information

MDPI

Article

Exponential Operations and an Aggregation Method for Single-Valued Neutrosophic Numbers in Decision Making

Zhikang Lu and Jun Ye *

Department of Electrical and Information Engineering, Shaoxing University, 508 Huancheng West Road, Shaoxing 312000, Zhejiang Province, China; luzk000@163.com
* Correspondence: yejun@usx.edu.cn; Tel.: +86-575-88327323

Academic Editor: Willy Susilo
Received: 22 April 2017; Accepted: 2 June 2017; Published: 7 June 2017

Abstract: As an extension of an intuitionistic fuzzy set, a single-valued neutrosophic set is described independently by the membership functions of its truth, indeterminacy, and falsity, which is a subclass of a neutrosophic set (NS). However, in existing exponential operations and their aggregation methods for neutrosophic numbers (NNs) (basic elements in NSs), the exponents (weights) are positive real numbers in unit intervals under neutrosophic decision-making environments. As a supplement, this paper defines new exponential operations of single-valued NNs (basic elements in a single-valued NS), where positive real numbers are used as the bases, and single-valued NNs are used as the exponents. Then, we propose a single-valued neutrosophic weighted exponential aggregation (SVNWEA) operator based on the exponential operational laws of single-valued NNs and the SVNWEA operator-based decision-making method. Finally, an illustrative example shows the applicability and rationality of the presented method. A comparison with a traditional method demonstrates that the new decision-making method is more appropriate and effective.

Keywords: single-valued neutrosophic set; exponential operational law; single-valued neutrosophic weighted exponential aggregation (SVNWEA) operator; decision making

1. Introduction

Real life problems often call for decision-making under uncertainty, meaning we have to make a choice based on incomplete and indeterminate data. To deal with indeterminate and inconsistent information, a neutrosophic set (NS) [1] can express and handle the indeterminate and inconsistent information, while an intuitionistic fuzzy set (IFS) and an interval-valued IFS cannot. In an NS, the membership functions of its truth, falsity, and indeterminacy are in a real standard or nonstandard interval $]^{-}0, 1^{+}[$. Because the NS concept was introduced by Smarandache [1] from a philosophical point of view, it implies the difficulty of real scientific and engineering applications. Thus, the concepts of a single-valued NS [1,2] and an interval NS [3] were presented as the NS subclasses to be easily used for actual applications. After that, a simplified NS [4] was presented as an NS subclass, including the concepts of single-valued and interval NSs, which is the extension of an IFS and an interval-valued IFS. Recently, single-valued and interval NSs, and simplified NSs have received great attention and obtained their applications [5–13]. Some basic operations [1–3] on NSs have been introduced, including "intersection", "union", "supplement", "multiplication", "addition", and so on. Then, Ye [4] defined some basic operations of simplified NSs, as well as weighted arithmetic and geometric averaging operators of simplified NSs for aggregating simplified neutrosophic information, and used the two aggregation operators for multiple attribute decision-making (MADM). However, Zhang et al. [13] found shortcomings in some of the operational laws of single-valued NSs, and improved some of the operational laws of interval NSs and some aggregation operators of interval NSs for MADM problems. Liu and Wang [14] further introduced normalized weighted Bonferroni mean operators of single-valued NSs for MADM problems. Furthermore, some generalized neutrosophic number Hamacher aggregation (GNNHA) operators were put forward by Liu et al. [15], and applied to group decision making. Ye [16] proposed interval neutrosophic ordered weighted aggregation operators, and the possibility degree ranking method for interval neutrosophic MADM problems. Liu and Wang [17] presented an interval neutrosophic prioritized ordered weighted averaging operator for MADM. Liu and Teng [18] further put forward a normal neutrosophic generalized weighted power averaging operator for MADM. Sun et al. [19] proposed an interval neutrosophic numbers Choquet integral operator for MADM. Zhao et al. [20] presented a generalized weighted aggregation operator for interval NSs and applied it to MADM. Furthermore, Ye [21] presented credibility-induced interval neutrosophic weighted arithmetic and geometric average operators, and a projection measure-based ranking method for interval neutrosophic numbers (NNs), and then applied them to interval neutrosophic MADM problems with credibility information. Wu et al. [22] introduced a cross-entropy and prioritized aggregation operator of simplified NNs (basic elements in a simplified NS) and applied them to MADM problems. Zhang et al. [23] presented an outranking method for multicriteria decision-making problems with interval-valued neutrosophic information.

However, in the existing literature, we notice that the basic element in the weighted geometric average operator of simplified NNs is composed of a positive real number and simplified NNs, including single-valued and interval NNs. Then, in an intuitionistic fuzzy number (IFN) (a basic element in IFS) environment, Gou et al. [24] defined exponential operational laws of IFNs, where the positive real numbers are used as the bases and IFNs are used as the exponents, and proposed an intuitionistic fuzzy exponential aggregation method for MADM problems. In existing simplified neutrosophic (including single-valued and interval neutrosophic) MADM problems, positive real numbers are used as the exponents (weights) in all the existing exponential operations and their aggregation methods for simplified NNs. In fact, when decision-makers give the weighted values of attributes importance in a complex decision-making process, the weighted values also imply incomplete, indeterminate, and inconsistent information, due to the indeterminacy and inconsistency of the decision-makers' thinking patterns for the complexity and ambiguity in the decision-making process. Then, the single-valued NN is concerned with truth, falsity, and indeterminacy degrees, while the IFN is only concerned with truth and falsity degrees. Hence, the single-valued NN contains more information than the IFN and extends the IFN concept. However, IFNs cannot express indeterminate and inconsistent information, while single-valued NNs can express incomplete, indeterminate, and inconsistent information, and are very suitable for the expression of truth, falsity, and indeterminacy degrees for the attribute weights in indeterminate decision-making situations. Unfortunately, all existing studies do not present the operational laws and aggregation operators using single-valued neutrosophic exponential weights in a single-valued neutrosophic environment. Motivated by the exponential operational law of IFNs and the corresponding aggregation method of intuitionistic fuzzy information [24], it is necessary that exponential operational laws of single-valued NNs and a corresponding single-valued neutrosophic aggregation method are developed as an important supplement to the existing simplified neutrosophic aggregation techniques. To present the operational laws and aggregation operators using single-valued neutrosophic exponential weights in a single-valued neutrosophic environment, this paper first defines new exponential operational laws of single-valued NSs and single-valued NNs, where positive real numbers are used as the bases and single-valued NSs and single-valued NNs are used as the exponents, and proposes a single-valued neutrosophic weighted exponential aggregation (SVNWEA) operator based on the exponential operational laws of single-valued NNs, and discusses its properties. Then, we develop a single-valued neutrosophic MADM method by using the SVNWEA operator. In the MADM problem with single-valued neutrosophic weight information, the data in the decision matrix that is given by decision-makers (DMs) are given by positive real numbers, and the attribute weights are provided by single-valued NNs. However, the traditional aggregation operators of single-valued NNs cannot deal with such a decision-making problem, while the proposed exponential aggregation operator of single-valued NNs can effectively solve this issue.

