



# Article Offset Free Tracking Predictive Control Based on Dynamic PLS Framework

# Jin Xin \* 🕑, Wang Yue and Luo Lin

School of Information and Control Engineering, Liaoning Shihua University, Fushun 113001, China; wangyue@lnpu.edu.cn (W.Y.); lin.l.csc@gmail.com (L.L.)

\* Correspondence: jinxin@lnpu.edu.cn; Tel.: +86-135-0043-1319

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**Abstract:** This paper develops an offset free tracking model predictive control based on a dynamic partial least square (PLS) framework. First, state space model is used as the inner model of PLS to describe the dynamic system, where subspace identification method is used to identify the inner model. Based on the obtained model, multiple independent model predictive control (MPC) controllers are designed. Due to the decoupling character of PLS, these controllers are running separately, which is suitable for distributed control framework. In addition, the increment of inner model output is considered in the cost function of MPC, which involves integral action in the controller. Hence, the offset free tracking performance is guaranteed. The results of an industry background simulation demonstrate the effectiveness of proposed method.

Keywords: partial least square; model predictive control; offset free control; distributed controller

## 1. Introduction

Nowadays, industrial production is becoming more complex and larger in scale. There are usually hundreds or thousands of input and output variables in this class of novel industrial system. It raises higher requirements for process control technology. With the development of cyber physical systems, more and more new technologies are applied to the industrial process control, such as network control [1], wireless technology [2] and embedded technology [3]. These technologies bring in innovations in the control system, which lead to conventional control theory not being applicable for the application of these new technologies. Model predictive control (MPC) [4] is the most widely used advanced control theory. It has three features: predictive model; rolling optimization, and; feedback correction. Due to the advantages of dealing with multivariable control and constraints [5], MPC has many successful applications. The process of industrial production is sometimes nonlinear. MPC designs for nonlinear system is also one of the hot spots of research. Many research results have formed a special course [6]. Many researchers combine other nonlinear method with MPC such as nonlinear identification [7], neural network [8] and fuzzy model [9] to deal with nonlinear control problem. However, with the increase of dimensionality of controlled variables, the computation load has been greatly increased. MPC is a typical centralized control method, it is not suitable for combing with new cyber physical system technologies such as those mentioned above.

On the other hand, with the development of computation and storage techniques, it became possible to obtain huge dimension and large number of industry production operation data. While the process is often restricted to move in a space of much lower dimension than the data indicate. Hence, many data reduction methods based on principal component extraction are available. Partial least square (PLS) is one of these methods, which is used to model the relationship between two blocks of data. It has been applied to many areas in the past decades, such as quality prediction, process monitoring and chemometrics [10]. PLS is not only able to extract principle component

multiple single input single output (SISO) subsystems. One can design controllers for each subsystem separately, which is suitable for the application of distribute control of cyber physical system. Kaspar and Ray [12,13] proposed a dynamic PLS framework by utilizing the PLS loading matrices to construct precompensators and postcompensatores. Chen and Cheng [14] proposed another dynamic PLS framework with the ARX model. Laurí et al. [15] proposed a PLS-based MPC relevant identification method. With these dynamic PLS models (DyPLS), many control schemes are applied. Kaspar and Ray proposed a proportion-integration-differentiation (PID) control scheme under this framework [12], and Chen and Cheng [14] designed multi-loop adaptive PID controllers based on a modified decoupling PLS framework. Hu et al. [16,17] proposed a multi-loop internal model controller in the dynamic PLS framework and achieved better performance for disturbance rejection. LÜ and Liang [18] proposed a multi-loop constrained MPC scheme.

