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Charging Method Selection of a Public Charging Station Using an Interval-Valued Picture Fuzzy Bidirectional Projection Based on VIKOR Method with Unknown Attribute Weights

Chittaranjan Shit and Ganesh Ghorai *

Department of Applied Mathematics, Vidyasagar University, Midnapore 721 102, India; rsmors_mathchitta@mail.vidyasagar.ac.in
* Correspondence: math.ganesh@mail.vidyasagar.ac.in

Abstract: Excessive use of fossil fuel-powered vehicles is a major problem for the entire world today, because of which greenhouse gases are increasing day by day. As a result, climate change and global warming have grown to be serious problems that affect both the environment and life on Earth. However, the effective way of reducing greenhouse gases is to use electric vehicles for commuting. The assessment and selection of the best possible way of charging an electric vehicle is a convoluted decision-making challenge due to the presence of assorted contradictory criteria. Additionally, individual decision makers' minds and insufficient data are obstacles to doing this. In this regard, interval-valued picture fuzzy sets have been considered as a compatible tool to handle vagueness. In this paper, a multi-attribute group decision-making problem with the bidirectional projection-based VlseKriterijumska Optimizacija I Kompromisno Resenje ($VIKOR$) method is considered where the weights are partially known. The objective weights of the attributes in this model are determined using the deviation-based approach. The compromised solution is also assessed using the $VIKOR$ approach. Both the interval-valued image fuzzy Schweizer–Sklar power weighted geometric operator and the interval-valued picture fuzzy Schweizer–Sklar power weighted averaging operator are used in this process. Lastly, a numerical example showing the most suitable way to charge an electric vehicle is given to demonstrate the suggested methodology. To evaluate the robustness and efficacy of the suggested strategy, a comparative analysis with current techniques and a sensitivity analysis of the parameters are also carried out.

Keywords: interval-valued picture fuzzy sets; Schweizer–Sklar operations; bidirectional projection measure; public charging stations; $VIKOR$

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1. Introduction

Climate change and global warming are significant challenges that have impacted the environment and life on Earth in the current situation. One of the main causes of climate change is GHG emissions. Air pollution and greenhouse gas (GHG) emissions from the fossil fuel-based conveyance sector have been higher than ever in recent years, especially in large, crowded cities. Electrified transportation is considered an acceptable solution to reduce GHG. Additionally, it is an alternative way to reduce other environmental impacts such as climate change and improve air quality. Compared with fossil fuel-based cars, electric vehicles (EVs) offer zero carbon emissions, excellent reliability, efficient performance, and low maintenance charges. In addition, EVs will provide the use of renewable energy systems and energy storage systems in public charging stations (PCSs).

Recently, some researchers have conducted some works on this issue. Acharige et al. [1] carried out an extensive analysis of charging technology, standards, architecture, and converter configurations for PCSs. Furthermore, He et al. [2] developed a thorough assessment system and method of charging networks for EVs under the traffic network and power grid. In addition, in the area of charging and discharging strategies [3], smart charging strategies [4], and control strategies [5] of EVs in PCSs have drawn great attention from authors. Further, climate change is having a major effect on the world economy. Extreme weather events and climate-related disasters have become increasingly expensive over time. Gentili [6] presented a list of the features shared by all complex systems involved in the 2030 Agenda and highlighted the reasons why there are certain limitations in the prediction of complex systems' behaviors.

Multi-attribute group decision-making $(MAGDM)$ problems are common in everyday decision-making scenarios. The goal of this method is to choose the best choice among the available options by taking into account a number of attributes. The growing complexity of the socioeconomic environment in many real-world scenarios brings with it a number of issues, including a lack of knowledge and information, uncertainty in the context of decision making, and challenges with information extraction. As a result, decision makers find it difficult to numerically represent the quality values that correspond to the options. However, expressing preference values using fuzzy numbers is more appropriate in decision-making problems. In recent years, theoretical studies of various $MAGDM$ problems have been proposed by authors [7–15].

An essential technique for addressing the uncertainty in decision-making challenges is fuzzy sets (FSs) [16]. Later, intuitionistic fuzzy sets (IFSs) [17], an extension of FSs, express decision making more effectively than FSs. Additionally, Atanassov [18] presented extended IFSs in which an element's membership values are interval numbers. This applied successfully in different areas of supplier selection [19–21], pattern recognition [22,23], data mining [24], medical diagnosis [25,26], and supply chain management [27,28]. However, there are some situations where IFSs or interval-valued intuitionistic fuzzy sets (IvVIFSs) do not work. For example, if a decision maker's chance to select an alternative is 0.4 and the chance to not select is 0.3, there remains a chance that he or she could be undecided. Cuong and Kreinovich [29,30] addressed this issue by introducing the neutral membership value of an element. Interval-valued picture fuzzy sets (IvVPFSs) [31], an extension of picture fuzzy sets (PFSs) and IvVIFSs, provide a solution to this problem. Thus, inspired by IvVPFSs, various authors have developed aggregation operators. The aggregation operators under IvVPFSs are applied in different fields, like supplier selection [32], enterprise resource management [33], employee selection [34], and air quality evaluation [35]. A linguistic variable is defined as a variable whose values are sentences in a natural or artificial language, according to Zadeh [36], who advocated the analysis of complex systems and decision processes. A fuzzy-based multicriteria decision-making technique was presented by Shahmohammad et al. [37] to address the trend, problems, and future directions in decision making. A detailed review of IvVPFSs is given in the literature (Section 3).

The remainder of the paper is structured as follows: Section 2 briefly reviews the fundamental concepts of PFSs, IvVPFSs, SS-TNs, SS-TCNs, and PA operators. In Section 3, we discuss a literature review of some existing works and also identify the research gap. In Section 4, we introduce several SS-TN and SS-TCN operations under IvVPFSs, along with the IvVPFSSPWA and IvVPFSSPWG operators and their associated properties. Additionally, we define various projection measures under IvVPFSs. Section 5 presents a bidirectional projection-based $VIKOR$ to address the $MAGDM$ problem with attribute values in the form of IvVPFNs. To validate the proposed approach, Section 6 provides a real-world example of selecting an EV charging technique. Section 7 includes both sensitivity analysis

and comparative analysis. Finally, Section 8 discusses the conclusions, limitations, and future research directions.

2. Preliminary Definitions

In this section, we briefly review the fundamental definitions and operations of PFSs, IvVPFSs, SS-TNs, and SS-TCNs, which are essential for the development of this paper.

Definition 1 ([30]). *On a universal set $\mathfrak{X}$, a PFSs is defined as a triplet, as follows:*

$$\mathfrak{P} = \{(\varkappa, \check{\mu}_{\mathfrak{P}}(\varkappa), \check{\eta}_{\mathfrak{P}}(\varkappa), \check{\nu}_{\mathfrak{P}}(\varkappa)) | \varkappa \in \mathfrak{X}\}, \tag{1}$$

where $\check{\mu}_{\mathfrak{P}}(\varkappa) : \mathfrak{X} \to [0,1]$, $\check{\eta}_{\mathfrak{P}}(\varkappa) : \mathfrak{X} \to [0,1]$, and $\check{\nu}_{\mathfrak{P}}(\varkappa) : \mathfrak{X} \to [0,1]$ are the positive, neutral, and negative membership functions, respectively. These functions satisfy the condition $\check{\mu}_{\mathfrak{P}}(\varkappa) + \check{\eta}_{\mathfrak{P}}(\varkappa) + \check{\nu}_{\mathfrak{P}}(\varkappa) \leq 1$. $\check{\pi}_{\mathfrak{P}}(\varkappa) = 1 - \check{\mu}_{\mathfrak{P}}(\varkappa) - \check{\eta}_{\mathfrak{P}}(\varkappa) - \check{\nu}_{\mathfrak{P}}(\varkappa)$ is called the indeterminacy of the PFSs.

Definition 2 ([30]). *On a universal set $\mathfrak{X}$, an IvPFS is defined as a triplet, as follows:*

$$\mathfrak{P} = \{(\varkappa, \check{\mu}_{\mathfrak{P}}(\varkappa), \check{\eta}_{\mathfrak{P}}(\varkappa), \check{\nu}_{\mathfrak{P}}(\varkappa)) | \varkappa \in \mathfrak{X}\}, \tag{2}$$

where $\check{\mu}_{\mathfrak{P}} = [\check{\mu}^l, \check{\mu}^u]$ is the PMD, $\check{\eta}_{\mathfrak{P}} = [\check{\eta}^l, \check{\eta}^u]$ is the NuMD, and $\check{\nu}_{\mathfrak{P}} = [\check{\nu}^l, \check{\nu}^u]$ is the NMD representing the interval numbers under the restrictions that $\sup \check{\mu}_{\mathfrak{P}}(\varkappa) + \sup \check{\eta}_{\mathfrak{P}}(\varkappa) + \sup \check{\nu}_{\mathfrak{P}}(\varkappa) \leq 1$. In short, $\mathfrak{P} = ([\check{\mu}^l, \check{\mu}^u], [\check{\eta}^l, \check{\eta}^u], [\check{\nu}^l, \check{\nu}^u])$ is called IvVPFN, and $\check{\pi}^l = 1 - \check{\mu}^u - \check{\eta}^u - \check{\nu}^u$, $\check{\pi}^u = 1 - \check{\mu}^l - \check{\eta}^l - \check{\nu}^l$ are called indeterminacy degree of IvVPFN.

Definition 3 ([38]). *The score function S of IvVPFN $\mathfrak{P} = ([\check{\mu}^l, \check{\mu}^u], [\check{\eta}^l, \check{\eta}^u], [\check{\nu}^l, \check{\nu}^u])$ is defined as*

$$S(\mathfrak{P}) = \frac{\check{\mu}^l(1 - \check{\eta}^l - \check{\nu}^l) + \check{\mu}^u(1 - \check{\eta}^u - \check{\nu}^u)}{3}, \tag{3}$$

where $S(\mathfrak{P}) \in [-1, 1]$.

Definition 4 ([39]). *The SS TN and TCN operations are given by*

$$\mathbb{T}(\check{\wp}_1, \check{\wp}_2) = (\check{\wp}_1^{\check{\tau}} + \check{\wp}_2^{\check{\tau}} - 1)^{\frac{1}{\check{\tau}}}, \tag{4}$$

$$\mathbb{S}(\check{\wp}_1, \check{\wp}_2) = 1 - \left((1 - \check{\wp}_1)^{\check{\tau}} + (1 - \check{\wp}_2)^{\check{\tau}} - 1\right)^{\frac{1}{\check{\tau}}}, \tag{5}$$

where $\check{\wp}_1, \check{\wp}_2 \in [0,1]$ and $\check{\tau} < 0$. Particularly, when $\check{\tau} = 0$, SS TN and TCN operations becomes algebraic TNs and TCNs, that is, $\mathbb{T}(\check{\wp}_1, \check{\wp}_2) = \check{\wp}_1\check{\wp}_2$ and $\mathbb{S}(\check{\wp}_1, \check{\wp}_2) = \check{\wp}_1 + \check{\wp}_2 - \check{\wp}_1\check{\wp}_2$.

Definition 5 ([40]). *Suppose $\mathfrak{X} = \{\check{\varkappa}_1, \check{\varkappa}_2, \ldots, \check{\varkappa}_k\}$ is a universe of discourse. Let $\mathfrak{P}_1 = ([\check{\mu}_1^l, \check{\mu}_1^u], [\check{\eta}_1^l, \check{\eta}_1^u], [\check{\nu}_1^l, \check{\nu}_1^u])$ and $\mathfrak{P}_2 = ([\check{\mu}_2^l, \check{\mu}_2^u], [\check{\eta}_2^l, \check{\eta}_2^u], [\check{\nu}_2^l, \check{\nu}_2^u])$ be any two IvVPFNs. Then the distance measure based on the Hellinger distance of four dimension is*

$$D(\mathfrak{P}_1, \mathfrak{P}_2) = \left(\frac{1}{4k} \sum_{\iota=1}^{k} \left(\begin{array}{l} |\sqrt{\check{\mu}_1^l} - \sqrt{\check{\mu}_2^l}| + |\sqrt{\check{\mu}_1^u} - \sqrt{\check{\mu}_2^u}| + |\sqrt{\check{\eta}_1^l} - \sqrt{\check{\eta}_2^l}| + |\sqrt{\check{\eta}_1^u} - \sqrt{\check{\eta}_2^u}| \\ + |\sqrt{\check{\nu}_1^l} - \sqrt{\check{\nu}_2^l}| + |\sqrt{\check{\nu}_1^u} - \sqrt{\check{\nu}_2^u}| + |\sqrt{\check{\pi}_{l_1}} - \sqrt{\check{\pi}_{l_2}}| + |\sqrt{\check{\pi}_{u_1}} - \sqrt{\check{\pi}_{u_2}}| \end{array} \right) \right). \tag{6}$$

Definition 6 ([41]). *Given a set of real arguments, $\mathfrak{U}_\iota$, $(\iota = 1, 2, \ldots, k)$, the PA operator is defined by*

$$PA(\mathfrak{U}_1, \mathfrak{U}_2, \ldots, \mathfrak{U}_k) = \frac{\sum_{\iota=1}^{k} (1 + T(\mathfrak{U}_\iota))\mathfrak{U}_\iota}{\sum_{\iota=1}^{k} (1 + T(\mathfrak{U}_\iota))}, \tag{7}$$

where $T(\mathfrak{U}_\iota) = \sum_{\substack{l=1 \\ l \neq \iota}}^{n} Sup(\mathfrak{U}_\iota, \mathfrak{U}_l)$, and the support for $\mathfrak{U}_\iota$ from $\mathfrak{U}_l$ is $Sup(\mathfrak{U}_\iota \mathfrak{U}_l)$, satisfying the following properties:

1. $Sup(\mathfrak{U}_\iota, \mathfrak{U}_l) = Sup(\mathfrak{U}_l, \mathfrak{U}_\iota)$;
2. $Sup(\mathfrak{U}_\iota, \mathfrak{U}_l) \in [0, 1]$;
3. $Sup(\mathfrak{U}_\iota, \mathfrak{U}_l) \geq Sup(\mathfrak{U}_m, \mathfrak{U}_n)$ *if* $|\mathfrak{U}_\iota - \mathfrak{U}_l| < |\mathfrak{U}_m - \mathfrak{U}_n|$.

3. Literature Review

We have divided the literature review into four parts. In the first part, we have elaborated on the use of decision making in the IvVPF context. The second part investigates the application of Schweizer–Sklar aggregation operations in different fuzzy set environments. The third part reviews the application of the *VIKOR* method. The proposed work's motivation is presented in the fourth subsection, which also lists the research gaps.

3.1. Decision-Making Techniques Using IvVPFs

According to Zadeh [16], FSs, which include an element's MD, are a fruitful way to deal with uncertainty in real-world situations. PFSs [29] and IvVPFs [31] are extended versions of FSs. PFSs contain PMD, NuMD, and NMD, which express more information than FSs. PMD, NuMD, and NMD are represented in IvVPFs using a subset of unit closed intervals. Several authors have used the idea of IvVPFs in numerous research areas. Khalil et al. [31] introduced some new operations on IvVPFSs and IvVPF soft sets. They used them to evaluate possible investment opportunities in a wealth management company. In order to choose suppliers for logistics service value concretion, Naeem et al. [32] suggested IvVPFSs' uncertain linguistic aggregation operators. Bobin et al. [34] used the Leipzig leadership model to select a suitable employee by applying the correlation coefficient. In order to evaluate the air quality index, Abolfathi and Ebadian [35] suggested several IvVPF frank aggregation operators and offered an enhanced score and accuracy function. Ma et al. [42] suggested design idea evaluations in the early phases of developing a new product. An IVPFPMSM operator and a WIVPFPMSM operator were proposed by [33]. (See Table 1).

Table 1. Summary of research on the context of IvVPF.

References	Methods	Developed Operators/Measures	*MADM*/*MAGDM*	Application
[31]	...	...	*MADM*	Investment opportunity selection
[43]	...	Similarity measures	*MADM*	Company strategy selection
[44]	...	...	*MADM*	Pattern recognition
[32]	...	Linguistic averaging and geometric operators	MCGDM	Supplier selection
[34]	TOPSIS	IVPFHSWAO and IVPFHSWGO operators	*MAGDM*	Employee selection
[35]	...	Weighted arithmetic and geometric operators	MDGAM	Air quality index
[42]	AHP	...	*MCDM*	Kano's customer satisfaction model
[33]	...	IVPFPMSM and WIVPFPMSM operators	*MCDM*	Enterprise resource management

3.2. Application of Schweizer–Sklar Operations on Different Fuzzy Environments

SS operations are a family of parametric equations used in the fields of fuzzy logic and decision-making theory. Schweizer and Sklar [39] introduced these operations, which involve a parameter that adjusts the degree of t-norm or t-conorm. These operations are flexible in situations when it is necessary to adjust the logical operators' strictness because they are parameterized. In recent years, these two operations have been applicable in different fields in the decision making. Liu and Wang [45] proposed some power SS aggregation operators under an IF environment, such as IVIFSSPWA operator and IVIFSSPWG operator. Additionally, they solved a *MAGDM* problem of technology electronic companies' supplier selection. Zhang [46] introduced an IVIFSSWA operator and defined some new operations under an intuitionistic fuzzy environment. Wei et al. [47] proposed different SS operators under Fermatean fuzzy information. They defined a novel entropy measure and solved a MCGDM issue of green supplier selection. Yang and Zhang [48] proposed a MULTIMOORA method including a PFSSPWA operator and a PFSSPWG operator and applied it to CO_2 geological storage site selection. A MCGDM problem of share market investment company selection proposed by Gayen et al. [49], also introduced introduced a DHq-ROFSSHA operator and a DHq-ROFSSHG operator. Garg et al. [50] proposed different prioritized aggregation operators under an IF environment and applied them to employee selection. Garg et al. [51] proposed four aggregation operators, namely, LDFSSPoA, LDFSSWPoA, LDFSSPoG, and LDFSSWPoG operators, and considered a *MADM* problem to select a green sustainable chain. Kalsoom et al. [52] proposed different aggregation operators under IvVIF information and applied them to green supplier selection. Garg et al. [53] introduced IFRSSWA and IFRSSWG operators to solve *MAGDM* problem and applied them to investment in foreign stock. (See Table 2).

Table 2. Summary on Schweizer–Sklar operations under different fuzzy environments.

References	Methods	Developed Operators/Measures	*MADM*/*MAGDM*	Application
[45]	...	IVIFSSPWA and IVIFSSPWG operators	*MAGDM*	Supplier selection
[46]	...	IVIFSSWA operator	*MADM*	Supplier selection
[47]	CoCoSo	FFSSWA, FFSSOWA, FFSSWG, and FFSSOWG operators	*MAGDM*	Green supplier selection
[48]	MULTIMOORA	PFSSPWA and PFSSPWG operators	*MCDM*	CO_2 geological storage site selection
[49]	...	DHq-ROFSSHA and DHq-ROFSSHG operators	MCGDM	Investment company selection
[50]	...	IFSSPA, IFSSPG, IFSSWPA, and IFSSWPG operators	*MADM*	Employee selection
[51]	...	LDFSSPoA, LDFSSWPoA, LDFSSPoG, and LDFSSWPoG operators	*MADM*	Green sustainable chain
[52]	...	CIVIFSSPA, CIVIFSSPG, CIVIFSSPOA, and CIVIFSSPOG operators	*MADM*	Green supplier selection
[53]	...	IFRSSWA and IFRSSWG operators	*MAGDM*	Investment in foreign stock
[54]	...	*CA-IFSSPA*, *CA-IFSSPOA*, *CA-IFSSPG*, and *CA-IFSSPOG operators*	*MADM*	Logistics facilitator selection

3.3. Application of Projection-Based *VIKOR* Method Under Different Fuzzy Environments

A *MCDM* strategy called the *VIKOR* method was proposed by Duckstein and Opricovic [55] to assist decision makers in ranking and picking options based on competing criteria. This approach is helpful when decision makers seek to identify a compromise solution. This approach has been widely used in a variety of fields, including sustainability [56], logistics [57], supplier selection [58], WET-PPP project [59,60], and construction projects[61]. The proposed projection-based compromising method for *MCDM*, as developed in this paper [62], effectively addresses the challenges of uncertainty in decision making by utilizing IvVIFSs to enhance the evaluation of alternatives through compre-

hensive compromising indices and comparative analyses. The projection model proposed by Wei et al. [63] measures the similarity between alternatives and a picture fuzzy ideal point using the angle cosine of the attribute vectors. They [64] provided a comprehensive representation of expert evaluations, capturing the complexity of safety assessments in construction projects. In addition, Narayanamoorthy et al. [65] proposed a $VIKOR$ method under IvVIF hesitant fuzzy entropy to sort out the industrial robot. Krishankumar et al. [66] proposed an extended $VIKOR$ method to incorporate IFSs and validated the result through a personal selection case. The spherical fuzzy AHP method facilitates a location-based dynamic pricing model for advertisement, optimizing cost based on user engagement and location relevance [67]. The integration of q-RIVOFS into the $VIKOR$ model represents a significant advancement in $MAGDM$, as it enhances the model's ability to handle complex and uncertain decision environments, thereby providing more effective evaluation results compared with traditional decision-making methods [58]. Recently, a new measure [68] was proposed to enhance the accuracy of evaluations in IvVIF contexts. Additionally, Yue [69] proposed a PFS projection-based $VIKOR$ to select software products with reliability. Peng et al. [70] proposed a bidirectional projection-based VIKOR under a PFS framework to select the new energy vehicle.