The rest of the paper is constructed below. Section 2 introduces some basic notation for single-valued NSs. Section 3 proposes some exponential operational laws of single-valued NSs and single-valued NNs. Section 4 presents the exponential aggregation operator of single-valued NNs by using the exponential operational laws of single-valued NNs, and discusses its properties. A MADM method is developed by using the SVNWEA operator in Section 5. Section 6 provides an illustrative example to show the applicability of the presented method. Section 7 contains some conclusions and future research directions.

2. Preliminaries of Single-Valued NSs

For the expression of indeterminate and inconsistent information in the real world, Smarandache [1] presented NSs from a philosophical point of view. For a NS B in a universe of discourse U, it is described independently by its truth, falsity, and indeterminacy membership functions $T_B(x)$, $F_B(x)$, and $I_B(x)$ in the real standard interval [0, 1] or the nonstandard interval $]^{-}0, 1^{+}[$, such that $T_B(x)$: $U \to\ ]^{-}0, 1^{+}[$, $F_B(x)$: $U \to\ ]^{-}0, 1^{+}[$, $I_B(x)$: $U \to\ ]^{-}0, 1^{+}[$, and $^{-}0 \le \sup T_B(x) + \sup I_B(x) + \sup F_B(x) \le 3^{+}$ for $x \in U$.

Obviously, an NS is difficult to apply in a practical problem since its three functions lie in the nonstandard interval $]^{-}0, 1^{+}[$. Thus, the concept of a single-valued NS was introduced by Smarandache [1] and Wang et al. [2]. When the three functions in an NS can be constrained in the real standard interval [0, 1] as an NS subclass, it is easily applied in real science and engineering areas. Then, the definition of a single-valued NS [1,2] is introduced below.

Definition 1. [1,2]. *Let U be a universe of discourse. A single-valued NS S in U is described independently by its truth, falsity, and indeterminacy membership functions $T_S(x)$, $F_S(x)$, $I_S(x)$, where $T_S(x)$, $I_S(x)$, $F_S(x) \in [0, 1]$ satisfy the condition $0 \le T_S(x) + I_S(x) + F_S(x) \le 3$ for $x \in U$. Then, the single-valued NS S can be denoted as $S = \{\langle x, T_S(x), I_S(x), F_S(x)\rangle | x \in U\}$.*

For convenience, a basic element $\langle x, T_S(x), I_S(x), F_S(x)\rangle$ in a single-valued NS S is denoted by $s = <T_s, I_s, F_s>$ for short, which is called a single-valued NN.

Suppose that two single-valued NNs are $s_1 = \langle T_{s_1}, I_{s_1}, F_{s_1}\rangle$ and $s_2 = \langle T_{s_2}, I_{s_2}, F_{s_2}\rangle$, then there are the following relations [2,13,14]:

(1) $s_1^c = \langle F_{s_1}, 1 - I_{s_1}, T_{s_1}\rangle$ (the complement of s_1);
(2) $s_1 \subseteq s_2$ if and only if $T_{s_1} \le T_{s_2}$, $I_{s_1} \ge I_{s_2}$, and $F_{s_1} \ge F_{s_2}$;
(3) $s_1 = s_2$ if and only if $s_2 \subseteq s_1$ and $s_1 \subseteq s_2$;
(4) $s_1 \oplus s_2 = \langle T_{s_1} + T_{s_2} - T_{s_1}T_{s_2}, I_{s_1}I_{s_2}, F_{s_1}F_{s_2}\rangle$;
(5) $s_1 \otimes s_2 = \langle T_{s_1}T_{s_2}, I_{s_1} + I_{s_2} - I_{s_1}I_{s_2}, F_{s_1} + F_{s_2} - F_{s_1}F_{s_2}\rangle$;
(6) $\rho s_1 = \left\langle 1 - (1 - T_{s_1})^{\rho}, (I_{s_1})^{\rho}, (F_{s_1})^{\rho}\right\rangle$ for $p > 0$; and
(7) $(s_1)^{\rho} = \left\langle (T_{s_1})^{\rho}, 1 - (1 - I_{s_1})^{\rho}, 1 - (1 - F_{s_1})^{\rho}\right\rangle$ for $p > 0$.

For any single-valued NN $s = <T_s, I_s, F_s>$, its score and accuracy functions [13,14] can be introduced, respectively, as follows:

$$P(s) = (2 + T_s - I_s - F_s)/3, \quad P(s) \in [0,1], \tag{1}$$

$$Q(s) = T_s - F_s, \quad Q(s) \in [-1,1]. \tag{2}$$

According to the two functions $P(s)$ and $Q(s)$, the comparison and ranking of two single-valued NNs is introduced by the following definition [13,14].

Definition 2. *Let* $s_1 = \langle T_{s_1}, I_{s_1}, F_{s_1} \rangle$ *and* $s_2 = \langle T_{s_2}, I_{s_2}, F_{s_2} \rangle$ *be two single-valued NNs. We can define the following ranking method:*

(1) If $P(s_1) > P(s_2)$, then $s_1 > s_2$;
(2) If $P(s_1) = P(s_2)$ and $Q(s_2) > Q(s_1)$, then $s_2 > s_1$;
(3) If $P(s_1) = P(s_2)$ and $Q(s_1) = Q(s_2)$, then $s_2 = s_1$.

Let $s_i = \langle T_{s_i}, I_{s_i}, F_{s_i} \rangle$ ($i = 1, 2, \ldots, n$) be a collection of single-valued NNs. Based on the weighted aggregation operators of interval NNs [13], the single-valued neutrosophic weighted arithmetic averaging operator (SVNWAA) and the single-valued neutrosophic weighted geometric averaging operator (SVNWGA) can be introduced as follows:

$$SVNWAA(s_1, s_2, \ldots, s_n) = \sum_{i=1}^{n} w_i s_i = \left\langle 1 - \prod_{i=1}^{n} (1 - T_{s_i})^{w_i}, \prod_{i=1}^{n} (I_{s_i})^{w_i}, \prod_{i=1}^{n} (F_{s_i})^{w_i} \right\rangle, \tag{3}$$

$$SVNWGA(s_1, s_2, \ldots, s_n) = \prod_{i=1}^{n} (s_i)^{w_i} = \left\langle \prod_{i=1}^{n} (T_{s_i})^{w_i}, 1 - \prod_{i=1}^{n} (1 - I_{s_i})^{w_i}, 1 - \prod_{i=1}^{n} (1 - F_{s_i})^{w_i} \right\rangle, \tag{4}$$

where w_i ($i = 1, 2, \ldots, n$) is the weight of s_i ($i = 1, 2, \ldots, n$), $w_i \in [0, 1]$, and $\sum_{i=1}^{n} w_i = 1$.