In practice it is often required to track a reference set-point, and achieve zero offset in the presence of persistent disturbances and plant-model mismatch. This type of problem is usually considered as an offset-free problem. To solve this problem, many methods have been proposed so far. One is to augment the state model with disturbance model, which can account for the presence of disturbance or for plant-model mismatch (an observer is usually used). Many algorithms have been developed based on it, for example, references [19-21]. González et al. [22] pointed out that if the observer does not reach accurately the stationary state, the model predictions are made on a mistaken stationary model. Another method is to describe the system in the so-called velocity form, see [23,24], where the state is composed by the state increments and the output error, while the manipulated variable is the control increment. The velocity form does not require the use of a state estimator and does not require to compute the steady state target for the plant state and control variables, but no stability results have been established for MPC algorithms based on the velocity form [23]. Pannocchia et al. [25] point out that velocity form model is a special case of disturbance model/observer in the offset-free linear MPC design. A third method is based on the internal model principle. With this method, an integral model is introduced to the system model, and MPC is designed to guarantee the stable of system and integral model. Then the effect of the disturbances is cancelled by the MPC optimization and the controlled variables are tracked at their set-points. This strategy has been followed in [26–29].

This paper focus on the offset free tracking performance of dynamic PLS control framework, and proposed a novel MPC control strategy. The paper organized as follows. In Section 2, the used dynamic PLS model and its modification are defined and described. Offset-free MPC in DyPLS is described. An industrial background simulation is given to demonstrate the merit of the proposed method. Conclusions are drawn in Section 5.

#### 2. Dynamic PLS Model Description

The origins of PLS are traced to Herman Wold's original non-linear iterative partial least square (NIPALS) algorithm [30]. The advantage of PLS lies in that it decomposes the multivariate regression problem into a series of uncorrelated univariate regression problems, and problems of non-square and ill-conditioned in the regression can be dealt with [11]. The conventional PLS consists of an outer model which extracts the latent variables, and an inner model to estimate the algebraic relationship between the input and output scores [31]. Considering scaled  $l \times m$  dimension input dataset X and a  $l \times n$  dimension output dataset Y, where l, m and n denote the number of sampling, input dimension and output dimension, respectively. The outer model is built by decomposing matrix X and Y as follows:

$$X = \sum_{\substack{r=1\\r=1}}^{R} t_r \boldsymbol{p}_r^{\mathrm{T}} + \boldsymbol{E}^* = \boldsymbol{T} \boldsymbol{P}^{\mathrm{T}} + \boldsymbol{E}^*$$

$$Y = \sum_{\substack{r=1\\r=1}}^{R} u_r \boldsymbol{q}_r^{\mathrm{T}} + \boldsymbol{F}^* = \boldsymbol{U} \boldsymbol{Q}^{\mathrm{T}} + \boldsymbol{F}^*$$
(1)

where *R* is the number of latent variables (due to the effect of dimensionality reduction of PLS.  $R \le \max(m, n)$  is met in most cases),  $p_r$  and  $q_r$  are the *r*-th loading vectors of loading matrices *P* and *Q*, respectively;  $t_r$  and  $u_r$  are the *r*-th score vectors of matrices *T* and *U*, respectively; and *E*<sup>\*</sup> and *F*<sup>\*</sup> are residual matrices of *X* and *Y*, respectively.

In inner model, the score matrices T and U are related by a diagonal matrix B (Equation (2)) which is obtained by least squares (LS) method.

$$u_r = b_r t_r$$

$$U = TB$$
(2)

where,  $b_r = \frac{u_r^T t_r}{t_r^T t_r}$ ;  $\mathbf{B} = (\mathbf{T}^T \mathbf{T})^{-1} \mathbf{T}^T \mathbf{U}$ ;  $\mathbf{U} = \text{diag}(u_1, u_2, \cdots, u_R)$ ;  $\mathbf{T} = \text{diag}(t_1, t_2, \cdots, t_R)$ . The conventional PLS is suitable for pure algebraic regression. It is not able to cope with dynamic

The conventional PLS is suitable for pure algebraic regression. It is not able to cope with dynamic system such as industry process systems. Researchers have proposed many different DyPLS models by introducing dynamic structures into PLS to describe the dynamic character, such as time-series terms [13], dynamic filters [32] or the ARX model [33].