3.4. Identified Gaps and Contributions

From the above analysis, it is clear that there is a notable gap in the application of Schweizer–Sklar operations, bidirectional projection, and $MAGDM$-based $VIKOR$ within the context of IvVPFSs. The following clearly describes the reasons behind studying this work:

- Selecting the most effective way to charge an EV in PCS is an important issue. In recent times, several researchers have developed mathematical models related to EVs. However, from the literature, we can see that the selection of the best charging procedure does not exist in the context of IvVPFSs. Thus, we have taken this problem in our work.
- In decision-making problems, subjective weights are considered based on the preferences and judgment of the decision makers. These weights reflect the importance of each criterion as prescribed by the individual. As a result, these vary from one decision maker to another. On the other side, a mathematical method using the data determines the objective weights. Therefore, the second reason is to calculate the combined weights using DBM.
- $VIKOR$ focuses on finding the compromise solution that balances the best and worst possible outcomes. It handles the conflicting criteria in the decision-making problem and evaluates the optimal alternatives. This method can be applied in both deterministic and fuzzy environments. It is found that in the literature, there is no research applying the $VIKOR$ method in the IvVPFSs. Therefore, our third motivation is to introduce the $VIKOR$ method under an IvVPF environment to solve the $MAGDM$ problem.
- The angle and distance between each alternative and the optimal solution are taken into account by the bidirectional projection measure. This motivates us to consider such type of measure in an IvVPF environment.
- Aggregation operators play an important role in decision making, particularly when multiple contradictory criteria are involved. In the literature, we have found various aggregation operators developed by scholars within the IvVPF environment. Some of these operators are parameter-free, while others include parameters. However, there is no paper addressing SS operations in an IvVPF environment. Therefore, it is essential to introduce the aggregation operators using SS-TN and SS-TCN operations.

The contributions of this paper are as follows:

1. To overcome the $MAGDM$ problem, a new VIKOR approach based on bidirectional projections is introduced in an IvVPF context. This established strategy allows us to identify the best option.
2. An IvVPFSSPWA operator and an IvVPFSSPWG operator are developed. Additionally, different properties of these operators are discussed.
3. To ascertain the criteria's objective weight in the context of the IvVPF setting, the DBM approach is presented. After that, the cumulative weight is calculated and applied to this $MAGDM$ issue.
4. Utilizing the proposed $VIKOR$ model to find the best possible way to charge EVs in the PCSs.

4. Schweizer–Sklar Operations for IvVPFNs

SS operations are a family of parametric equations used in decision making. These operations involve a parameter that adjusts the degree of TNs and TCNs. In the context of IvVPFSs, these operations can be applied to handle the vagueness and uncertainty in decision making. In this section, we have defined some SS operations as follows:

Definition 7. *Suppose for any three IvVPFNs,* $\mathfrak{P} = ([\breve{\mu}^l, \breve{\mu}^u], [\breve{\eta}^l, \breve{\eta}^u], [\breve{\nu}^l, \breve{\nu}^u])$, $\mathfrak{P}_1 = ([\breve{\mu}_1^l, \breve{\mu}_1^u], [\breve{\eta}_1^l, \breve{\eta}_1^u], [\breve{\nu}_1^l, \breve{\nu}_1^u])$, *and* $\mathfrak{P}_2 = ([\breve{\mu}_2^l, \breve{\mu}_2^u], [\breve{\eta}_2^l, \breve{\eta}_2^u], [\breve{\nu}_2^l, \breve{\nu}_2^u])$, *the following operations are defined based on SS TNs and TCNs:*

$$\mathfrak{P}_1 \oplus \mathfrak{P}_2 = \left(\begin{array}{l} \left[1 - \left((1-\breve{\mu}_1^l)^{\breve{\tau}} + (1-\breve{\mu}_2^l)^{\breve{\tau}} - 1 \right)^{\frac{1}{\breve{\tau}}}, 1 - \left((1-\breve{\mu}_1^u)^{\breve{\tau}} + (1-\breve{\mu}_2^u)^{\breve{\tau}} - 1 \right)^{\frac{1}{\breve{\tau}}} \right], \\ \left[(\breve{\eta}_1^{l^{\breve{\tau}}} + \breve{\eta}_2^{l^{\breve{\tau}}} - 1)^{\frac{1}{\breve{\tau}}}, (\breve{\eta}_1^{u^{\breve{\tau}}} + \breve{\eta}_2^{u^{\breve{\tau}}} - 1)^{\frac{1}{\breve{\tau}}} \right], \left[(\breve{\nu}_1^{l^{\breve{\tau}}} + \breve{\eta}_2^{l^{\breve{\tau}}} - 1)^{\frac{1}{\breve{\tau}}}, (\breve{\nu}_1^{u^{\breve{\tau}}} + \breve{\nu}_2^{u^{\breve{\tau}}} - 1)^{\frac{1}{\breve{\tau}}} \right] \end{array} \right),$$

$$\mathfrak{P}_1 \otimes \mathfrak{P}_2 = \left(\begin{array}{l} \left[(\breve{\mu}_1^{l^{\breve{\tau}}} + \breve{\mu}_2^{l^{\breve{\tau}}} - 1)^{\frac{1}{\breve{\tau}}}, (\breve{\mu}_1^{u^{\breve{\tau}}} + \breve{\mu}_2^{u^{\breve{\tau}}} - 1)^{\frac{1}{\breve{\tau}}} \right], \left[1 - \left((1-\breve{\eta}_1^l)^{\breve{\tau}} + (1-\breve{\eta}_2^l)^{\breve{\tau}} - 1 \right)^{\frac{1}{\breve{\tau}}}, \right. \\ \left. 1 - \left((1-\breve{\eta}_1^u)^{\breve{\tau}} + (1-\breve{\eta}_2^u)^{\breve{\tau}} - 1 \right)^{\frac{1}{\breve{\tau}}} \right], \left[1 - \left((1-\breve{\nu}_1^l)^{\breve{\tau}} + (1-\breve{\nu}_2^l)^{\breve{\tau}} - 1 \right)^{\frac{1}{\breve{\tau}}}, \right. \\ \left. 1 - \left((1-\breve{\nu}_1^u)^{\breve{\tau}} + (1-\breve{\nu}_2^u)^{\breve{\tau}} - 1 \right)^{\frac{1}{\breve{\tau}}} \right] \end{array} \right),$$

$$\breve{\phi}\mathfrak{P} = \left(\begin{array}{l} \left[1 - \left(\breve{\phi}(1-\breve{\mu}^l)^{\breve{\tau}} - (\breve{\phi}-1) \right)^{\frac{1}{\breve{\tau}}}, 1 - \left(\breve{\phi}(1-\breve{\mu}^u)^{\breve{\tau}} - (\breve{\phi}-1) \right)^{\frac{1}{\breve{\tau}}} \right], \\ \left[\left(\breve{\phi}\breve{\eta}^{l^{\breve{\tau}}} - (\breve{\phi}-1) \right)^{\frac{1}{\breve{\tau}}}, \left(\breve{\phi}\breve{\eta}^{u^{\breve{\tau}}} - (\breve{\phi}-1) \right)^{\frac{1}{\breve{\tau}}} \right], \\ \left[\left(\breve{\phi}\breve{\nu}^{l^{\breve{\tau}}} - (\breve{\phi}-1) \right)^{\frac{1}{\breve{\tau}}}, \left(\breve{\phi}\breve{\nu}^{u^{\breve{\tau}}} - (\breve{\phi}-1) \right)^{\frac{1}{\breve{\tau}}} \right], \end{array} \right) \breve{\phi} > 0,$$

$$\mathfrak{P}^{\breve{\varphi}} = \begin{pmatrix} \left[\left(\breve{\varphi}\breve{\mu}^{l^{\breve{\tau}}} - (\breve{\varphi}-1)\right)^{\frac{1}{\breve{\tau}}}, \left(\breve{\varphi}\breve{\eta}^{u^{\breve{\tau}}} - (\breve{\varphi}-1)\right)^{\frac{1}{\breve{\tau}}}\right], \left[1-\left(\breve{\varphi}(1-\breve{\eta}^{l})^{\breve{\tau}} - (\breve{\varphi}-1)\right)^{\frac{1}{\breve{\tau}}},\right. \\ \left. 1-\left(\breve{\varphi}(1-\breve{\eta}^{u})^{\breve{\tau}} - (\breve{\varphi}-1)\right)^{\frac{1}{\breve{\tau}}}\right], \left[1-\left(\breve{\varphi}(1-\breve{\nu}^{l})^{\breve{\tau}} - (\breve{\varphi}-1)\right)^{\frac{1}{\breve{\tau}}}, \right. \\ \left. 1-\left(\breve{\varphi}(1-\breve{\nu}^{u^{\breve{\tau}}}) - (\breve{\varphi}-1)\right)^{\frac{1}{\breve{\tau}}}\right] \end{pmatrix} \breve{\varphi} > 0.$$

Theorem 1. *Suppose* $\mathfrak{P}_1$ *and* $\mathfrak{P}_2$ *are any two IVPFNs. Then the following properties hold:*

1. $\mathfrak{P}_1 \oplus \mathfrak{P}_2 = \mathfrak{P}_2 \oplus \mathfrak{P}_1$,
2. $\mathfrak{P}_1 \otimes \mathfrak{P}_2 = \mathfrak{P}_2 \otimes \mathfrak{P}_1$,
3. $\breve{\varphi}(\mathfrak{P}_1 \oplus \mathfrak{P}_2) = \breve{\varphi}\mathfrak{P}_1 \oplus \breve{\varphi}\mathfrak{P}_2$, $\breve{\varphi} > 0$,
4. $\breve{\varphi}_1\mathfrak{P}_1 \oplus \breve{\varphi}_2\mathfrak{P}_1 = (\breve{\varphi}_1 + \breve{\varphi}_2)\mathfrak{P}_1$, $\breve{\varphi}_1, \breve{\varphi}_2 > 0$,
5. $\mathfrak{P}_1^{\breve{\varphi}} \otimes \mathfrak{P}_2^{\breve{\varphi}} = (\mathfrak{P}_1 \otimes \mathfrak{P}_2)^{\breve{\varphi}}$, $\breve{\varphi} > 0$,
6. $\mathfrak{P}_1^{\breve{\varphi}_1} \otimes \mathfrak{P}_1^{\breve{\varphi}_2} = \mathfrak{P}_1^{\breve{\varphi}_1+\breve{\varphi}_2}$, $\breve{\varphi}_1, \breve{\varphi}_2 > 0$.

Proof. We have to prove 1 and 3. The remaining cases follow a similar pattern.

1. We know that

$$\mathfrak{P}_1 \oplus \mathfrak{P}_2 = \begin{pmatrix} \left[1-\left((1-\breve{\mu}_1^{l})^{\breve{\tau}} + (1-\breve{\mu}_2^{l})^{\breve{\tau}} - 1\right)^{\frac{1}{\breve{\tau}}}, \right. \\ \left. 1-\left((1-\breve{\mu}_1^{u})^{\breve{\tau}} + (1-\breve{\mu}_2^{u})^{\breve{\tau}} - 1\right)^{\frac{1}{\breve{\tau}}}\right], \\ \left[(\breve{\eta}_1^{l^{\breve{\tau}}} + \breve{\eta}_2^{l^{\breve{\tau}}} - 1)^{\frac{1}{\breve{\tau}}}, (\breve{\eta}_1^{u^{\breve{\tau}}} + \breve{\eta}_2^{u^{\breve{\tau}}} - 1)\right], \\ \left[(\breve{\nu}_1^{l^{\breve{\tau}}} + \breve{\eta}_2^{l^{\breve{\tau}}} - 1)^{\frac{1}{\breve{\tau}}}, (\breve{\nu}_1^{u^{\breve{\tau}}} + \breve{\nu}_2^{u^{\breve{\tau}}} - 1)\right] \end{pmatrix}$$

$$= \begin{pmatrix} \left[1-\left((1-\breve{\mu}_2^{l})^{\breve{\tau}} + (1-\breve{\mu}_1^{l})^{\breve{\tau}} - 1\right)^{\frac{1}{\breve{\tau}}}, \right. \\ \left. 1-\left((1-\breve{\mu}_2^{u})^{\breve{\tau}} + (1-\breve{\mu}_1^{u})^{\breve{\tau}} - 1\right)^{\frac{1}{\breve{\tau}}}\right], \\ \left[(\breve{\eta}_2^{l^{\breve{\tau}}} + \breve{\eta}_1^{l^{\breve{\tau}}} - 1)^{\frac{1}{\breve{\tau}}}, (\breve{\eta}_2^{u^{\breve{\tau}}} + \breve{\eta}_1^{u^{\breve{\tau}}} - 1)\right], \\ \left[(\breve{\nu}_2^{l^{\breve{\tau}}} + \breve{\eta}_1^{l^{\breve{\tau}}} - 1)^{\frac{1}{\breve{\tau}}}, (\breve{\nu}_2^{u^{\breve{\tau}}} + \breve{\nu}_1^{u^{\breve{\tau}}} - 1)\right] \end{pmatrix} = \mathfrak{P}_2 \oplus \mathfrak{P}_1.$$

3. We have to prove $\breve{\varphi}(\mathfrak{P}_1 \oplus \mathfrak{P}_2) = \breve{\varphi}\mathfrak{P}_1 \oplus \breve{\varphi}\mathfrak{P}_2$. To avoid complexity, we prove this component-wise. Therefore, the first component of $\breve{\varphi}(\mathfrak{P}_1 \oplus \mathfrak{P}_2)$ should be of the form

$$1-\left(\breve{\varphi}\left(1-\left(1-\left((1-\breve{\mu}_1^{l})^{\breve{\tau}} + (1-\breve{\mu}_2^{l})^{\breve{\tau}} - 1\right)^{\frac{1}{\breve{\tau}}}\right)\right)^{\breve{\tau}} - (\breve{\varphi}-1)\right)^{\frac{1}{\breve{\tau}}}$$

$$= 1-\left(\breve{\varphi}\left((1-\breve{\mu}_1^{l})^{\breve{\tau}} + (1-\breve{\mu}_2^{l})^{\breve{\tau}} - 1\right) - (\breve{\varphi}-1)\right)^{\frac{1}{\breve{\tau}}}$$

$$= 1-\left(\breve{\varphi}\left((1-\breve{\mu}_1^{l})^{\breve{\tau}} + (1-\breve{\mu}_2^{l})^{\breve{\tau}}\right) - 2\breve{\varphi} + 1\right)^{\frac{1}{\breve{\tau}}}.$$

Similarly, the first component of $\breve{\varphi}\mathfrak{P}_1 \oplus \breve{\varphi}\mathfrak{P}_2$ is of the form

$$1 - \left(\breve{\varphi}\left(1-\mu_1^l\right)^{\breve{\tau}} - (\breve{\varphi}-1) + \breve{\varphi}\left(1-\mu_2^l\right)^{\breve{\tau}} - (\breve{\varphi}-1) - 1\right)^{\frac{1}{\breve{\tau}}}$$

$$= 1 - \left(\breve{\varphi}\left((1-\breve{\mu}_1^l)^{\breve{\tau}} + (1-\breve{\mu}_2^l)^{\breve{\tau}}\right) - 2\breve{\varphi} + 1\right)^{\frac{1}{\breve{\tau}}}.$$

Thus, we can prove the other component in a similar way; hence, the result. □

4.1. IvVPF Power SS Aggregation Operators

In light of the operational rule of IvVPFNs concerning Schweizer–Sklar operations as detailed in Section 4, coupled with the benefits of the power aggregation operator, we will now present several novel aggregation operators, like the IvVPFSSPA operator, IvVPFSSPG operator, IvVPFSSPWA operator, and IvVPFSSPWG operator.

Definition 8. *Suppose* $\mathfrak{P}_\iota = \left([\breve{\mu}_\iota^l, \breve{\mu}_\iota^u], [\breve{\eta}_\iota^l, \breve{\eta}_\iota^u], [\breve{\nu}_\iota^l, \breve{\nu}_\iota^u]\right)$, $(\iota = 1, 2, \ldots, k)$ *is a collection of IvVPFNs and* $IvVPFSSPA : \psi^n \to \psi$ *is such that*

$$IvVPFSSPA(\mathfrak{P}_1, \mathfrak{P}_2, \ldots, \mathfrak{P}_n) = \frac{\oplus_{\iota=1}^k \left((1+T(\mathfrak{P}_\iota)).{}_{ss}\mathfrak{P}_\iota\right)}{\sum\limits_{\iota=1}^k (1+T(\mathfrak{P}_\iota))}, \tag{8}$$

where ψ *is the collection of all IvVPFNs in a universal set* $\mathfrak{X}$ *and* $T(\mathfrak{P}_\iota) = \sum\limits_{\substack{l=1 \\ l\neq\iota}}^{k} sup(\mathfrak{P}_\iota, \mathfrak{P}_l)$. *Then an IvVPFSSPA function is called an IvVPFSSPA operator.*

Theorem 2. *Suppose that* $\mathfrak{P}_\iota = \left([\breve{\mu}_\iota^l, \breve{\mu}_\iota^u], [\breve{\eta}_\iota^l, \breve{\eta}_\iota^u], [\breve{\nu}_\iota^l, \breve{\nu}_\iota^u]\right)$, $(\iota = 1, 2, \ldots, k)$ *is a collection of IvVPFNs and* $\breve{\tau} < 0$. *Then utilizing Equation (8), the aggregated result is again a IvVPFN given by*

$$IvVPFSSPA(\mathfrak{P}_1, \mathfrak{P}_2, \ldots, \mathfrak{P}_k) = \begin{pmatrix} \left[1 - \left(\sum\limits_{\iota=1}^k \breve{\omega}_\iota (1-\breve{\mu}_\iota^l)^{\breve{\tau}}\right)^{\frac{1}{\breve{\tau}}}, 1 - \left(\sum\limits_{\iota=1}^k \breve{\omega}_\iota (1-\breve{\mu}_\iota^u)^{\breve{\tau}}\right)^{\frac{1}{\breve{\tau}}}\right], \\ \left[\left(\sum\limits_{\iota=1}^k \breve{\omega}_\iota \breve{\eta}_\iota^{l^{\breve{\tau}}}\right)^{\frac{1}{\breve{\tau}}}, \left(\sum\limits_{\iota=1}^k \breve{\omega}_\iota \breve{\eta}^{u_\iota^{\breve{\tau}}}\right)^{\frac{1}{\breve{\tau}}}\right], \\ \left[\left(\sum\limits_{\iota=1}^k \breve{\omega}_\iota \breve{\nu}_\iota^{l^{\breve{\tau}}}\right)^{\frac{1}{\breve{\tau}}}, \left(\sum\limits_{\iota=1}^k \breve{\omega}_\iota \breve{\nu}^{u_\iota^{\breve{\tau}}}\right)^{\frac{1}{\breve{\tau}}}\right] \end{pmatrix} \tag{9}$$

where $\breve{\omega}_\iota = \frac{(1+T(\mathfrak{P}_\iota))}{\sum\limits_{\iota=1}^k (1+T(\mathfrak{P}_\iota))}$ *is the integrated weights for* $\iota = 1, 2, \ldots, k$.