3. Exponential Operational Laws of Single-Valued NSs and Single-Valued NNs

As a supplement to the existing operational laws of single-valued NSs, this section defines new exponential operational laws of single-valued NSs and single-valued NNs, where the positive real numbers are used as the bases and the single-valued NSs and single-valued NNs are used as the exponents.

Definition 3. *Let S = {<x, $T_S(x)$, $I_S(x)$, $F_S(x)$> | x ∈ U} be a single-valued NS in a universe of discourse U. Then, an exponential operational law of the single-valued NS S is defined as*

$$\mu^S = \begin{cases} \left\{ \left\langle x, \mu^{1-T_S(x)}, 1 - \mu^{I_S(x)}, 1 - \mu^{F_S(x)} \right\rangle \middle| x \in U \right\}, & \mu \in (0,1) \\ \left\{ \left\langle x, (1/\mu)^{1-T_S(x)}, 1 - (1/\mu)^{I_S(x)}, 1 - (1/\mu)^{F_S(x)} \right\rangle \middle| x \in U \right\}, & \mu \geq 1 \end{cases} \tag{5}$$

where $T_S(x) \in [0, 1]$, $I_S(x) \in [0, 1]$, $F_S(x) \in [0, 1]$, and $0 \leq T_S(x) + I_S(x) + F_S(x) \leq 3$ for $x \in U$. Obviously, μ^S is also a single-valued NS. Let us discuss the following two cases:

(1) If $\mu \in [0, 1]$, the membership functions of the truth, indeterminacy, and falsity are$\mu^{1-T_S(x)}[0,1]$, $\mu^{I_S(x)}[0,1]$, and $\mu^{F_S(x)}[0,1]$ for any $x \in U$, respectively. Thus, $\left\{ \left\langle x, \mu^{1-T_S(x)}, 1 - \mu^{I_S(x)}, 1 - \mu^{F_S(x)} \right\rangle \middle| x \in U \right\}$ is a single-valued NS.
(2) If $\mu \geq 1$, then there is $0 \leq 1/\mu \leq 1$. It is obvious that $\left\{ \left\langle x, (1/\mu)^{1-T_S(x)}, 1 - (1/\mu)^{I_S(x)}, 1 - (1/\mu)^{F_S(x)} \right\rangle \middle| x \in U \right\}$ is also a single-valued NS.

Similarly, we can also propose an operational law for a single-valued NN.

Definition 4. *Let s = <T_s, I_s, F_s> be a single-valued NN, then an exponential operational law of the single-valued NN s is defined as follows:*

$$\mu^s = \begin{cases} \langle \mu^{1-T_s}, 1 - \mu^{I_s}, 1 - \mu^{F_s} \rangle, & \mu \in (0,1) \\ \left\langle (1/\mu)^{1-T_s}, 1 - (1/\mu)^{I_s}, 1 - (1/\mu)^{F_s} \right\rangle, & \mu \geq 1 \end{cases} \tag{6}$$

It is obvious that μ^s is also a single-valued NN. Let us consider the following example.

Example 1. *Assume that a single-valued NN s is s = <0.5, 0.3, 0.2>, and μ_1 = 0.6 and μ_2 = 5 are two real numbers. Then, according to Definition 4, we can obtain*

$$\mu_1^s = 0.6^{<0.5,0.3,0.2>} = \left\langle 0.6^{1-0.5}, 1-0.6^{0.3}, 1-0.6^{0.2} \right\rangle = \langle 0.7746,\ 0.1421,\ 0.0971 \rangle,$$

$$\mu_2^s = 5^{<0.5,0.3,0.2>} = \left\langle (1/5)^{1-0.5}, 1-(1/5)^{0.3}, 1-(1/5)^{0.2} \right\rangle = \langle 0.4472,\ 0.3830,\ 0.2752 \rangle.$$

In the following, the properties of the exponential operational laws of single-valued NNs only are discussed when $\mu \in (0, 1)$ because the properties of μ^s for $\mu \in (0, 1)$ are almost the same as $\mu \geq 1$.

Theorem 1. *Let $s_1 = \langle T_{s_1}, I_{s_1}, F_{s_1} \rangle$ and $s_2 = \langle T_{s_2}, I_{s_2}, F_{s_2} \rangle$ be two single-valued NNs, and $\mu \in (0, 1)$. Then, there are the following commutative laws:*

(1) $\mu^{s_2} \oplus \mu^{s_1} = \mu^{s_1} \oplus \mu^{s_2}$;
(2) $\mu^{s_2} \otimes \mu^{s_1} = \mu^{s_1} \otimes \mu^{s_2}$.

Obviously, the commutative laws are true. Their proofs are omitted here.

Theorem 2. *Let $s_1 = \langle T_{s_1}, I_{s_1}, F_{s_1} \rangle$, $s_2 = \langle T_{s_2}, I_{s_2}, F_{s_2} \rangle$, and $s_3 = \langle T_{s_3}, I_{s_3}, F_{s_3} \rangle$ be three single-valued NNs, and $\mu \in (0, 1)$. Then, there are the following associative laws:*

(1) $(\mu^{s_2} \oplus \mu^{s_3}) \oplus \mu^{s_1} = \mu^{s_2} \oplus (\mu^{s_3} \oplus \mu^{s_1})$;
(2) $(\mu^{s_2} \otimes \mu^{s_3}) \otimes \mu^{s_1} = \mu^{s_2} \otimes (\mu^{s_3} \otimes \mu^{s_1})$.

Obviously, the associative laws are also true. Their proofs are omitted here.