In this paper, state space model is used as PLS inner model to describe the dynamic character. For the *r*-th latent variable, inner state space model can be expressed as follows:

$$\begin{aligned} \mathbf{x}_r(k+1) &= \mathbf{A}_r \mathbf{x}_r(k) + \mathbf{B}_r t_r(k) \\ u_r(k) &= \mathbf{C}_r \mathbf{x}_r(k) \end{aligned} \tag{3}$$

The state space model can be obtained by subspace identification method [34], and the dimension of parameters  $A_r$ ,  $B_r$ ,  $C_r$  can be determined by model output error method. The structure of dynamic PLS with state space model is shown in Figure 1, where,  $W_x$  and  $W_y$  are scaling matrices of X and Y.



Figure 1. Structure of dynamic partial least square (DyPLS) models with state space model.

## 3. Predictive Controller Design

#### 3.1. Dynamic PLS Control Framework

Shown in Figure 2 is the control structure with dynamic PLS model proposed by Kaspar and Ray [13]. Based on the DyPLS model proposed in Section 2, this scheme employs the PLS loading matrices P and Q as precompensators and postcompensators to connect the actual process. With the loading matrices, inputs and outputs of the system are mapped into the latent variable space. Controllers designed in the latent variable space use input scores to control output scores. Since the multiple input multiple output (MIMO) system is decoupled into multiple single input single output (SISO) subsystems in latent variable space, the original MIMO controller problem is decomposed into multiple independent SISO control problems. A variety of conventional controls scheme could be involved in this control framework [13]. In this framework, each controller in the latent variable space responds synchronously to the error between score variable u and its set-point, which is  $Y_{set}$  mapped

into the latent variable space. Generated controller output *T* is back mapped into origin space and then is used as the input of the system.



Figure 2. PLS control framework proposed by Kaspar and Ray.

In Figure 2,  $G_{ci}(i = 1, \dots, R)$ ,  $G_p$  and  $G_d$  are the transfer function matrices of the controller in latent variable space, the controlled plant and disturbances, respectively;  $Q^+$  is the appropriate inverse of Q.  $Y_{set}$  is the set-point sequence of process; D is the disturbance sequence.  $\Delta = 1 - z^{-1}$ .

## 3.2. Equation of Prediction

The *r*-th inner model Equation (3) can be considered in incremental forms as follows:

$$\Delta \mathbf{x}_r(k+1) = \mathbf{A}_r \Delta \mathbf{x}_r(k) + \mathbf{B}_r \Delta t_r(k)$$
  

$$\Delta u_r(k) = \mathbf{C}_r \Delta \mathbf{x}_r(k)$$
(4)

Let  $N_{p,r}$  and  $N_{c,r}$  denote prediction horizon and control horizon for the *r*-th controller, respectively. Considering Equation (3), the  $N_{p,r}$  step ahead prediction of state with  $N_{c,r}$  control step are

$$\begin{bmatrix} \hat{\mathbf{x}}_{r}(k+1|k) \\ \vdots \\ \hat{\mathbf{x}}_{r}(k+N_{c,r}|k) \\ \vdots \\ \hat{\mathbf{x}}_{r}(k+N_{c,r}+1|k) \\ \vdots \\ \hat{\mathbf{x}}_{r}(k+N_{p,r}|k) \end{bmatrix} = \begin{bmatrix} A_{r} \\ \vdots \\ A_{r}^{N_{c,r}} \\ A_{r}^{N_{c,r}+1} \\ \vdots \\ A_{r}^{N_{p,r}} \end{bmatrix} \mathbf{x}_{r}(k) + \begin{bmatrix} B_{r} & 0 \\ \vdots \\ \sum_{\substack{r=0 \\ i=0}}^{N_{c,r}} A_{r}^{i}B_{r} \\ \vdots \\ \sum_{\substack{r=0 \\ i=0}}^{N_{c,r}-1} A_{r}^{i}B_{r} \\ \vdots \\ \sum_{\substack{r=0 \\ i=0}}^{N_{c,r}} A_{r}^{i}B_{r} \\ \vdots \\ \sum_{\substack{r=0 \\ i=0}}^{N_{c,r}-1} A_{r}^{i}B_{r} \\ \vdots \\ \sum_{\substack{r=0 \\ i=0}}^{N_{c,r}} A_{r}^{i}B_{r} \\ \vdots \\ \sum_{\substack{r=0 \\ i=0}}^{N_{c,r}-1} A_{r}^{i}B_{r} \\ \vdots \\ \sum_{\substack{r=0 \\ i=0}^{N_{c,r}-1} A_{r}^$$