To prove Equation (9), we initially establish the validity of the following equation for any value of $\breve{\omega} = (\breve{\omega}_1, \breve{\omega}_2, \ldots, \breve{\omega}_k)^T$ without imposing any constraints on $\breve{\omega}$.

$$IvVPFSSPA(\mathfrak{P}_1, \mathfrak{P}_2, \ldots, \mathfrak{P}_k) = \begin{pmatrix} \left[1 - \left(\sum\limits_{\iota=1}^k \breve{\omega}_\iota (1-\breve{\mu}_\iota^l)^{\breve{\tau}} - \sum\limits_{\iota=1}^k \breve{\omega}_\iota + 1\right)^{\frac{1}{\breve{\tau}}}, \right. \\ \left. 1 - \left(\sum\limits_{\iota=1}^k \breve{\omega}_\iota (1-\breve{\mu}_\iota^u)^{\breve{\tau}} - \sum\limits_{\iota=1}^k \breve{\omega}_\iota + 1\right)^{\frac{1}{\breve{\tau}}}\right], \\ \left[\left(\sum\limits_{\iota=1}^k \breve{\omega}_\iota \breve{\eta}_\iota^{l^{\breve{\tau}}} - \sum\limits_{\iota=1}^k \breve{\omega}_\iota + 1\right)^{\frac{1}{\breve{\tau}}}, \left(\sum\limits_{\iota=1}^k \breve{\omega}_\iota \breve{\eta}_\iota^{u^{\breve{\tau}}} - \sum\limits_{\iota=1}^k \breve{\omega}_\iota + 1\right)^{\frac{1}{\breve{\tau}}}\right], \\ \left[\left(\sum\limits_{\iota=1}^k \breve{\omega}_\iota \breve{\nu}_\iota^{l^{\breve{\tau}}} - \sum\limits_{\iota=1}^k \breve{\omega}_\iota + 1\right)^{\frac{1}{\breve{\tau}}}, \left(\sum\limits_{\iota=1}^k \breve{\omega}_\iota \breve{\nu}_\iota^{u^{\breve{\tau}}} - \sum\limits_{\iota=1}^k \breve{\omega}_\iota + 1\right)^{\frac{1}{\breve{\tau}}}\right] \end{pmatrix} \tag{10}$$

The following is a proof of Equation (10) using the following mathematical induction formula:

Proof. First, we can see how Equation (8) is transformed as

$$\begin{aligned} IvVPFSSPA(\mathfrak{P}_1, \mathfrak{P}_2, \ldots, \mathfrak{P}_k) &= \frac{\oplus_{\iota=1}^{k}\left((1+T(\mathfrak{P}_\iota))_{\cdot ss}\mathfrak{P}_\iota\right)}{\sum\limits_{\iota=1}^{k}(1+T(\mathfrak{P}_\iota))} \\ &= \oplus_{\iota=1}^{k}\left(\frac{(1+T(\mathfrak{P}_\iota))}{\sum\limits_{\iota=1}^{k}(1+T(\mathfrak{P}_\iota))}{}_{\cdot ss}\mathfrak{P}_\iota\right) \\ &= \oplus_{\iota=1}^{k}\breve{\omega}_{\iota\cdot ss}\mathfrak{P}_\iota, \end{aligned}$$

where $\breve{\omega}_\iota = \frac{(1+T(\mathfrak{P}_\iota))}{\sum\limits_{\iota=1}^{k}(1+T(\mathfrak{P}_\iota))}$. According to the operational rule in Definition 7, we have

$$\breve{\omega}_{\iota\cdot ss}\mathfrak{P}_\iota = \left(\begin{array}{c} \left[1-\left(\breve{\omega}_\iota(1-\breve{\mu}_\iota^l)^{\breve{\tau}}-(\breve{\omega}_\iota-1)\right)^{\frac{1}{\breve{\tau}}}, 1-\left(\breve{\omega}_\iota(1-\breve{\mu}_\iota^u)^{\breve{\tau}}-(\breve{\omega}_\iota-1)\right)^{\frac{1}{\breve{\tau}}}\right], \\ \left[\left(\breve{\omega}_\iota\breve{\eta}_\iota^{l^{\breve{\tau}}}-(\breve{\omega}_\iota-1)\right)^{\frac{1}{\breve{\tau}}}, \left(\breve{\omega}_\iota\breve{\eta}_\iota^{u^{\breve{\tau}}}-(\breve{\omega}_\iota-1)\right)^{\frac{1}{\breve{\tau}}}\right], \\ \left[\left(\breve{\omega}_\iota\breve{\nu}_\iota^{l^{\breve{\tau}}}-(\breve{\omega}_\iota-1)\right)^{\frac{1}{\breve{\tau}}}, \left(\breve{\omega}_\iota\breve{\nu}_\iota^{u^{\breve{\tau}}}-(\breve{\omega}_\iota-1)\right)^{\frac{1}{\breve{\tau}}}\right] \end{array}\right).$$

When $k = 2$, we have

$$\breve{\omega}_1.\mathfrak{P}_1 = \left(\begin{array}{c} \left[1-\left(\breve{\omega}_1(1-\breve{\mu}_1^l)^{\breve{\tau}}-(\breve{\omega}_1-1)\right)^{\frac{1}{\breve{\tau}}}, 1-\left(\breve{\omega}_1(1-\breve{\mu}_1^u)^{\breve{\tau}}-(\breve{\omega}_1-1)\right)^{\frac{1}{\breve{\tau}}}\right], \\ \left[\left(\breve{\omega}_1\breve{\eta}_1^{l^{\breve{\tau}}}-(\breve{\omega}_1-1)\right)^{\frac{1}{\breve{\tau}}}, \left(\breve{\omega}_1\breve{\eta}_1^{u^{\breve{\tau}}}-(\breve{\omega}_1-1)\right)^{\frac{1}{\breve{\tau}}}\right], \\ \left[\left(\breve{\omega}_1\breve{\nu}_1^{l^{\breve{\tau}}}-(\breve{\omega}_1-1)\right)^{\frac{1}{\breve{\tau}}}, \left(\breve{\omega}_1\breve{\nu}_1^{u^{\breve{\tau}}}-(\breve{\omega}_1-1)\right)^{\frac{1}{\breve{\tau}}}\right] \end{array}\right)$$

and

$$\breve{\omega}_2.\mathfrak{P}_2 = \left(\begin{array}{c} \left[1-\left(\breve{\omega}_2(1-\breve{\mu}_2^l)^{\breve{\tau}}-(\breve{\omega}_2-1)\right)^{\frac{1}{\breve{\tau}}}, 1-\left(\breve{\omega}_2(1-\breve{\mu}_2^u)^{\breve{\tau}}-(\breve{\omega}_2-1)\right)^{\frac{1}{\breve{\tau}}}\right], \\ \left[\left(\breve{\omega}_2\breve{\eta}_2^{l^{\breve{\tau}}}-(\breve{\omega}_2-1)\right)^{\frac{1}{\breve{\tau}}}, \left(\breve{\omega}_2\breve{\eta}_2^{u^{\breve{\tau}}}-(\breve{\omega}_2-1)\right)^{\frac{1}{\breve{\tau}}}\right], \\ \left[\left(\breve{\omega}_2\breve{\nu}_2^{l^{\breve{\tau}}}-(\breve{\omega}_2-1)\right)^{\frac{1}{\breve{\tau}}}, \left(\breve{\omega}_2\breve{\nu}_2^{u^{\breve{\tau}}}-(\breve{\omega}_2-1)\right)^{\frac{1}{\breve{\tau}}}\right] \end{array}\right).$$

$\therefore \breve{\omega}_1.\mathfrak{P}_1 \oplus_{ss} \breve{\omega}_2.\mathfrak{P}_2$

$$= \begin{pmatrix} \left[1-\left(\sum_{\iota=1}^{2}\breve{\omega}_\iota(1-\breve{\mu}_\iota^l)^{\breve{\tau}}-\sum_{\iota=1}^{2}\breve{\omega}_\iota+1\right)^{\frac{1}{\breve{\tau}}}, 1-\left(\sum_{\iota=1}^{2}\breve{\omega}_\iota(1-\breve{\mu}_\iota^u)^{\breve{\tau}}-\sum_{\iota=1}^{2}\breve{\omega}_\iota+1\right)^{\frac{1}{\breve{\tau}}}\right], \\ \left[\left(\sum_{\iota=1}^{2}\breve{\omega}_\iota\breve{\eta}_\iota^{l^{\breve{\tau}}}-\sum_{\iota=1}^{2}\breve{\omega}_\iota+1\right)^{\frac{1}{\breve{\tau}}}, \left(\sum_{\iota=1}^{2}\breve{\omega}_\iota\breve{\eta}_\iota^{u^{\breve{\tau}}}-\sum_{\iota=1}^{2}\breve{\omega}_\iota+1\right)^{\frac{1}{\breve{\tau}}}\right], \\ \left[\left(\sum_{\iota=1}^{2}\breve{\omega}_\iota\breve{\nu}_\iota^{l^{\breve{\tau}}}-\sum_{\iota=1}^{2}\breve{\omega}_\iota+1\right)^{\frac{1}{\breve{\tau}}}, \left(\sum_{\iota=1}^{2}\breve{\omega}_\iota\breve{\nu}_\iota^{u^{\breve{\tau}}}-\sum_{\iota=1}^{2}\breve{\omega}_\iota+1\right)^{\frac{1}{\breve{\tau}}}\right] \end{pmatrix}.$$

Thus, when $k = 2$ (Equation (10)) is true for $k = m$, we assume that the equation is true. Then we have

$$IvVPFSSPA(\mathfrak{P}_1, \mathfrak{P}_2, \ldots, \mathfrak{P}_m)$$

$$= \begin{pmatrix} \left[1-\left(\sum_{\iota=1}^{m}\breve{\omega}_\iota(1-\breve{\mu}_\iota^l)^{\breve{\tau}}-\sum_{\iota=1}^{m}\breve{\omega}_\iota+1\right)^{\frac{1}{\breve{\tau}}}, \right. \\ \left. 1-\left(\sum_{\iota=1}^{m}\breve{\omega}_\iota(1-\breve{\mu}_\iota^u)^{\breve{\tau}}-\sum_{\iota=1}^{m}\breve{\omega}_\iota+1\right)^{\frac{1}{\breve{\tau}}}\right], \\ \left[\left(\sum_{\iota=1}^{m}\breve{\omega}_\iota\breve{\eta}_\iota^{l^{\breve{\tau}}}-\sum_{\iota=1}^{m}\breve{\omega}_\iota+1\right)^{\frac{1}{\breve{\tau}}}, \left(\sum_{\iota=1}^{m}\breve{\omega}_\iota\breve{\eta}_\iota^{u^{\breve{\tau}}}-\sum_{\iota=1}^{m}\breve{\omega}_\iota+1\right)^{\frac{1}{\breve{\tau}}}\right], \\ \left[\left(\sum_{\iota=1}^{m}\breve{\omega}_\iota\breve{\nu}_\iota^{l^{\breve{\tau}}}-\sum_{\iota=1}^{m}\breve{\omega}_\iota+1\right)^{\frac{1}{\breve{\tau}}}, \left(\sum_{\iota=1}^{m}\breve{\omega}_\iota\breve{\nu}_\iota^{u^{\breve{\tau}}}-\sum_{\iota=1}^{m}\breve{\omega}_\iota+1\right)^{\frac{1}{\breve{\tau}}}\right] \end{pmatrix}. \quad (11)$$

Now when $k = m + 1$, we have

$$\breve{\omega}_{m+1\cdot ss}\mathfrak{P}_{m+1} = \begin{pmatrix} \left[1-\left(\breve{\omega}_{m+1}(1-\breve{\mu}_{m+1}^l)^{\breve{\tau}}-(\breve{\omega}_{m+1}-1)\right)^{\frac{1}{\breve{\tau}}}, \right. \\ \left. 1-\left(\breve{\omega}_{m+1}(1-\breve{\mu}_{m+1}^u)^{\breve{\tau}}-(\breve{\omega}_{m+1}-1)\right)^{\frac{1}{\breve{\tau}}}\right], \\ \left[\left(\breve{\omega}_{m+1}\breve{\eta}_{m+1}^{l^{\breve{\tau}}}-(\breve{\omega}_{m+1}-1)\right)^{\frac{1}{\breve{\tau}}}, \left(\breve{\omega}_{m+1}\breve{\eta}_{m+1}^{u^{\breve{\tau}}}-(\breve{\omega}_{m+1}-1)\right)^{\frac{1}{\breve{\tau}}}\right], \\ \left[\left(\breve{\omega}_{m+1}\breve{\nu}_{m+1}^{l^{\breve{\tau}}}-(\breve{\omega}_{m+1}-1)\right)^{\frac{1}{\breve{\tau}}}, \left(\breve{\omega}_{m+1}\breve{\nu}_{m+1}^{u^{\breve{\tau}}}-(\breve{\omega}_{m+1}-1)\right)^{\frac{1}{\breve{\tau}}}\right] \end{pmatrix}.$$

$$\begin{aligned} \therefore IvVPFSSPA(\mathfrak{P}_1, \mathfrak{P}_2, \ldots, \mathfrak{P}_m, \mathfrak{P}_{m+1}) &= IvVPFSSPA(\mathfrak{P}_1, \mathfrak{P}_2, \ldots, \mathfrak{P}_m) \oplus \breve{\omega}_{m+1}\mathfrak{P}_{m+1} \\ &= \oplus_{k=1}^{m}\breve{\omega}_k\mathfrak{P}_k \oplus \breve{\omega}_{m+1}\mathfrak{P}_{m+1} \end{aligned}$$

$$= \begin{pmatrix} \left[1-\left(\sum_{\iota=1}^{m}\breve{\omega}_\iota(1-\breve{\mu}_\iota^l)^{\breve{\tau}}-\sum_{\iota=1}^{m}\breve{\omega}_\iota+1\right)^{\frac{1}{\breve{\tau}}}, 1-\left(\sum_{\iota=1}^{m}\breve{\omega}_\iota(1-\breve{\mu}_\iota^u)^{\breve{\tau}}-\sum_{\iota=1}^{m}\breve{\omega}_\iota+1\right)^{\frac{1}{\breve{\tau}}}\right], \\ \left[\left(\sum_{\iota=1}^{m}\breve{\omega}_\iota\breve{\eta}_\iota^{l^{\breve{\tau}}}-\sum_{\iota=1}^{m}\breve{\omega}_\iota+1\right)^{\frac{1}{\breve{\tau}}}, \left(\sum_{\iota=1}^{m}\breve{\omega}_\iota\breve{\eta}_\iota^{u^{\breve{\tau}}}-\sum_{\iota=1}^{m}\breve{\omega}_\iota+1\right)^{\frac{1}{\breve{\tau}}}\right], \\ \left[\left(\sum_{\iota=1}^{m}\breve{\omega}_\iota\breve{\nu}_\iota^{l^{\breve{\tau}}}-\sum_{\iota=1}^{m}\breve{\omega}_\iota+1\right)^{\frac{1}{\breve{\tau}}}, \left(\sum_{\iota=1}^{m}\breve{\omega}_\iota\breve{\nu}_\iota^{u^{\breve{\tau}}}-\sum_{\iota=1}^{m}\breve{\omega}_\iota+1\right)^{\frac{1}{\breve{\tau}}}\right] \end{pmatrix}$$

$$\oplus \begin{pmatrix} \left[1-\left(\breve{\omega}_{m+1}(1-\breve{\mu}_{m+1}^l)^{\breve{\tau}}-(\breve{\omega}_{m+1}-1)\right)^{\frac{1}{\breve{\tau}}}, 1-\left(\breve{\omega}_{m+1}(1-\breve{\mu}_{m+1}^u)^{\breve{\tau}}-(\breve{\omega}_{m+1}-1)\right)^{\frac{1}{\breve{\tau}}}\right], \\ \left[\left(\breve{\omega}_{m+1}\breve{\eta}_{m+1}^{l^{\breve{\tau}}}-(\breve{\omega}_{m+1}-1)\right)^{\frac{1}{\breve{\tau}}}, \left(\breve{\omega}_{m+1}\breve{\eta}_{m+1}^{u^{\breve{\tau}}}-(\breve{\omega}_{m+1}-1)\right)^{\frac{1}{\breve{\tau}}}\right], \\ \left[\left(\breve{\omega}_{m+1}\breve{\nu}_{m+1}^{l^{\breve{\tau}}}-(\breve{\omega}_{m+1}-1)\right)^{\frac{1}{\breve{\tau}}}, \left(\breve{\omega}_{m+1}\breve{\nu}_{m+1}^{u^{\breve{\tau}}}-(\breve{\omega}_{m+1}-1)\right)^{\frac{1}{\breve{\tau}}}\right] \end{pmatrix}$$

$$= \begin{pmatrix} \left[1-\left(\sum_{\iota=1}^{m+1}\breve{\omega}_\iota(1-\breve{\mu}_\iota^l)^{\breve{\tau}}-\sum_{\iota=1}^{m+1}\breve{\omega}_\iota+1\right)^{\frac{1}{\breve{\tau}}}, 1-\left(\sum_{\iota=1}^{m+1}\breve{\omega}_\iota(1-\breve{\mu}_\iota^u)^{\breve{\tau}}-\sum_{\iota=1}^{m+1}\breve{\omega}_\iota+1\right)^{\frac{1}{\breve{\tau}}}\right], \\ \left[\left(\sum_{\iota=1}^{m+1}\breve{\omega}_\iota\breve{\eta}_\iota^{l^{\breve{\tau}}}-\sum_{\iota=1}^{m+1}\breve{\omega}_\iota+1\right)^{\frac{1}{\breve{\tau}}}, \left(\sum_{\iota=1}^{m+1}\breve{\omega}_\iota\breve{\eta}_\iota^{u^{\breve{\tau}}}-\sum_{\iota=1}^{m+1}\breve{\omega}_\iota+1\right)^{\frac{1}{\breve{\tau}}}\right], \\ \left[\left(\sum_{\iota=1}^{m+1}\breve{\omega}_\iota\breve{\nu}_\iota^{l^{\breve{\tau}}}-\sum_{\iota=1}^{m+1}\breve{\omega}_\iota+1\right)^{\frac{1}{\breve{\tau}}}, \left(\sum_{\iota=1}^{m+1}\breve{\omega}_\iota\breve{\nu}_\iota^{u^{\breve{\tau}}}-\sum_{\iota=1}^{m+1}\breve{\omega}_\iota+1\right)^{\frac{1}{\breve{\tau}}}\right] \end{pmatrix}$$

Therefore, Equation (10) is true for $k = m+1$. Therefore, we can conclude that Equation (10) is true for all k based on the rule of mathematical induction. Since Equation (10) is true for any constraint of $\breve{\omega}$, Equation (9) is true. □

Example 1. *Suppose* $\mathfrak{P}_1 = ([0.63, 0.71], [0.12, 0.17], [0.08, 0.11])$, $\mathfrak{P}_2 = ([0.63, 0.66], [0.17, 0.22], [0.10, 0.11])$, *and* $\mathfrak{P}_3 = ([0.58, 0.61], [0.19, 0.23], [0.15, 0.16])$ *are three IvVPFNs. Then by using Equation* (6), *the distance is calculated as* $D(\mathfrak{P}_1, \mathfrak{P}_2) = 0.02353$, $D(\mathfrak{P}_1, \mathfrak{P}_3) = 0.0544$, *and* $D(\mathfrak{P}_2, \mathfrak{P}_3) = 0.03084$. *Thus, the support values are* $Sup(\mathfrak{P}_1, \mathfrak{P}_2) = 0.97647$, $Sup(\mathfrak{P}_1, \mathfrak{P}_3) = 0.9456$, *and* $Sup(\mathfrak{P}_2, \mathfrak{P}_3) = 0.96913$. *Now* $T(\mathfrak{P}_1) = Sup(\mathfrak{P}_1, \mathfrak{P}_2) + Sup(\mathfrak{P}_1, \mathfrak{P}_3) = 0.97647 + 0.9456 = 1.92207$; *similarly,* $T(\mathfrak{P}_2) = 1.9456$ *and* $T(\mathfrak{P}_3) = 1.91473$.

$$\begin{aligned} \therefore \breve{\omega}_1 &= \frac{(1+T(\mathfrak{P}_1))}{\sum_{\iota=1}^{3}(1+T(\mathfrak{P}_\iota))} \\ &= \frac{(1+1.92207)}{(1+1.92207)+(1+1.9456)+(1+1.91473)} \\ &= 0.3327. \end{aligned}$$

Similarly, $\breve{\omega}_2 = 0.3354$, $\breve{\omega}_3 = 0.3319$.

$$\therefore IvVPFSSA(\mathfrak{P}_1, \mathfrak{P}_2, \mathfrak{P}_3) = ([0.615, 0.667], [0.162, 0.208], [0.111, 0.128]).$$

Theorem 3. (*Idempotency*) *Suppose* $\mathfrak{P}_\iota = ([\breve{\mu}_\iota^l, \breve{\mu}_\iota^u], [\breve{\eta}_\iota^l, \breve{\eta}_\iota^u], [\breve{\nu}_\iota^l, \breve{\nu}_\iota^u])$, $(\iota = 1, 2, \ldots, k)$ *is a collection of IvVPFNs and* $\breve{\tau} < 0$. *If* $\mathfrak{P}_\iota = \mathfrak{P} = ([\breve{\mu}^l, \breve{\mu}^u], [\breve{\eta}^l, \breve{\eta}^u], [\breve{\nu}^l, \breve{\nu}^u])$, $\iota = 1, 2, \ldots, k$, *then*

$$IvVPFSSPA(\mathfrak{P}_1, \mathfrak{P}_2, \ldots, \mathfrak{P}_k) = \mathfrak{P}. \tag{12}$$