Theorem 3. *Let s = $<T_s, I_s, F_s>$ be a single-valued NN. If $\mu_1 \geq \mu_2$, then one can obtain $(\mu_1)^s \geq (\mu_2)^s$ for μ_1, $\mu_2 \in (0, 1)$ and $(\mu_1)^s \leq (\mu_2)^s$ for μ_1, $\mu_2 \geq 1$.*

Proof. When $\mu_1 \geq \mu_2$ and μ_1, $\mu_2 \in (0, 1)$, based on Definition 4, we can obtain

$$(\mu_1)^s = \left\langle (\mu_1)^{1-T_s}, 1-(\mu_1)^{I_s}, 1-(\mu_1)^{F_s} \right\rangle \text{and} (\mu_2)^s = \left\langle (\mu_2)^{1-T_s}, 1-(\mu_2)^{I_s}, 1-(\mu_2)^{F_s} \right\rangle.$$

Since $(\mu_1)^{1-T_s} \geq (\mu_2)^{1-T_s}$, $1-(\mu_1)^{I_s} \leq 1-(\mu_2)^{I_s}$, and $1-(\mu_1)^{F_s} \leq 1-(\mu_2)^{F_s}$ for $\mu_1 \geq \mu_2$, their scores are$P((\mu_1)^s) = \left[(\mu_1)^{1-T_s} + 2 - (1-(\mu_1)^{I_s}) - (1-(\mu_1)^{F_s})\right]/3 = \left[(\mu_1)^{1-T_s} + (\mu_1)^{I_s} + (\mu_1)^{F_s}\right]/3$ and $P((\mu_2)^s) = \left[(\mu_2)^{1-T_s} + 2 - (1-(\mu_2)^{I_s}) - (1-(\mu_2)^{F_s})\right]/3 = \left[(\mu_2)^{1-T_s} + (\mu_2)^{I_s} + (\mu_2)^{F_s}\right]/3$. Thus, there are the following two cases:

(a) If $P((\mu_2)^s) > P((\mu_1)^s)$, then $(\mu_2)^s > (\mu_1)^s$.
(b) If $P((\mu_2)^s) = P((\mu_1)^s)$, then we can only obtain $(\mu_1)^{1-T_s} = (\mu_2)^{1-T_s}$, $1-(\mu_1)^{I_s} = 1-(\mu_2)^{I_s}$, and $1-(\mu_1)^{F_s} = 1-(\mu_2)^{F_s}$for $\mu_1 = \mu_2$, i.e., $(\mu_1)^s = (\mu_2)^s$.

According to the above two cases, there is $(\mu_1)^s \geq (\mu_2)^s$. Then, when μ_1, $\mu_2 \geq 1$ and $\mu_1 \geq \mu_2$, we can know $0 < 1/\mu_1 \leq 1/\mu_2 \leq 1$. As discussed above, we can obtain $(\mu_1)^s \leq (\mu_2)^s$. This completes the proof. □

Meanwhile, μ^s implies some special values when μ takes some values:

(1) If μ = 1, then $\mu^s = \langle \mu^{1-T_s}, 1-\mu^{I_s}, 1-\mu^{F_s} \rangle = \langle 1, 0, 0 \rangle$ for every single-valued NN s;
(2) If s = <1, 0, 0>, then $\mu^s = \langle \mu^{1-T_s}, 1-\mu^{I_s}, 1-\mu^{F_s} \rangle = \langle 1, 0, 0 \rangle$ for every value of μ;
(3) If s = <0, 1, 1>, then $\mu^s = \langle \mu^{1-T_s}, 1-\mu^{I_s}, 1-\mu^{F_s} \rangle = \langle \mu, 1-\mu, 1-\mu \rangle$ for every value of μ.

4. Single-Valued Neutrosophic Weighted Exponential Aggregation Operator

Based on Definition 4, this section develops a SVNWEA operator, where the bases are a collection of positive real numbers of μ_i (i =1, 2, . . . , n) and the exponents are a collection of single-valued NNs for $s_i = \langle T_{s_i}, I_{s_i}, F_{s_i} \rangle$ (i =1, 2, . . . , n).

Definition 5. *Let $s_i = \langle T_{s_i}, I_{s_i}, F_{s_i} \rangle$ for i = 1, 2, . . . , n be a collection of single-valued NNs, $\mu_i \in$ (0, 1) for i = 1, 2, . . . , n, and SVNWEA: $\Omega^n \to \Omega$. If the function SVNWEA is defined as*

$$SVNWEA(s_1, s_2, \cdots, s_n) = \prod_{i=1}^{n} (\mu_i)^{s_i}, \tag{7}$$

then the function SVNWEA is called a SVNWEA operator, where s_i (i = 1, 2, . . . , n) is the exponential weight of μ_i (i = 1, 2, . . . , n).

Theorem 4. *Let $s_i = \langle T_{s_i}, I_{s_i}, F_{s_i} \rangle$ for i = 1, 2, . . . , n be a collection of single-valued NNs. Then the aggregated value of the SVNWEA operator is a single-valued NN, where*

$$SVNWEA(s_1, s_2, \cdots, s_n) = \left\langle \prod_{i=1}^{n} (\mu_i)^{1-T_{s_i}}, 1 - \prod_{i=1}^{n} (\mu_i)^{I_{s_i}}, 1 - \prod_{i=1}^{n} (\mu_i)^{F_{s_i}} \right\rangle \tag{8}$$

and s_i (i = 1, 2, . . . , n) is the exponential weight of μ_i (i = 1, 2, . . . , n).

Proof. By using mathematical induction, we can prove Equation (8).

(1) When n = 2, we have

$$\begin{aligned} & SVNWEA(s_1, s_2) = (\mu_1)^{s_1} \otimes (\mu_2)^{s_1} \\ & = \left\langle \begin{array}{l} (\mu_1)^{1-T_{s_1}} (\mu_2)^{1-T_{s_2}}, 1 - (\mu_1)^{I_{s_1}} + 1 - (\mu_2)^{I_{s_2}} - \left(1 - (\mu_1)^{I_{s_1}}\right)\left(1 - (\mu_2)^{I_{s_2}}\right), \\ 1 - (\mu_1)^{F_{s_1}} + 1 - (\mu_2)^{F_{s_2}} - \left(1 - (\mu_1)^{F_{s_1}}\right)\left(1 - (\mu_2)^{F_{s_2}}\right) \end{array} \right\rangle \\ & = \left\langle \prod_{i=1}^{2} (\mu_i)^{1-T_{s_i}}, 1 - \prod_{i=1}^{2} (\mu_i)^{I_{s_i}}, 1 - \prod_{i=1}^{2} (\mu_i)^{F_{s_i}} \right\rangle. \end{aligned} \tag{9}$$

(2) When n = k, according to Equation (8) there is the following formula:

$$SVNWEA(s_1, s_2, \cdots, s_k) = \left\langle \prod_{i=1}^{k} (\mu_i)^{1-T_{s_i}}, 1 - \prod_{i=1}^{k} (\mu_i)^{I_{s_i}}, 1 - \prod_{i=1}^{k} (\mu_i)^{F_{s_i}} \right\rangle. \tag{10}$$

