3\}) = 0.0141$ $m(\{\theta_1, \theta_2\}) = 0.0423$ $m(\Theta) = 0.0011$	$\begin{split} m(\{\theta_1\}) &= \textbf{0.5317} \\ m(\{\theta_2\}) &= 0.4070 \\ m(\{\theta_3\}) &= 0.0170 \\ m(\{\theta_1, \theta_2\}) &= 0.0443 \end{split}$	$m(\{\theta_1\}) = 0.5969$ $m(\{\theta_2\}) = 0.3596$ $m(\{\theta_3\}) = 0.0137$ $m(\{\theta_1, \theta_2\}) = 0.0296$ $m(\Theta) = 0.0002$	$m(\{\theta_1\}) = 0.6923$ $m(\{\theta_2\}) = 0.2832$ $m(\{\theta_3\}) = 0.0100$ $m(\{\theta_1, \theta_2\}) = 0.0142$ $m(\Theta) = 0.0002$

Table 2. Combination results of different evidence bodies.

# 5. Conclusions

In this paper, a new method was proposed to evaluate the reliability of evidence sources. The concept of supporting degree was introduced, based on the similarity degree of BPAs. Based on the relation between the BPAs and the IFSs, the supporting degree between BPAs can be obtained with the help of intuitionistic fuzzy similarity measures. Then the reliability of evidence sources can be evaluated by normalizing the total supporting degree of each BPA. To cope with the combination of conflicting information, we developed a new combination method. In the new combination method, the original BPAs were modified, based on the reliability of evidence sources and the discounting operation. The modified BPAs were combined by Dempster's combination rule. Illustrative examples have been presented to validate the proposed evaluation method and combination method. It has been shown that the evaluation of evidence reliability can assign a low reliability to the BPA that is different from others. By the evidence discounting based on evidence reliability, the influence of unreliable BPAs on the final result will be reduced. Comparison with other methods indicates that the proposed method can deal well with the conflicting information in information fusion. Moreover, the proposed method is more cautious, which is helpful for decision making.

This paper proposed an attempt to evaluate evidence reliability in the framework of IFSs. More investigation on the relation between BPAs and IFSs, the search of a more effective and reasonable supporting degree for the BPAs, and the modification of classical combination rules could be the focus of future research.

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