Proof. We know that by Equation (9),
$IvVPFSSPA(\mathfrak{P}_1, \mathfrak{P}_2, \ldots, \mathfrak{P}_k)$

$$\begin{aligned}
&= \begin{pmatrix} \left[1 - \left(\sum_{\iota=1}^{k} \breve{\omega}_\iota (1 - \breve{\mu}_\iota^l)^{\breve{\tau}}\right)^{\frac{1}{\breve{\tau}}}, 1 - \left(\sum_{\iota=1}^{k} \breve{\omega}_\iota (1 - \breve{\mu}_\iota^u)^{\breve{\tau}}\right)^{\frac{1}{\breve{\tau}}}\right], \\ \left[\left(\sum_{\iota=1}^{k} \breve{\omega}_\iota \breve{\eta}_\iota^{l^{\breve{\tau}}}\right)^{\frac{1}{\breve{\tau}}}, \left(\sum_{\iota=1}^{k} \breve{\omega}_\iota \breve{\eta}_\iota^{u^{\breve{\tau}}}\right)^{\frac{1}{\breve{\tau}}}\right], \\ \left[\left(\sum_{\iota=1}^{k} \breve{\omega}_\iota \breve{\nu}_\iota^{l^{\breve{\tau}}}\right)^{\frac{1}{\breve{\tau}}}, \left(\sum_{\iota=1}^{k} \breve{\omega}_\iota \breve{\nu}_\iota^{u^{\breve{\tau}}}\right)^{\frac{1}{\breve{\tau}}}\right] \end{pmatrix} \\
&= \begin{pmatrix} \left[1 - \left(\sum_{\iota=1}^{k} \breve{\omega}_\iota (1 - \breve{\mu}^l)^{\breve{\tau}}\right)^{\frac{1}{\breve{\tau}}}, 1 - \left(\sum_{\iota=1}^{k} \breve{\omega}_\iota (1 - \breve{\mu}^u)^{\breve{\tau}}\right)^{\frac{1}{\breve{\tau}}}\right], \\ \left[\left(\sum_{\iota=1}^{k} \breve{\omega}_\iota \breve{\eta}^{l^{\breve{\tau}}}\right)^{\frac{1}{\breve{\tau}}}, \left(\sum_{\iota=1}^{k} \breve{\omega}_\iota \breve{\eta}^{u^{\breve{\tau}}}\right)^{\frac{1}{\breve{\tau}}}\right], \\ \left[\left(\sum_{\iota=1}^{k} \breve{\omega}_\iota \breve{\nu}^{l^{\breve{\tau}}}\right)^{\frac{1}{\breve{\tau}}}, \left(\sum_{\iota=1}^{k} \breve{\omega}_\iota \breve{\nu}^{u^{\breve{\tau}}}\right)^{\frac{1}{\breve{\tau}}}\right] \end{pmatrix} \\
&= \begin{pmatrix} \left[1 - \left((1 - \breve{\mu}^{l^{\breve{\tau}}})\right)^{\frac{1}{\breve{\tau}}}, 1 - \left((1 - \breve{\mu}^{u^{\breve{\tau}}})\right)^{\frac{1}{\breve{\tau}}}\right], \\ \left[(\breve{\eta}^{l^{\breve{\tau}}})^{\frac{1}{\breve{\tau}}}, (\breve{\eta}^{u^{\breve{\tau}}})^{\frac{1}{\breve{\tau}}}\right], \left[(\breve{\nu}^{l^{\breve{\tau}}})^{\frac{1}{\breve{\tau}}}, (\breve{\nu}^{u^{\breve{\tau}}})^{\frac{1}{\breve{\tau}}}\right] \end{pmatrix} \\
&= ([\breve{\mu}^l, \breve{\mu}^u], [\breve{\eta}^l, \breve{\eta}^u], [\breve{\nu}^l, \breve{\nu}^u]) \\
&= \mathfrak{P}.
\end{aligned}$$

Hence, it is proved. □

Theorem 4. (*Boundedness*) *Suppose* $\mathfrak{P}_\iota = \left([\breve{\mu}_\iota^l, \breve{\mu}_\iota^u], [\breve{\eta}_\iota^l, \breve{\eta}_\iota^u], [\breve{\nu}_\iota^l, \breve{\nu}_\iota^u]\right)$, $(\iota = 1, 2, \ldots, k)$ *is a collection of IvVPFNs,* $\breve{\tau} < 0$ *and* $\mathfrak{P}^+ = \left([\max_\iota \breve{\mu}_\iota^l, \max_\iota \breve{\mu}_\iota^u], [\min_\iota \breve{\eta}_\iota^l, \min_\iota \breve{\eta}_\iota^u], [\min_\iota \breve{\nu}_\iota^l, \min_\iota \breve{\nu}_\iota^u]\right)$, $\mathfrak{P}^- = \left([\min_\iota \breve{\mu}_\iota^l, \min_\iota \breve{\mu}_\iota^u], [\max_\iota \breve{\eta}_\iota^l, \max_\iota \breve{\eta}_\iota^u], [\max_\iota \breve{\nu}_\iota^l, \max_\iota \breve{\nu}_\iota^u]\right)$. *Then*

$$\mathfrak{P}^- \leq IvVPFSSPA(\mathfrak{P}_1, \mathfrak{P}_2, \ldots, \mathfrak{P}_k) \leq \mathfrak{P}^+. \tag{13}$$

Proof. We know that for $\iota = 1, 2, \ldots, k$

$$\begin{aligned}
&\min_\iota \breve{\mu}_\iota^l \leq \breve{\mu}_\iota^l \leq \max_\iota \breve{\mu}_\iota^l \\
&\Rightarrow 1 - \min_\iota \breve{\mu}_\iota^l \geq 1 - \breve{\mu}_\iota^l \geq 1 - \max_\iota \breve{\mu}_\iota^l \\
&\Rightarrow \breve{\omega}_\iota (1 - \min_\iota \breve{\mu}_\iota^l)^{\breve{\tau}} \leq \breve{\omega}_\iota (1 - \breve{\mu}_\iota^l)^{\breve{\tau}} \leq \breve{\omega}_\iota (1 - \max_\iota \breve{\mu}_\iota^l)^{\breve{\tau}}, \ \breve{\tau} < 0 \\
&\Rightarrow \left(\sum_{\iota=1}^{k} \breve{\omega}_\iota (1 - \min_\iota \breve{\mu}_\iota^l)^{\breve{\tau}}\right)^{\frac{1}{\breve{\tau}}} \geq \left(\sum_{\iota=1}^{k} \breve{\omega}_\iota (1 - \breve{\mu}_\iota^l)^{\breve{\tau}}\right)^{\frac{1}{\breve{\tau}}} \geq \left(\sum_{\iota=1}^{k} \breve{\omega}_\iota (1 - \max_\iota \breve{\mu}_\iota^l)^{\breve{\tau}}\right)^{\frac{1}{\breve{\tau}}}
\end{aligned}$$

$$\Rightarrow 1-\left(\sum_{\iota=1}^{k}\check{\omega}_{\iota}(1-\min_{\iota}\check{\mu}_{\iota}^{l})^{\check{\tau}}\right)^{\frac{1}{\check{\tau}}}\leq 1-\left(\sum_{\iota=1}^{k}\check{\omega}_{\iota}(1-\check{\mu}_{\iota}^{l})^{\check{\tau}}\right)^{\frac{1}{\check{\tau}}}\leq 1-\left(\sum_{\iota=1}^{k}\check{\omega}_{\iota}(1-\max_{\iota}\check{\mu}_{\iota}^{l})^{\check{\tau}}\right)^{\frac{1}{\check{\tau}}}$$

$$\Rightarrow \min_{\iota}\check{\mu}_{\iota}^{l}\leq 1-\left(\sum_{\iota=1}^{k}\check{\omega}_{\iota}(1-\check{\mu}_{\iota}^{l})^{\check{\tau}}\right)^{\frac{1}{\check{\tau}}}\leq \max_{\iota}\check{\mu}_{\iota}^{l}.$$

Similarly, we have the following relation:

$$\min_{\iota}\check{\mu}_{\iota}^{u}\leq 1-\left(\sum_{\iota=1}^{k}\check{\omega}_{\iota}(1-\check{\mu}_{\iota}^{u})^{\check{\tau}}\right)^{\frac{1}{\check{\tau}}}\leq \max_{\iota}\check{\mu}_{\iota}^{u}.$$

Again for neutrality,

$$\min_{\iota}\check{\eta}_{\iota}^{l}\leq\check{\eta}_{\iota}^{l}\leq\max_{\iota}\check{\eta}_{\iota}^{l}$$

$$\Rightarrow\left(\min_{\iota}\check{\eta}_{\iota}^{l}\right)^{\check{\tau}}\geq\check{\eta}_{\iota}^{l}\geq\left(\max_{\iota}\check{\eta}_{\iota}^{l^{\check{\tau}}}\right)^{\check{\tau}},\ \check{\tau}<0$$

$$\Rightarrow\sum_{\iota=1}^{k}\check{\omega}_{\iota}\left(\min_{\iota}\check{\eta}_{\iota}^{l}\right)^{\check{\tau}}\geq\sum_{\iota=1}^{k}\check{\omega}_{\iota}\check{\eta}_{\iota}^{l^{\check{\tau}}}\geq\sum_{\iota=1}^{k}\check{\omega}_{\iota}\left(\max_{\iota}\check{\eta}_{\iota}^{l}\right)^{\check{\tau}}$$

$$\Rightarrow\left(\sum_{\iota=1}^{k}\check{\omega}_{\iota}\left(\min_{\iota}\check{\eta}_{\iota}^{l}\right)^{\check{\tau}}\right)^{\frac{1}{\check{\tau}}}\leq\left(\sum_{\iota=1}^{k}\check{\omega}_{\iota}\check{\eta}_{\iota}^{l^{\check{\tau}}}\right)^{\frac{1}{\check{\tau}}}\leq\left(\sum_{\iota=1}^{k}\check{\omega}_{\iota}\left(\max_{\iota}\check{\eta}_{\iota}^{l}\right)^{\check{\tau}}\right)^{\frac{1}{\check{\tau}}}$$

$$\Rightarrow\min_{\iota}\check{\eta}_{\iota}^{l}\leq\left(\sum_{\iota=1}^{k}\check{\omega}_{\iota}\check{\eta}_{\iota}^{l^{\check{\tau}}}\right)^{\frac{1}{\check{\tau}}}\leq\max_{\iota}\check{\eta}_{\iota}^{l}.$$

Similarly, we can obtain the other relation

$$\min_{\iota}\check{\eta}_{\iota}^{u}\leq\left(\sum_{\iota=1}^{k}\check{\omega}_{\iota}\check{\eta}_{\iota}^{u^{\check{\tau}}}\right)^{\frac{1}{\check{\tau}}}\leq\max_{\iota}\check{\eta}_{\iota}^{u},$$

$$\min_{\iota}\check{\nu}_{\iota}^{l}\leq\left(\sum_{\iota=1}^{k}\check{\omega}_{\iota}\check{\nu}_{\iota}^{l^{\check{\tau}}}\right)^{\frac{1}{\check{\tau}}}\leq\max_{\iota}\check{\nu}_{\iota}^{l},$$

$$\min_{\iota}\check{\nu}_{\iota}^{u}\leq\left(\sum_{\iota=1}^{k}\check{\omega}_{\iota}\check{\nu}_{\iota}^{u^{\check{\tau}}}\right)^{\frac{1}{\check{\tau}}}\leq\max_{\iota}\check{\nu}_{\iota}^{u}$$

□

Theorem 5. (*Monotonicity*) *Suppose* $\mathfrak{P}_{\iota}=\left([\check{\mu}_{\iota}^{l},\check{\mu}_{\iota}^{u}],[\check{\eta}_{\iota}^{l},\check{\eta}_{\iota}^{u}],[\check{\nu}_{\iota}^{l},\check{\nu}_{\iota}^{u}]\right)$ *and* $\mathfrak{P}_{\iota}'=\left([\check{\mu}_{\iota}^{l'},\check{\mu}_{\iota}^{u'}],[\check{\eta}_{\iota}^{l'},\check{\eta}_{\iota}^{u'}],[\check{\nu}_{\iota}^{l'},\check{\nu}_{\iota}^{u'}]\right)$ *are two collections of IvVPFNs. If* $\mathfrak{P}_{\iota}\leq\mathfrak{P}_{\iota}'$, *then*

$$IvVPFSSPA(\mathfrak{P}_{1},\mathfrak{P}_{2},\ldots\mathfrak{P}_{\iota})\leq IvVPFSSPA(\mathfrak{P}_{1}',\mathfrak{P}_{2}',\ldots\mathfrak{P}_{\iota}').\tag{14}$$

Proof. The proof is straightforward. □

Definition 9. *Suppose* $\mathfrak{P}_\iota = \left([\breve{\mu}_\iota^l, \breve{\mu}_\iota^u], [\breve{\eta}_\iota^l, \breve{\eta}_\iota^u], [\breve{\nu}_\iota^l, \breve{\nu}_\iota^u]\right)$, $(\iota = 1, 2, \ldots, k)$ *is a collection of IvVPFNs and* $IvVPFSSPG : \psi^k \to \psi$ *is such that*

$$IvVPFSSPG(\mathfrak{P}_1, \mathfrak{P}_2, \ldots, \mathfrak{P}_k) = \mathfrak{P}_\iota^{\frac{\otimes_{\iota=1}^{k}\left((1+T(\mathfrak{P}_\iota))\right)}{\sum_{\iota=1}^{k}(1+T(\mathfrak{P}_\iota))}}, \tag{15}$$

where ψ *is the collection of all IvVPFNs in a universal set* $\mathfrak{X}$ *and* $T(\mathfrak{P}_\iota) = \sum_{\substack{l=1 \\ l\neq\iota}}^{k} sup(\mathfrak{P}_\iota, \mathfrak{P}_l)$. *Then IvVPFSSPG is called interval-valued picture fuzzy Schweizer–Sklar power geometric aggregation operator.*

Theorem 6. *Suppose* $\mathfrak{P}_\iota = \left([\breve{\mu}_\iota^l, \breve{\mu}_\iota^u], [\breve{\eta}_\iota^l, \breve{\eta}_\iota^u], [\breve{\nu}_\iota^l, \breve{\nu}_\iota^u]\right)$, $(\iota = 1, 2, \ldots, k)$ *is a collection of IvVPFNs and* $\breve{\tau} < 0$. *Then, utilizing Equation (15), the aggregated result is again an IvVPFN given by*

$$IvVPFSSPG(\mathfrak{P}_1, \mathfrak{P}_2, \ldots, \mathfrak{P}_k) = \left(\begin{array}{c} \left[\left(\sum_{\iota=1}^{k} \breve{\omega}_\iota {\breve{\mu}_\iota^l}^{\breve{\tau}} \right)^{\frac{1}{\breve{\tau}}}, \left(\sum_{\iota=1}^{k} \breve{\omega}_\iota {\breve{\mu}_\iota^u}^{\breve{\tau}} \right)^{\frac{1}{\breve{\tau}}} \right], \\ \left[1 - \left(\sum_{\iota=1}^{k} \breve{\omega}_\iota (1-\breve{\eta}_\iota^l)^{\breve{\tau}} \right)^{\frac{1}{\breve{\tau}}}, 1 - \left(\sum_{\iota=1}^{k} \breve{\omega}_\iota (1-\breve{\eta}_\iota^u)^{\breve{\tau}} \right)^{\frac{1}{\breve{\tau}}} \right], \\ \left[1 - \left(\sum_{\iota=1}^{k} \breve{\omega}_\iota (1-\breve{\nu}_\iota^l)^{\breve{\tau}} \right)^{\frac{1}{\breve{\tau}}}, 1 - \left(\sum_{\iota=1}^{k} \breve{\omega}_\iota (1-\breve{\nu}_\iota^u)^{\breve{\tau}} \right)^{\frac{1}{\breve{\tau}}} \right] \end{array} \right), \tag{16}$$

where $\breve{\omega}_\iota = \frac{(1+T(\mathfrak{P}_\iota))}{\sum_{\iota=1}^{k}(1+T(\mathfrak{P}_\iota))}$ *is the integrated weights for* $\iota = 1, 2, \ldots, k$.

Proof. The proof is identical to Theorem 2. □

Theorem 7. $(Idempotency)$ *Suppose* $\mathfrak{P}_\iota = \left([\breve{\mu}_\iota^l, \breve{\mu}_\iota^u], [\breve{\eta}_\iota^l, \breve{\eta}_\iota^u], [\breve{\nu}_\iota^l, \breve{\nu}_\iota^u]\right)$, $(\iota = 1, 2, \ldots, k)$ *is a collection of IvVPFNs and* $\breve{\tau} < 0$. *If* $\mathfrak{P}_\iota = \mathfrak{P} = \left([\breve{\mu}^l, \breve{\mu}^u], [\breve{\eta}^l, \breve{\eta}^u], [\breve{\nu}^l, \breve{\nu}^u]\right)$, $\iota = 1, 2, \ldots, k$, *then*

$$IvVPFSSPG(\mathfrak{P}_1, \mathfrak{P}_2, \ldots, \mathfrak{P}_k) = \mathfrak{P} \tag{17}$$

Proof. The proof is identical to Theorem 3. □

Theorem 8. $(Boundedness)$ *Suppose* $\mathfrak{P}_\iota = \left([\breve{\mu}_\iota^l, \breve{\mu}_\iota^u], [\breve{\eta}_\iota^l, \breve{\eta}_\iota^u], [\breve{\nu}_\iota^l, \breve{\nu}_\iota^u]\right)$, $(\iota = 1, 2, \ldots, k)$ *is a collection of IvVPFNs,* $\breve{\tau} < 0$, *and* $\mathfrak{P}^+ = \left([\max_\iota \breve{\mu}_\iota^l, \max_\iota \breve{\mu}_\iota^u], [\min_\iota \breve{\eta}_\iota^l, \min_\iota \breve{\eta}_\iota^u], [\min_\iota \breve{\nu}_\iota^l, \min_\iota \breve{\nu}_\iota^u]\right)$, $\mathfrak{P}^- = \left([\min_\iota \breve{\mu}_\iota^l, \min_\iota \breve{\mu}_\iota^u], [\max_\iota \breve{\eta}_\iota^l, \max_\iota \breve{\eta}_\iota^u], [\max_\iota \breve{\nu}_\iota^l, \max_\iota \breve{\nu}_\iota^u]\right)$. *Then*

$$\mathfrak{P}^- \leq IvVPFSSPG(\mathfrak{P}_1, \mathfrak{P}_2, \ldots, \mathfrak{P}_k) \leq \mathfrak{P}^+. \tag{18}$$

Proof. The proof is identical to Theorem 4. □

4.2. IvPFS Bidirectional Projection Measure

In this section, motivated by the existing work under PF, IvVIFSs [64,68], the concept of IvVPFS projection measure, normalized projection measure, bidirectional projection measure, and related properties are discussed, which are used in the evaluation of an EV public charging station.

Definition 10. *Let* $\mathfrak{P} = ([\breve{\mu}_j^l, \breve{\mu}_j^u], [\breve{\eta}_j^l, \breve{\eta}_j^u], [\breve{\nu}_j^l, \breve{\nu}_j^u])$, $(j = 1, 2, \ldots, n)$ *be a collection of n IvVPFNs. Then the module is defined by*

$$\|\mathfrak{P}\| = \left(\sum_{j=1}^{n}\left((\breve{\mu}_j^l)^2 + (\breve{\mu}_j^u)^2 + (\breve{\eta}_j^l)^2 + (\breve{\eta}_j^u)^2 + (\breve{\nu}_j^l)^2 + (\breve{\nu}_j^u)^2 + (\breve{\pi}_j^l)^2 + (\breve{\pi}_j^u)^2\right)\right)^{\frac{1}{2}}. \quad (19)$$

Definition 11. *Let* $\mathfrak{P} = ([\breve{\mu}_j^l, \breve{\mu}_j^u], [\breve{\eta}_j^l, \breve{\eta}_j^u], [\breve{\nu}_j^l, \breve{\nu}_j^u])$ *and* $\mathfrak{P}' = ([\breve{\mu}_j'^l, \breve{\mu}_j'^u], [\breve{\eta}_j'^l, \breve{\eta}_j'^u], [\breve{\nu}_j'^l, \breve{\nu}_j'^u])$ *be two collections of IvVPFNs; then the inner product is defined by*

$$\mathfrak{P} \cdot \mathfrak{P}' = \sum_{j=1}^{n}\left(\breve{\mu}_j^l\breve{\mu}_j'^l + \breve{\mu}_j^u\breve{\mu}_j'^u + \breve{\eta}_j^l\breve{\eta}_j'^l + \breve{\eta}_j^u\breve{\eta}_j'^u + \breve{\nu}_j^l\breve{\nu}_j'^l + \breve{\nu}_j^u\breve{\nu}_j'^u + \breve{\pi}_j^l\breve{\pi}_j'^l + \breve{\pi}_j^u\breve{\pi}_j'^u\right). \quad (20)$$

Definition 12. *Given two collections of IvVPFNs,* $\mathfrak{P}$ *and* $\mathfrak{P}'$*, the cosine of the included angle between* $\mathfrak{P}$ *and* $\mathfrak{P}'$ *is defined by*

$$\cos(\mathfrak{P}, \mathfrak{P}') = \frac{\mathfrak{P} \cdot \mathfrak{P}'}{\|\mathfrak{P}\|\|\mathfrak{P}'\|}. \quad (21)$$

Definition 13. *Let* $\mathfrak{P} = ([\breve{\mu}_j^l, \breve{\mu}_j^u], [\breve{\eta}_j^l, \breve{\eta}_j^u], [\breve{\nu}_j^l, \breve{\nu}_j^u])$ *and* $\mathfrak{P}' = ([\breve{\mu}_j'^l, \breve{\mu}_j'^u], [\breve{\eta}_j'^l, \breve{\eta}_j'^u], [\breve{\nu}_j'^l, \breve{\nu}_j'^u])$ *be two collections of IvVPFNs. Then*

$$Prj_{\mathfrak{P}'}(\mathfrak{P}) = \|\mathfrak{P}\|\cos(\mathfrak{P}, \mathfrak{P}') = \frac{\mathfrak{P}\cdot\mathfrak{P}'}{\|\mathfrak{P}'\|}, \quad (22)$$

$$Prj_{\mathfrak{P}}(\mathfrak{P}') = \|\mathfrak{P}'\|\cos(\mathfrak{P}, \mathfrak{P}') = \frac{\mathfrak{P}\cdot\mathfrak{P}'}{\|\mathfrak{P}\|}, \quad (23)$$

are called projection measures of $\mathfrak{P}$ *on* $\mathfrak{P}'$ *and projection measures of* $\mathfrak{P}'$ *on* $\mathfrak{P}$*, respectively.*

Definition 14. *Let* $\mathfrak{P} = ([\breve{\mu}_j^l, \breve{\mu}_j^u], [\breve{\eta}_j^l, \breve{\eta}_j^u], [\breve{\nu}_j^l, \breve{\nu}_j^u])$ *and* $\mathfrak{P}_j' = ([\breve{\mu}_j'^l, \breve{\mu}_j'^u], [\breve{\eta}_j'^l, \breve{\eta}_j'^u], [\breve{\nu}_j'^l, \breve{\nu}_j'^u])$ *be two collections of IvVPFNs. Then*

$$Nprj_{\mathfrak{P}'}(\mathfrak{P}) = \frac{\min\{\mathfrak{P}\cdot\mathfrak{P}', \|\mathfrak{P}\|^2, \|\mathfrak{P}'\|^2\}}{\max\{\mathfrak{P}\cdot\mathfrak{P}', \|\mathfrak{P}\|^2, \|\mathfrak{P}'\|^2\}} \quad (24)$$

is called normalized projection measure of $\mathfrak{P}$ *on* $\mathfrak{P}'$*.*

Definition 15. *Let* $\mathfrak{P} = ([\breve{\mu}_j^l, \breve{\mu}_j^u], [\breve{\eta}_j^l, \breve{\eta}_j^u], [\breve{\nu}_j^l, \breve{\nu}_j^u])$ *and* $\mathfrak{P}' = ([\breve{\mu}_j'^l, \breve{\mu}_j'^u], [\breve{\eta}_j'^l, \breve{\eta}_j'^u], [\breve{\nu}_j'^l, \breve{\nu}_j'^u])$ *be two collection of IvVPFNs. Then the IvVPF bidirectional projection measure is defined as*

$$Bprj_{\mathfrak{P}'}(\mathfrak{P}) = \frac{\|\mathfrak{P}\|\|\mathfrak{P}'\|}{\|\mathfrak{P}\|\|\mathfrak{P}'\| + \left|\|\mathfrak{P}'\| - \|\mathfrak{P}\|\right|(\mathfrak{P}\cdot\mathfrak{P}')}. \quad (25)$$

Obviously, the larger value of the bidirectional measure represents the closeness of the two collections of IvVPFNs.