(3) When n = k+1, we have the following results based on the operational laws of Definition 4 and combining (9) and (10)

$$\begin{aligned} & SVNWEA(s_1, s_2, \cdots, s_k, s_{k+1}) = \left\langle \prod_{i=1}^{k} (\mu_i)^{1-T_{s_i}}, 1 - \prod_{i=1}^{k} (\mu_i)^{I_{s_i}}, 1 - \prod_{i=1}^{k} (\mu_i)^{F_{s_i}} \right\rangle \otimes (\mu_{k+1})^{s_{k+1}} \\ & = \left\langle \prod_{i=1}^{k+1} (\mu_i)^{1-T_{s_i}}, 1 - \prod_{i=1}^{k+1} (\mu_i)^{I_{s_i}}, 1 - \prod_{i=1}^{k+1} (\mu_i)^{F_{s_i}} \right\rangle. \end{aligned}$$

Therefore, for the above results we can obtain that Equation (8) holds for any n.
Thus, the proof is completed. □

It is obvious that the SVNWEA operator contains the following properties:

(1) Boundedness: Let $s_i = \langle T_{s_i}, I_{s_i}, F_{s_i} \rangle$ (i = 1, 2, ..., n) be a collection of single-valued NNs, and let $s_{\min} = \left\langle \min_i(T_{s_i}), \max_i(I_{s_i}), \max_i(F_{s_i}) \right\rangle$, $s_{\max} = \left\langle \max_i(T_{s_i}), \min_i(I_{s_i}), \min_i(F_{s_i}) \right\rangle$ for i = 1, 2, ..., n, $s^+ = SVNWEA(s_{\max}, s_{\max}, \cdots, s_{\max}) = \left\langle \prod_{i=1}^{n} (\mu_i)^{1-\max_i(T_{s_i})}, 1 - \prod_{i=1}^{n} (\mu_i)^{\min_i(I_{s_i})}, 1 - \prod_{i=1}^{n} (\mu_i)^{\min_i(F_{s_i})} \right\rangle$, $s^- = SVNWEA(s_{\min}, s_{\min}, \cdots, s_{\min}) = \left\langle \prod_{i=1}^{n} (\mu_i)^{1-\min_i(T_{s_i})}, 1 - \prod_{i=1}^{n} (\mu_i)^{\max_i(I_{s_i})}, 1 - \prod_{i=1}^{n} (\mu_i)^{\max_i(F_{s_i})} \right\rangle$, then there is $s^- \leq SVNWEA(s_1, s_2, \cdots, s_n) \leq s^+$.

(2) Monotonicity: Let $s_i = \langle T_{s_i}, I_{s_i}, F_{s_i} \rangle$ and $s_i^* = \left\langle T_{s_i^*}, I_{s_i^*}, F_{s_i^*} \right\rangle$for i = 1, 2, ..., n be two collections of single-valued NNs. If $s_i \leq s_i^*$, then $SVNWEA(s_1, s_2, \cdots, s_n) \leq SVNWEA(s_1^*, s_2^*, \cdots, s_n^*)$.

Proof. (1) For any i, we have $\min_i(T_{s_i}) \leq T_{s_i} \leq \max_i(T_{s_i})$, $\min_i(I_{s_i}) \leq I_{s_i} \leq \max_i(I_{s_i})$, and $\min_i(F_{s_i}) \leq F_{s_i} \leq \max_i(F_{s_i})$. Then $\prod_{i=1}^{n} (\mu_i)^{1-T_{s_i}} \geq \prod_{i=1}^{n} (\mu_i)^{1-\min_i(T_{s_i})}$, $\prod_{i=1}^{n} (\mu_i)^{1-T_{s_i}} \leq \prod_{i=1}^{n} (\mu_i)^{1-\max_i(T_{s_i})}$, $1 - \prod_{i=1}^{n} (\mu_i)^{I_{s_i}} \leq 1 - \prod_{i=1}^{n} (\mu_i)^{\max_i(I_{s_i})}$, $1 - \prod_{i=1}^{n} (\mu_i)^{I_{s_i}} \geq 1 - \prod_{i=1}^{n} (\mu_i)^{\min_i(I_{s_i})}$, $1 - \prod_{i=1}^{n} (\mu_i)^{F_{s_i}} \leq 1 - \prod_{i=1}^{n} (\mu_i)^{\max_i(F_{s_i})}$, and $1 - \prod_{i=1}^{n} (\mu_i)^{F_{s_i}} \geq 1 - \prod_{i=1}^{n} (\mu_i)^{\min_i(F_{s_i})}$.

Let $SVNWEA(s_1, s_2, \cdots, s_n) = s = \langle T_s, I_s, F_s \rangle$, $s^- = \langle T_{s^-}, I_{s^-}, F_{s^-} \rangle$, and $s^+ = \langle T_{s^+}, I_{s^+}, F_{s^+} \rangle$, then their score values are as follows:

$$\begin{aligned} &P(s) = (2 + T_s - I_s - F_s)/3 = \tfrac{1}{3}\left[\prod_{i=1}^{n} (\mu_i)^{1-T_{s_i}} + \prod_{i=1}^{n} (\mu_i)^{I_{s_i}} + \prod_{i=1}^{n} (\mu_i)^{F_{s_i}}\right] \\ &\geq P(s^-) = (2 + T_{s^-} - I_{s^-} - F_{s^-})/3 = \tfrac{1}{3}\left[\prod_{i=1}^{n} (\mu_i)^{1-\min_i(T_{s_i})} + \prod_{i=1}^{n} (\mu_i)^{\max_i(I_{s_i})} + \prod_{i=1}^{n} (\mu_i)^{\max_i(I_{s_i})}\right] \end{aligned},$$

$$\begin{aligned} &P(s) = (2 + T_s - I_s - F_s)/3 = \tfrac{1}{3}\left[\prod_{i=1}^{n} (\mu_i)^{1-T_{s_i}} + \prod_{i=1}^{n} (\mu_i)^{I_{s_i}} + \prod_{i=1}^{n} (\mu_i)^{F_{s_i}}\right] \\ &\leq P(s^+) = (2 + T_{s^+} - I_{s^+} - F_{s^+})/3 = \tfrac{1}{3}\left[\prod_{i=1}^{n} (\mu_i)^{1-\max_i(T_{s_i})} + \prod_{i=1}^{n} (\mu_i)^{\min_i(I_{s_i})} + \prod_{i=1}^{n} (\mu_i)^{\min_i(I_{s_i})}\right] \end{aligned}.$$

Thus, there are the following three cases:

(a) If $P(s^-) < P(s) < P(s^+)$, then $s^- < SVNWEA(s_1, s_2, \cdots, s_n) < s^+$ holds obviously.