Similarly, with Equation (4), the  $N_{p,r}$  step ahead prediction of incremental state with  $N_{c,r}$  control step are

$$\begin{array}{c} \Delta \hat{\mathbf{x}}_{r}(k+1|k) \\ \vdots \\ \Delta \hat{\mathbf{x}}_{r}(k+N_{c,r}|k) \\ \Delta \hat{\mathbf{x}}_{r}(k+N_{c,r}+1|k) \\ \vdots \\ \Delta \hat{\mathbf{x}}_{r}(k+N_{p,r}|k) \end{array} \right] = \begin{bmatrix} A_{r} \\ \vdots \\ A_{r}^{N_{c,r}} \\ A_{r}^{N_{c,r}+1} \\ \vdots \\ A_{r}^{N_{p,r}} \end{bmatrix} \mathbf{x}_{r}(k) + \begin{bmatrix} B_{r} & 0 \\ \vdots & \ddots & \vdots \\ \sum_{\substack{N_{c,r}-1 \\ N_{c,r}} A_{r}^{i}B_{r} & \cdots & B_{r} \\ \sum_{\substack{i=0 \\ i=0}}^{N_{c,r}} A_{r}^{i}B_{r} & \cdots & \sum_{i=0}^{1} A_{r}^{i}B_{r} \\ \vdots & \vdots & \vdots \\ N_{p,r}-1 & N_{p,r}-N_{c,r} \\ \sum_{i=0}^{N_{p,r}-N_{c,r}} A_{r}^{i}B_{r} & \cdots & \sum_{i=0}^{1} A_{r}^{i}B_{r} \end{bmatrix} \begin{bmatrix} \Delta \hat{f}_{r}(k|k) \\ \vdots \\ \Delta \hat{f}_{r}(k+N_{c,r}-1|k) \end{bmatrix}$$
(6)

From Equation (5) and considering Equation (3), the estimates of future of inner model output are

$$\begin{bmatrix} \hat{u}_{r}(k+1|k) \\ \vdots \\ \hat{u}_{r}(k+N_{c,r}|k) \\ \hat{u}_{r}(k+N_{c,r}+1|k) \\ \vdots \\ \hat{u}_{r}(k+N_{p,r}|k) \end{bmatrix} = \begin{bmatrix} C_{r}A_{r} \\ \vdots \\ C_{r}A_{r}^{N_{c,r}} \\ C_{r}A_{r}^{N_{c,r}} \\ C_{r}A_{r}^{N_{c,r}} \end{bmatrix} x_{r}(k) + \begin{bmatrix} C_{r}B_{r} \\ \vdots \\ \sum_{\substack{N_{c,r} \\ i=0 \\ N_{c,r}} C_{r}A_{r}^{i}B_{r} \\ \vdots \\ N_{p,r}-1 \\ \sum_{i=0} C_{r}A_{r}^{i}B_{r} \end{bmatrix} t_{r}(k-1) + \begin{bmatrix} C_{r}B_{r} & 0 \\ \vdots & \ddots & \vdots \\ \sum_{\substack{N_{c,r} \\ N_{c,r}} C_{r}A_{r}^{i}B_{r} & \cdots & C_{r}B_{r} \\ \vdots & \vdots \\ N_{p,r}-1 \\ \sum_{i=0} C_{r}A_{r}^{i}B_{r} \end{bmatrix} \cdot (7)$$