Some advantages of using the IvVPF bidirectional projection measure in decision making are that it considers both the directions and the angles between the objects. It can handle various types of data, including fuzzy numbers. Additionally, it can accurately measure the closeness between alternatives and the ideal solution. However, some disadvantages of this measure are that it is time-consuming and highly sensitive and has limited application.

Example 2. *Let* $\mathfrak{P} = (([0, 0.1], [0.1, 0.2], [0.2, 0.3]); ([0.2, 0.4], [0, 0.2], [0.1, 0.3]); ([0.3, 0.4], [0.1, 0.3], [0.2, 0.3]))$ *and* $\mathfrak{P}' = (([0, 0.2], [0, 0.3], [0.1, 0.2]); ([0.1, 0.3], [0.1, 0.2], [0, 0.1]); ([0.2, 0.3],$

$[0, 0.2], [0, 0.3]))$ *be two collections of IvVPFNs. Then* $\|\mathfrak{P}\| = 1.523$, $\|\mathfrak{P}'\| = 1.726$, $\mathfrak{P} \cdot \mathfrak{P}' = 2.420$.

$$\begin{aligned} \therefore Prj_{\mathfrak{P}'}(\mathfrak{P}) &= 1.402 \\ Prj_{\mathfrak{P}}(\mathfrak{P}') &= 1.590 \\ Nprj_{\mathfrak{P}'}(\mathfrak{P}) &= 0.779 \\ Bprj_{\mathfrak{P}'}(\mathfrak{P}) &= 0.843. \end{aligned}$$

5. Materials and Methods

This section introduces a $MAGDM$ problem under an IvVPF environment by integrating the deviation-based method (DBM) and the $VIKOR$ method and establishes an evaluation framework for an EV public charging station. A useful method for determining the subjective weight of the characteristic is the DBM [71]. The advantage of the DBM is its capacity to evaluate the criteria's accuracy using the weights that the decision experts assigned. The $VIKOR$ approach is also highly helpful because it is easy to use and yields a compromise solution that is as close to the ideal solution as possible. However, there are some shortcomings using the DBM and $VIKOR$ methods. The $VIKOR$ method requires the assignment of weights to criteria and involves complexity in handling a large number of alternatives. The DBM, however, is unable to handle the uncertainty. IvVPFSs provide an effective tool that helps solve complex decision-making problems due to their PMD, NuMD, and NMD. Simpler, more sensible, and more efficient solutions will be obtained by combining the aggregation operator, DBM, and IvVPF bidirectional projection-based $VIKOR$. Therefore, in order to assess the alternatives' preference order, we present the $VIKOR$ approach based on IvVPF bidirectional projection. There are three stages in the evaluation process: aggregated decision matrix preparation, DBM for weight determination, and IvVPF bidirectional projection-based $VIKOR$. The key concept of the proposed framework is depicted in Figure 1. The following goes into further information regarding the methodology.

1. Suppose $\{\check{\mathcal{I}}_1, \check{\mathcal{I}}_2, \ldots, \check{\mathcal{I}}_m\}$ is m alternatives, $\{\check{\mathcal{J}}_1, \check{\mathcal{J}}_2, \ldots, \check{\mathcal{J}}_n\}$ is n attributes with weight $(w_1, w_2, \ldots, w_n)$ such that $0 \leq w_k \leq 1$, $\sum_{k=1}^{n} w_k = 1$. To evaluate these alternatives under the attributes, there are t decision makers (DMs) $(\mathfrak{E}^1, \mathfrak{E}^2, \ldots, \mathfrak{E}^t)$ with weights $(\check{\omega}_1, \check{\omega}_2, \ldots, \check{\omega}_t)$ such that $0 \leq \check{\omega}_t \leq 1$, $\sum_{k=1}^{t} \check{\omega}_k = 1$.
2. Assume that $E^t = [e_{ij}^t]_{m \times n}$ is the linguistic decision matrix provided by the t th expert, where the linguistic variable e_{ij}^t represents the evaluation of the ith concerning the j th attribute according to the t th decision expert. Let $([\check{\mu}_{ij}^{tl}, \check{\mu}_{ij}^{tu}], [\check{\eta}_{ij}^{tl}, \check{\eta}_{ij}^{tu}], [\check{\nu}_{ij}^{tl}, \check{\nu}_{ij}^{tu}])$ be the IvVPFNs corresponding to the linguistic variable e_{ij}^t, which are shown in Table 3.
3. Use the following equation to normalize the decision matrices:

$$\bar{E}^t = \begin{cases} e_{ij'}^t & for\ beneficial\ attributes \quad \check{\mathcal{J}}_j \\ \left(e_{ij}^t\right)^c, & for\ non\text{-}beneficial\ attributes \quad \check{\mathcal{J}}_j \end{cases}$$
$$for\ i = 1, 2, \ldots, m; j = 1, 2, \ldots, n \tag{26}$$

4. Using decision expert weights $\check{\omega}_j$ and IvVPFSSPWA or IvVPFSSPWG operators, compute the aggregated decision matrix $E = [e_{ij}]_{m \times n}$, where $e_{ij} = ([\check{\mu}_{ij}^l, \check{\mu}_{ij}^u], [\check{\eta}_{ij}^l, \check{\eta}_{ij}^u], [\check{\nu}_{ij}^l, \check{\nu}_{ij}^u])$.

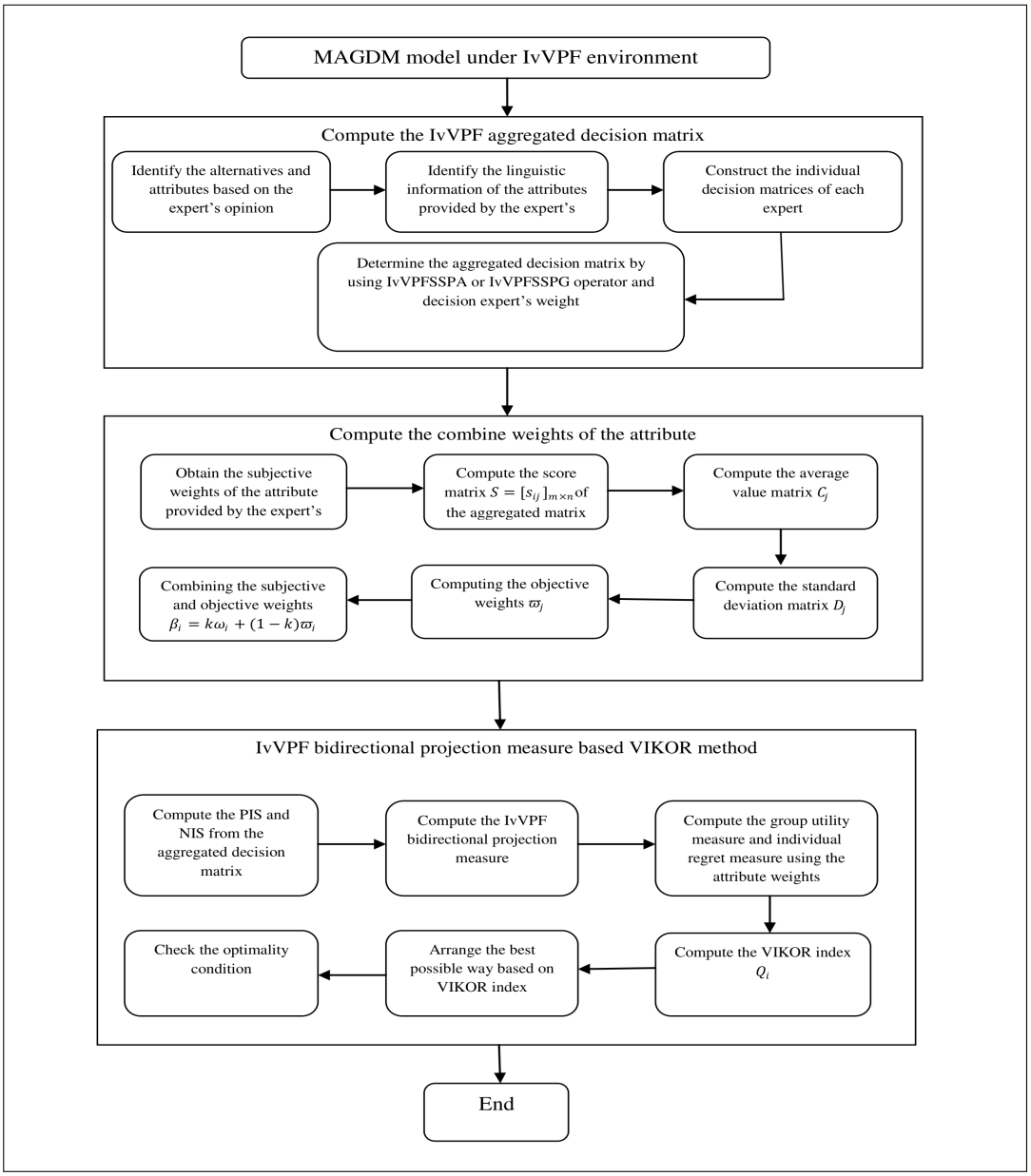


Figure 1. Methodology.

Table 3. IvVPF seven-point scale.

Linguistic Term	Mark	IvVPFN
Very good	$\mathcal{VG}$	([0.70, 0.75], [0.10, 0.15], [0.05, 0.09])
Good	$\mathcal{G}$	([0.68, 0.71], [0.12, 0.17], [0.08, 0.11])
Medium good	$\mathcal{MG}$	([0.63, 0.66], [0.17, 0.22], [0.10, 0.11])
Medium	$\mathcal{M}$	([0.58, 0.61], [0.19, 0.23], [0.15, 0.16])
Medium poor	$\mathcal{MP}$	([0.48, 0.51], [0.29, 0.32], [0.13, 0.15])
Poor	$\mathcal{P}$	([0.38, 0.46], [0.49, 0.52], [0.01, 0.02])
Very poor	$\mathcal{VP}$	([0.18, 0.30], [0.61, 0.62], [0.03, 0.07])

We determine the attribute's weight in this stage. We initially acquired the expert's subjective weight before applying the DBM in the manner described below:

1. The subjective weight of the attributes is obtained by consulting the decision expert. Let $\breve{\omega}_j$ represent the jth attribute's subjective weight.
2. The following phases make up the IvVPF environment-based DBM technique for determining the objective weight of the attributes.

(a) The score matrix $\check{S} = [s_{ij}]_{m\times n}$ of E is calculated by using Definition 3, where $s_{ij} = S(e_{ij})$.

(b) Equation (27) is used to calculate the average matrix $\check{C} = [C_j]_{1\times n}$, where

$$\check{C}_j = \frac{1}{m}\sum_{i=1}^{m} s_{ij}. \tag{27}$$

(c) The standard deviation matrix $\check{D} = [\check{D}_j]_{1\times n}$ is calculated by the following equation:

$$\check{D}_j = \sqrt{\frac{1}{m}\sum_{i=1}^{m}(s_{ij} - \check{C}_j)^2}. \tag{28}$$

(d) Use Equation (29) to determine the attribute's objective weight

$$\varpi_j = \frac{\check{D}_j}{\sum_{j=1}^{n}\check{D}_j}. \tag{29}$$

3. Let β_j be the combined weight, which is obtained by combining the subjective weight $(\check{\omega}_j)$ and the objective weight (ϖ_j), where

$$\beta_j = k\check{\omega}_j + (1-k)\varpi_j, \tag{30}$$

in which the aggregating coefficient, k, can be any value in the range $[0, 1]$.

By measuring the solution's proximity to the ideal solution, the $VIKOR$ approach is used to determine which is the best option. This strategy has grown in popularity because it facilitates decision making by successfully generating compromise solutions. The $VIKOR$ technique is applied in this step to determine the most efficient EV charging method. Listed below are the steps in the IvVPF $VIKOR$ approach:

1. For every criterion, find the negative ideal solution (NIS) and positive ideal solution (PIS). The following formulas are used to calculate the PIS and NIS:
For beneficial attributes:

$$\begin{aligned} P^+ &= \left([\max_i \check{\mu}_{ij}^l, \max_i \check{\mu}_{ij}^u], [\min_i \check{\eta}_{ij}^l, \min_i \check{\eta}_{ij}^u], [\min_i \check{\nu}_{ij}^l, \min_i \check{\nu}_{ij}^u]\right), \\ P^- &= \left([\min_i \check{\mu}_{ij}^l, \min_i \check{\mu}_{ij}^u], [\max_i \check{\eta}_{ij}^l, \max_i \check{\eta}_{ij}^u], [\max_i \check{\nu}_{ij}^l, \max_i \check{\nu}_{ij}^u]\right) \end{aligned} \tag{31}$$

For non-beneficial attributes:

$$\begin{aligned} P^+ &= \left([\min_i \check{\mu}_{ij}^l, \min_i \check{\mu}_{ij}^u], [\max_i \check{\eta}_{ij}^l, \max_i \check{\eta}_{ij}^u], [\max_i \check{\nu}_{ij}^l, \max_i \check{\nu}_{ij}^u]\right), \\ P^- &= \left([\max_i \check{\mu}_{ij}^l, \max_i \check{\mu}_{ij}^u], [\min_i \check{\eta}_{ij}^l, \min_i \check{\eta}_{ij}^u], [\min_i \check{\nu}_{ij}^l, \min_i \check{\nu}_{ij}^u]\right) \end{aligned} \tag{32}$$

2. Calculate the group utility measure (GUM) and individual regret measure (IRM) by using Equation (25), and the expression for GUM and IRM can be obtained as shown in the following Equations (33) and (34).

$$\mathcal{S}_i = \sum_{j=1}^{n}\beta_j\left(1 - \frac{Bprj_{P_i}(P^-)}{Bprj_{P_i}(P^-) + Bprj_{P^+}(P_i)}\right) \tag{33}$$

$$\mathcal{R}_i = \max_i \left\{ \beta_j \left(1 - \frac{Bprj_{P_i}(P^-)}{Bprj_{P_i}(P^-) + Bprj_{P^+}(P_i)} \right) \right\} \quad (34)$$

3. Calculate the compromise evaluation (CE) by using the following Equation (35):

$$Q_i = \theta \left(\frac{\mathcal{S}_i - \mathcal{S}^+}{\mathcal{S}^- - \mathcal{S}^+} \right) + (1-\theta) \left(\frac{\mathcal{R}_i - \mathcal{R}^+}{\mathcal{R}^- - \mathcal{R}^+} \right), \quad (35)$$

where $S^- = \max_i\{\mathcal{S}_i\}$, $S^+ = \min_i\{\mathcal{S}_i\}$, $R^- = \max_i\{\mathcal{R}_i\}$, and $R^+ = \min_i\{\mathcal{R}_i\}$. Here, θ is a balancing parameter. It shows how much importance is given to the strategy GUM with the best overall benefit, while $1-\theta$, the complement of θ, shows how much importance is given to the IRM. If θ is more than 0.5, it means that the evaluation will focus more on the overall benefit, which most decision makers agree with. If θ is less than 0.5, it means that the evaluation will focus more on the IRM. To balance both, θ is set to 0.5.

4. The most favorable choice satisfies the following criterion and is chosen based on the smaller values of Q_i, as follows:
   (a) Acceptable advantage: $\check{\mathcal{I}}_{(1)}$ and $\check{\mathcal{I}}_{(2)}$ are the best and second-best options, respectively, ranked by Q_i under the conditions $Q(\check{\mathcal{I}}_{(2)}) - Q(\check{\mathcal{I}}_{(1)}) \geq \frac{1}{m-1}$.
   (b) Acceptable stability in decision making: $\mathcal{S}_i$ or $\mathcal{R}_i$ must also rank the alternative $\check{\mathcal{I}}_{(1)}$ as the best.

   If any of the aforementioned conditions are not met, a set of alternative options can be considered as a compromise solution based on the following rules:
   (a) If Condition 4b is not met, both $\check{\mathcal{I}}_{(1)}$ and $\check{\mathcal{I}}_{(2)}$ will serve as a compromise solution.
   (b) If Condition 4a is not met, then $\check{\mathcal{I}}_{(1)}, \check{\mathcal{I}}_{(2)} \ldots \check{\mathcal{I}}_{(r)}$ will be considered compromise solutions, where r is the maximum value such that $Q(\check{\mathcal{I}}_{(r)}) - Q(\check{\mathcal{I}}_{(1)}) < \frac{1}{m-1}$.

6. Results

India exhibits a keen interest in mitigating its GHG emissions. To this end, the nation is concentrating on multiple sectors characterized by elevated GHG emissions. The transportation sector, with a particular emphasis on road transportation, represents a substantial contributor to GHG emissions, and addressing this concern necessitates the pivotal role of EVs. In order to boost EV manufacturing and acceptance, the Indian government has implemented various policies. It is seen that, nowadays, the number of EV users is increasing day by day in India. The total number of public charging stations is 12,146 till date 02.02.2024 as per the information received from the Ministry of Power (https://pib.gov.in/PressReleaseIframePage.aspx?PRID=2003003 (accessed on 23 January 2025)). By the end of 2030, the government plans to install a total of 46,397 EV public charging stations in nine major cities. The Ministry of Power published rules and guidelines for a charging infrastructure, outlining the precise duties and obligations of the state and federal stakeholders. Currently, the numbers of operational PCSs in different states are given in Figure 2.