(b) If $P(s) = P(s^-)$, then there is $2 + T_s - I_s - F_s = 2 + T_{s^-} - I_{s^-} - F_{s^-}$. Thus, we can obtain $T_s = T_{s^-}$, $F_s = F_{s^-}$, and $I_s = I_{s^-}$. Hence, there is $Q(s) = T_s - F_s = T_{s^-} - F_{s^-} = Q(s^-)$. Based on Definition 2, we have $s = SVNWEA(s_1, s_2, \cdots, s_n) = s^-$.

(c) If $P(s) = P(s^+)$, then there is $2 + T_s - I_s - F_s = 2 + T_{s^+} - I_{s^+} - F_{s^+}$. Thus, we can obtain $T_s = T_{s^+}$, $F_s = F_{s^+}$, and $I_s = I_{s^+}$. Hence, there is $Q(s) = T_s - F_s = T_{s^+} - F_{s^+} = Q(s^+)$. Based on Definition 2, we have $s = SVNWEA(s_1, s_2, \cdots, s_n) = s^+$.

Based on the above three cases, there is $s^- \leq SVNWEA(s_1, s_2, \cdots, s_n) \leq s^+$.

(2) If $s_i \leq s_i^*$, this implies $T_{s_i} \leq T_{s_i^*}$, $I_{s_i} \geq I_{s_i^*}$, and $F_{s_i} \geq F_{s_i^*}$ for any i. Then, we have $\mu^{1-T_{s_i}} \leq \mu^{1-T_{s_i^*}}$, $\mu^{I_{s_i}} \geq \mu^{I_{s_i^*}}$, and $\mu^{F_{s_i}} \geq \mu^{F_{s_i^*}}$. Hence, there are $\prod_{i=1}^{n} \mu^{1-T_{s_i}} \leq \prod_{i=1}^{n} \mu^{1-T_{s_i^*}}$, $1 - \prod_{i=1}^{n} \mu^{I_{s_i}} \leq 1 - \prod_{i=1}^{n} \mu^{I_{s_i^*}}$, and $1 - \prod_{i=1}^{n} \mu^{F_{s_i}} \leq 1 - \prod_{i=1}^{n} \mu^{F_{s_i^*}}$.

Let $SVNWEA(s_1, s_2, \cdots, s_n) = s = \langle T_s, I_s, F_s \rangle$ and $s^* = \langle T_{s^*}, I_{s^*}, F_{s^*} \rangle$. Then, according to Equation (1), the score values are as follows:

$$\begin{aligned} P(s) &= (2+T_s-I_s-F_s)/3 = \frac{1}{3}\left[\prod_{i=1}^{n}\mu^{1-T_{s_i}}+\prod_{i=1}^{n}\mu^{I_{s_i}}+\prod_{i=1}^{n}\mu^{F_{s_i}}\right] \\ &\le P(s^*) = (2+T_{s^*}-I_{s^*}-F_{s^*})/3 = \frac{1}{3}\left[\prod_{i=1}^{n}\mu^{1-T_{s_i^*}}+\prod_{i=1}^{n}\mu^{I_{s_i^*}}+\prod_{i=1}^{n}\mu^{F_{s_i^*}}\right] \end{aligned}.$$

Hence, there are the following two cases:

(a) If $P(s) < P(s^*)$, then there is $SVNWEA(s_1, s_2, \cdots, s_n) < SVNWEA(s_1^*, s_2^*, \cdots, s_n^*)$.

(b) If $P(s) = P(s^*)$, then there is $2 + T_s - I_s - F_s = 2 + T_{s^*} - I_s - F_{s^*}$. Thus, we can obtain $T_s = T_{s^*}$, $F_s = F_{s^*}$, and $I_s = I_{s^*}$. Hence, there is $Q(s) = T_s - F_s = T_{s^*} - F_{s^*} = Q(s^*)$. Based on Definition 2, we have $SVNWEA(s_1, s_2, \cdots, s_n) = SVNWEA(s_1^*, s_2^*, \cdots, s_n^*)$.

According to the above two cases, there is $SVNWEA(s_1, s_2, \cdots, s_n) \le SVNWEA(s_1^*, s_2^*, \cdots, s_n^*)$. Therefore, we complete the proofs. □

5. MADM Method Based on the SVNWEA Operator

By the SVNWEA operator, we can deal with some MADM problems, where the weight of an attribute can be expressed as a single-valued NN s_i (i = 1, 2, ..., n), and $\mu_i \in (0, 1)$ (i = 1, 2, ..., n) can be represented as the attribute value for an alternative. Thus, we can establish a MADM method.

In a MADM problem, let $B = \{B_1, B_2, \ldots, B_m\}$ be a set of alternatives and $R = \{R_1, R_2, \ldots, R_n\}$ be a set of attributes. The suitable judgment (satisfaction evaluation) of an attribute R_i (i = 1, 2, ..., n) for an alternative B_j (j = 1, 2, ..., m) is given by decision-makers, and then the evaluation values are expressed by a positive real/fuzzy number $\mu_{ji} \in (0, 1)$ (j = 1, 2, ..., m; i = 1, 2, ..., n). Therefore, a decision matrix $D = (\mu_{ji})_{m \times n}$ can be established. Then, the single-valued NN $s_i = \langle T_{s_i}, I_{s_i}, F_{s_i}\rangle$ is given as the weight of the attribute R_i (i = 1, 2, ..., n), where $T_{s_i} \in [0, 1]$ indicates the degree to which a DM prefers the attribute R_i, $I_{s_i} \in [0, 1]$ indicates the indeterminate degree to which a DM prefers/does not prefer the attribute R_i, and $F_{s_i} \in [0, 1]$ indicates the degree to which a DM does not prefer the attribute R_i. Then, the decision-making steps are described below:

Step 1: Use Equation (8) to get the overall attribute value d_j = SVNWEA($s_1, s_2, \ldots, s_n$) (j = 1, 2, ..., m) for the alternatives B_j (j = 1, 2, ..., m).

Step 2: Utilize Equation (1) to calculate the score values of $P(d_j)$ (j = 1, 2, ..., m). Then the accuracy degrees of $Q(d_i)$ and $Q(d_j)$ are calculated if the two score values $P(d_i)$ and $P(d_j)$ are equal.

Step 3: According to the score values (the accuracy degrees), the alternatives are ranked and the best one is selected.

Step 4: End.

6. Illustrative Example

This section gives an illustrative example of the selection problem of global suppliers for a manufacturing company as the single-valued neutrosophic MADM problem to show the application and rationality of the presented decision-making method based on the SVNWEA operator.

Let us consider a manufacturing company, which needs to choose the best global supplier corresponding to the core competencies of suppliers. The manufacturing company provides a set of four suppliers $B = \{B_1, B_2, B_3, B_4\}$, which need to satisfy the four attributes: (1) R_1 (the level of technology innovation); (2) R_2 (reputation); (3) R_3 (the ability of management); and (4) R_4 (the level of service).