Equation (7) can be written in matrix notation as follows:

$$\hat{\boldsymbol{u}}_r(k) = \Phi_r \boldsymbol{x}_r(k) + \boldsymbol{\Psi}_r \boldsymbol{u}_r(k) + \boldsymbol{\Gamma}_r \Delta \hat{\boldsymbol{t}}_r(k)$$
(8)

where 
$$\hat{u}_{r}(k) = [\hat{u}_{r}(k+1|k), \cdots, \hat{u}_{r}(k+N_{p,r}|k)]^{\mathrm{T}}, \Delta \hat{t}_{r}(k) = [\Delta \hat{t}_{r}(k|k), \cdots, \Delta \hat{t}_{r}(k+N_{c,r}-1|k)]^{\mathrm{T}},$$
  

$$\Phi_{r} = [C_{r}A_{r}, \cdots C_{r}A_{r}^{N_{p,r}}]^{\mathrm{T}}, \Psi_{r} = [C_{r}B_{r}, \cdots, \sum_{i=0}^{N_{p,r}-1} C_{r}A_{r}^{i}B_{r}], \Gamma_{r} = \begin{bmatrix} C_{r}A_{r}B_{r} & \cdots & C_{r}B_{r} \\ \sum_{i=0}^{N_{c,r}} C_{r}A_{r}^{i}B_{r} & \cdots & \sum_{i=0}^{1} C_{r}A_{r}^{i}B_{r} \\ \sum_{i=0}^{N_{c,r}} C_{r}A_{r}^{i}B_{r} & \cdots & \sum_{i=0}^{1} C_{r}A_{r}^{i}B_{r} \\ \vdots & \vdots & \vdots \\ N_{p,r}-1 & N_{p,r}-N_{c,r} \\ \sum_{i=0}^{N_{p,r}-N_{c,r}} C_{r}A_{r}^{i}B_{r} & \cdots & \sum_{i=0}^{1} C_{r}A_{r}^{i}B_{r} \end{bmatrix}.$$

Similarly, from Equation (6) and considering Equation (4), the estimates of future of inner model output are

$$\begin{bmatrix} \Delta \hat{u}_{r}(k+1|k) \\ \vdots \\ \Delta \hat{u}_{r}(k+N_{c,r}|k) \\ \Delta \hat{u}_{r}(k+N_{c,r}+1|k) \\ \vdots \\ \Delta \hat{u}_{r}(k+N_{p,r}|k) \end{bmatrix} = \begin{bmatrix} C_{r}A_{r} \\ \vdots \\ C_{r}A_{r}^{N_{c,r}} \\ C_{r}A_{r}^{N_{c,r}+1} \\ \vdots \\ C_{r}A_{r}^{N_{p,r}} \end{bmatrix} \mathbf{x}_{r}(k) + \begin{bmatrix} C_{r}B_{r} & 0 \\ \vdots & \ddots & \vdots \\ \sum_{\substack{N_{c,r}-1 \\ N_{c,r}} \\ C_{r}A_{r}^{i}B_{r} & \cdots & C_{r}B_{r} \\ \sum_{i=0}^{N_{c,r}} C_{r}A_{r}^{i}B_{r} & \cdots & \sum_{i=0}^{1} C_{r}A_{r}^{i}B_{r} \\ \vdots & \vdots \\ \sum_{i=0}^{N_{p,r}-1} C_{r}A_{r}^{i}B_{r} & \cdots & \sum_{i=0}^{1} C_{r}A_{r}^{i}B_{r} \end{bmatrix} \begin{bmatrix} \Delta \hat{t}_{r}(k|k) \\ \vdots \\ \Delta \hat{t}_{r}(k+N_{c,r}-1|k) \end{bmatrix}$$
(9)

Equation (9) can be written in matrix notation as follows:

$$\Delta \hat{\boldsymbol{u}}_r(k) = \Phi_r \Delta \boldsymbol{x}_r(k) + \boldsymbol{\Gamma}_r \Delta \hat{\boldsymbol{t}}_r(k) \tag{10}$$

where  $\Delta \hat{u}_r(k) = [\Delta \hat{u}_r(k+1|k), \cdots, \Delta \hat{u}_r(k+N_{p,r}|k)]^{\mathrm{T}}$ .