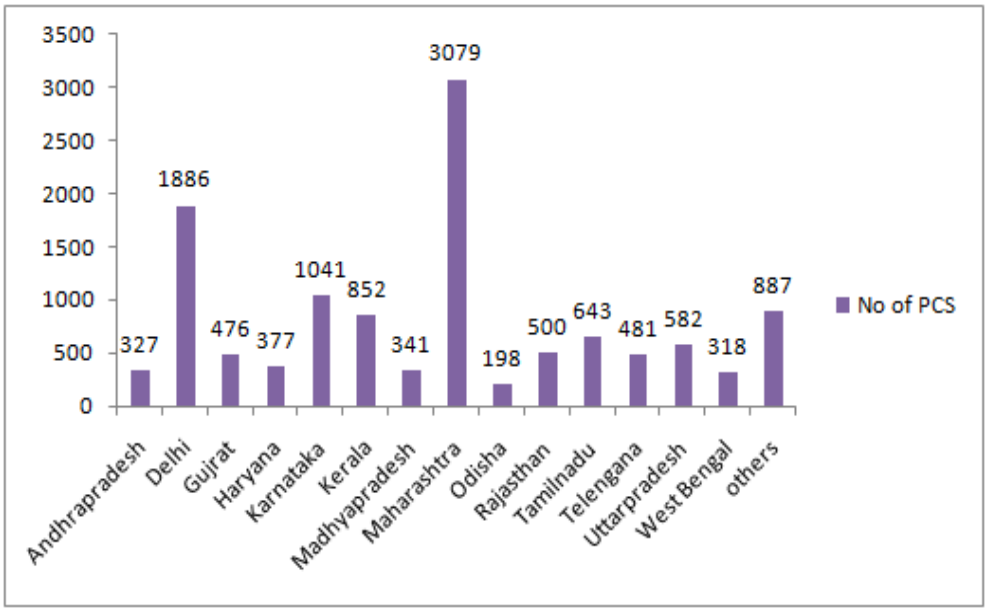


Figure 2. Numbers of operational PCS in different states.

6.1. Definition of Alternatives

The government of India is enthusiastic to find the most systematic way to manage how EV public charging stations are performing. The National Board of EV Public Charging Stations (NBEVPCS) has been established by the Indian Ministry of Power, which constitutes three experts. Now, there are several ways available to the government of India for evaluating public charging stations. According to expert opinions, we have discussed about the alternatives given in Table 4.

Table 4. Experts' suggested alternatives.

Alternatives	Description
($\check{\mathcal{I}}_1$): Level 1 charging (120 volts AC)	This is the simplest form of charging. This charging system is using a standard household electrical outlet. Level 1 charging technology is incredibly slow, with a charging range of only two to five miles per hour. When fast charging options are few in rural locations, this technique can be used for overnight charging at home.
($\check{\mathcal{I}}_2$): Level 2 charging (240 volts AC)	This is the dedicated form of charging. This facility is installed by a certified electrician. These charging stations are typically located in public spaces like parking lots, office buildings, and apartment complexes. In comparison with Level 1 charging, Level 2 chargers offer faster charging. This technique provides electricity with approximately 10 to 25 miles each charging hour.
($\check{\mathcal{I}}_3$): DC fast charging	This technology, also known as rapid charging, operates at a higher voltage (480 volts DC). These technologies can charge an EV faster than Level 1 or Level 2. These chargers can provide up to 60–80% battery charge in just 20–30 min. Generally, these charging stations are found along highways and in commercial areas.
($\check{\mathcal{I}}_4$): Wireless charging	Generally, this technology is known as inductive charging, which is free from physical cables and plugs. These stations use electromagnetic fields to transfer energy.
($\check{\mathcal{I}}_5$): Solar-powered charging	Solar energy is used in certain EV charging stations to produce electricity, offering EV users a sustainable and renewable energy source. These systems can either connect to the grid, sending extra energy back into it, or store it in batteries for use when the weather is cloudy.

6.2. Definition of Attributes

From the discussion in Table 4, the experts have suggested seven attributes. The alternatives will be arranged under these attributes. The attributes are given in Table 5. In this study, it is considered that $\check{\mathcal{J}}_1$, $\check{\mathcal{J}}_2$, $\check{\mathcal{J}}_3$, $\check{\mathcal{J}}_4$, $\check{\mathcal{J}}_6$, $\check{\mathcal{J}}_7$ are beneficial attributes and $\check{\mathcal{J}}_5$ is a non-beneficial attribute.

Table 5. The attributes' information.

Attributes	Description
($\check{\mathcal{J}}_1$): Accessibility	EV users should be able to find public charging stations easily along their journey. The stations should be near places where people are already moving, like shopping malls, offices, and highways.
($\check{\mathcal{J}}_2$): Compatibility	EV charging stations should have different types of cords that plug into all kinds of EVs. In this way, no confusion arises with regard to what kind of EV users have. They will be able to plug it in and charge it up. There are different types of cords, like CHAdeMO, CCS, and even Tesla super chargers.
($\check{\mathcal{J}}_3$): Power capacity	EV charging should have sufficient power capacity to charge an EV at a minimum amount of time. Fast charging reduces the waiting time of EV users.
($\check{\mathcal{J}}_4$): Reliability	For EV users, it is crucial that charging stations are available whenever they need to charge up their EV. Regular maintenance and servicing are essential to minimize the damage of EV.
($\check{\mathcal{J}}_5$): Safety	EV charging stations should follow the safety standards to protect users, EVs, and the surrounding infrastructure from electrical hazards.
($\check{\mathcal{J}}_6$): User experience	Charging stations should be user-friendly with an easy and clear picture or words showing users what to do. The sign should be easy to understand to users. There should also be light, a place to sit, a washroom, and a roof to keep users out of the rain or sun.
($\check{\mathcal{J}}_7$): Environmentally friendly	Charging stations should be powered by renewable energy so that greenhouse gas emissions are minimized.

6.3. Implementation of the Method

Our goal in this section is to show how the established model can be applied. In order to accomplish this, we have developed our suggested technique to address the numerical challenge of determining the best method for charging EVs. Our suggested method involves a step-by-step calculation to determine the numerical experiment's result.

1. This step determines the aggregated decision matrix by using an IvVPFSSWA aggregation operator.
   (a) Three decision experts $\mathfrak{E}^t$, $(t = 1, 2, 3)$ with weight $(0.5, 0.2, 0.3)$ are consulted, and they choose five alternatives and seven conflicting attributes.
   (b) Using the IvVPF seven-point scale (provided in Table 3), we create a unique decision matrix for each decision expert from Table 6.
   (c) Equation (26) should be used to normalize each decision matrix.
   (d) We utilize the decision expert weights $(0.5, 0.2, 0.3)$ and Equation (9) to construct the cumulative decision matrix, which is given in Table 7.

Table 6. Experts' decision in linguistic terms.

Matrix	Alternative	$\check{\mathcal{J}}_1$	$\check{\mathcal{J}}_2$	$\check{\mathcal{J}}_3$	$\check{\mathcal{J}}_4$	$\check{\mathcal{J}}_5$	$\check{\mathcal{J}}_6$	$\check{\mathcal{J}}_7$
$\mathfrak{E}^1$	$\check{\mathcal{I}}_1$	$\mathcal{VG}$	$\mathcal{G}$	$\mathcal{VG}$	$\mathcal{VG}$	$\mathcal{VP}$	$\mathcal{P}$	$\mathcal{MP}$
	$\check{\mathcal{I}}_2$	$\mathcal{P}$	$\mathcal{VG}$	$\mathcal{VG}$	$\mathcal{G}$	$\mathcal{MG}$	$\mathcal{P}$	$\mathcal{G}$
	$\check{\mathcal{I}}_3$	$\mathcal{MG}$	$\mathcal{G}$	$\mathcal{G}$	$\mathcal{P}$	$\mathcal{MP}$	$\mathcal{MG}$	$\mathcal{G}$
	$\check{\mathcal{I}}_4$	$\mathcal{MG}$	$\mathcal{G}$	$\mathcal{MG}$	$\mathcal{MG}$	$\mathcal{G}$	$\mathcal{G}$	$\mathcal{G}$
	$\check{\mathcal{I}}_5$	$\mathcal{MP}$	$\mathcal{MP}$	$\mathcal{MP}$	$\mathcal{MP}$	$\mathcal{MG}$	$\mathcal{VP}$	$\mathcal{MG}$
$\mathfrak{E}^2$	$\check{\mathcal{I}}_1$	$\mathcal{VG}$	$\mathcal{G}$	$\mathcal{G}$	$\mathcal{VG}$	$\mathcal{M}$	$\mathcal{G}$	$\mathcal{MG}$
	$\check{\mathcal{I}}_2$	$\mathcal{G}$	$\mathcal{VG}$	$\mathcal{VG}$	$\mathcal{G}$	$\mathcal{MG}$	$\mathcal{VG}$	$\mathcal{G}$
	$\check{\mathcal{I}}_3$	$\mathcal{MG}$	$\mathcal{G}$	$\mathcal{G}$	$\mathcal{G}$	$\mathcal{VG}$	$\mathcal{G}$	$\mathcal{G}$
	$\check{\mathcal{I}}_4$	$\mathcal{MG}$	$\mathcal{MG}$	$\mathcal{G}$	$\mathcal{M}$	$\mathcal{G}$	$\mathcal{G}$	$\mathcal{MG}$
	$\check{\mathcal{I}}_5$	$\mathcal{G}$	$\mathcal{MG}$	$\mathcal{MG}$	$\mathcal{G}$	$\mathcal{VG}$	$\mathcal{VG}$	$\mathcal{M}$
$\mathfrak{E}^3$	$\check{\mathcal{I}}_1$	$\mathcal{G}$	$\mathcal{G}$	$\mathcal{VG}$	$\mathcal{G}$	$\mathcal{VP}$	$\mathcal{P}$	$\mathcal{G}$
	$\check{\mathcal{I}}_2$	$\mathcal{G}$	$\mathcal{VG}$	$\mathcal{VG}$	$\mathcal{VG}$	$\mathcal{MG}$	$\mathcal{MG}$	$\mathcal{G}$
	$\check{\mathcal{I}}_3$	$\mathcal{MG}$	$\mathcal{VG}$	$\mathcal{MG}$	$\mathcal{MG}$	$\mathcal{G}$	$\mathcal{G}$	$\mathcal{VG}$
	$\check{\mathcal{I}}_4$	$\mathcal{MG}$	$\mathcal{MG}$	$\mathcal{VP}$	$\mathcal{VP}$	$\mathcal{G}$	$\mathcal{G}$	$\mathcal{G}$
	$\check{\mathcal{I}}_5$	$\mathcal{G}$	$\mathcal{MG}$	$\mathcal{P}$	$\mathcal{P}$	$\mathcal{MG}$	$\mathcal{P}$	$\mathcal{MG}$

Table 7. Aggregated decision matrix.

	$\mathcal{J}_1$	$\mathcal{J}_2$	$\mathcal{J}_3$	$\mathcal{J}_4$	$\mathcal{J}_5$	$\mathcal{J}_6$	$\mathcal{J}_7$
$\check{\mathcal{I}}_1$	$\begin{pmatrix}[0.6944,0.7398],\\ [0.1049,0.1552],\\ [0.0553,0.0948]\end{pmatrix}$	$\begin{pmatrix}[0.6800,0.7100],\\ [0.1200,0.1700],\\ [0.0800,0.1100]\end{pmatrix}$	$\begin{pmatrix}[0.6963,0.7434],\\ [0.1032,0.1534],\\ [0.0533,0.0931]\end{pmatrix}$	$\begin{pmatrix}[0.6944,0.7398],\\ [0.1049,0.1552],\\ [0.0553,0.0948]\end{pmatrix}$	$\begin{pmatrix}[0.0569,0.0895],\\ [0.3657,0.4180],\\ [0.1979,0.3245]\end{pmatrix}$	$\begin{pmatrix}[0.4990,0.5555],\\ [0.2443,0.3217],\\ [0.0111,0.0222]\end{pmatrix}$	$\begin{pmatrix}[0.6001,0.6323],\\ [0.1720,0.2268],\\ [0.1017,0.1253]\end{pmatrix}$
$\check{\mathcal{I}}_2$	$\begin{pmatrix}[0.6004,0.6409],\\ [0.1632,0.2265],\\ [0.0142,0.0281]\end{pmatrix}$	$\begin{pmatrix}[0.7000,0.7500],\\ [0.1000,0.1500],\\ [0.0500,0.0900]\end{pmatrix}$	$\begin{pmatrix}[0.7000,0.7500],\\ [0.1000,0.1500],\\ [0.0500,0.0900]\end{pmatrix}$	$\begin{pmatrix}[0.6864,0.7239],\\ [0.1128,0.1632],\\ [0.0661,0.1027]\end{pmatrix}$	$\begin{pmatrix}[0.1000,0.1100],\\ [0.1700,0.2200],\\ [0.6300,0.6600]\end{pmatrix}$	$\begin{pmatrix}[0.5833,0.6365],\\ [0.1745,0.2417],\\ [0.0141,0.0279]\end{pmatrix}$	$\begin{pmatrix}[0.6800,0.7100],\\ [0.1200,0.1700],\\ [0.0800,0.1100]\end{pmatrix}$
$\check{\mathcal{I}}_3$	$\begin{pmatrix}[0.6300,0.6600],\\ [0.1700,0.2200],\\ [0.1000,0.1100]\end{pmatrix}$	$\begin{pmatrix}[0.6864,0.7239],\\ [0.1128,0.1632],\\ [0.0661,0.1027]\end{pmatrix}$	$\begin{pmatrix}[0.6672,0.6974],\\ [0.1301,0.1812],\\ [0.0847,0.1100]\end{pmatrix}$	$\begin{pmatrix}[0.5734,0.6150],\\ [0.1934,0.2568],\\ [0.0142,0.0281]\end{pmatrix}$	$\begin{pmatrix}[0.1005,0.1268],\\ [0.1457,0.2029],\\ [0.5577,0.5910]\end{pmatrix}$	$\begin{pmatrix}[0.6578,0.6881],\\ [0.1386,0.1902],\\ [0.0883,0.1100]\end{pmatrix}$	$\begin{pmatrix}[0.6864,0.7239],\\ [0.1128,0.1632],\\ [0.0661,0.1027]\end{pmatrix}$
$\check{\mathcal{I}}_4$	$\begin{pmatrix}[0.6300,0.6600],\\ [0.1700,0.2200],\\ [0.1000,0.1100]\end{pmatrix}$	$\begin{pmatrix}[0.6576,0.6879],\\ [0.1387,0.1903],\\ [0.0884,0.1100]\end{pmatrix}$	$\begin{pmatrix}[0.5955,0.6320],\\ [0.1758,0.2342],\\ [0.0497,0.0921]\end{pmatrix}$	$\begin{pmatrix}[0.5648,0.6018],\\ [0.2045,0.2578],\\ [0.0511,0.0957]\end{pmatrix}$	$\begin{pmatrix}[0.0800,0.1100],\\ [0.1200,0.1700],\\ [0.6800,0.7100]\end{pmatrix}$	$\begin{pmatrix}[0.6800,0.7100],\\ [0.1200,0.1700],\\ [0.0800,0.1100]\end{pmatrix}$	$\begin{pmatrix}[0.6717,0.7018],\\ [0.1265,0.1772],\\ [0.0830,0.1100]\end{pmatrix}$
$\check{\mathcal{I}}_5$	$\begin{pmatrix}[0.6156,0.6480],\\ [0.1562,0.2116],\\ [0.0961,0.1252]\end{pmatrix}$	$\begin{pmatrix}[0.5742,0.6055],\\ [0.2070,0.2560],\\ [0.1120,0.1253]\end{pmatrix}$	$\begin{pmatrix}[0.5059,0.5457],\\ [0.2658,0.3138],\\ [0.0182,0.0359]\end{pmatrix}$	$\begin{pmatrix}[0.5329,0.5731],\\ [0.2178,0.2782],\\ [0.0182,0.0358]\end{pmatrix}$	$\begin{pmatrix}[0.0907,0.1061],\\ [0.1450,0.1985],\\ [0.6422,0.6752]\end{pmatrix}$	$\begin{pmatrix}[0.4831,0.5634],\\ [0.2106,0.2470],\\ [0.0388,0.0813]\end{pmatrix}$	$\begin{pmatrix}[0.6215,0.6516],\\ [0.1735,0.2219],\\ [0.1060,0.1162]\end{pmatrix}$

2. We ascertain the attribute's combined weights in this stage. First, we ask the decision expert for the subjective weights of the attributes. Next, we determine the objective weights in relation to IvVPFNs using the DBM.
   (a) The subjective weight of all attributes are considered as

$$\check{\omega}_j = (0.17, 0.12, 0.24, 0.08, 0.14, 0.06, 0.19).$$

   (b) Here, using the DBM, we obtain the following result.
      i. We utilize Definition 3 of a score function and determine the score matrix as follows:

$$\check{S} = \begin{bmatrix} 0.3793 & 0.3517 & 0.3825 & 0.3793 & 0.0160 & 0.2453 & 0.2818 \\ 0.3239 & 0.3883 & 0.3883 & 0.3650 & 0.0111 & 0.3127 & 0.3517 \\ 0.3007 & 0.3650 & 0.3394 & 0.2980 & 0.0186 & 0.3300 & 0.3650 \\ 0.3007 & 0.3299 & 0.2957 & 0.2698 & 0.0097 & 0.3517 & 0.3437 \\ 0.2967 & 0.2552 & 0.2390 & 0.2668 & 0.0109 & 0.2470 & 0.2930 \end{bmatrix}.$$

      ii. The average value matrix C is determined by Equation (27) as follows:

$$\check{C} = \begin{bmatrix} 0.3203 & 0.3380 & 0.3290 & 0.3158 & 0.0133 & 0.2974 & 0.3271 \end{bmatrix}.$$

      iii. The standard deviation matrix D is determined by Equation (28) as follows:

$$\check{D} = \begin{bmatrix} 0.0311 & 0.0455 & 0.0560 & 0.0475 & 0.0034 & 0.0436 & 0.0333 \end{bmatrix}.$$

      iv. The objective weight of the attributes is determined by Equation (29) as follows:

$$\omega_j = \begin{bmatrix} 0.1193 & 0.1749 & 0.2151 & 0.1825 & 0.0132 & 0.1674 & 0.1277 \end{bmatrix}.$$

   (c) We consider $k = 0.5$ to determine the combined weight of the attribute by the following Equation (30):

$$\beta_j = \begin{bmatrix} 0.1446 & 0.1474 & 0.2276 & 0.1312 & 0.0766 & 0.1137 & 0.1588 \end{bmatrix}.$$

3. This step determines the $VIKOR$ method.

(a) Utilizing Equations (31) and (32), find the PIS and NIS.

$$\begin{aligned} P^{+} = \Big(& \big([0.6944, 0.7398], [0.1049, 0.1552], [0.0142, 0.0281] \big); \\ & \big([0.7000, 0.7500], [0.1000, 0.1500], [0.0500, 0.0900] \big); \\ & \big([0.7000, 0.7500], [0.1000, 0.1500], [0.0182, 0.0359] \big); \\ & \big([0.6944, 0.7398], [0.1049, 0.1552], [0.0142, 0.0281] \big); \\ & \big([0.0569, 0.0895], [0.3658, 0.4181], [0.6800, 0.7100] \big); \\ & \big([0.6800, 0.7100], [0.1200, 0.1700], [0.0111, 0.0222] \big); \\ & \big([0.6864, 0.7239], [0.1128, 0.1632], [0.0661, 0.1027] \big) \Big) \end{aligned}$$

$$\begin{aligned} P^{-} = \Big(& \big([0.6004, 0.6409], [0.1700, 0.2265], [0.1000, 0.1250] \big); \\ & \big([0.5742, 0.6055], [0.2070, 0.2560], [0.1120, 0.1253] \big); \\ & \big([0.5059, 0.5457], [0.2658, 0.3138], [0.0847, 0.1100] \big); \\ & \big([0.5329, 0.5731], [0.2178, 0.2782], [0.0661, 0.1027] \big); \\ & \big([0.1005, 0.1268], [0.1200, 0.1700], [0.1979, 0.3245] \big); \\ & \big([0.4811, 0.5555], [0.2443, 0.3218], [0.0883, 0.1100] \big); \\ & \big([0.6001, 0.6323], [0.1735, 0.2268], [0.1060, 0.1253] \big) \Big). \end{aligned}$$

(b) Equations (33) and (34) compute the GUM and IRM values in the manner shown in Table 8.

(c) The CE values are (taking $\theta = 0.5$) calculated by using Equation (35), as shown in Table 9.

(d) We see that the best alternative is $\check{\mathcal{I}}_5$. The ranking order is $\check{\mathcal{I}}_5 \succ \check{\mathcal{I}}_4 \succ \check{\mathcal{I}}_3 \succ \check{\mathcal{I}}_1 \succ \check{\mathcal{I}}_2$. Additionally, Tables 8 and 9 demonstrate that the results provided meet the following requirement:

i. $Q(\check{\mathcal{I}}_{(4)}) - Q(\check{\mathcal{I}}_{(5)}) = 0.4394 > \frac{1}{5-1} = 0.25$.

ii. The optimal solution is $\check{\mathcal{I}}_5$ ranked by $\mathcal{S}_i$ and $\mathcal{R}_i$ (Table 8).

Table 8. Values of $\mathcal{S}_i$, $\mathcal{R}_i$, and Q_i.

	$\check{\mathcal{I}}_1$	$\check{\mathcal{I}}_2$	$\check{\mathcal{I}}_3$	$\check{\mathcal{I}}_4$	$\check{\mathcal{I}}_5$
$\mathcal{S}_i$	0.5104	0.5177	0.5085	0.4987	0.4748
$\mathcal{R}_i$	0.1221	0.1227	0.1177	0.1110	0.1055
Q_i	0.8969	1.0000	0.7489	0.4394	0.0000

Table 9. Prioritizing options according to $\mathcal{S}_i$, $\mathcal{R}_i$, and Q_i.