Then, the suitable judgment (satisfaction evaluation) of an attribute R_i (i = 1, 2, 3, 4) for an alternative B_j (j = 1, 2, 3, 4) is given by decision-makers, and then the evaluation values are represented by the positive real/fuzzy values of $\mu_{ji} \in (0, 1)$ (j, i = 1, 2, 3, 4), which are established by the following decision matrix D:

$$D = \begin{bmatrix} 0.6 & 0.7 & 0.8 & 0.7 \\ 0.6 & 0.8 & 0.7 & 0.9 \\ 0.7 & 0.6 & 0.8 & 0.7 \\ 0.8 & 0.7 & 0.6 & 0.8 \end{bmatrix}.$$

Then, the single-valued neutrosophic weight vector for the four attributes is $S = (s_1, s_2, s_3, s_4)$ = (<0.6, 0.2, 0.4>, <0.7, 0.1, 0.2>, <0.7, 0.2, 0.1>, <0.8, 0.3, 0.2>).

The proposed MADM method is applied to solve the supplier selecting problem, and then its decision-making steps are given as follows:

Step 1: Use Equation (8) to calculate the overall value of attributes for each supplier B_j (j = 1, 2, 3, 4): When j = 1, we can obtain

$$\begin{aligned} d_1 &= SVNWEA(s_1, s_2, s_3, s_4) = \left\langle \prod_{i=1}^{4} (\mu_{1i})^{1-T_{s_i}}, 1 - \prod_{i=1}^{4} (\mu_{1i})^{I_{s_i}}, 1 - \prod_{i=1}^{4} (\mu_{1i})^{F_{s_i}} \right\rangle . \\ &= \langle 0.6379,\ 0.2513,\ 0.3088 \rangle \end{aligned}$$

By the similar calculation, we can obtain the rest of the results:

d_2 = <0.6708, 0.2034, 0.2634>, d_3 = <0.6478, 0.2397, 0.2872>, and d_4 = <0.6743, 0.2207, 0.2261>.

Step 2: Compute the score values of d_j (j = 1, 2, 3, 4) by Equation (1):

$$P(d_1) = 0.6926,\ P(d_2) = 0.7346,\ P(d_3) = 0.7070, \text{ and } P(d_4) = 0.7425.$$

Step 3: Since the ranking order of the score values is $P(d_4) > P(d_2) > P(d_3) > P(d_1)$, the ranking order of the four alternatives is $B_4 > B_2 > B_3 > B_1$. Therefore, B_4 is the best supplier among the four suppliers.

For a convenient comparison, we use the single-valued neutrosophic weighted arithmetic averaging (SVNWAA) operator of Equation (3) to solve the decision-making problem.

Step 1: Use Equation (3) to compute the overall value of attributes for each supplier B_j (j = 1, 2, 3, 4):

When j = 1, we can obtain

$$\begin{aligned} d_1{'} &= SVNWAA(s_1, s_2, s_3, s_4) = \sum_{i=1}^{4} s_i \mu_{1i} = \left\langle 1 - \prod_{i=1}^{n} (1 - T_{s_i})^{\mu_{1i}}, \prod_{i=1}^{n} (I_{s_i})^{\mu_{1i}}, \prod_{i=1}^{n} (F_{s_i})^{\mu_{1i}} \right\rangle \\ &= \langle 0.9693,\ 0.0090,\ 0.0096 \rangle \end{aligned}$$

Similarly, we can calculate the overall attribute values of the rest of the suppliers for B_j (j = 2, 3, 4):

$d_2{'}$ = <0.9777, 0.0066, 0.0075>, $d_3{'}$ = <0.9684, 0.0097, 0.0103>, and $d_4{'}$ = <0.9723, 0.0080, 0.0108>.

Step 2: Compute the score values of $d_j{'}$ (j = 1, 2, 3, 4) by Equation (1) as follows:

$P(d_1{'}) = 0.9835$, $P(d_2{'}) = 0.9879$, $P(d_3{'}) = 0.9828$, and $P(d_4{'}) = 0.9845$.

Step 3: Since the ranking order of the four score values is $P(d_2{'}) > P(d_4{'}) > P(d_1{'}) > P(d_3{'})$, the ranking order of the four alternatives is $B_2 > B_4 > B_1 > B_3$ and the best supplier is B_2.

For comparative convenience between the two methods, their results are shown in Table 1.

Table 1. The results of the two decision-making methods.

Aggregated Method	Score Value	Ranking
SVNWEA operator	$P(d_1) = 0.6926$, $P(d_2) = 0.7346$, $P(d_3) = 0.7070$, $P(d_4) = 0.7425$	$B_4 > B_2 > B_3 > B_1$
SVNWAA operator	$P(d_1{'}) = 0.9835$, $P(d_2{'}) = 0.9879$, $P(d_3{'}) = 0.9828$, $P(d_4{'}) = 0.9845$	$B_2 > B_4 > B_1 > B_3$

From the results of Table 1, we can give the comparative analyses between the SVNWEA operator and the SVNWAA operator as follows:

(1) In Step 1, the SVNWEA operator needs to utilize the attribute weight of the single-valued NN s_i and the characteristic value $\mu_{ji} \in (0, 1)$ of an attribute R_i for an alternative B_j; while the SVNWAA operator needs to exchange the roles of s_j and μ_{ji} to use it, by the attribute weight $\mu_{ji} \in (0, 1)$ and the characteristic value s_i of an attribute R_i for an alternative B_j. Therefore, the SVNWAA operator used in this case is unreasonable while the SVNWEA operator used in this case is reasonable, because we do not change the positions and the meanings of the attribute values and the weights.

(2) The two ranking orders derived by using the SVNWEA operator and the SVNWAA operator are obviously different. The main reason is that the positions and meanings of the attribute values and the weights are necessarily exchanged, respectively, which may result in unreasonable decision-making results.

Furthermore, compared with the existing method introduced in an IFN environment [24], our method uses single-valued neutrosophic weights, which contain truth, falsity, and indeterminacy degrees, and can deal with the incomplete, indeterminate, and inconsistent problems in a decision-making process; the existing method [24] uses intuitionistic fuzzy weights, which contain truth and falsity degrees, and cannot handle the incomplete, indeterminate, and inconsistent decision-making problems. Since an IFN is only a special case of a single-valued NN, our method is the extension of the existing method [24], and then the existing method [24] is only a special case of our method.

Besides, existing NN decision-making methods based on NN weighted aggregation operators also cannot deal with decision-making problems using single-valued neutrosophic exponential weights as presented in this paper.