## 3.3. Offset-Free Control

In order to get offset-free tracking performance, it should give zero value for the error between set-point and system output as

$$\lim_{k \to \infty} e(k) = \lim_{k \to \infty} u_{\text{set}_r}(k) - u_r(k) \to 0$$
(11)

According to the final value theorem applied to the control system, it is necessary to provide at least one integrator action in the control loop. A typical cost function of MPC is the 2-norm form as follows:

$$J_r = \sum_{j=1}^{N_{p,r}} \|u_{\text{set}_r}(k+j) - \hat{u}_r(k+j|k)\|_{Q_u}^2 + \sum_{j=1}^{N_{c,r}} \|\Delta \hat{t}_r(k+j-1|k)\|_{Q_t}^2$$
(12)

where  $u_{set_r}$  is the *r*-th component of  $u_{set}$ .  $Q_u$  and  $Q_t$  are weighting diagonal matrices. Equation (12) does not add any integrator action. One can modify Equation (12) as follows:

$$J_r = \sum_{j=1}^{N_{p,r}} \|u_{\text{set}_r}(k+j) - \hat{u}_r(k+j|k)\|_{Q_u}^2 + \sum_{j=1}^{N_{c,r}} \|\Delta \hat{t}_r(k+j-1|k)\|_{Q_t}^2 + \sum_{j=1}^{N_{p,r}} \|\Delta \hat{u}_r(k+j|k)\|_{Q_{\Delta u}}^2$$
(13)

Equation (13) can be written in matrix notation as follows:

$$J_r = \|\boldsymbol{u}_{\text{set}_r} - \hat{\boldsymbol{u}}_r(k)\|_{\boldsymbol{Q}_u}^2 + \|\Delta \hat{\boldsymbol{t}}_r(k)\|_{\boldsymbol{Q}_t}^2 + \|\Delta \hat{\boldsymbol{u}}_r(k)\|_{\boldsymbol{Q}_{\Delta u}}^2$$
(14)

The cost function Equation (14) contains integrator action for increments  $\Delta \hat{u}_r(k)$ . To optimize the cost function Equation (14), one can get analytical solution of  $\Delta \hat{u}_r(k)$  as

$$\Delta \hat{\boldsymbol{t}}_r(k) = -\left[\boldsymbol{\Gamma}_r^{\mathrm{T}}(\boldsymbol{Q}_u + \boldsymbol{Q}_{\Delta u})\boldsymbol{\Gamma}_r + \boldsymbol{Q}_t\right]^{-1}\boldsymbol{\Gamma}_r^{\mathrm{T}}\left[(\boldsymbol{Q}_u + \boldsymbol{Q}_{\Delta u})\boldsymbol{\Phi}_r\boldsymbol{x}_r(k) - \boldsymbol{Q}_{\Delta u}\boldsymbol{\Phi}_r\boldsymbol{x}_r(k-1) + \boldsymbol{Q}_u(\boldsymbol{\Psi}_r\boldsymbol{u}_r(k) - \boldsymbol{u}_{\mathrm{set}})\right]$$
(15)

One can calculate the control low with Equation (15) offline, so the proposed method will not dramatically increase the online calculation burden. Comparing with conventional MPC, there is only one more polynomial  $x_r(k-1)$  in the proposed method.

The structure of proposed controller is shown in Figure 3. It is a distributed form which is similar as the distributed model predictive control form [35]. Each controller is a SISO controller, hence the control burden is lower than the conventional MIMO MPC. Hence, this control method is suitable for multiple controllers be embedded in to different low cost control nodes. In each node, there are independent mapping matrices  $Q^+W_y^{-1}$  and  $W_x^{-1}P$ . There is another advantage of this structure. For controllers are operating separately, if one controller is broken down, other controllers can still make the system having a part control action.