Index	Ranking Order
$\mathcal{S}_i$	$\check{\mathcal{I}}_5 \succ \check{\mathcal{I}}_4 \succ \check{\mathcal{I}}_3 \succ \check{\mathcal{I}}_1 \succ \check{\mathcal{I}}_2$
$\mathcal{R}_i$	$\check{\mathcal{I}}_5 \succ \check{\mathcal{I}}_4 \succ \check{\mathcal{I}}_3 \succ \check{\mathcal{I}}_1 \succ \check{\mathcal{I}}_2$
Q_i	$\check{\mathcal{I}}_5 \succ \check{\mathcal{I}}_4 \succ \check{\mathcal{I}}_3 \succ \check{\mathcal{I}}_1 \succ \check{\mathcal{I}}_2$

7. Discussion

7.1. Sensitivity Analysis

In this paper, three parameters ($\check{\tau}$, k, and θ) are included in the suggested technique. Sensitivity analysis was performed to confirm the suggested approach. The following method may be used to perform the sensitivity analysis based on the parameters $\check{\tau}$, the aggregating coefficient k, and the decision coefficient θ.

1. Analyze the influence of the parameter $\check{\tau}$ and k on criterion weights.
   In step 2b, we introduce a method for determining the criterion weights. Here, we primarily explore how the parameter $\check{\tau}$ and k affect the calculation of these criterion weights, and then examine their interrelationship as shown in Figure 3. When $\check{\tau}$ varies from -1 to -10 with step size -1, we can see that there are several changes in the weights. We can observe that $\check{\mathcal{J}}_1$, $\check{\mathcal{J}}_5$, and $\check{\mathcal{J}}_7$ gradually increase as $\check{\mathcal{J}}_4$ and $\check{\mathcal{J}}_6$ gradually decrease. The values of $\check{\mathcal{J}}_2$ and $\check{\mathcal{J}}_3$ are fluctuating as the variation of $\check{\tau}$.

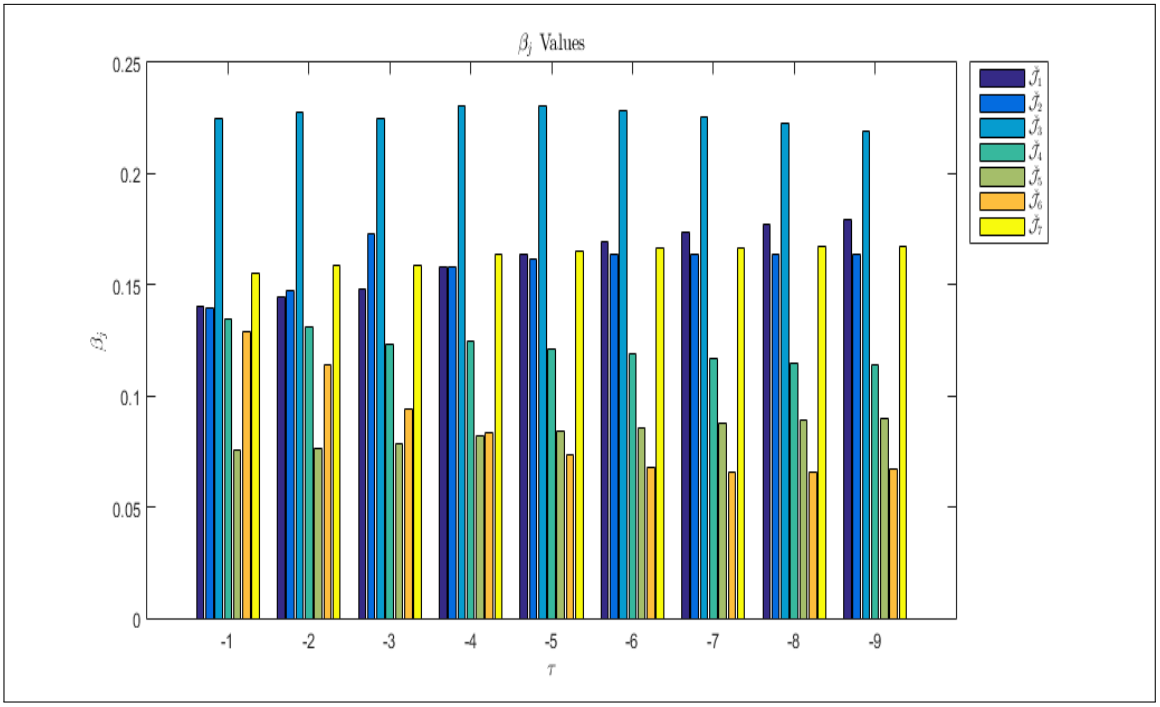


Figure 3. Trend of criterion weights changing when $\check{\tau}$ changes.

Again, when k varies from 0.1 to 0.9 with step size 0.1, we can observe that $\check{\mathcal{J}}_1$, $\check{\mathcal{J}}_3$, $\check{\mathcal{J}}_5$, and $\check{\mathcal{J}}_7$ gradually increase and $\check{\mathcal{J}}_2$, $\check{\mathcal{J}}_4$, and $\check{\mathcal{J}}_6$ gradually decrease. Thus, we can say that the parameter $\check{\tau}$ and k directly affect the criterion weights, as shown in Figure 4.

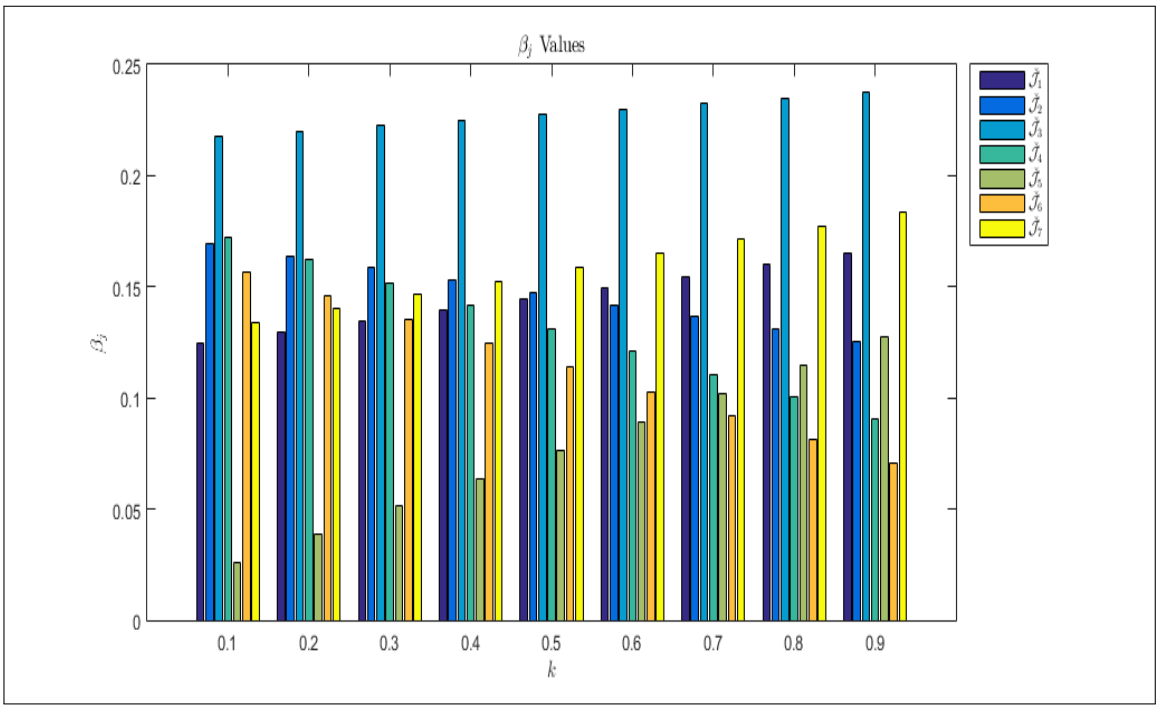
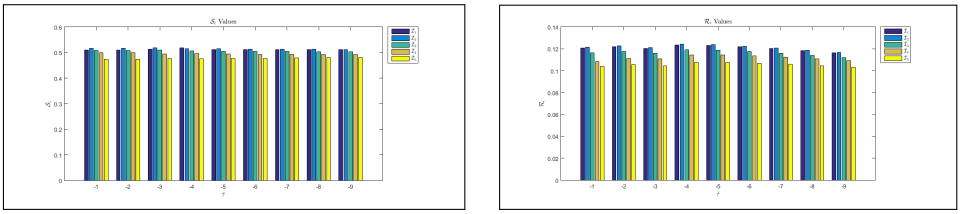


Figure 4. Trend of criterion weights changing when k changes.

2. Examine how the parameters affect the values of $\mathcal{S}_i$, $\mathcal{R}_i$, and Q_i.
The GUM $\mathcal{S}_i$ and IRM $\mathcal{R}_i$, which are displayed in Figures 5a,b and 6a,b, are indirectly impacted by the values of $\check{\tau}$ and k. It has a direct impact on the criterion weights.

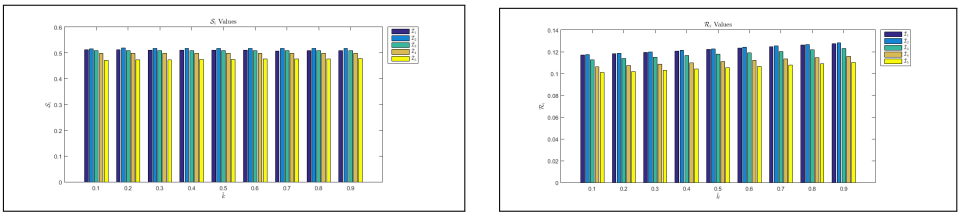
   (a) When we take $k = 0.5$ and $\theta = 0.5$ and varies $\check{\tau}$ from -1 to -9 with step size -1, we obtain different values of $\mathcal{S}_i$ and $\mathcal{R}_i$ (Figure 5a,b). We have seen that for $-3 \leq \check{\tau} \leq -1$, the utility values $\mathcal{S}_1$, $\mathcal{S}_2$, $\mathcal{S}_3$, and $\mathcal{S}_5$ are decreased as $\check{\tau}$ increases and $\mathcal{S}_4$ increases as $\check{\tau}$ increases. For $-9 \leq \check{\tau} \leq -4$, the utility measure values $\mathcal{S}_1$, $\mathcal{S}_2$, $\mathcal{S}_3$, and $\mathcal{S}_4$ increase as $\check{\tau}$ increases and $\mathcal{S}_5$ decreases as $\check{\tau}$ increases. For $-9 \leq \check{\tau} \leq -4$, the IRM values $\mathcal{R}_1$, $\mathcal{R}_2$, $\mathcal{R}_3$, $\mathcal{R}_4$, and $\mathcal{R}_5$ increase as $\check{\tau}$ increases.



(**a**) Change trend of $\mathcal{S}_i$ (**b**) Change trend of $\mathcal{R}_i$

Figure 5. Trend of $\mathcal{S}_i$ and $\mathcal{R}_i$ as $\check{\tau}$ changes.

   (b) By setting $\check{\tau} = -2$, $\theta = 0.5$ and varying k from 0.1 to 0.9 with step size 0.1, we obtain different values of $\mathcal{S}_i$ and $\mathcal{R}_i$ (Figure 6a,b). We have seen that the values of $\mathcal{S}_1$, $\mathcal{S}_2$, $\mathcal{S}_3$, and $\mathcal{S}_4$ are decreasing as the values of k are increasing and the values of $\mathcal{S}_5$ are increasing as the values of k are increasing. Again the values of all $\mathcal{R}_i$ are increasing as the values of k are increasing.



(**a**) Change trend of $\mathcal{S}_i$ (**b**) Change trend of $\mathcal{R}_i$

Figure 6. Trend of $\mathcal{S}_i$ and $\mathcal{R}_i$ as k changes.

(c) Figure 7 illustrates the change trend of Q_i when we set the parameters k and θ to start at 0.1 and raise them step-wise by 0.1. In the range $(0, 1)$, for k and θ, the values of Q_1 declined as k and θ grew. The values of Q_3 and Q_4 declined as k grew for $k \in (0, 1)$. The values of Q_3 and Q_4 rose in proportion to the values of $\theta \in (0, 1)$. As k and θ rose, the values of Q_2 and Q_5 remained unchanged.

(d) We set the parameter $\check{\tau}$ to start at -1 and decrease it step-wise by -1 and the parameter k to start at 0.1 and increase it step-wise by 0.1. We observe the change trend of the values Q_i, as shown in Figures 8 and 9. For $\check{\tau} < 0$, the values of Q_1 increase as $\check{\tau}$ decreases. For $\check{\tau} < -2$ and $\check{\tau} < -3$, the values of Q_3 and Q_4 decrease, respectively. For $\check{\tau} < 0$, the values of Q_2 and Q_5 remain unaltered as $\check{\tau}$ decreases.

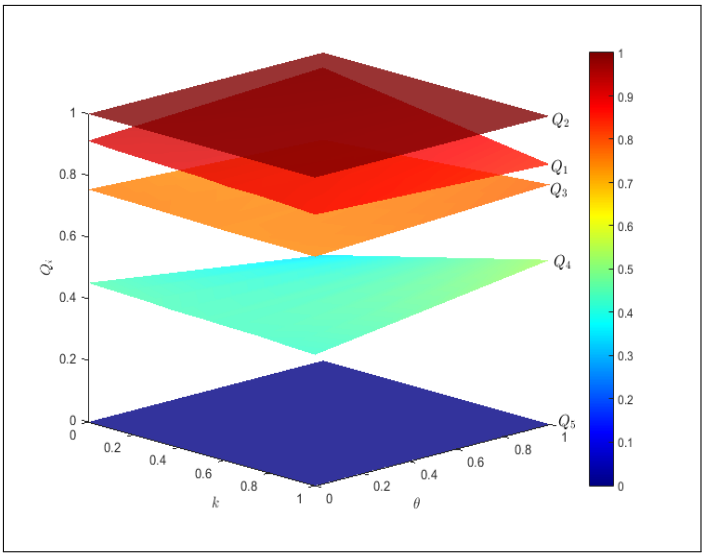


Figure 7. Q_i values with varying k and θ values.

3. Examine how the parameters effect the final sorting.
Using the suggested IvVPF decision-making framework, several values of $\check{\tau} < 0$, $k \in (0, 1)$, and $\theta \in (0, 1)$ were taken into account to show the dependability of the final ranking result of alternatives. Tables 10–12 present the findings of the sensitivity analysis.
First, we have investigated the impact of the parameter $\check{\tau}$ on the ultimate ranking. We have taken the parameter $\check{\tau}$ from -1 up to -100 with $k = 0.5$ and $\theta = 0.5$, and we have achieved a distinct compromise evaluation Q_i of alternatives. Table 10 presents the findings. The ultimate ranking, as we have seen, is $\check{\mathcal{I}}_5 \succ \check{\mathcal{I}}_4 \succ \check{\mathcal{I}}_3 \succ \check{\mathcal{I}}_1 \succ \check{\mathcal{I}}_2$. We did not obtain the final ranking for $\check{\tau} = -30$ and $\check{\tau} = -40$. For $\check{\tau} = -50$ to -100, the final ranking is $\check{\mathcal{I}}_4 \succ \check{\mathcal{I}}_5 \succ \check{\mathcal{I}}_3 \succ \check{\mathcal{I}}_2 \succ \check{\mathcal{I}}_1$.
The ranking of the alternatives stays $\check{\mathcal{I}}5 \succ \check{\mathcal{I}}4 \succ \check{\mathcal{I}}3 \succ \check{\mathcal{I}}1 \succ \check{\mathcal{I}}_2$ as indicated in Table 11 when the parameter k is changed from 0.1 to 0.9 while keeping $\check{\tau}$ and θ as constants.

This suggests that the parameter k has little to no effect on identifying the best course of action.

At last, we have examined how the decision coefficient θ affects the final values of Q_i. When the parameter θ is varied from 0.1 to 0.9 while keeping $\check{\tau} = -2$ and $k = 0.5$ fixed, the final ranking of the alternatives is $\check{\mathcal{I}}5 \succ \check{\mathcal{I}}4 \succ \check{\mathcal{I}}3 \succ \check{\mathcal{I}}1 \succ \check{\mathcal{I}}_2$ (see Table 12). Our finding indicates that for the parameter θ, there is no change in decision-making result.

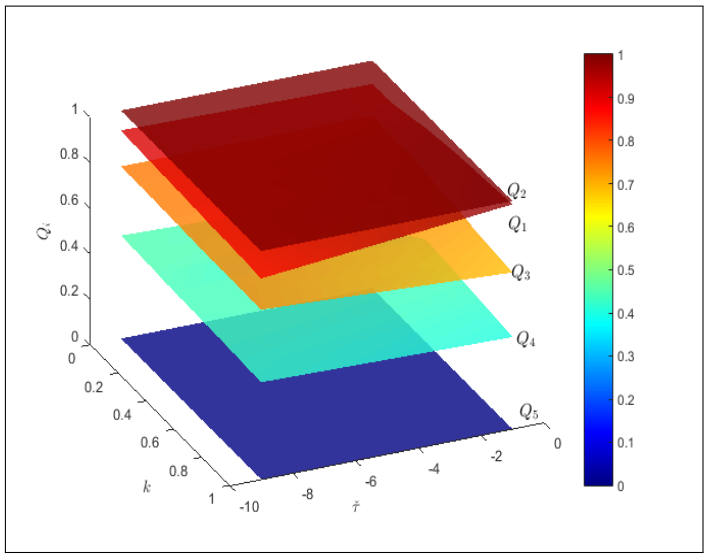


Figure 8. Q_i values with varying k and $\check{\tau}$.

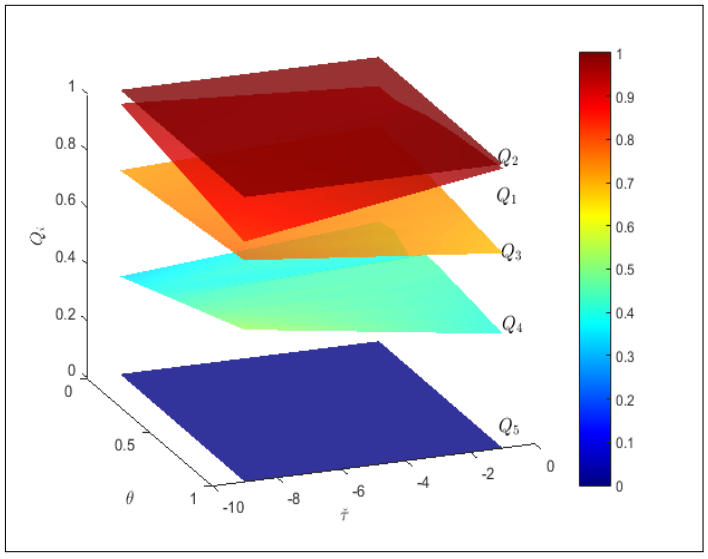


Figure 9. Q_i values with varying $\check{\tau}$ and θ.

Table 10. Findings of parameter $\check{\tau}$ when $k = 0.5$ and $\theta = 0.5$.