Obviously, using single-valued neutrosophic weights can make the decision-making process more appropriate and effective, and more suitably reflect the real decision-making process. Therefore, our method not only extends the existing methods, but also provides a new way for decision-makers in a single-valued neutrosophic environment.

7. Conclusions

This paper proposed new exponential operational laws of single-valued NSs and single-valued NNs as a useful supplement to the existing operational laws of single-valued NSs and single-valued NNs. Then, we presented the SVNWEA operator for aggregating single-valued neutrosophic information, and discussed its properties. Next, we established the MADM method by use of the SVNWEA operator to solve MADM problems with single-valued neutrosophic exponential weights. Finally, an illustrative example was provided to show the applicability and rationality of the presented method. In future work, we shall further extend the developed method to interval NNs and apply it to other fields, such as medical diagnosis and image processing.

Acknowledgments: This paper is supported by the National Natural Science Foundation of China (No. 71471172).

Author Contributions: Jun Ye proposed the exponential operational laws of single-valued NSs and single-valued NNs, and the SVNWEA operator; Zhikang Lu presented the MADM method by use of the SVNWEA operator to solve MADM problems with single-valued neutrosophic exponential weights and provided the calculation and analysis of the illustrative example; Jun Ye and Zhikang Lu wrote the paper.

Conflicts of Interest: The authors declare that we have no conflict of interest regarding the publication of this paper.

References

1. Smarandache, F. *Neutrosophy: Neutrosophic Probability, Set, and Logic*; American Research Press: Rehoboth, DE, USA, 1998.
2. Wang, H.; Smarandache, F.; Zhang, Y.Q.; Sunderraman, R. *Interval Neutrosophic Sets and Logic: Theory and Applications in Computing*; Hexis: Phoenix, AZ, USA, 2005.

3. Wang, H.; Smarandache, F.; Zhang, Y.Q.; Sunderraman, R. Single valued neutrosophic sets. *Multisp. Multistruct.* **2010**, *4*, 410–413.
4. Ye, J. A multicriteria decision-making method using aggregation operators for simplified neutrosophic sets. *J. Intell. Fuzzy Syst.* **2014**, *26*, 2459–2466.
5. Ye, J. Multicriteria decision-making method using the correlation coefficient under single-value neutrosophic environment. *Int. J. Gen. Syst.* **2013**, *42*, 386–394. [CrossRef]
6. Ye, J. Similarity measures between interval neutrosophic sets and their applications in multicriteria decision making. *J. Intell. Fuzzy Syst.* **2014**, *26*, 165–172.
7. Ye, J. Single valued neutrosophic cross-entropy for multicriteria decision making problems. *Appl. Math. Model.* **2014**, *38*, 1170–1175. [CrossRef]
8. Peng, J.J.; Wang, J.Q.; Zhang, H.Y.; Chen, X.H. An outranking approach for multi-criteria decision-making problems with simplified neutrosophic sets. *Appl. Soft Comput.* **2014**, *25*, 336–346. [CrossRef]
9. Peng, J.J.; Wang, J.Q.; Wang, J.; Zhang, H.Y.; Chen, X.H. Simplified neutrosophic sets and their applications in multi-criteria group decision-making problems. *Int. J. Syst. Sci.* **2016**, *47*, 2342–2358. [CrossRef]
10. Majumdar, P.; Samant, S.K. On similarity and entropy of neutrosophic sets. *J. Intell. Fuzzy Syst.* **2014**, *26*, 1245–1252.
11. Sahin, R.; Kucuk, A. Subsethood measure for single valued neutrosophic sets. *J. Intell. Fuzzy Syst.* **2015**, *29*, 525–530. [CrossRef]
12. Biswas, P.; Pramanik, S.; Giri, B.C. TOPSIS method for multi-attribute group decision-making under single-valued neutrosophic environment. *Neural Comput. Appl.* **2016**, *27*, 727–737. [CrossRef]
13. Zhang, H.Y.; Wang, J.Q.; Chen, X.H. Interval neutrosophic sets and their application in multicriteria decision making problems. *Sci. World J.* **2014**, 645953. [CrossRef] [PubMed]
14. Liu, P.D.; Wang, Y.M. Multiple attribute decision making method based on single-valued neutrosophic normalized weighted Bonferroni mean. *Neural Comput. Appl.* **2014**, *25*, 2001–2010. [CrossRef]
15. Liu, P.D.; Chu, Y.C.; Li, Y.W.; Chen, Y.B. Some generalized neutrosophic number Hamacher aggregation operators and their application to group decision making. *J. Intell. Fuzzy Syst.* **2014**, *16*, 242–255.
16. Ye, J. Multiple attribute decision-making method based on the possibility degree ranking method and ordered weighted aggregation operators of interval neutrosophic numbers. *J. Intell. Fuzzy Syst.* **2015**, *28*, 1307–1317.
17. Liu, P.D.; Wang, Y.M. Interval neutrosophic prioritized OWA operator and its application to multiple attribute decision making. *J. Syst. Sci. Complex.* **2015**, *29*, 681–697. [CrossRef]
18. Liu, P.D.; Teng, F. Multiple attribute decision making method based on normal neutrosophic generalized weighted power averaging operator. *Int. J. Mach. Learn. Cybern.* **2015**. [CrossRef]
19. Sun, H.X.; Yang, H.X.; Wu, J.Z.; Yao, O.Y. Interval neutrosophic numbers Choquet integral operator for multi-criteria decision making. *J. Intell. Fuzzy Syst.* **2015**, *28*, 2443–2455. [CrossRef]
20. Zhao, A.W.; Du, J.G.; Guan, H.J. Interval valued neutrosophic sets and multi-attribute decision-making based on generalized weighted aggregation operator. *J. Intell. Fuzzy Syst.* **2015**, *29*, 2697–2706.
21. Ye, J. Interval neutrosophic multiple attribute decision-making method with credibility information. *Int. J. Fuzzy Syst.* **2016**, *18*, 914–923. [CrossRef]
22. Wu, X.H.; Wang, J.Q.; Peng, J.J.; Chen, X.H. Cross-entropy and prioritized aggregation operator with simplified neutrosophic sets and their application in multi-criteria decision-making problems. *Int. J. Fuzzy Syst.* **2016**. [CrossRef]
23. Zhang, H.Y.; Wang, J.Q.; Chen, X.H. An outranking approach for multi-criteria decision-making problems with interval-valued neutrosophic sets. *Neural Comput. Appl.* **2016**, *27*, 615–627. [CrossRef]
24. Gou, X.J.; Xu, Z.S.; Lei, Q. New operational laws and aggregation method of intuitionistic fuzzy information. *J. Intell. Fuzzy Syst.* **2016**, *30*, 129–141. [CrossRef]