Figure 3. The structure of proposed method in distributed form.

## 3.4. Stability Analysis

Suppose the prediction horizon and control horizon of MPC in latent space are equal, and prediction horizon for different latent variable controllers are also equal. Equation (13) can be reformed as

$$J_r = \sum_{j=1}^{N_p} \|u_{\text{set}_r}(k+j) - \hat{u}_r(k+j|k)\|_{Q_u}^2 + \sum_{j=1}^{N_p} \|\Delta \hat{t}_r(k+j-1|k)\|_{Q_t}^2 + \sum_{j=1}^{N_p} \|\Delta \hat{u}_r(k+j|k)\|_{Q_{\Delta u}}^2$$
(16)

According to Equations (8) and (13), Equation (16) is a function of state variables and increment input prediction, as

$$J_r = \sum_{j=1}^{N_p} L(\hat{x}_r(k+j|k), \Delta \hat{t}_r(k+j-1|k))$$
(17)

Suppose  $\{\Delta \hat{t}_r(k|k), \dots, \Delta \hat{t}_r(k+N_p-1|k)\}$  is the optimal sequence of solution Equation (17). At every time t,  $\Delta \hat{t}_r(k|k)$  is added to control rate at the previous time and acts on the system. Let  $J_r^*(k)$  and  $\{\hat{x}_r^*(k|k), \dots, \hat{x}_r^*(k+N_p-1|k)\}$  denote the optimal performance index and optimal state corresponding to this control action. If dynamic PLS can fully describe the actual system and there is no disturbance in system,  $x_r(k+1) = \hat{x}_r^*(k+1|k)$ . According to Equation (4), one can get  $\hat{t}_r(k+N_p|k) = 0$ . When  $\hat{x}_r(k+N_p|k) = 0$ , then  $\hat{x}_r(k+N_p+1|k) = 0$ . Hence, a feasible control sequence at time t is  $\{\Delta \hat{t}_r(k+1|k), \dots, \Delta \hat{t}_r(k+N_p-1|k), 0\}$ . A performance index corresponding to it is

$$J_r(k+1) = J_r^*(k) - L(\hat{x}_r(k+1|k), \Delta \hat{t}_r(k|k)) \le J_r^*(k)$$
(18)

 $J_r(k+1)$  is not less than  $J_r^*(k+1)$ . Hence

$$J_r^*(k+1) \le J_r^*(k) - L(\hat{x}_r(k+1|k), \Delta \hat{t}_r(k|k)) \le J_r^*(k)$$
(19)

Hence, the performance index of each controller in latent space is monotonically decreasing. That is to say that latent output  $u_r(k)$  is bounded. In other words, when  $k \to \infty$ ,  $J_r^*(k+1) = J_r^*(k)$ . According to Equation (1), the system output in original space is a linear combination of  $u_r(k)$ . When  $u_r(k)$  is stable, the original space output is stable.

## 4. Case Study

The efficiency of the proposed method is demonstrated by carrying out simulation study on polyethylene reaction, which is described by Embirucu and Fontes [36]. Polyethylene is the largest synthetic polymer in terms of production, and stirred-tank reactor is a significant part of it. Ziegler-Natta and Phillips catalysis can be used in this reaction. In these both catalysis, the system is a typical MIMO system with nine inputs and seven outputs. In this study case, Phillips catalysis is chosen. And the first three equations are extracted in this simulation, which are described as follows:

$$\begin{aligned} &(1 - 0.9021q^{-1})y_1(k) = (0.9283 - 0.8350q^{-1})x_1(k) \\ &(1 - 0.9067q^{-1})y_2(k) = (0.8415 - 0.7664q^{-1})x_1(k) + (0.6873 - 0.6023q^{-1})x_2(k) \\ &(1 - 0.8932q^{-1})y_3(k) = (0.8591 - 0.7536q^{-1})x_1(k) + (0.8097 - 0.7066q^{-1})x_3(k) + 0.0081x_4(k) \end{aligned}$$

The description of input and output variables are illustrated in Table 1. This system is a typically non-square system with four inputs and three outputs.