$\check{\tau}$	Q_i					Final Ranking
	$\check{\mathcal{I}}_1$	$\check{\mathcal{I}}_2$	$\check{\mathcal{I}}_3$	$\check{\mathcal{I}}_4$	$\check{\mathcal{I}}_5$	
-1	0.8964	1	0.7564	0.4211	0	$\check{\mathcal{I}}_5 \succ \check{\mathcal{I}}_4 \succ \check{\mathcal{I}}_3 \succ \check{\mathcal{I}}_1 \succ \check{\mathcal{I}}_2$
-2	0.8969	1	0.7489	0.4394	0	$\check{\mathcal{I}}_5 \succ \check{\mathcal{I}}_4 \succ \check{\mathcal{I}}_3 \succ \check{\mathcal{I}}_1 \succ \check{\mathcal{I}}_2$
-3	0.9168	1	0.7525	0.3868	0	$\check{\mathcal{I}}_5 \succ \check{\mathcal{I}}_4 \succ \check{\mathcal{I}}_3 \succ \check{\mathcal{I}}_1 \succ \check{\mathcal{I}}_2$
-4	0.9391	1	0.7417	0.4501	0	$\check{\mathcal{I}}_5 \succ \check{\mathcal{I}}_4 \succ \check{\mathcal{I}}_3 \succ \check{\mathcal{I}}_1 \succ \check{\mathcal{I}}_2$

Table 10. *Cont.*

$\breve{\tau}$	Q_i					**Final Ranking**
	$\breve{\mathcal{I}}_1$	$\breve{\mathcal{I}}_2$	$\breve{\mathcal{I}}_3$	$\breve{\mathcal{I}}_4$	$\breve{\mathcal{I}}_5$	
−5	0.9521	1	0.7300	0.4442	0	$\breve{\mathcal{I}}_5 \succ \breve{\mathcal{I}}_4 \succ \breve{\mathcal{I}}_3 \succ \breve{\mathcal{I}}_1 \succ \breve{\mathcal{I}}_2$
−6	0.9617	1	0.7180	0.4346	0	$\breve{\mathcal{I}}_5 \succ \breve{\mathcal{I}}_4 \succ \breve{\mathcal{I}}_3 \succ \breve{\mathcal{I}}_1 \succ \breve{\mathcal{I}}_2$
−7	0.9706	1	0.7069	0.4235	0	$\breve{\mathcal{I}}_5 \succ \breve{\mathcal{I}}_4 \succ \breve{\mathcal{I}}_3 \succ \breve{\mathcal{I}}_1 \succ \breve{\mathcal{I}}_2$
−8	0.9780	1	0.6966	0.4124	0	$\breve{\mathcal{I}}_5 \succ \breve{\mathcal{I}}_4 \succ \breve{\mathcal{I}}_3 \succ \breve{\mathcal{I}}_1 \succ \breve{\mathcal{I}}_2$
−9	0.9838	0.9997	0.6879	0.4025	0	$\breve{\mathcal{I}}_5 \succ \breve{\mathcal{I}}_4 \succ \breve{\mathcal{I}}_3 \succ \breve{\mathcal{I}}_1 \succ \breve{\mathcal{I}}_2$
−10	0.9839	0.9953	0.6743	0.3894	0	$\breve{\mathcal{I}}_5 \succ \breve{\mathcal{I}}_4 \succ \breve{\mathcal{I}}_3 \succ \breve{\mathcal{I}}_1 \succ \breve{\mathcal{I}}_2$
−20	0.9854	0.9624	0.0.5886	0.0.2935	0	$\breve{\mathcal{I}}_5 \succ \breve{\mathcal{I}}_4 \succ \breve{\mathcal{I}}_3 \succ \breve{\mathcal{I}}_1 \succ \breve{\mathcal{I}}_2$
−30	1	0.9065	0.4011	0.0753	0.1326	Not evaluated
−40	1	0.7320	0.3447	0.0662	0.2456	Not evaluated
−50	1	0.7404	0.2928	0.0542	0.2704	$\breve{\mathcal{I}}_4 \succ \breve{\mathcal{I}}_5 \succ \breve{\mathcal{I}}_3 \succ \breve{\mathcal{I}}_2 \succ \breve{\mathcal{I}}_1$
−60	1	0.7406	0.2882	0.0490	0.2740	$\breve{\mathcal{I}}_4 \succ \breve{\mathcal{I}}_5 \succ \breve{\mathcal{I}}_3 \succ \breve{\mathcal{I}}_2 \succ \breve{\mathcal{I}}_1$
−70	1	0.7409	0.2874	0.0450	0.2770	$\breve{\mathcal{I}}_4 \succ \breve{\mathcal{I}}_5 \succ \breve{\mathcal{I}}_3 \succ \breve{\mathcal{I}}_2 \succ \breve{\mathcal{I}}_1$
−80	1	0.7414	0.2870	0.0418	0.2785	$\breve{\mathcal{I}}_4 \succ \breve{\mathcal{I}}_5 \succ \breve{\mathcal{I}}_3 \succ \breve{\mathcal{I}}_2 \succ \breve{\mathcal{I}}_1$
−90	1	0.7417	0.2866	0.0395	0.2801	$\breve{\mathcal{I}}_4 \succ \breve{\mathcal{I}}_5 \succ \breve{\mathcal{I}}_3 \succ \breve{\mathcal{I}}_2 \succ \breve{\mathcal{I}}_1$
−100	1	0.7423	0.2867	0.0379	0.2812	$\breve{\mathcal{I}}_4 \succ \breve{\mathcal{I}}_5 \succ \breve{\mathcal{I}}_3 \succ \breve{\mathcal{I}}_2 \succ \breve{\mathcal{I}}_1$

Table 11. Findings of parameter k when $\breve{\tau} = -2$ and $\theta = 0.5$.

k	Q_i					**Final Ranking**
	$\breve{\mathcal{I}}_1$	$\breve{\mathcal{I}}_2$	$\breve{\mathcal{I}}_3$	$\breve{\mathcal{I}}_4$	$\breve{\mathcal{I}}_5$	
0.1	0.9130	1	0.7550	0.4517	0	$\breve{\mathcal{I}}_5 \succ \breve{\mathcal{I}}_4 \succ \breve{\mathcal{I}}_3 \succ \breve{\mathcal{I}}_1 \succ \breve{\mathcal{I}}_2$
0.2	0.9065	1	0.7534	0.4483	0	$\breve{\mathcal{I}}_5 \succ \breve{\mathcal{I}}_4 \succ \breve{\mathcal{I}}_3 \succ \breve{\mathcal{I}}_1 \succ \breve{\mathcal{I}}_2$
0.3	0.9040	1	0.7522	0.4453	0	$\breve{\mathcal{I}}_5 \succ \breve{\mathcal{I}}_4 \succ \breve{\mathcal{I}}_3 \succ \breve{\mathcal{I}}_1 \succ \breve{\mathcal{I}}_2$
0.4	0.9002	1	0.7497	0.4396	0	$\breve{\mathcal{I}}_5 \succ \breve{\mathcal{I}}_4 \succ \breve{\mathcal{I}}_3 \succ \breve{\mathcal{I}}_1 \succ \breve{\mathcal{I}}_2$
0.5	0.8975	1	0.7474	0.4384	0	$\breve{\mathcal{I}}_5 \succ \breve{\mathcal{I}}_4 \succ \breve{\mathcal{I}}_3 \succ \breve{\mathcal{I}}_1 \succ \breve{\mathcal{I}}_2$
0.6	0.8904	1	0.7449	0.4354	0	$\breve{\mathcal{I}}_5 \succ \breve{\mathcal{I}}_4 \succ \breve{\mathcal{I}}_3 \succ \breve{\mathcal{I}}_1 \succ \breve{\mathcal{I}}_2$
0.7	0.8903	1	0.7454	0.4339	0	$\breve{\mathcal{I}}_5 \succ \breve{\mathcal{I}}_4 \succ \breve{\mathcal{I}}_3 \succ \breve{\mathcal{I}}_1 \succ \breve{\mathcal{I}}_2$
0.8	0.8856	1	0.7447	0.4285	0	$\breve{\mathcal{I}}_5 \succ \breve{\mathcal{I}}_4 \succ \breve{\mathcal{I}}_3 \succ \breve{\mathcal{I}}_1 \succ \breve{\mathcal{I}}_2$
0.9	0.8795	1	0.7430	0.4245	0	$\breve{\mathcal{I}}_5 \succ \breve{\mathcal{I}}_4 \succ \breve{\mathcal{I}}_3 \succ \breve{\mathcal{I}}_1 \succ \breve{\mathcal{I}}_2$

Table 12. Findings of parameter θ when $k = 0.5$ and $\breve{\tau} = -2$.

θ	Q_i					**Final Ranking**
	$\breve{\mathcal{I}}_1$	$\breve{\mathcal{I}}_2$	$\breve{\mathcal{I}}_3$	$\breve{\mathcal{I}}_4$	$\breve{\mathcal{I}}_5$	
0.1	0.9516	1	0.7169	0.3435	0	$\breve{\mathcal{I}}_5 \succ \breve{\mathcal{I}}_4 \succ \breve{\mathcal{I}}_3 \succ \breve{\mathcal{I}}_1 \succ \breve{\mathcal{I}}_2$
0.2	0.9381	1	0.7246	0.3672	0	$\breve{\mathcal{I}}_5 \succ \breve{\mathcal{I}}_4 \succ \breve{\mathcal{I}}_3 \succ \breve{\mathcal{I}}_1 \succ \breve{\mathcal{I}}_2$
0.3	0.9245	1	0.7322	0.3910	0	$\breve{\mathcal{I}}_5 \succ \breve{\mathcal{I}}_4 \succ \breve{\mathcal{I}}_3 \succ \breve{\mathcal{I}}_1 \succ \breve{\mathcal{I}}_2$
0.4	0.9110	1	0.7398	0.4147	0	$\breve{\mathcal{I}}_5 \succ \breve{\mathcal{I}}_4 \succ \breve{\mathcal{I}}_3 \succ \breve{\mathcal{I}}_1 \succ \breve{\mathcal{I}}_2$
0.5	0.8975	1	0.7474	0.4384	0	$\breve{\mathcal{I}}_5 \succ \breve{\mathcal{I}}_4 \succ \breve{\mathcal{I}}_3 \succ \breve{\mathcal{I}}_1 \succ \breve{\mathcal{I}}_2$
0.6	0.8839	1	0.7550	0.4622	0	$\breve{\mathcal{I}}_5 \succ \breve{\mathcal{I}}_4 \succ \breve{\mathcal{I}}_3 \succ \breve{\mathcal{I}}_1 \succ \breve{\mathcal{I}}_2$
0.7	0.8704	1	0.7627	0.4859	0	$\breve{\mathcal{I}}_5 \succ \breve{\mathcal{I}}_4 \succ \breve{\mathcal{I}}_3 \succ \breve{\mathcal{I}}_1 \succ \breve{\mathcal{I}}_2$
0.8	0.8569	1	0.7703	0.5096	0	$\breve{\mathcal{I}}_5 \succ \breve{\mathcal{I}}_4 \succ \breve{\mathcal{I}}_3 \succ \breve{\mathcal{I}}_1 \succ \breve{\mathcal{I}}_2$
0.9	0.8434	1	0.7779	0.5334	0	$\breve{\mathcal{I}}_5 \succ \breve{\mathcal{I}}_4 \succ \breve{\mathcal{I}}_3 \succ \breve{\mathcal{I}}_1 \succ \breve{\mathcal{I}}_2$

7.2. Comparative Analysis

In this section, we have demonstrated a comparison of the proposed approach with different IvVPF aggregation operators and different IvVPF projection measures. First, we have compared our proposed operator with the IVPFFWA and IVPFFWG operators mentioned in [35]. We have calculated the aggregated result considering the value of the parameter $\lambda = 2$. The result shows that for the IVPFFWA operator, the ranking of the alternatives is $\check{\mathcal{I}}_2 \succ \check{\mathcal{I}}_1 \succ \check{\mathcal{I}}_3 \succ \check{\mathcal{I}}_4 \succ \check{\mathcal{I}}_5$, and for the IVPFFWG operator, the ranking is $\check{\mathcal{I}}_3 \succ \check{\mathcal{I}}_2 \succ \check{\mathcal{I}}_1 \succ \check{\mathcal{I}}_4 \succ \check{\mathcal{I}}_5$. Next, we have analyzed IVPFAAWA and IVPFAAWG operators, taking $\upsilon = 1$ (from [72]). Here, the IVPFAAWA operator ranks the alternative as $\check{\mathcal{I}}_2 \succ \check{\mathcal{I}}_1 \succ \check{\mathcal{I}}_3 \succ \check{\mathcal{I}}_4 \succ \check{\mathcal{I}}_5$, and the IVPFAAWG operator ranks them as $\check{\mathcal{I}}_3 \succ \check{\mathcal{I}}_2 \succ \check{\mathcal{I}}_1 \succ \check{\mathcal{I}}_4 \succ \check{\mathcal{I}}_5$. Additionally, utilizing the IVPFWA operator [73], the ranking is $\check{\mathcal{I}}_2 \succ \check{\mathcal{I}}_1 \succ \check{\mathcal{I}}_3 \succ \check{\mathcal{I}}_4 \succ \check{\mathcal{I}}_5$, and for the IVPFWG operator, the ranking is $\check{\mathcal{I}}_3 \succ \check{\mathcal{I}}_2 \succ \check{\mathcal{I}}_1 \succ \check{\mathcal{I}}_4 \succ \check{\mathcal{I}}_5$. Across all the methods, the optimal alternative is consistently either $\check{\mathcal{I}}_2$ or $\check{\mathcal{I}}_3$. The comparison results are shown in Figure 10 and Table 13. Additionally, we have compared the ranking of the alternative with the IvVPF projection measure and the IvVPF normalized projection measure (see Figure 11 and Table 14). From Figures 10 and 11, it can be concluded that the suggested method gives the best optimal result.

Table 13. Comparison with different aggregation operators.

Method	Operator	Score Values					Ranking
		$\check{\mathcal{I}}_1$	$\check{\mathcal{I}}_2$	$\check{\mathcal{I}}_3$	$\check{\mathcal{I}}_4$	$\check{\mathcal{I}}_5$	
[35]	IVPFFWA ($\lambda = 2$)	0.3365	0.3705	0.3311	0.2615	0.1461	$\check{\mathcal{I}}_2 \succ \check{\mathcal{I}}_1 \succ \check{\mathcal{I}}_3 \succ \check{\mathcal{I}}_4 \succ \check{\mathcal{I}}_5$
	IVPFFWG ($\lambda = 2$)	0.1214	0.1491	0.1536	0.0319	−0.0550	$\check{\mathcal{I}}_3 \succ \check{\mathcal{I}}_2 \succ \check{\mathcal{I}}_1 \succ \check{\mathcal{I}}_4 \succ \check{\mathcal{I}}_5$
[72]	IVPFAAWA ($\upsilon = 1$)	0.2995	0.3140	0.2965	0.2670	0.2198	$\check{\mathcal{I}}_2 \succ \check{\mathcal{I}}_1 \succ \check{\mathcal{I}}_3 \succ \check{\mathcal{I}}_4 \succ \check{\mathcal{I}}_5$
	IVPFAAWG ($\upsilon = 1$)	0.1955	0.2082	0.2113	0.1661	0.1410	$\check{\mathcal{I}}_3 \succ \check{\mathcal{I}}_2 \succ \check{\mathcal{I}}_1 \succ \check{\mathcal{I}}_4 \succ \check{\mathcal{I}}_5$
[73]	IVPFWA	0.3451	0.3788	0.3380	0.2711	0.1555	$\check{\mathcal{I}}_2 \succ \check{\mathcal{I}}_1 \succ \check{\mathcal{I}}_3 \succ \check{\mathcal{I}}_4 \succ \check{\mathcal{I}}_5$
	IVPFWG	0.0965	0.1238	0.1320	0.0045	−0.0757	$\check{\mathcal{I}}_3 \succ \check{\mathcal{I}}_2 \succ \check{\mathcal{I}}_1 \succ \check{\mathcal{I}}_4 \succ \check{\mathcal{I}}_5$
Method	Operator	*VIKOR* Index Q_i					Ranking
		$\check{\mathcal{I}}_1$	$\check{\mathcal{I}}_2$	$\check{\mathcal{I}}_3$	$\check{\mathcal{I}}_4$	$\check{\mathcal{I}}_5$	
Proposed operator	IvVPFSSPA($\check{\tau} = -2$)	0.8969	1	0.7489	0.4394	0	$\check{\mathcal{I}}_5 \succ \check{\mathcal{I}}_4 \succ \check{\mathcal{I}}_3 \succ \check{\mathcal{I}}_1 \succ \check{\mathcal{I}}_2$
	IvVPFSSPG ($\check{\tau} = -6$)	0.9384	1	0.7253	0.2632	0	$\check{\mathcal{I}}_5 \succ \check{\mathcal{I}}_4 \succ \check{\mathcal{I}}_3 \succ \check{\mathcal{I}}_1 \succ \check{\mathcal{I}}_2$

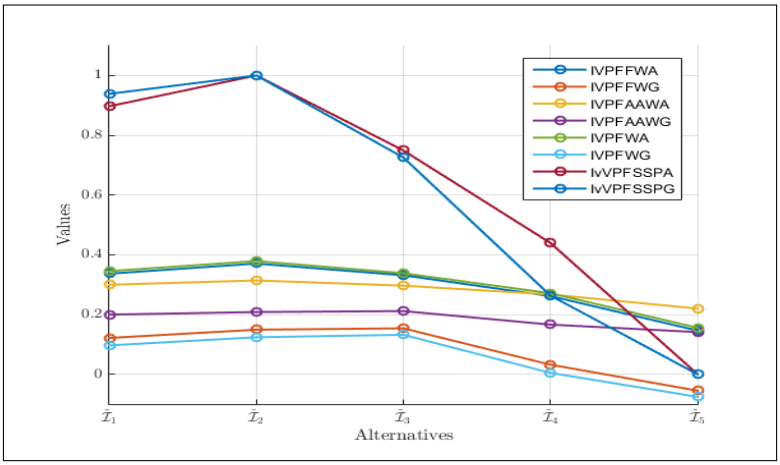


Figure 10. Comparison of suggested operators with several operators.

Table 14. Comparison with different projection measures.

Projection	*VIKOR* Index Q_i					Ranking
	$\check{\mathcal{I}}_1$	$\check{\mathcal{I}}_2$	$\check{\mathcal{I}}_3$	$\check{\mathcal{I}}_4$	$\check{\mathcal{I}}_5$	
IvVPF projection measure	0.6923	1	0.6997	0.4935	0	$\check{\mathcal{I}}_5 \succ \check{\mathcal{I}}_4 \succ \check{\mathcal{I}}_1 \succ \check{\mathcal{I}}_3 \succ \check{\mathcal{I}}_2$
IvVPF normalized projection measure	0.8611	1	0.7549	0.4641	0	$\check{\mathcal{I}}_5 \succ \check{\mathcal{I}}_4 \succ \check{\mathcal{I}}_3 \succ \check{\mathcal{I}}_1 \succ \check{\mathcal{I}}_2$
IvVPF bidirectional projection measure	0.8969	1	0.7489	0.4394	0	$\check{\mathcal{I}}_5 \succ \check{\mathcal{I}}_4 \succ \check{\mathcal{I}}_3 \succ \check{\mathcal{I}}_1 \succ \check{\mathcal{I}}_2$

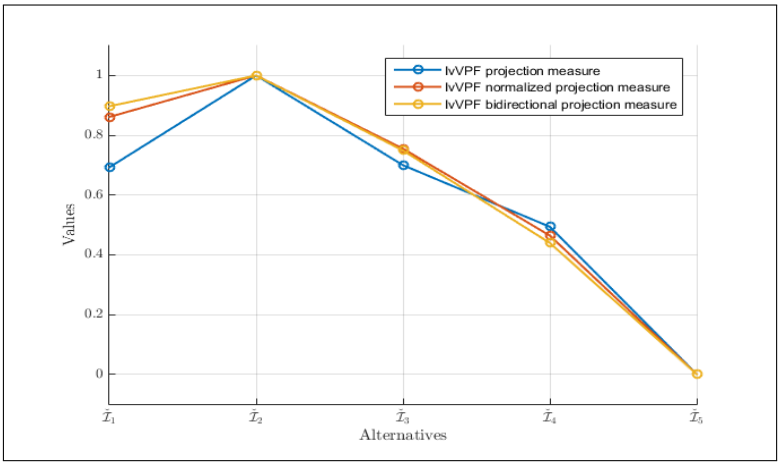


Figure 11. Comparison of different IvVPF projection measures.

8. Conclusions

In this paper, we have explored an IvVPF framework-based *MAGDM* approach to evaluate the best sustainable way to charge EVs in PCSs where the criterion's weight is known to some extent. To do this, we have developed two aggregation operators (IvVPFSSPWA and IvVPFSSPWG) to aggregate the IvVPF information. In our suggested method, we have ascertained the objective weight of the criteria by using the DBM and the proposed IvVPF bidirectional projection-based *VIKOR* approach. Then, to accomplish the applicability of the proposed methodology, we have considered a numerical instance. From the results and discussion, we have found that the most effective way is $\check{\mathcal{I}}_5$ (solar-powered charging) to reduce GHG emissions and save the climate. Next, we have conducted sensitivity analysis of the used parameters $\check{\tau}$, k, and θ. Additionally, a comparative analysis has been conducted to demonstrate the feasibility of the result.

However, it is crucial to acknowledge that the proposed decision making has certain limitations, which are detailed as follows:

- From the beginning, it is assumed that the decision makers are optimistic about the outcome of the decision-making result. Therefore, it is not suitable for every *MAGDM* problem.
- This method fails when one decision maker makes decisions, while others disagree.
- The suggested method's computation procedure is very complex. However, if we are able to make a computer program for this method, then it will be easy to solve and save time.

Thus, further research is required to assess the preferences of decision makers in a *MAGDM* problem within an IvVPF environment. Additionally, we will apply this method in large-scale group decision making. In the future, we will apply this in the learning management selection problem, risk management problem, etc.

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Abbreviations

The following abbreviations are used in this manuscript:

Notations	full form
FSs	fuzzy sets
IFSs	intuitionistic fuzzy sets
PFSs	picture fuzzy sets
MD	membership degree
PMD	positive membership degree
NuMD	neutral membership degree
IvVIFSs	interval-valued intuitionistic fuzzy sets
IvVPFSs	interval-valued picture fuzzy sets
IvVPFNs	interval-valued picture fuzzy numbers
SS-TNs	Schweizer—Sklar triangular norms
SS-TCNs	Schweizer–Sklar triangular conorms
VIKOR	VlseKriterijumska Optimizacija I Kompromisno Resenje
MAGDM	multi-attribute group decision making
IvVPFSSPWA	interval-valued picture fuzzy Schweizer–Sklar power-weighted averaging aggregation
IvVPFSSPWG	interval-valued picture fuzzy Schweizer–Sklar power-weighted geometric aggregation
EVs	electric vehicles
PCSs	public charging stations
GHG	greenhouse gas
Prj	projection
Nprj	normalized projection
Bprj	bidirectional projection

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