	Symbol	Description	
input	<i>x</i> <sub>1</sub>	monomer feed flow	
	<i>x</i> <sub>2</sub>	solvent(n-hexane) feed flow	
	$x_3$	catalyst feed flow	
	$x_4$	gas recycle/monomer feed ratio	
output	$y_1$	production	
	$y_2$	slurry polymer	
	$y_3$	catalyst efficiency	

Table 1. The description of input and output variables.

In order to build a well-described dynamic PLS model, four input random step signals magnitudes ranging between -3.5 and 3.5 (shown in Figure 4) are applied to excite the system. In this simulation,

a white noise with deviation 0.01 and zero mean is added to input data. The system responses are collected as a part of modeling data (solid line shown in Figure 5). Modeling results are shown in the Figure 5 with dotted line and marked with "DyPLS". In the DyPLS model, the number of latent variables is three, and the number of state variables of the inner model is five. The integral of squared error between system responses and modeling results of three outputs are  $4.2531 \times 10^3$ ,  $5.5489 \times 10^3$  and 243.64, respectively. One can see that the DyPLS did not match process well, that is because some of useful information are ignored due to the iterative modeling mechanism of PLS (according to Equation (1), there is always *F*<sup>\*</sup> and *E*<sup>\*</sup>).



Figure 4. Input data set for polyethylene reaction.



Figure 5. Output data set and modeling results.

The total simulation horizon for the control comparison is 900. Step signal, ramp signal and sinusoidal signal are used as the set-point of outputs to simulate the common reference signals in industry. In this case study, the conventional MPC, conventional MPC in dynamic PLS and the proposed method are compared together. Control results are shown in Figures 6–11. Shown in Figures 7, 9 and 11 are the control error between system outputs and set-points of two methods. In order to evaluate and compare the control performance, the integral of squared error (ISE) between set-points and system outputs is used, results are shown in Table 2. In this case study, Matlab R2012a is

used. The configuration of computer is i5 2.6 GHz CPU and 8 GB RAM. Computing time of the three methods are shown in Table 2. From the simulation results, one can conclude that the proposed method has less tracking error than the conventional methods. And the computing time of PLS framework is significantly lower than that of conventional MPC.



Figure 6. Control results of conventional model predictive control (MPC) in dynamic PLS.



Figure 7. Control error of conventional MPC.



Figure 8. Control results of conventional MPC in dynamic PLS.



Figure 9. Control error of conventional MPC in dynamic PLS.



Figure 10. Control results of offset free MPC in dynamic PLS.



Figure 11. Control error of offset free MPC in dynamic PLS.

	Conventional MPC	Conventional MPC in Dynamic PLS	Proposed Method
ISE of $y_1$	37.33	6.38	5.70
ISE of $y_2$	52.21	9.26	7.80
ISE of $y_3$	31.98	14.73	9.08
Computing time (ms)	87.23	63.87	64.37

Table 2. Integral of squared error (ISE) and computing time of three methods.

Shown in Figure 12 is the control result for third controller is broken. One can see that even if a controller is broken, the proposed method can still keep the system output within a certain range.



Figure 12. Control results for one controller broken.

### 5. Conclusions

In this paper, a MPC controller design in the DyPLS framework with an offset-free mechanism was proposed. First, a state space model-based DyPLS model is identified. Based on the obtained model, multiple sub-MPCs are designed. With the integral action in the proposed MPC, offset-free reference tracking performance is guaranteed. With the decoupling character of PLS, the proposed method can be considered as distributed form. It can reduce the calculation burden and is suitable for being embedded into distributed control systems such as network control systems. Furthermore, due to the independently running character of controllers, the proposed method can retain partial control performance for the system when some controllers are broken down.

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